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# Beyond Ricardo: Assignment Models in International Trade

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## Abstract

International trade has experienced a Ricardian revival. In this article, we offer a user guide to assignment models, which we refer to as Ricardo-Roy (R-R) models, that have contributed to this revival.

## 1. INTRODUCTION

International trade has experienced a Ricardian revival. For almost 200 years, David Ricardo's theory of comparative advantage has been perceived as a useful pedagogical tool with little empirical content. The seminal work of Eaton & Kortum (2002) has shattered this perception and led to a boom in quantitative work in the field, nicely surveyed in Eaton & Kortum (2012). As part of this Ricardian revival, trade economists have also developed assignment models that incorporate multiple factors of production into Ricardo's original model. In recent years, these models have been used to study a broad set of issues, ranging from the impact of trade on the distribution of earnings to its mitigating effect on the consequences of climate change in agricultural markets. The goal of this article is to offer a user guide to these multifactor generalizations of the Ricardian model, which we refer to as Ricardo-Roy (R-R) models.

By an R-R model, we formally mean a trade model in which production functions are linear, as in the original Ricardian model, but one in which countries may be endowed with more than one factor, as in the Roy model. Total output in any given sector and country, for example, wine in Portugal, can thus be expressed as

$$Q(\text{Wine, Portugal}) = \sum_f A(f, \text{Wine, Portugal})L(f, \text{Wine, Portugal}),$$

where  $A(f, \text{Wine, Portugal})$  denotes the productivity of factor  $f$ , if employed in the wine sector in Portugal, and  $L(f, \text{Wine, Portugal})$  denotes the employment of that factor. When the number of factors in each country is equal to 1, the R-R model collapses to the Ricardian model. Depending on the particular application, different factors may correspond to different types of labor, capital, or land, whereas different sectors may correspond to different industries, occupations, or tasks. But regardless of the particular application, the key feature of R-R models is that factors' marginal products, and hence marginal rates of technical substitution, are constant. As a result, comparative advantage—i.e., relative differences in productivity—drives the assignment of factors to sectors around the world.

The first part of our article uses R-R models to revisit a number of classical questions in the field. Among other things, we discuss how cross-country differences in technologies and factor endowments affect the pattern of international trade, as in Costinot (2009), as well as how changes in the economic environment—including opening up to trade—affect factor allocation and factor prices, as in Costinot & Vogel (2010). Answering these questions in the context of an R-R model requires new tools and techniques. Because of the linearity of the production function, corner solutions in R-R models are the norm rather than the exception. Hence, the main issue when solving for competitive equilibria is to characterize the extensive margin, that is, the set of sectors to which a given factor should be assigned. Fortunately, standard mathematical notions and results from the monotone comparative statics literature, such as log-supermodularity and Milgrom & Shannon's (1994) monotonicity theorem, are well suited to deal with this and other related issues. We briefly review these mathematical tools in Section 2.

Compared to previous neoclassical trade models, R-R models offer a useful compromise. They are more general than Ricardian models, which makes them amenable to the study of how factor endowments shape international specialization as well as the distributional consequences of trade, yet because marginal rates of technical substitution are constant, they remain significantly more tractable than general neoclassical trade models with arbitrary numbers of goods and factors. Predictions derived in such general models tend to be either weak or unintuitive. For example, the “friends and enemies” result of Jones & Scheinkman (1977) states that a rise in the price of some good causes a disproportionately larger increase in the price of some factor, but depending on the number of goods and factors, it may or may not lead to a disproportionately larger decrease in the

price of some other factor. A common theme in that older literature, reviewed by Ethier (1984), is that predictions in high-dimensional environments hinge on the answer to one fairly abstract question: Are there more goods than factors in the world?

In Section 3, we demonstrate that R-R models deliver sharp predictions in economies with large numbers of goods and factors. First, they offer variations of classical theorems (e.g., factor price equalization, Rybczynski, and Stolper-Samuelson theorems) whose empirical content is no weaker than their famous counterparts in the two-good-two-factor Heckscher-Ohlin model. Second, R-R models offer new predictions regarding the impact of changes in the distribution of prices, factor endowments, or factor demands with no counterparts in the two-good-two-factor Heckscher-Ohlin model. These theoretical results are useful because they open the door for general equilibrium analyses of recent phenomena that have been documented in the labor and public finance literatures, but would otherwise fall outside the scope of standard trade theory. These recent phenomena include changes in inequality at the top of the income distribution as well as wage and job polarization (see, e.g., Piketty & Saez 2003, Autor et al. 2008, and Goos & Manning 2007, respectively).

Section 4 presents various extensions of R-R models. We first introduce imperfect competition, as in Sampson (2014). When good markets are monopolistically competitive à la Melitz (2003), we show how the same tools and techniques can also shed light on the relationship between firm heterogeneity, worker heterogeneity, and international trade. We also incorporate sequential production, as in Costinot et al. (2013), to study how vertical specialization shapes inequality and the interdependence of nations. We conclude by discussing a number of generalizations and variations of the basic linear production functions at the core of R-R models.

The last two sections focus on quantitative and empirical work. In Section 5, we emphasize parametric applications of R-R models using generalized extreme value (GEV) distributions of productivity shocks. We draw a distinction between models that feature unobserved heterogeneity across goods, as in the influential Ricardian model of Eaton & Kortum (2002), and models that feature unobserved heterogeneity across factors, as in the more recent work of Lagakos & Waugh (2013), Hsieh et al. (2013), and Burstein et al. (2014). In both cases, we discuss how to conduct counterfactual and welfare analysis and highlight the key differences associated with these two distinct approaches. In Section 6, we turn to nonparametric applications of R-R models to agricultural markets based on detailed microlevel data from the Food and Agriculture Organization's (FAO's) Global Agro-Ecological Zones (GAEZ) project. These nonparametric applications include empirical tests of Ricardo's theory of comparative advantage (Costinot & Donaldson 2012), the measurement of the gains from economic integration (Costinot & Donaldson 2014), and a quantitative analysis of the consequences of climate change (Costinot et al. 2015a).

R-R models are related to an older literature on linear programming in economics (see Dorfman et al. 1958). Because production functions are linear in R-R models, solving for efficient allocations in such models amounts to solving linear programming problems, an observation made by Whiting (1953) in the context of the Ricardian model. Ruffin (1988) was the first to point out that multiple-factor generalizations of the Ricardian model may provide a useful alternative to Heckscher-Ohlin models with arbitrary neoclassical production functions. He offers a number of examples with two countries and two or three factors in which simpler theorems about trade, welfare, and factor payments can be derived. A similar idea is presented by Ohnsorge & Trefler (2007), who use the log-normal specification of the Roy model to derive variations of the Rybczynski and Heckscher-Ohlin theorems in economies with heterogeneous workers.

Although labor markets are not the only possible application of R-R models, they are an important one. Assignment models, in general, and the Roy model, in particular, have been fruitfully applied by labor economists to study the effect of self-selection on the distribution of earnings as well as the assignment of workers to tasks (see, e.g., Roy 1951, Heckman & Sedlacek 1985, Borjas

1987, Heckman & Honoré 1990, Teulings 1995, Teulings 2005, Acemoglu & Autor 2011). Sattinger (1993) provides an early survey of that literature that clarifies the relationship between the Roy model and other assignment models. Some of these alternative assignment models, such as Becker (1973), Lucas (1978), and Garicano (2000), have also been fruitfully applied in an open-economy context to study the effects of international trade and offshoring on heterogeneous workers or entrepreneurs (see, e.g., Grossman & Maggi 2000, Kremer & Maskin 2006, Antràs et al. 2006, Nocke & Yeaple 2008, Monte 2011, Grossman et al. 2013). Similar to R-R models, these alternative assignment models can be thought of as very simple neoclassical models—in the sense that very strong assumptions on the complementarity between factors of production are imposed—which makes them well suited to the study of economies with a large number of factors of production. Antràs & Rossi-Hansberg (2009) and Garicano & Rossi-Hansberg (2015), in this journal, as well as Grossman (2013) offer nice overviews of recent work in this area.

## 2. THE MATHEMATICS OF COMPARATIVE ADVANTAGE

The premise of Ricardo’s theory of comparative advantage is that some individuals or countries are relatively more productive in some activities than in others. In his famous example, England is relatively better than Portugal at producing cloth than wine. Assuming that labor is the only factor of production in each country and that technology is subject to constant returns to scale, the previous statement can be expressed as

$$\frac{A(\text{Cloth, England})}{A(\text{Wine, England})} \geq \frac{A(\text{Cloth, Portugal})}{A(\text{Wine, Portugal})}, \tag{1}$$

where  $A(\cdot, \cdot)$  denotes labor productivity in a given sector and country. According to the inequality given in Equation 1, England has a comparative advantage in cloth, and if the inequality did not hold, it would have a comparative advantage in wine.

Now let us move beyond Ricardo’s example and consider a world economy with any number of goods and countries. How would one generalize the inequality in Equation 1 to formalize the notion that some countries may have a comparative advantage in some sectors? A fruitful way to proceed is to assume that each country and sector can be described by a scalar,  $\gamma$  and  $\sigma$ , respectively. For instance,  $\gamma$  and  $\sigma$  may reflect the quality of a country’s financial institutions and the dependence of a sector on external financing, as in Matsuyama (2005); the level of rigidities in a country’s labor market and the volatility of sectoral productivity or demand shocks, as in Melitz & Cunat (2012); or, more generally, the level of development of a country and the technological intensity of a sector, as in Krugman (1986). In such environments, statements about the comparative advantage of high- $\gamma$  countries in high- $\sigma$  sectors can still be expressed as

$$\frac{A(\sigma', \gamma')}{A(\sigma, \gamma')} \geq \frac{A(\sigma', \gamma)}{A(\sigma, \gamma)}, \text{ for all } \sigma' \geq \sigma \text{ and } \gamma' \geq \gamma. \tag{2}$$

Mathematically, the inequality in Equation 2 implies that  $A$  is log-supermodular in  $(\sigma, \gamma)$ . This particular form of complementarity captures the idea that increasing one variable is relatively more important when the other variable is high and is intimately related to the notion of comparative advantage introduced by Ricardo.

The previous idea easily extends to situations in which  $\sigma$  and  $\gamma$  are multidimensional. For any  $\mathbf{x}, \mathbf{x}' \in \mathbb{R}^n$ , let  $\max(\mathbf{x}, \mathbf{x}')$  be the vector of  $\mathbb{R}^n$  whose  $i$ -th component is  $\max(x_i, x'_i)$ , and let  $\min(\mathbf{x}, \mathbf{x}')$  be the vector whose  $i$ -th component is  $\min(x_i, x'_i)$ . Given the previous notation, a function  $g: \mathbb{R}^n \rightarrow \mathbb{R}^+$  is log-supermodular if, for all  $\mathbf{x}, \mathbf{x}' \in \mathbb{R}^n$ ,



$$g(\max(\mathbf{x}, \mathbf{x}')) \cdot g(\min(\mathbf{x}, \mathbf{x}')) \geq g(\mathbf{x}) \cdot g(\mathbf{x}').$$

The inequality in Equation 2 corresponds to the special case in which  $\mathbf{x} \equiv (\sigma', \gamma)$  and  $\mathbf{x}' \equiv (\sigma, \gamma')$ . Accordingly,  $g$  is log-supermodular if and only if, for all  $i$  and  $j$ ,  $g$  is supermodular in  $(x_i, x_j)$  when regarded as a function of the arguments  $(x_i, x_j)$  only. If  $g$  also is twice differentiable, then the latter condition is equivalent to  $(\partial^2 \ln g) / (\partial x_i \partial x_j) \geq 0$ . If the above inequality holds with a strict inequality, we say that  $g$  is strictly log-supermodular, and if the above inequality is reversed, we say that  $g$  is log-submodular.

Most of our theoretical results build on three properties of log-supermodular functions.

**Property 1:** If  $g, b: \mathbb{R}^n \rightarrow \mathbb{R}^+$  are log-supermodular, then  $gb$  is log-supermodular.

**Property 2:** If  $g: \mathbb{R}^n \rightarrow \mathbb{R}^+$  is log-supermodular, then  $G(\mathbf{x}_{-i}) \equiv \int g(x_i, \mathbf{x}_{-i}) dx_i$  is log-supermodular, where  $\mathbf{x}_{-i}$  denotes the vector  $\mathbf{x}$  with the  $i$ -th component removed.

**Property 3:** If  $g: \mathbb{R}^n \rightarrow \mathbb{R}^+$  is log-supermodular, then  $x_i^*(\mathbf{x}_{-i}) \equiv \arg \max_{x_i \in \mathbb{R}} g(x_i, \mathbf{x}_{-i})$  is increasing in  $\mathbf{x}_{-i}$  with respect to the component-wise order.

Properties 1 and 2 state that log-supermodularity is preserved by multiplication and integration. Property 1 derives from the definition of log-supermodularity. A general proof of Property 2 can be found in Karlin & Rinott (1980). Because log-supermodularity is a strong form of complementarity—stronger than quasi-supermodularity and the single-crossing property—Property 3 derives from Milgrom & Shannon's (1994) monotonicity theorem. Note that in Property 3,  $x_i^*(\mathbf{x}_{-i})$  may not be a singleton. If it is not a singleton, the monotonicity of  $x_i^*(\mathbf{x}_{-i})$  is expressed in terms of the strong set order. In this case, both the supremum and the infimum of the set  $x_i^*(\mathbf{x}_{-i})$  are increasing in  $\mathbf{x}_{-i}$ .

As we demonstrate next, log-supermodularity offers a powerful way to parameterize cross-country differences in technology, preferences, and endowments to study their implications for the global allocation of factors and the distribution of earnings.

### 3. THE R-R MODEL

In this section, we introduce our baseline version of the R-R model and derive cross-sectional and comparative static predictions in this environment.

#### 3.1. Assumptions

Consider a world economy with many countries indexed by  $\gamma \in \Gamma \subset \mathbb{R}^3$ . The vector of country characteristics,  $\gamma$ , comprises a technology shifter,  $\gamma_A$ , a taste shifter,  $\gamma_D$ , and a factor endowment shifter,  $\gamma_L$ . These three variables capture all potential sources of international specialization. Each country is populated by a representative agent endowed with multiple factors indexed by  $\omega \in \Omega \subset \mathbb{R}$ . The representative agent has homothetic preferences over multiple goods or sectors indexed by  $\sigma \in \Sigma \subset \mathbb{R}$ . All markets are perfectly competitive, and all goods are freely traded across countries.<sup>1</sup>  $p(\sigma)$  denotes the world price of good  $\sigma$ . Factors are immobile across countries and

<sup>1</sup>Free trade in goods plays the same role in an R-R model as in a standard neoclassical trade model. It is crucial for R-R versions of the factor price equalization and the Heckscher-Ohlin theorems, which rely on good prices being equalized around the world. It plays no role for predictions such as the Stolper-Samuelson theorem, which hold for arbitrary vectors of good prices. Similar to the Ricardian model, trade costs can be easily incorporated into R-R models when productivity shocks are drawn from a GEV distribution, as in Eaton & Kortum (2002). We discuss such parametric applications in Section 5.

perfectly mobile across sectors.  $L(\omega, \gamma_L) \geq 0$  denotes the inelastic supply of factor  $\omega$  in country  $\gamma$ , and  $w(\omega, \gamma)$  denotes the price of factor  $\omega$  in country  $\gamma$ .

The defining feature of R-R models is that production functions are linear. Output of good  $\sigma$  in country  $\gamma$  is given by

$$Q(\sigma, \gamma) = \int_{\Omega} A(\omega, \sigma, \gamma_A) L(\omega, \sigma, \gamma) d\omega, \tag{3}$$

where  $A(\omega, \sigma, \gamma_A) \geq 0$  denotes the exogenous productivity of factor  $\omega$  in country  $\gamma$  if employed in sector  $\sigma$ , and  $L(\omega, \sigma, \gamma)$  denotes the endogenous quantity of factor  $\omega$  used to produce good  $\sigma$  in country  $\gamma$ .<sup>2</sup>

The Ricardian model corresponds to the special case in which there is only one factor of production in each country. In this situation, the production possibility frontier in any country  $\gamma$  reduces to a straight line (see Figure 1a). In an R-R model more generally, countries may be endowed with multiple factors of production, leading to kinks in the production possibility frontier (see Figure 1b). As the number of factors goes to infinity, the production possibility frontier becomes smooth, as in a standard Heckscher-Ohlin or specific factor model (see Figure 1c).<sup>3</sup>

This is an important observation. Holding the number of factors fixed, an R-R model with a linear production function is necessarily more restrictive than a standard neoclassical trade model. But the number of factors need not be fixed. In particular, an R-R model with a continuum of factors does not impose more a priori restrictions on the data than a Heckscher-Ohlin model with two factors. To take an analogy from the literature on discrete choice models in industrial organization, assuming that a continuum of heterogeneous consumers have constant marginal rates of substitution may not lead to different implications for aggregate demand than assuming a representative agent with a general utility function (see, e.g., Anderson et al. 1992). We come back to related issues in Section 5 when discussing parametric applications of R-R models.

### 3.2. Competitive Equilibrium

In a competitive equilibrium, consumers maximize utility, firms maximize profits, and markets clear.

**3.2.1. Consumers.** Let  $D(p, I(\gamma)|\sigma, \gamma_D)$  denote the Marshallian demand for good  $\sigma$  in country  $\gamma$  as a function of the schedule of world prices,  $p \equiv \{p(\sigma)\}$ , and the income of country  $\gamma$ 's representative agent,  $I(\gamma) \equiv \int_{\Omega} w(\omega, \gamma) L(\omega, \gamma_L) d\omega$ . By definition of the Marshallian demand, utility maximization requires the consumption of good  $\sigma$  in country  $\gamma$  to satisfy

$$D(\sigma, \gamma) = D(p, I(\gamma)|\sigma, \gamma_D). \tag{4}$$

<sup>2</sup>There may be a continuum or a discrete number of factors in  $\Omega$ . Whenever the integral sign  $\int_{\Omega}$  appears, one should therefore think of a Lebesgue integral. If there is a finite number of factors,  $\int_{\Omega}$  is simply equivalent to  $\sum_{\Omega}$ . Integrals over country and sector characteristics should be interpreted in a similar manner.

<sup>3</sup>In an Arrow-Debreu economy, of which R-R models are special cases, one can always think of factors located in different countries as different factors. In the absence of trade costs and cross-country differences in preferences, the closed economy of an R-R model with  $N$  factors is therefore equivalent to the world economy of a Ricardian model with  $N$  countries.





Figure 1

Production possibility frontiers in the Ricardo-Roy (R-R) model with two goods and  $N = 1, 2, \infty$  factors.

**3.2.2. Firms.** For future reference, it is useful to start by studying the cost-minimization problem of a representative firm, which is a necessary condition for profit maximization. By Equation 3, the unit-cost function of a firm producing good  $\sigma$  in country  $\gamma$  is given by

$$c(\sigma, \gamma) \equiv \min_{l(\omega, \sigma, \gamma) \geq 0} \int_{\Omega} w(\omega, \gamma) l(\omega, \sigma, \gamma) d\omega$$

$$\text{s.t. } \int_{\Omega} A(\omega, \sigma, \gamma_A) l(\omega, \sigma, \gamma) d\omega \geq 1.$$

The linearity of the production function immediately implies

$$c(\sigma, \gamma) = \min_{\omega \in \Omega} \{w(\omega, \gamma) / A(\omega, \sigma, \gamma_A)\}. \tag{5}$$

In turn, the set of factors,  $\Omega(\sigma, \gamma) \equiv \{\omega \in \Omega : L(\omega, \sigma, \gamma) > 0\}$ , demanded by firms producing good  $\sigma$  in country  $\gamma$  satisfies

$$\Omega(\sigma, \gamma) \subset \arg \min_{\omega \in \Omega} \{w(\omega, \gamma) / A(\omega, \sigma, \gamma_A)\}. \tag{6}$$

Having characterized the unit-cost function of a representative firm, its profit function can be expressed as  $\pi(\sigma, \gamma) \equiv \max_{q \geq 0} \{p(\sigma)q - c(\sigma, \gamma)q\}$ . Profit maximization then requires

$$p(\sigma) \leq c(\sigma, \gamma), \text{ with equality if } \Omega(\sigma, \gamma) \neq \emptyset. \tag{7}$$

**3.2.3. Market clearing.** Factor and good market clearing finally require

$$\int_{\Sigma} L(\omega, \sigma, \gamma) d\sigma = L(\omega, \gamma_L), \text{ for all } \omega, \gamma, \tag{8}$$

$$\int_{\Gamma} D(\sigma, \gamma) d\gamma = \int_{\Gamma} Q(\sigma, \gamma) d\gamma, \text{ for all } \sigma. \tag{9}$$

To summarize, a competitive equilibrium corresponds to consumption,  $D : \Sigma \times \Gamma \rightarrow \mathbb{R}^+$ , output,  $Q : \Sigma \times \Gamma \rightarrow \mathbb{R}^+$ , factor allocation,  $L : \Omega \times \Sigma \times \Gamma \rightarrow \mathbb{R}^+$ , good prices,  $p : \Sigma \rightarrow \mathbb{R}^+$ , and factor prices,  $w : \Omega \times \Gamma \rightarrow \mathbb{R}^+$ , such that Equations 3–9 hold.

### 3.3. Cross-Sectional Predictions

In this section, we follow Costinot (2009) and focus on the cross-sectional predictions of an R-R model. Formally, we take good prices,  $p(\sigma)$ , as given and explore how factor allocation, factor



prices, and aggregate output vary across countries and industries in a competitive equilibrium. Accordingly, demand considerations and the good market clearing conditions (Equations 4 and 8) play no role here.

**3.3.1. Factor allocation.** A central question in assignment models is, Who works where? In the context of an R-R model, one may be interested in characterizing the set of workers employed in particular sectors or, conversely, the set of goods produced by particular countries. To make progress on these issues, a common practice in the literature is to impose restrictions on technology that generate positive assortative matching (PAM).

**Assumption 1:**  $A(\omega, \sigma, \gamma_A)$  is strictly log-supermodular in  $(\omega, \sigma)$  and  $(\sigma, \gamma_A)$ .

According to Assumption 1, high- $\omega$  factors are relatively more productive in high- $\sigma$  sectors, and high- $\gamma$  countries are relatively more productive in high- $\sigma$  sectors. A simple example of a log-supermodular function is  $A(\omega, \sigma, \gamma_A) \equiv \exp(\omega\sigma)$  or  $\exp(\sigma\gamma_A)$ , as in Krugman (1986), Teulings (1995), and Ohnsorge & Trefler (2007).<sup>4</sup>

By Property 3, Assumption 1 implies that  $\arg \min_{\omega \in \Omega} \{w(\omega, \gamma)/A(\omega, \sigma, \gamma_A)\}$  is increasing in  $\sigma$  in any country  $\gamma$ . Because the log-supermodularity of  $A$  in  $(\omega, \sigma)$  is strict, one can further show that for any pair of sectors,  $\sigma \neq \sigma'$ , there can be at most one factor  $\omega_0$  such that  $\omega_0 \in \arg \min_{\omega \in \Omega} \{w(\omega, \gamma)/A(\omega, \sigma, \gamma_A)\} \cap \arg \min_{\omega \in \Omega} \{w(\omega, \gamma)/A(\omega, \sigma', \gamma_A)\}$ . Combining the two previous observations, we obtain our first result.

**PAM (I):** Suppose that Assumption 1 holds. Then, for any country  $\gamma$ ,  $\Omega(\sigma, \gamma)$  is increasing in  $\sigma$ .

This is intuitive. In a competitive equilibrium, high- $\omega$  factors should be employed in the high- $\sigma$  sectors in which they have a comparative advantage.

We can follow a similar strategy to analyze patterns of international specialization. Let  $\Sigma(\omega, \gamma) \equiv \{\sigma \in \Sigma : L(\omega, \sigma, \gamma) > 0\}$  denote the set of sectors in which factor  $\omega$  is employed in country  $\gamma$ . The conditions in Equations 5 and 7 imply that the value of the marginal product of a factor  $\omega$  in any sector  $\sigma$  should be weakly less than its price,

$$p(\sigma)A(\omega, \sigma, \gamma_A) \leq w(\omega, \gamma), \text{ for all } \sigma, \tag{10}$$

with equality if factor  $\omega$  is employed in that sector,  $\omega \in \Omega(\sigma, \gamma)$ . Because  $\sigma \in \Sigma(\omega, \gamma)$  if and only if  $\omega \in \Omega(\sigma, \gamma)$ , the condition in Equation 10 further implies that

$$\Sigma(\omega, \gamma) \subset \arg \max_{\sigma \in \Sigma} \{p(\sigma)A(\omega, \sigma, \gamma_A)\}. \tag{11}$$

This condition states that factors from any country should be employed in the sector that maximizes the value of their marginal product, an expression of the efficiency of perfectly competitive markets.

<sup>4</sup>The strict log-supermodularity of  $A(\omega, \sigma, \gamma_A)$  in  $(\omega, \sigma)$  formally rules out the possibility that two distinct factors are perfect substitutes across all sectors. At a theoretical level, this restriction is purely semantic. If two workers differ only in terms of their absolute advantage, one can always refer to them as one factor and let the efficiency units with which they are endowed vary. This is the convention we adopt in this article. Because we assume the existence of a representative agent, the distribution of these efficiency units is irrelevant for any of our theoretical results and omitted. At an empirical level, however, one should keep in mind that the distribution of earnings depends both on the schedule of prices per efficiency units, which we refer to as the factor price,  $w(\omega, \gamma)$ , and on the distribution of these efficiency units.





Starting from the condition in Equation 11 and using the exact same logic as above, we obtain the following prediction.

**PAM (II):** Suppose that Assumption 1 holds. Then, for any factor  $\omega$ ,  $\Sigma(\omega, \boldsymbol{\gamma})$  is increasing in  $\gamma_A$ .

In a competitive equilibrium, there must also be a ladder of countries, with high- $\boldsymbol{\gamma}$  countries in high- $\sigma$  sectors. This is the prediction at the heart of many Ricardian models, such as the technology gap model developed by Krugman (1986), as well as many models in the institutions and trade literature reviewed by Nunn & Trefler (2015). As discussed by Costinot (2009), Assumption 1 is critical for such patterns in the sense that it cannot be dispensed with for PAM to arise in all economic environments satisfying the assumptions of Section 3.1.<sup>5</sup>

An important special case in the literature is that in which  $\Sigma(\omega, \boldsymbol{\gamma})$  is a singleton. This corresponds to a situation in which each factor is assigned only to one sector. A sufficient condition for  $\Sigma(\omega, \boldsymbol{\gamma})$  to be a singleton is that  $\arg \max_{\sigma \in \Sigma} \{p(\sigma)A(\omega, \sigma, \gamma_A)\}$  is itself a singleton. Graphically, this situation arises when production occurs at a vertex of the production possibility frontier in Figure 1. As the numbers of factors and hence vertices increase, this restriction becomes increasingly milder. If there is a continuum of factors, then  $\Sigma(\omega, \boldsymbol{\gamma})$  must be a singleton for almost all  $\omega$ .<sup>6</sup>

**3.3.2. Factor prices.** The conditions given in Equations 5 and 7 also have strong implications for the distribution of factor prices within and between countries. Starting from the condition given in Equation 10 and noting that there must exist a  $\sigma$  such that  $\omega \in \Omega(\sigma, \boldsymbol{\gamma})$  by factor market clearing, we obtain

$$w(\omega, \boldsymbol{\gamma}) = \max_{\sigma \in \Sigma} \{p(\sigma)A(\omega, \sigma, \gamma_A)\}. \quad (12)$$

Now consider two countries with the same technology,  $\gamma_A = \gamma'_A$ , but with potentially different endowments and preferences. Equation 12 immediately implies  $w(\omega, \boldsymbol{\gamma}) = w(\omega, \boldsymbol{\gamma}') \equiv w(\omega)$ . In other words, we always have factor price equalization (FPE), as originally noted by Ruffin (1988) and summarized below.

**R-R FPE theorem:** If there are no technological differences between countries, then factor prices are equalized under free trade,  $w(\omega, \boldsymbol{\gamma}) = w(\omega)$  for all  $\boldsymbol{\gamma}$ .

For all subsequent results, we restrict ourselves to an economy with a continuum of factors. As discussed above, this implies that  $\Sigma(\omega, \boldsymbol{\gamma})$  is a singleton. Thus, the allocation of factors to sectors can be summarized by a matching function,  $M$ , such that  $\Sigma(\omega, \boldsymbol{\gamma}) = \{M(\omega, \boldsymbol{\gamma})\}$ . In Section 3.4, we also add the assumption of a continuum of goods.

Under the assumption of a continuum of factors, we can analyze the distribution of factor prices within each country by differentiating Equation 12 with respect to  $\omega$ . By the envelope theorem, we must have

<sup>5</sup>The symmetry between PAM (I) and PAM (II) should not be surprising. As discussed above, factors in different countries can always be defined as different factors in an Arrow-Debreu economy. Under this alternative interpretation, PAM within countries and PAM between countries are two sides of the same coin.

<sup>6</sup>This is true regardless of whether there is a finite number of goods or a continuum of goods with finite measure. To see this, note for any  $\omega \neq \omega'$ , the overlap between  $\Sigma(\omega, \boldsymbol{\gamma})$  and  $\Sigma(\omega', \boldsymbol{\gamma})$  must be measure zero under Assumption 1. So if the set of factors for which  $\Sigma(\omega, \boldsymbol{\gamma})$  is not a singleton had strictly positive measure, the set of goods to which they are assigned would have to have infinite measure.

$$\frac{d \ln w(\omega, \boldsymbol{\gamma})}{d\omega} = \frac{\partial \ln A(\omega, M(\omega, \boldsymbol{\gamma}), \boldsymbol{\gamma}_A)}{\partial \omega}. \quad (13)$$

Equation 13 is one of the key equilibrium conditions used in our comparative static analysis. Intuitively, if two distinct factors,  $\omega_1$  and  $\omega_2$ , were to be employed in the same sector  $\sigma$ , then their relative prices should exactly equal their relative productivities,  $w(\omega_1, \boldsymbol{\gamma})/w(\omega_2, \boldsymbol{\gamma}) = A(\omega_1, \sigma, \boldsymbol{\gamma}_A)/A(\omega_2, \sigma, \boldsymbol{\gamma}_A)$  or, in logs,

$$\Delta \ln w(\omega_1, \boldsymbol{\gamma}) - \Delta \ln w(\omega_2, \boldsymbol{\gamma}) = \Delta \ln A(\omega_1, \sigma, \boldsymbol{\gamma}_A) - \Delta \ln A(\omega_2, \sigma, \boldsymbol{\gamma}_A).$$

Equation 13 expands on this observation by using the fact that reallocations of factors across sectors must have second-order effects on the value of a factor's marginal product.<sup>7</sup>

Finally, whereas Equation 12 relies on perfect competition in good markets, through the first-order condition (Equation 7), Equation 13 does not. The condition given in Equation 5 and factor market clearing alone imply that  $w(\omega, \boldsymbol{\gamma}) = \max_{\sigma \in \Sigma} \{c(\sigma, \boldsymbol{\gamma})A(\omega, \sigma, \boldsymbol{\gamma}_A)\}$ . Starting from this expression and invoking the envelope theorem, we again obtain Equation 13. This observation will play a central role in extending R-R models to environments with imperfectly competitive good markets.

**3.3.3. Aggregate output.** Above we establish that Assumption 1 imposes PAM. PAM, however, only imposes a restriction on the extensive margin of employment, that is, whether a factor should be employed in a sector in a particular country; it does not impose any restriction on the intensive margin of employment and, in turn, aggregate output.

To derive cross-sectional predictions about aggregate output, we now impose the following restriction on the distribution of factor endowments.

**Assumption 2:**  $L(\omega, \boldsymbol{\gamma}_L)$  is log-supermodular.

For any pair of countries,  $\boldsymbol{\gamma}'_L \geq \boldsymbol{\gamma}_L$ , and factors,  $\omega' \geq \omega$ , such that  $L(\omega, \boldsymbol{\gamma}_L), L(\omega, \boldsymbol{\gamma}'_L) \neq 0$ , Assumption 2 implies

$$\frac{L(\omega', \boldsymbol{\gamma}'_L)}{L(\omega, \boldsymbol{\gamma}'_L)} \geq \frac{L(\omega', \boldsymbol{\gamma}_L)}{L(\omega, \boldsymbol{\gamma}_L)}.$$

According to Assumption 2, high- $\boldsymbol{\gamma}_L$  countries are relatively abundant in high- $\omega$  factors. Formally, it is equivalent to the assumption that the densities of countries' factor endowments can be ranked in terms of monotone likelihood ratio dominance. Milgrom (1981) offers many examples of density functions that satisfy this assumption, including the normal (with mean  $\boldsymbol{\gamma}_L$ ) and the uniform (on  $[0, \boldsymbol{\gamma}_L]$ ). This is the natural generalization of the notion of skill abundance in a two-factor model. Note that Assumption 2 also allows us to consider situations in which different sets of factor are available in countries  $\boldsymbol{\gamma}$  and  $\boldsymbol{\gamma}'$ . In such situations, the highest- $\omega$  factor must be in country  $\boldsymbol{\gamma}$  and the lowest- $\omega$  factor in country  $\boldsymbol{\gamma}'$ .

<sup>7</sup>Here, we implicitly assume that  $w(\cdot, \boldsymbol{\gamma})$  is differentiable. In economies with a continuum of goods, this property follows from assuming that  $A(\omega, \sigma, \boldsymbol{\gamma})$  is differentiable. In economies with a discrete number of goods,  $w(\cdot, \boldsymbol{\gamma})$  would necessarily feature a discrete number of kinks. In such environments, the envelope theorem of Milgrom & Segal (2002) provides a strict generalization of Equation 13.

Because  $\Sigma(\omega, \gamma)$  is a singleton, employment of a factor  $\omega$  in a particular sector  $\sigma$  must now be equal to the total endowment of that factor,  $L(\omega, \gamma_L)$ , whenever  $\omega \in \Omega(\sigma, \gamma)$ . Thus, output of good  $\sigma$  can be expressed as

$$Q(\sigma, \gamma) = \int_{\Omega(\sigma, \gamma)} A(\omega, \sigma, \gamma_A) L(\omega, \gamma_L) d\omega.$$

If there are no technological differences between countries, FPE further implies that the allocation of factors to sectors must be the same in all countries,  $\Omega(\sigma, \gamma) \equiv \Omega(\sigma)$ , so that the previous expression simplifies to

$$Q(\sigma, \gamma) = \int_{\Omega(\sigma)} A(\omega, \sigma, \gamma_A) L(\omega, \gamma_L) d\omega. \quad (14)$$

Using Equation 14 together with PAM and Properties 1 and 2, which imply that log-supermodularity is preserved by multiplication and integration, Costinot (2009) establishes the following Rybczynski-type result.

**R-R Rybczynski theorem:** Suppose that Assumptions 1 and 2 hold. Then  $Q(\sigma, \gamma)$  is log-supermodular in  $(\sigma, \gamma_L)$ .

For any pair of goods,  $\sigma \leq \sigma'$ , and any pair of countries with identical technology,  $\gamma_A = \gamma'_A$ , but different endowments,  $\gamma_L \leq \gamma'_L$ , the previous property implies that

$$\frac{Q(\sigma', \gamma')}{Q(\sigma, \gamma')} \geq \frac{Q(\sigma', \gamma)}{Q(\sigma, \gamma)}.$$

In other words, the country that is relatively more abundant in the high- $\omega$  factors (i.e., country  $\gamma'$ ) produces relatively more in the sector that is intensive in those factors under PAM (i.e., sector  $\sigma'$ ). This is akin to the predictions of the Rybczynski theorem in a two-by-two Heckscher-Ohlin model. Here, however, the previous prediction holds for an arbitrarily large number of goods and factors. If one further assumes that countries have identical preferences,  $\gamma_D = \gamma'_D$ , the Rybczynski theorem above implies that high- $\gamma_L$  countries are net exporters of high- $\sigma$  goods, whereas low- $\gamma_L$  countries are net exporters of low- $\sigma_L$  goods, in line with the predictions of the two-by-two Heckscher-Ohlin theorem, a point emphasized by Ohnsorge & Trefler (2007).

As shown by Costinot (2009), one can use a similar logic to establish that aggregate employment and aggregate revenue in a country and sector must also be log-supermodular functions of  $(\sigma, \gamma_L)$ . Using US data on cities' skill distributions, sectors' skill intensities, and cities' sectoral employment, Davis & Dingel (2013) provide empirical support for such predictions.

### 3.4. Comparative Static Predictions

The goal of this subsection is to go from cross-sectional predictions to comparative static predictions about the effects of various shocks on factor allocation and factor prices. We start by revisiting the Stolper-Samuelson theorem, which emphasizes shocks to good prices. We then turn to the consequences of factor endowment and taste shocks.<sup>8</sup> Following Costinot & Vogel (2010),

<sup>8</sup>If one reinterprets goods as tasks used to produce a unique final good, as in Costinot & Vogel (2010), then taste shocks can also be interpreted as technological shocks to that final good production function.

we do so in the case of a continuum of both goods and factors:  $\Sigma = [\underline{\sigma}, \bar{\sigma}]$  and  $\Omega = [\underline{\omega}, \bar{\omega}]$ . Under mild regularity conditions on productivity, endowments, and demand functions, this guarantees that the schedule of factor prices and the matching function are differentiable, which we assume throughout. Comparative static results in the discrete case can be found in Costinot & Vogel (2009).

**3.4.1. Price shocks.** Consider a small open economy whose characteristics  $\gamma$  are held fixed, whereas country characteristics in the rest of the world, which we summarize by  $\phi$ , are subject to a shock. Using this parameterization, a foreign shock to technology, tastes, or factor endowments simply corresponds to a change from  $\phi$  to  $\phi'$ . In a neoclassical environment, foreign shocks only affect the small open economy  $\gamma$  through their effects on world prices. To make that relationship explicit here, we now let  $p(\sigma, \phi)$  denote the world price of good  $\sigma$  as a function of foreign characteristics  $\phi$ .

In line with the analysis of Section 3.3, we restrict ourselves to foreign shocks that satisfy the following restriction.

**Assumption 3:**  $p(\sigma, \phi)$  is log-supermodular in  $(\sigma, \phi)$ .

For any pair of goods,  $\sigma' \geq \sigma$ , a shock from  $\phi$  to  $\phi' \geq \phi$  corresponds to an increase in the relative price of good  $\sigma'$ , which is the good intensive in high- $\omega$  factors under PAM. In the context of the two-by-two Heckscher-Ohlin model, the Stolper-Samuelson theorem predicts that the relative price of the skill-intensive good should lead to an increase in the relative price of skilled workers. We now demonstrate that in an R-R model, a similar prediction extends to economies with an arbitrarily large number of goods and factors.

For the purposes of this subsection, and this subsection only, we let  $w(\cdot, \gamma, \phi)$  and  $M(\cdot, \gamma, \phi)$  denote the schedule of factor prices and the matching function in country  $\gamma$  as a function of the foreign shock,  $\phi$ . Using this notation, we can rewrite Equation 12 as

$$w(\omega, \gamma, \phi) = \max_{\sigma \in \Sigma} \{A(\omega, \sigma, \gamma_A) p(\sigma, \phi)\}.$$

Starting from the previous equation and invoking the envelope theorem, now with respect to a change in  $\phi$ , we obtain

$$\frac{d \ln w(\omega, \gamma, \phi)}{d \phi} = \frac{\partial \ln p(M(\omega, \gamma, \phi), \phi)}{\partial \phi}. \tag{15}$$

Because PAM implies that  $M$  is increasing in  $\omega$ , Assumption 3 further implies that

$$\frac{d}{d \omega} \left( \frac{\partial \ln p(M(\omega, \gamma, \phi), \phi)}{\partial \phi} \right) = \frac{dM(\omega, \gamma, \phi)}{d \omega} \frac{\partial^2 \ln p(M(\omega, \gamma, \phi), \phi)}{\partial \sigma \partial \phi} \geq 0.$$

Combining the previous inequality with Equation 15, we obtain the following Stolper-Samuelson-type result.

**R-R Stolper-Samuelson theorem:** Suppose that Assumptions 1 and 3 hold. Then  $w(\omega, \gamma, \phi)$  is log-supermodular in  $(\omega, \phi)$ .

Economically speaking, the previous result states that an increase in the relative price of high- $\sigma$  goods (caused by a shock from  $\phi$  to  $\phi'$ ) must be accompanied by an increase in the relative price of high- $\omega$  factors (which tend to be employed in the production of these goods). The intuition is again



simple. Take two factors,  $\omega' \geq \omega$ , employed in two sectors,  $\sigma' \geq \sigma$ , before the shock. If both factors were to remain employed in the same sector after the shock, then the change in their relative prices would just be equal to the change in the relative prices of the goods they produce,

$$\ln \left[ \frac{w(\omega', \gamma, \phi')}{w(\omega, \gamma, \phi)} \right] - \ln \left[ \frac{w(\omega', \gamma, \phi)}{w(\omega, \gamma, \phi)} \right] = \ln \left[ \frac{p(\sigma', \phi')}{p(\sigma, \phi')} \right] - \ln \left[ \frac{p(\sigma', \phi)}{p(\sigma, \phi)} \right].$$

Hence, an increase in the relative price of good  $\sigma'$  would mechanically increase the relative price of factor  $\omega'$ . Similar to Section 3.3.2, the previous Stolper-Samuelson-type result expands on this observation by using the fact that factor reallocations across sectors must have second-order effects on the value of a factor's marginal product.

Under the assumption that the small open economy is fully diversified, both before and after the shock, the previous result further implies the existence of a factor  $\omega^* \in (\underline{\omega}, \bar{\omega})$  such that real factor returns decrease for all factors below  $\omega^*$  and increase for all factors above  $\omega^*$ . In other words, a foreign shock must create winners and losers. Intuitively, factor  $\underline{\omega}$  must lose because it keeps producing good  $\underline{\sigma}$ , whose price decreases relative to all other prices. Conversely, factor  $\bar{\omega}$  must win because it keeps producing good  $\bar{\sigma}$ , whose price increases relative to all other prices.

**3.4.2. Endowment and taste shocks.** We proceed in two steps. We first study the consequences of endowment and taste shocks in a closed economy. Using the fact that the free trade equilibrium reproduces the integrated equilibrium, we then discuss how these comparative static results under autarky can be used to study the effects of opening up to trade.

Consider a closed economy with characteristic  $\gamma$ . A competitive equilibrium under autarky corresponds to  $(D^a, Q^a, L^a, p^a, w^a)$  such that Equations 3–8 hold, and the good market clearing condition (Equation 9) is given by

$$D^a(\sigma, \gamma) = Q^a(\sigma, \gamma), \text{ for all } \sigma \text{ and } \gamma. \tag{16}$$

We start by expressing the competitive equilibrium of a closed economy in a compact form as a system of two differential equations in the schedule of factor prices,  $w^a$ , and the matching function,  $M^a$ .

Given PAM, the factor market clearing condition (Equation 8) can be rearranged as

$$\int_{\underline{\sigma}}^{M^a(\omega, \gamma)} \frac{Q^a(\sigma, \gamma)}{A((M^a)^{-1}(\sigma, \gamma), \sigma, \gamma_A)} d\sigma = \int_{\underline{\omega}}^{\omega} L(v, \gamma_L) dv, \text{ for all } \omega. \tag{17}$$

From utility maximization and the good market clearing condition (Equations 4 and 16), we also know that

$$Q^a(\sigma, \gamma) = D^a(p^a, I^a(\gamma) | \sigma, \gamma_D).$$

Substituting into Equation 17 and differentiating with respect to  $\omega$ , we obtain after rearrangements

$$\frac{dM^a(\omega, \gamma)}{d\omega} = \frac{A(\omega, M^a(\omega, \gamma), \gamma_A)L(\omega, \gamma_L)}{D(p^a, I^a(\gamma) | M^a(\omega, \gamma), \gamma_D)}. \tag{18}$$

In a competitive equilibrium, the slope of the matching function is set such that factor supply equals factor demand. The higher the supply of a given factor,  $L(\omega, \gamma_L)$ , relative to its demand,



$D(p^a, I^a(\boldsymbol{\gamma})|M^a(\omega, \boldsymbol{\gamma}), \gamma_D)/A(\omega, M^a(\omega, \boldsymbol{\gamma}), \gamma_A)$ , the faster it should get assigned to sectors for markets to clear.

Costinot & Vogel (2010) derive a number of comparative static predictions in the case in which demand functions are constant elasticity of substitution (CES):

$$D(p, I(\boldsymbol{\gamma})|\sigma, \gamma_D) = \frac{B(\sigma, \gamma_D)p^{-\varepsilon}(\sigma)I(\boldsymbol{\gamma})}{P^{1-\varepsilon}(\gamma_D)}, \tag{19}$$

where  $B(\sigma, \gamma_D)$  is a demand shifter of good  $\sigma$ , and  $P(\gamma_D) = \left(\int_{\Sigma} B(\sigma, \gamma_D)p^{1-\varepsilon}(\sigma)d\sigma\right)^{1/(1-\varepsilon)}$  denotes the CES price index. In the rest of this article, we refer to an economy in which Equation 19 holds as a CES economy. In such an economy, normalizing the CES price index to 1, Equation 18 can be rearranged as

$$\frac{dM^a(\omega, \boldsymbol{\gamma})}{d\omega} = \frac{A^{1-\varepsilon}(\omega, M^a(\omega, \boldsymbol{\gamma}), \gamma_A)(w^a(\omega, \boldsymbol{\gamma}))^\varepsilon L(\omega, \gamma_L)}{B(M^a(\omega, \boldsymbol{\gamma}), \gamma_D) \int_{\Omega} w^a(\omega', \boldsymbol{\gamma})L(\omega', \gamma_L)d\omega'}, \tag{20}$$

where  $p^a(M^a(\omega, \boldsymbol{\gamma})) = w^a(\omega, \boldsymbol{\gamma})/A(\omega, M^a(\omega, \boldsymbol{\gamma}), \gamma_A)$ , by the conditions given in Equations 5 and 7, and  $I^a(\boldsymbol{\gamma}) = \int_{\Omega} w^a(\omega', \boldsymbol{\gamma})L(\omega', \gamma_L)d\omega'$ .

Equations 13 and 20 offer a system of two differential equations in  $(M^a, w^a)$ . The characterization of a competitive equilibrium is completed by the two boundary conditions,  $M^a(\underline{\omega}, \boldsymbol{\gamma}) = \underline{\sigma}$  and  $M^a(\bar{\omega}, \boldsymbol{\gamma}) = \bar{\sigma}$ , which state that the lowest and highest factors should be employed in the lowest and highest sectors, an implication of PAM.

Given Equations 13 and 20, one can study how shocks to factor supply and factor demand, parameterized as changes in  $\gamma_L$  and  $\gamma_D$ , respectively, affect factor allocation,  $M^a(\omega, \boldsymbol{\gamma})$ , and factor prices,  $w^a(\omega, \boldsymbol{\gamma})$ . As we did for technology and factor endowments, we impose the following restriction on how demand shocks,  $\gamma_D$ , affect the relative consumption of various goods.

**Assumption 4:**  $B(\sigma, \gamma_D)$  is log-submodular in  $(\sigma, \gamma_D)$ .

Given Equation 19, Assumption 4 implies that an increase in  $\gamma_D$  lowers the relative demand for high- $\sigma$  goods.<sup>9</sup> For any pair of goods,  $\sigma \geq \sigma'$ , and countries,  $\gamma'_D \geq \gamma_D$ , such that  $B(\sigma, \gamma_D), B(\sigma, \gamma'_D) \neq 0$ , we must have

$$\frac{D(p, I(\boldsymbol{\gamma}')|\sigma', \gamma'_D)}{D(p, I(\boldsymbol{\gamma}')|\sigma, \gamma'_D)} \leq \frac{D(p, I(\boldsymbol{\gamma})|\sigma', \gamma_D)}{D(p, I(\boldsymbol{\gamma})|\sigma, \gamma_D)}.$$

In this environment, Costinot & Vogel (2010) establish the two following comparative static results about factor allocation and factor prices.

**Comparative statics I (factor allocation):** Suppose that Assumptions 1, 2, and 4 hold in a CES economy under autarky. Then  $M^a(\omega, \boldsymbol{\gamma})$  is decreasing in  $\gamma_D$  and  $\gamma_L$ .

**Comparative statics II (factor prices):** Suppose that Assumptions 1, 2, and 4 hold in a CES economy under autarky. Then  $w^a(\omega, \boldsymbol{\gamma})$  is log-submodular in  $(\omega, \gamma_D)$  and  $(\omega, \gamma_L)$ .

<sup>9</sup>Assuming log-submodularity rather than log-supermodularity is purely expositional. This convention guarantees that changes in  $\gamma_L$  and  $\gamma_D$  have symmetric effects on factor allocation and factor prices.



Consider first an endowment shock from  $\gamma_L$  to  $\gamma'_L \geq \gamma_L$ . By Assumption 2, this corresponds to an increase in the relative supply of high- $\omega$  factors. In the new equilibrium, this must be accompanied by an increase in the set of sectors employing higher- $\omega$  factors, which is achieved by a downward shift in the matching function. Having established that the matching function must shift down, one can then use Equation 13 to sign the effect of a change in the relative factor supply on relative factor prices:

$$\frac{d^2 \ln w^a(\omega, \gamma)}{d\gamma_L d\omega} = \frac{dM^a(\omega, \gamma)}{d\gamma_L} \frac{\partial^2 \ln A(\omega, M^a(\omega, \gamma), \gamma_A)}{d\gamma_L d\omega} \leq 0,$$

where the previous inequality uses  $(dM^a(\omega, \gamma))/d\gamma_L \leq 0$  and  $\partial^2 \ln A(\omega, M^a(\omega, \gamma), \gamma_A)/d\gamma_L d\omega \geq 0$  by Assumption 1. As intuition would suggest, if the relative supply of high- $\omega$  factors goes up, their relative price must go down.

The intuition regarding the effect of a taste shock is similar. By Assumption 4, an increase in  $\gamma_D$  corresponds to a decrease in the relative demand for high- $\sigma$  goods. This change in factor demand must be accompanied by factors moving into lower- $\sigma$  sectors, which explains why  $M^a(\omega, \gamma)$  is decreasing in  $\gamma_D$ . Conditional on the change in the matching function, the effects on relative factor prices are the same as in the case of a shock to factor endowments. If factors move into lower- $\sigma$  sectors in which low- $\omega$  factors have a comparative advantage, low- $\omega$  factors will be relatively better off.

As shown in Costinot & Vogel (2010), the same approach can be used to study richer endowment and taste shocks (e.g., shocks that disproportionately affect middle factors or sectors). Although the economic forces at play are similar to those presented here, such extensions are important as they allow for the analysis of recent labor market phenomena such as job and wage polarization, as emphasized by Acemoglu & Autor (2011).

To go from the previous closed-economy results to the effect of opening up to trade, we can use the fact that under FPE, the free trade equilibrium replicates the integrated equilibrium. Hence, in the absence of technological differences across countries, factor allocation and prices in any country  $\gamma$ ,  $M(\omega, \gamma)$  and  $w(\omega, \gamma)$ , must be equal to those of a fictitious world economy under autarky,  $M^a(\omega, \gamma^w)$  and  $w^a(\omega, \gamma^w)$ , with

$$\frac{dM^a(\omega, \gamma^w)}{d\omega} = \frac{A^{1-\varepsilon}(\omega, M^a(\omega, \gamma^w), \gamma_A^w) (w^a(\omega, \gamma^w))^\varepsilon L(\omega, \gamma_L^w)}{B(M^a(\omega, \gamma^w), \gamma_D^w) \int_{\Omega} w^a(\omega', \gamma^w) L(\omega', \gamma_L^w) d\omega'}, \quad (21)$$

$$\frac{d \ln w^a(\omega, \gamma^w)}{d\omega} = \frac{\partial \ln A(\omega, M^a(\omega, \gamma^w), \gamma_A^w)}{d\omega}. \quad (22)$$

In the previous system of equations,  $\gamma_A^w$  corresponds to the technological parameter common across countries, whereas  $\gamma_L^w$  and  $\gamma_D^w$  are implicitly defined such that

$$L(\omega, \gamma_L^w) = \int_{\Gamma} L(\omega, \gamma_L) d\gamma_L,$$

$$B(\sigma, \gamma_D^w) = \int_{\Gamma} \frac{\int_{\Omega} w^a(\omega, \gamma^w) L(\omega, \gamma_L) d\omega}{\int_{\Omega} w^a(\omega, \gamma^w) L(\omega, \gamma_L^w) d\omega} P^{\varepsilon-1}(\gamma) B(\sigma, \gamma_D) d\gamma_D.$$

In the two-country case, one can check that if  $\gamma \leq \gamma'$ , then  $\gamma^w \in [\gamma, \gamma']$ . This simple observation implies that the consequences of opening up to trade in country  $\gamma$  are isomorphic to an increase in  $\gamma_D$  and  $\gamma_L$  under autarky, with effects on factor allocation and factor prices as described above. Trade will lead to sector downgrading for all factors (i.e., a downward shift in the matching function) and to a pervasive decrease in the relative price of high- $\omega$  factors. The opposite is true in country  $\gamma'$ . Similar to the case of a closed economy, the previous logic can also be used to study the effects of trade integration between countries that differ in terms of diversity, as emphasized by Grossman & Maggi (2000).

We conclude by pointing out that although we have presented the above comparative static results as closed-economy results in an R-R model with a continuum of factors, they can always be interpreted as open-economy results in a Ricardian model with a continuum of countries, as in Matsuyama (1996) and Yanagawa (1996). To do so, one simply needs to define factors in different countries as different factors. Under this interpretation, the previous results can be used, for instance, to shed light on the impact of growth in a subset of countries on patterns of specialization, as captured by the matching function, and the world income distribution, as captured by the schedule of factor prices.

## 4. THEORETICAL EXTENSIONS

The baseline R-R model presented above is special along two dimensions: Good markets are perfectly competitive, and production functions are linear. In this section, we relax these assumptions about market structure and technology and show how to apply the tools and techniques introduced in Section 3 to these alternative environments.

### 4.1. Monopolistic Competition

We first follow Sampson (2014) and introduce monopolistic competition with firm-level heterogeneity à la Melitz (2003) into an otherwise standard R-R model.<sup>10</sup> We focus on a world economy comprising  $n + 1$  symmetric countries and omit for now the vector of country characteristics  $\gamma$ . Goods markets are monopolistically competitive, and preferences are CES over a continuum of symmetric varieties. There is an unbounded pool of potential entrants that are ex ante identical. To enter, a firm incurs a sunk cost,  $f_e > 0$ . Entry costs and all other fixed costs are proportional to the CES price index, which we normalize to 1. Upon entry, a firm randomly draws a blueprint with characteristic  $\sigma$  from a distribution  $G$ . If the firm incurs an additional fixed cost  $f > 0$ , it can produce a differentiated variety for the domestic market using the same linear production function as in Section 3.1,

$$q(\sigma) = \int_{\Omega} A(\omega, \sigma) l(\omega, \sigma) d\omega,$$

where  $A(\omega, \sigma)$  denotes the productivity of the firm if it were to hire  $l(\omega, \sigma)$  units of factor  $\omega \in [\underline{\omega}, \bar{\omega}]$ . We further assume that  $A(\omega, \sigma)$  is strictly increasing in  $\sigma$  so that  $\sigma$  is an index of firm-level productivity. The production function in Melitz (2003) corresponds to the special case in which there is only one factor of production and  $A(\omega, \sigma) \equiv \sigma$ . Finally, in order to export, a firm must incur a fixed cost  $f_x \geq 0$  per market and a per unit iceberg trade cost  $\tau \geq 1$ .

<sup>10</sup>Other recent papers introducing monopolistic competition into an R-R model include Edwards & Perroni (2014), Gaubert (2014), and Grossman & Helpman (2014).



Similar to Section 3.2, consumers maximize their utility, firms maximize their profits, and markets clear. The key difference is that firms have market power. Thus, profit maximization now requires marginal cost to be equal to marginal revenue rather than price,

$$\frac{dr(q, \sigma)}{dq} = \frac{w(\omega)}{A(\omega, \sigma)},$$

$$\frac{dr_x(q_x, \sigma)}{dq_x} = \frac{\tau w(\omega)}{A(\omega, \sigma)},$$

where  $r(q, \sigma)$  and  $r_x(q_x, \sigma)$  denote a firm's revenue if it sells  $q > 0$  and  $q_x > 0$  units in the domestic and foreign markets, respectively. In contrast, the cost-minimization problem of the firm is unchanged. Given the linearity of the production function, the conditions in Equations 5 and 6 must still hold. Under Assumption 1, this immediately implies that we must have PAM in this alternative environment: High- $\omega$  factors will be employed in high- $\sigma$  firms. Because high- $\sigma$  firms will also be larger in terms of sales and are more likely to be exporters, as in Melitz (2003), R-R models with monopolistic competition therefore provide simple microfoundations for the well-documented firm-size and exporter wage premia.<sup>11</sup>

As discussed in Section 3.3, because Equation 5 still holds, we must also have  $w(\omega) = \max_{\sigma \in \Sigma} \{A(\omega, \sigma)c(\sigma)\}$ . By the envelope theorem, this implies

$$\frac{d \ln w(\omega)}{d\omega} = \frac{\partial \ln A(\omega, M(\omega))}{\partial \omega},$$

exactly as in the baseline R-R model. Combining the goods and factor market clearing conditions, which are unchanged, one can then use the same strategy as in Section 3.4 to show that

$$\frac{dM(\omega)}{d\omega} = \frac{A(\omega, M(\omega))L^w(\omega)}{D^w(p, E^w|M(\omega))},$$

where  $L^w(\omega)$  denotes the world endowment of factor  $\omega$ ;  $E^w$  denotes world expenditure, which includes spending by both consumers and firms; and  $D^w(p, E^w|\sigma)$  denotes world absorption for  $\sigma$  varieties. In short, the two key differential equations characterizing factor prices and the matching function remain unchanged under monopolistic competition.

Of course, one should not infer from the previous observation that monopolistically competitive models do not have new implications. In the present environment, world absorption,  $D^w(p, E^w|\sigma)$ , depends both on the level of variable trade costs,  $\tau$ , and on the fixed costs,  $f_e$ ,  $f$ , and  $f_x$ , which determine the entry and exit decisions of firms across markets. This opens up new and interesting channels through which trade integration—modeled as a change in  $\tau$ ,  $f_x$ , or  $n$ —may affect the distribution of earnings.

Let  $\underline{\sigma}$  denote the productivity cutoff above which firms choose to produce, and  $\underline{\sigma}_x$  denote the productivity cutoff above which they choose to export. Under the assumption that preferences are CES, one can then express world demand for  $\sigma$  varieties as

<sup>11</sup>Yeaple (2005) provides an early example of a monopolistically competitive model with firm and worker heterogeneity in which PAM arises under Assumption 1. Alternative microfoundations for the firm-size and exporter wage premia based on extensions of Melitz (2003) with imperfectly competitive labor markets can be found, for example, in Davidson et al. (2008), Helpman et al. (2010), and Egger & Kreckemeier (2012).

$$D^w(p, E^w | \sigma) = \frac{B^w(\sigma, \gamma_D^w) p^{-\varepsilon}(\sigma) g(\sigma) E^w}{\int_{\Sigma} B^w(\sigma', \gamma_D^w) p^{1-\varepsilon}(\sigma') g(\sigma') d\sigma'}$$

where world demand characteristics,  $\gamma_D^w$ , and the demand shifter for  $\sigma$  varieties,  $B^w(\sigma, \gamma_D^w)$ , are such that

$$\gamma_D^w \equiv (\tau, n, \underline{\sigma}, \underline{\sigma}_x),$$

$$B^w(\sigma, \gamma_D^w) \equiv \left(1 + n\tau^{1-\varepsilon} \mathbb{I}_{[\underline{\sigma}_x, \infty)}(\sigma)\right) \mathbb{I}_{[\underline{\sigma}, \infty)}(\sigma).$$

In the previous expression,  $\mathbb{I}_{[\underline{\sigma}, \infty)}(\cdot)$  and  $\mathbb{I}_{[\underline{\sigma}_x, \infty)}(\cdot)$  are indicator functions that capture the selection of different firms into domestic production and export, respectively. Although the cutoffs  $\underline{\sigma}$  and  $\underline{\sigma}_x$  are themselves endogenous objects that depend on fixed and variable trade costs through standard zero-profit conditions, it will be convenient to study how trade integration shapes inequality in two steps: (a) treat the demand shifters,  $B^w$ , as functions of  $\sigma$  and  $\gamma_D^w \equiv (\tau, n, \underline{\sigma}, \underline{\sigma}_x)$ , and (b) analyze how  $\underline{\sigma}$  and  $\underline{\sigma}_x$  vary with  $\tau, n, f$ , and  $f_x$ .

To apply the results of Section 3.4.2 in this environment, one only needs to check that  $B^w$  is log-submodular in  $(\sigma, \tau)$  and log-supermodular in  $(\sigma, n)$ ,  $(\sigma, \underline{\sigma})$ , and  $(\sigma, \underline{\sigma}_x)$ , which is a matter of simple algebra. From our previous analysis, we then obtain that ceteris paribus, a decrease in trade costs,  $\tau$ , or an increase in the number of countries,  $n$ , and the productivity cutoffs,  $\underline{\sigma}$  and  $\underline{\sigma}_x$ , should lead to an upward shift in the matching function and a pervasive increase in the relative price of high- $\omega$  factors around the world.

This is an important difference between the models in Sections 3 and 4.1. Whereas trade integration in Section 3.4 leads to opposite effects at home and abroad—because endowments and demand in the integrated economy lie in between the endowments and demand in the two countries—selection effects à la Melitz (2003) imply that trade integration—modeled as a reduction in trade costs or an increase in the number of countries—has the same effects on the distribution of earnings around the world. Intuitively, if trade integration increases the relative demand for high- $\sigma$  firms everywhere, it must also increase the relative price of the high- $\omega$  factors that are employed in these firms.

As alluded to above, the total effect of a change in variable trade costs or the number of countries is more subtle. In addition to their direct effects, they also indirectly affect entry and exit decisions, which is reflected in changes in  $\underline{\sigma}$  and  $\underline{\sigma}_x$ . This last effect tends to work in the opposite direction: When variable trade costs fall, this lowers the export cutoff,  $\underline{\sigma}_x$ , which then increases the relative demand of firms below that cutoff. Sampson (2014) analyzes these countervailing forces and provides further extensions, including endogenous technology adoption as in Yeaple (2005).

## 4.2. Vertical Specialization

Up to this point, we have focused on the implications of R-R models for trade in goods. Although these goods may be intermediate goods or tasks, the previous analysis abstracts from global supply chains in which countries specialize in different stages of a good’s production sequence, a phenomenon that Hummels et al. (2001) refer to as vertical specialization. Building on earlier work by Dixit & Grossman (1982) and Sanyal (1983), Costinot et al. (2012b, 2013) develop variants of R-R models with sequential production to study how vertical specialization shapes



inequality and the interdependence of nations. We now briefly describe their framework and summarize their results.<sup>12</sup>

There is a unique final good whose production requires a continuum of stages  $\sigma \in [\underline{\sigma}, \bar{\sigma}]$ . At the end of each stage  $\sigma$ , firms can use factors of production and the input from that stage to perform the next stage,  $\sigma + \delta\sigma$ . If firms from country  $\gamma$  combine  $Q(\omega, \sigma, \gamma)$  units of intermediate good  $\sigma$  with  $L(\omega, \sigma, \gamma)$  units of factor  $\omega$  for all  $\omega \in \Omega$ , then the total output of intermediate good  $\sigma + \delta\sigma$  in country  $\gamma$  is equal to

$$Q(\sigma + \delta\sigma, \gamma) = \int_{\Omega} A(\omega, \gamma_A) \min\{Q(\omega, \sigma, \gamma), L(\omega, \sigma, \gamma)/\delta\sigma\} d\omega, \quad (23)$$

where total factor productivity,  $A(\omega, \gamma_A)$ , is such that

$$A(\omega, \gamma_A) \equiv \exp(-\lambda(\omega, \gamma_A)/\delta\sigma). \quad (24)$$

$\lambda(\omega, \gamma_A)$  can be interpreted as the constant Poisson rate at which mistakes occur along a given supply chain, as in Sobel (1992) and Kremer (1993). At any given stage, the likelihood of such mistakes may depend on the quality of workers and machines, indexed by  $\omega$ , as well as the quality of infrastructure and institutions in a country, indexed by  $\gamma_A$ , but is assumed to be constant across stages.

When  $\lambda(\omega, \gamma_A)$  is strictly decreasing in  $\omega$  and  $\gamma_A$ , so that high- $\omega$  factors and high- $\gamma_A$  countries have an absolute advantage in all stages, there must be vertical specialization in any free trade equilibrium with more productive factors or countries specializing in later stages of production. Mathematically, PAM arises for the same reason as it did in earlier sections. By Equations 23 and 24, the cumulative amount of factor  $\omega$  necessary to produce all stages from  $\underline{\sigma}$  to  $\sigma$  in country  $\gamma$  is equal to  $\exp((\sigma - \underline{\sigma})\lambda(\omega, \gamma_A))$ , which is log-submodular in both  $(\omega, \sigma)$  and  $(\sigma, \gamma_A)$ . Because of the sequential nature of production, absolute productivity differences are a source of comparative advantage.<sup>13</sup>

Once PAM has been established, competitive equilibria can still be described as a system of differential equations that jointly characterize the schedule of factor prices and the matching function. In a Ricardian version of this model, with only one factor of production per country, Costinot et al. (2013) use this system to contrast the effects of technological change in countries located at the bottom and the top of a supply chain. In a two-country version of this model with a continuum of factors, Costinot et al. (2012b) use a similar approach to analyze the consequences of trade integration between countries with different factor endowments. Although the effects of trade integration on the matching function are the same as in Section 3.4, sequential production leads to new and richer predictions about the effects of trade on inequality. Namely, standard Stolper-Samuelson forces operate at the bottom of the chain, but the opposite is true at the top.

### 4.3. Other Extensions

In the baseline R-R model, as well as the previous extensions, factors of production are characterized by their exogenous productivity in various economic activities. In practice, productivity

<sup>12</sup>Yi (2003, 2010) and Johnson & Moxnes (2013) offer examples of quantitative work using Ricardian models with sequential production. The implications of contractual imperfections in such environments are explored by Antràs & Chor (2013).

<sup>13</sup>Costinot et al. (2013) also study cases in which the rate of mistakes, and hence factor productivity, varies across stages. If the stage-varying Poisson rate,  $\lambda(\omega, \sigma, \gamma_A)$ , is submodular in  $(\omega, \sigma)$  and  $(\sigma, \gamma_A)$ , then factor productivity is log-supermodular in these variables and PAM still applies.

may be neither exogenous nor the only source of heterogeneity among factors. The marginal product of labor may vary with the stock of capital; workers may have different preferences over working conditions; and workers may vary in terms of how costly it is for them to acquire skills. Fortunately, such considerations can all be incorporated into an R-R model.

As shown in the **Supplemental Appendix** (follow the Supplemental Material link from the Annual Reviews home page at <http://www.annualreviews.org>), the tools and techniques of Section 3 can be used to derive similar cross-sectional and comparative static predictions in economies with the following:

1. Factor complementarity, if output of good  $\sigma$  in country  $\gamma$  is given by

$$Q(\sigma, \gamma) = F[K^{\text{agg}}(\sigma, \gamma), L^{\text{agg}}(\sigma, \gamma)|\sigma, \gamma],$$

where  $F(\cdot, \cdot|\sigma, \gamma)$  is a constant returns to scale production function;  $K^{\text{agg}}(\sigma, \gamma)$  and  $L^{\text{agg}}(\sigma, \gamma)$  denote the aggregate amounts of capital and labor, respectively, with

$$L^{\text{agg}}(\sigma, \gamma) = \int_{\Omega} A(\omega, \sigma, \gamma_A)L(\omega, \sigma, \gamma)d\omega.$$

2. Heterogeneous preferences, if the utility of a worker with characteristic  $\omega$  receiving a wage  $w^c(\sigma, \gamma)$  in sector  $\sigma$  and country  $\gamma$  is given by

$$V(\omega, \sigma, \gamma) \equiv w^c(\sigma, \gamma)U(\omega, \sigma, \gamma_U),$$

where  $\gamma_U$  is a new exogenous preference shifter, and  $U(\omega, \sigma, \gamma_U)$  is strictly log-supermodular in  $(\omega, \sigma)$  and  $(\sigma, \gamma_U)$ .<sup>14</sup>

3. Endogenous skills, if firms from country  $\gamma$  need to pay  $S(\omega, \sigma, \gamma_S) > 0$  to train a worker of type  $\omega$  in sector  $\sigma$ —with learning costs proportional to the consumer price index—and  $S(\omega, \sigma, \gamma_S)$  is strictly submodular in  $(\omega, \sigma)$  and  $(\sigma, \gamma_S)$ .<sup>15</sup>

## 5. PARAMETRIC APPLICATIONS

The two previous sections derive a number of sharp cross-sectional and comparative static predictions, especially in economies with a continuum of goods and factors. In the data, however, one always observes a discrete number of factors and sectors. Furthermore, PAM never perfectly holds for these observed groups of factors and sectors; all factors are likely to be employed in all sectors, albeit with different intensity.

One way to bridge the gap between theory and data is to maintain the assumption that there is a continuum of factors or goods that are perfectly observed by consumers and firms, but add the assumption that the econometrician observes only coarser measures of these characteristics. Under these assumptions, there may therefore be unobserved heterogeneity, from the point of view of the econometrician, within a given group of factors or goods.

<sup>14</sup>When thinking about heterogeneity in preferences, a natural interpretation of  $\sigma$  is location rather than industry. In practice, different individuals may choose to live in different cities because they value their various amenities differently. With this interpretation in mind, R-R models also provide a useful framework to analyze the relationship between migration and trade, both within and between countries.

<sup>15</sup>This is the approach followed by Blanchard & Willmann (2013).



Several papers in the trade literature follow the previous approach. The empirical content of such papers then crucially depends on the distributional assumptions imposed on unobserved heterogeneity across goods or factors. By far the most common assumption in the existing literature is to assume GEV distributions of productivity, as in the influential work of Eaton & Kortum (2002).<sup>16</sup> Sections 5.1 and 5.2 discuss the implications of GEV distributions of productivity shocks across goods and factors, respectively.

### 5.1. Unobserved Productivity Shocks Across Goods

Consider first an R-R model with a discrete number of factors and countries and a continuum of goods. For notational convenience, true characteristics, which are perfectly observed by all market participants, are now indexed by  $(\omega^*, \sigma^*, \gamma^*)$ , whereas characteristics observed by the econometrician are indexed by  $(\omega, \sigma, \gamma)$ . We maintain this convention throughout this section. Here, factor and country characteristics are perfectly observed by the econometrician,  $\omega^* = \omega$  and  $\gamma^* = \gamma$ , but good characteristics are not,  $\sigma^* \neq \sigma$ . Specifically, for each observed value of  $\sigma$ , we assume that there exists a continuum of goods  $\sigma^* \in [0, 1]$  with the same observable characteristic and refer to this measure one of goods as an industry. In practice, the pair of observables  $(\omega, \gamma)$  may refer to “worker with a college degree from the United States,” whereas the observable  $\sigma$  may measure “the share of the total wage bill associated with workers with a college degree” in a given industry. In this case, unobserved heterogeneity across goods may reflect that goods with a different unobservable characteristic,  $\sigma^*$ , but an identical observable characteristic,  $\sigma$ , may employ different types of workers  $\omega$  in a competitive equilibrium.

Factor productivity,  $A(\omega^*, \sigma^*, \gamma_A^*)$ , is independently drawn across all factors, goods, and countries from a Fréchet distribution. Given observables  $(\omega, \sigma, \gamma)$ , we assume that

$$\Pr\{A(\omega, \sigma^*, \gamma_A) \leq a | \sigma\} = \exp\left(-[a/T(\omega, \sigma, \gamma_A)]^{-\theta(\sigma)}\right), \quad (25)$$

where  $T(\omega, \sigma, \gamma_A) \geq 0$  and  $\theta(\sigma) > \varepsilon(\sigma) - 1$ , with  $\varepsilon(\sigma)$  the elasticity of demand (introduced below). The first parameter,  $T(\omega, \sigma, \gamma_A)$ , is a locational shifter that can be thought of as the fundamental productivity of a given factor and country in an industry. The second parameter,  $\theta(\sigma) > 1$ , is a shape parameter that captures the extent of intra-industry heterogeneity and is assumed to be constant across factors and countries. The Ricardian model developed by Eaton & Kortum (2002) corresponds to the special case in which there is only one factor (i.e., a unique  $\omega$ ) and one industry (i.e., a unique  $\sigma$ ). Multi-industry extensions considered by, for example, Levchenko & Zhang (2011), Costinot et al. (2012a), and Caliendo & Parro (2015) correspond to cases in which  $\sigma$  can take multiple values. Here we further allow for multiple factors within each country.

In line with the existing literature, we assume a two-tier utility function in which the upper level is Cobb-Douglas—across industries with observables  $\sigma$ —and the lower level is CES—across goods  $\sigma^*$  within the same industry. Specifically, the demand function for a good  $\sigma^*$  with observable characteristic  $\sigma$  is given by

<sup>16</sup>This distributional assumption is standard in the analysis of discrete choice models in industrial organization (see, e.g., McFadden 1973, Berry 1994), as well as in the matching literature (see, e.g., Choo & Siow 2006). Following the seminal work of Roy (1951), numerous papers in the labor literature have focused instead on environments in which the distribution of worker skills is log-normally distributed (see, e.g., Heckman & Sedlacek 1985). In the international trade literature, Ohnsorge & Trefler (2007) also impose log-normality. Liu & Trefler (2011) propose an alternative empirical approach based on linearized versions of the Roy model’s estimating equations.

$$D(p, I(\boldsymbol{\gamma})|\sigma^*, \gamma_D) = \frac{B(\sigma, \gamma_D)p^{-\varepsilon(\sigma)}(\sigma^*)I(\boldsymbol{\gamma})}{P^{1-\varepsilon(\sigma)}(\sigma)}, \quad (26)$$

where  $P(\sigma) \equiv \left( \int_0^1 p^{1-\varepsilon(\sigma)}(\sigma^*) d\sigma^* \right)^{1/(1-\varepsilon(\sigma))}$  denotes the industry-level price index, and  $B(\sigma, \gamma_D) \in [0, 1]$  now refers to the exogenous share of expenditure on all goods with observable characteristic  $\sigma$ . The rest of the assumptions are the same as in Section 3.

Because none of the equilibrium conditions derived in Section 3.2 depends on either the number of goods, factors, and countries or the distribution of productivity across goods, factors, and countries, a competitive equilibrium remains characterized by the same system of equations. Given the specific distributional assumptions imposed by Equation 25, however, one can further show that for an industry with observable characteristic  $\sigma$ , the probability that factor  $\omega$  in country  $\boldsymbol{\gamma}$  offers the minimum cost of producing a good  $\sigma^*$  is given by

$$\pi(\omega, \boldsymbol{\gamma}|\sigma) = \frac{[w(\omega, \boldsymbol{\gamma})/T(\omega, \sigma, \gamma_A)]^{-\theta(\sigma)}}{\sum_{\boldsymbol{\gamma}'} \sum_{\omega'} [w(\omega', \boldsymbol{\gamma}')/T(\omega', \sigma, \gamma'_A)]^{-\theta(\sigma)}}. \quad (27)$$

As is well known, Equation 25 also implies that, conditional on offering the minimum cost of producing a good, the distribution of unit costs across goods  $\sigma^*$  is the same for all factors and countries. Thus,  $\pi(\omega, \boldsymbol{\gamma}|\sigma)$  is also equal to the share of expenditure on goods with observable characteristics  $\sigma$  that are produced using factor  $\omega$  in country  $\boldsymbol{\gamma}$ .<sup>17</sup>

Using the previous observation and Equation 26, which implies that a share  $B(\sigma, \gamma_D)$  of total expenditure in country  $\boldsymbol{\gamma}$  is allocated to industry  $\sigma$ , one can rearrange the factor market clearing condition (Equation 8) as

$$w(\omega, \boldsymbol{\gamma})L(\omega, \gamma_L) = \sum_{\boldsymbol{\gamma}'} \sum_{\sigma} \pi(\omega, \boldsymbol{\gamma}|\sigma)B(\sigma, \gamma'_D)I(\boldsymbol{\gamma}'), \text{ for all } \omega, \boldsymbol{\gamma}. \quad (28)$$

Equations 27 and 28 uniquely pin down the schedule of factor prices  $w(\omega, \boldsymbol{\gamma})$  up to a choice of numéraire. To go from good prices to factor prices, one can then use the fact that the lower-level utility is CES. Under this assumption, Equation 25 implies that the CES price index associated with goods with observable characteristic  $\sigma$  is given by

$$P(\sigma) = \chi(\sigma) \times \left( \sum_{\boldsymbol{\gamma}} \sum_{\omega} [w(\omega, \boldsymbol{\gamma})/T(\omega, \sigma, \gamma_A)]^{-\theta(\sigma)} \right)^{-1/\theta(\sigma)}, \quad (29)$$

with  $\chi(\sigma) \equiv \left( \Gamma \left( \frac{\theta(\sigma) + 1 - \varepsilon(\sigma)}{\theta(\sigma)} \right) \right)^{1/(1-\varepsilon(\sigma))}$ .<sup>18</sup> Knowledge of factor prices,  $w(\omega, \boldsymbol{\gamma})$ , and price indices,  $P(\sigma)$ , is then sufficient to conduct welfare analysis.

<sup>17</sup>In line with the analysis of Section 3, we assume that all goods are freely traded. Therefore, shares of expenditures are constant across all importing countries, as can be seen from Equation 27. As shown by Eaton & Kortum (2002) and mentioned in Section 3.1, it is easy to introduce iceberg trade costs in such an environment. It is equally easy to introduce them in the R-R models considered in Section 5.2.

<sup>18</sup> $\Gamma(\cdot)$  denotes the gamma function, i.e.,  $\Gamma(t) = \int_0^\infty u^{t-1} \exp(-u) du$ .



Compared to Section 3.3, Equation 27 implies a weaker form of PAM in the cross section. Consider the following stochastic version of Assumption 1.

**Assumption 1 (Fréchet):**  $T(\omega, \sigma, \gamma_A)$  is strictly log-supermodular in  $(\omega, \sigma)$  and  $(\sigma, \gamma_A)$ .

This new version of Assumption 1 also captures the idea that high- $\omega$  factors and high- $\gamma_A$  countries are relatively more productive in high- $\sigma$  industries, but it does not require this to be true for all goods  $\sigma^*$  within an industry. Starting from Equation 27, one can check that if this version of Assumption 1 holds and  $\theta(\sigma)$  does not vary across industries,  $\theta(\sigma) \equiv \theta$ , then  $\pi(\omega, \gamma|\sigma)$  is also strictly log-supermodular in  $(\omega, \sigma)$  and  $(\sigma, \gamma_A)$ . This implies that high- $\omega$  factors and high- $\gamma_A$  countries should tend to sell relatively more in high- $\sigma$  industries. Because of Equation 25 and the assumption of a continuum of goods with the same observable characteristic  $\sigma$ , all factors in all countries will now be used in all industries. But for a given factor,  $\omega$ , if one compares the distribution of sales across industries of two countries such that  $\gamma'_A \geq \gamma_A$ , then  $\pi(\omega, \gamma'|\cdot)$  must dominate  $\pi(\omega, \gamma|\cdot)$  in terms of the monotone likelihood ratio property. This is the idea behind the revealed measure of comparative advantage developed in Costinot et al. (2012a).

Equation 27 also creates a tight connection between good prices and factor prices. Combining Equations 27 and 29 yields

$$w(\omega, \gamma) = P(\sigma)T(\omega, \sigma, \gamma_A) \left[ \chi(\sigma)^{\theta(\sigma)} \pi(\omega, \gamma|\sigma) \right]^{-1/\theta(\sigma)}, \text{ for all } \sigma. \quad (30)$$

This is akin to Equation 12 with two important differences. First,  $P(\sigma)$  corresponds to the CES price index of all goods with characteristics  $\sigma$ . Second, the term  $[\chi(\sigma)^{\theta(\sigma)} \pi(\omega, \gamma|\sigma)]^{-1/\theta(\sigma)}$  adjusts for the effect of self-selection of factors from different countries across industries, which creates a wedge between fundamental productivity,  $T(\omega, \sigma, \gamma_A)$ , and average productivity.

Last but not least, starting from Equations 27 and 28, one can use the exact hat algebra, as in Dekle et al. (2008) and other quantitative trade models discussed in Costinot & Rodríguez-Clare (2015), to conduct comparative static and welfare analysis. Although few analytical results are available in this environment, counterfactual simulations can be performed using only estimates of  $\theta(\sigma)$  that can be obtained from the value of trade elasticities in a gravity equation. In the next subsection, we discuss how richer analytical results can be obtained when one goes from unobserved heterogeneity across goods to unobserved heterogeneity across factors.

## 5.2. Unobserved Productivity Shocks Across Factors

Consider now the polar case of an R-R model with a discrete number of sectors and countries and a continuum of factors. Compared to Section 5.1, sector and country characteristics are perfectly observed by the econometrician,  $\sigma^* = \sigma$  and  $\gamma^* = \gamma$ , but factor characteristics are not,  $\omega^* \neq \omega$ . In line with the analysis of the previous subsection, for each value of  $\omega$ , we assume that there exists a continuum of factors  $\omega^* \in [0, 1]$  with the same observable characteristic. In this environment, factors with a different unobservable characteristic,  $\omega^*$ , but an identical observable characteristic,  $\omega$ , may therefore be allocated to sectors with different characteristics  $\sigma$  in a competitive equilibrium.

The distributional assumption imposed on factor productivity,  $A(\omega^*, \sigma^*, \gamma_A^*)$ , is similar to the one imposed in Section 5.1. Factor productivity remains independently drawn across all factors, goods, and countries from a Fréchet distribution, but given observables  $(\omega, \sigma, \gamma)$ , we now assume that

$$\Pr\{A(\omega^*, \sigma, \gamma_A) \leq a|\omega\} = \exp\left(-[a/T(\omega, \sigma, \gamma_A)]^{-\theta(\omega, \gamma_A)}\right), \quad (31)$$

with  $T(\omega, \sigma, \gamma_A) \geq 0$  and  $\theta(\omega, \gamma_A) > 1$ . Besides the fact that unobserved heterogeneity now derives from  $\omega^*$  rather than  $\sigma^*$ , the only difference between Equations 25 and 31 is that the shape parameter  $\theta$  is now specific to a group of factors within a country. Fréchet distributions of productivity shocks across factors have been imposed in the recent closed-economy models of Lagakos & Waugh (2013) and Hsieh et al. (2013), as well as the open-economy models of Burstein et al. (2014), Costinot et al. (2015a), and Fajgelbaum & Redding (2014).

In line with earlier sections, we restrict ourselves to CES demand functions. Here again, a competitive equilibrium is characterized by the same system of equations as in Section 3.2. Similarly, for a factor with observable characteristic  $\omega$  in country  $\gamma$ , the probability that the value of the marginal product is maximized in sector  $\sigma$  is given by

$$\pi(\sigma|\omega, \gamma) = \frac{[p(\sigma)T(\omega, \sigma, \gamma_A)]^{\theta(\omega, \gamma_A)}}{\sum_{\sigma'} [p(\sigma')T(\omega, \sigma', \gamma_A)]^{\theta(\omega, \gamma_A)}}. \quad (32)$$

Because of Equation 31, the distribution of prices for factors with observable characteristic  $\omega$  in country  $\gamma$  is now the same across all sectors  $\sigma$ . Hence,  $\pi(\sigma|\omega, \gamma)$  is also the share of income of factors from country  $\gamma$  with observable characteristic  $\omega$  that is earned in sector  $\sigma$ . Together with the assumption of CES demand functions, this implies that the goods market clearing condition (Equation 9) can be rearranged as

$$\sum_{\gamma} B(\sigma, \gamma_D)(p(\sigma))^{1-\varepsilon} I(\gamma) = \sum_{\gamma'} \sum_{\omega} \bar{w}(\omega, \gamma') \pi(\sigma|\omega, \gamma') L(\omega, \gamma'), \text{ for all } \sigma, \quad (33)$$

where  $\bar{w}(\omega, \gamma)$  denotes the average return across all factors  $\omega^*$  from country  $\gamma$  with observable characteristic  $\omega$ , which is constant across all sectors, and the CES price index has been normalized to 1. Finally, Equation 31 implies that  $\bar{w}(\omega, \gamma)$  satisfies

$$\bar{w}(\omega, \gamma) = \chi(\omega, \gamma_A) \times \left( \sum_{\sigma} [p(\sigma)T(\omega, \sigma, \gamma_A)]^{\theta(\omega, \gamma_A)} \right)^{1/\theta(\omega, \gamma_A)}, \quad (34)$$

where  $\chi(\omega, \gamma_A) \equiv \Gamma\left(\frac{\theta(\omega, \gamma_A)-1}{\theta(\omega, \gamma_A)}\right)$ . Equations 32–34 fully pin down good prices,  $p(\sigma)$ , and average factor prices,  $\bar{w}(\omega, \gamma)$ , up to a normalization.

Before studying the differences between the models of Sections 5.1 and 5.2, it is useful to understand their similarities. Like Equation 27 in Section 5.1, Equation 32 provides a simple relationship between fundamental productivities,  $T$ 's, shape parameters,  $\theta$ 's, and factor allocations,  $\pi$ 's.<sup>19</sup> Under the Fréchet version of Assumption 1 and the restriction that  $\theta(\omega, \gamma_A) \equiv \theta$ , Equation 32 also generates a weaker form of PAM:  $\pi(\sigma|\omega, \gamma)$  is strictly log-supermodular in  $(\omega, \sigma)$  and  $(\sigma, \gamma_A)$ . In terms of price predictions, Equations 32 and 34 imply that

<sup>19</sup>Hsieh et al. (2013) and Burstein et al. (2014) use this relationship, estimates of  $\theta$ 's, and data on factor allocation to recover the productivity of different worker groups across occupations as well as the implicit taxes or wedges they may face.





$$\bar{w}(\omega, \gamma) = p(\sigma)T(\omega, \sigma, \gamma_A) \left[ \chi(\omega, \gamma_A)^{-\theta(\omega, \gamma_A)} \pi(\sigma | \omega, \gamma) \right]^{-1/\theta(\omega, \gamma_A)}, \text{ for all } \sigma, \quad (35)$$

which again highlights the importance of self-selection. Furthermore, given the CES-like functional forms in Equations 32–34, one can still use the exact algebra of Dekle et al. (2008) to conduct comparative static and welfare analysis in this environment, an approach implemented by Burstein et al. (2014).<sup>20</sup>

Given these similarities, it would be tempting to view this subsection as a simple relabeling of factors and goods in Section 5.1. It is not. The equilibrium system of equations here (Equations 32–34) cannot be obtained by permutations of  $\sigma$ ,  $\omega$ , and  $\gamma$  in Equations 27–29. To take one example,  $\bar{w}(\omega, \gamma)$  here is an arithmetic average, whereas  $P(\sigma)$  in Section 5.1 is not. Such distinctions are consequential. Whereas we have very few analytical results about the effects of trade integration or other shocks in the environment considered in Section 5.1, the **Supplemental Appendix** establishes that the FPE theorem, the Rybczynski theorem, the Stolper-Samuelson theorem, as well as the comparative static results in Section 3.4, all generalize to the present environment.

Similar results can be derived in an environment with unobserved preference shocks, as in Artuc et al. (2010), Dix-Carneiro (2014), and Redding (2014). Such extensions are important as they provide a natural framework to pursue quantitative work on the consequences of international trade in economies with imperfect factor mobility across sectors and locations. We also discuss them in detail in the **Supplemental Appendix**.

## 6. NONPARAMETRIC APPLICATIONS

Ideally, when applying R-R models to the analysis of counterfactual questions, one would like to have information about the productivity of all factors in all sectors and countries. Unfortunately, such detailed information is rarely available. Typically, researchers may be able to observe factual productivity, that is, the productivity of factors in the sectors in which they are currently employed, but not their counterfactual productivity, that is, their productivity in the sectors to which they would be reallocated in response to a counterfactual shock.

In Section 5, we discuss parametric applications of R-R models that deal with the previous issue by assuming that unobserved factor productivity is drawn from Fréchet distributions. We now turn to alternative applications of R-R models that aim to dispense as much as possible with functional-form assumptions. The basic idea behind these applications is to focus on agriculture, a sector of the economy in which the scientific knowledge of how essential inputs such as water, soil, and climatic conditions map into outputs is uniquely well understood. As a result, agronomists are able to predict with great accuracy how productive various parcels of land would be were they to be used to grow any one set of crops. In short, both factual and counterfactual productivities are known in this particular context.

All applications in this section are based on detailed microlevel data from the FAO's GAEZ project. This data set records potential crop yields for millions of 5-arcminute grid cells covering Earth's surface (see Nunn & Qian 2011 and Costinot & Donaldson 2014 for additional information). In Section 6.1, we first illustrate how the GAEZ data can be used to test the cross-sectional implications of Ricardo's theory of comparative advantage. In Sections 6.2 and 6.3, we then discuss how the same agronomic data can be used to measure the historical gains from

<sup>20</sup>Galle et al. (2014) follow a similar strategy in a quantitative trade model that combines both unobserved good and factor heterogeneity drawn from Fréchet distributions.

economic integration as well as to quantify the consequences of climate change in agricultural markets, respectively.

### 6.1. Testing Ricardo’s Theory of Comparative Advantage

Ricardo’s theory of comparative advantage, upon which R-R models build, is one of the oldest and most distinguished theories in economics. However, it is a difficult theory to confront to the data. As already noted by Deardorff (1984, p. 476),

Problems arise, however, most having to do with the observability of [productivity by industry and country]. The . . . problem is implicit in the Ricardian model itself . . . [because] the model implies complete specialization in equilibrium. . . This in turn means that the differences in labor requirements cannot be observed, since imported goods will almost never be produced in the importing country.

Accordingly, empirical work on the cross-sectional implications of the Ricardian model has proceeded by imposing ad hoc specifications, as in MacDougall (1951), Stern (1962), Balassa (1963), and Golub & Hsieh (2000), or strong distributional assumptions on the extent of unobserved labor productivity, as in Costinot et al. (2012a).

Costinot & Donaldson (2012) propose an alternative empirical strategy that relies instead on productivity and price data from the FAO for 17 major agricultural crops and 55 major agricultural countries. As discussed above, the GAEZ project reports total production capacity in tons per hectare for each 5-arcminute grid cell within these 55 countries. Costinot & Donaldson (2012) interpret each grid cell as a distinct factor  $\omega$  and total production capacity and the area of each grid cell as the empirical counterparts of  $A(\omega, \sigma, \gamma_A)$  and  $L(\omega, \gamma_L)$ , respectively. Under the assumption that  $\Sigma(\omega, \gamma)$  is a singleton,<sup>21</sup> the aggregate output of crop  $\sigma$  in country  $\gamma$  is given as

$$Q(\sigma, \gamma) = \int_{\Omega(\sigma, \gamma)} A(\omega, \sigma, \gamma_A) L(\omega, \gamma_L) d\omega, \tag{36}$$

where  $\Omega(\sigma, \gamma)$  is the set of factors allocated to crop  $\sigma$  in country  $\gamma$ ,

$$\Omega(\sigma, \gamma) = \left\{ \omega \in \Omega \mid \frac{A(\omega, \sigma, \gamma_A)}{A(\omega, \sigma', \gamma_A)} > \frac{p(\sigma', \gamma)}{p(\sigma, \gamma)} \text{ for all } \sigma' \neq \sigma \right\}. \tag{37}$$

In the previous expression,  $p(\sigma, \gamma)$  denotes the local price of crop  $\sigma$  in country  $\gamma$ , which is allowed to vary across countries because of transportation costs or taxes and subsidies.

Combining Equations 36 and 37 with productivity and price data, Costinot & Donaldson (2012) then compute predicted output levels for all crops and countries and compare them to those observed in the data. Their empirical results show that the output levels predicted by Ricardo’s theory of comparative advantage agree reasonably well with actual data on worldwide agricultural production.

<sup>21</sup>This assumption is satisfied for all but two grid cells in Brazil.



## 6.2. Measuring the Gains from Economic Integration

In Section 6.1, we show how to use data on agricultural output, productivity, and prices to test the cross-sectional predictions of R-R models. Alternatively, in the absence of price data, one could use the cross-sectional predictions of R-R models, together with agricultural output and productivity data, to back out model-consistent prices. This is the approach followed by Costinot & Donaldson (2014) in the context of US agricultural markets from 1880 to 1997.

Combining agronomic data from the GAEZ project with historical data from the US Census, the authors first demonstrate how to identify the spatial distribution of crop prices across US counties over time, while still allowing for crop-county-year-specific productivity shocks. By comparing local crop prices, estimated from the model, and prices in major wholesale markets, observed in historical data, the authors then estimate trade costs for each US county from 1880 to 1997. Finally, armed with the previous estimates, they compute the aggregate productivity gains due to changes in trade costs over that time period, which they refer to as the gains from economic integration.

In terms of the empirical validity of R-R models, Costinot & Donaldson (2014) find a robust, positive correlation between county-level prices inferred from the model and historical state-level prices, which are the most disaggregated price data available over the long time period. Because the estimation of local prices in Costinot & Donaldson (2014) does not use any direct information on prices—it relies instead on information about historical output levels and modern productivity—these additional empirical results also suggest that R-R models have significant explanatory power in the data.

## 6.3. Quantifying the Consequences of Climate Change

Another attractive feature of the GAEZ data set is that potential crop yields are available under various climate change scenarios. Hence, using both pre- and postclimate change crop yields as inputs into an R-R model, one can explore the macrolevel consequences of climate change in agricultural markets. This is the starting point of the quantitative analysis in Costinot et al. (2015a).

The authors develop an R-R model that combines detailed microlevel data with otherwise parsimonious parametric assumptions. Compared to the applications reviewed in Sections 6.1 and 6.2, it features observed productivity heterogeneity across grid cells, as reported in the GAEZ data set, but also unobserved productivity heterogeneity within grid cells, which is assumed to be drawn from a Fréchet distribution across a continuum of parcels. Closed-form solutions for factor allocation and aggregate output levels are therefore akin to those presented in Section 5.2, which dramatically simplifies the computation of competitive equilibria in an environment with 10 crops, 50 countries, and 1.7 million grid cells.

Using this trade model, Costinot et al. (2015a) quantify the impact of climate change on agricultural markets. This new type of computational general equilibrium models—based on rich microlevel data and a tight connection between theory and empirics—could be applied to many other outstanding questions in the field. Sotelo (2013) provides an interesting example. By combining the GAEZ data with a theoretical framework similar to the one in Costinot et al. (2015a), he quantifies the contribution of trade frictions to the low agricultural productivity of Peru.

## 7. CONCLUDING REMARKS

In this article, we provide a user guide to multifactor generalizations of the Ricardian model of international trade, which we refer to as R-R models. We present tools and techniques for

theoretical, quantitative, and empirical work in this class of assignment models. We also review a number of applications of these models. We conclude by discussing several open questions and promising avenues for future research.

On the theoretical front, we describe how the baseline R-R model could be extended along a number of dimensions, including the introduction of monopolistic competition and sequential production. R-R models could also accommodate many of the other features that have previously been studied in a Ricardian environment, such as external economies of scale, as in Ethier (1982), Grossman & Rossi-Hansberg (2010), and Matsuyama (2013), or learning by doing, as in Krugman (1987) and Young (1991). R-R models could also be used to study the relationship between trade and labor market frictions, perhaps building on the closed-economy models of Teulings (2003) and Teulings & Gautier (2004), or endogenous technological change, as in the work of Acemoglu (2002). In an R-R model, technological change may be biased toward particular tasks rather than factors, as in Atkinson & Stiglitz (1969), with factors endogenously reallocating between these tasks. It would also be interesting to explore the implications of R-R models for trade policy, from both a positive and a normative perspective. Such models may provide a useful alternative to the traditional specific-factor model. The methodology developed by Costinot et al. (2015b) to characterize optimal trade taxes in a Ricardian model should prove helpful in this more general environment.

On the empirical front, R-R models are particularly well suited to study the distributional consequences of international trade. Building on the important work of Artuc et al. (2010), one could imagine using variations of the parametric models and methods presented in Section 5 to quantify the importance of previous trade liberalization episodes across sectors and regions. These simple static models may offer a useful compromise between reduced-form work, as in Goldberg & Pavcnik (2007), and richer dynamic structural models, as in Dix-Carneiro (2014). As shown in Sampson (2014), R-R models can also accommodate firm-level heterogeneity in imperfectly competitive markets. One could also imagine using parametric versions of such R-R models to help analyze the increasing number of matched employer-employee data sets that have become available.

Of course, future empirical and quantitative work does not have to be limited to parametric applications. It is well known that GEV distributions impose strong restrictions on own-price and cross-price elasticities. Galichon & Salanié (2012) offer an example of how to relax these distributional assumptions in one-to-one matching models. It may be interesting to follow a similar approach when studying the effects of international trade in local labor markets. Finally, agricultural trade policy is one of the most salient issues in today's world economy. R-R models and the GAEZ data described in Section 6 offer a unique opportunity to shed light on this key policy issue. What are the effects on the Common Agricultural Policy on the misallocation of resources in European agricultural markets? How do agricultural trade policies set in developed economies affect households in developing countries? These are central questions about which we know fairly little but that could be addressed using R-R models. In sum, much remains to be done.

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