ADDENDUM TO: INTERMEDIATED TRADE

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December 27, 2010

Abstract

This addendum provides the proofs of Propositions 4-6 in Section 6 of our main paper.

Throughout this addendum, we restrict ourselves to equilibria in which the intermediation level and the agents' expected lifetime utilities immediately jump to their new steady state values after W- and M-integration. Like in our main paper, one can show that such equilibria always exist. For expositional purposes, we also ignore issues related to the endogenous separation of matched farmer-trader pairs under M-integration. Details are available upon request.

1 Large Northern Traders

1.1 Assumptions

Compared to the model described in Section 5 of our main paper, we assume that each Northern trader active in the South now belongs to one of n trading companies. The number of Northern trading companies n is exogenously given, but each company can costlessly adjust the measure $x > 0$ of traders that it employs in the South. For simplicity, we restrict ourselves to symmetric equilibria in which all Northern trading companies have the same size at all points in time. The common size of Northern trading companies as well as the measure of Southern traders active in the South are endogenously determined through the following free entry conditions:

$$V_{T^N}^U = V_{T^S}^U = 0, \tag{1}$$

The matching between each individual members of the trading company and Southern farmers is as described in Section 5 of our main paper. Bargaining, however, now proceeds under the common knowledge that if a farmer refuses to trade with a given member of a trading company, other members of that trading company will stop intermediating on her behalf until she has been matched with a Southern trader or another Northern trading company.[1] Hence, the Nash bargaining consumption levels of a Southern farmer-Northern trader match with good i, $(C_{F_i}, S_{F_i}, C_{T_i}, S_{T_i})$, now solves

$$\begin{array}{ll} \max\limits_{C_{F_i},S_{F_i},C_{T_i},S_{T_i}} & \left(V_{T_i^N}^M - V_{T^N}^U\right)^{\overline{\beta}} \left(V_{F_i^N}^M - \underline{V}_F^U\right)^{1-\overline{\beta}} \\ \text{s.t.} & pC_{F_i} + S_{F_i} + pC_{T_i} + S_{T_i} \leq (p/a_C) \cdot \mathcal{I}_C + (1/a_S)(1 - \mathcal{I}_C), \end{array}$$

where $\underline{V}_F^U$ is the expected lifetime utility of an unmatched Southern farmer *if* she were to refuse to trade. Compared to Section 5 of our main paper, the key difference lies in the fact that $\underline{V}_F^U$ is now different from the expected lifetime utility, V_F^U, of an unmatched farmer who has never refused to trade (or has already been matched with another trader). The rest of the model is as described in our main paper. In particular, we maintain the assumption that $\overline{\beta} > \beta$ and $\tau > \tau^*$.

[1] We are thus assuming that matching has a "cleansing" effect on a farmer's past behavior. Thus once a punished farmer has been matched with another trader, either from the North or the South, he can no longer be recognized by the Northern trading company that had previously ostracized him. While this assumption is admittedly ad-hoc, it considerably simplifies the analysis below. Since newly matched farmers can no longer be punished, traders' margins are independent of farmers' history.

1.2 Predictions

In the present environment the steady state values of the intermediation level and the expected lifetime utilities of the different agents can therefore be written as:

$$
\begin{aligned}
rV_F^U &= \mu_F\left(\theta^L\right)\left[\phi\left(V_{F^N}^M - V_F^U\right) + (1-\phi)\left(V_{F^S}^M - V_F^U\right)\right] && (2)\\
r\underline{V}_F^U &= \mu_F\left(\theta^L\right)\left[\left(\frac{n-1}{n}\right)\phi\left(V_{F^N}^M - \underline{V}_F^U\right) + (1-\phi)\left(V_{F^S}^M - \underline{V}_F^U\right)\right] && (3)\\
rV_{F^N}^M &= \left(1-\alpha^N\right)v(p^W) + \lambda\left(V_F^U - V_{F^N}^M\right), && (4)\\
rV_{F^S}^M &= \left(1-\alpha^S\right)v(p^W) + \lambda\left(V_F^U - V_{F^S}^M\right), && (5)\\
rV_{T^N}^U &= -\tau^* + \mu_T\left(\theta^L\right)\left(V_{T^N}^M - V_{T^N}^U\right), && (6)\\
rV_{T^N}^M &= \alpha^N v(p^W) - \tau^* + \lambda\left(V_{T^N}^U - V_{T^N}^M\right), && (7)\\
rV_{T^S}^U &= -\tau + \mu_T\left(\theta^L\right)\left(V_{T^S}^M - V_{T^S}^U\right), && (8)\\
rV_{T^S}^M &= \alpha^S v(p^W) - \tau + \lambda\left(V_{T^S}^U - V_{T^S}^M\right). && (9)
\end{aligned}
$$

where $\phi \equiv u_{T^N}/\left[u_{T^N} + u_{T^S}\right]$ is the share of unmatched Northern traders active in the South; θ^L is the intermediation level in the Southern island in the presence of large Northern traders; and all other variables are defined in the same way as in our main paper. Using the previous expressions and equation (1), we can establish the following proposition.

Proposition 4 *The equilibrium under M-integration with large Northern traders is isomorphic to an equilibrium under M-integration with infinitesimally small Northern traders with primitive bargaining power*

$$\beta^L \equiv \frac{\left[r + \mu_F\left(\theta^L\right)\right]\overline{\beta}}{r + \mu_F\left(\theta^L\right)\left[\overline{\beta} + \left(\frac{n-1}{n}\right)\left(1-\overline{\beta}\right)\right]} > \overline{\beta}.$$

Proof. Equations (6)-(9) imply

$$
\begin{aligned}
V_{T^S}^M - V_{T^S}^U &= \frac{\alpha^S v\left(p^W\right)}{r + \lambda + \mu_T\left(\theta^L\right)}, && (10)\\
V_{T^N}^M - V_{T^N}^U &= \frac{\alpha^N v\left(p^W\right)}{r + \lambda + \mu_T\left(\theta^L\right)}. && (11)
\end{aligned}
$$

Similarly, equations (2)-(4) imply

$$
\begin{aligned}
V_{F^S}^M - V_F^U &= \frac{\left(1-\alpha^S\right)(r+\lambda) + \phi\mu_F\left(\theta^L\right)\left(\alpha^N - \alpha^S\right)}{(r+\lambda)\left[r + \lambda + \mu_F\left(\theta^L\right)\right]} \cdot v(p^W) && (12)\\
V_{F^N}^M - \underline{V}_F^U &= \frac{\left[(r+\lambda)\left(1-\alpha^N\right) - (1-\phi)\,\mu_F\left(\theta^L\right)\left(\alpha^N - \alpha^S\right)\right]\left[r + \mu_F\left(\theta^L\right)\right]}{(r+\lambda)\left[r + \lambda + \mu_F\left(\theta^L\right)\right]\left[r + \mu_F\left(\theta^L\right)\left(\frac{n-\phi}{n}\right)\right]} \cdot v(p^W) && (13)
\end{aligned}
$$

From Nash bargaining, we know that

$$(1-\beta)\left(V_{T^S}^M - V_{T^S}^U\right) = \beta\left(V_{F^S}^M - V_F^U\right), \tag{14}$$
$$(1-\overline{\beta})\left(V_{T^N}^M - V_{T^N}^U\right) = \overline{\beta}\left(V_{F^N}^M - \underline{V}_F^U\right). \tag{15}$$

Combining equations (10)-(15), we get, after simple rearrangements,

$$\alpha^S = \left(\frac{\beta}{1-\beta}\right)\left[\frac{r+\lambda+\mu_T\left(\theta^L\right)}{r+\lambda+\mu_F\left(\theta^L\right)}\right]\left[1-\alpha^S+\frac{\phi\mu_F\left(\theta^L\right)}{(r+\lambda)}\left(\alpha^N-\alpha^S\right)\right],$$
$$\alpha^N = \left(\frac{\overline{\beta}}{1-\overline{\beta}}\right)\left[\frac{r+\lambda+\mu_T\left(\theta^L\right)}{r+\lambda+\mu_F\left(\theta^L\right)}\right]\left[\frac{r+\mu_F\left(\theta^L\right)}{r+\mu_F\left(\theta^L\right)\left(\frac{n-\phi}{n}\right)}\right]\left[1-\alpha^N+\frac{(\phi-1)\,\mu_F\left(\theta^L\right)}{(r+\lambda)}\left(\alpha^N-\alpha^S\right)\right].$$

The two previous equations immediately imply that $\alpha^N > \alpha^S$. To see this, suppose that $\alpha^N \le \alpha^S$. Since $\overline{\beta} > \beta$, the two previous equations then require

$$1-\alpha^S+\frac{\phi\mu_F\left(\theta^L\right)}{(r+\lambda)}\left(\alpha^N-\alpha^S\right) > 1-\alpha^N+\frac{(\phi-1)\,\mu_F\left(\theta^L\right)}{(r+\lambda)}\left(\alpha^N-\alpha^S\right),$$

which implies $\alpha^N > \alpha^S$ and contradicts $\alpha^N \le \alpha^S$. Since $\alpha^N > \alpha^S$ and $\tau^* < \tau$, the same logic as in our main paper implies $\phi = 1$ under M-integration. The share of surplus α^N accruing to Northern traders therefore simplifies to

$$\alpha^N = \frac{\overline{\beta}\left[r+\lambda+\mu_T\left(\theta^L\right)\right]\left[r+\mu_F\left(\theta^L\right)\right]}{\left(1-\overline{\beta}\right)\left[r+\lambda+\mu_F\left(\theta^L\right)\right]\left[r+\mu_F\left(\theta^L\right)\left(\frac{n-1}{n}\right)\right]+\overline{\beta}\left[r+\lambda+\mu_T\left(\theta^L\right)\right]\left[r+\mu_F\left(\theta^L\right)\right]}. \tag{16}$$

By equations (1), (6), and (11), we also know the intermediation level satisfies

$$\frac{\alpha^N v\left(p^W\right)}{r+\lambda+\mu_T\left(\theta^L\right)} = \frac{\tau^*}{\mu_T\left(\theta^L\right)}. \tag{17}$$

Equations (16) and (17) implicitly determine the Northern traders' margin, α^N, and the intermediation level, θ^L. To conclude, let us define $\beta^L \equiv \frac{\left[r+\mu_F\left(\theta^L\right)\right]\overline{\beta}}{r+\mu_F\left(\theta^L\right)\left[\overline{\beta}+\left(\frac{n-1}{n}\right)\left(1-\overline{\beta}\right)\right]} > \overline{\beta}$. After simple algebra, we can rearrange equation (16) as

$$\alpha^N = \frac{\left[\mu_T\left(\theta^L\right)+r+\lambda\right]\beta^L}{r+\lambda+\left(1-\beta^L\right)\mu_F\left(\theta^L\right)+\mu_T\left(\theta^L\right)\beta^L}. \tag{18}$$

Combining equations (17) and (18), we finally get

$$\alpha^N = \beta^L - \frac{\left(1-\beta^L\right)\left(\theta^L-1\right)\tau^*}{v\left(p^W\right)},$$
$$\frac{v\left(p^W\right)-\tau^*}{\tau^*} = \frac{r+\lambda+\left(1-\beta^L\right)\mu_F\left(\theta^L\right)}{\beta^L\mu_T\left(\theta^L\right)},$$

which are just the counterparts of equations (36) and (37) in our main paper. Thus the traders Northern traders' margin, α^N, and the intermediation level, θ^L, are identical to those that would prevail in an equilibrium with infinitesimally small Northern traders with primitive bargaining power β^L. Given equations (2)-(9), the expected lifetime utilities of all agents are identical as well. **QED** ■

2 Endogenous Number of Farmers, Exogenous Number of Traders

2.1 Assumptions

Preferences, technology, matching, and bargaining are as described in Section 2 of our main paper. Compared to our main paper, we assume that an exogenous measure N_T of the island inhabitants are traders, and for simplicity, that these traders can be connected to Walrasian markets at zero cost, $\tau = 0$. Conversely, we assume that there is a large pool of potential farmers who can decide at any point in time to become active or inactive. As in Section 2 of our main paper, active farmers get zero utility per period when unmatched, but stand to obtain some remuneration when matched with a trader. By contrast, inactive farmers are now involved in a non-market activity that generates a constant expected lifetime utility, V_F^*, e.g., subsistence agriculture. We assume that the pool of potential traders is large enough to ensure that the measure of farmers operating on the island, N_F, is not constrained by population size and that some agents are always involved in subsistence agriculture. Hence, in equilibrium, N_F is endogenously pinned down by the farmers' indifference condition:

$$V_F^U = V_F^*. \tag{19}$$

The rest of the model is as described in our main paper. In particular, we assume that $\overline{\beta} > \beta$ when analyzing the consequences of M-integration.

2.2 Predictions

For future reference, let us first describe the Bellman equations characterizing the expected lifetime utilities of the different agents in our economy. Under autarky and W-integration, since $\tau = 0$, we simply have

$$\begin{aligned} rV_F^U &= \mu_F(\theta)\left(V_F^M - V_F^U\right), & (20)\\ rV_F^M &= (1-\alpha)\, v(p) + \lambda\left(V_F^U - V_F^M\right), & (21)\\ rV_T^U &= \mu_T(\theta)\left(V_T^M - V_T^U\right), & (22)\\ rV_T^M &= \alpha v(p) + \lambda\left(V_T^U - V_T^M\right). & (23) \end{aligned}$$

Under M-integration, the Bellman equations depends on the share $\phi \equiv u_{T^N}/\left[u_{T^N}+u_{T^S}\right]$ of unmatched Northern traders active in the South:

$$\begin{aligned}
rV_F^U &= \mu_F\left(\theta^M\right)\left[\phi V_{F^N}^M+(1-\phi)V_{F^S}^M-V_F^U\right], &(24)\\
rV_{F^S}^M &= \left(1-\alpha^S\right)v(p^W)+\lambda\left(V_F^U-V_{F^S}^M\right), &(25)\\
rV_{F^N}^M &= \left(1-\alpha^N\right)v(p^W)+\lambda\left(V_F^U-V_{F^N}^M\right), &(26)\\
rV_{T^N}^U &= \mu_T\left(\theta^M\right)\left(V_{T^N}^M-V_{T^N}^U\right), &(27)\\
rV_{T^N}^M &= \alpha^N v(p^W)+\lambda\left(V_{T^N}^U-V_{T^N}^M\right), &(28)\\
rV_{T^S}^U &= \mu_T\left(\theta^M\right)\left(V_{T^S}^M-V_{T^S}^U\right), &(29)\\
rV_{T^S}^M &= \alpha^S v(p^W)+\lambda\left(V_{T^S}^U-V_{T^S}^M\right). &(30)
\end{aligned}$$

where θ^M is the intermediation level in the Southern island under M-integration and all other variables are defined in the same way as in our main paper. Using the previous expressions and equation (19), we can establish the following proposition.

Proposition 5 *Suppose that there is an endogenous number of farmers and an exogenous number of traders. Then W-integration worsens the farmers' terms of trade, increases the traders' terms of trade, and makes all agents (weakly) better off. By contrast, M-integration always creates winners and losers and may decrease aggregate welfare.*

Proof. We decompose our proof into three parts. First, we compute the traders' margins and the level of intermediation under autarky. Second, we analyze the consequences of W-integration. Third, we analyze the consequences of M-integration.

Traders' margins and intermediation level. Since farmers play the same role as traders in our main paper, we can use the same logic as in Appendix A to compute the traders' margin, α, and the intermediation level, θ, under autarky. By equations (20)-(23), we know that

$$\begin{aligned}
V_T^M-V_T^U &= \frac{\alpha v\left(p\right)}{r+\lambda+\mu_T\left(\theta\right)} &(31)\\
V_F^M-V_F^U &= \frac{\left(1-\alpha\right)v\left(p\right)}{r+\lambda+\mu_F\left(\theta\right)} &(32)
\end{aligned}$$

We also know that Nash bargaining implies

$$\beta\left(V_T^M-V_T^U\right)=\left(1-\beta\right)\left(V_F^M-V_F^U\right). \tag{33}$$

Combining equations (31)-(32), we obtain

$$\alpha=\beta-\frac{\beta\left(V_F^M-V_F^U\right)\left[\mu_F\left(\theta\right)-\mu_T\left(\theta\right)\right]}{v\left(p\right)}. \tag{34}$$

By equation (19) and (20), we also know that

$$V_F^M - V_F^U = \frac{rV_F^*}{\mu_F(\theta)}. \tag{35}$$

Equations (34) and (35) imply

$$\alpha = \beta - \frac{\beta r V_F^* (\theta - 1)}{\theta v(p)}. \tag{36}$$

Combining equations (32), (35), and (36), we finally get

$$\frac{v(p) - rV_F^*}{rV_F^*} = \frac{r + \lambda + \beta \mu_T(\theta)}{(1-\beta)\mu_F(\theta)}. \tag{37}$$

Equations (36) and (37) are the counterparts of equation (17) and (20) in our main paper. For future reference, note that equations (36) and (37) further imply

$$\alpha = \beta \cdot \left[\frac{r + \mu_T(\theta) + \lambda}{r + \lambda + (1-\beta)\mu_F(\theta) + \beta \mu_T(\theta)} \right]. \tag{38}$$

Consequences of W-integration. We are now ready to discuss the consequences of W-integration. Like in our main paper, upon integration, the relative price of coffee will jump to $p^W = a_C^*/a_S^*$. Accordingly, all Southern farmers involved in market activities will immediately specialize in coffee production, which will raise the indirect utility all matched farmer-trader pairs from $v(p)$ to $v\left(p^W\right) > v(p)$. By equations (37) and (38), this will decrease the level of intermediation $\theta^W < \theta$, which will, in turn, worsen the farmers' terms of trade and improve the traders' terms of trade. Using the same logic as in our main paper, it is then easy to check that all agents are better off under W-integration.

Consequences of M-integration. Let us now turn to the consequences of M-integration. Combining equation (19) with our Bellman equations, (24)-(30), and invoking Nash bargaining, we can compute the Southern and Northern traders' share, α^S and α^N, and the intermediation level, θ^M, as we did in the proof of Proposition 4. Simple algebra leads to

$$\alpha^S = \beta \cdot \left[\frac{r + \lambda + \mu_T\left(\theta^M\right)}{r + \lambda + \beta \mu_T\left(\theta^M\right)} \right] \left[\frac{v(p^W) - rV_F^*}{v(p^W)} \right], \tag{39}$$

$$\alpha^N = \bar{\beta} \cdot \left[\frac{r + \lambda + \mu_T\left(\theta^M\right)}{r + \lambda + \bar{\beta} \mu_T\left(\theta^M\right)} \right] \left[\frac{v(p^W) - rV_F^*}{v(p^W)} \right], \tag{40}$$

$$\frac{v(p^W) - rV_F^*}{rV_F^*} = \left[\phi \frac{\left(1-\bar{\beta}\right)\mu_F\left(\theta^M\right)}{r + \lambda + \bar{\beta}\mu_T\left(\theta^M\right)} + (1-\phi) \frac{(1-\beta)\mu_F\left(\theta^M\right)}{r + \lambda + \beta \mu_T\left(\theta^M\right)} \right]^{-1}. \tag{41}$$

Note that $\bar{\beta} > \beta$ implies $\alpha^N > \alpha^S$ as well as $\theta^M \geq \theta^W$ with strict inequality if $\phi > 0$. Using equations (19), (24), (25), (26), (29) and(30), we can also express the expected lifetime utilities of the different agents as follows:

$$V_{F^S}^M = \frac{\left(1-\alpha^S\right) v(p^W) + \lambda V_F^*}{r+\lambda}, \quad (42)$$

$$V_{F^N}^M = \frac{\left(1-\alpha^N\right) v(p^W) + \lambda V_F^*}{r+\lambda}, \quad (43)$$

$$V_{T^S}^U = \frac{\alpha^S \mu_T\left(\theta^M\right) v\left(p^W\right)}{r\left[r+\lambda+\mu_T\left(\theta^M\right)\right]}, \quad (44)$$

$$V_{T^S}^M = \frac{\alpha^S\left[r+\mu_T\left(\theta^M\right)\right] v(p^W)}{r\left[r+\lambda+\mu_T\left(\theta^M\right)\right]}. \quad (45)$$

Equations (42), (44) and (45), together with Nash bargaining further imply

$$V_{F^S}^M = V_F^* + (1-\beta)\left[\frac{v(p^W) - rV_F^* - rV_{T^S}^U}{r+\lambda}\right]. \quad (46)$$

Equation (46) implies that M-integration always creates winners and losers: if $\Delta V_{T^S}^U > 0$, then $\Delta V_{F^S}^M < 0$ and vice versa. To establish that M-integration may also decrease aggregate welfare, we first note that at any date t before M-integration and after W-integration, aggregate welfare is given by

$$W(t) = u_F(t) V_F^U(t) + u_T(t) V_T^U(t) + \left[N_F - u_F(t)\right]\left[V_F^M(t) + V_T^M(t)\right].$$

By equations (19), (21), and (23), we know that

$$V_F^U(t) = V_F^*,$$

$$V_F^M(t) + V_T^M(t) = \frac{v(p^W) + \lambda\left[V_F^* + V_T^U(t)\right]}{r+\lambda}.$$

Thus aggregate welfare can be rearranged as

$$W(t) = V_{T^S}^U(t)\left[u_{T^S}(t) + \frac{\lambda\left[N_F - u_F(t)\right]}{r+\lambda}\right] + V_F^*\left[u_F(t) + \frac{\lambda\left[N_F - u_F(t)\right]}{r+\lambda}\right] + \frac{\left[N_F - u_F(t)\right] v(p^W)}{r+\lambda}.$$

Since $v\left(p^W\right)$ is not affected by M-integration and $u_{T^S}(t)$ and $u_F(t)$ are predetermined at date t, the previous expression implies that changes in social welfare caused by M-integration, ΔW, must reflect changes in the expected lifetime utility of unmatched traders, $\Delta V_{T^S}^U$. By equations (39) and (44), we know that the expected lifetime utility of unmatched Southern traders after M-integration is given by

$$V_{T^S}^U = \frac{\beta \mu_T\left(\theta^M\right)\left[v(p^W) - rV_F^*\right]}{r\left[r+\lambda+\beta\mu_T\left(\theta^M\right)\right]}.$$

Before M-integration, the exact same logic implies

$$V_{T^S}^U = \frac{\beta \mu_T\left(\theta^W\right)\left[v(p^W) - rV_F^*\right]}{r\left[r+\lambda+\beta\mu_T\left(\theta^W\right)\right]}.$$

Since $\theta^M \geq \theta^W$, the two previous equations implies $\Delta V_{TS}^U \leq 0$, and in turn, $\Delta W \leq 0$. To conclude note that if a positive measure of Northern traders are active in the South, then we necessarily have $\theta^M > \theta^W$, and in turn, $\Delta W < 0$. Since one can always find Northern productivity levels, $1/a_C^*$ and $1/a_S^*$, and Northern market institutions, β^* and τ^*, such that the expected lifetime utility V_T^* of unmatched Northern traders is lower than their expected lifetime utility in the South in the absence of Northern traders, $\left(V_{T^N}^U\right)_{\phi=0}$, M-integration may decrease aggregate welfare. **QED** ■

3 Occupational Choices

3.1 Assumptions

Compared to Section 2 in our main paper, we now assume that the island is inhabited by a measure L of agents who, at any point in time, can either become farmers or traders. For simplicity, we also assume that traders can be connected to Walrasian markets at zero cost, $\tau = 0$, as in our previous extension. The rest of our model is unchanged. In terms of equilibrium conditions, the key difference between the model described in our main paper and the present one is that the expected lifetime utility of unmatched farmers and traders must now satisfy:

$$\begin{aligned} V_T^U &\geq V_F^U, \text{ if } N_T > 0, \\ V_F^U &\geq V_T^U, \text{ if } N_F > 0. \end{aligned}$$

This is the counterpart of the free entry condition, equation (10), in our original model. In any non-degenerate equilibrium with both types of agent being active, the previous conditions, of course, imply that agents must be indifferent between becoming a farmer or a trader:

$$V_T^U = V_F^U, \text{ if } N_T, N_F > 0. \tag{47}$$

The rest of the model is as described in our main paper. In particular, we assume that $\overline{\beta} > \beta$ when analyzing the consequences of M-integration.

3.2 Predictions

Like in the previous extension, the Bellman equations characterizing the expected lifetime utilities of the different agents in our economy under autarky and W-integration are given by equations (20)-(23) and their expected lifetime utilities under M-integration are given by equations (24)-(30). Using the previous equations and equation (47), we can establish the following proposition.

Proposition 6 *Suppose that all agents can become farmers or traders at any point in time. Then, W-integration does not affect the farmers' and traders' terms of trade and makes all agents better off. By contrast, M-integration may create winners and losers and decrease aggregate welfare.*

Proof. We again decompose our proof into three parts. First, we compute the traders' margins and the level of intermediation under autarky. Second, we analyze the consequences of W-integration.

Third, we analyze the consequences of M-integration.

Traders' margins and intermediation level. Let us first compute the level of intermediation θ under autarky. Combining equations (20) and (22) with equation (47), which must hold under autarky, we obtain

$$\mu_F(\theta)\left(V_F^M - V_F^U\right) = \mu_T(\theta)\left(V_T^M - V_T^U\right).$$

By Nash Bargaining, we know that

$$(1-\beta)\left(V_T^M - V_T^U\right) = \beta\left(V_F^M - V_F^U\right).$$

Combining the two previous expressions we obtain

$$\theta = \frac{\beta}{1-\beta}. \tag{48}$$

Let us now turn to the expected lifetime utilities of the different agents under autarky. Equations (20) and (21) imply

$$rV_F^U = \frac{(1-\alpha)\mu_F(\theta)v(p)}{r+\lambda+\mu_F(\theta)}, \tag{49}$$

$$rV_F^M = \frac{(1-\alpha)[r+\mu_F(\theta)]v(p)}{r+\lambda+\mu_F(\theta)}. \tag{50}$$

Similarly, equations (22) and (23) imply

$$rV_T^U = \frac{\alpha\mu_T(\theta)v(p)}{r+\lambda+\mu_T(\theta)}, \tag{51}$$

$$rV_T^M = \frac{\alpha[r+\mu_T(\theta)]v(p)}{r+\lambda+\mu_T(\theta)}. \tag{52}$$

Combining equations (47), (48), (49) and (51), we can express the share α accruing to traders as

$$\frac{\alpha}{1-\alpha} = \frac{\theta[r+\lambda+\mu_T(\theta)]}{[r+\lambda+\mu_F(\theta)]}. \tag{53}$$

For future reference, note that using equation (48), the previous expression can also be rearranged as

$$\alpha = \frac{\beta\left[r+\lambda+\mu_T\left(\frac{\beta}{1-\beta}\right)\right]}{r+\lambda+(1-\beta)\mu_F\left(\frac{\beta}{1-\beta}\right)+\beta\mu_T\left(\frac{\beta}{1-\beta}\right)}. \tag{54}$$

Consequences of W-integration. We are now ready to discuss the consequences of W-integration. Like in our main paper, upon integration, the relative price of coffee will jump to $p^W = a_C^*/a_S^*$. Accordingly, all Southern farmers will immediately specialize in coffee production, which will raise the indirect utility all matched farmer-trader pairs from $v(p)$ to $v\left(p^W\right) > v(p)$. By equation (48), W-integration therefore leaves the level of intermediation unchanged, $\theta^W = \theta$, and by equation (53), the traders, margins, $\alpha^W = \alpha$. Thus W-integration does not affect the farmers' and traders' terms of trade. Finally, since $v\left(p^W\right) > v(p)$, $\theta^W = \theta$, and $\alpha^W = \alpha$, equations (49)-(52) immediately imply that W-integration makes all agents better off.

Consequences of M-integration. Let us now turn to the consequences of M-integration. For simplicity, we restrict ourselves to a situation in which Northern productivity levels, $1/a_C^*$ and $1/a_S^*$, and Northern market institutions, β^* and τ^*, are such that the expected lifetime utility V_T^* of unmatched Northern traders searching in the North, both before and after M-integration, is exactly equal to the expected lifetime utility $\left(V_T^U\right)^W$ of a Southern trader under W-integration. In what follows we will posit an equilibrium in which all Southern agents become farmers after M-integration. Thus the share of unmatched Northern traders $\phi = 1$. We will then verify at the end of our proof that there indeed exist parameter values such that it is optimal for all Southern agents to become farmers after M-integration.

Let us start by computing the Northern traders' margins, α^N, and the intermediation level, θ^M, as we did in the proof of Proposition 4. Combining equations (24), (26), (27), and (28), and invoking Nash bargaining, we obtain after simple algebra

$$\alpha^N = \overline{\beta} + \frac{\left(1-\overline{\beta}\right) r V_T^* \left(1-\theta^M\right)}{v\left(p^W\right)}. \tag{55}$$

$$\frac{rV_T^* - \overline{\beta} v\left(p^W\right)}{rV_T^*} = \left(1-\overline{\beta}\right)\left(1-\theta^M\right) - \frac{r+\lambda}{\mu_T\left(\theta^M\right)}. \tag{56}$$

where V_T^* is the expected lifetime utility of a Northern trader searching in the North. For future reference, note that θ^M is decreasing in V_T^* and increasing in $v\left(p^W\right)$ and $\overline{\beta}$. Note also that if $\overline{\beta} = \beta$ and $V_T^* = \left(V_T^U\right)^W$, then $\theta^M = \theta^W$. Thus under our assumptions that $V_T^* = \left(V_T^U\right)^W$ and $\overline{\beta} > \beta$, the level of intermediation will increase in the South under M-integration in the posited equilibrium.

Compared to the proof of Proposition 5, it is convenient to show first that M-integration may decrease aggregate welfare, and that in those circumstances, it may also lead to distributional conflicts. At any date t before M-integration and after W-integration, aggregate welfare is given by

$$W(t) = u_F(t) V_F^U(t) + u_T(t) V_T^U(t) + \left[N_F - u_F(t)\right]\left[V_F^M(t) + V_T^M(t)\right].$$

By equations (47), (21), and (23), we know that

$$\begin{aligned} V_T^U(t) &= V_F^U(t), \\ V_F^M(t) + V_T^M(t) &= \frac{v(p^W) + \lambda\left[V_F^U(t) + V_T^U(t)\right]}{r+\lambda}. \end{aligned}$$

Thus aggregate welfare can be rearranged as

$$W(t) = V_F^U(t)\left[u_F(t) + u_T(t) + \frac{2\lambda\left[N_F - u_F(t)\right]}{r+\lambda}\right] + \frac{\left[N_F - u_F(t)\right] v(p^W)}{r+\lambda}.$$

Since $v\left(p^W\right)$ is not affected by M-integration and $u_T\left(t\right)$ and $u_F\left(t\right)$ are predetermined at date t, the previous expression implies that if all Southern agents become farmers after M-integration, which, as mentioned earlier, we will check at the end of this proof, changes in social welfare caused by M-integration, ΔW, must reflect changes in the expected lifetime utility of unmatched farmers, ΔV_F^U. By equations (20) and (21), we know that the change in the steady state value of the expected lifetime utility of unmatched farmers caused by M-integration is such that

$$\frac{r\Delta V_F^U}{v(p^W)} = \frac{\mu_F\left(\theta^M\right)\left(1-\alpha^N\right)}{r+\lambda+\mu_F\left(\theta^M\right)} - \frac{\mu_F\left(\theta^W\right)\left(1-\alpha^W\right)}{r+\lambda+\mu_F\left(\theta^W\right)}. \tag{57}$$

Like in our main paper, we will now show that $\Delta V_F^U < 0$ if $\overline{\beta} > \beta > \varepsilon$. To do so we establish the following lemma.

Lemma 1 *Suppose that only Northern traders are active in the South, then* $dV_F^U/d\overline{\beta} \geq 0$ *if and only if* $\overline{\beta} \leq \varepsilon \equiv d\ln m\left(u_F, u_T\right)/\,d\ln u_T$.

Proof. We proceed in two steps.

Step 1: *If only Northern traders are active in the South, then the expected lifetime utility of unmatched Southern farmers satisfy*

$$V_F^U = \frac{\left(1-\overline{\beta}\right)\theta^M}{\overline{\beta}} V_T^*. \tag{58}$$

By equations (20) and (21), we know that

$$rV_F^U = \frac{\mu_F\left(\theta^M\right)\left(1-\alpha^N\right)v\left(p^W\right)}{r+\lambda+\mu_F\left(\theta^D\right)}.$$

Combining this expression with equation (55), we obtain

$$rV_F^U = \frac{\left(1-\overline{\beta}\right)\mu_F\left(\theta^M\right)\left[1-\frac{rV_T^*\left(1-\theta^M\right)}{v(p^W)}\right]v\left(p^W\right)}{r+\lambda+\mu_F\left(\theta^M\right)}. \tag{59}$$

After simple algebra, equations (56) and (59) imply

$$rV_F^U = \frac{\left(1-\overline{\beta}\right)\mu_F\left(\theta^M\right)v\left(p^W\right)}{r+\lambda+\overline{\beta}\mu_T\left(\theta^M\right)+\left(1-\overline{\beta}\right)\mu_F\left(\theta^M\right)},$$

which can be rearranged as

$$rV_F^U = \left[\frac{\left(1-\overline{\beta}\right)\theta^M}{\overline{\beta}}\right]\cdot\left[\frac{\overline{\beta}v\left(p^W\right)}{\frac{(r+\lambda)}{\mu_T\left(\theta^M\right)}+\overline{\beta}+\left(1-\overline{\beta}\right)\theta^M}\right].$$

Equation (58) directly derives from the previous expression and equation (56).

Step 2: *If only Northern traders are active in the South, then the expected lifetime utility of unmatched Southern farmers satisfy*

$$\frac{dV_F^U}{d\overline{\beta}} = \frac{V_T^* \theta^M}{\overline{\beta}^2} \left[\frac{(r+\lambda)\left(\varepsilon - \overline{\beta}\right)}{(r+\lambda)(1-\varepsilon) + \left(1-\overline{\beta}\right) \mu_F\left(\theta^M\right)} \right], \tag{60}$$

with $(r+\lambda)(1-\varepsilon) + \left(1-\overline{\beta}\right) \mu_F\left(\theta^M\right) > 0$.

By differentiating equation (58), we get

$$\frac{dV_F^U}{d\overline{\beta}} = \frac{V_T^* \theta^M}{\left(\overline{\beta}\right)^2} \left[\left(1-\overline{\beta}\right) \frac{d\ln\theta^M}{d\ln\overline{\beta}} - 1 \right]. \tag{61}$$

By directly differentiating equation (56), it is easy to check that

$$\frac{d\ln\theta^M}{d\ln\overline{\beta}} = \frac{r+\lambda+\mu_F\left(\theta^M\right)}{(r+\lambda)(1-\varepsilon) + \left(1-\overline{\beta}\right) \mu_F\left(\theta^M\right)}, \tag{62}$$

where $(r+\lambda)(1-\varepsilon) + \left(1-\overline{\beta}\right) \mu_F\left(\theta^M\right) > 0$ since θ^M is increasing in $\overline{\beta}$. Equation (60) directly derives from equations (61) and (62). By Step 2, if only Northern traders are active in the South, then $dV_F^U/d\overline{\beta} \geq 0$ if and only if $\overline{\beta} \leq \varepsilon$. **QED**

Lemma 1 states that starting from an equilibrium in which only Northern traders are active in the South, a hypothetical increase in the bargaining power of Northern traders decreases aggregate welfare if $\overline{\beta} > \varepsilon \equiv d\ln m(u_F, u_T)/d\ln u_T$. A direct corollary of Lemma 1 is that if M-integration leads to the exit of all unmatched Southern traders, as posited in this equilibrium, then it must also decrease aggregate welfare in the South whenever $\overline{\beta} > \beta > \varepsilon$. We now demonstrate that there indeed exist parameter values such that $\overline{\beta} > \beta > \varepsilon$ and only Northern traders are active under M-integration. To do so we build on the following lemma.

Lemma 2 *Suppose that only Northern traders are active in the South. Then there exist* $\delta > 0$ *and* $\eta > 0$ *such that* $V_F^U > V_{TS}^U$ *if* $\beta \in (\varepsilon, \varepsilon + \delta)$ *and* $\overline{\beta} \in (\beta, \beta + \eta)$*, where* V_{TS}^U *is the expected lifetime utility that a Southern agent would get if he were to become a trader after M-integration.*

Proof. By equations (20)-(23), we know that

$$rV_F^U = \frac{\mu_F\left(\theta^M\right)\left(1-\alpha^N\right) v\left(p^W\right)}{r+\lambda+\mu_F\left(\theta^M\right)} \tag{63}$$

$$rV_{F^N}^M = \frac{\left[r+\mu_F\left(\theta^M\right)\right]\left(1-\alpha^N\right) v\left(p^W\right)}{r+\lambda+\mu_F\left(\theta^M\right)} \tag{64}$$

$$rV_{TS}^U = \frac{\mu_T\left(\theta^M\right)\alpha^S v\left(p^W\right)}{r+\lambda+\mu_T\left(\theta^M\right)} \tag{65}$$

$$rV_{TS}^M = \frac{\left[r+\mu_T\left(\theta^M\right)\right]\alpha^S v\left(p^W\right)}{r+\lambda+\mu_T\left(\theta^M\right)} \tag{66}$$

Equations (63) and (65) imply

$$\frac{V^U_{T^S}}{V^U_F} = \left[\frac{r+\lambda+\mu_F\left(\theta^M\right)}{r+\lambda+\mu_T\left(\theta^M\right)}\right]\left(\frac{1}{\theta^M}\right)\left(\frac{\alpha^S}{1-\alpha^N}\right) \tag{67}$$

Let us now express α^S as a function of α^N. By equations (22) and (23), we know that

$$\left[r+\lambda+\mu_T\left(\theta^M\right)\right]\left(V^M_{T^S}-V^U_{T^S}\right) = \alpha^S v\left(p^W\right). \tag{68}$$

Similarly, by equations (20) and (21), we know that

$$(r+\lambda)\left(V^M_{F^S}-V^U_F\right) = \left(1-\alpha^S\right)v\left(p^W\right) - \mu_F\left(\theta^M\right)\left(V^M_{F^N}-V^U_F\right). \tag{69}$$

We also know that Nash bargaining implies

$$(1-\beta)\left(V^M_{T^S}-V^U_{T^S}\right) = \beta\left(V^M_{F^S}-V^U_F\right). \tag{70}$$

Multiplying equation (68) by $(1-\beta)$ and equation (69) by β and subtracting, we get

$$\alpha^S = \beta + \frac{(1-\beta)\,\mu_T\left(\theta^M\right)\left(V^M_{T^S}-V^U_{T^S}\right) - \beta\mu_F\left(\theta^D\right)\left(V^M_{F^N}-V^U_F\right)}{v\left(p^W\right)}. \tag{71}$$

Combining equations (63)-(66) and (71), we get

$$\alpha^S = \beta\left[\frac{r+\lambda+\mu_T\left(\theta^M\right)}{r+\lambda+\mu_F\left(\theta^M\right)}\right]\left[\frac{r+\lambda+\alpha^N\mu_F\left(\theta^M\right)}{r+\lambda+\beta\mu_T\left(\theta^M\right)}\right].$$

Together with equation (67), this implies

$$\frac{V^U_{T^S}}{V^U_F} = \left(\frac{\beta}{\theta^D}\right)\left(\frac{r+\lambda+\alpha^N\mu_F\left(\theta^D\right)}{\left(1-\alpha^N\right)\left[r+\lambda+\beta\mu_T\left(\theta^D\right)\right]}\right). \tag{72}$$

By equation (63), we know that

$$\alpha^N = 1 - \frac{rV^U_F\left[r+\lambda+\mu_F\left(\theta^D\right)\right]}{v\left(p^W\right)\mu_F\left(\theta^D\right)}.$$

Combining this expression with equation (72), we obtain

$$\frac{V^U_{T^S}}{V^U_F} = \left(\frac{\beta\mu_T\left(\theta^D\right)}{r+\lambda+\beta\mu_T\left(\theta^D\right)}\right)\left(\frac{v\left(p^W\right)}{rV^U_F}-1\right).$$

This implies

$$\frac{d\ln\left(V_{T^S}^U/V_F^U\right)}{d\overline{\beta}} = \frac{d\ln\left\{\beta\mu_T\left(\theta^M\right)/\left[r+\lambda+\beta\mu_T\left(\theta^M\right)\right]\right\}}{d\theta^M}\left(\frac{d\theta^M}{d\overline{\beta}}\right) + \frac{d\ln\left(\frac{v\left(p^W\right)}{rV_F^U}-1\right)}{dV_F^U}\left(\frac{dV_F^U}{d\overline{\beta}}\right).$$

It is easy to check that $\frac{d\ln\left\{\beta\mu_T\left(\theta^M\right)/\left[r+\lambda+\beta\mu_T\left(\theta^M\right)\right]\right\}}{d\theta^M} < 0$ and $\frac{d\ln\left(\frac{v\left(p^W\right)}{rV_F^U}-1\right)}{dV_F^U} < 0$. Since $\frac{d\theta^M}{d\overline{\beta}} > 0$ and $\left(\frac{dV_F^U}{d\overline{\beta}}\right)_{\overline{\beta}=\varepsilon} = 0$, we therefore have

$$\left(\frac{d\ln\left(V_{T^S}^U/V_F^U\right)}{d\overline{\beta}}\right)_{\overline{\beta}=\varepsilon} < 0.$$

Lemma 2 derives from the previous inequality and the fact that $V_{T^S}^U = V_F^U$ if $\overline{\beta} = \beta = \varepsilon$. **QED**

Lemma 2 implies that starting from β close enough to ε, there exists an equilibrium in which only Northern traders are active in the South under M-integration if $\overline{\beta}$ is also close enough to β. Combining this observation with Lemma 1, we have demonstrated the existence of an equilibrium such that aggregate welfare is strictly lower under M-integration than under W-integration.

To conclude our proof we now demonstrate that M-integration may also create winners and losers. The argument is similar to the proof of Lemma 2. By equation (29), we know that

$$V_{T^S}^M = \frac{\left[r+\mu_T\left(\theta^M\right)\right]}{\mu_T\left(\theta^M\right)}V_F^U.$$

Differentiating we therefore have

$$\frac{d\ln V_{T^S}^M}{d\overline{\beta}} = \frac{d\ln\left\{\left[r+\mu_T\left(\theta^M\right)\right]/\mu_T\left(\theta^M\right)\right\}}{d\theta^M}\left(\frac{d\theta^M}{d\overline{\beta}}\right) + \frac{d\ln V_F^U}{d\ln\overline{\beta}}.$$

It is easy to check that $\frac{d\ln\left\{\left[r+\mu_T\left(\theta^M\right)\right]/\mu_T\left(\theta^M\right)\right\}}{d\theta^M} > 0$. Since $\frac{d\theta^M}{d\overline{\beta}} > 0$, $\frac{dV_F^U}{d\overline{\beta}} < 0$ if $\overline{\beta} > \varepsilon$, and $\left(\frac{dV_F^U}{d\overline{\beta}}\right)_{\overline{\beta}=\varepsilon} = 0$, there must exist $\delta' > 0$ and $\eta' > 0$ such that $\frac{d\ln V_{TS}^M}{d\overline{\beta}} > 0$ and $\frac{d\ln V_F^U}{d\ln\overline{\beta}} < 0$ if $\beta \in (\varepsilon, \varepsilon+\delta')$ and $\overline{\beta} \in (\beta, \beta+\eta')$. The fact that M-integration may create winners and losers directly follow from this observation. **QED** ■

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