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AN ELEMENTARY THEORY OF COMPARATIVE ADVANTAGE

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AN ELEMENTARY THEORY OF COMPARATIVE ADVANTAGE

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Comparative advantage, whether driven by technology or factor endowment, is at the core of neoclassical trade theory. Using tools from the mathematics of complementarity, this paper offers a simple yet unifying perspective on the fundamental forces that shape comparative advantage. The main results characterize sufficient conditions on factor productivity and factor supply to predict patterns of international specialization in a multifactor generalization of the Ricardian model which we refer to as an "elementary neoclassical economy." These conditions, which hold for an arbitrarily large number of countries, goods, and factors, generalize and extend many results from the previous trade literature. They also offer new insights about the joint effects of technology and factor endowments on international specialization.

KEYWORDS: Comparative advantage, neoclassical trade theory, log-supermodularity.

1. INTRODUCTION

COMPARATIVE ADVANTAGE, whether driven by technology or factor endowment, is at the core of neoclassical trade theory. Using tools from the mathematics of complementarity, this paper offers a simple yet unifying perspective on the fundamental forces that shape comparative advantage in economies with an arbitrarily large number of countries, goods, and factors.

Section 2 offers a review of some basic definitions and results in the mathematics of complementarity. Our analysis emphasizes one key property: log-supermodularity. Broadly speaking, the log-supermodularity of a multivariate function captures the idea that increasing one variable is *relatively more* important when the other variables are high. To fix ideas, consider the following statement. Countries with better financial systems produce *relatively more* in sectors with higher financial requirements. The formal counterpart to this statement is that aggregate output is log-supermodular in the quality of countries' financial systems and the level of sectors' financial requirements. In a trade context, log-supermodularity provides a powerful way to conceptualize the relationship between technology, factor endowment, and international specialization, as we will soon demonstrate.

Section 3 describes our theoretical framework. We develop a multifactor generalization of the Ricardian model with an arbitrary number of countries

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and sectors which we refer to as an "elementary neoclassical economy." Factors of production are immobile across countries and perfectly mobile across sectors. Each country, sector, and factor is associated with a distinct characteristic denoted γ, σ, and ω, respectively. For instance, γ may capture the quality of a country's educational system, σ may capture the skill intensity of a sector, and ω may capture the number of years of education of a worker. The two primitives of our model are (i) factor productivity, $q(\omega, \sigma, \gamma)$, which may vary across countries and sectors, and (ii) factor supply, $f(\omega, \gamma)$, which may vary across countries. They reflect the two sources of comparative advantage in a neoclassical environment: technology and factor endowment.

In this paper, we derive three sets of results on the pattern of international specialization. Section 4 focuses on the case in which only technological differences are a source of comparative advantage. Formally, we assume that $q(\omega, \sigma, \gamma) \equiv h(\omega)a(\sigma, \gamma)$. Under this restriction, our general model reduces to a standard Ricardian model. In this environment, we show that if $a(\sigma, \gamma)$ is log-supermodular, then aggregate output $Q(\sigma, \gamma)$ is log-supermodular as well. Economically speaking, if high-γ countries are relatively more productive in high-σ sectors, then they should produce relatively more in these sectors. This first result has played an important, albeit implicit, role in many applications and extensions of the Ricardian model. It is at the heart, for example, of the recent literature on institutions and trade; see, for example, Acemoglu, Antras, and Helpman (2007), Costinot (2007), Cuñat and Melitz (2006), Levchenko (2007), Matsuyama (2005), Nunn (2007), and Vogel (2007). At a formal level, these papers all share the same fundamental objective: providing microtheoretical foundations for the log-supermodularity of factor productivity with respect to countries' "institutional quality" and sectors' "institutional dependence," whatever those characteristics may be.

Section 5 analyzes the polar case in which factor productivity varies across countries in a Hicks-neutral way, $q(\omega, \sigma, \gamma) \equiv a(\gamma)h(\omega, \sigma)$. Hence, only factor endowment differences are a source of comparative advantage. This particular version of our model is a simple generalization of Ruffin (1988). In this environment, we show that if $f(\omega, \gamma)$ and $h(\omega, \sigma)$ are log-supermodular, then aggregate output $Q(\sigma, \gamma)$ also is log-supermodular. The basic logic is intuitive. On the one hand, high-γ countries have relatively more high-ω factors. On the other hand, high-ω factors are more likely to be employed in high-σ sectors because they are relatively more productive in these sectors. This explains why high-γ countries should produce relatively more in high-σ sectors. As in the Ricardian case, log-supermodularity provides the mathematical apparatus to make these "relatively more" statements precise.

As we later discuss, this second set of results can be used to establish the robustness of many qualitative insights from the literature on heterogeneity and trade. Whether they focus on worker heterogeneity or firm heterogeneity à la Melitz (2003), previous insights typically rely on strong functional forms which guarantee explicit closed form solutions. For example, Ohnsorge and

Trefler (2004) assumed that distributions of worker skills are log-normal, while Helpman, Melitz, and Yeaple (2004) and Antras and Helpman (2004) assumed that distributions of firm productivity are Pareto. Our results formally show that assuming the log-supermodularity of $f(\omega, \gamma)$ is critical for many of their results, whereas assuming log-normal and Pareto distributions is not.

Section 6 considers elementary neoclassical economies in which both factor endowment and technological differences are sources of comparative advantage. In these economies, we show that unless strong functional form restrictions are imposed, robust predictions about international specialization can only be derived in the two most extreme sectors. In general, the log-supermodularity of $f(\omega, \gamma)$ and $q(\omega, \sigma, \gamma)$ is not sufficient to derive the log-supermodularity of aggregate output. In the presence of complementarities between factor and sector characteristics, which are necessary for factor endowments to affect comparative advantage, the indirect impact of Ricardian technological differences on the assignment of factors to sectors may dominate its direct impact on factor productivity. This is an important observation that highlights the potential caveats of combining insights from distinct models without a generalizing framework.

Although we are, to the best of our knowledge, the first ones to emphasize the role of log-supermodularity in a trade context, this property has been used previously in many areas of economics, including auction theory (Milgrom and Weber (1982)), monotone comparative statics under uncertainty (Jewitt (1987) and Athey (2002)), and matching (Shimer and Smith (2000)). From a mathematical standpoint, Jewitt's (1987) and Athey's (2002) works are most closely related to our paper. In particular, the fact that log-supermodularity is preserved by multiplication and integration is, like in Jewitt (1987) and Athey (2002), at the core of our analysis.[2] In this respect, our contribution is to show that this mathematical property also has natural and useful applications for international trade.

The theory of comparative advantage presented in this paper is attractive for two reasons. The first one is that it allows us to consider both sources of comparative advantage, technology and factor endowment—within a unifying yet highly tractable framework. This is important not only for generalizing results from the previous literature, but also because factor endowment in practice coexists with technology and institutional differences. Indeed, they often have the same causes (see, e.g., Acemoglu (1998)). The second reason is dimensionality. For pedagogical purposes, neoclassical trade theory is usually taught using simple models with a small number of countries, goods, and factors. The two most

[2]This close mathematical connection notwithstanding, our results are not about monotone comparative statics. In this paper, we are interested in the cross-sectional variation of aggregate output within a given equilibrium, not changes in aggregate output across equilibria. In particular, the fact that all countries face the same prices within a given free trade equilibrium is crucial for our results.

celebrated examples are the Ricardian model—with one factor, two goods, and two countries—and the Heckscher–Ohlin model—with two factors, two goods, and two countries.[3] In these simple models, differences in either technology or factor endowments have strong implications for the pattern of international specialization. Unfortunately, strong results do not generally survive in environments with higher dimensionality (see, e.g., Ethier (1984) and Deardorff (2007)). By contrast, our predictions hold for an arbitrarily large number of countries, goods, and factors. In this respect, our paper is closely related to Deardorff (1980). Compared to Deardorff's (1980) law of comparative advantage, our main results are less general in that we restrict ourselves to a multifactor generalization of the Ricardian model under free trade, but they are stronger in that they apply to any pair of goods and derive from restrictions on the model's primitives—factor productivity and factor supply—rather than autarky prices.

Finally, we believe that our general approach could also be useful outside international trade. The basic structure of our model is central to many models with agent heterogeneity. At the core of these models, there are populations of agents sorting across occupations. As we argue in our concluding remarks, whatever these categories may refer to in practice, they often are the formal counterparts to countries, factors, and sectors in our theory.

2. LOG-SUPERMODULARITY

Our analysis emphasizes one particular form of complementarity: log-supermodularity.[4] Since this concept is not widely used in the trade literature, we begin with a review of some basic definitions and results. Topkis (1998) and Athey (2002) offer an excellent overview and additional references.

2.1. *Definition*

Let $X = \prod_{i=1}^{n} X_i$, where each X_i is totally ordered. For any $x, x' \in X$, we say that $x \geq x'$ if $x_i \geq x'_i$ for all $i = 1, \ldots, n$. We let $\max(x, x')$ be the vector of X whose ith component is $\max(x_i, x'_i)$ and let $\min(x, x')$ be the vector whose ith component is $\min(x_i, x'_i)$. Finally, we denote by x_{-i} the vector x with the ith component removed. With the previous notations, log-supermodularity can be defined as follows.

DEFINITION 1: A function $g : X \to \mathbb{R}^+$ is log-supermodular if for all x, $x' \in X$, $g(\max(x, x')) \cdot g(\min(x, x')) \geq g(x) \cdot g(x')$.

[3]Davis (1995) offered a simple combination of both models with three goods, two factors, and two countries.

[4]In the statistics literature, Karlin (1968) referred to log-supermodularity as total positivity of order 2.

If g is strictly positive, then g is log-supermodular if and only if $\ln g$ is supermodular. This means that if g also is twice differentiable, then g is log-supermodular in (x_i, x_j) if and only if $\partial^2 \ln g/\partial x_i \partial x_j \geq 0$. To get more intuition about the form of complementarities that log-supermodularity captures, consider $g: X_1 \times X_2 \to \mathbb{R}^+$. For every $x_1' \geq x_1''$, $x_2' \geq x_2''$, the log-supermodularity of g in (x_1, x_2) implies that

$$g(x_1', x_2') \cdot g(x_1'', x_2'') \geq g(x_1', x_2'') \cdot g(x_1'', x_2').$$

If g is strictly positive, this can be rearranged as

$$g(x_1', x_2')/g(x_1'', x_2') \geq g(x_1', x_2'')/g(x_1'', x_2'').$$

Thus, the relative returns to increasing the first variable, x_1, are increasing in the second variable, x_2. This is equivalent to the monotone likelihood ratio property; see Milgrom (1981). In a trade context, this property may capture the fact that high-x_1 countries are relatively more productive in high-x_2 sectors, as in the Ricardian model, or that high-x_1 countries are relatively more abundant in high-x_2 factors, as in the Heckscher–Ohlin model.

2.2. *Results*

Most of our analysis builds on the following two results:

LEMMA 1: *If $g, h: X \to \mathbb{R}^+$ are log-supermodular, then gh is log-supermodular.*

LEMMA 2: *Let μ_i be a σ-finite measure on X_i. If $g: X \to \mathbb{R}^+$ is log-supermodular and integrable, then $G(x_{-i}) = \int_{X_i} g(x)\, d\mu_i(x_i)$ is log-supermodular.*

In other words, log-supermodularity is preserved by multiplication and integration. Lemma 1 directly derives from Definition 1. Proofs of Lemma 2 can be found in Lehmann (1955) for the bivariate case, and Ahlswede and Daykin (1978) and Karlin and Rinott (1980) for the multivariate case. In the rest of this paper, we assume that whenever integrals appear, requirements of integrability and measurability are met.

3. THEORETICAL FRAMEWORK

We consider a world economy comprising $c = 1, \ldots, C$ countries with characteristics $\gamma^c \in \Gamma$, $s = 1, \ldots, S$ goods or sectors with characteristics $\sigma^s \in \Sigma$, and multiple factors of production indexed by their characteristics $\omega \in \Omega$, where Γ, Σ, and Ω are totally ordered sets.[5] We let μ be a σ-finite measure on Ω.

[5]We could allow for the existence of countries and sectors whose characteristics cannot be ordered. In this case, our results would simply apply to the subset of countries and sectors with

The number of factors in Ω may be continuous or discrete. Factors of production are immobile across countries and perfectly mobile across sectors. $f(\omega, \gamma^c) \geq 0$ denotes the inelastic supply of factor ω in country c. Factors of production are perfect substitutes within each country and sector, but vary in their productivity $q(\omega, \sigma^s, \gamma^c) \geq 0$. In country c and sector s, aggregate output is given by

$$Q(\sigma^s, \gamma^c) = \int_{\Omega} q(\omega, \sigma^s, \gamma^c) l(\omega, \sigma^s, \gamma^c)\, d\mu(\omega), \tag{1}$$

where $l(\omega, \sigma^s, \gamma^c)$ is the quantity of factor ω allocated to sector s in country c.

At this point, we wish to be clear that our theoretical framework is more general than a Ricardian model in that it allows multiple factors of production, but is less general than a standard neoclassical model in that it rules out imperfect substitutability between these factors within each sector. We come back briefly to the relationship between our model and the Heckscher–Ohlin model in Section 5.

Throughout this paper, we focus on the supply side of this economy under free trade. Our goal is to determine how the cross-sectional variation of our two primitives, $q(\omega, \sigma^s, \gamma^c)$ and $f(\omega, \gamma^c)$, affects the cross-sectional variation of aggregate output, $Q(\sigma^s, \gamma^c)$, taking world prices $p(\sigma^s) > 0$ as given. To this end, we follow the dual approach of Dixit and Norman (1980).

DEFINITION 2: $l(\cdot, \cdot, \cdot)$ is an efficient allocation if, for all $c = 1, \dots, C$, it solves the revenue maximization problem

$$\max_{l(\cdot,\cdot,\gamma^c)} \sum_{s=1}^{S} p(\sigma^s) Q(\sigma^s, \gamma^c) \tag{2}$$

$$\text{subject to} \quad \sum_{s=1}^{S} l(\omega, \sigma^s, \gamma^c) \leq f(\omega, \gamma^c) \quad \text{for } \mu\text{-almost all } \omega \in \Omega.$$

According to Definition 2, $l(\cdot, \cdot, \cdot)$ is an efficient allocation if it is feasible and it maximizes the value of national output at given prices in all countries. Since there are constant returns to scale, a competitive equilibrium with a large number of profit-maximizing firms would lead to an efficient allocation.

Because of the linearity of aggregate output, efficient allocations are easy to characterize. Unlike more general neoclassical models, the marginal return $r(\omega, \sigma^s, \gamma^c)$ of factor ω in sector s and country c is independent of the alloca-

ordered characteristics. By contrast, our analysis crucially relies on the fact that Ω is totally ordered.

tion of factors in that sector: $r(\omega, \sigma^s, \gamma^c) = p(\sigma^s)q(\omega, \sigma^s, \gamma^c)$.[6] As a result, we can solve problem (2) factor-by-factor the same way we would solve the revenue maximization problem in a simple Ricardian model. In any country c, almost all factors ω should be employed in the sector(s) where $p(\sigma^s)q(\omega, \sigma^s, \gamma^c)$ is maximum.

In the rest of this paper, we restrict ourselves to environments where problem (2) admits a unique solution.

ASSUMPTION 0: *The solution to the revenue maximization problem* (2) *is unique for all* $c = 1, \dots, C$ *and* μ*-almost all* $\omega \in \Omega$.

By our previous discussion, Assumption 0 requires $p(\sigma^s)q(\omega, \sigma^s, \gamma^c)$ to be maximized in a *single* sector for almost all factors and all countries. Since Assumption 0 plays a crucial role in our analysis, it is important to understand why and in which circumstances it is more likely to be satisfied.

At a formal level, Assumption 0 is an implicit restriction on the demand side of the world economy, which requires world consumption to be at a vertex of the world production possibility frontier. This is illustrated in Figure 1 in the case of an economy with one factor, two goods, and two countries or, equivalently, two factors, two goods, and one country. Ceteris paribus, the more vertices there are on the world production possibility frontier, the milder that restriction on preferences becomes. From an economic standpoint, this means

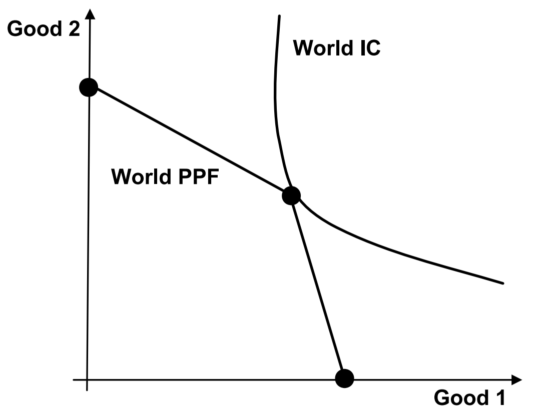

FIGURE 1.—Uniqueness of the efficient allocation.

[6]This would no longer be true, for example, if aggregate production functions were Cobb–Douglas, $Q(\sigma^s, \gamma^c) = \exp\{\int_\Omega \alpha(\omega) \ln l(\omega, \sigma^s, \gamma^c)\, d\mu(\omega)\}$. Under this assumption, marginal returns would be given by $r(\omega, \sigma^s, \gamma^c) = p(\sigma^s)\alpha(\omega)Q(\sigma^s, \gamma^c)/l(\omega, \sigma^s, \gamma^c)$, which would depend on the price, $p(\sigma^s)$, and exogenous technological characteristics, $\alpha(\omega)$, but also $Q(\sigma^s, \gamma^c)/l(\omega, \sigma^s, \gamma^c)$.

that Assumption 0 is more likely to be satisfied in economies with the following attributes:

(i) *A large number of countries*, as in the Ricardian models developed by Becker (1952), Matsuyama (1996), and Yanagawa (1996).

(ii) *A large number of factors*, as in the trade models with worker heterogeneity developed by Grossman and Maggi (2000), Grossman (2004), and Ohnsorge and Trefler (2004).

In particular, if there is a continuum of distinct factors in the economy, then Assumption 0 is generically true. Although prices are endogenous objects which may adjust to equalize marginal returns across sectors, a finite number of prices cannot, in general, equalize the returns of an infinite numbers of factors.[7]

Throughout this paper, we maintain Assumption 0, which allows us to express aggregate output under an efficient allocation as follows.

PROPOSITION 1: *Suppose that Assumption 0 holds. Then, for all $c = 1, \dots, C$ and $s = 1, \dots, S$, aggregate output under an efficient allocation is given by*

$$Q(\sigma^s, \gamma^c) = \int_{\Omega(\sigma^s, \gamma^c)} q(\omega, \sigma^s, \gamma^c) f(\omega, \gamma^c)\, d\mu(\omega), \tag{3}$$

where $\Omega(\sigma^s, \gamma^c)$ is the set of factors allocated to sector s in country c:

$$\Omega(\sigma^s, \gamma^c) = \left\{ \omega \in \Omega \middle| r(\omega, \sigma^s, \gamma^c) > \max_{s' \neq s} r\left(\omega, \sigma^{s'}, \gamma^c\right) \right\}. \tag{4}$$

From now on, we refer to a world economy where Equations (3) and (4) hold as an *elementary neoclassical economy*. The rest of our paper offers sufficient conditions to make predictions on the pattern of international specialization in this environment.

4. SOURCE OF COMPARATIVE ADVANTAGE (I): TECHNOLOGY

4.1. *Definition*

We first consider economies in which factor productivity satisfies

$$q(\omega, \sigma, \gamma) \equiv h(\omega) a(\sigma, \gamma) \tag{5}$$

with $h(\omega) > 0$ and $a(\sigma, \gamma) \geq 0$. Equation (5) allows only Ricardian technological differences across countries. Since a is a function of σ and γ, some countries may be relatively more productive in some sectors than others. By contrast, factors may not be relatively more productive in some sectors than

[7]Finally, note that Assumption 0 also is trivially satisfied in Ricardian models with Armington preferences; see, for example, Acemoglu and Ventura (2002). In those models, since $q(\omega, \sigma^s, \gamma^c)$ is strictly positive in a single sector, $p(\sigma^s) q(\omega, \sigma^s, \gamma^c)$ is maximized in a single sector as well.

others: if factor ω' is twice as productive as factor ω in a given sector, then it is twice as productive in all of them.

DEFINITION 3: An elementary neoclassical economy is a R-economy if Equation (5) holds.

In a R-economy, there are no "real" differences across factors of production. If there exists $\omega \in \Omega$ such that $r(\omega, \sigma^s, \gamma^c) > \max_{s' \neq s} r(\omega, \sigma^{s'}, \gamma^c)$, then $r(\omega', \sigma^s, \gamma^c) > \max_{s' \neq s} r(\omega', \sigma^{s'}, \gamma^c)$ for all $\omega' \in \Omega$. Hence, a R-economy is isomorphic to a standard Ricardian model. In this environment, Assumption 0 directly implies

$$\Omega(\sigma^s, \gamma^c) = \Omega \quad \text{or} \quad \emptyset.$$

Since there are no real differences across factors of production in a R-economy, their marginal returns are maximized in the same sector. As a result, countries only produce one good.[8] Given this restriction, our analysis of the Ricardian model is similar in terms of scope to the analysis of Jones (1961).[9]

4.2. *Assumption*

To make predictions on the pattern of international specialization in a R-economy, we make the following assumption:

ASSUMPTION 1: *$a(\sigma, \gamma)$ is log-supermodular.*

Assumption 1 states that high-γ countries are relatively more productive in high-σ sectors. For any pair of countries, c_1 and c_2, and goods, s_1 and s_2, such that $\gamma^{c_1} \geq \gamma^{c_2}$, $\sigma^{s_1} \geq \sigma^{s_2}$, $a(\sigma^{s_1}, \gamma^{c_2}) \neq 0$, and $a(\sigma^{s_2}, \gamma^{c_2}) \neq 0$, Assumption 1 implies

$$\frac{a(\sigma^{s_1}, \gamma^{c_1})}{a(\sigma^{s_1}, \gamma^{c_2})} \geq \frac{a(\sigma^{s_2}, \gamma^{c_1})}{a(\sigma^{s_2}, \gamma^{c_2})}.$$

This is the standard inequality at the heart of the Ricardian model. The log-supermodularity of a simply requires that it holds for any ordered pairs of country and sector characteristics.

[8]Of course, this stark implication of Assumption 0 will no longer be true in elementary neoclassical economies with more than one factor of production.

[9]Section 4.4 briefly discusses how our results generalize to Ricardian environments where countries produce more than one good.

4.3. *Predictions*

The main result of this section can be stated as follows.

THEOREM 1: *In a R-economy, Assumption 1 implies $Q(\sigma,\gamma)$ log-supermodular.*

The formal proof as well as all subsequent proofs can be found in the Appendix. The argument is simple. If $Q(\sigma,\gamma)$ were not log-supermodular, then one could find a pair of countries and sectors such that the marginal returns of factors of production in the low-σ sector would be (i) strictly higher in the high-γ country and (ii) strictly lower in the low-γ country. Under free trade, this is precisely what the log-supermodularity of a precludes.

Theorem 1 imposes strong restrictions on the pattern of international specialization. If a country with characteristic γ_1 specializes in a sector with characteristic σ_1, then a country with characteristic $\gamma_2 < \gamma_1$ cannot specialize in a sector with characteristic $\sigma_2 > \sigma_1$. In other words, there must be a ladder of countries such that higher-γ countries produce higher-σ goods.

COROLLARY 1: *Suppose that Assumption 1 holds in a R-economy. Then high-γ countries specialize in high-σ sectors.*

So far, we have shown that Assumption 1 is sufficient to make predictions on the pattern of international specialization in a R-economy. Conversely, we can show that Assumption 1 cannot be dispensed with if the log-supermodularity of Q is to hold in *all* R-economies. To see this, consider a two-sector R-economy. In this environment, if a were not log-supermodular, then one could find a high-γ country in which the marginal returns of factors of production would be strictly higher in the low-σ sector and a low-γ country in which the marginal returns of factors of production would be strictly higher in the high-σ sector. Therefore, the high-γ country would specialize in the low-σ sector and the low-γ country in the high-σ sector, which would contradict the log-supermodularity of Q.

4.4. *Relation to the Previous Literature*

Making predictions on the pattern of international specialization in a Ricardian model with a large number of goods and countries is knowingly difficult. Deardorff (2007) noted that "Jones (1961) seems to have done about as well as one can, showing that an efficient assignment of countries to goods will minimize the product of their unit labor requirements." We have just shown that by imposing the log-supermodularity of factor productivity across countries and sectors, one can generate much stronger predictions. The reason is simple.

Using our notations and taking logs, Jones (1961) stated that an efficient assignment of countries to goods must solve

$$\max \sum \ln a(\sigma, \gamma).$$

Corollary 1 merely points out that the solution to this assignment problem exhibits positive assortative matching if $\ln a(\sigma, \gamma)$ is supermodular (see, e.g., Becker (1973), Kremer (1993), and Legros and Newman (2002)).

Though we have restricted ourselves in this section to the case where each country only produces one good, the formal connection between the Ricardian model and assignment models holds more generally. In Dornbusch, Fischer, and Samuelson (1977), for example, both countries produce a continuum of goods, but the pattern of international specialization still reflects the optimal assignment of goods to countries. Formally, let $\Sigma(\gamma) \equiv \{\sigma \in \Sigma | l(\omega, \sigma, \gamma) > 0$ for some $\omega \in \Omega\}$ be the set of goods produced in a country with characteristic γ. Without Assumption 0, this set may not be a singleton. However, using the same logic as in Theorem 1, it is easy to show that if $a(\sigma, \gamma)$ is strictly log-supermodular, then $\Sigma(\gamma)$ must be increasing in the strong set order. Put simply, high-γ countries must specialize in high-σ sectors, as previously stated in Corollary 1.

In light of this discussion, it should not be surprising that log-supermodularity has played an important, albeit implicit, role in many applications and extensions of the Ricardian model. In his "technology gap" model of international trade, Krugman (1986) assumed, using our notation, that labor productivity in country c and sector s is given by $a(\sigma^s, \gamma^c) \equiv \exp(\sigma^s \gamma^c)$, where σ^s is an index of good s's technological intensity and γ^c is a measure of country c's closeness to the world technological frontier. Since $\partial^2 \ln a / \partial\sigma\, \partial\gamma > 0$, this functional form satisfies Assumption 1, which is the critical sufficient condition for Krugman's (1986) results to hold.

Log-supermodularity also is at the heart of the recent literature on institutions and trade; see, for example, Acemoglu, Antras, and Helpman (2007), Costinot (2007), Cuñat and Melitz (2006), Levchenko (2007), Matsuyama (2005), Nunn (2007), and Vogel (2007). These papers have shown that cross-country differences in institutions may give rise to a pattern of comparative advantage, even in the absence of true technological differences. Though the aforementioned papers differ in terms of the institutional characteristics they focus on—from credit market imperfections to rigidities in the labor market—they share the same fundamental objective: providing microtheoretical foundations for the log-supermodularity of factor productivity with respect to countries' "institutional quality" and sectors' "institutional dependence," whatever those characteristics may be.

5. SOURCE OF COMPARATIVE ADVANTAGE (II): FACTOR ENDOWMENT

5.1. *Definition*

We now turn to the case where q satisfies

$$q(\omega, \sigma, \gamma) \equiv a(\gamma)h(\omega, \sigma) \tag{6}$$

with $a(\gamma) > 0$ and $h(\omega, \sigma) \geq 0$. Equation (6) allows factor productivity to vary across countries, but only in a Hicks-neutral way.[10] Therefore, there are no Ricardian technological differences in this environment.

DEFINITION 4: An elementary neoclassical economy is a F-economy if Equation (6) holds.

The key feature of a F-economy is that the set of factors allocated to a given sector is the same in all countries. Because of free trade and Hicks-neutrality, we have

$$\begin{aligned} \Omega(\sigma^s, \gamma^c) &\equiv \widetilde{\Omega}(\sigma^s) \\ &= \left\{\omega \in \Omega \middle| p(\sigma^s)h(\omega, \sigma^s) > \max_{s' \neq s} p\big(\sigma^{s'}\big)h\big(\omega, \sigma^{s'}\big)\right\}. \end{aligned} \tag{7}$$

In a F-economy, the assignment function $\Omega(\sigma^s, \gamma^c)$ does not vary across countries. Hence, patterns of international specialization may only arise from cross-country differences in factor endowments, $f(\omega, \gamma^c)$.

5.2. *Assumptions*

To make predictions on the pattern of international specialization in a F-economy, we make two assumptions:

ASSUMPTION 2: *$f(\omega, \gamma)$ is log-supermodular.*

Assumption 2 states that high-γ countries are relatively more abundant in high-ω factors. For any pair of countries, c_1 and c_2, and factors, ω_1 and ω_2, such that $\gamma^{c_1} \geq \gamma^{c_2}$, $\omega_1 \geq \omega_2$, and $f(\omega_2, \gamma^{c_1}), f(\omega_2, \gamma^{c_2}) \neq 0$, Assumption 2 implies

$$\frac{f(\omega_1, \gamma^{c_1})}{f(\omega_2, \gamma^{c_1})} \geq \frac{f(\omega_1, \gamma^{c_2})}{f(\omega_2, \gamma^{c_2})}.$$

As previously mentioned, this is equivalent to the assumption that the densities of countries' factor endowments, $f_c(\omega) \equiv f(\omega, \gamma_c)$, can be ranked in terms of

[10]One could easily extend the analysis of this section to the case where $q(\omega, \sigma, \gamma) \equiv a(\omega, \gamma)h(\omega, \sigma)$. Here, changes in $a(\omega, \gamma)$ are isomorphic to changes in $f(\omega, \gamma)$.

monotone likelihood ratio dominance. Milgrom (1981) offered many examples of density functions that satisfy this assumption, including the normal (with mean γ) and the uniform (on $[0, \gamma]$).

ASSUMPTION 3: *$h(\omega, \sigma)$ is log-supermodular.*

Assumption 3 states that high-ω factors are relatively more productive in high-σ sectors, irrespective of the country where they are located. In our model, Assumptions 2 and 3 play the same role as the ordinal assumptions on factor abundance and factor intensity, respectively, in the $2 \times 2 \times 2$ Heckscher–Ohlin model.

5.3. *Predictions*

The main result of this section can be stated as follows.

THEOREM 2: *In a F-economy, Assumptions 2 and 3 imply $Q(\sigma, \gamma)$ log-supermodular.*

The proof relies on Lemmas 1 and 2, but the broad logic is intuitive. If $h(\omega, \sigma)$ satisfies Assumption 3, then high-ω factors are assigned to high-σ sectors under an efficient allocation. If, in addition, $f(\omega, \gamma)$ satisfies Assumption 2, then a high value of γ raises the likelihood of high values of ω relative to low values of ω. This increases the likelihood that a given factor is allocated to high-σ sectors and, in turn, increases the relative output of these sectors. This, in a nutshell, explains why $Q(\sigma, \gamma)$ is log-supermodular.

Now consider a pair of countries, c_1 and c_2, producing a pair of goods, s_1 and s_2, with $\gamma^{c_1} \geq \gamma^{c_2}$ and $\sigma^{s_1} \geq \sigma^{s_2}$. Theorem 2 implies $Q^{s_1 c_1}/Q^{s_1 c_2} \geq Q^{s_2 c_1}/Q^{s_2 c_2}$, where $Q^{sc} \equiv Q(\sigma^s, \gamma^c)$. Still considering the pair of countries, c_1 and c_2, and applying Theorem 1 to an arbitrary subset of J goods, we obtain the following corollary.

COROLLARY 2: *In a F-economy where Assumptions 2 and 3 hold, if two countries produce J goods, with $\gamma^{c_1} \geq \gamma^{c_2}$ and $\sigma^{s_1} \geq \cdots \geq \sigma^{s_J}$, then the high-$\gamma$ country tends to specialize in the high-σ sectors*:

$$\frac{Q^{s_1 c_1}}{Q^{s_1 c_2}} \geq \cdots \geq \frac{Q^{s_J c_1}}{Q^{s_J c_2}}.$$

In a F-economy, the assignment function $\widetilde{\Omega}(\sigma^s)$ is the the same in all countries. As a result, cross-country differences in factor endowments are mechanically reflected in their patterns of specialization. With Hicks-neutral technological differences around the world, a country produces relatively more—compared to other countries—in sectors in which a relatively higher share of its factors selects. Corollary 2 operationalizes that idea by showing that Assumptions 2 and 3 are sufficient to characterize the "sectors in which a relatively higher share of [a country's] factors selects" and, in turn, the pattern of international specialization.

Compared to standard Heckscher–Ohlin predictions, Corollary 2 is a strong result. Even with an arbitrarily large number of goods and factors, it offers predictions on the cross-sectional variation of aggregate output rather than the factor content of trade. To get a better sense of where this strong result comes from, consider a Heckscher–Ohlin model with two factors, K and L, and multiple sectors with different factor intensities, $(K/L)^s$. Because of constant returns to scale, such a model can always be rearranged as an elementary neoclassical economy in which $\omega \equiv (K/L)^s$. The key difference between this model and ours, however, is that its restrictions are about aggregate endowments of capital and labor, not the full distribution of $(K/L)^s$. If there only are two sectors, the two sets of restrictions are equivalent,[11] but with more than two sectors, restrictions on the full distribution are stronger and, hence, lead to stronger predictions.

Corollary 2 also shows that imperfect competition and product differentiation, which are common assumptions in the empirical trade literature, are not necessary to derive "smooth" predictions about aggregate output in environments with multiple countries, goods, and factors.[12] In Romalis (2004), for example, similar predictions are derived in an economy with two factors, two countries, and a continuum of goods because of monopolistic competition and non-factor-price equalization. In a F-economy, markets are perfectly competitive and factor price equalization holds. Yet, the log-supermodularity of h in (ω, σ) creates a strong enough connection between factor and sector characteristics—namely, positive assortative matching—to guarantee that countries should produce relatively more in the sectors that use their abundant factors intensively.

Finally, it is worth pointing out that Assumption 3 only matters indirectly through its impact on $\widetilde{\Omega}(\sigma^s)$. In the proof of Theorem 2, once we have established that high-ω factors are assigned to high-σ sectors, restrictions on

[11]In the two-sector case, the supply of the two factors, $\omega^1 \equiv (K/L)^1$ and $\omega^2 \equiv (K/L)^2$, must be given by

$$V(\omega^1) = \frac{K - L(K/L)^2}{(K/L)^1 - (K/L)^2} \quad \text{and} \quad V(\omega^2) = \frac{K - L(K/L)^1}{(K/L)^2 - (K/L)^1},$$

respectively. Hence there is a one-to-one mapping between K and L, on the one hand, and $V(\omega^1)$ and $V(\omega^2)$, on the other hand.

[12]Eaton and Kortum (2002) made a related point in a Ricardian environment, showing that a gravity equation can be derived under perfect competition in a multicountry–multisector economy. The mechanism emphasized by our model, however, is very different. Unlike Eaton and Kortum (2002), it relies on the efficient assignment of heterogeneous factors across sectors rather than random productivity shocks.

$h(\omega,\sigma)$ are irrelevant. Therefore, Assumptions 2 and 3 also imply that aggregate employment, $L(\sigma^s,\gamma^c)\equiv\int_{\widetilde{\Omega}(\sigma^s)}f(\omega,\gamma^c)\,d\omega$, and aggregate revenues, $R(\sigma^s,\gamma^c)\equiv\int_{\widetilde{\Omega}(\sigma^s)}r(\omega,\sigma^s)f(\omega,\gamma^c)\,d\omega$, are log-supermodular in a F-economy.

COROLLARY 3: *In a F-economy where Assumptions 2 and 3 hold, if two countries produce J goods, with $\gamma^{c_1}\geq\gamma^{c_2}$ and $\sigma^{s_1}\geq\cdots\geq\sigma^{s_J}$, then aggregate employment and aggregate revenues follow the same pattern of specialization as aggregate output*:

$$\frac{L^{s_1c_1}}{L^{s_1c_2}}\geq\cdots\geq\frac{L^{s_Jc_1}}{L^{s_Jc_2}}\quad\textit{and}\quad\frac{R^{s_1c_1}}{R^{s_1c_2}}\geq\cdots\geq\frac{R^{s_Jc_1}}{R^{s_Jc_2}}.$$

Corollary 3 is attractive from an empirical standpoint. To test such predictions on the pattern of international specialization, one is free to use aggregate data on either output, employment, or revenues. Moreover, our predictions all are ordinal in nature. This means that one does not need to observe the true country and sector characteristics to confront them with the data; any monotonic transformation of γ and σ will do.

5.4. *Minimal Sufficient Conditions*

We have just shown that Assumptions 2 and 3 are sufficient conditions to predict the pattern of international specialization in a F-economy. This raises one obvious question: Are there weaker properties on $f(\omega,\gamma^c)$ and $h(\omega,\sigma^s)$ that may also lead to the log-supermodularity of $Q(\sigma^s,\gamma^c)$? The short answer is, like in Section 4, that Assumptions 2 and 3 cannot be dispensed with if one wants to make predictions in *all* F-economies. To address that question formally, we follow the strategy of Athey (2002) and make the following statement:

DEFINITION 5: H_1 and H_2 are a minimal pair of sufficient conditions for a given conclusion C if (i) C holds whenever H_2 does, if and only if H_1 holds and (ii) C holds whenever H_1 does, if and only if H_2 holds.

Definition 5 states that if H_1 and H_2 are a minimal pair of sufficient conditions, then one cannot weaken either H_1 or H_2 without imposing further assumptions on the model. Note that this does not mean that a given conclusion C holds if and only if H_1 and H_2 are satisfied. It simply means that without one or the other, the conclusion C may not hold in all environments. In the next theorem, we show that the log-supermodularity of $f(\omega,\gamma^c)$ and $h(\omega,\sigma^s)$ is a minimal pair of sufficient conditions to predict the log-supermodularity of $Q(\sigma^s,\gamma^c)$ in all F-economies.

THEOREM 3: *In a F-economy, Assumptions 2 and 3 are a minimal pair of sufficient conditions for $Q(\sigma^s,\gamma^c)$ to be log-supermodular.*

From Theorem 2, we already know that Assumptions 2 and 3 are a pair of sufficient conditions. To establish that this pair is minimal, we need to prove the existence of F-economies in which Assumption 3 (resp. Assumption 2) and the log-supermodularity of $Q(\sigma^s, \gamma^c)$ imply Assumption 2 (resp. Assumption 3). First, we construct a F-economy with sector-specific factors, that is, an economy where high-ω factors can only produce in one high-σ sector. In this environment, if $Q(\sigma^s, \gamma^c)$ is log-supermodular, then $f(\omega, \gamma^c)$ automatically is log-supermodular. Second, we construct a F-economy with country-specific factors, that is, an economy where high-ω factors can only be found in one high-γ country. Using the same logic, we show that the log-supermodularity of $Q(\sigma^s, \gamma^c)$ implies the log-supermodularity of $h(\omega, \sigma^s)$.

5.5. *Relation to the Previous Literature*

The model presented in this section is a generalization of the $r \times n$ Ricardian model of Ruffin (1988) that allows for a continuum of factors and Hicks-neutral technological differences. Whereas Ruffin (1988) offered analytical results in the two-good–two-factor case and the two-good–three-factor case, our results hold for an arbitrarily large number of goods and factors. Like in the Ricardian case, imposing the log-supermodularity of the primitives of our model is crucial to predict the pattern of international specialization in economies with higher dimensionality.

Ohnsorge and Trefler (2004) have developed an elegant variation of Ruffin's (1988) model with a continuum of workers.[13] To derive closed form solutions, they assumed that workers' endowments of human capital are normally distributed and that worker productivity takes an exponential form. In the working paper version of this paper (Costinot (2007)) we demonstrated that their model can be described as a F-economy. Once the signs of the cross-derivatives of $\ln f$ and $\ln h$ have been computed using their functional forms, their results on the pattern of international specialization (Ohnsorge and Trefler (2004, Theorem 1, p. 15, Theorem 3, p. 19, and Theorem 5, p. 27)) are implications of Corollary 2. By identifying log-supermodularity as the critical sufficient condition for their results to hold, our analysis demonstrates not only the possibility of their results, but also their robustness.

Interestingly, the forces that shape the pattern of international specialization in a F-economy also play a central role in determining the prevalence of organizational forms in trade models with firm-level heterogeneity à la Melitz (2003), most notably Helpman, Melitz, and Yeaple (2004) and Antras and Helpman (2004). The formal relationship between these papers and ours is discussed in detail in Costinot (2007). To map these models into our general framework, one needs to reinterpret each factor as a firm with productivity ω,

[13] Ohnsorge and Trefler (2007) considered the case of a continuum of goods and workers.

each country as an industry with characteristic γ, and each sector as an organization with characteristic σ. Then total sales $Q(\sigma, \gamma)$ by firms with a σ organization in a γ industry can be expressed as $Q(\sigma, \gamma) = \int_{\widetilde{\Omega}(\sigma)} h(\omega, \sigma) f(\omega, \gamma)\, d\omega$, where $\widetilde{\Omega}(\sigma) = \{\omega \in \Omega | r(\omega, \sigma) > \max_{\sigma' \neq \sigma} r(\omega, \sigma')\}$. Under this interpretation, $h(\omega, \sigma)$ and $r(\omega, \sigma)$ are the sales and profits of a firm with productivity ω and organization σ, and $f(\omega, \gamma)$ is density of productivity levels in sector γ.[14] Like in Theorem 2, the assignment function $\widetilde{\Omega}(\sigma)$ is the same across industries. As a result, differences in the distribution of firm productivity are mechanically reflected in the prevalence of various organizational forms. This explains why industries with relatively more productive firms have relatively more sales associated with the organization that more productive firms select, whether it is FDI or vertical integration.[15]

6. MULTIPLE SOURCES OF COMPARATIVE ADVANTAGE

6.1. *A Simple Generalization With Factor-Augmenting Technological Differences*

Before offering a general analysis of economies with factor endowment and Ricardian technological differences, we present a simple generalization of a F-economy that allows for factor-augmenting technological differences. Formally, we assume that q satisfies

$$q(\omega, \sigma, \gamma) = a(\gamma) h[\omega + t(\gamma), \sigma], \tag{8}$$

where $\omega + t(\gamma) \in \Omega$ for all $\omega \in \Omega$ and $\gamma \in \Gamma$. For example, one can think of ω as the log of workers' number of years of education and of $t(\gamma)$ as a measure of the quality of the educational system in a given country. If $t(\gamma) \equiv 0$, then we are back to the Hicks-neutral case, but if $t(\gamma) \neq 0$, Equation (8) introduces the possibility of technological complementarities between country and sector characteristics like in a R-economy.

DEFINITION 6: An elementary neoclassical economy is a F_a-economy if Equation (8) holds.

[14] The previous models and ours only differ in terms of market structure. In the previous models, monopolistic competition implies $r(\omega, \sigma) \neq p(\sigma) q(\omega, \sigma, \gamma)$. In Costinot (2007), we showed that our analysis carries over to that environment if $r(\omega, \sigma)$ satisfies a single crossing property. In a F-economy, the only role of Assumption 3 is to guarantee that $p(\sigma) q(\omega, \sigma, \gamma)$ satisfies a single crossing property for all $p(\sigma)$.

[15] Antras and Helpman (2004, footnote 10, p. 571) also recognized the existence of a connection between the mechanism at work in their model and Melitz (2003) and Helpman, Melitz, and Yeaple (2004). However, they did not discuss the critical assumptions on which this logic depends. Our analysis shows, for example, that assuming the log-supermodularity of $f(\omega, \gamma)$ is critical, whereas assuming Pareto distributions is not.

F_a-economies are interesting because they offer a simple way to combine factor endowment and Ricardian technological differences. Formally, F_a-economies are equivalent to F-economies up to a change of variable, $\widetilde{\omega} \equiv \omega + t(\gamma)$. Once endowments have been expressed in the same efficiency units across countries, aggregate output is equal to

$$Q(\sigma^s, \gamma^c) = \int_{\widetilde{\Omega}(\sigma^s)} a(\gamma^c) h(\omega, \sigma^s) \widetilde{f}(\omega, \gamma^c)\, d\mu(\omega), \tag{9}$$

where $\widetilde{f}(\omega, \gamma^c) \equiv f[\omega - t(\gamma^c), \gamma^c]$ and $\widetilde{\Omega}(\sigma^s)$ is given by Equation (7). Therefore, if we can show that $\widetilde{f}(\omega, \gamma^c)$ is log-supermodular, then we can still use Theorem 2 to predict the pattern of international specialization. The next theorem offers sufficient conditions on f and t that allow us to do so.

THEOREM 4: *Consider a F_a-economy where Assumptions 2 and 3 hold. If t is increasing in γ and f is strictly positive and log-concave in ω, then $Q(\sigma, \gamma)$ is log-supermodular.*

Like log-supermodularity, log-concavity is satisfied by many standard distributions including the uniform, normal, and extreme value distributions (see Bagnoli and Bergstrom (2005)). The monotonicity of t and the log-concavity of f guarantee that technological differences reinforce the pattern of international specialization driven by factor endowment differences, Assumptions 2 and 3. Even in the absence of true cross-country differences in factor supplies, they imply that high-γ countries are relatively more abundant, now in efficiency units, in the high-ω factors.[16] The pattern of international specialization in a F_a-economy follows.

Finally, note that the previous results are stronger than standard Heckscher–Ohlin predictions with factor-augmenting productivity differences (see, e.g., Trefler (1993)). Like in a F-economy, our approach leads to predictions on the cross-sectional variation of aggregate output rather than the factor content of trade.

6.2. *General Technological Differences*

We have just shown that the predictions of Sections 4 and 5 may extend to an environment with both Ricardian and factor-endowment sources of comparative advantage, albeit under strong functional form restrictions. We now turn to the case in which q satisfies the following assumption.

[16]It is worth emphasizing that Theorem 4 crucially relies on the linear relationship between ω and $t(\gamma)$. We could, in principle, generalize the nature of factor-augmenting technological differences by assuming that $q(\omega, \sigma^s, \gamma^c) = a(\gamma^c)h[t(\omega, \gamma^c), \sigma^s]$. This generalized version of a F_a-economy would still be equivalent to a F-economy up to a change of variable, but predicting the log-supermodularity of Q would then require strong regularity conditions on f—as strong as the restrictions on t are weak .

ASSUMPTION 4: *$q(\omega, \sigma, \gamma)$ is log-supermodular.*

Assumption 4 is a strict generalization of the assumptions used in R- and F-economies.[17] Compared to a R-economy, it allows complementarities between ω and σ; compared to a F-economy, it allows complementarities between σ and γ. Hence, both factor endowment and technological differences can, in principle, determine the pattern of international specialization. The question we want to ask is, "Does Assumption 4 put enough structure on the nature of technological differences to predict the pattern of international specialization across all countries and sectors of an elementary neoclassical economy?" Perhaps surprisingly, the answer is no. Under Assumption 4, robust predictions about international specialization can only be derived in the two most extreme sectors, as we now demonstrate.

6.2.1. *A Counterexample*

We start by offering a counterexample in which factor supply satisfies Assumption 2, factor productivity satisfies Assumption 4, and yet aggregate output is not log-supermodular.

Consider an elementary neoclassical economy comprising two countries with characteristics $\gamma^2 > \gamma^1 > 0$, $S > 2$ sectors with characteristics $\sigma^S \equiv (\sigma_1^S, \sigma_2^S) > \cdots > \sigma^1 \equiv (\sigma_1^1, \sigma_2^1) > 0$, and a continuum of factors $\omega \in [0, 1]$. Factor productivity and factor supply are given by

$$q(\omega, \sigma, \gamma) \equiv \omega^{\sigma_1} \gamma^{\sigma_2}, \tag{10}$$

$$f(\omega, \gamma) \equiv 1. \tag{11}$$

By Equations (10) and (11), Assumptions 4 and 2 are trivially satisfied. For any $c = 1, 2$ and $s = 1, \ldots, S - 1$, we denote by $\omega^*_{c,s}$ the factor whose marginal return is equalized between sector s and sector $s + 1$ in country c. By Equation (10), we have

$$\omega^*_{c,s} = (\gamma^c)^{-\lambda^s} \cdot \left[\frac{p(\sigma^s)}{p(\sigma^{s+1})}\right]^{1/(\sigma_1^{s+1} - \sigma_1^s)}, \tag{12}$$

where $\lambda^s \equiv (\sigma_2^{s+1} - \sigma_2^s)/(\sigma_1^{s+1} - \sigma_1^s)$. For any $\gamma^2 > \gamma^1 > 0$ and $\sigma^S > \cdots > \sigma^1 > 0$, there exist $p(\sigma^S) > \cdots > p(\sigma^1) > 0$ such that $1 > \omega^*_{c,S} > \cdots > \omega^*_{c,1} > 0$ for $c = 1, 2$. Assuming that the previous series of inequalities holds, we can write

[17] Strictly speaking, Assumption 4 is not a generalization of the assumptions used in a F_a-economy. According to Equation (8), if h is log-supermodular and t is increasing, then q is log-supermodular in (ω, σ) and (σ, γ), but not necessarily log-supermodular in (ω, γ). However, as mentioned in Section 6.1, there exists a change of variable $\widetilde{\omega} \equiv \omega + t(\gamma)$ such that a F_a-economy reduces to a F-economy.

$\Omega(\sigma^s, \gamma^c) = (\omega^*_{c,s-1}, \omega^*_{c,s})$, with the convention that $\omega^*_{c,-1} = 0$ and $\omega^*_{c,S+1} = 1$. Now combining this observation with Equations (3), (4), (10), and (11), we obtain

$$(13) \qquad Q(\sigma^s, \gamma^c) = (\gamma^c)^{\sigma_2^s} \cdot \left[\frac{(\omega^*_{c,s})^{\sigma_1^s+1} - (\omega^*_{c,s-1})^{\sigma_1^s+1}}{\sigma_1^s + 1}\right].$$

Now take $1 < s_1 < s_2 < S$ such that $\lambda^{s_1} = \lambda^{s_1-1} = \lambda^{s_2} = \lambda^{s_2-1} = 1$. Equations (12) and (13) imply $Q(\sigma^{s_1}, \gamma^1)Q(\sigma^{s_2}, \gamma^2) \geq Q(\sigma^{s_1}, \gamma^2)Q(\sigma^{s_2}, \gamma^1) \Leftrightarrow (\gamma^2/\gamma^1)^{(\sigma_2^{s_2}-\sigma_2^{s_1})-(\sigma_1^{s_2}-\sigma_1^{s_1})} \geq 1$. Since $\gamma^2 > \gamma^1 > 0$, Q is not log-supermodular for $\sigma_2^{s_2} - \sigma_2^{s_1} < \sigma_1^{s_2} - \sigma_1^{s_1}$.[18]

What went wrong? The problem comes from the restrictions (or lack thereof) that Assumption 4 imposes on the variations of the assignment function, $\Omega(\sigma, \gamma)$. In a R-economy, Equation (5) guarantees that $\Omega(\sigma, \gamma)$ is either Ω or $\emptyset$. In a F-economy, Equation (6) guarantees that $\Omega(\sigma, \gamma)$ is invariant across countries. Here, Equation (10) merely requires that (i) $1 > \omega^*_{c,S} > \cdots > \omega^*_{c,1} > 0$ for $c = 1, 2$, by the log-supermodularity of q in (ω, σ), and that (ii) $\omega^*_{1,s} > \omega^*_{2,s}$ for $s = 1, \ldots, S-1$, by the log-supermodularity of q in (σ, γ). The latter condition opens up the possibility for country 2 to produce relatively less in the high-σ sectors. Though the log-supermodularity of q in (σ, γ) makes identical factors relatively more productive in the high-σ sectors in country 2, it also leads to lower-ω factors to be assigned in these sectors under an efficient allocation. Since lower-ω factors are relatively less productive in the high-σ sectors by the log-supermodularity of q in (ω, σ), this indirect effect tends to reduce the relative output of high-σ sectors in country 2. Our counterexample simply offers conditions under which the indirect effect of Ricardian technological differences on factor assignment dominates their direct effect on factor productivity: $\sigma_1^{s_2} - \sigma_1^{s_1} > \sigma_2^{s_2} - \sigma_2^{s_1}$.

At a formal level, our complementarity approach breaks down because it requires conditions that are too strong. To predict patterns of specialization using Lemmas 1 and 2, we now need $\mathbb{1}_{\Omega(\sigma,\gamma)}(\omega)$ to be log-supermodular. Assuming that q is log-supermodular in (σ, γ) and (ω, σ) guarantees that $\mathbb{1}_{\Omega(\sigma,\gamma)}(\omega)$ is log-supermodular in (σ, γ) and (ω, σ), as in R- and F-economies, respectively. Both results are the counterparts of predictions about the monotonicity of solutions of assignment problems in the monotone comparative statics literature (see, e.g., Topkis (1998)). Unfortunately, there is no underlying assignment problem such that the log-supermodularity of q in (ω, γ) implies the log-supermodularity of $\mathbb{1}_{\Omega(\sigma,\gamma)}(\omega)$ in (ω, γ). On the contrary, if q is log-supermodular in (σ, γ) and (ω, σ), then $\mathbb{1}_{\Omega(\sigma,\gamma)}(\omega)$ must be log-submodular in

[18]The conditions $\sigma^S > \cdots > \sigma^1 > 0$, $\lambda^{s_1} = \lambda^{s_1-1} = \lambda^{s_2} = \lambda^{s_2-1} = 1$, and $\sigma_2^{s_2} - \sigma_2^{s_1} < \sigma_1^{s_2} - \sigma_1^{s_1}$ all are satisfied, for example, if $S = 6$, $s_1 = 2$, $s_2 = 5$, $(\sigma_1^1, \ldots, \sigma_1^6) = (1, 2, 3, 5, 6, 7)$, and $(\sigma_2^1, \ldots, \sigma_2^6) = (1, 2, 3, 4, 5, 6)$.

(ω, γ). This is what we refer to as the "indirect effect of Ricardian technological differences on factor assignment."

As a careful reader may have already noticed, our counterexample is based purely on technological considerations. Although the fact that factor endowment differences *can* be a source of comparative advantage is crucial, our argument does not rely on factor-endowment differences per se: $f(\omega, \gamma^1) = f(\omega, \gamma^2) = 1$. Instead, it relies on the fact that for factor endowments to be a source of comparative advantage, sectors must vary in their factor intensity, that is, q must be log-supermodular in (ω, σ). In our counterexample, the log-supermodularity of q in both (σ, γ) and (ω, σ) is the only issue.

6.2.2. *Robust Predictions*

When only one source of comparative advantage is present, Theorems 1 and 2 show that the log-supermodularity of the primitives of our model implies the log-supermodularity of aggregate output. These are strong results that apply to all countries and all sectors. Theorem 4 offers similar predictions in the presence of multiple sources of comparative advantage if additional functional form restrictions are imposed.

The previous counterexample shows, unfortunately, that these strong results do not hold generally. This is an important observation that highlights the potential caveats of combining insights from distinct models without a generalizing framework. In a R-economy, if high-γ countries are relatively more productive in high-σ sectors, then they produce relatively more in these sectors. In a F-economy, if high-γ countries are relatively more abundant in high-ω factors and high-ω factors are relatively more productive in high-σ sectors, then the same pattern of international specialization arises. In our counterexample, these assumptions on technology and factor endowments all are satisfied, yet high-γ countries do not produce relatively more in all high-σ sectors.[19]

Our last theorem offers weaker, but robust predictions on the cross-sectional variation of aggregate output in this general environment.

THEOREM 5: *Let* $\underline{\sigma} \equiv \min_{1\leq s\leq S} \sigma^s$ *and* $\overline{\sigma} \equiv \max_{1\leq s\leq S} \sigma^s$. *In an elementary neoclassical economy, Assumptions 2 and 4 imply* $Q(\underline{\sigma}, \gamma^{c_1})Q(\overline{\sigma}, \gamma^{c_2}) \geq Q(\underline{\sigma}, \gamma^{c_2})Q(\overline{\sigma}, \gamma^{c_1})$ *for any pair of countries such that* $\gamma^{c_1} \leq \gamma^{c_2}$.

According to Theorem 5, if both factor endowment and technological differences are sources of comparative advantage, the pattern of international specialization is unambiguous only in the two most extreme sectors. The formal argument combines the main ideas of Theorems 1 and 2. Holding the assignment function $\Omega(\sigma, \gamma)$ constant across countries, the log-supermodularity

[19]In addition, the previous counterexample shows that assuming $q(\omega, \sigma, \gamma) \equiv h(\omega, \sigma)a(\sigma, \gamma)$ with h and a log-supermodular is not sufficient to derive the log-supermodularity of Q.

of f and q implies that high-γ countries produce relatively more in the high-σ sectors, as in a F-economy. Since q is log-supermodular in σ and γ, the cross-country variation in $\Omega(\sigma, \gamma)$ then reinforces this effect in the two most extreme sectors. As in a R-economy, a given factor ω is more likely to be found in high-σ sectors in high-γ countries. This implies that a given sector σ is more likely to be assigned low-ω factors in high-γ countries, and, in turn, that $\Omega(\underline{\sigma}, \gamma^{c_2}) \subseteq \Omega(\underline{\sigma}, \gamma^{c_1})$ and $\Omega(\overline{\sigma}, \gamma^{c_1}) \subseteq \Omega(\overline{\sigma}, \gamma^{c_2})$. In other words, Ricardian technological differences necessarily lead to more factors in the highest-σ sector in high-γ countries and more factors in the lowest-σ sector in low-γ countries, thereby strengthening the pattern of international specialization driven by factor endowment differences.

Our final result is reminiscent of the Rybczynski results derived by Jones and Scheinkman (1977) in a standard neoclassical model with an arbitrary number of goods and factors. They showed, among other things, that if factor prices are constant across countries, as in our model, then an increase in the endowment of one factor must decrease output in one sector and increase output more than proportionally in another sector. An important difference between this result and ours is that Theorem 5 clearly identifies the two most extreme sectors as those being affected by changes in factor endowments.[20]

7. CONCLUDING REMARKS

The present paper has developed an elementary theory of comparative advantage. Our theory emphasizes an intimate relationship between log-supermodularity and comparative advantage, whether driven by technology or factor endowment. If factor productivity and/or factor supply are log-supermodular, then many sharp predictions on the pattern of international specialization can be derived in economies with an arbitrarily large number of countries, goods, and factors.

While we have focused on the determinants of international specialization, we believe that our general results could also be useful outside international trade. The basic structure of our model—Equations (3) and (4)—is central to many models with agent heterogeneity. These models may focus on different aggregate variables and different market structures, but at the core, there are "agents" with characteristics ω, these agents belong to "populations" with density $f(\omega, \gamma)$ and they sort into "occupations" with characteristics σ based on their returns $r(\omega, \sigma, \gamma)$. In public finance, agents may be taxpayers with different income sorting across different cities; in corporate finance, agents may

[20]Theorem 5 also is related, though less closely, to recent work by Zhu and Trefler (2005), Costinot and Komunjer (2007), Chor (2008), and Morrow (2008). For empirical purposes, the previous papers derived predictions on the pattern of international specialization in economies with Ricardian differences and multiple factors of productions. These predictions, however, are based on assumptions on factor prices, rather than primitive assumptions on factor supply.

be firms with different productivity choosing different financial instruments; in development economics, agents may be individuals with different wealth choosing whether or not to become entrepreneurs; in labor economics, agents may be migrants with different levels of education sorting across destinations; and in organization economics, agents may be workers with different skills sorting across different layers of a hierarchy.[21] In any of these circumstances, our analysis demonstrates that log-supermodularity potentially has important implications for the cross-sectional variation of aggregate variables across populations and occupations.

APPENDIX: PROOFS

PROOF OF THEOREM 1: We proceed by contradiction. Consider $\sigma \geq \sigma'$ and $\gamma \geq \gamma'$. Suppose that $Q(\sigma', \gamma)Q(\sigma, \gamma') > Q(\sigma, \gamma)Q(\sigma', \gamma')$. This implies $Q(\sigma', \gamma) > 0$ and $Q(\sigma, \gamma') > 0$ with $\sigma \neq \sigma'$ and $\gamma \neq \gamma'$. Since $Q(\sigma', \gamma) > 0$, Equations (3) and (4) further imply the existence of $\omega \in \Omega$ such that $r(\omega, \sigma', \gamma) > r(\omega, \sigma, \gamma)$. Using Equation (5), we can simplify the previous inequality into $p(\sigma')a(\sigma', \gamma) > p(\sigma)a(\sigma, \gamma)$. Since $Q(\sigma, \gamma') > 0$, similar reasoning implies $p(\sigma)a(\sigma, \gamma') > p(\sigma')a(\sigma', \gamma')$. Combining the previous inequalities, we obtain $a(\sigma', \gamma)a(\sigma, \gamma') > a(\sigma, \gamma)a(\sigma', \gamma')$, which contradicts Assumption 1. *Q.E.D.*

PROOF OF THEOREM 2: Let $\widetilde{h}(\omega, \sigma) \equiv \mathbb{1}_{\widetilde{\Omega}(\sigma)}(\omega) \cdot h(\omega, \sigma)$. We first show that $\widetilde{h}(\omega, \sigma)$ is log-supermodular. We proceed by contradiction. Consider $\omega \geq \omega'$ and $\sigma \geq \sigma'$. Suppose that $\widetilde{h}(\omega, \sigma')\widetilde{h}(\omega', \sigma) > \widetilde{h}(\omega, \sigma)\widetilde{h}(\omega', \sigma')$. This implies $\omega \in \widetilde{\Omega}(\sigma')$ and $\omega' \in \widetilde{\Omega}(\sigma)$ with $\omega \neq \omega'$ and $\sigma \neq \sigma'$. Using Equation (7), we then get $p(\sigma')h(\omega, \sigma') > p(\sigma)h(\omega, \sigma)$ and $p(\sigma)h(\omega', \sigma) > p(\sigma')h(\omega', \sigma')$. The two previous inequalities imply $h(\omega, \sigma')h(\omega', \sigma) > h(\omega, \sigma)h(\omega', \sigma')$, which contradicts Assumption 3. By Equations (3), (6), and (7), we have $Q(\sigma, \gamma) = \int \widetilde{h}(\omega, \sigma)a(\gamma)f(\omega, \gamma)\,d\omega$. We have just shown that $\widetilde{h}(\omega, \sigma)$ is log-supermodular. $a(\gamma)$ is trivially log-supermodular. By Assumption 2, we know that $f(\omega, \gamma)$ is log-supermodular. Theorem 2 derives from these three observations and the fact that log-supermodularity is preserved by multiplication and integration, by Lemmas 1 and 2. *Q.E.D.*

PROOF OF THEOREM 3: Theorem 2 shows that Assumptions 2 and 3 are sufficient conditions. That they are a minimal pair is proved by two lemmas.

[21]See, for example, Epple and Romer (1991), Champonnois (2007), Banerjee and Newman (1993), Grogger and Hanson (2008), and Garicano and Rossi-Hansberg (2006), respectively.

LEMMA 3: *If $Q(\sigma, \gamma)$ is log-supermodular in any F-economy where Assumption 3 holds, then Assumption 2 holds.*

PROOF: We proceed by contradiction. Suppose that there exist $\omega \geq \omega'$ and $\gamma \geq \gamma'$ such that

$$f(\omega, \gamma')f(\omega', \gamma) > f(\omega, \gamma)f(\omega', \gamma'). \tag{14}$$

Inequality (14) implies $\omega \neq \omega'$. Now consider a F-economy with two factors, ω and ω', two countries with characteristics γ and γ', and two sectors with characteristics $\sigma > \sigma'$, such that

$$a(\gamma) = a(\gamma') = 1, \tag{15}$$

$$h(\omega, \sigma) = h(\omega', \sigma') = 1, \tag{16}$$

$$h(\omega, \sigma') = h(\omega', \sigma) = 0, \tag{17}$$

which is possible since $\omega \neq \omega'$ and $\sigma \neq \sigma'$. Combining Equations (15), (16), and (17) with Equations (3), (6), and (7) and inequality (14), we get $Q(\sigma, \gamma')Q(\sigma', \gamma) > Q(\sigma, \gamma)Q(\sigma', \gamma')$. By Equations (16) and (17), Assumption 3 is satisfied—a contradiction. *Q.E.D.*

LEMMA 4: *If $Q(\sigma, \gamma)$ is log-supermodular in any F-economy where Assumption 2 holds, then Assumption 3 holds.*

PROOF: We proceed by contradiction. Suppose that there exist $\omega \geq \omega'$ and $\sigma \geq \sigma'$ such that

$$h(\omega, \sigma')h(\omega', \sigma) > h(\omega, \sigma)h(\omega', \sigma'). \tag{18}$$

Inequality (18) implies $\omega \neq \omega'$, $h(\omega, \sigma') > 0$, and $h(\omega', \sigma) > 0$. This further implies the existence of $p(\sigma) > 0$ and $p(\sigma') > 0$ such that

$$\frac{h(\omega', \sigma)}{h(\omega', \sigma')} > \frac{p(\sigma')}{p(\sigma)} > \frac{h(\omega, \sigma)}{h(\omega, \sigma')}, \tag{19}$$

where $\frac{h(\omega',\sigma)}{h(\omega',\sigma')} = +\infty$ if $h(\omega', \sigma') = 0$. Now consider a F-economy with two factors, ω and ω', two countries with characteristics $\gamma > \gamma'$, and two sectors with characteristics σ and σ', such that prices satisfy inequality (19) and

$$a(\gamma) = a(\gamma') = 1, \tag{20}$$

$$f(\omega, \gamma) = f(\omega', \gamma') = 1, \tag{21}$$

$$f(\omega, \gamma') = f(\omega', \gamma) = 0, \tag{22}$$

which is possible since $\omega \neq \omega'$ and $\gamma \neq \gamma'$. Inequality (19) implies $p(\sigma') \times h(\omega, \sigma') > p(\sigma)h(\omega, \sigma)$ and $p(\sigma)h(\omega', \sigma) > p(\sigma')h(\omega', \sigma')$. Combining these two inequalities with Equations (3), (6), (7), (20), (21), and (22), we get $Q(\sigma, \gamma')Q(\sigma', \gamma) > Q(\sigma, \gamma)Q(\sigma', \gamma')$. By Equations (21) and (22), Assumption 2 is satisfied—a contradiction. *Q.E.D.*

PROOF OF THEOREM 4: We first show that if $f(\omega, \gamma)$ is log-supermodular, strictly positive, and log-concave in ω, and t is increasing in γ, then $\widetilde{f}(\omega, \gamma) \equiv f[\omega - t(\gamma), \gamma]$ is log-supermodular. We proceed by contradiction. Suppose that there exist $\omega \geq \omega'$ and $\gamma \geq \gamma'$ such that

$$f[\omega' - t(\gamma), \gamma]f[\omega - t(\gamma'), \gamma'] > f[\omega - t(\gamma), \gamma]f[\omega' - t(\gamma'), \gamma']. \tag{23}$$

Since f is log-supermodular in (ω, γ), we know that

$$f[\omega - t(\gamma'), \gamma]f[\omega' - t(\gamma'), \gamma'] \geq f[\omega' - t(\gamma'), \gamma]f[\omega - t(\gamma'), \gamma']. \tag{24}$$

Combining inequalities (23) and (24) and the fact that $f > 0$, we get

$$f[\omega' - t(\gamma), \gamma]f[\omega - t(\gamma'), \gamma] > f[\omega - t(\gamma), \gamma]f[\omega' - t(\gamma'), \gamma]. \tag{25}$$

Since t is increasing in γ, inequality (25) implies that $f(\cdot, \gamma)$ cannot be of Polya frequency of order 2, which contradicts f log-concave in ω; see An (1998). At this point, we know that aggregate output is given by Equations (7) and (9), that $\widetilde{f}(\omega, \gamma)$ is log-supermodular, and that $h(\omega, \sigma)$ is log-supermodular by Assumption 2. Thus, we can invoke Theorem 2, which implies $Q(\sigma, \gamma)$ is log-supermodular. *Q.E.D.*

PROOF OF THEOREM 5: We use the following lemma.

LEMMA 5: *If $q(\omega, \sigma, \gamma)$ satisfies Assumption 4, then $\Omega(\underline{\sigma}, \gamma^{c_2}) \subseteq \Omega(\underline{\sigma}, \gamma^{c_1})$ and $\Omega(\overline{\sigma}, \gamma^{c_1}) \subseteq \Omega(\overline{\sigma}, \gamma^{c_2})$ for any $\gamma^{c_2} \geq \gamma^{c_1}$.*

PROOF: Without loss of generality, suppose that $\underline{\sigma} = \sigma^1$. We only show that Assumption 4 implies $\Omega(\underline{\sigma}, \gamma^{c_2}) \subseteq \Omega(\underline{\sigma}, \gamma^{c_1})$ for any $\gamma^{c_2} \geq \gamma^{c_1}$. The argument for $\Omega(\overline{\sigma}, \gamma^{c_1}) \subseteq \Omega(\overline{\sigma}, \gamma^{c_2})$ is similar. We proceed by contradiction. Take $\omega \in \Omega(\underline{\sigma}, \gamma^{c_2})$. By Equation (4), we have $p(\underline{\sigma})q(\omega, \underline{\sigma}, \gamma^{c_2}) > \max_{s\neq 1} p(\sigma^s)q(\omega, \sigma^s, \gamma^{c_2})$. Now suppose that $\omega \notin \Omega(\underline{\sigma}, \gamma^{c_1})$. By Equation (4), we also have $\max_{s\neq 1} p(\sigma^s)q(\omega, \sigma^s, \gamma^{c_1}) > p(\underline{\sigma})q(\omega, \underline{\sigma}, \gamma^{c_1})$. Let $s^* \equiv \arg\max_{s\neq 1} p(\sigma^s)q(\omega, \sigma^s, \gamma^{c_1})$. Combining the two previous inequalities, we get $q(\omega, \underline{\sigma}, \gamma^{c_2})q(\omega, \sigma^{s^*}, \gamma^{c_1}) > q(\omega, \sigma^{s^*}, \gamma^{c_2})q(\omega, \underline{\sigma}, \gamma^{c_1})$. Since $\sigma^{s^*} \geq \underline{\sigma}$ and $\gamma^{c_2} \geq \gamma^{c_1}$, this contradicts Assumption 4. *Q.E.D.*

PROOF OF THEOREM 5—Continued: Fix $\gamma^{c_1} \in \Gamma$ and define $\widetilde{Q}(\sigma, \gamma)$ such that

$$\widetilde{Q}(\sigma, \gamma) \equiv \int_{\Omega(\sigma, \gamma^{c_1})} q(\omega, \sigma, \gamma)f(\omega, \gamma)\, d\mu(\omega) \tag{26}$$

for all $\sigma \in \Sigma$ and $\gamma \in \Gamma$. We can use the same reasoning as in Theorem 2 to show that $\widetilde{Q}(\sigma, \gamma)$ is log-supermodular. By Assumption 4, $\mathbb{1}_{\Omega(\sigma,\gamma^{c_1})}(\omega)$ is log-supermodular in (ω, σ). By Assumptions 2 and 4, $q(\omega, \sigma, \gamma)$ and $f(\omega, \gamma)$ also are log-supermodular. Hence, $\widetilde{Q}(\sigma, \gamma)$ is log-supermodular by Lemmas 1 and 2, which implies

$$\widetilde{Q}(\underline{\sigma}, \gamma^{c_1})\widetilde{Q}(\overline{\sigma}, \gamma^{c_2}) \geq \widetilde{Q}(\underline{\sigma}, \gamma^{c_2})\widetilde{Q}(\overline{\sigma}, \gamma^{c_1}) \tag{27}$$

for any $\gamma^{c_2} \geq \gamma^{c_1}$. By Equation (26), we have $\widetilde{Q}(\underline{\sigma}, \gamma^{c_1}) = Q(\underline{\sigma}, \gamma^{c_1})$ and $\widetilde{Q}(\overline{\sigma}, \gamma^{c_1}) = Q(\overline{\sigma}, \gamma^{c_1})$. Since $q(\omega, \sigma, \gamma) \geq 0$ and $f(\omega, \gamma) \geq 0$, we also have $\widetilde{Q}(\underline{\sigma}, \gamma^{c_2}) \geq Q(\underline{\sigma}, \gamma^{c_2})$ and $Q(\overline{\sigma}, \gamma^{c_2}) \geq \widetilde{Q}(\overline{\sigma}, \gamma^{c_2})$ by Lemma 5. Combining the previous conditions with inequality (27), we get $Q(\underline{\sigma}, \gamma^{c_1})Q(\overline{\sigma}, \gamma^{c_2}) \geq Q(\underline{\sigma}, \gamma^{c_2})Q(\overline{\sigma}, \gamma^{c_1})$ for any $\gamma^{c_2} \geq \gamma^{c_1}$. *Q.E.D.*

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