ASYMMETRIC INFORMATION, BARGAINING, AND UNEMPLOYMENT FLUCTUATIONS*

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We construct a dynamic general equilibrium model where wages are determined by bilateral bargaining and the firm has superior information. The asymmetry of information introduces unemployment fluctuations and *dynamic wage sluggishness*. Because the information of the firm only is revealed gradually, wages fall slowly in response to a negative shock and unemployment exhibits additional persistence. It is shown that high job destruction will generally be followed by a period of higher than average job destruction, that the presence of common shocks introduces an informational externality, and that bargaining is an inefficient method of wage determination compared to implicit contracts.

1. INTRODUCTION

Unemployment both in the United States and Europe exhibits considerable persistence: a period of high unemployment is usually followed by further periods of higher than average unemployment (see for instance Jaeger and Parkinson 1994). Existing theories have difficulty in explaining this high level of persistence. In this paper I argue that dynamic bargains and asymmetric information in the wage determination process can be an additional source of persistence in unemployment fluctuations.

The main intuition is simple: in noncompetitive labor markets, both workers and firms receive rents from the employment relation. However, if one of the parties is imperfectly informed about the size of the total rents to be distributed, their demand can be excessively high leading to a premature termination of the employment relation. This feature in our model will lead to the destruction of jobs. However, in a dynamic bargaining framework, agreements often take time and the relevant information is only revealed gradually. Thus when a bad shock hits the economy, the severity of this shock is only revealed slowly and this creates what we call *dynamic wage sluggishness*. Wages are not only high (relative to the marginal product of labor) now but also in the future periods because, with dynamic bargaining, not all the relevant information about the severity of the shock is revealed immediately. Therefore a temporary shock will not only lead to jobs being

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TABLE 1

Job Creation = 0.019939 + 0.59116 JC(-1)
(3.5293)   (5.6724)
Job Destruction = 0.03791 + 0.67948 JD(-1)
(5.2540)   (7.2956)
Sample. 72, Quarter 4 to 88, Quarter 4
Source. Davis, Haltiwanger, and Schuh (1995).
Dummies for second, third and fourth quarters are used. No further lags are significant.

T-statistics in parentheses.

destroyed in the current period but also in the periods to come. In other words, when information relevant to the formation of wages is only spread slowly, the adverse effects of this shock will also be spread out over time.

To capture these features we will consider a dynamic general equilibrium model in which the marginal product of labor is only observed by firms. Jobs are created when unemployed workers are matched with new firms. Yet the job can be destroyed if the worker and the firm fail to agree on the wage rate. We will analyze the behavior of this economy in the presence of aggregate and idiosyncratic shocks to the marginal product of labor and show that the presence of private information will lead to *amplification* of the impact of shocks on employment fluctuations and to additional *persistence*.

This channel of persistence differs from the existing ones in the literature; the most usual channel of persistence in the literature relies on sluggish job creation. A shock increases the unemployment level and the unemployed workers cannot easily get back into jobs. This can be because of search (e.g. Pissarides 1985, Wright 1986) or due to insiders who ask for higher wages and prevent an expansion of employment (e.g. Lindbeck and Snower 1988, Blanchard and Summers 1986) or because unemployed workers lose their skills (Pissarides 1992). Nevertheless simple regressions of job creation and destruction on their respective past values show equal degree of persistence (Table 1).[2]

It seems plausible that part of our failure to account for successive periods of high unemployment may be due to our inability to explain this high degree of persistence in job destruction. Recent models that endogenize job destruction such as Mortensen and Pissarides (1994) and Caballero and Hammour (1994) can potentially generate an aggregate flow of job destruction that is persistent. Essentially, a negative aggregate shock first leads to a large amount of job destruction and then the flow of job destruction may remain higher than before because each job is more likely to fall below the cut-off level of firm-specific productivity. However, an important feature of these models is that still most of the persistence has to come from job creation because job destruction is always a *forward-looking* (jump) variable whereas job creation is the backward-looking variable. In contrast, the additional degree of persistence in our model is provided because, due to the slow revelation of information, job destruction becomes a *backward-looking* variable.

The model developed in this paper has some related precedents: the game forms we use are similar to those studied by Fudenberg, Levine, and Tirole (1985), (1987)

[2] Since the job destruction series is highly nonlinear, the notion of persistence has to be interpreted carefully.

and Hart and Tirole (1988). The contribution by Grossman, Hart, and Maskin (1983) is also closely related to our model. They construct a general equilibrium model with incomplete information and show that involuntary unemployment would result. Our paper differs from theirs in so far as we are using a dynamic bargaining framework for wage determination and this enables us to discuss the slow revelation of information and how this would contribute to the persistence of unemployment fluctuations. Pissarides (1985) and (1987) analyze equilibrium in a search economy and obtain unemployment fluctuations in response to productivity shocks but the effects are obtained through the arrival rate of new firms (which we hold constant) and persistence is provided by search. Wright (1986) combines search and signal extraction and obtains similar fluctuations. However in Wright's paper all persistence is provided by search and the only role of imperfect information is to cause changes in employment. Smith (1989) also constructs a Real Business Cycle model whereby workers are heterogenous but observationally equivalent, his results are similar in that the private information of workers lead to unemployment, but again not to persistence.

An important issue for imperfect information general equilibrium models is whether the relevant information will be revealed by aggregate variables (see for instance Grossman, Hart, and Maskin 1983) and undo the effects that are derived from imperfect information. Our dynamic framework is useful here because aggregate variables are weighted averages of relevant aggregate information of different periods and unless the whole past history of unobserved aggregate variables is already in the agents' information set, each different component of relevant information cannot be deduced. We also argue that in the absence of a Walrasian auctioneer, the revelation of information by prices and aggregate variables will be slower.

The plan of the paper is as follows. Section 2 lays down the basic model. Section 3 compares decentralized bargaining to optimal implicit contracts as a method of wage formation and shows that bargaining induces additional contractual incompleteness and increases the inefficiency. Section 4 analyzes the consequences of allowing private agents to observe a larger set of aggregate variables and in particular discusses whether these variables reveal sufficient information to undo the effects discussed here. Section 5 considers some extensions. Section 6 concludes and the Appendix contains the proofs.

2. THE MODEL

A. *Description of the Economy.* The economy consists of a continuum of infinitely lived workers normalized to 1. There is no birth nor death. In each period a constant number of firms arrive to the market and post vacancies to be matched with unemployed workers. Firms are potentially infinitely lived but may become obsolete and cease to exist. Each firm is assumed to be able to employ only one worker and the labor services offered by this worker are indivisible.[3] The measure of

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[3] This assumption leads to the result that all fluctuations are in the number of employees rather than employment hours. Relaxing this assumption would lead to a mixture of the two.

generated matches is denoted by $x(u)u$ where u is the unemployment rate and in this section we will assume that $x(u)$, the probability that an unemployed worker will find a match, is constant and equal to x. Generation t firms employ a vintage of technology that becomes available at time t. We assume that the productivity of each vintage of technology is independently drawn from the same distribution. Thus there is a new invention that takes place every period (such as the cotton textile, steam engine, computers, superconductors) but it is not known in advance how profitable it will be. Moreover, there is firm level heterogeneity and how successful each firm will be in implementing this technology is uncertain too. As a result, there will be a common and an idiosyncratic component in the productivity of each firm but once this productivity is determined, it remains the same until the firm disappears. It also follows from the above assumptions that productivity is stationary in the sense that the productivity of each firm (of every generation) has the same unconditional distribution.[4] It is assumed that after the match with an unemployed worker, the firm finds out about its productivity, implying that at the wage determination stage the firm has superior information about the value of the worker.

Both the worker and the firm maximize the expected value of their discounted future income and have a discount factor equal to δ. Wage determination takes the form of bargaining in which the worker makes all the offers (this is to avoid multiplicity of equilibria that would arise if the informed party, the firm, also makes offers but does not change our results in any crucial way). The offer of the worker at each stage is a wage demand. If the firm accepts this, it employs the worker at this wage rate in all future periods until it ceases to exist. Thus the firm and the worker sign an enforceable long-term contract at the agreed wage rate (it is intuitive to suppose that after an agreement the firm and the worker enter into a long-term relationship and no more inefficiency arises). Following an agreement there is nevertheless an exogenous probability, s, in every period that the firm will become obsolete and quit the market because a new technology makes it unprofitable (the assumption that the probability s is independent of the profitability of the firm is very convenient but not crucial for our argument). After a separation the worker joins the unemployment pool and looks for a new match. On the other hand, if the offer of the worker is refused by the firm, no production takes place in period t and provided that the pair is not separated, the worker makes a new offer in period $t + 1$.[5] The probability that the firm will become obsolete is $q + s$ after a disagree-

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[4] A justified criticism is that new technologies will be more productive than old ones thus productivity will not be stationary as assumed. However this does not pose a serious problem for our analysis because what is important is deviation from expected profitability. This is an advantage compared to most real business cycle (RBC) models that need "technical regress" to explain recessions since we only need "less technical progress than expected."

[5] Our analysis would remain unchanged if some lower level of production takes place while the firm and the worker are together but before long-term agreement is reached. The advantage of this version would be that a job would be clearly created after a match and if a long-term agreement is not reached and the pair separate, this can be more easily interpreted as a job destruction. The model in the text however has the advantage of requiring less notation but we will still refer to all separations as job destruction. The important feature for our results is that jobs face higher uncertainty at the early stages and this feature receives support from the data: Davis and Haltiwanger (1992) find that newly created jobs are more likely to be destroyed and Dunne, Roberts, and Samuelson (1989) find that young plants are more likely to die.

ment. Thus there is an additional probability, $q > 0$, of separation following dis-
agreement. Further if agreement is not reached in T periods, the firm again
becomes obsolete and the worker returns to unemployment. These last two assump-
tions can be justified by an argument similar to Hart's (1989) that a nonproducing
firm risks losing its clientele and hence faces a higher probability of becoming
unprofitable.[6]

We now make a very restrictive assumption that the worker cannot quit the
relationship even if he becomes sufficiently pessimistic. This is in order to prevent
the multiplicity of equilibria shown to exist by Fudenberg, Levine, and Tirole (1987)
when the worker is free to exercise his outside option. In Section 5B an infinite
horizon bargaining problem, where the worker has the option to quit the relation-
ship at any stage, will be discussed. We will then show that the end point of
bargaining, T, assumed exogenous in this section, will be endogenously determined
and as assumed here the outside option will not be exercised before T. Thus this
restrictive assumption will be justified later. Moreover, from the later discussion, it
can be seen that our main results would hold with much more general bargaining
models. The key feature of our model is that incomplete information leads to
inefficient separations and the revelation of relevant information is gradual and
takes time.

Finally we make some assumptions about the macroeconomic environment. In
our economy there is no money; each bargain takes place on an isolated island and
agents can not observe the outcome of other bargains, the aggregate output that is
produced nor the unemployment rate. This assumption, which implies that the
worker is unable to obtain additional information about the productivity of the firm
from aggregate variables, is obviously quite restrictive and will be relaxed in Section
4. Also note that as the probability of a match for a worker is independent of
macroeconomic conditions and as the productivity of each generation of technology
is independently drawn, the expected value of workers' outside opportunity will be
constant and its expected value is denoted by R.

Firm i can produce y_i units of output per period if it employs the worker with
which it is matched (i.e. the firm's productivity is constant over time). It is assumed
that y_i has a continuous unconditional distribution function, $F(y)$, with support
$[y^{min}, y^{max}]$. However, the distribution of productivity conditional on the realization
of generation t technology is different. The aggregate productivity of vintage t
technology is denoted by z_t and the conditional distribution of each firm's productiv-
ity is denoted by $H(y|z_t)$. It is assumed that $H(\cdot|z')$ first-order stochastically
dominates $H(\cdot|z)$ for all $z' > z$. As z_t is not in the information set of the worker, as
far as the worker is concerned the distribution of y_i is $F(y)$. It can be asked in this
context what constitutes z_t. Shocks such as oil price or other raw material price
changes are likely to be in the information set of the workers and are thus not good
candidates. And yet, machines used in each period would have an expected level of
quality and they will usually be below or above this level which is ex ante not in the
workers' (nor firms') information set. These variations around expected quality will
be captured by the aggregate productivity shock, z_t. Thus in terms of our earlier

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[6] Naturally, this story makes more sense when firms sell nonhomogenous products but it is
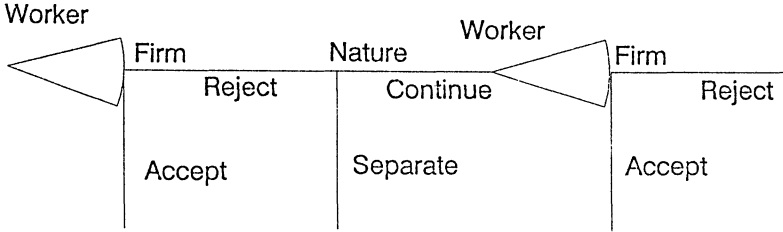assumed, for simplicity, that all products are homogenous.

Worker

Firm   Nature   Worker   Firm

Reject   Continue   Reject

Accept   Separate   Accept

motivation, the worker may observe that all generation t firms use the steam engine but does not know how profitable the steam engine will be (z_t) nor how successful the firm it has matched with will be in using this steam engine (y_i).

B. *Wage Determination.* We start characterizing the equilibrium of this economy by analyzing the bargaining problem between a worker and a firm, with the game tree given in Figure 1. We will be looking for the Perfect Bayesian Equilibrium of this game. This equilibrium concept is defined as a set of history-contingent wage demands by the worker, an acceptance rule by the firm and an updating rule for the beliefs (about the type of the firm) of the worker. The strategies of the worker and the firm must be in equilibrium given the beliefs (the updating rule) and these beliefs must be consistent with (i.e. derived from) the optimal strategies of the firm. We first state the successive skimming lemma proved by Fudenberg, Levine, and Tirole (1985) which will be very useful in analyzing this game. Our model is slightly different from theirs but generalizing the lemma is straightforward, so the proof is omitted.

LEMMA 1. *If a firm with productivity y^* is indifferent between accepting the current offer and waiting for one more period, then all firms with $y > y^*$ will strictly prefer to accept the current offer.*

Lemma 1 implies that after a rejection of a wage demand, the worker deduces that the productivity of the firm, y, must be lower than a certain cut-off level, y^*. We also need to introduce the following notation. $F^\tau(y)$ is the distribution function to which the worker believes the productivity of the firm belongs at the τth stage of bargaining. In other words, F^τ represents the updated beliefs of the worker after $\tau - 1$ rejections. The wage demand that the worker plans to make at the τth stage of bargaining is denoted by w^τ. We also define y^τ as the level of productivity at which the firm is just indifferent between accepting w^τ and waiting for $w^{\tau+1}$. This implies, from Lemma 1, that all $y > y^\tau$ will accept w^τ.

By definition, the beliefs of the worker at time T are given by F^T and the worker knows that the firm will accept all wage offers $w \le y$. And, if this last demand is rejected, he will become unemployed which has expected return R. Thus the

optimal wage demand, w^T, will be given by

$$(1) \qquad w^T \in \text{argmax}_w\{F^T(w)R + [1 - F^T(w)]g(w)\}$$

where $g(w)$ is the expected future returns if the wage demand w is accepted. Thus

$$(2) \quad g(w) = (1-s)w + sR + \delta(1-s)^2 w + \delta s(1-s)R + \delta^2(1-s)^3 w + \dots$$

$$= \frac{(1-s)w}{1 - \delta(1-s)} + \frac{sR}{1 - \delta(1-s)}.$$

At time $T - 1$, the firm can either accept the wage demand of the worker (i.e. w^{T-1}) or reject and wait for w^T. If it accepts the wage offer, its pay-off is

$$(3) \qquad \frac{y - w^{T-1}}{1 - \delta(1-s)}.$$

If it rejects the wage offer, then at the last stage of bargaining it has the option to accept. However, with probability $q + s$, the relationship will break up and the firm will receive zero pay-off. Thus its pay-off in this case is

$$(4) \qquad \delta(1 - s - q) \times \max\left\{\frac{y - w^T}{1 - \delta(1-s)}, 0\right\}.$$

Therefore, the cut-off level y^{T-1} is obtained by setting (3) equal to (4) which gives

$$(5) \quad y^{T-1} = \begin{cases} y^{\min} & \text{if } \dfrac{w^{T-1} - \delta(1-s-q)w^T}{1 - \delta(1-s-q)} < y^{\min} \\[2ex] y^{\max} & \text{if } \dfrac{w^{T-1} - \delta(1-s-q)w^T}{1 - \delta(1-s-q)} > y^{\max} \\[2ex] \dfrac{w^{T-1} - \delta(1-s-q)w^T}{1 - \delta(1-s-q)} & \text{otherwise}. \end{cases}$$

When the worker makes his wage demand at $T - 1$, he takes into account the influence that this will have on the rest of the game (i.e. it will change the acceptance rule of the firm, therefore the inference that the worker can draw from the firm's rejection and hence his optimal wage demand in period T). The worker therefore chooses

$$(6) \quad w^{T-1} \in \text{argmax}_w\Big\{[1 - F^{T-1}(y^{T-1})]g(w) + [F^{T-1}(y^{T-1}) - F^{T-1}(y^{T-1})]$$

$$\times [(1 - s - q)\delta g(w^T) + (s + q)\delta R] + F^{T-1}(w^T)\delta^2 R\Big\}$$

where w^T and y^{T-1} are evaluated as "functions" of w^{T-1} through (1) and (5). The intuition for this equation is as follows. The worker evaluates everything with his current beliefs, so F^{T-1} is used. If $y \geq y^{T-1}$, then the firm will accept the wage demand w^{T-1} now (note that y^{T-1} depends on w^{T-1} through (5)). If y is less than y^{T-1}, then the wage demand at $T-1$ will be refused. But if y is greater than w^T, then an agreement will be reached in the last period provided that a separation does not take place (which has probability $1 - s - q$). Finally if a separation takes place or if y is less than w^T, the worker will take his reservation return, R. Note that given the specification of the game, strategies are independent of aggregate economic conditions; they only depend upon the history of the game between the worker and the firm. This is due to two of our assumptions: (1) that the probability of a match for a worker is independent of the unemployment rate in the economy, (2) that the worker observes neither aggregate unemployment and output nor the outcome of other bargains to form beliefs about the aggregate productivity shock, z_t.

Finally, we need to determine R, the reservation return of the worker. When the worker is unemployed, he finds a match with probability x in which case he receives the expected return from a match which we denote by r. With probability $(1 - x)$ he remains unemployed in which case he gets the return from unemployment, which is R. Therefore

(7) $$R = \delta[xr + (1 - x)R]$$

$$R = \frac{\delta xr}{1 - \delta(1 - x)}.$$

Note that neither R nor r is time-dependent because the level of unemployment does not affect the probability of a match and the aggregate productivity shock z_t, is assumed to be serially uncorrelated. The expression for r will in general be quite involved (it is in fact equal to the maximized value of the expected utility of the worker at the first stage of bargaining). To illustrate we can give the relevant expression for $T = 2$ denoted by r^2:

(8) $$r^2 = [1 - F(y^1)]g(w^1) + \delta[F(y^1) - F(w^2)]$$

$$\times [(1 - s - q)g(w^2) + (s + q)R] + F(w^2)\delta^2 R$$

where, following our notation, w^1 and w^2 are the equilibrium wage demands in the first and second periods, y^1 is the level of productivity above which the wage demand of the worker is accepted at the first stage of bargaining. The intuition for this equation is exactly the same as for (6).

To prove that an equilibrium exists in this game is not entirely straightforward. By Lemma 1, the subjective probability distribution at T is equal to $F(y)$ truncated at a certain point and it is continuous because $F(y)$ is so. Therefore the worker faces a straightforward maximization problem at T, and as the wage demand is chosen from a compact set, the correspondence of maximizing values is nonempty, compact-valued and upper hemi-continuous (uhc) by Berge's Maximum Theorem (see for

example Stokey, Lucas, and Prescott 1989). However, we cannot select a continuous function from a uhc correspondence and show that the correspondence of maximizing values is nonempty and uhc at the next stage. Instead we follow Leininger (1986) in relaxing the requirements of the Maximum Theorem. We show that we can always select a function from a uhc correspondence that preserves the uhc property at the next step. This result, which is a novel generalization of Berge's Maximum Theorem and which can be more generally useful in establishing the existence of equilibrium in sequential games, is stated in Lemma 2 and the proof is in the Appendix.

LEMMA 2. *Let* $f: XxYxZ \rightarrow \mathscr{R}$ *be a continuous function,* $\Omega: Z \rightarrow X$ *be a continuous and compact valued correspondence and* $\Gamma: XxZ \rightarrow Y$ *be a compact valued and uhc correspondence in x and z, then*

$$\phi(z) = \{x: f(x, y, z) = \max f(x, y, z)\}$$

$$x \in \Omega(z), y \in \Gamma(x, z)$$

is nonempty, compact valued and uhc in z.

Using Lemma 2 we can prove that an equilibrium exists for the game between the firm and the worker and that equilibrium strategies have "nice" properties. This is done in Proposition 1 (a proof that is general enough to cover the cases considered in Section 4 and 5 is given in the Appendix). Before we state the proposition, let us also define $w^\tau(b^\tau)$ as the correspondence to which the equilibrium wage demands at the τ th stage belong and b^τ as the vector of all variables exogenous to the decisions of the worker and the firm at the τ th stage of bargaining.

PROPOSITION 1. *In the game played between a firm and a worker, a Perfect Bayesian Equilibrium exists and* $w^\tau(b^\tau)$ *is nonempty, compact valued and uhc in* b^τ *for all* τ *from* 1 *to* T.[7]

C. *Dynamic Equilibrium of the Economy.* The law of motion of unemployment rate is given as

$$(9) \qquad u_{t+1} = s(1 - u_t) + d_t + (1 - x)u_t.$$

Next period's unemployment consists of there components: the break up of jobs which happens at rate s for all pairs that are matched together (natural job destruction), the inflow because of additional separations during bargaining, which we denote by d_t (inefficient job destruction; an additional proportion q of pairs who

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[7] We can also see from the proof of Proposition 1 that the backward induction that describes the Perfect Bayesian Equilibrium of this game is unique. However this does not guarantee uniqueness of the equilibrium because a maximization problem such as (1) or (6) may have multiple solutions. Fudenberg, Levine, and Tirole (1985) have shown that in the infinite horizon version of this game without reservation returns and with some restrictions on the distribution function, the equilibrium is generically unique.

disagreed and those who were unable to agree in T periods) and finally, the workers who are unemployed this period and fail to find a job in the next, which is given by $(1-x)u_t$. Hence the two kinds of job destruction and job creation are taking place simultaneously in this economy as is the case in the data (e.g. Davis and Haltiwanger 1990, 1992). Rearranging (9),

$$(10) \qquad\qquad u_{t+1} = s + (1 - x - s)u_t + d_t.$$

There will be fluctuations in the unemployment rate as d_t changes, thus aggregate fluctuations in this economy occur because the amount of (inefficient) job destruction is variable. Yet if aggregate productivity, z_t, does not change, the conditional distribution of each firm's productivity will be constant. By a Law of Large Numbers argument, d_t too will be constant and unemployment will converge to a "steady state" value (although job destruction and job creation would still be taking place simultaneously). Therefore, the driving force of unemployment fluctuations in this economy is z_t. We thus define the steady state of this economy conditional upon a value of z. In other words, we say that the economy is in steady state if $z_t = z$ and the unemployment rate is given by $u_t = u$ for all t. The steady state unemployment rate conditional upon z, is therefore given by

$$(11) \qquad\qquad u^*(z) = \frac{s + d(z)}{s + x}$$

where u^* and d are written as functions of z.

Inspection of (10) leads to a number of observations. First, provided that $x + s$ is less than 1, unemployment rate exhibits persistence in the sense that if we increase d_t from its steady state value $d(z)$ and then reduce it back to $d(z)$ next period, this will increase not only u_t but also u_{t+1} above the steady state level of unemployment, $u^*(z)$. Yet by holding all future values of d_t constant at their steady state value, we are deriving all persistence from search (i.e. from sluggish job creation). The reason why unemployment remains high after an initial rise is that it takes time for separated workers to find jobs.[8]

Secondly, and more importantly for our purposes, incomplete information introduces additional persistence. In other words, if $z_t < z$, then $d_t > d(z)$, and we will also have $d_{t+1} > d(z)$, i.e. higher separations (job destruction). We can see this through the following argument. Suppose $z_t < z$ and $z_{t+j} = z$ for all $j \geq 1$. This implies that $d_t > d(z)$ because there are more disagreements and a proportion q of these disagreements end in separations. However, some of the generation t firms who failed to agree in period t will also fail to agree in period $t + 1$. The probability of disagreement in period $t + 1$ is $\Pr[y < y^2]$ where y^2 is the cut-off level of productivity for the second stage of bargaining. However by definition, as z falls, the

[8] It is useful to note that although this channel of persistence is not related to the asymmetry of information, in this model there would be no unemployment fluctuations if information were complete. Thus, the incompleteness of information is the source of all unemployment fluctuations.

conditional distribution of y, $H(y|z)$ shifts to the left, thus $\Pr[y < y^2|z'] > \Pr[y < y^2|z]$ for all $z' < z$. Therefore, after $z_t < z$ we have $d_{t+1} > d(z)$.

The intuition for this additional persistence is as follows: the superior information of the firm is relevant for a number of periods but in the first stage of bargaining only part of this information is revealed, i.e. whether y is less than y^1 or not. As a result, wages do not perfectly adjust to an adverse shock leading to "dynamic wage sluggishness." Because after an adverse shock wages are higher than they should be for a number of periods, separations (job destruction) occur at a higher rate than average. This intuition survives beyond the specific bargaining game we have chosen to illustrate it. The crucial requirement for incomplete information to lead to additional persistence is that agreement should take place overtime, rather than in an instant, while agents grope towards the changed situation.

The incomplete information channel of persistence is "short lived"; after T periods there will no longer be an effect from the realization of z_t because all the information will have been revealed or become no longer relevant. Therefore, because $1 - x - s$ is less than 1, if $z_{t+j} = z$ for all $j \geq 1$, the unemployment rate will return to its steady state value and we can conclude that the steady state equilibrium of this economy is globally stable. From the above discussion, wage demands are independent of the aggregate level of unemployment. So wages do not fluctuate much, but are still procyclical: when z_t is low, agreements will be delayed and as $w^\tau > w^{\tau+1}$, *average agreed wages* will be low. When z_t is low, unemployment is high and we are in a "recession" thus giving us procyclical real wages. Yet the close association between wages and the marginal product of labor no longer holds, making wages much less procyclical than implied by a simple (i.e. full information and competitive) Real Business Cycle model. In practice we can identify two channels through which wages respond to economic conditions. First, wages may change in response to changes in the productivity of the firm and second, they may respond to aggregate economic conditions, for example by falling when unemployment rises. The second channel is not present as we assume the matching probability to be independent of the amount of unemployment in the market (we allow this channel to work in the extensions). The first mechanism, on the other hand, works only imperfectly. To see this consider the special case with $T = 1$; the optimal wage demand will be given by $w = (1 + R)/2$ and if there is no agreement, the relationship will end. Therefore wages do not respond to aggregate productivity, z_t. However for $T > 1$, this channel works to some extent and wages become procyclical. Related to the delay in information revelation, it is also interesting to note that wages lag the movement of unemployment over the cycle since they respond to information that becomes available only slowly.

We can summarize the results of this section in Proposition 2. (The existence of steady state equilibrium under more general conditions then here, covering the extensions of Sections 4 and 5 is provided in the Appendix.)

PROPOSITION 2. *Under the assumption that $x + s < 1$, given the equilibrium of the bargaining game, this economy has a unique and globally stable steady state unemployment level. Unemployment, output and wages fluctuate around this steady state in*

response to changes in z_t. Fluctuations exhibit persistence due to search and incomplete information effects.

D. *Efficiency.* Obviously, there are substantial inefficiencies in this model. Not all potentially productive relationships are formed due to the existence of search imperfections and once a match is formed agreement can be delayed or a separation may result because of asymmetric information. We thus say that the economy is unable to achieve the first best. However, it can also be asked whether a Social Planner can improve the welfare of the agents in this economy, when she is subject to the same search and incomplete information constraints. In our model the answer to this question is in the affirmative and we refer to this as constrained inefficiency. There are two sources leading to this additional inefficiency. The first is that bargaining is not an efficient way of determining wages under incomplete information (in contrast to the complete information case mentioned above). This is discussed in the next section of the paper. The second is that the existence of an aggregate productivity shock, z_t, unobserved by workers, introduces an *informational externality*. To see this suppose that there are two workers bargaining with their separate firms and that one pair bargains first and the other immediately after the first pair. If the second worker does not see the outcome of the first bargain, his optimal strategy is described by our above analysis. However, if he sees whether the other pair agreed or not at the first stage, he can update his beliefs about the aggregate productivity shock. Therefore, he will be better off if he can condition his wage demands on the outcome of the other bargain.[9] It thus follows that the decentralized equilibrium of this economy can be improved upon if this informational externality can be exploited. There may be various ways of achieving this. First, instead of decentralized bargaining, the economy can move to a coordinated bargaining system.[10] Second, bargains can be organized in a sequential order and the results of previous bargains can be made public.[11] This may give a rationale for why many countries do not have perfectly synchronized wage determination rounds despite the fact that synchronization is often argued to reduce inflationary wage pressure.

3. COMPARISON WITH OPTIMAL CONTRACTS

The above results are similar to those obtained in the asymmetric information optimal contracts literature in the sense that in both frameworks, firms with lower

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[9] This will be achieved to some extent when we allow workers to observe aggregate unemployment and output but not perfectly.

[10] In Section 5B we will argue that observed variables will not transmit much information about current dated variables because of the absence of a coordinating agent. To have coordinating bargaining can be in this context recognized as introducing such an agent.

[11] The technical details of this argument are developed in Section IV of Acemoglu (1992). It has to be noted that this system would not internalize all the effects of the informational externality and a similar inefficiency to those encountered in the herding models (e.g. Banerjee 1992) will remain.

productivity are obliged to have lower employment than the first-best level.[12] In this section we compare bargaining to optimal contracts as a framework of wage determination under asymmetric information.

Let $V(c)$ be the return to the firm from a contract c, $S(V^*)$ be the set of all contracts satisfying the incentive compatibility constraint (i.e. the firm's behavior cannot be conditioned upon its private information) and yielding expected utility greater than or equal to V^*, i.e. $V(c) \geq V^*$. Also let c^* be an optimal contract in $S(V^*)$ and $U(c)$ be the expected utility of the worker when contract c is signed. It follows that $\forall c \in S(V^*)$, $U(c^*) \geq U(c)$. We also define c^b as a contract that implements a Perfect Bayesian Equilibrium of the bargaining game. Before comparing contracts and bargaining we also need to note that, in our analysis, we restricted the worker not to make an offer such as "You can employ me at w_1 for all future periods or at w_2 in alternating periods." In other words, we assumed that if the worker works at time t, he will also work at time $t+1$ unless the firm becomes obsolete. If we do not impose the same restriction on the contracts as well, they will achieve more than bargaining. However, our argument is that there is an economically more interesting mechanism which makes bargaining less efficient. In order to highlight this mechanism, we only allow contracts which necessarily employ the worker at $t+1$ if they did so at t.

PROPOSITION 3. *There exist an incentive compatible contract c^{**} such that $V(c^{**}) \geq V(c^b)$ and $U(c^{**}) \geq U(c^b)$ where the inequalities are strict at least for some parameterizations.*

This proposition states that contracts are socially more efficient than bargaining. The intuition of this proposition is simple: contracts enable the worker to commit to a wage demand path such as, "I ask for w^1 now and I will ask for w^2 next period." However, with bargaining, the worker cannot commit to such a path because w^2 must be ex-post optimal given the information revealed by the rejection of the firm.[13] This imposes $T-1$ additional constraints. Therefore, the choice set is smaller under bargaining and contracts are more efficient. Another way of viewing this result is to remark that bargaining introduces some contractual incompleteness which increases inefficiency; in bargaining, future wage demands cannot be determined now and will depend on the information revealed by the firm's rejection decisions and therefore the firm will be more willing to reject now in order to

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[12] The contracts we have in mind are such that the worker and the firm get together at $t = 0$, and determine a complete schedule of wage and employment levels (or probabilities). The firm chooses a particular point on this schedule after finding out about its productivity. For details, see Grossman and Hart (1983), Hart (1983). All the contracts we refer to are assumed to be enforceable.

[13] So far, we have assumed that enforceable contracts could be written after the bargain, motivated by the observation that once agreement is reached the worker and the firm enter a long-term relationship and avoid further inefficiencies. In the absence of this, the worker will not settle for the same wage rate in the second period but ask for a higher wage after the first stage of production. In this case we need to allow more complicated long-term contracts or concentrate on short-term contracts (see for example Hart and Tirole 1988) but again similar results will be obtained.

reduce future wage demands. Thus bargaining gives similar results to contracts but contracts can implement equilibria that bargaining cannot implement, hence bargaining leads to more inefficiency and to slower revelation of information which in our model leads to dynamic wage sluggishness and to additional persistence in unemployment. Of course, although implicit contracts are more efficient, bargaining often arises because of the enforcement problems associated with implicit contracts.

4. INFORMATION REVELATION BY AGGREGATE VARIABLES

In the analysis of Section 2, it was assumed that workers did not observe aggregate variables, therefore only updated their beliefs about the productivity of their firm from the firm's rejection decisions. This is obviously unrealistic and it should be analyzed how larger information sets that include aggregate variables would change our results. The first point to note is that workers will never be able to learn about the idiosyncratic component of their firm's productivity from observing aggregate variables. Therefore the maximum information that will be revealed from aggregates (or observing neighbors) is to discover z_t. If workers know the value of z_t, there will still be separations due to the idiosyncratic uncertainty, but from the law of large numbers, the number of jobs destroyed would be constant in each period. As a result, there would be no unemployment fluctuations in this model and no additional persistence from incomplete information in general. Therefore the question we need to address in this section is whether aggregate variables that workers observe at time t would fully reveal z_t. We will argue that there are two reasons for z_t not to be fully revealed.

Before we proceed further, we first need to note that in the present case, our analysis in Section 2 would still apply but $F^\tau(y)$, the updated probability distribution of the worker needs to be changed to $F^\tau(y|\Omega_t)$ which denotes the subjective probability distribution of the worker about the productivity of the firm at the τth stage of bargaining and when the worker has observed all the variables in the information set Ω_t. We can rewrite equations (1) to (8) in this way and all our analysis will be unchanged (recall that all our proofs are general enough to deal with this case, see the Appendix). These equations will give us a new level of separations at time t, d_t, and the question is whether d_t depends on z_t (aggregate fluctuations) and whether it depends on past values of z_t (persistence in job destruction). It is straightforward to see that if $z_t \in \Omega_t$ there will be no aggregate fluctuations. Next, it is also immediate that if $z_{t-k} \in \Omega_t$ for all $k > 0$, the relevant aggregate information is revealed immediately and the presence of incomplete information will not lead to additional persistence. In the rest of this section we will investigate whether under plausible conditions, Ω_t would contain current and past values of z_t.

To start with, consider a simple version of our model with $T = 1$. In this case, workers would demand wages recognizing that if their first demand is refused they will become unemployed. Now if each agent observes the aggregate unemployment rate in the last period, u_{t-1}, then they will be able to deduce the number of matches this period, thus all agents in this economy will be able to calculate the optimal wage demand of each worker (which is the same across workers). They can therefore construct a mapping from z_t to aggregate unemployment, u_t. If z_t is high, unem-

ployment will be low, and so on. Moreover, under our assumption of first-order stochastic dominance ranking on $H(\cdot\,|z_t)$, this mapping will be one-to-one. Now if the workers also observe u_t, they will naturally be able to deduce the level of z_t. In this case provided that they can condition their wage demands on the current aggregate unemployment rate (or some other current dated aggregate variable), they will be able to avoid the aggregate uncertainty and z_t will have no influence on aggregate unemployment fluctuations (however, see below why it is not trivial to condition wage demands on current dated aggregate variables). This simple example makes it clear that the one-to-one mapping from z_t to unemployment (or some other aggregate variable) is the key for full revelation of the aggregate uncertainty. Again in this simple example if u_{t-1} happened not to be in the workers' information set, the mapping from z_t to u_t would not be one-to-one and observing u_t would not fully reveal the relevant aggregate information. To see this, it is sufficient to consider an example. A worker may observe a high value of u_t but this can be due to a very low value of z_t combined with a moderate level of unemployment last period or a moderate value of z_t and a very high value of past unemployment. Now, observing u_t is still informative but no longer fully revealing.

Next consider another example with $T = 2$. In this case current unemployment would depend on past level of unemployment and, the two components of job destruction: first, jobs destroyed from current matches and second, jobs destroyed from matches of the previous period. Also suppose that workers observe u_{t-1}. Is u_t fully revealing about z_t? The answer is no because now there is no unique mapping from z_t to u_t unless we condition it on the amount of job destruction from the matches of the previous period or on z_{t-1}. This can be generalized quite simply. For the current dated aggregate variables to be fully revealing, we need z_{t-k} to be in the information set of all workers for all $k \leq T - 1$. However, in a dynamic world in which workers move and new entrants come to the market (and many other features also missing from our formal model because of obvious reasons), it is a very strong assumption to suppose that all past aggregate incomplete information problems have been solved. Thus under plausible conditions, all past uncertainty will not have been fully revealed and this will prevent the existence of a one-to-one mapping from the set of current aggregate variables to z_t. This argument does not only establish that incomplete information will cause fluctuations in our model, but also because past information has not been fully revealed, as in our basic model, relevant information is still *revealed gradually* thus incomplete information contributes to persistence in unemployment fluctuations.

An additional argument can also be made for the limited degree of information transmission. Even when all the relevant information can be revealed by aggregate variables, for this not to have an effect on the behavior of the aggregate economy, we need this information to be fed into the current decision rules of agents. In the presence of a Walrasian auctioneer who coordinates trade, this is done in a straightforward way. Each agent submits a demand schedule to the auctioneer conditional on the realization of the aggregate variables. For instance, I demand the wage w_1 if aggregate unemployment is u_1 but a different wage w_2 if the aggregate unemployment rate is u_2. However, the distinguishing feature of our economy is that the auctioneer does not exist. Each agent has to make a wage demand and it

is only as a result of these wage demands that the aggregate unemployment level is determined. In the absence of a coordinating agent, it is not possible to condition current wage demands on the current realization of the unemployment rate. This is because for the aggregate unemployment rate to be determined, the firms have to decide whether they accept the wage demands of their workers; therefore the exact wage demands of the workers need to be determined first. Consequently in general the relevant information will only be fed into decision rules with some delay, thus the transmission of information will be slowed down.

This idea can be formalized in a very simple way. We can define a relation P such that aPb means a precedes b where a and b refer to decisions by distinct set of agents in the economy. "Precede" in this context means that decision b can be conditioned upon the *realization* of decision a. In a centralized trading system such as a market coordinated by a Walrasian auctioneer, P does not need to be anti-symmetric; aPb and bPa can be simultaneously true since the auctioneer can determine both at the same time. Unemployment is only determined from individual decision rules but individual decision rules can be conditioned upon the unemployment rate of the same period because agents can submit schedules that map the aggregate unemployment rate (and other aggregate variables) to their decisions and the auctioneer finds a general equilibrium as a fixed point vector of all these decision rules. However, in an environment without an auctioneer, this procedure does not seem plausible and motivated by this, we define a decentralized trading system as an environment in which P is anti-symmetric for any two arguments that refer to different sets of agents. If aPb, then bPa cannot be true. Now in our example take "a" to be individual decisions and "b" to be the aggregate unemployment rate, thus aPb must be true since without knowing the individual decision rules we cannot determine how many separations will occur. Thus it is impossible at the same time to condition individual agreements on the current unemployment rate. Therefore, the current decision rules of workers and firms can only depend on the realization of the past unemployment rate. These considerations will further limit the degree to which relevant information will be revealed and more importantly will be used by the agents in the decentralized equilibrium.

Overall we can argue that under plausible conditions, not all the relevant aggregate information will be revealed and fed into decisions rules in an economy like ours and the effects discussed in Section 2 will survive even in the presence of larger information sets than considered there.

5.   EXTENSIONS

A.   *A More General Search Technology.*   We now assume that the probability of a match for an unemployed worker is given by $x(u)$ where $x(\cdot)$ is continuous and differentiable with a negative first derivative and has an elasticity smaller than 1, i.e.

$$(12) \qquad\qquad -\frac{x'(u)u}{x(u)} < 1$$

for all values of u. Under this assumption persistence effects that arise from search continue to exist. In this setting, equation (9) becomes

(13) $$u_{t+1} = s(1 - u_t) + d_t + [1 - x(u_t)]u_t$$

and we can see that holding disagreements constant, future unemployment is higher the higher current unemployment is, i.e. from (12)

(14) $$\left. \frac{du_{t+1}}{du_t} \right|_{d_t=d} = 1 - s - x'(u_t)u_t - x(u_t) > 0.$$

Workers will now be able to obtain some signals about the economic environment from their matching experience. The probability of a match is high when unemployment is low; thus a worker who does not observe aggregate unemployment will form expectations about the level of aggregate unemployment from his matching history. For example, if a worker is matched as soon as he becomes unemployed, he will deduce that the unemployment rate, u_t, is low and, judging his reservation return relatively high, will make higher wage demands. On the other hand a worker who receives no matches for a few periods will believe unemployment to be high and will make more moderate wage demands upon being matched with a firm. Also, as unemployment now affects matching probabilities and so outside opportunities, the second channel, mentioned in Section 2D, through which aggregate economic conditions influence wages will function, albeit only imperfectly. In the Appendix, we prove that a steady state equilibrium exists, with this general search technology (as well as when a vector of aggregate variables, k_t, is in the information set of the workers).[14]

   B.   *Infinite Horizon Bargaining with Outside Options.*   The assumptions on the bargaining game used in Section 2 were quite restrictive. First, we imposed an end period to the bargaining game. Second, we have not allowed the worker to opt out of the relationship and join the unemployment pool. Although these assumptions considerably simplified the analysis and the exposition, it will be instructive to investigate whether any of our results have specifically followed from these. A more natural bargaining game to analyze would be one where the horizon is infinite and the worker can choose to take his outside option at any stage. This is the game that we will discuss in this section. We will see that the equilibrium of the present game is the same as the equilibrium of the game analyzed in Section 2, when T is chosen appropriately.
   Rather than solve this game fully we will borrow from the analysis of Fudenberg, Levine, and Tirole (1987). Also for simplification, we set $q = s = 0$ and take R

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[14] Another possible extension to the search technology is to allow on-the-job search (see Acemoglu 1992). This does not change any of our results but enables us to separate quits from separations. As the probability of a match on-the-job can be plausibly assumed to be decreasing in unemployment, this extension would give us procyclical quits and countercyclical involuntary separations.

constant as in Section 2. Two results that are important for us follow from their analysis:

(1) There exist multiple equilibria.

(2) As long as $R > g(y^{min})$, at some point, the worker will become sufficiently pessimistic and quit the relationship.

The existence of multiple equilibria poses some problems as well as raises some interesting issues. It can be asked whether shifts in the relevant equilibrium do contribute to the cyclical fluctuations of the economy. This is certainly possible but there is not much we can say about it either. A more natural approach would be to suppose that once one of the possible equilibria is chosen, a change of equilibrium thereafter is unlikely. However, point (2) above implies that once we are in one of these equilibria, the firm and the worker will bargain up to a point until which the outside option is not exercised and at that point the worker switches. Thus we can call this point T and the qualitative results from all of these equilibria are the same as our basic model analyzed in Section 2, with the difference that T is now endogenously determined.[15]

6. CONCLUSION

We have analyzed a dynamic general equilibrium in which wages are determined through bargaining. Because the firm has superior information, delays and inefficient separations lead to output and employment fluctuations. This persistence of fluctuations is derived from the incomplete information channel which introduces "dynamic wage sluggishness," as well as the more conventional search channel. A feature of this dynamic wage sluggishness compared to conventional channels of persistence is that it implies persistence in job destruction not only in job creation.

Our model has close links with the RBC models such as Kydland and Prescott (1982) and Long and Plosser (1983) since we are concerned with the dynamics of the aggregate economy in response to an exogenous driving force. However, the economic and persistence mechanisms are different and the presence of missing markets breaks the link between real wages and the marginal product of labour. This reduces real wage fluctuations while increasing the cyclical variability of employment. Missing markets also make the equilibrium inefficient. Further, given the informational and technological restrictions, a Social Planner can still improve upon the decentralized equilibrium by exploiting the informational externalities that are present. We have also emphasized the similarity of our results to optimal contracts and shown that bargaining is a less efficient way of wage and employment determination. However, it may nevertheless arise because of enforceability problems associated with complicated implicit contracts.

As our model was mainly concerned with economic and persistence mechanisms derived from incomplete information bargaining embedded in a dynamic general equilibrium model, we have not discussed the driving force of the cycle in great detail and interpreted it as a common productivity shock. However, the same set-up

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[15] Fudenberg, Levine, and Tirole (1987) also show that a "no-switching" equilibrium may exist but only when the lower support of the distribution, y^{min}, is sufficiently high.

can be used to generate economic fluctuations in response to real demand shocks or even changes in "animal spirits" as long as all the effects are not in everyone's information set. In particular, the persistence channel provided by incomplete information may be important in explaining the sluggish response of the real economy to unanticipated demand shocks.

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APPENDIX

PROOF OF LEMMA 2.   Take an arbitrary z. A function $y = h(x, z)$, upper semi-continuous (usc) in x can always be selected from $\Gamma(x, z)$. Let $g(x, z) = f(x, h(x, z), z)$. Then $g(x, z)$ is *usc* in x and x belongs to the continuous and compact valued correspondence $\Omega(z)$, therefore a maximum exists and $\phi(z)$ is nonempty.

Now take $z_n \to z$, $x_n \in \phi(z_n)$ and $x_n \to x$. By definition, there exists $y_n \in \Gamma(x_n, z_n)$ such that

(A1) $f(x_n, y_n, z_n) \geq f(x', y', z'_n) \ \forall x' \in \Omega(z_n), y' \in \Gamma(x', z_n)$.

Γ is *uhc* in x and z, therefore, there exists $y \in \Gamma(x, z)$ such that $y_n \to y$. Thus take limits in (A1) and use the continuity of f, we have

(A2) $\limsup f(x_n, y_n, z_n) = f(x, y, z) \geq f(x', y', z) \ \forall x' \in \phi(z), y' \in \Gamma(x', z_n)$.

Therefore, $x \in \phi(z)$ and ϕ is *uhc* in z.

Finally, $\phi \subseteq \Omega$ and Ω is compact so ϕ is bounded. Also, take $x_n \to x$ and $x_n \in \phi(z)$. By the above argument, $\lim f(x_n, y_n, z) = f(x, y, z)$ as f is continuous and $x \in \phi(z)$ and ϕ is closed, therefore compact valued. As z was arbitrary ϕ is nonempty, *uhc* and compact valued for all z.   Q.E.D.

PROOF OF PROPOSITION 1.   At T the worker maximizes (1) in the text choosing w from the nonempty compact set $[y^{T-1}, y^{\max}]$. From Lemma 1, F^T is continuous in y^{T-1} (and trivially in $w^{T-1}, y^{T-2}, \ldots$ since it does not directly depend on these). Therefore, the correspondence to which maximizers of (1) belong to, $w^T(b^T)$, is nonempty, compact valued and *uhc* in $b^T = (y^{T-1}, w^{T-1}, y^{T-2}, \ldots, R)$ by Berge's Maximum Theorem. Equation (5) in the text gives y^{T-1} as a function of w^{T-1} and w^T, thus we can substitute this function in $w^T(b^T)$ and obtain a nonempty and compact-valued correspondence, ϕ^T, which is also *uhc* in $w^{T-1}, y^{T-2}, w^{T-2}, \ldots$ and R. At $T-1$ the worker maximizes (6) which is continuous in w^T, w^{T-1}, y^{T-1} and R choosing w^T from the set $[y^{T-2}, y^{\max}]$ and w^{T-1} from the set $[y^{T-1}, y^{\max}]$ subject to (1) and (5). We can substitute from (5) for y^{T-1} and as (5) is continuous the function to be maximized is continuous in $w^T, y^{T-2}, w^{T-1}, \ldots$ and R and it is subject to the constraint that $w^{T-1} \in \phi^T(y^{T-2}, w^{T-2}, \ldots, R)$ defined above. Therefore Lemma 2

implies that $w^{T-1}(b^{T-1})$ is nonempty, compact valued and *uhc* in $b^{T-1} = (y^{T-2}, w^{T-2}, \ldots, R)$. We can repeat this argument recursively and conclude that $w^\tau(b^\tau)$ is nonempty and that it is compact valued and *uhc* in b^τ for all τ. The fact that the optimal strategy correspondence is nonempty also implies that an equilibrium exists.   Q.E.D.

PROOF OF PROPOSITION 3. The optional contract is chosen as a result of a maximization problem subject to incentive compatibility constraints and subject to the restriction that the firm gets a certain level of utility. Thus if we denote the maximand by $G(w^1, w^2, \ldots, w^T)$, the optimal contract, c^*, is chosen as T wages $\{w^{1c}, w^{2c}, \ldots, w^{Tc}\}$ such that

(A3) $\{w^{1c}, w^{2c}, \ldots, w^{Tc}\} \in \text{argmax } G(w^1, w^2, \ldots, w^T)$

subject to $T-1$ incentive compatibility constraints which take the form of equations similar to (5) in the text and that $V(c^*) \geq V^*$. However from the proof of Proposition 1, the solution of the bargaining problem is given by maximizing (A3) subject to the above constraints and $T-1$ additional constraints

(A4) $w^\tau \in w^\tau(y^{\tau-1}, w^{\tau-1}, \ldots)$

for $\tau = 2, \ldots, T$, which say that future wage demands must be ex post optimal for the worker given the information revealed by the rejection decisions of the firm, e.g. (1) in the text.

Therefore defining the set of contracts to which c^b belongs to as S^b, we can see that $S^b \subset S(V(c^b))$, where, by definition, the firm gets return $V(c^b)$ in the equilibrium of the bargaining game and strict inclusion holds because of the $T-1$ additional constraints. For the optimal contract in $S(V(c^b))$, c^*, it is true that $U(c^*) \geq U(c)$, $\forall c \in S(V(c^b))$, therefore $V(c^*) \geq V(c^b)$. Because of the strict inclusion, the inequalities will be strict at least for some problems.[16] Therefore, we can always find c^{**} such that $V(c^{**}) \geq V(c^b)$ and $U(c^{**}) \geq U(c^b)$ and the strict inequalities will hold at least for some parameterizations.   Q.E.D.

PROOF (*Existence of Steady State in the General Case*). To establish the existence of a steady state we need to show that equation (9) has a fixed point $u_t = u_{t+1}$ when we hold $z_t = z$ as before (i.e. the right-hand side of (9) as a correspondence in u should have a fixed point). The only complication now is that equilibrium strategies and reservation returns depend on u_t (and possibly some other aggregate variables). Therefore, d_t will vary with u_t as well as z and other variables. We will establish the existence of a fixed-point using Kakutani (1941)'s Fixed-Point Theorem, thus we need to show that d defined as a correspondence is *uhc* in u and convex-valued.

First note that the proof of Proposition 1 goes through if we replace R by the expectation of reservation returns conditional on available information at time

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[16] It is trivial, but takes a long time to construct an example for this.

t, $E[R_t|\Omega_t]$.[17] Therefore as the optimal wage demands are *uhc* in $E[R_{t+j}|\Omega_t]$, if $E[R_{t+j}|\Omega_t]$ is *uhc* in u_t, so will the optimal wage demands be. Now assuming that $u_t \in \Omega_t$, we can simply write

$$(A5) \qquad E[R_t|\Omega_t] = \delta\{x(u_t)E[r_t|\Omega_t] + (1 - x(u_t)E[R_{t+1}|\Omega_t])\}$$

where r_t is the return from a match at time t and it is time dependent because reservation returns are so. As $x(\cdot)$ is continuous, we only need to show that r_t is *uhc* in $u_t, u_{t+1}, \ldots$. As above in the text we only give the expression of r_t^2, the expected return from a match when $T = 2$, now subscripted by t to denote a generation t match:

$$(A6) \quad E[r_t^2|\Omega_t] = (1 - F(y_t^1|\Omega_t))g(w_t^1|\Omega_t)$$

$$+ \delta(F(y_t^1|\Omega_t) - F(w_t^2|\Omega_t))((1 - s - q)g(w_t^2|\Omega_t)(w_t^2)$$

$$+ (s + q)E[R_{t+1}|\Omega_t]) + F(w_t^2|\Omega_t)\delta^2E[R_{t+2}|\Omega_t]$$

where all variables now depend on t and thus are subscripted by t. For example, beliefs may depend on t (more precisely on Ω_t but we have not written it explicitly in order not to make the notation even more complicated) because the agent may be able to form expectations of z_t from observing U_t. Future expected returns ($g(w)$) depend on t because reservation returns are time varying. However we can see that r_t^2 (and in general r_t for all values of T) is continuous in w_t^1, w_t^2 and y_t^1. Since these are compact valued and *uhc* in all exogenous variables from Proposition 1, r_t is compact valued and *uhc* in $u_t, u_{t+1}, \ldots$. Therefore, the number of separations at time t, d_t, is compact valued and *uhc* in $u_t, u_{t+1}, \ldots$ (thus in u).

Finally we need to show that it is convex valued. Suppose not, then for some u (such that $u_{t+j} = u$ for all $j \geq 0$) and given the value of z, there can be d_1 or d_2 separations but not $\lambda d_1 + (1 - \lambda)d_2$ separations for some value of λ between 0 and 1. However, the statement that there can be d_1 (d_2) separations means that there exist an equilibrium of the wage determination game which will lead to d_1 (d_2) separations. We can choose λ proportion of pairs to play the first equilibrium and $(1 - \lambda)$ proportion to play the second, which will give us $\lambda d_1 + (1 - \lambda)d_2$ separation. Therefore, the mapping that gives d as a "function" of u is compact valued, *uhc* and convex valued and by Kakutani's (1941) paper a fixed-point exists.   Q.E.D.

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[17] We are just referring to one pair's bargaining problem and Ω_t is the worker's information set at time t.

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