The Network Origins of Large Economic Downturns*

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Abstract

This paper shows that large economic downturns may result from the propagation of microeconomic shocks over the input-output linkages across different firms or sectors within the economy. Building on the framework of Acemoglu et al. (2012), we argue that the economy's input-output structure can fundamentally reshape the distribution of aggregate output, increasing the likelihood of large downturns from infinitesimal to substantial. More specifically, we show that an economy with non-trivial intersectoral input-output linkages that is subject to thin-tailed productivity shocks may exhibit deep recessions as frequently as economies that are subject to heavy-tailed shocks. Moreover, we show that in the presence of input-output linkages, aggregate volatility is not necessarily a sufficient statistic for the likelihood of large downturns. Rather, depending on the shape of the distribution of the idiosyncratic shocks, different features of the economy's input-output network may be of first-order importance. Finally, our results establish that the effects of the economy's input-output structure and the nature of the idiosyncratic firm-level shocks on aggregate output are not separable, in the sense that the likelihood of large economic downturns is determined by the interplay between the two.

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1 Introduction

Understanding the origins of large economic downturns such as the Great Depression has been one of the central questions in macroeconomics, dating back to the early days of the field.

Two leading explanations for the origins of such "macroeconomic tail risks" are popular. The first maintains that sizable fluctuations in aggregate economic activity are the result of large "exogenous" shocks that impact a wide range of firms, consumers or workers within the economy. The real business cycles literature, pioneered by Kydland and Prescott (1982), provides a prominent example of this approach, taking the origins of all economic fluctuations to be exogenous technology shocks.[1] The more recent versions of this approach also account for large economic downturns and slowdowns, including the Great Depression, by relying on aggregate technology shocks or other stochastic factors that affect the efficiency of the macroeconomy (Cooley and Prescott, 1995). Several other leading accounts of the Great Depression follow Friedman and Schwartz (1963) in emphasizing the role of large monetary shocks as its main underlying cause. Relatedly, the recent literature on "rare disasters", building on Rietz (1988) and pioneered by Barro (2006), models the occurrence of (largely exogenous) economic disasters and studies their implications for asset prices (see also Gabaix (2012)).

The second explanation, instead, seeks to explain large economic downturns as a result of the amplification of small initial shocks due to the presence of capital, labor or product market imperfections. For example, Kiyotaki and Moore (1997) show that dynamic interactions between credit limits and asset prices can turn small, temporary shocks into large, persistent fluctuations in aggregate output.[2] Relatedly, Diamond (1982), Bryant (1983) and Cooper and John (1988), *inter alia*, show how labor or product market interactions can lead to macroeconomic multiplicity of equilibria, creating substantial fragility in the face of small shocks.

In this paper, we provide a third, complementary explanation for the origins of large economic downturns. We argue that large drops in aggregate economic activity may result from the propagation of microeconomic shocks through the input-output linkages across different firms or sectors within the economy. Building on the framework developed by Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), we establish that the presence of intersectoral input-output linkages can fundamentally reshape the distribution of aggregate output, increasing the likelihood of large downturns from infinitesimal to substantial.

As a benchmark, we first study the class of *balanced* economies, in which all sectors take roughly symmetric roles as input suppliers to others. We show that the likelihood of a large downturn in this class of economies decays exponentially fast in the number of sectors, regardless of the distribution of firm-level shocks. Our result thus implies that, absent any other amplification mechanisms or aggregate shocks, the likelihood of a large drop in aggregate output in a balanced economy is

[1] See King and Rebelo (1999) for a survey of this literature and its evolution.

[2] See Bernanke, Gertler, and Gilchrist (1999) for an overview.

infinitesimal.

Our result for balanced economies is essentially an "irrelevance result", showing that large economic downturns are equally unlikely in this class of economies, irrespective of the distribution of the idiosyncratic shocks. Our subsequent results, however, establish that if the input-output structure of the economy is not balanced, the likelihood of a large recession is no longer independent of the nature of the sectoral-level disturbances and may be substantial. In particular, we show that depending on the economy's underlying network structure, the frequency of large GDP contractions is highly sensitive to the shape of the distribution of microeconomic shocks. To characterize this dependence, we analyze the likelihood of such contractions for different shock distributions.

First, we focus on the case that microeconomic shocks are normally distributed and show that the probability of a large downturn is determined by the standard deviation of log GDP which we refer to as "aggregate volatility". In other words, economies that exhibit high levels of aggregate volatility are also more likely to experience deeper and more frequent recessions at the face of normally distributed shocks. Even though intuitive, such an equivalence does not necessarily hold if the shocks are not normally distributed. In particular, we show that when microeconomic shocks are drawn from distributions with exponential tails, aggregate volatility is no longer a sufficient statistic for the likelihood of large economic downturns. Rather, it is the relative importance of the most dominant supplier in the economy which determines the frequency of downturns.

The significance of this observation lies in part in its contrast with a result of Acemoglu et al. (2012), who show that under fairly general conditions, aggregate output is asymptotically normally distributed, irrespective of the distribution of microeconomic shocks. In other words, our results establish that two economies with different shock distributions may experience large recessions with significantly different frequencies, even when aggregate output in both economies has the same volatility and asymptotic distribution. This observation underscores the importance of separately studying the determinants of large economic downturns, as such macroeconomic tail risks may vary significantly even across economies in which aggregate output converges to the same exact asymptotic distribution at identical rates.

The rest of the paper characterizes how the underlying input-output network structure of the economy reshapes the distribution of aggregate output. We show that an economy with non-trivial input-output linkages that is subject to thin-tailed (e.g., exponentially distributed) productivity shocks may exhibit deep recessions as frequently as an economy without linkages that is subject to heavy-tailed, Pareto distributed shocks. In this sense, our results provide a novel solution to what Bernanke, Gertler, and Gilchrist (1996) refer to as the "small shocks, large cycles puzzle" by arguing that the interaction between the underlying network structure of the economy and the shape of the productivity shocks is of first-order importance in determining the nature of aggregate fluctuations.

Our paper belongs to the literature that studies the microeconomic origins of aggregate fluctuations. Gabaix (2011), for instance, argues that firm-level idiosyncratic shocks may translate into fluctuations at the aggregate level if the firm size distribution is sufficiently heavy-tailed (in the sense that the largest firms contribute disproportionally to aggregate output). Nirei (2006), on the other hand, demonstrates that following a threshold rule (such as the (S, s) policy) by the firms may create positive feedback loops that can amplify firm-level disturbances into aggregate effects. Other studies in this literature include Jovanovic (1987), Durlauf (1993), Horvath (1998, 2000), and Dupor (1999). On the empirical side, Carvalho and Gabaix (forthcoming) explore whether changes in the microeconomic composition of the economy during the post-war period can account for the Great Moderation, while Foerster, Sarte, and Watson (2011) study the relative importance of aggregate and micro-level shocks in the volatility of industrial production.

As already mentioned, the current paper is most closely related to the recent work of Acemoglu et al. (2012), who also study the role of the economy's input-output structure in translating microeconomic shocks into aggregate fluctuations. In particular, they provide a comprehensive study of how intersectoral input-output relationships affect the volatility of aggregate output. In contrast, rather than using GDP volatility as the measure of aggregate fluctuations, the focus of the current paper is on the likelihood of large economic downturns. Our results establish that not only aggregate volatility may not be a particularly useful measure for the frequency and depth of large recessions, but also that it is the interaction between the shape of the firm-level shock distributions and the structure of the input-output network which is of first-order importance.

The rest of the paper is organized as follows. We present the model and provide a characterization of the economy's aggregate output in Section 2. Our main results are presented in Section 3. Section 4 concludes. All proofs are provided in the Appendix.

2 Model

The model is a static variant of the multi-sector model of Long and Plosser (1983), analyzed by Acemoglu et al. (2012). Consider a static economy consisting of n competitive sectors denoted by $\{1, 2, \ldots, n\}$, each of which producing a distinct product. Each product can be either consumed by a mass of consumers or used as an input for production of other goods. Firms in each sector employ Cobb-Douglas production technologies with constant returns to scale that transform labor and intermediate goods to final products. The production at each sector is subject to some idiosyncratic Harrod-neutral productivity shock. More specifically, the output of sector i, which we denote by x_i, is equal to

$$x_i = z_i^\beta \ell_i^\beta \prod_{j=1}^{n} x_{ij}^{(1-\beta)w_{ij}}, \tag{1}$$

where z_i is the corresponding productivity shock, ℓ_i is the amount of labor hired by the firms in sector i, x_{ij} is the amount of good j used for production of good i, and $\beta \in (0,1)$ is the share of labor in production. The exponent $w_{ij} \geq 0$ in (1) captures the share of good j in the production technology of good i. In particular, a higher w_{ij} means that good j is more important in producing i, whereas $w_{ij} = 0$ implies that good j is not a required input for the production of good i. Note that the assumption that firms employ constant returns to scale technologies implies that $\sum_{j=1}^{n} w_{ij} = 1$ for all i.

The productivity shocks z_i are independent and identically distributed across sectors. We denote the common cumulative distribution function of $\epsilon_i = \log(z_i)$ by F, which we assume to be symmetric around the origin and with full support over $\mathbb{R}$. With some abuse of terminology, we refer to ϵ_i as the productivity shock to sector i.

In addition to the firms, the economy is populated with a unit mass of identical consumers. Each consumer is endowed with one unit of labor which can be hired by the firms for the purpose of production. We assume that the representative consumer has Cobb-Douglas preferences over the n goods produced in the economy. In particular,

$$u(c_1, \ldots, c_n) = A\left(\prod_{j=1}^{n} c_i\right)^{1/n},$$

where A is a normalization constant and c_i is the amount of good i consumed.

The input-output relationships between different sectors within the economy can be summarized by the matrix $W = [w_{ij}]$, which we refer to as the *input-output matrix*. Alternatively, one can capture the intersectoral trade patterns by the means of a directed, weighted graph on n vertices. Each vertex of this graph, which we refer to as the *input-output network*, corresponds to a sector and a directed edge (j, i) with weight $w_{ij} > 0$ is present from vertex j to vertex i if sector i uses good j is an input for production. Thus, the input-output matrix W corresponds to the (weighted) adjacency matrix of the underlying input-output network. Given their equivalence, we use the two concepts interchangeably. We define the degree of sector i as the share of i's output in the input supply of the entire economy, normalized by $1-\beta$; that is, $d_i = \sum_{j=1}^{n} w_{ji}$. We also define the following related concept:

Definition 1. The *influence vector* of an economy with the corresponding input-output matrix W is a vector v such that for all i,

$$v_i = \beta/n + (1-\beta)\sum_{j=1}^{n} v_j w_{ji}. \tag{2}$$

The elements of the influence vector, which is also known as the *Bonacich centrality vector* (Bonacich, 1987), provide an intuitive measure of different sectors' importance as input suppliers in the economy. In particular, according to (2), the influence of sector i is defined, recursively, as a linear function of the centralities of its immediate downstream customers. Hence, a sector is considered more important if it is an input supplier to other important suppliers in the economy.

The competitive equilibrium of the economy described above is defined in the usual way: it consists of a collection of prices and quantities such that (i) the representative consumer maximizes her utility; (ii) the representative firm in each sector maximizes its profits while taking the prices and the wage as given; and (iii) all markets clear.

The following result, which is proved in Acemoglu et al. (2012), characterizes the aggregate equilibrium output of the economy as a function of the pattern of intersectoral trades and the sectors' idiosyncratic productivity shocks.

Proposition 1. *The logarithm of real value added in the economy is equal to*

$$y \equiv \log(\mathrm{GDP}) = \sum_{i=1}^{n} v_i \epsilon_i,$$

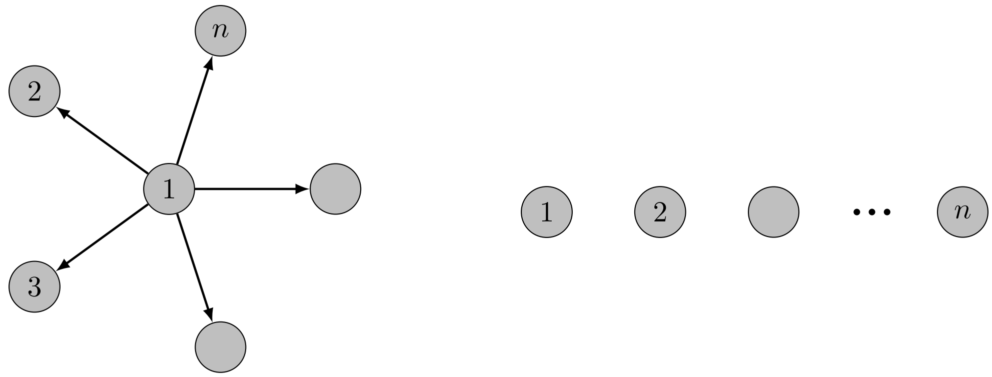
where v_i is the i-th element of the economy's influence vector.

This result highlights that the overall output of the economy is closely related to the pattern of input-output relationships between different sectors. In particular, it shows that the logarithm of the real value added, which for simplicity we refer to as *aggregate output*, is a convex combination of the sectors' (log) productivity shocks, with the weights given by the corresponding centralities. Proposition 1 thus establishes that productivity shocks to sectors that take more central positions in the input-output network play more significant roles in determining the level of aggregate economic activity. This is due to the fact that in the presence of interconnections between firms, a shock to sector i propagates over the input-output network to i's downstream customers, the customers of its customers and so forth, leading to an aggregate effect that goes beyond its effect on i's productivity.

Relatedly, Proposition 1 also shows that the likelihood of large economic downturns is not independent of the economy's input-output structure. Rather, given that the shocks can propagate over the input-output network, certain economies may exhibit more frequent and deeper recessions than others. The following example clarifies such a possibility.

Example 1. Consider the input-output network structure depicted in Figure 1(a). All sectors in this economy use the output of sector 1 as an intermediate good for production, that is $w_{i1} = 1$ for all i. The dominant role of the firms in sector 1 as input suppliers is also reflected in the influence vector corresponding to the economy's input-output matrix. In particular, it is easy to verify that the centrality of sector 1 is equal to $v_1 = 1 - \beta(n-1)/n$, which is greater than $v_i = \beta/n$, the centralities of sectors $i \neq 1$. Hence, by Proposition 1, and for large enough n, a large negative productivity shock to sector 1 translates into an essentially one-for-one drop in the economy's aggregate output. This is intuitive in view of the above discussion: whereas the effect of a shock to a peripheral sector remains confined to that sector, the shocks to sector 1 propagate to all other sectors in the economy, leading to a large aggregate effect.

In contrast, large downturns are much less likely in the economy depicted in Figure 1(b), which consists of n non-interacting sectors. In particular, given the absence of any dominant supplier, the economy exhibits large output drops only when a significant fraction of the sectors are hit with large negative productivity shocks; an event which occurs with a much smaller probability.



(a) An economy with a star input-output structure (b) A collection of non-interacting sectors

Figure 1. The network representation of two economies.

In the remainder of this paper, we study how the propagation of microeconomic shocks over the economy's input-output network may lead to large drops in the overall economic activity.

3 Input-Output Networks and Large Downturns

As highlighted by Proposition 1, the interconnections between different sectors may function as a propagation mechanism of idiosyncratic shocks throughout the economy. In this section, we study whether and to what extent the presence of such intersectoral input-output relations can lead to large aggregate fluctuations.

To answer these questions in a systematic way, we adopt the framework developed by Acemoglu et al. (2012): we focus on an infinite sequence of economies indexed by the number of sectors and study the distributional properties of the aggregate output as $n \to \infty$. More formally, we consider a sequence $\{W_n\}_{n\in\mathbb{N}}$, in which W_n corresponds to the input-output matrix of an economy consisting of n sectors.[3] We take other features of the economy, such as the distribution of shocks, F, and the share of labor in production, β, fixed throughout the sequence. Given a sequence of economies $\{W_n\}_{n\in\mathbb{N}}$, we denote the corresponding sequences of aggregate outputs and influence vectors by $\{y_n\}_{n\in\mathbb{N}}$ and $\{v_n\}_{n\in\mathbb{N}}$, respectively, and use d_{in} to denote the degree of sector i in the n-th economy.

One natural measure of economic fluctuations at the aggregate level is the standard deviation of the economy's aggregate output, which following Acemoglu et al. (2012), we refer to as the *aggregate volatility* of the economy. In view of Proposition 1, for any given sequence of economies $\{W_n\}_{n\in\mathbb{N}}$, aggregate volatility is equal to

$$(\operatorname{var} y_n)^{1/2} = \sigma \|v_n\|_2,^{4} \tag{3}$$

where σ is the standard deviation of the sector-specific idiosyncratic shocks ϵ_i. The above equality

[3]The elements of this sequence can be interpreted as corresponding to different levels of disaggregation of the same economy.

[4]Throughout the paper, we use $\|\xi\|_p$ to denote the p-norm of vector $\xi \in \mathbb{R}^n$, that is, $\|\xi\|_p = \left(\sum_{i=1}^n |\xi_i|^p\right)^{1/p}$ for $p \in [1,\infty)$, and $\|\xi\|_\infty = \max_i |\xi_i|$ for $p = \infty$.

immediately implies that aggregate volatility converges to zero at rate $\|v_n\|_2$ as $n \to \infty$. Therefore, the economy exhibits higher levels of aggregate fluctuations (measured in terms aggregate volatility), the slower the Euclidean norm of the corresponding influence vector converges to zero.

Acemoglu et al. (2012) provide a comprehensive study of the behavior of aggregate volatility in terms of the structural properties of the input-output network. They show that depending on the economy's input-output structure, it is possible for the aggregate volatility to decay at a rate slower than $1/\sqrt{n}$, in contrast to what is predicted by the law of large numbers. In other words, the presence of input-output relationships between different sectors can transform idiosyncratic firm- or sector-level shocks to fluctuations at the aggregate level. Furthermore, they show that the extent of asymmetry in the role of different sectors as input suppliers is closely associated with the economy's aggregate volatility. In particular, high variability in the sectors' roles as suppliers of intermediate goods (measured in terms of the heterogeneity in their first-order and second-order degrees) leads to a higher level of aggregate volatility.

Even though aggregate volatility is a highly useful notion of fluctuations at the aggregate level, in essence, it only captures the nature of fluctuations "near the mean". This raises the possibility that the standard deviation or variance of the log value added may not be informative about the likelihood and frequency of large drops in the GDP. In fact, two economies may experience large downturns with significantly different frequencies, despite the fact that they exhibit identical limiting behavior in terms of aggregate volatility (as well as asymptotic distribution). We illustrate such a possibility in the next example.

Before doing so, however, we need to introduce a metric for measuring and quantifying the likelihood of large economic downturns. Even though there is no such standard metric — reflecting in part the fact that, despite its importance, there is relatively little systematic analysis of large economic downturns and macroeconomic tail risk — a natural measure is the likelihood that aggregate output falls below a given threshold. More specifically, for a given sequence of economies $\{W_n\}_{n\in\mathbb{N}}$, we consider the limiting behavior of $\mathbb{P}(y_n < -c)$ for some positive constant $c > 0$ as $n \to \infty$.

Example 2. Consider a sequence of economies $\{W_n\}_{n\in\mathbb{N}}$ with the corresponding input-output network depicted in Figure 2. As the figure suggests, sector 1 is the sole supplier for k_n many sectors, while the rest of the sectors do not rely on sector 1's output for production.

It is easy to verify that as long as the degree of sector 1 does not grow too fast with n, aggregate volatility (the standard deviation of log GDP) decays to zero at rate $1/\sqrt{n}$. More specifically, as long as $k_n = o(\sqrt{n})$, equation (3) implies that $(\mathrm{var}\, y_n)^{1/2} \sim 1/\sqrt{n}$.[5] Moreover, Theorem 1 of Acemoglu et al. (2012) guarantees that, regardless of the distribution of the shocks, aggregate output is asymptotically normally distributed; that is, $\sqrt{n}y_n$ converges in distribution to a normal random variable with zero mean and variance σ^2. Thus, in essence, a variant of the central limit theorem applies, no matter what the common distribution of idiosyncratic sectoral-level shocks is.

[5] Given two sequences of positive numbers $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$, we write $a_n = o(b_n)$ if $\lim_{n\to} a_n/b_n = 0$.

However, the likelihood of a large drop in aggregate output is not invariant to the distribution of microeconomic shocks. Recall that by Proposition 1, aggregate output is equals $\sum_{i=1}^{n} v_{in}\epsilon_i$. Thus, as long as k_n grows with n — which guarantees that v_{1n} decays to zero at a rate slower than $1/n$ — the realization of a large, negative shock to sector 1 would lead to a proportionally large drop in aggregate output. As a consequence, the distributional properties of the microeconomic shocks play a central role in the likelihood of large downturns. In fact, as we will show in Propositions 3 and 4, large downturns may arise with significantly different frequencies depending on whether the shocks have normal or exponential distributions. This is despite the fact that both the aggregate volatility and the asymptotic distribution of aggregate output — which is normal by the central limit theorem — are invariant with respect to the distribution of the shocks.

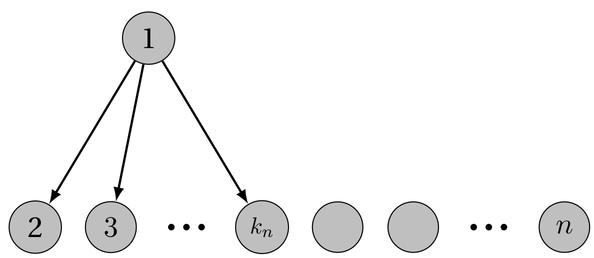


Figure 2. Firm 1 is the input supplier to a fraction k_n/n of the firms in the economy.

This example highlights that macroeconomic tail risks have very different network origins than aggregate volatility measured in terms of the standard deviation of log GDP. Motivated by this example and general discussion, in the remainder of the paper we provide a characterization of the relationship between the likelihood of large downturns on the one hand, and the distribution of idiosyncratic shocks and the economy's underlying network structure on the other.

3.1 Balanced Economies: Where No Tail Risks Arise

Before turning to the impact of the economy's network structure on the likelihood of tail risks, we present a class of economies in which tail risks are highly unlikely. This class of economies — which serve as a benchmark for our latter results — have balanced input-output structures in the sense that the shares of all sectors' output in the input supply of the entire economy are of the same order. More formally,

Definition 2. A sequence of economies $\{W_n\}_{n\in\mathbb{N}}$ is *balanced* if $\max_i d_{in} \sim 1$.[6]

Thus, in balanced structures, there is a limit to the extent of asymmetry in the roles of different sectors as input suppliers, in the sense that the degree of no sector increases unboundedly as $n \to \infty$. Hence, balanced structures lie at the opposite end of the spectrum from the star network

[6]Given two sequences of positive numbers $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$, we write $a_n \sim b_n$ as $n \to \infty$ if $0 < \liminf_{n\to\infty} a_n/b_n \leq \limsup_{n\to\infty} a_n/b_n < \infty$.

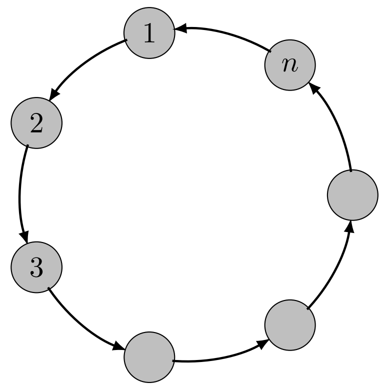


Figure 3. A balanced economy with a ring input-output structure.

structure depicted in Figure 1(a), in which one sector is the sole supplier of all other sectors within the economy. Figures 1(b) and 3 depict two instances of economies with balanced structures. We have the following result:

Proposition 2. *Consider a sequence of balanced economies* $\{W_n\}_{n\in\mathbb{N}}$ *and suppose that* $\mathbb{E}|\epsilon_i|^k < \infty$ *for all positive integers* k. *Then, there exists* $\bar{\beta} \in (0,1)$ *such that for* $\beta \geq \bar{\beta}$ *and for any constant* $c > 0$,

$$|\log \mathbb{P}(y_n < -c)| \sim n \tag{4}$$

as $n \to \infty$, *regardless of the distribution of the shocks.*

Thus, as long as the economy's input-output structure is balanced and the distribution of productivity shocks has thin enough tails (to the extent that all moments are finite), the likelihood of large drops in the economy's aggregate output decays as $e^{-\lambda n + o(n)}$ for some positive constant λ. In other words, large economic downturns become exponentially unlikely as $n \to \infty$.

Note that the rate established in (4) is the same across *all* balanced economies. Therefore, as long as there is little asymmetry in the sectors' roles as input suppliers, the likelihood of sharp drops in the GDP is equal to that of the economy consisting of non-interacting sectors, depicted in Figure 1(b). This observation thus means that intersectoral input-output linkages play no significant role in amplifying the risk of large economic downturns as long as the underlying production network is balanced. In this sense, Proposition 2 is in line with the results of Acemoglu et al. (2012) who establish that balanced economies also exhibit the least amount of aggregate volatility.

3.2 Normal Shocks

Proposition 2 is essentially an irrelevance result, suggesting that large economic downturns are equally unlikely in all balanced economies, regardless of the distributions of the shocks. Nevertheless, as illustrated by Example 1, the presence of non-balanced input-output network structures may have a non-trivial effect on the likelihood of large drops in the economy's aggregate output. In the remainder of this section, we study whether and how the interaction between the shape of the idiosyncratic shocks' distribution and the economy's network structure determines the frequency of large economic downturns. Our next result answers this question in the presence of normally distributed productivity shocks.

Proposition 3. *Consider a sequence of economies $\{W_n\}_{n\in\mathbb{N}}$. If the productivity shocks are normally distributed, then, for any constant $c > 0$,*

$$|\log \mathbb{P}\left(y_n < -c\right)| \sim 1/\left\|v_n\right\|_2^2 \tag{5}$$

as $n \to \infty$.

The above result establishes that in general, and unlike the special case of balanced structures, the intersectoral input-output structure plays a defining role in determining the likelihood of large drops in aggregate output. In particular, as long as $\|v_n\|_2$ vanishes at a rate slower than $1/\sqrt{n}$, the probability of large downturns decays at a subexponential rate.[7] Furthermore, Proposition 3 shows that in the presence of normally distributed shocks, the large deviation probability decays at a rate determined by the Euclidean norm of the influence vector — the same quantity that measures aggregate volatility. Hence, economies that exhibit high levels of aggregate volatility are also more likely to experience deeper and more frequent recessions.

Proposition 3 formalizes the intuition discussed in Example 1. It is easy to verify that the Euclidean norm of the aggregate output in a sequence of economies with a star network structure (depicted in Figure 1(a)) is bounded away from zero for all n — i.e., $\|v_n\|_2 \sim 1$ as $n \to \infty$. Hence, by the previous result, the probability that aggregate output falls below the threshold $-c$ remain positive, regardless of the size of the economy. Thus, in contrast with the predication of Proposition 2 for balanced networks, the possibility of propagation of shocks from the central sector 1 to the rest of the sectors means that the economy can experience much deeper recessions originated from microeconomic idiosyncratic shocks.

3.3 Exponential-Tailed Shocks

As already mentioned, Proposition 3 establishes that in the presence of normally distributed shocks, large economic downturns are more likely in economies that have a higher level of aggregate volatility. Even though intuitive, such an equivalence does not necessarily hold if the shocks are not normally distributed. Instead, as we show next, the interplay of the shape of the distribution of the productivity shocks on the one hand, and the input-output structure on the other, has non-trivial implications for the likelihood of large economic downturns.

To illustrate this point, we next consider the case that the idiosyncratic productivity shocks have a distribution with exponential tails, in the sense that the tail probabilities decay exponentially fast. More formally:

[7] On the other hand, when the sequence of economies is balanced, one can verify that $\|v_n\|_2 \sim 1/\sqrt{n}$, which means that $|\log \mathbb{P}(y_n < -c)|$ scales with n, as predicted by Proposition 2.

Definition 3. A random variable with distribution function F has *exponential tails* if

$$\big|\log(1-F(t))\big| \sim t.$$

Thus, if the counter-cumulative probability distribution is such that $1 - F(t) = L(t)e^{-\gamma t}$ for some constant $\gamma > 0$ and some sub-exponential function $L(t)$ — such as a polynomial — then the corresponding random variable has an exponential tail.[8] Clearly, distributions belonging to this class have heavier tails than the normal distribution. We have the following result:

Proposition 4. *Consider a sequence of economies $\{W_n\}_{n\in\mathbb{N}}$ and suppose that the productivity shocks have exponential tails. Then, for any constant $c > 0$,*

$$|\log \mathbb{P}\left(y_n < -c\right)| \sim 1/\|v_n\|_\infty \tag{6}$$

as $n \to \infty$.

The significance of the above result is threefold. First, it shows that in contrast to the case that the shocks are normally distributed, aggregate volatility is no longer a sufficient statistic for the likelihood of large downturns. Rather, in the presence of exponentially distributed shocks, it is the centrality of the most central sector in the economy's input-output network that is of first-order importance.

Second, it shows that the likelihood of large downturns can be significantly higher at the face of exponentially distributed shocks compared to the case that the shocks are normally distributed. This is due to the fact that by Hölder's inequality (Steele, 2004, p. 135), $\|v_n\|_2^2 \leq \|v_n\|_\infty$, implying that as $n \to \infty$, the expression on the right-hand side of (6) grows at a slower rate than the corresponding expression in (5).

Finally, as we show in Example 3 below and discuss in greater detail in the next subsection, Proposition 4 also implies that the economy's input-output network can fundamentally reshape the distribution of aggregate output, in the sense that it may translate relatively thin-tailed idiosyncratic shocks into aggregate effects that could only arise due to large, heavy-tailed shocks when the network is absent. In other words, observing large economic downturns frequently may not be due to the effect of shocks that are drawn from some heavy-tailed distribution, but rather, be a consequence of the propagation of shocks drawn from a relatively thin-tailed distributions over the economy's network structure.

The following example illustrates the contrast between Propositions 3 and 4.

Example 2 (continued). Recall the sequence of economies $\{W_n\}_{n\in\mathbb{N}}$ depicted in Figure 2 and assume that $k_n \sim \log n$. It is easy to verify that $\|v_n\|_2 \sim 1/\sqrt{n}$, whereas $\|v_n\|_\infty \sim (\log n)/n$. Proposition 3 thus implies that in the presence of normally distributed shocks, the likelihood of a large downturn satisfies $|\log \mathbb{P}(y_n < -c)| \sim n$; an exponential rate of decay identical to that of a balanced economy.[9]

[8] The Laplace and hyperbolic distributions are examples of probability distributions with exponential tails.

[9] Notice, however, that the economy depicted in Figure 2 is not balanced as long as k_n grows with n.

On the other hand, if the productivity shocks have exponential tails, Proposition 4 implies that the likelihood of large downturns satisfies $|\log \mathbb{P}(y_n < -c)| \sim n/(\log n)$, corresponding to a much slower (in fact, subexponential) rate of decay.

Thus, even though the economy depicted in Figure 2 behaves similar to a balanced network at the face of normally distributed shocks, the likelihood of a large drop in aggregate output is considerably higher if the shocks are exponentially distributed. Note that, as we mentioned earlier, this is despite the facts that (i) aggregate volatility decays at rate $1/\sqrt{n}$; and (ii) aggregate output scaled by aggregate volatility, $\sqrt{n}y_n$, is asymptotically normally distributed, *regardless of the distribution of the shocks.*

This example thus highlights the intricate dependence of the extent of propagation of shocks in non-balanced economies on the shape of their distribution. Indeed, the same input-output structure can lead to dramatically different outcomes at the aggregate level depending on the nature the microeconomic idiosyncratic shocks. In contrast, as Proposition 2 shows, the likelihood of a large contraction in aggregate output is the same in an economy with a balanced structure, regardless of the distribution of the shocks.[10]

3.4 The Network Origins of Macroeconomic Tail Risks

In this subsection, we show that the economy's input-output structure may translate relatively thin-tailed idiosyncratic shocks into aggregate effects that can only arise due to heavy-tailed disturbances in the absence of the network. In other words, we show that increasing the extent of asymmetry in the economy's underlying network structure has the same exact effect on the size of macroeconomic tail risks as subjecting the firms to shocks with heavier tails.

To this end, we characterize the likelihood of large downturns when the firms are subjects to shocks drawn from a *stable distribution* with parameter $\alpha \in (1, 2)$. Distributions in this class have Pareto tails (Zolotarev, 1986). In particular,

$$1 - F(t) \sim t^{-\alpha}. \tag{7}$$

Note that a smaller α corresponds to a distribution with heavier tails, according to which large shocks are more likely.[11] We have the following result:

Proposition 5. *Consider a sequence of economies $\{W_n\}_{n\in\mathbb{N}}$ and suppose that the productivity shocks have a stable distribution with parameter $\alpha \in (1, 2)$. Then, for any constant $c > 0$,*

$$\mathbb{P}\left(y_n < -c\right) \sim \|v_n\|_\alpha^\alpha \tag{8}$$

as $n \to \infty$.

[10] Both exponential and normal distributions have finite moments and hence, satisfy the assumption of Proposition 2.
[11] Note that this class of distributions have a well-defined mean.

In line with our previous results, the above proposition shows that the shape of the distribution of idiosyncratic productivity shocks plays a central role in determining the likelihood of large downturns. Furthermore, given that $\|v_n\|_\alpha^\alpha \leq \|v_n\|_{\hat{\alpha}}^{\hat{\alpha}}$ for $\alpha < \hat{\alpha}$, Proposition 5 also implies that large downturns are significantly more likely, the heavier the tail of the shock distribution is.

The following example illustrates how the underlying network of the economy can turn thin-tailed microeconomic shocks into aggregate effects that are essentially heavy-tailed.

Example 3. Consider the sequence of economies $\{W_n\}_{n\in\mathbb{N}}$ depicted in Figure 2 and assume that $k_n \sim n/(\log n)$. It is easy to verify that for such an economy $\|v_n\|_\infty \sim 1/\log n$, and hence, by Proposition 4, the likelihood of a large downturn in the presence of exponentially distributed shocks satisfies

$$|\log \mathbb{P}(y_n < -c)| \sim \log n. \tag{9}$$

In other words, as $n \to \infty$, the likelihood of large drops in the economy's aggregate output decays at rate $\lambda/n + o(1/n)$ for some positive constant λ.

Next, consider the sequence of economies $\{\tilde{W}_n\}_{n\in\mathbb{N}}$ depicted in Figure 1(b), in which firms do not rely on one another as input suppliers for production. In such an economy, aggregate output is simply the unweighted average of firm-level shocks, that is $\tilde{y}_n = (1/n)\sum_{i=1}^n \epsilon_i$. Furthermore, suppose that rather than being exponentially distributed, the sector-specific productivity shocks have a heavy-tailed, stable distribution with parameter $\alpha \in (1,2)$. By Proposition 5, we have

$$|\log \mathbb{P}(\tilde{y}_n < -c)| \sim \log n. \tag{10}$$

Comparing (10) with (9) implies that the probability of extreme events in an economy with non-interacting sectors subject to heavy-tailed shocks decays at the same exact rate as in an economy with a non-trivial structure that is subject to thin-tailed, exponentially distributed shocks.

The above example thus underscores a central insight of our paper: large economic downturns may occur regularly not necessarily due to (aggregate or idiosyncratic) shocks that are drawn from heavy-tailed distributions, but rather as a consequence of the interplay of relatively thin-tailed distributions with the economy's network structure. In other words, the propagation of productivity shocks over the input-output network of the economy can lead to aggregate effects that may appear to be due to heavy-tailed disturbances, even when the shocks themselves are drawn from distributions that are relatively thin-tailed.

The following corollary to Proposition 5 further emphasizes this insight.

Corollary 1. *Suppose that the productivity shocks have a stable distribution with parameter* $\alpha \in (1,2)$. *Furthermore, suppose that the firms' centralities follow a power law with parameter* $\theta \in (1/\alpha, 1)$, *that is* $v_{in} = r_n i^{-\theta}$ *for some normalization constant* r_n. *Then,*

$$\mathbb{P}\left(y_n < -c\right) \sim n^{-\alpha(1-\theta)}, \tag{11}$$

as $n \to \infty$.

A first implication of the above corollary is that increasing θ makes large downturns more likely. Note that θ measures the extent of asymmetry in the sectors' roles as input suppliers in the economy: a larger θ means that some sectors take significantly more central positions than others in the input-output network of the economy. On the other hand, the extreme case that of $\theta = 0$ corresponds to a balanced structure. Thus, large recessions are more likely, the more asymmetric or "unbalanced" the underlying network structure is.

More importantly, increasing the extent of asymmetry in the underlying input-output structure of the economy (measured in terms of a higher θ) has the same exact effect on the likelihood of large downturns as making the firms subject to shocks of heavier tails (measured in terms of a smaller α). Thus, in line with Example 3, this observation underscores how the input-output linkages between different sectors play as significant of a role in determining the frequency of large economic downturns as the nature of the shocks.

4 Conclusions

This paper shows that input-output linkages between different firms or sectors within the economy can have a significant effect on the frequency and depth of large economic downturns. By focusing on a sequence of economies, we study the likelihood of such downturns as a function of the underlying network structure of the economy and the nature of idiosyncratic microeconomic shocks.

We first establish that aggregate volatility is not necessarily a sufficient statistic for the likelihood of large recessions. Depending on the shape of the distribution of the idiosyncratic shocks, different features of the economy's input-output network may be of first-order importance. Furthermore, we show that the effects of the intersectoral input-output linkages and the nature of the idiosyncratic shocks on aggregate output are not separable. Rather, the likelihood of large economic downturns is determined by the interplay between the two. More specifically, we show that even though some economies exhibit the same large deviation properties as the balanced structures in the presence of normally distributed shocks, they may experience significantly more frequent downturns (compared to balanced economies) when shocks are exponentially distributed.

Finally, our results establish that the economy's input-output structure can fundamentally reshape the distribution of aggregate output, increasing the likelihood of large downturns from infinitesimal to substantial. In particular, we show that an economy with non-trivial intersectoral input-output linkages that is subject to thin-tailed productivity shocks may exhibit deep recessions as frequently as economies that are subject to shocks with significantly heavier tails. In this sense, our results provide a novel solution to what Bernanke et al. (1996) refer to as the "small shocks, large cycles puzzle" by arguing that the interaction between the underlying network structure of the economy and the shape of the microeconomic shocks' distribution may lead to sizable fluctuations at the aggregate level.

A Proofs

Notation Throughout the proofs, we use the following notation. Given two sequences of positive real numbers $\{a_n\}_{n\in\mathbb{N}}$ and $\{b_n\}_{n\in\mathbb{N}}$, we write $a_n = O(b_n)$, if they satisfy $\limsup_{n\to\infty} a_n/b_n < \infty$, whereas $a_n = \Omega(b_n)$ if $\liminf_{n\to\infty} a_n/b_n > 0$. On the other hand, $a_n = o(b_n)$ means that $\lim_{n\to\infty} a_n/b_n = 0$. Finally, we write $a_n \sim b_n$, if $a_n = O(b_n)$ and $a_n = \Omega(b_n)$ hold simultaneously.

Proof of Proposition 2

We first state and prove a simple lemma.

Lemma 1. *For any sequence of balanced economies $\{W_n\}_{n\in\mathbb{N}}$, there exist $\bar{\beta} \in (0,1)$ and positive constants q and Q such that for $\beta \geq \bar{\beta}$,*

$$q \leq nv_{in} \leq Q \tag{12}$$

for all i and n.

Proof. To prove the lower bound, note that by (2), $v_{in} \geq \beta/n$ for all i. Thus, one can simply choose $q = \beta$. On the other hand, to prove the upper bound in (12), note that (2) also implies

$$v_{in} \leq \beta/n + (1-\beta)(\max_i v_{in})(\max_i \sum_{j=1}^{n} w_{ji}).$$

Therefore,

$$\|v_n\|_\infty \leq \frac{\beta}{n} + k(1-\beta)\|v_n\|_\infty,$$

where k is a positive constant independent of n. In deriving the above inequality we are using the fact that in any sequence of balanced economies, $d_{in} = \max_i \sum_{j=1}^{n} w_{ji} \sim 1$. Thus, as long as $\beta > (k-1)/k$, we have $\|v_n\|_\infty \leq Q/n$, where $Q = \beta/(1-k(1-\beta))$, completing the proof. □

Proof of Proposition 2 We first show that $\limsup_{n\to\infty}(1/n)\,|\log \mathbb{P}(y_n < -c)| < \infty$. To this end, note that $\sum_{i=1}^{n} v_{in} = 1$, which implies that if $\epsilon_i < -c$ for all i, then $y_n < -c$. Therefore,

$$[F(-c)]^n \leq \mathbb{P}(y_n < -c),$$

and as a result,

$$\limsup_{n\to\infty} \frac{1}{n}\,|\log \mathbb{P}(y_n < -c)| < \infty. \tag{13}$$

Next we show that $\liminf_{n\to\infty}(1/n)\,|\log \mathbb{P}(y_n < -c)| > 0$. Using Chernoff's inequality, we have

$$\mathbb{P}(y_n < -c) \leq e^{-n\delta c}\mathbb{E}\left(e^{n\delta y_n}\right) = e^{-n\delta c}\prod_{i=1}^{n}\mathbb{E}\left(e^{n\delta v_{in}\epsilon_i}\right),$$

for any $\delta \geq 0$. Taking logarithms from both sides of the above inequality implies

$$\log \mathbb{P}(y_n < -c) \leq -n\delta c + \sum_{i=1}^{n} g(n\delta v_{in})$$

where $g(t) = \log \mathbb{E}[\exp(t\epsilon_i)]$ is the cumulant-generating function corresponding to random variable ϵ_i. Thus, by Lemma 1,

$$\begin{aligned} \frac{1}{n} \log \mathbb{P}(y_n < -c) &\leq -\delta c + \max_{\delta q \leq t \leq \delta Q} g(t) \\ &\leq -\Lambda(\delta), \end{aligned}$$

where $\Lambda(\delta) = \delta c - \max_{0 \leq t \leq \delta Q} g(t)$. Note that $\Lambda(0) = 0$. Furthermore, by the envelope theorem, $\Lambda'(0) > 0$. Consequently, it is immediate that there exists small enough $\delta > 0$ such that $\Lambda(\delta) > 0$, and therefore,

$$\liminf_{n \to \infty} \frac{1}{n} \left| \log \mathbb{P}(y_n < -c) \right| \geq \Lambda(\delta) > 0. \tag{14}$$

Combining (14) with (13) completes the proof. □

Proof of Proposition 3

Proposition 1 implies that aggregate output y_n is a convex combination of the firms' productivity shocks. Given that the shocks ϵ_i are normally distributed, it is immediate that aggregate output is normally distributed with mean zero and standard deviation $\sigma \|v_n\|_2$, where σ is the standard deviation of the idiosyncratic shocks. In other words, $y_n \sim \mathcal{N}(0, \sigma^2 \|v_n\|_2^2)$. Hence,

$$\mathbb{P}(y_n < -c) = 1 - \Phi\left(\frac{c}{\sigma \|v_n\|_2}\right),$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal.

If $\|v_n\|_2 \sim 1$, the above equality shows that the statement of the proposition holds trivially. In particular, if $\|v_n\|_2$ does not decay to zero, then the event $\{y_n < -c\}$ occurs with some positive probability even as $n \to 0$. This is simply a consequence of the fact that $\mathrm{var}(y_n) = \|v_n\|_2^2$. Thus, for the rest of the proof we assume that $\|v_n\|_2 = o(1)$. On the other hand, if $\phi(\cdot)$ denotes the density function of the standard normal, it is well-known that

$$\lim_{t \to \infty} \frac{t[1 - \Phi(t)]}{\phi(t)} = 1.$$ [12]

Therefore,

$$\mathbb{P}\left(y_n < -c\right) \sim \frac{\phi\left(c/\sigma \|v_n\|_2\right)}{c/\sigma \|v_n\|_2}.$$

Taking logarithms from both sides establishes the result. □

[12] See for example, Grimmett and Stirzaker (2001, p. 98).

Proof of Proposition 4

First suppose that the sequence of economies are such that $\|v_n\|_\infty \sim 1$. The fact that $\|\zeta\|_\infty \leq \|\zeta\|_2$ for any vector ζ implies that $\mathrm{var}(y_n) = \|v_n\|_2^2$ does not decay to zero either. Therefore, the event $\{y_n < -c\}$ occurs with some positive probability even as $n \to 0$, establishing the result. Thus, for the rest of the proof, we assume that $\|v_n\|_\infty = o(1)$.

We first show that when the productivity shocks ϵ_i have a common symmetric distribution $F(\cdot)$ with an exponential tail, then $\limsup_{n\to\infty} -\|v_n\|_\infty \log \mathbb{P}(y_n < -c) < \infty$. Note that if $v_{in}\epsilon_i < -c$ and $\sum_{j\neq i} v_{jn}\epsilon_j < 0$ hold for some i, then $y_n < -c$. Therefore, by the independence and symmetry assumptions, we have

$$\mathbb{P}(y_n < -c) \geq \frac{1}{2}\mathbb{P}\left(\epsilon_i \|v_n\|_\infty < -c\right) = \frac{1}{2}\Big[1 - F\left(c/\|v_n\|_\infty\right)\Big],$$

which implies

$$\limsup_{n\to\infty} \|v_n\|_\infty \cdot |\log \mathbb{P}(y_n < -c)| \leq \limsup_{n\to\infty} \|v_n\|_\infty \cdot \Big|\log\big[1 - F\left(c/\|v_n\|_\infty\right)\big]\Big|.$$

Given the assumptions that $\|v_n\|_\infty = o(1)$ and $F(\cdot)$ has exponential tails, the right-hand side of the above inequality is finite. Therefore,

$$\limsup_{n\to\infty} \|v_n\|_\infty \cdot |\log \mathbb{P}(y_n < -c)| < \infty. \tag{15}$$

We next show that $\liminf_{n\to\infty} \|v_n\|_\infty \cdot |\log \mathbb{P}(y_n < -c)| > 0$. To establish this, we compute an upper bound for the generating function of ϵ_i, and use Chernoff's inequality to bound the tail event probability $\mathbb{P}(y_n < -c)$.[13] However, we first remark that if $F(\cdot)$ has an exponential tail, then there exists a strictly positive constant γ such that

$$1 - F(t) < e^{-\gamma t} \tag{16}$$

for all $t > 0$. This is due to the fact that the function $-(1/t)\log[1 - F(t)]$ is always positive for $t > 0$ and has a strictly positive limit inferior.

We now proceed with the proof. Note that by symmetry of the distributions, and for $k \geq 2$ we have

$$\frac{1}{2}\mathbb{E}\left|\epsilon_i\right|^k = \int_0^\infty t^k dF(t) = \int_0^\infty k t^{k-1}\left(1 - F(t)\right)dt$$

where we have used integration by parts and the fact that

$$0 \leq \lim_{t\to\infty} t^k\left(1 - F(t)\right) = \lim_{t\to\infty} \exp\left[k\log(t) + \log(1 - F(t))\right] = 0;$$

a consequence of the exponential tail assumption. Thus, by (16), there exists a positive constant $r = 1/\gamma$ such that

$$\frac{1}{2}\mathbb{E}|\epsilon_i|^k \leq \int_0^\infty k t^{k-1} e^{-t/r} dt = r^k k!$$

[13] For a similar argument, see, e.g., Teicher (1984).

for all $k \geq 2$. Therefore, for $\delta < 1/(r\|v_n\|_\infty)$ and for all i, we have

$$\begin{aligned}\mathbb{E}\left(e^{\delta v_{in}\epsilon_i}\right) &= 1 + \sum_{k=2}^{\infty} \frac{(\delta v_{in})^k}{k!} \mathbb{E}\left(\epsilon_i^k\right) \\ &\leq 1 + 2\sum_{k=2}^{\infty} (\delta r v_{in})^k.\end{aligned}$$

The above inequality implies that

$$\mathbb{E}\left(e^{\delta v_{in}\epsilon_i}\right) \leq 1 + \frac{2(\delta r v_{in})^2}{1 - \delta r v_{in}} \leq \exp\left(\frac{2(\delta r v_{in})^2}{1 - \delta r \|v_n\|_\infty}\right). \tag{17}$$

Using (17), we can now compute an upper bound for the large deviation probability. In particular, from Chernoff's inequality, we have

$$\mathbb{P}(y_n < -c) \leq e^{-\delta c}\mathbb{E}\left(e^{\delta y_n}\right) = e^{-\delta c}\prod_{i=1}^{n} \mathbb{E}\left(e^{\delta v_{in}\epsilon_i}\right),$$

implying that

$$\log \mathbb{P}(y_n < -c) \leq -\delta c + \sum_{i=1}^{n} \frac{2(\delta r v_{in})^2}{1 - \delta r \|v_n\|_\infty} = -\delta c + \frac{2(\delta r \|v_n\|_2)^2}{1 - \delta r \|v_n\|_\infty}.$$

Letting $\delta = c/(4r^2\|v_n\|_2^2 + rc\|v_n\|_\infty)$ leads to[14]

$$\log \mathbb{P}(y_n < -c) \leq \frac{-c^2}{8r^2\|v_n\|_2^2 + 2rc\|v_n\|_\infty} \leq \frac{-c^2}{2r\|v_n\|_\infty(4r + c)},$$

where we have used the fact that $\|v_n\|_\infty \geq \|v_n\|_2^2$. Therefore,

$$\liminf_{n\to\infty} \|v_n\|_\infty \cdot |\log \mathbb{P}(y_n < -c)| \geq \frac{c^2}{8r^2 + 2rc} > 0. \tag{18}$$

Combining (15) and (18) completes the proof. □

Proof of Proposition 5

Recall from (2) that aggregate output of economy W_n is equal to $y_n = \sum_{i=1}^n v_{in}\epsilon_i$. On the other hand, it is well-know that if if ϵ_i are independent and have the common stable distribution with parameter α, then

$$\sum_{i=1}^{n} v_{in}\epsilon_i =_d \left(\sum_{i=1}^{n} v_{in}^\alpha\right)^{1/\alpha} \cdot \epsilon_1.$$[15]

Therefore, for any given $c > 0$,

$$\mathbb{P}(y_n < -c) = \mathbb{P}(\|v_n\|_\alpha \cdot \epsilon_1 < -c) = F(-c/\|v_n\|_\alpha).$$

[14] Note that this choice of δ satisfies $\delta r\|v_n\|_\infty < 1$, the condition required for deriving (17).
[15] See for example, Zolotarev (1986).

Note that if $\|v_n\|_\alpha$ is bounded away from zero for infinitely many n, then the result is immediately obtained. On the other hand, if $\|v_n\|_\alpha \to 0$ as $n \to \infty$, then the symmetry of the probability distribution F and the tail property (7) imply

$$\mathbb{P}(y_n < -c) = 1 - F(c/\|v_n\|_\alpha) \sim \|v_n\|_\alpha^\alpha,$$

completing the proof. □

Proof of Corollary 1

By construction, the elements of the influence vector v_n should add up to one, that is

$$r_n \sum_{i=1}^{n} i^{-\theta} = 1.$$

Therefore, as $n \to \infty$,

$$r_n \sim n^{\theta-1}$$

where we are using the assumption that $\theta < 1$. On the other hand, Proposition 5 implies that the probability of large downturns satisfies $\mathbb{P}(y_n < -c) \sim \|v_n\|_\alpha^\alpha$, and as a consequence,

$$\mathbb{P}(y_n < -c) = \sum_{i=1}^{n} v_{in}^\alpha \sim n^{\alpha(\theta-1)} \sum_{i=1}^{n} i^{-\alpha\theta}.$$

Finally, note that as long as $\alpha\theta > 1$, the series $\sum_{i=1}^{n} i^{-\alpha\theta}$ converges to a finite number, proving the result. □

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