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EQUILIBRIUM BIAS OF TECHNOLOGY

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This paper presents three sets of results about equilibrium bias of technology. First, I show that when the menu of technological possibilities only allows for factor-augmenting technologies, the increase in the supply of a factor induces technological change *relatively biased* toward that factor—meaning that the induced technological change increases the relative marginal product of the factor becoming more abundant. Moreover, this induced bias can be strong enough to make the relative marginal product of a factor increasing in response to an increase in its supply, thus leading to an *upward-sloping* relative demand curve. I also show that these results about relative bias do not generalize when more general menus of technological possibilities are considered. Second, I prove that under mild assumptions, the increase in the supply of a factor induces technological change that is *absolutely biased* toward that factor—meaning that it increases its marginal product at given factor proportions. The third and most important result in the paper establishes the possibility of and conditions for *strong absolute equilibrium bias*—whereby the price (marginal product) of a factor *increases* in response to an increase in its supply. I prove that, under some regularity conditions, there will be strong absolute equilibrium bias *if and only if* the aggregate production function of the economy fails to be jointly concave in factors and technology. This type of failure of joint concavity is possible in economies where equilibrium factor demands and technologies result from the decisions of different agents.

KEYWORDS: Biased technology, economic growth, endogenous technical change, innovation, nonconvexity.

1. INTRODUCTION

DESPITE THE GENERALLY AGREED UPON IMPORTANCE of technological progress for economic growth and a large and influential literature on technological progress,[2] the determinants of the *direction* and *bias* of technological change are not well understood. An analysis of equilibrium bias of technology is important for a number of reasons. First, in most situations, technical change is not neutral: it benefits some factors of production, while directly or indirectly reducing the compensation of others. This possibility is illustrated both by the distributional impact of the major technologies introduced during the Industrial Revolution and by the effects of technological change on the structure

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[2]See, among others, Reinganum (1981, 1985), Spence (1984), and Grossman and Shapiro (1987) in the industrial organization literature and Romer (1990), Segerstrom, Anant, and Dinopoulos (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), and Stokey (1991) in the economic growth literature.

of wages during the past half century or so.[3] The bias of technological change determines its distributional implications, for example, which groups are the winners and which will be the losers from technological progress, and thus the willingness of different groups to embrace new technologies. Second, the bias of technology determines how factor prices respond to changes in factor supplies, for example, whether equilibrium factor demand curves are downward-sloping as in basic producer theory. Third, understanding the determinants of innovation requires an analysis of the bias and direction of new technologies, for example, for evaluating whether lines of previous innovations or technologies will be exploited in the future and the potential compatibility between old and new technologies.[4]

Recent research has focused on the *relative bias of technology*—defined as the impact of technology on relative factor prices at given factor proportions.[5] For example, using a growth model with two factors, factor-augmenting technologies and a constant elasticity of substitution between factors, Acemoglu (2002) showed that when a factor becomes more abundant, technology becomes endogenously more (relatively) biased toward that factor. Moreover, this induced bias could be strong enough that relative demand curves for factors could be *upward-sloping*, rather than downward-sloping as in models with exogenous technology. These results were derived in the context of versions of the endogenous growth models of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), and as such, they incorporated a number of specific features. Investigating whether these results are an artifact of the assumptions imposed in this class of models is important not only for obtaining general theorems about equilibrium bias, but also because without such an investigation, our understanding of the forces that determine the nature of technological progress would be incomplete.

This paper presents three sets of results about equilibrium bias of technology.

First, the results concerning relative bias mentioned above extend to a more general environment with factor-augmenting technologies. In particular, it is shown that in this more general environment an increase in the abundance of a factor always makes technology relatively biased toward this factor (*weak relative equilibrium bias*). Moreover, if the (local) elasticity of substitution between factors is sufficiently large, the relative demand curves for factors are upward-sloping (*strong relative equilibrium bias*).[6] It is also shown, however,

[3]On the biases and distributional effects of the technologies introduced during the Industrial Revolution, see Mantoux (1961) or Mokyr (1990). On the skill bias of more recent technologies, see, for example, Autor, Krueger, and Katz (1998).

[4]See, for example, Farrell and Saloner (1985) and Katz and Shapiro (1985).

[5]Equivalently, relative bias can be described as referring to cost-minimizing relative factor demands at a given factor price ratio. The two definitions of relative bias are equivalent for the purposes of this paper.

[6]The difference between weak and strong bias can be alternatively expressed as follows: weak bias refers to how factor prices or demands change at *given* factor proportions when there is an

that these results do not extend further; once the menu of possibilities includes non-factor-augmenting technologies, it is easy to construct examples where an increase in the abundance of a factor induces technology to be biased against this factor.

A more natural concept may be *absolute* bias, which has not received much attention in the literature. A technology is *absolutely biased* toward a factor if it increases its marginal product. While a study of relative bias is relevant to a range of questions (including those concerning inequality), the study of absolute bias is important for understanding how factor demands respond to changes in factor supplies and for the implications of technological change for the level of factor returns. The *second* major result in this paper is that an increase in the abundance of a factor always induces a change in technology that is absolutely biased toward this factor (*weak absolute equilibrium bias*). The intuition for this result is simple and is related to a type of "market size effect": when a factor becomes more abundant, technologies that make use of this factor become more valuable, ensuring a change in the direction of technological change toward increasing the demand and thus the equilibrium price of this factor (at given factor proportions).

The *third* major result in this paper is the most important and relates to *strong absolute equilibrium bias*. Strong absolute equilibrium bias refers to a situation in which an increase in the abundance of a factor induces sufficient bias to increase its marginal product (price), making the endogenous-technology factor demand curve upward-sloping. Theorem 4 establishes that, under some regularity conditions, there will be strong absolute equilibrium bias if and only if the production possibilities set of the economy is nonconvex at the equilibrium point.

To describethis result at a heuristic level, let θ represent the technology choices and let Z denote the only factor in the economy. Suppose that θ is decided by a technology producer (which may sell this technology or intermediate goods embodying this technology to firms), while firms make the employment decisions for factor Z, taking the available technology θ as given. A (general) equilibrium requires both the technology producer and firms to maximize their profits. Under certain conditions to be specified below, the profits of the technology producer can be represented as a transformation of the net aggregate production function of the economy $F(Z, \theta)$ (inclusive of the costs of producing new technologies). A point (Z^*, θ^*) is an equilibrium if it is an optimum both for the technology producer (i.e., $F(Z^*, \theta^*) \geq F(Z^*, \theta')$ for all feasible θ') and for final good producers (i.e., $F(Z^*, \theta^*) - w_Z Z^* \geq F(Z', \theta^*) - w_Z Z'$

induced change in technology (i.e., an adjustment in technology to the equilibrium level corresponding to the new factor proportions). Strong bias, on the other hand, refers to the change in factor prices or demand at the new factor proportions after the induced change in technology. It therefore consists of the direct effect of the change in factor proportions on (relative) factor prices at given technology *plus* the impact of the induced technology.

for all feasible Z', where w_Z is the price of factor Z). Although $F(Z, \theta)$ will be concave in Z and in θ at an equilibrium point (Z^*, θ^*), it need not be *jointly* concave in (Z, θ). Instead, (Z^*, θ^*) could be a "quasi-saddle point," meaning that there exists a direction in the (Z, θ) plane in which both $F(Z, \theta)$ and $F(Z, \theta) - w_Z Z$ increase (see footnote 30 below). The essence of Theorem 4 is that whenever $F(Z, \theta)$ fails to be jointly concave in (Z, θ) at an equilibrium (Z^*, θ^*) (or equivalently whenever (Z^*, θ^*) is a "quasi-saddle point"), there will be strong absolute equilibrium bias and, conversely, if $F(Z, \theta)$ is jointly concave in (Z, θ) at (Z^*, θ^*), then there cannot be strong absolute equilibrium bias. Intuitively, when (Z^*, θ^*) is a quasi-saddle point, a change in Z induces the technology producer to develop technologies that move the economy in the (Z, θ) direction that increases $F(Z, \theta)$ and $F(Z, \theta) - w_Z Z$, and this translates into a higher price for Z (higher w_Z). When $F(Z, \theta)$ is jointly concave at (Z^*, θ^*), this is not possible and an increase in Z necessarily reduces its equilibrium price.

This discussion illustrates three important points. The first is the intimate link between nonconvexity and strong equilibrium bias—failure of joint concavity is necessary (and essentially sufficient) for strong absolute equilibrium bias. The second is that in a competitive equilibrium, where the first and second welfare theorems hold, strong absolute equilibrium bias is not possible, because such an environment would ensure that (Z^*, θ^*) is a maximum. Third and related is that strong absolute equilibrium bias is not a pathological phenomenon. Once we consider noncompetitive environments, the equilibrium point (Z^*, θ^*) is often a quasi-saddle point; (Nash) equilibrium only requires each player to optimize and does not imply joint optimization.

The early literature on technological bias includes the work on induced innovations by, among others, Hicks (1932), Kennedy (1964), Samuelson (1965), Drandakis and Phelps (1965), Nordhaus (1973), and Binswanger and Ruttan (1978), although these papers did not specify microfounded models of technological change and, consequently, did not obtain results related to weak or strong bias. The more recent literature includes Acemoglu (1998, 2002, 2003a, 2003b), Acemoglu and Zilibotti (2001), Kiley (1999), Caselli and Coleman (2004), Xu (2001), Gancia (2003), Thoenig and Verdier (2003), Ragot (2003), Duranton (2004), Benabou (2005), and Jones (2005).[7] Specialized versions of the results on relative bias presented here are contained in Acemoglu (1998, 2002). The more general results on relative bias, the relationship between relative bias and factor-augmenting technologies, the results on absolute bias, and

[7]The focus of the first papers in this literature, Acemoglu (1998) and Kiley (1999), was to investigate when and why technology could be biased toward skilled workers. Later Acemoglu (2003b) and Jones (2005) studied similar ideas to investigate why technical change may be purely labor-augmenting. Acemoglu (2003a), Xu (2001), Gancia (2003), and Thoenig and Verdier (2003) used versions of this framework to investigate the effect of international trade on the bias of technology.

the results that link strong absolute bias and nonconvexity have not been featured in any of these papers.

The results on weak absolute bias are also related to the LeChatelier principle (see, for example, Samuelson (1947, 1960), Milgrom and Roberts (1996), and Roberts (1999)). The LeChatelier principle states that long-run demand curves of profit-maximizing firms (which allow adjustment in all factors) are more elastic than short-run demand curves (which hold the employment level of other factors constant). Results about the endogenous bias of technology may be viewed as equilibrium versions of the LeChatelier principle. The main difference is that the focus here is on the effect of changes in factor supplies on *general equilibrium* outcomes, rather than the partial equilibrium/maximization focus of the LeChatelier principle. The above discussion illustrates that the general equilibrium structure is responsible for the possibility of strong equilibrium bias (because a firm's demand curve for a factor can never be upward-sloping; see, e.g., Mas-Colell, Whinston, and Green (1995, Proposition 5.C.2)). Equivalently, as discussed above, strong bias requires technology and factor demands to be chosen by different agents.

The rest of the paper is organized as follows. Section 2 describes a number of alternative environments, with different market structures and assumptions on technology choice, and shows that the determination of equilibrium bias in these different economies boils down to the same problem. Section 3 provides a generalization of existing relative bias results. Section 4 presents the results on weak absolute equilibrium bias. Section 5 contains the main theorem of the paper, which establishes the possibility of strong absolute equilibrium bias and demonstrates the relationship between nonconvexities and strong bias. Section 6 concludes.

2. THE BASIC ENVIRONMENTS

Consider a static economy consisting of a unique final good and $N+1$ factors of production, Z and $L = (L_1, \ldots, L_N)$. All agents' preferences are defined over the consumption of the final good and all factors are supplied inelastically, with supplies denoted by $\bar{Z} \in \mathbb{R}_+$ and $\bar{L} \in \mathbb{R}_+^N$. Throughout I will focus on comparative statics with respect to changes in the supply of factor Z, while holding the supply of other factors, $\bar{L}$, constant. The economy consists of a continuum of firms (final good producers) denoted by the set $\mathcal{F}$, each with an identical production function. Without loss of any generality let us normalize the measure of $\mathcal{F}$, $|\mathcal{F}|$, to 1. The price of the final good is also normalized to 1.[8]

I first describe technology choice in four different economic environments. All these environments will lead to a similar structure for the determination of

[8]Because all agents' preferences are defined over the final good, ownership of firms is not important for the equilibrium allocations. In particular, firms will always maximize profits independently of their exact ownership structure. For this reason, I do not specify the ownership structure of firms in what follows.

equilibrium bias, but will have different implications for the convexity of the aggregate production set and thus for strong equilibrium bias (see below for formal definitions).

The first, Economy D (for *decentralized*), is a decentralized competitive economy in which technologies are chosen by firms themselves. In some ways, in this economy, technology choice can be interpreted as choice of just another set of factors and the entire analysis can be conducted in terms of technology adoption.[9]

The second, Economy C (for *centralized*), features a socially run research firm that chooses the technology. The third and the fourth environments, Economies M and O (for *monopoly* and *oligopoly*) are more standard in analyses of technological progress, and allow either a monopoly or a set of oligopolies to produce and sell new technologies (or machines embodying these technologies) to final good producers.

2.1. *Economy D—Decentralized Equilibrium*

In the first environment, Economy D, all markets are competitive and technology is decided by each firm separately. Each firm $i \in \mathcal{F}$ has access to a production function

$$Y^i = G(Z^i, L^i, \theta^i), \tag{1}$$

where $Z^i \in \mathcal{Z} \subset \mathbb{R}_+$, $L^i \in \mathcal{L} \subset \mathbb{R}_+^N$, and $\theta^i \in \Theta \subset \mathbb{R}^K$ is the measure of technology. G is a real-valued production function, which I take to be twice continuously differentiable in (Z^i, L^i, θ^i) throughout its domain $\mathcal{Z} \times \mathcal{L} \times \Theta$. The cost of technology $\theta \in \Theta$ in terms of final goods is $C(\theta)$, which is also taken to be twice continuously differentiable.[10]

Each final good producer (firm) maximizes profits,

$$\max_{Z^i \in \mathcal{Z}, L^i \in \mathcal{L}, \theta^i \in \Theta} \pi(Z^i, L^i, \theta^i) = G(Z^i, L^i, \theta^i) - w_Z Z^i - \sum_{j=1}^{N} w_{Lj} L_j^i - C(\theta^i), \tag{2}$$

where w_Z is the price of factor Z and w_{Lj} is the price of factor L_j for $j = 1, \dots, N$, all taken as given by the firm. The vector of prices for factors L is

[9]See Boldrin and Levine (2001, 2004) and Quah (2002) for other models of technological change in competitive economies.

[10]Throughout, the cost of creating (producing) new technologies is in terms of the final good. This is only to highlight the novel results in the paper. If the "technology sector" has different factor intensities than the final good sector, there will be an additional—indirect—effect, because changes in factor supplies will influence the costs of creating new technologies. Nevertheless, this will not affect our main results (as long as producing different types of technologies does not require different factor intensities).

denoted by w_L. Because there is a total supply $\bar{Z}$ of factor Z and a total supply $\bar{L}_j$ of L_j, market clearing requires

$$\int_{i\in\mathcal{F}} Z^i\,di \le \bar{Z} \quad \text{and} \quad \int_{i\in\mathcal{F}} L^i_j\,di \le \bar{L}_j \quad \text{for} \quad j = 1, \ldots, N. \tag{3}$$

DEFINITION 1: An equilibrium in Economy D is a set of decisions $\{Z^i, L^i, \theta^i\}_{i\in\mathcal{F}}$ and factor prices (w_Z, w_L) such that $\{Z^i, L^i, \theta^i\}_{i\in\mathcal{F}}$ solve (2) given prices (w_Z, w_L) and (3) holds.

I refer to any θ^i that is part of the set of equilibrium allocations, $\{Z^i, L^i, \theta^i\}_{i\in\mathcal{F}}$, as *equilibrium technology*. For notational convenience let us define the "net production function"

$$F(Z^i, L^i, \theta^i) \equiv G(Z^i, L^i, \theta^i) - C(\theta^i). \tag{4}$$

ASSUMPTION 1: *Either $F(Z^i, L^i, \theta^i)$ is jointly strictly concave in (Z^i, L^i, θ^i) and increasing in (Z^i, L^i), and $\mathcal{Z}$, $\mathcal{L}$, and Θ are convex or $F(Z^i, L^i, \theta^i)$ is increasing in (Z^i, L^i) and exhibits constant returns to scale in (Z^i, L^i, θ^i), and we have $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$.*

Assumption 1 is restrictive, because it requires concavity (strict concavity or constant returns to scale) jointly in the factors of production and technology.[11] Such an assumption is necessary for a competitive equilibrium to exist; the other economic environments considered below will relax this assumption. It is worth emphasizing that the notation $F(Z^i, L^i, \theta^i)$ does *not* imply that there are no costs of technology adoption (recall (4)).

PROPOSITION 1: *Suppose Assumption 1 holds. Then any equilibrium technology θ in Economy D is a solution to*

$$\max_{\theta'\in\Theta} F(\bar{Z}, \bar{L}, \theta'), \tag{5}$$

and any solution to this problem is an equilibrium technology.

The proof uses standard arguments and can be found in Acemoglu (2005).

Proposition 1, which establishes the first and second welfare theorems for this environment, enables us to focus on a simple maximization problem for the determination of equilibrium technology. An important implication of this proposition is that the equilibrium is a Pareto optimum (and vice versa) and corresponds to a maximum of F in the entire vector (Z^i, L^i, θ^i).

[11] It is also possible to allow for mixtures of constant returns to scale and strict convexity. However, because this is not essential for the focus here, I simplify the notation by imposing Assumption 1.

It is also straightforward to see that equilibrium factor prices in this economy are equal to the marginal products of the G or the F function. That is, $w_Z = \partial G(\bar{Z}, \bar{L}, \theta)/\partial Z = \partial F(\bar{Z}, \bar{L}, \theta)/\partial Z$ and $w_{Lj} = \partial G(\bar{Z}, \bar{L}, \theta)/\partial L_j = \partial F(\bar{Z}, \bar{L}, \theta)/\partial L_j$ for $j = 1, \ldots, N$, where θ is the equilibrium technology choice (and where the second equalities follow in view of (4)).

2.2. *Economy C—Centralized Equilibrium*

In this economy, each firm again has access to the production function (1), with $Z^i \in \mathcal{Z} \subset \mathbb{R}_+$, $L^i \in \mathcal{L} \subset \mathbb{R}_+^N$, and $\theta^i \in \Theta \subset \mathbb{R}^K$. In addition, each firm has free access to technology θ provided by a centralized (socially run) research firm. This research firm can create any technology θ at cost $C(\theta)$ from the available technology menu Θ. Once created, this technology is nonrival, nonexcludable, and available to any firm. In addition, to further simplify the analysis, I assume that the research firm can only choose one technology, which might be, for example, because of the necessity of standardization across firms.[12]

All factor markets are again competitive. Consequently, given the technology offer of θ from the research firm, the maximization problem of each final good producer is

$$\max_{Z^i \in \mathcal{Z}, L^i \in \mathcal{L}} \pi(Z^i, L^i, \theta) = G(Z^i, L^i, \theta) - w_Z Z^i - \sum_{j=1}^{N} w_{Lj} L_j^i. \tag{6}$$

Notice that in contrast to Economy D, final good producers are only maximizing with respect to (Z^i, L^i), not with respect to θ^i, which will be determined by the centralized research firm.

The objective of the research firm is to maximize total net output:

$$\max_{\theta \in \Theta} \Pi(\theta) = \int_0^1 G(Z^i, L^i, \theta)\, di - C(\theta). \tag{7}$$

DEFINITION 2: An equilibrium in Economy C is a set of firm decisions $\{Z^i, L^i\}_{i \in \mathcal{F}}$, technology choice θ, and factor prices (w_Z, w_L) such that $\{Z^i, L^i\}_{i \in \mathcal{F}}$ solve (6) given (w_Z, w_L) and θ, (3) holds, and the technology choice for the research firm, θ, maximizes (7).

[12]In general, a social planner may want to create two different technologies, say θ_1 and θ_2, and provide one technology to a subset of firms and the other to the rest. This strategy could be optimal if $C(\theta)$ were sufficiently small (so that duplication costs are not too large). In the current environment, this is generally not possible because of nonexcludability; all firms would want to use the technology that is superior. To simplify the discussion, I assume that choosing two separate technologies from the menu is not possible (see the further discussion in Acemoglu (2005)).

We now impose a weaker version of Assumption 1, which only requires concavity in (Z, L):

ASSUMPTION 2: *Either $G(Z^i, L^i, \theta^i)$ is jointly strictly concave and increasing in (Z^i, L^i), and $\mathcal{Z}$ and $\mathcal{L}$ are convex or $G(Z^i, L^i, \theta^i)$ is increasing and exhibits constant returns to scale in (Z^i, L^i), and we have $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$.*

PROPOSITION 2: *Suppose Assumption 2 holds. Then any equilibrium technology θ in Economy C is a solution to*

$$\max_{\theta' \in \Theta} F(\bar{Z}, \bar{L}, \theta') \equiv G(\bar{Z}, \bar{L}, \theta') - C(\theta') \tag{8}$$

and any solution to this problem is an equilibrium technology.

The proof again uses standard arguments and is contained in Acemoglu (2005).

This proposition shows that technology choice in Economy C is identical to that in Economy D. Nevertheless, the equilibrium of this economy need not be a social optimum. To obtain the social optima, we would need to allow the centralized research firm to act as a social planner and decide not only the technology θ, but also the allocation of factors to firms (potentially operating some firms at a larger scale and shutting down others). However, because in this environment, once created, technologies are nonexcludable, all firms have access to them and will be active.

For our purposes, the more important difference is that although in Economy D the function $F(\bar{Z}, \bar{L}, \theta)$ is jointly concave in (Z, θ), the same is *not* true in Economy C.

Finally, note that as in Economy D, equilibrium factor prices are given by $w_Z = \partial G(\bar{Z}, \bar{L}, \theta)/\partial Z = \partial F(\bar{Z}, \bar{L}, \theta)/\partial Z$ and $w_{Lj} = \partial G(\bar{Z}, \bar{L}, \theta)/\partial L_j = \partial F(\bar{Z}, \bar{L}, \theta)/\partial L_j$ for $j = 1, \dots, N$.

2.3. *Economy M—Monopoly Equilibrium*

The next environment features a monopolist supplying technologies to final good producers. I take the simplest structure to deliver results similar to Propositions 1 and 2.[13] There is a unique final good and each firm has access to the production function

$$Y^i = \alpha^{-\alpha}(1-\alpha)^{-1}[G(Z^i, L^i, \theta^i)]^{\alpha} q(\theta^i)^{1-\alpha}. \tag{9}$$

This is similar to (1), except that $G(Z^i, L^i, \theta^i)$ is now a subcomponent of the production function, which depends on θ^i, the technology used by the firm.

[13] Appendix A in Acemoglu (2005) analyzes the same environment as in Economy C with a monopolist provider of technologies that can charge nonlinear prices.

Assumption 2 now applies to this subcomponent. The subcomponent G needs to be combined with an intermediate good embodying technology θ^i, denoted by $q(\theta^i)$—conditioned on θ^i to emphasize that it embodies technology θ^i. This intermediate good is supplied by the monopolist. The term $\alpha^{-\alpha}(1-\alpha)^{-1}$ is a convenient normalization. This structure is similar to models of endogenous technology (e.g., Romer (1990), Grossman and Helpman (1991), or Aghion and Howitt (1992, 1998)), but is somewhat more general because it does not impose that technology necessarily takes a factor-augmenting form. I continue to assume that $Z^i \in \mathcal{Z} \subset \mathbb{R}_+$, $L^i \in \mathcal{L} \subset \mathbb{R}_+^N$ and that G is twice continuously differentiable in (Z^i, L^i, θ^i).

The monopolist can create technology θ at cost $C(\theta)$ from the technology menu. Once θ is created, the technology monopolist can produce the intermediate good embodying technology θ at constant per unit cost normalized to $1-\alpha$ unit of the final good (this is also a convenient normalization). It can then set a (linear) price per unit of the intermediate good of type θ, denoted by χ.

All factor markets are competitive, and each firm takes the available technology, θ, and the price of the intermediate good embodying this technology, χ, as given and maximizes

$$\max_{\substack{Z^i\in\mathcal{Z},L^i\in\mathcal{L},\\ q(\theta)\geq 0}} \pi(Z^i, L^i, q(\theta) \mid \theta, \chi) = \alpha^{-\alpha}(1-\alpha)^{-1}[G(Z^i, L^i, \theta)]^{\alpha} q(\theta)^{1-\alpha} - w_Z Z^i - \sum_{j=1}^{N} w_{Lj} L_j^i - \chi q(\theta), \tag{10}$$

which gives the following simple inverse demand for intermediate of type θ as a function of its price, χ, and the factor employment levels of the firm as

$$q^i(\theta, \chi, Z^i, L^i) = \alpha^{-1} G(Z^i, L^i, \theta)\chi^{-1/\alpha}. \tag{11}$$

The problem of the monopolist is to maximize its profits,

$$\max_{\theta,\chi,[q^i(\theta,\chi,Z^i,L^i)]_{i\in\mathcal{F}}} \Pi = (\chi - (1-\alpha)) \int_{i\in\mathcal{F}} q^i(\theta, \chi, Z^i, L^i)\, di - C(\theta) \tag{12}$$

subject to (11). Therefore, an equilibrium in this economy can be defined as follows:

DEFINITION 3: An equilibrium in Economy M is a set of firm decisions $\{Z^i, L^i, q^i(\theta, \chi, Z^i, L^i)\}_{i\in\mathcal{F}}$, technology choice θ, and factor prices (w_Z, w_L) such that $\{Z^i, L^i, q^i(\theta, \chi, Z^i, L^i)\}_{i\in\mathcal{F}}$ solve (10) given (w_Z, w_L) and technology θ, (3) holds, and the technology choice and pricing decision for the monopolist, (θ, χ), maximize (12) subject to (11).

This definition emphasizes that, as in Economy C, factor demands and technology are decided by different agents (the former by the final good producers; the latter by the technology monopolist).

To characterize the equilibrium, note that (11) defines a constant elasticity demand curve, so the profit-maximizing price of the monopolist is given by the standard monopoly markup over marginal cost and is equal to $\chi = 1$. Consequently, $q^i(\theta) = q^i(\theta, \chi = 1, \bar{Z}, \bar{L}) = \alpha^{-1} G(\bar{Z}, \bar{L}, \theta)$ for all $i \in \mathcal{F}$. Substituting this into (12), the profits and the maximization problem of the monopolist can be expressed as

$$\max_{\theta \in \Theta} \Pi(\theta) = G(\bar{Z}, \bar{L}, \theta) - C(\theta). \tag{13}$$

Thus we have established the following proposition (proof in the text):

PROPOSITION 3: *Suppose Assumption 2 holds. Then any equilibrium technology θ in Economy M is a solution to*

$$\max_{\theta' \in \Theta} F(\bar{Z}, \bar{L}, \theta') \equiv G(\bar{Z}, \bar{L}, \theta') - C(\theta')$$

and any solution to this problem is an equilibrium technology.

Relative to Economies D and C, the presence of the monopoly markup implies greater distortions in this economy.[14] Nevertheless, once again equilibrium technology in Economy M is a solution to a problem identical to that in Economy D or C, that of maximizing $F(\bar{Z}, \bar{L}, \theta) \equiv G(\bar{Z}, \bar{L}, \theta) - C(\theta)$ as in (4). As in Economy C, $F(\bar{Z}, \bar{L}, \theta)$ need not be concave in (Z, θ), even in the neighborhood of the equilibrium.

Finally, it can be verified that in this economy, equilibrium factor prices are given by $w_Z = (1-\alpha)^{-1} \partial G(\bar{Z}, \bar{L}, \theta)/\partial Z$ and $w_{Lj} = (1-\alpha)^{-1} \partial G(\bar{Z}, \bar{L}, \theta)/\partial L_j$, which are proportional to the derivatives of the G or the F function defined in (4). To facilitate comparison with Economies D and C, with a slight abuse of terminology I will refer to the derivatives of the G or the F function as the "equilibrium factor prices," even in Economy M.

2.4. *Economy O—Oligopoly Equilibrium*

Finally, similar results can also be obtained when a number of different firms supply complementary or competing technologies. In this case, some

[14]For example, it can be verified that taking the behavior of the final good producers as given, the socially optimal allocation in this case would maximize $(1-\alpha)^{-1/\alpha} G(\bar{Z}, \bar{L}, \theta^i) - C(\theta)$ rather than $G(\bar{Z}, \bar{L}, \theta^i) - C(\theta)$.

more structure needs to be imposed to ensure tractability. Let θ^i be the vector $\theta^i \equiv (\theta^i_1, \dots, \theta^i_S)$, and suppose that output is now given by

$$(14) \qquad Y^i = \alpha^{-\alpha}(1-\alpha)^{-1}[G(Z^i, L^i, \theta^i)]^\alpha \sum_{s=1}^{S} q_s(\theta^i_s)^{1-\alpha},$$

where $\theta^i_s \in \Theta_s \subset \mathbb{R}^{K_s}$ is a technology supplied by technology producer $s = 1, \dots, S$, and $q_s(\theta^i_s)$ is an intermediate good (or machine) produced and sold by technology producer s, which embodies technology θ^i_s.[15] Factor markets are again competitive, and a maximization problem similar to (10) gives the inverse demand functions for intermediates as

$$(15) \qquad q^i_s(\theta, \chi_s, Z^i, L^i) = \alpha^{-1} G(Z^i, L^i, \theta)\chi_s^{-1/\alpha},$$

where χ_s is the price charged for intermediate good $q_s(\theta^i_s)$ by technology producer $s = 1, \dots, S$.

Let the cost of creating technology θ_s be $C_s(\theta_s)$, where $C_s(\cdot)$ is twice continuously differentiable for $s = 1, \dots, S$. The cost of producing each unit of any intermediate good is again normalized to $1 - \alpha$.

DEFINITION 4: An equilibrium in Economy O is a set of firm decisions $\{Z^i, L^i, [q^i_s(\theta, \chi_s, Z^i, L^i)]_{s=1}^S\}_{i\in\mathcal{F}}$, technology choices $(\theta_1, \dots, \theta_S)$, and factor prices (w_Z, w_L) such that $\{Z^i, L^i, [q^i_s(\theta, \chi_s, Z^i, L^i)]_{s=1}^S\}_{i\in\mathcal{F}}$ maximize firm profits given (w_Z, w_L) and the technology vector $(\theta_1, \dots, \theta_S)$, (3) holds, and the technology choice and pricing decision for technology producer $s = 1, \dots, S$, (θ_s, χ_s), maximize its profits subject to (15).

The profit-maximization problem of each technology producer is similar to (12) and implies a profit-maximizing price for intermediate goods equal to $\chi_s = 1$ for any $\theta_s \in \Theta_s$ and each $s = 1, \dots, S$. Consequently, with the same steps as in the previous subsection, each technology producer will solve the problem,

$$(16) \qquad \max_{\theta_s \in \Theta_s} \Pi_s(\theta_s) = G(\bar{Z}, \bar{L}, \theta_1, \dots, \theta_s, \dots, \theta_S) - C_s(\theta_s).$$

This establishes the following proposition (proof in the text):

PROPOSITION 4: *Suppose Assumption 2 holds. Then any equilibrium technology in Economy O is a vector $(\theta^*_1, \dots, \theta^*_S)$ such that θ^*_s is solution to*

$$\max_{\theta_s \in \Theta_s} G(\bar{Z}, \bar{L}, \theta^*_1, \dots, \theta_s, \dots, \theta^*_S) - C_s(\theta_s)$$

[15] A potentially unappealing feature of the production function (14) is that technology θ_s might affect productivity even when $q_s(\theta_s) = 0$. This can be avoided by writing the production function separately for the cases in which $q_s(\theta_s) = 0$ for some s as $G(\bar{Z}, \bar{L}, \theta_1, \dots, \theta_s = 0, \dots, \theta_S)$. In practice, this is not an issue, because as equation (15) shows, final good producers will always choose $q_s(\theta_s) > 0$ for all s. I therefore do not to introduce the additional notation.

for each $s = 1, \dots, S$, *and any such vector gives an equilibrium technology*.

The parallels between this result and Propositions 1–3 are evident. The major difference is that the equilibrium is no longer given by a simple maximization problem, but as a fixed point of a set of maximization problems. Nevertheless, this has little effect on the results below (and when it does, I will discuss Economy O separately). In the special case where $\partial^2 G/\partial\theta_s\,\partial\theta_{s'} = 0$ for all s and s' (which is the case, for example, in the product variety models as in Romer (1990) or Grossman and Helpman (1991)), the equilibrium can again be represented as a solution to a unique maximization problem—that of maximizing $G(\bar{Z}, \bar{L}, \theta_1, \dots, \theta_s, \dots, \theta_S) - \sum_{s=1}^{S} C_s(\theta_s)$.

Two additional remarks about Economy O are useful:

REMARK 1: Economy O can also be used to model "monopolistic competition," whereby the number of active firms is determined by a zero profit condition. In particular, assume that to be active each technology producer needs to pay some fixed cost $\bar{c}_s > 0$. Then in equilibrium, a subset of the S oligopolists will be active. Without loss of any generality, let this subset be $\{1, \dots, S'\}$. Then the equilibrium problem is simply $\max_{\theta_s \in \Theta_s} G(\bar{Z}, \bar{L}, \theta_1^*, \dots, \theta_s, \dots, \theta_{S'}^*, 0, \dots, 0) - C_s(\theta_s)$ for $1 \le s \le S'$. All the local results below apply when the set of active firms is taken as given, and the global results also apply when the set of active firms changes in response to changes in the supply of a factor.

REMARK 2: It is also possible to allow competition between technology producers for supplying the same technology. For example, suppose that for each s, there exists an "innovative" firm, with cost of creating new technologies equal to $C_s(\theta_s)$ and marginal cost of producing each unit of the intermediate good equal to $1 - \alpha$ as above, and there also exists a set $\mathcal{M}_s$ of "fringe" firms that can copy this technology without any cost and compete to supply intermediates embodying this technology to final good producers. Each fringe firm $r \in \mathcal{M}_r$ faces a marginal cost of producing intermediates equal to $\xi_r > 1 - \alpha$. Then it is straightforward to verify that in equilibrium the innovative firm will be the sole supplier of the intermediate good charging the price $\chi_s = \max\langle 1, \min_{r \in \mathcal{M}_s}\{\xi_r\}\rangle$, that is, possibly a limit price. In this case, only the constant in front of the profit function will be affected and all of the results apply as before.

3. RELATIVE EQUILIBRIUM BIAS

The previous section established that in a number of different environments, with different market structures and conceptions of technology choice, the characterization of equilibrium technology boils down to the maximization of some function $F(\bar{Z}, \bar{L}, \theta)$, where $\bar{Z}$ and $\bar{L}$ are the factor supplies in the economy. In this and the next two sections, I make use of this characterization to derive a number of results about equilibrium bias of technology.

This section analyzes relative equilibrium bias, and I focus on a more specialized economy with only two factors, that is, $N = 1$. Moreover, Θ is a convex compact subset of $\mathbb{R}^K$ (with the jth component of θ denoted by θ_j). I continue to assume that the production function $G : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}^K \to \mathbb{R}_+$ is twice continuously differentiable in (Z, L, θ).[16]

Recall that, in a two-factor economy, *relative equilibrium bias* is defined as the effect of technology on the marginal product (price) of a factor relative to the marginal product (price) of the other factor. Denote the marginal products of the two factors by

$$w_Z(Z, L, \theta) = \frac{\partial G(Z, L, \theta)}{\partial Z} \quad \text{and} \quad w_L(Z, L, \theta) = \frac{\partial G(Z, L, \theta)}{\partial L}$$

when employment levels (factor proportions) are given by (Z, L) and the technology is θ. From the twice differentiability of G, these marginal products are also differentiable functions of Z and L. In the remainder, I suppress the arguments of the derivatives when this causes no confusion (i.e., I use $\partial G/\partial Z$ instead of $\partial G(\bar{Z}, \bar{L}, \theta)/\partial Z$, etc.).

DEFINITION 5: An increase in technology θ_j for $j = 1, \dots, K$ is *relatively biased* toward factor Z at $(\bar{Z}, \bar{L}, \theta) \in \mathcal{Z} \times \mathcal{L} \times \Theta$ if $\partial(w_Z/w_L)/\partial\theta_j \geq 0$.

Definition 5 simply expresses what it means for a technology to be relatively biased toward a factor (similarly, a decrease in θ_j is relatively biased toward factor Z if the derivative in Definition 5 is nonpositive). From this definition, it is clear that (weak) relative equilibrium bias corresponds to a change in technology θ in a direction biased toward Z *in response to an increase* in $\bar{Z}$ (or $\bar{Z}/\bar{L}$); this is stated in the next definition.[17]

DEFINITION 6: Denote the equilibrium technology at factor supplies $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$ by $\theta^*(\bar{Z}, \bar{L})$ and assume that $\partial\theta_j^*/\partial Z$ exists at $(\bar{Z}, \bar{L})$ for all $j = 1, \dots, K$. Then there is *weak relative equilibrium bias* at $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$ if

$$\sum_{j=1}^{K} \frac{\partial(w_Z/w_L)}{\partial\theta_j} \frac{\partial\theta_j^*}{\partial Z} \geq 0. \tag{17}$$

[16]Because $C(\theta)$ is twice continuously differentiable, $F(\bar{L}, \bar{Z}, \theta) \equiv G(\bar{L}, \bar{Z}, \theta) - C(\theta)$ is also twice continuously differentiable. Throughout I will refer to both G and F as "the production function" of the economy.

[17]Throughout this section, I focus on changes in the supply of factor Z, which is also equivalent to a change in relative supplies Z/L (with L kept constant). Moreover, I denote the change in equilibrium technology by $\partial\theta_j/\partial Z$ rather than $d\theta_j/dZ$ because θ is not generally a function of only Z. I reserve the notation $d(w_Z/w_L)/dZ$ to denote the total change in relative (or absolute) wages, which includes the technological adjustment, and contrast this with the partial change $\partial(w_Z/w_L)/\partial Z$, which holds technology constant (see, for example, (18)).

Weak relative equilibrium bias requires the (overall) change in technology in response to an increase in $\bar{Z}$ to be biased toward Z at the point $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$ for which $\partial\theta_j^*/\partial Z$ exists for all j. The condition that $\partial\theta_j^*/\partial Z$ exists for all j used in this definition will be further discussed below (see the discussion after Theorem 2 in the next section).

The next definition introduces the more stringent concept of strong relative equilibrium bias, which requires that in response to an increase in $\bar{Z}$, technology changes so much that the overall effect (after the induced change in technology) is to increase the relative price of Z.

DEFINITION 7: Denote the equilibrium technology at factor supplies $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$ by $\theta^*(\bar{Z}, \bar{L})$, and assume that $\partial\theta_j^*/\partial Z$ exists at $(\bar{Z}, \bar{L})$ for all $j = 1, \dots, K$. Then there is *strong relative equilibrium bias* at $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$ if

$$\frac{d(w_Z/w_L)}{dZ} = \frac{\partial(w_Z/w_L)}{\partial Z} + \sum_{j=1}^{K} \frac{\partial(w_Z/w_L)}{\partial\theta_j}\frac{\partial\theta_j^*}{\partial Z} > 0. \tag{18}$$

By comparing the latter two definitions, it is clear that there will be strong relative equilibrium bias if (17) is large enough to dominate the direct (negative) effect of the increase in relative supplies on relative wages (the first term in (18)).

Before deriving the main results of this section, it is useful to clarify the notions introduced so far using an example, which captures the main findings in Acemoglu (1998, 2002), but in the context of Economy C, M, or O studied above rather than in the endogenous growth setup of the original papers.[18]

EXAMPLE 1—Relative Equilibrium Bias: Suppose that

$$G(Z, L, \theta) = \left[\gamma(A_Z Z)^{(\sigma-1)/\sigma} + (1-\gamma)(A_L L)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \tag{19}$$

where $\theta = (A_Z, A_L) \in \Theta = \mathbb{R}_+^2$. In particular, A_Z and A_L are two separate factor-augmenting technology terms, $\gamma \in (0, 1)$, and $\sigma \in [0, \infty]$ is the elasticity of substitution between the two factors. When $\sigma = \infty$, the two factors are perfect substitutes, and the production function is linear. When $\sigma = 1$, the production function is Cobb–Douglas; when $\sigma = 0$, there is no substitution between the two factors and the production function is Leontief.

The relative marginal product of Z is given by

$$\frac{w_Z}{w_L} = \frac{\gamma}{1-\gamma}\left(\frac{A_Z}{A_L}\right)^{(\sigma-1)/\sigma}\left(\frac{\bar{Z}}{\bar{L}}\right)^{-1/\sigma}, \tag{20}$$

[18]Related static models of the direction of technology choice have also been considered by Caselli and Coleman (2004) and Jones (2005).

which is decreasing in the relative supply of Z, $\bar{Z}/\bar{L}$. This is the usual substitution effect, leading to a downward-sloping relative demand curve (with exogenous technology). This expression also makes it clear that the measure of relative bias toward Z will correspond to $\bar{\theta} \equiv (A_Z/A_L)^{(\sigma-1)/\sigma}$, because higher levels of $\bar{\theta}$ increase the marginal product of Z relative to L for all values of σ (recall Definition 5). It is important that the bias toward factor Z is $(A_Z/A_L)^{(\sigma-1)/\sigma}$, not A_Z/A_L (A_Z/A_L is the ratio of Z-augmenting to L-augmenting technology). When $\sigma > 1$, an increase in A_Z/A_L increases the relative marginal product of Z, while when $\sigma < 1$, an increase in A_Z/A_L reduces the relative marginal product of Z. Suppose also that the costs of producing new technologies are $\eta_Z A_Z^{1+\delta}$ and $\eta_L A_L^{1+\delta}$, where $\delta > 0$. Despite the fact that $\delta > 0$ introduces diminishing returns in the choice of technology, the aggregate production possibilities set of this economy is potentially nonconvex, because there is choice both over the factors of production, Z and L, and the technologies, A_Z and A_L (see below). From Proposition 2 or 3, equilibrium technology in Economy C or M is given by the solution to the strictly concave maximization problem

$$\max_{A_Z, A_L} \left[\gamma(A_Z\bar{Z})^{(\sigma-1)/\sigma} + (1-\gamma)(A_L\bar{L})^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} - \eta_Z A_Z^{1+\delta} - \eta_L A_L^{1+\delta}. \tag{21}$$

(When considering Economy O, Proposition 4 implies a similar maximization problem, with the only difference that A_Z and A_L maximize (21) individually, which leads to identical results, because (21) is jointly concave in A_Z and A_L.)

Now, taking the ratio of the first-order conditions with respect to A_Z and A_L, and denoting the equilibrium values by asterisks (*), the solution to this problem yields

$$\frac{A_Z^*}{A_L^*} = \left(\frac{\eta_Z}{\eta_L}\right)^{-\sigma/(1+\sigma\delta)} \left(\frac{\gamma}{1-\gamma}\right)^{\sigma/(1+\sigma\delta)} \left(\frac{\bar{Z}}{\bar{L}}\right)^{(\sigma-1)/(1+\sigma\delta)}. \tag{22}$$

This equation can also be expressed in an alternative form useful for Theorem 1:

$$\frac{\partial \ln(A_Z^*/A_L^*)}{\partial \ln(Z/L)} = \frac{\sigma-1}{1+\sigma\delta}. \tag{23}$$

There will be weak equilibrium bias if the expression in Definition 6 is nonnegative. Using (20) (and preparing for Theorem 1), we can express the condition for weak relative equilibrium bias as[19]

$$(24)\qquad \Delta\left(\frac{w_Z}{w_L}\right) \equiv \frac{\partial \ln(w_Z/w_L)}{\partial \ln(A_Z/A_L)} \frac{\partial \ln(A_Z^*/A_L^*)}{\partial \ln(Z/L)} \geq 0.$$

Using (20) and (23), we obtain

$$\Delta\left(\frac{w_Z}{w_L}\right) = \frac{\sigma-1}{\sigma} \times \frac{\sigma-1}{1+\sigma\delta} = \frac{(\sigma-1)^2}{(1+\sigma\delta)\sigma} \geq 0,$$

which is always nonnegative, thus establishing that there is always *weak relative equilibrium bias*.[20] Intuitively, when $\sigma > 1$, an increase in $\bar{Z}/\bar{L}$ increases A_Z^*/A_L^*, which in turn raises w_Z/w_L at given factor proportions. In contrast, when $\sigma < 1$, an increase in $\bar{Z}/\bar{L}$ reduces A_Z^*/A_L^*, but in this case, an increase in A_Z^*/A_L^* is relatively biased against factor Z, so the decrease in A_Z^*/A_L^* again raises w_Z/w_L.

Next, to investigate the conditions under which there is strong relative equilibrium bias, let us use Definition 7 (again in log form) and check whether

$$\frac{\partial \ln(w_Z/w_L)}{\partial \ln(Z/L)} + \frac{\partial \ln(w_Z/w_L)}{\partial \ln(A_Z^*/A_L^*)} \frac{\partial \ln(A_Z^*/A_L^*)}{\partial \ln(Z/L)} > 0.$$

From (20) and (24), this condition is equivalent to

$$(25)\qquad -\frac{1}{\sigma} + \frac{(\sigma-1)^2}{(1+\sigma\delta)\sigma} = \frac{\sigma-2-\delta}{1+\sigma\delta}$$

being strictly positive. Therefore, when $\sigma > 2+\delta$, the relative demand curve for Z is *upward-sloping* and there is strong relative equilibrium bias.[21] This

[19]Expressing everything in terms of log changes rather than level changes is simply for convenience (and also useful for Theorem 1). In particular, note that for $x > 0$ and $a > 0$, $\partial x/\partial a \gtreqless 0$ if and only if $\partial \ln x/\partial \ln a \gtreqless 0$. Moreover, because L is constant, $\partial x/\partial(Z/L) = (\partial x/\partial Z)L$, so $\Delta(w_Z/w_L) \geq 0$ if and only if $\sum_{j=1}^{K} \partial(w_Z/w_L)/\partial\theta_j \times \partial\theta_j^*/\partial Z \geq 0$ (as required by Definition 6).

[20] Alternatively, the same result follows by looking directly at the measure of relative bias toward Z, $\bar{\theta} \equiv (A_Z/A_L)^{(\sigma-1)/\sigma}$. Substituting for (22), we have

$$\bar{\theta} = \left(\frac{\eta_Z}{\eta_L}\right)^{-(\sigma-1)/(1+\sigma\delta)} \left(\frac{\gamma}{1-\gamma}\right)^{(\sigma-1)/(1+\sigma\delta)} \left(\frac{\bar{Z}}{\bar{L}}\right)^{(\sigma-1)^2/((1+\sigma\delta)\sigma)},$$

which is always nondecreasing in $\bar{Z}/\bar{L}$.

[21]In Acemoglu (2003a), the condition for upward-sloping relative demand curves was $\sigma > 2-\delta'$ for some other parameter $\delta' > 0$. The reason is that in that context, as in many endogenous growth models, the technology allowed for knowledge spillovers and the parameter δ' measured how much a particular type of technology benefits from past innovations in the same line, adding another degree of nonconvexity. Here a higher value of the parameter δ makes the aggre-

result can be also obtained more directly by substituting for A_Z^*/A_L^* from (22) into (20) to obtain

$$\frac{w_Z}{w_L}=\left(\frac{\eta_Z}{\eta_L}\right)^{-(\sigma-1)/(1+\sigma\delta)}\left(\frac{\gamma}{1-\gamma}\right)^{(\sigma+\sigma\delta)/(1+\sigma\delta)}\left(\frac{\bar{Z}}{\bar{L}}\right)^{(\sigma-2-\delta)/(1+\sigma\delta)},$$

and, thus,

$$\frac{d\ln(w_Z/w_L)}{d\ln(Z/L)}=\frac{\sigma-2-\delta}{1+\sigma\delta},$$

which confirms the result of strong relative equilibrium bias when $\sigma > 2+\delta$ shown in (25).[22]

This example thereforeillustrates the possibility of both weak and strong relative bias. In particular, technological change induced in response to an increase in Z is always (weakly) relatively biased toward Z and, moreover, if the condition $\sigma > 2+\delta$ is satisfied, there is also strong relative bias. This example also corresponds to the most general result that exists in the literature (see Acemoglu (2002)). Nevertheless, the structure of the economy is quite special

gate technology of the economy more "convex" and thus makes upward-sloping relative demand curves less likely.

[22]Anticipating Theorem 4, we can note that strong relative equilibrium bias, that is, the condition $\sigma > 2+\delta$, is also related to nonconvexity, but to the nonconvexity of a "modified production function" rather than that of the original G or F functions—this is because we are dealing with relative, *not* absolute, bias. In particular, consider the (net) modified production function, which is made up of the relative factor share of Z (i.e., $w_Z\bar{Z}/w_L\bar{L}$) minus the relative costs of technologies for the two factors:

$$\begin{aligned}\tilde{F}(\bar{Z},\bar{L},\theta)&=\frac{\sigma}{\sigma-1}\frac{w_Z\bar{Z}}{w_L\bar{L}}-\frac{1}{1+\delta}\left(\frac{\eta_Z}{\eta_L}\right)\left(\frac{A_Z}{A_L}\right)^{1+\delta}\\&=\frac{\gamma\sigma}{(1-\gamma)(\sigma-1)}\left(\frac{A_Z}{A_L}\right)^{(\sigma-1)/\sigma}\left(\frac{\bar{Z}}{\bar{L}}\right)^{(\sigma-1)/\sigma}-\frac{1}{1+\delta}\left(\frac{\eta_Z}{\eta_L}\right)\left(\frac{A_Z}{A_L}\right)^{1+\delta}.\end{aligned}$$

The first-order condition with respect to A_Z/A_L gives the equilibrium relative technology A_Z^*/A_L^* as in (22). Evaluating the second-order conditions at the equilibrium A_Z^*/A_L^*, we find that $(\partial^2\tilde{F}/\partial(\bar{Z}/\bar{L})^2)\times(\partial^2\tilde{F}/\partial(A_Z/A_L)^2)-(\partial^2\tilde{F}/\partial(\bar{Z}/\bar{L})\,\partial(A_Z/A_L))^2\geq 0$ if and only if $\sigma\leq 2+\delta$. In other words, there will be strong relative equilibrium bias if this modified production function fails to be jointly concave in $\bar{Z}/\bar{L}$ and A_Z/A_L at the equilibrium point A_Z^*/A_L^*. We will see the parallel between this result and Theorem 4 below.

This discussion also provides the intuition for the condition $\sigma > 2+\delta$ necessary for strong bias. First, a greater elasticity of substitution, σ, makes it more likely that the modified production function, $\tilde{F}$, is nonconvex. Second, a higher δ, which corresponds to greater convexity in the costs of generating new technologies, makes $\tilde{F}$ more convex. Finally, there is a "2" in this condition, because the first term in $\tilde{F}$—the relative factor share $w_Z Z/w_L L$—is jointly concave in $\bar{Z}/\bar{L}$ and A_Z/A_L when $\sigma\leq 2$.

(e.g., it uses specific aggregate production and cost functions). I next extend this result to a more general setup with factor-augmenting technologies. Before stating this result, recall that a function $f(x, y)$ is homothetic in x and y, if $(\partial f(x, y)/\partial x)/(\partial f(x, y)/\partial y)$ is a function of only x/y for all x and y.

THEOREM 1—Relative Equilibrium Bias with Factor-Augmenting Technologies: *Consider Economy C, M, or O with two factors,* $(Z, L) \in \mathcal{Z} \times \mathcal{L} \subset \mathbb{R}_+^2$, *and two factor-augmenting technologies,* $(A_Z, A_L) \in \mathbb{R}_+^2$, *such that the production function is* $G(A_Z Z, A_L L)$. *Assume that* G *is twice continuously differentiable, concave and homothetic in its two arguments, and that the cost of producing technologies* A_Z *and* A_L, $C(A_Z, A_L)$, *is also twice continuously differentiable, strictly convex, and homothetic in* A_Z *and* A_L. *Denote the first derivatives of* $C(A_Z, A_L)$ *by* C_Z *and* C_L. *Let* σ *be the (local) elasticity of substitution between* Z *and* L *defined by*

$$\sigma = -\frac{\partial \ln(Z/L)}{\partial \ln(w_Z/w_L)}\bigg|_{A_Z/A_L},$$

and let

$$\delta = \frac{\partial \ln(C_Z/C_L)}{\partial \ln(A_Z/A_L)}.$$

Finally, suppose that factor supplies are given by $(\bar{Z}, \bar{L})$, *and denote equilibrium technologies by* (A_Z^*, A_L^*) *and equilibrium factor prices by* w_Z *and* w_L. *Then we have that, for all* $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$,

$$\frac{\partial \ln(A_Z^*/A_L^*)}{\partial \ln(Z/L)} = \frac{\sigma - 1}{1 + \sigma\delta} \tag{26}$$

and

$$\frac{\partial \ln(w_Z/w_L)}{\partial \ln(A_Z/A_L)} \frac{\partial \ln(A_Z^*/A_L^*)}{\partial \ln(Z/L)} \geq 0, \tag{27}$$

so that there is always weak relative equilibrium bias. Moreover,

$$\frac{d \ln(w_Z/w_L)}{d \ln(Z/L)} = \frac{\sigma - 2 - \delta}{1 + \sigma\delta}, \tag{28}$$

so that there is strong relative equilibrium bias if and only if $\sigma - 2 - \delta > 0$.

PROOF: By Propositions 2 or 3, an equilibrium (A_Z^*, A_L^*) in Economy C or M maximizes $G(A_Z \bar{Z}, A_L \bar{L}) - C(A_Z, A_L)$. Moreover, by Proposition 4, A_Z^* and A_L^* individually maximize the same function in Economy O. In either case, the maximization problem is strictly concave in A_Z and A_L, and the first-order conditions are necessary and sufficient. Taking their ratio gives

$$\frac{\bar{Z}}{\bar{L}}\frac{\partial G/\partial Z}{\partial G/\partial L}=\frac{C_Z}{C_L}.$$

Recalling the definition of marginal products and multiplying both sides by A_Z^*/A_L^*, we obtain

$$\frac{\bar{Z}}{\bar{L}}\frac{w_Z}{w_L}=\frac{A_Z^*}{A_L^*}\frac{C_Z}{C_L}. \tag{29}$$

Because G is homothetic, w_Z/w_L is only a function of Z/L and A_Z/A_L. Moreover, because C is homothetic, C_Z/C_L is also only a function of A_Z/A_L, and δ in the theorem is well defined. Using these facts, taking logs in (29), and differentiating totally with respect to $\ln(Z/L)$ gives

$$\begin{aligned}&\left(1+\frac{\partial\ln(C_Z/C_L)}{\partial\ln(A_Z/A_L)}\right)\frac{\partial\ln(A_Z^*/A_L^*)}{\partial\ln(Z/L)}\\&\quad=\left.\frac{\partial\ln(w_Z/w_L)}{\partial\ln(Z/L)}\right|_{A_Z^*/A_L^*}+1+\left.\frac{\partial\ln(w_Z/w_L)}{\partial\ln(A_Z/A_L)}\right|_{\bar{Z}/\bar{L}}\frac{\partial\ln(A_Z^*/A_L^*)}{\partial\ln(Z/L)}.\end{aligned} \tag{30}$$

The definition of σ then implies

$$\frac{\partial\ln(w_Z/w_L)}{\partial\ln(A_Z/A_L)}=\frac{\sigma-1}{\sigma}. \tag{31}$$

Substituting (31) into (30), rearranging, and recalling the definitions of δ and σ, we obtain

$$\frac{\partial\ln(A_Z^*/A_L^*)}{\partial\ln(Z/L)}=\frac{\sigma-1}{1+\sigma\delta}$$

as in (26). Then (27) immediately follows by combining this with (31) and, by the same argument as in Example 1, this establishes weak equilibrium bias.

Equation (28) and the fact that there is strong relative equilibrium bias if and only if $\sigma>2+\delta$ follow from (26) by noting that

$$\begin{aligned}\frac{d\ln(w_Z/w_L)}{d\ln(Z/L)}&=-\frac{1}{\sigma}+\frac{\sigma-1}{\sigma}\frac{\partial\ln(A_Z^*/A_L^*)}{\partial\ln(Z/L)}\\&=\frac{\sigma-2-\delta}{1+\sigma\delta}.\end{aligned}$$

Q.E.D.

This theorem shows that the insights from Example 1 generalize in a fairly natural way as long as the potential menu of technological possibilities only consists of two technologies, one augmenting Z and the other augmenting L (and also, as long as G and C are homothetic). The only difference is that the parameter δ and the elasticity of substitution σ are no longer constants, but are functions of A_L, A_Z, $\bar{L}$, and $\bar{Z}$, so changes in factor supplies will have effects that depend on the local elasticity of substitution and the local value of δ. Nevertheless, the change in A_Z/A_L (or in $(A_Z/A_L)^{(\sigma-1)/\sigma}$ as in Example 1) induced by an increase in $\bar{Z}$ is always relatively biased toward Z, and there is strong equilibrium relative bias if $\sigma > 2+\delta$.

Theorem 1 establishes both the presence of *weak relative equilibrium bias* and the possibility of *strong relative equilibrium bias* (when the local elasticity of substitution between factors, σ, is sufficiently high and the parameter δ is relatively low). The intuition for both weak and strong relative bias is related to the "market size effect," whereby an increase in the relative abundance of a factor increases the market size for technologies that complement that factor and makes their development more profitable (see Acemoglu (1998)). The magnitude of the market size effect, and thus whether there is weak or strong relative bias, is also closely related to whether the aggregate production possibilities set of the economy is convex as illustrated in Example 1 (recall footnote 22).

The assumption that the production function ought to take the form $G(A_Z Z, A_L L)$, with two factor-augmenting technologies, *cannot be dispensed with* in Theorem 1. When it is relaxed, an increase in the supply of factor Z may induce a change in technology that is relatively biased against this factor. This is illustrated by the following two examples. For simplicity, both examples focus on economies with a single dimension of technology. The first one considers a homothetic, constant returns production function G and shows that, even in this case, a direct choice over the elasticity of substitution may lead to endogenous technological change biased against the factor that is becoming more abundant.[23] The second one shows that the form of the production function in Theorem 1, $G(A_Z Z, A_L L)$, is also important for the result.

EXAMPLE 2—Counterexample I: Suppose that

$$G(Z, L, \theta) = [Z^\theta + L^\theta]^{1/\theta} \tag{32}$$

and that the cost of technology creation, $C(\theta)$, defined over $\Theta = [a, b]$ is convex and twice continuously differentiable over the entire Θ. The choice of θ again maximizes $F(Z, L, \theta) \equiv G(Z, L, \theta) - C(\theta)$; thus we have $\partial G(\bar{Z}, \bar{L}, \theta^*)/\partial\theta - \partial C(\theta^*)/\partial\theta = 0$ and $\partial^2 G(\bar{Z}, \bar{L}, \theta^*)/\partial\theta^2 - \partial^2 C(\theta^*)/\partial\theta^2 < 0$. From Definition 6, a counterexample would correspond to a situation where

$$\Delta\left(\frac{w_Z}{w_L}\right) \equiv \frac{\partial(w_Z/w_L)}{\partial\theta}\frac{\partial\theta^*}{\partial Z} = -\frac{\partial(w_Z/w_L)}{\partial\theta}\frac{\partial^2 F/\partial\theta\,\partial Z}{\partial^2 F/\partial\theta^2} < 0. \tag{33}$$

[23]See Benabou (2005) for a model of endogenous choice of the elasticity of substitution as a function of the inequality of human capital among workers in the economy.

The equality in (33) follows from the implicit function theorem. Moreover, from (32), $w_Z/w_L = (Z/L)^{\theta-1}$, which is increasing in θ as long as $Z > L$ (making higher θ relatively biased toward Z). It can be verified easily (see Acemoglu (2005) for details) that when $C(\cdot)$ is chosen such that θ^* is sufficiently small (e.g., $\bar{L} = 1$, $\bar{Z} = 2$, $\theta^* = 0.1$), we also have $\partial^2 F(\bar{Z}, \bar{L}, \theta^*)/\partial\theta\,\partial Z < 0$. From the second-order conditions, $\partial^2 F/\partial\theta^2 < 0$, so that in this case $(\partial^2 F/\partial\theta\,\partial Z) \times (\partial^2 F/\partial\theta^2) > 0$ and an increase in Z/L reduces θ^*, creating a change in technology relatively biased against Z.

EXAMPLE 3—Counterexample II[24]: Suppose that

$$G(Z, L, \theta) = Z\theta + L\theta^2$$

and that the cost of creating new technologies is given by $C_0\theta^2/2$ with $C_0 > 0$ for all $\theta \in \Theta = \mathbb{R}$ and $L \in \mathcal{L} \subset (0, C_0/2)$. The equilibrium technology θ^* is given by

$$\theta^*(\bar{Z}, \bar{L}) = \frac{\bar{Z}}{C_0 - 2\bar{L}},$$

which is increasing in $\bar{Z}$ for any $\bar{L} \in \mathcal{L}$. The relative price of factor Z is given by $w_Z(\theta)/w_L(\theta) = \theta^{-1}$, which is clearly decreasing in θ. An increase in $\bar{Z}$ therefore induces technological change relatively biased against Z.

These examples show that a result similar to Theorem 1 no longer holds when the menu of technologies available to the society does not take the simple form with one technology augmenting factor Z and the other augmenting factor L.[25] Although this type of factor-augmenting technology is an interesting and empirically important special case, one may be interested in more general theorems that apply without imposing a specific structure on the interaction between technologies and the factors of production. More general results would be particularly useful for analyses of technology choices related to shifts from one type of organizational form or production system to another, such as those experienced during recent decades, during the emergence of the American system of manufacturing, or during the Industrial Revolution. These shifts transform the way the whole production process is organized and thus directly affect the elasticity of substitution between factors.

The next section presents a much more general theorem for absolute bias. In fact, Examples 2 and 3 already hint at this possibility. The reason why induced

[24]I thank Rabah Amir for suggesting an example along these lines.

[25]Nevertheless, it is also important to emphasize that these examples do *not* imply that with the general menu of technologies, changes in relative supplies will cause technical change that is relatively biased against the more abundant factor. In many cases, weak equilibrium bias will still apply, but without imposing more structure, we do not have a general theorem.

technology (in response to an increase in $\bar{Z}$) is not relatively biased toward Z in both examples is that the induced change in technology increases w_Z (at given factor proportions), but it has an even larger (positive) effect on the marginal product of the other factor, w_L.[26]

4. ABSOLUTE EQUILIBRIUM BIAS

This section shows that, under mild assumptions, an increase in the supply of a factor will induce a change in technology in a direction *absolutely biased* toward that factor. The problem of equilibrium technology choice is again equivalent to

$$\max_{\theta\in\Theta} F(\bar{Z}, \bar{L}, \theta), \tag{34}$$

where $\bar{L}$ denotes the supply of other inputs and $\bar{Z}$ denotes the supply of Z. Let us assume that Θ is a convex compact subset of $\mathbb{R}^K$ for some $K \geq 1$ and that F is twice differentiable in (Z, θ). As before, the marginal product (or price) of factor Z is $w_Z = \partial F/\partial Z$, which implies that w_Z is continuously differentiable in θ.

DEFINITION 8: An increase in technology θ_j for $j = 1, \ldots, K$ is *absolutely biased* towards factor Z at $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$ if $\partial w_Z/\partial \theta_j \geq 0$.

Conversely, we could define a decrease in technology θ as absolutely biased toward factor Z if the same derivative is nonpositive. Notice also that this definition requires the bias for only small changes in technology and only at the *current* factor proportions $(\bar{Z}, \bar{L})$.

DEFINITION 9: Denote the equilibrium technology at factor supplies $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$ by $\theta^*(\bar{Z}, \bar{L})$ and assume that $\partial \theta_j^*/\partial Z$ exists at $(\bar{Z}, \bar{L})$ for all $j = 1, \ldots, K$. Then there is *weak absolute equilibrium bias* at $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$ if

$$\sum_{j=1}^{K} \frac{\partial w_Z}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial Z} \geq 0. \tag{35}$$

[26]To see this more explicitly, note that $\partial^2 F/\partial\theta\,\partial Z = \partial w_Z/\partial\theta$ and rewrite (33) as

$$\Delta\left(\frac{w_Z}{w_L}\right) \equiv -\frac{\partial(w_Z/w_L)}{\partial\theta} \frac{\partial w_Z/\partial\theta}{\partial^2 F/\partial\theta^2}.$$

When w_L is constant, this is equivalent to (38) in the proof of Theorem 2 in the next section and is always nonnegative. However, as Examples 2 and 3 show, a large effect of θ on w_L can reverse this result.

This definition requires the induced change in technology resulting from an increase in $\bar{Z}$ to increase the marginal product of factor Z. As in Definition 6 for relative equilibrium bias, this definition also requires $\partial\theta_j^*/\partial Z$ to exist for all j. The next theorem will also be stated under this assumption, which can alternatively be replaced by Assumption A1 below.

THEOREM 2—Weak Absolute Equilibrium Bias: *Consider Economy D, C, or M. Suppose that Θ is a convex subset of $\mathbb{R}^K$ and that $F(Z, L, \theta)$ is twice continuously differentiable in (Z, θ). Let the equilibrium technology at factor supplies $(\bar{Z}, \bar{L})$ be $\theta^*(\bar{Z}, \bar{L})$, and assume that $\theta^*(\bar{Z}, \bar{L})$ is in the interior of Θ and that $\partial\theta_j^*/\partial Z$ exists at $(\bar{Z}, \bar{L})$ for all $j = 1, \ldots, K$. Then there is* weak absolute equilibrium bias *at all $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$, that is,*

$$\sum_{j=1}^{K} \frac{\partial w_Z}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial Z} \geq 0 \quad \textit{for all } (\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}, \tag{36}$$

with strict inequality if $\partial\theta_j^*/\partial Z \neq 0$ for some $j = 1, \ldots, K$.

PROOF: The proof follows from the implicit function theorem. For expositional clarity, I first present the case where $\theta \in \Theta \subset \mathbb{R}$. Because $\Theta \subset \mathbb{R}$ and θ^* is in the interior of Θ, we have $\partial F/\partial\theta = 0$ and $\partial^2 F/\partial\theta^2 \leq 0$. Because $\partial\theta^*/\partial Z$ exists at $(\bar{Z}, \bar{L})$ by hypothesis, from the implicit function theorem it must be equal to

$$\frac{\partial \theta^*}{\partial Z} = -\frac{\partial^2 F/\partial\theta\,\partial Z}{\partial^2 F/\partial\theta^2} = -\frac{\partial w_Z/\partial\theta}{\partial^2 F/\partial\theta^2}, \tag{37}$$

so we must have $\partial^2 F/\partial\theta^2 \neq 0$, that is, $\partial^2 F/\partial\theta^2 < 0$. This in turn implies

$$\frac{\partial w_Z}{\partial\theta} \frac{\partial\theta^*}{\partial Z} = -\frac{(\partial w_Z/\partial\theta)^2}{\partial^2 F/\partial\theta^2} \geq 0, \tag{38}$$

establishing the weak inequality in (36). Moreover, if $\partial\theta^*/\partial Z \neq 0$, then from (37), $\partial w_Z/\partial\theta \neq 0$, so (38) holds with strict inequality, establishing the result for the case in which $\Theta \subset \mathbb{R}$.

Next, let us look at the general case where $\theta \in \Theta \subset \mathbb{R}^K$ with $K > 1$. For a matrix (vector) v, let v' denote its transpose. Define Δw_Z as the change in w_Z resulting from the induced change in θ (at given factor proportions) as in (35),

$$\begin{aligned} \Delta w_Z &\equiv \sum_{j=1}^{K} \frac{\partial w_Z}{\partial\theta_j} \frac{\partial\theta_j^*}{\partial Z} \\ &= [\nabla_\theta w_Z]'[\nabla_Z \theta^*] \\ &= [\nabla^2_{\theta Z} F]'[\nabla_Z \theta^*], \end{aligned} \tag{39}$$

where $[\nabla_\theta w_Z]$ is a $K \times 1$ vector of changes in w_Z in response to each component of $\theta \in \Theta \subset \mathbb{R}^K$ and $[\nabla_Z \theta^*]$ is the gradient of θ with respect to Z, that is, a $K \times 1$ vector of changes in each component of θ in response to the change in $\bar{Z}$. The second line in (39) uses the fact that w_Z is the derivative of the F function, so $[\nabla^2_{\theta Z} F]$ is also the $K \times 1$ vector of changes in w_Z in response to each component of θ. Because $\partial \theta^*_j / \partial Z$ exists at $(\bar{Z}, \bar{L})$ for all j, the gradient $\nabla_Z \theta^*$ also exists and from the implicit function theorem (i.e., from differentiating the technology equilibrium condition $\nabla_\theta F = 0$), it satisfies

$$[\nabla_Z \theta^*]' = -[\nabla^2_{\theta Z} F]' [\nabla^2_{\theta\theta} F]^{-1},$$

where $\nabla^2_{\theta\theta} F$ is the $K \times K$ Hessian of F with respect to θ. The fact that θ^* is a solution to the maximization problem (34) implies that $\nabla^2_{\theta\theta} F$ is negative semidefinite. That $\nabla_Z \theta^*$ exists then implies that $\nabla^2_{\theta\theta} F$ is nonsingular and thus negative definite. Because it is a Hessian, it is also symmetric. Therefore, its inverse $[\nabla^2_{\theta\theta} F]^{-1}$ is also symmetric and negative definite. Substituting in (39), we obtain

$$\Delta w_Z = -[\nabla^2_{\theta Z} F]' [\nabla^2_{\theta\theta} F]^{-1} [\nabla^2_{\theta Z} F] \geq 0,$$

which establishes (36) for the case in which $\Theta \subset \mathbb{R}^K$.

By the definition of a negative definite matrix B, $x' B x < 0$ for all $x \neq 0$, so to establish the strict inequality in (36) in this case, it suffices that one element of $\nabla_Z \theta^*$ is nonzero, that is, $\partial \theta^*_j / \partial Z \neq 0$ for one $j = 1, \ldots, K$, completing the proof. *Q.E.D.*

This theorem therefore shows that once we shift our focus to absolute bias, there is a fairly general result. Under very mild assumptions, technological change induced by a change in factor supplies will be biased toward the factor that has become more abundant.

REMARK 3: The assumption that θ^* is in the interior of Θ can be relaxed. In this case, for some components of θ, the first-order conditions may hold as strict inequalities. Given the continuous differentiability of F, these strict inequalities will continue to hold in response to a small change in Z, and thus there would be no change in these components of θ. This case is still covered by the theorem, because (36) is stated with a weak inequality.

REMARK 4: Theorem 2 is stated for Economies D, C, and M. It can be verified that it also applies to Economy O as long as $\nabla^2_{\theta\theta} F$ is negative definite at the equilibrium technology θ^*. Because in this economy, θ^* is not chosen by a single firm, but its different components are decided by different technology producers, negative-definiteness of $\nabla^2_{\theta\theta} F$ is not guaranteed and needs to be imposed as an additional assumption. A sufficient condition is that F is concave in θ, but in fact much less is necessary, because negative-definiteness of $\nabla^2_{\theta\theta} F$

at θ^* requires neither F to be concave in θ nor the vector θ^* to be a global maximum for the technology producers considered jointly.

There is a clear parallel between Theorem 2 and the LeChatelier principle, because we can think of the change in technology as happening in the "long run," in which case Theorem 2 states that long-run changes in marginal products (factor prices) will be less than those in the short run because of induced technological change. However, there are also some important differences. First, this theorem concerns how marginal products (or prices) change as a result of induced technological changes resulting from changes in factor supplies rather than the elasticity of short-run and long-run demand curves. Second, it applies to the equilibrium of an economy, not to the maximization problem of a single firm. This last distinction will become central in the next section.

Theorem 2 is stated and proved under the assumption that $\partial\theta_j^*/\partial Z$ exists at $(\bar{Z}, \bar{L})$ for all $j = 1, \dots, K$. This assumption entails two restrictions. The first is the usual nonsingularity requirement to enable an application of the implicit function theorem, i.e., that the Hessian of F with respect to θ, $\nabla^2_{\theta\theta}F$, is nonsingular at the point θ^* (see, for example, Rudin (1964, Theorem 9.18), or Simon and Blume (1994, Theorem 15.2)). The second is more subtle; because we have not made global concavity assumptions (except in Economy D), a small change in Z may shift the technology choice from one local optimum to another, thus essentially making $\partial\theta_j^*/\partial Z$ infinite (or undefined). This possibility is also ruled out by this assumption. In fact, the assumption that $\partial\theta_j/\partial Z$ exists at $(\bar{Z}, \bar{L})$ can be replaced by the following assumption:

ASSUMPTION A1: *$\nabla^2_{\theta\theta}F$ is nonsingular, and there exists $\epsilon > 0$ such that for all $\theta' \in \Theta$ with $\partial F(\bar{Z}, \bar{L}, \theta')/\partial\theta = 0$ and $\theta' \neq \theta^*(\bar{Z}, \bar{L})$, we have $F(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L})) - F(\bar{Z}, \bar{L}, \theta') > \epsilon$.*

The second part of the assumption ensures that the peaks of the function $F(\bar{Z}, \bar{L}, \theta)$ in θ are "well separated" in the sense that in response to a small change in factor supplies, there will not be a "shift" in the global optimum of θ from one local optimum to another.[27] Consequently, Assumption A1 is equiv-

[27]More explicitly, suppose that the maximization problem (34) has multiple local maxima, and denote the set of these maxima at factor proportions $(\bar{Z}, \bar{L})$ by $\Theta^m(\bar{Z}, \bar{L})$. All of these solutions satisfy the first-order necessary conditions of problem (34). Suppose $\hat{\theta}(\bar{Z}, \bar{L})$ is a vector that satisfies these first-order necessary conditions. Given the nonsingularity assumption (first part of Assumption A1), the implicit function theorem can be applied to $\hat{\theta}(\bar{Z}, \bar{L})$. However, this does not guarantee that $\partial\theta^*/\partial Z$ exists, because θ^* corresponds to the global maximum of (34) and, loosely speaking, the change in Z may "shift" the global maximum from $\hat{\theta}(\bar{Z}, \bar{L})$ to some other $\tilde{\theta}(\bar{Z}, \bar{L}) \in \Theta^m(\bar{Z}, \bar{L})$. The second part of Assumption A1 rules this possibility out by imposing that one of the solutions to the first-order necessary conditions gives *uniformly* higher value, so

alent to assuming that $\partial\theta_j^*/\partial Z$ exists at $(\bar{Z}, \bar{L})$ for all j. A straightforward condition to ensure that Assumption A1 is satisfied is to assume that F is strictly quasi-concave in θ, although this is considerably stronger than Assumption A1. Because it is more intuitive to directly assume that the derivatives $\partial\theta_j^*/\partial Z$ exist rather than imposing Assumption A1, I state the relevant theorems under this direct assumption. However, depending on taste, Assumption A1 can be substituted in Theorem 2 and the subsequent theorems.

An important shortcoming of Theorem 2 should be noted here. As Definition 9 makes clear, weak absolute equilibrium bias is a local phenomenon. For example, an increase in $\bar{Z}$ may change θ^* in a direction biased toward Z at $(\bar{Z}, \bar{L})$, but this change may be biased against Z at some different factor proportions, say $(\bar{Z}', \bar{L}')$. This is illustrated in the next example:

EXAMPLE 4—No Global Bias: Suppose that $F(Z, \theta) = Z + (Z - Z^2/8)\theta - C(\theta)$, and $Z \in \mathcal{Z} = [0, 6]$ and $\Theta = [0, 2]$ so that F is everywhere increasing in Z. Suppose also that $C(\theta)$ is a strictly convex and continuously differentiable function with $C'(0) = 0$ and $C'(2) = \infty$ (where C' denotes C's derivative). $F(Z, \theta)$ satisfies all the conditions of Theorem 2 at all points $Z \in \mathcal{Z} = [0, 6]$ (because F is strictly concave in θ everywhere on $\mathcal{Z} \times \Theta = [0, 6] \times [0, 2]$).

Now consider $\bar{Z} = 1$ and $\bar{Z}' = 5$ as two potential supply levels of factor Z. It can be easily verified that $\theta^*(1)$ satisfies $C'(\theta^*(1)) = 7/8$, while $\theta^*(5)$ is given by $C'(\theta^*(5)) = 15/8$. The strict convexity of $C(\theta)$ implies that $\theta^*(5) > \theta^*(1)$. Moreover, $w_Z(Z, \theta) = 1 + (1 - Z/4)\theta$, so

$$w_Z(5, \theta^*(5)) = 1 - \theta^*(5)/4 < 1 - \theta^*(1)/4 = w_Z(5, \theta^*(1)).$$

Similar examples can be constructed for any $\bar{Z}' > \bar{Z}$.

This exampleshows that it may not be possible to "splice" local absolutely biased changes so as to obtain global absolute bias (say between two levels $\bar{Z}$ and $\bar{Z}'$), because a change in technology that is absolutely biased toward Z at $\bar{Z}$ may be biased against this factor at $\bar{Z}'$ (see footnote 31 below for further discussion). Nevertheless, it is possible to obtain global results by imposing further structure to rule out "reversals" in the direction of bias of technologies. In particular, similar to Milgrom and Roberts' (1996) generalization of the LeChatelier principle, a global version of Theorem 2 can be obtained by imposing a form of "supermodularity." This is done in detail in Acemoglu (2005); here I simply give the main result:

DEFINITION 10: Let θ^* be the equilibrium technology choice in an economy with factor supplies $(\bar{Z}, \bar{L})$. Then there is *global absolute equilibrium* bias if for

that a small (infinitesimal) change in Z cannot induce a shift from one element of $\Theta^m(\bar{Z}, \bar{L})$ to another.

any $\bar{Z}', \bar{Z} \in \mathcal{Z}$, $\bar{Z}' \geq \bar{Z}$ implies that

$$w_Z(\tilde{Z}, \bar{L}, \theta^*(\bar{Z}', \bar{L})) \geq w_Z(\tilde{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L})) \quad \text{for all } \tilde{Z} \in \mathcal{Z} \text{ and } \bar{L} \in \mathcal{L}.$$

There are two notions of "globality" in this definition. First, the increase from $\bar{Z}$ to $\bar{Z}'$ is not limited to small changes. Second, the change in technology induced by this increase is required to raise the price of factor Z for all $\tilde{Z} \in \mathcal{Z}$. The proof of Theorem 3 shows that these two notions of globality are closely related. Now we state another definition (see, e.g., Topkis (1998)):

DEFINITIONS 11: Let $x = (x_1, \dots, x_n)$ be a vector in $X \subset \mathbb{R}^n$ and suppose that the real-valued function $f(x)$ is twice continuously differentiable in x. Then $f(x)$ is *supermodular* on X if and only if $\partial^2 f(x)/\partial x_i \partial x_{i'} \geq 0$ for all $x \in X$ and for all $i \neq i'$.

Let X and T be partially ordered sets. Then a function $f(x, t)$ defined on a subset S of $X \times T$ has *increasing differences* (*strict increasing differences*) in (x, t), if for all $t'' > t$, $f(x, t'') - f(x, t)$ is nondecreasing (increasing) in x.

THEOREM 3—Global Equilibrium Bias: *Suppose that Θ is a lattice, let $\bar{\mathcal{Z}}$ be the convex hull of $\mathcal{Z}$, let $\theta^*(\bar{Z}, \bar{L})$ be the equilibrium technology at factor proportions $(\bar{Z}, \bar{L})$, and suppose that $F(Z, L, \theta)$ is continuously differentiable in Z, supermodular in θ on Θ for all $Z \in \bar{\mathcal{Z}}$ and $L \in \mathcal{L}$, and exhibits strictly increasing differences in (Z, θ) on $\bar{\mathcal{Z}} \times \Theta$ for all $L \in \mathcal{L}$, then there is* global absolute equilibrium bias, *that is, for any $\bar{Z}', \bar{Z} \in \mathcal{Z}$, $\bar{Z}' \geq \bar{Z}$ implies*

$$\theta^*(\bar{Z}', \bar{L}) \geq \theta^*(\bar{Z}, \bar{L}) \quad \textit{for all } \bar{L} \in \mathcal{L}$$

and

$$w_Z(\tilde{Z}, \bar{L}, \theta^*(\bar{Z}', \bar{L})) \geq w_Z(\tilde{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L})) \quad \textit{for all } \tilde{Z} \in \mathcal{Z} \textit{ and } \bar{L} \in \mathcal{L}, \tag{40}$$

with strict inequality if $\theta^*(\bar{Z}', \bar{L}) \neq \theta^*(\bar{Z}, \bar{L})$.

PROOF: Given the continuity and the supermodularity of $F(Z, L, \theta)$ on $\bar{\mathcal{Z}} \times \Theta$, and the fact that Θ is a lattice and $\mathcal{Z}$ is a subset of $\mathbb{R}$ (and therefore also a lattice), Topkis' monotonicity theorem implies that the set of equilibrium technologies is a nonempty, compact, and complete sublattice of Θ (see Theorems 2.7.1, 2.8.1, 2.8.4, 2.8.6 and Corollary 2.7.1 in Topkis (1998)). Moreover, supermodularity of F in θ and strict increasing differences in (Z, θ) imply that $\bar{Z}' \geq \bar{Z} \implies \theta^*(\bar{Z}', \bar{L}) \geq \theta^*(\bar{Z}, \bar{L})$ for all $\bar{L} \in \mathcal{L}$. Next, (strict) increasing differences of $F(Z, L, \theta)$ in (Z, θ) on $\bar{\mathcal{Z}} \times \Theta$ imply that $\partial F(\tilde{Z}, \bar{L}, \theta)/\partial Z$

is increasing in θ for all $\tilde{Z} \in [\bar{Z}, \bar{Z}'] \subset \bar{\mathcal{Z}}$. Because $w_Z(\tilde{Z}, \bar{L}, \theta^*(\bar{Z}', \bar{L})) = \partial F(\tilde{Z}, \bar{L}, \theta^*(\bar{Z}', \bar{L}))/\partial Z$, the weak inequality in (40) follows. The fact that F exhibits strict increasing differences in (Z, θ) then establishes the strict inequality when $\theta^*(\bar{Z}', \bar{L}) \neq \theta^*(\bar{Z}, \bar{L})$. *Q.E.D.*

The fact that $\theta^*(\bar{Z}', \bar{L}) \geq \theta^*(\bar{Z}, \bar{L})$ (say, rather than $\theta^*(\bar{Z}', \bar{L}) \leq \theta^*(\bar{Z}, \bar{L})$) is not important, because the order over the set Θ is not specified. It could be that as $\bar{Z}$ increases, some measure of technology t declines. However, in this case, this measure would correspond to a type of technology biased *against* factor Z. If so, we can simply change the order over this parameter, for example, we can consider changes in $\tilde{t} = -t$ rather than t.

5. STRONG ABSOLUTE EQUILIBRIUM BIAS

The results in Section 4 concern "weak" bias in the sense that they compare marginal products at a given level of factor supplies (in response to a change in θ induced by a change in Z). This section turns to "strong" bias. It provides conditions under which equilibrium bias will be strong in the sense that once technology has adjusted, the increase in the supply of factor Z will *increase* its marginal product (price). It will also clarify the close connection between nonconvexity of the aggregate of production possibilities set and strong bias. As noted in the Introduction, an analysis of strong bias is particularly important because it clarifies the central role of the equilibrium structure in the analysis here (recall that strong bias and the resulting upward-sloping factor demand curves are not possible in basic production theory).

DEFINITION 12: Denote the equilibrium technology at factor supplies $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$ by $\theta^*(\bar{Z}, \bar{L})$ and suppose that $\partial \theta_j^*/\partial Z$ exists at $(\bar{Z}, \bar{L})$ for all $j = 1, \ldots, K$. Then there is *strong absolute equilibrium bias* at $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$ if

$$\frac{dw_Z}{dZ} = \frac{\partial w_Z}{\partial Z} + \sum_{j=1}^{K} \frac{\partial w_Z}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial Z} > 0.$$

In this definition, dw_Z/dZ denotes the total derivative, while $\partial w_Z/\partial Z$ denotes the partial derivative holding $\theta = \theta^*(\bar{Z}, \bar{L})$. Recall also that if F is jointly concave in (Z, θ) at $(Z, \theta^*(\bar{Z}, \bar{L}))$, its Hessian with respect to (Z, θ), $\nabla^2 F_{(Z,\theta)(Z,\theta)}$, is negative semidefinite at this point (though negative semidefiniteness is not sufficient for local joint concavity). The main theorem of this section (and of the paper) is the following:

THEOREM 4—Nonconvexity and Strong Bias: *Consider Economy D, C, or M. Suppose that Θ is a convex subset of $\mathbb{R}^K$, suppose that F is twice continuously differentiable in (Z, θ), let $\theta^*(\bar{Z}, \bar{L})$ be the equilibrium technology at factor supplies $(\bar{Z}, \bar{L})$, and assume that θ^* is in the interior of Θ and that $\partial \theta_j^*(\bar{Z}, \bar{L})/\partial Z$ exists* *at* $(\bar{Z}, \bar{L})$ *for all* $j = 1, \ldots, K$. *Then there is* strong absolute equilibrium bias *at* $(\bar{Z}, \bar{L})$ *if and only if* $F(Z, L, \theta)$*'s Hessian in* (Z, θ), $\nabla^2 F_{(Z,\theta)(Z,\theta)}$, is not negative semidefinite *at* $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$.

PROOF: Let us start with the case where $\Theta \subset \mathbb{R}$. Because, by hypothesis, θ^* is in the interior of Θ, as in the proof of Theorem 2, we have $\partial F/\partial\theta = 0$, $\partial^2 F/\partial\theta^2 \leq 0$, and (37). Substituting (37) into the definition for dw_Z/dZ and recalling that $\partial w_Z/\partial Z = \partial^2 F/\partial Z^2$, we have the condition for strong absolute equilibrium bias as

$$\begin{aligned}\frac{dw_Z}{dZ} &= \frac{\partial w_Z}{\partial Z} + \frac{\partial w_Z}{\partial \theta}\frac{\partial \theta^*}{\partial Z}\\ &= \frac{\partial^2 F}{\partial Z^2} - \frac{(\partial^2 F/\partial\theta\,\partial Z)^2}{\partial^2 F/\partial\theta^2} > 0.\end{aligned}$$

From Assumption 1 or 2, F is concave in Z, so $\partial^2 F/\partial Z^2 \leq 0$, and from the fact that θ^* is a solution to (34) and $\partial\theta^*/\partial Z$ exists, we also have $\partial^2 F/\partial\theta^2 < 0$. Then the fact that F's Hessian, $\nabla^2 F_{(Z,\theta)(Z,\theta)}$, is not negative semidefinite at $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$ implies that

$$\text{(41)}\qquad \frac{\partial^2 F}{\partial Z^2} \times \frac{\partial^2 F}{\partial \theta^2} < \left(\frac{\partial^2 F}{\partial Z\,\partial Z\theta}\right)^2.$$

Because, at the optimal technology choice, $\partial^2 F/\partial\theta^2 < 0$, this immediately yields $dw_Z/dZ > 0$, establishing strong absolute bias at $(\bar{Z}, \bar{L}, \theta(\bar{Z}, \bar{L}))$ as claimed in the theorem.

Conversely, if $\nabla^2 F_{(Z,\theta)(Z,\theta)}$ is negative semidefinite at $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$, then (41) does not hold and this together with $\partial^2 F/\partial\theta^2 < 0$ implies that $dw_Z/dZ \leq 0$.

For the general case where $\Theta \subset \mathbb{R}^K$, the overall change in the price of factor Z is

$$\text{(42)}\qquad \frac{dw_Z}{dZ} = \frac{\partial^2 F}{\partial Z^2} - [\nabla^2_{\theta Z} F]'[\nabla^2_{\theta\theta} F]^{-1}[\nabla^2_{\theta Z} F].$$

Again by the same arguments, $\partial^2 F/\partial Z^2 \leq 0$ and $\nabla^2_{\theta\theta} F$ is negative definite and symmetric (which implies that its inverse $[\nabla^2_{\theta\theta} F]^{-1}$ is also negative definite and symmetric). Lemma 1 in the Appendix shows that if Q is an $(n-1) \times (n-1)$ symmetric negative definite matrix, with inverse denoted by Q^{-1}, b is a scalar, and v is an $(n-1) \times 1$ column vector, then an $n \times n$ matrix

$$B = \begin{pmatrix} Q & v \\ v' & b \end{pmatrix}$$

is negative semidefinite *if and only if* $b - v'Q^{-1}v \leq 0$. Let us now apply this lemma with $B = [\nabla^2 F_{(Z,\theta)(Z,\theta)}]$, $b = \partial^2 F/\partial Z^2$, $Q = [\nabla^2_{\theta\theta} F]$, and $v = [\nabla^2_{\theta Z} F]$, so that (42) evaluated at $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$ is equal to $b - v'Q^{-1}v$. Lemma 1 then implies that if $\nabla^2 F_{(Z,\theta)(Z,\theta)}$ is not negative semidefinite at $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$, then $b - v'Q^{-1}v > 0$, so that $dw_Z/dZ > 0$ and there is strong bias at $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$.

Conversely, from Lemma 1, if $\nabla^2 F_{(Z,\theta)(Z,\theta)}$ is negative semidefinite at $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$, then $b - v'Q^{-1}v \leq 0$ and $dw_Z/dZ \leq 0$, so that there is no strong bias at $(\bar{Z}, \bar{L}, \theta^*(\bar{Z}, \bar{L}))$. *Q.E.D.*

There are two important results in this theorem. *First*, it demonstrates the possibility of strong absolute equilibrium bias, whereby the induced response of technology is sufficiently pronounced that the endogenous-technology factor demand curves are upward- rather than downward-sloping. Moreover, the conditions necessary for strong absolute equilibrium bias are not very restrictive (see also Theorem 5 below).[28] This theorem, therefore, contrasts with the standard results from basic producer theory, where factor demand curves are always downward-sloping (e.g., Mas-Colell, Whinston, and Green (1995, Proposition 5.C.2)). Second, the theorem establishes that there will be strong absolute equilibrium bias *if and only if* the Hessian of the function $F(Z, L, \theta)$ with respect to (Z, θ) fails to be negative semidefinite, which loosely corresponds to F failing to be jointly concave in (Z, θ). It therefore highlights the importance of nonconvexities in generating strong bias. In this sense, strong absolute equilibrium bias is a manifestation of a strong form of the market size effect discussed above.

REMARK 5: The assumption that $\theta^*(\bar{Z}, \bar{L})$ is in the interior of Θ in Theorem 4 is adopted to obtain the "if and only if" result. When $\theta^*(\bar{Z}, \bar{L})$ is at the boundary of Θ, strong equilibrium bias is again possible, but for the reasons pointed out in Remark 3, failure of negative semidefiniteness is no longer sufficient. Furthermore, Theorem 4 is stated for Economy D, C, or M. As discussed in Remark 4, it also applies to Economy O under the additional assumption that $\nabla^2_{\theta\theta} F$ is negative definite at θ^*. If $\nabla^2_{\theta\theta} F$ is not negative definite, then the aggregate production possibilities set will be nonconvex, but there may not be strong bias. Nevertheless, Economy O is not less likely to exhibit strong bias than the other economies, because the additional nonconvexity resulting from oligopolistic competition may create another force toward strong bias.

[28]It is straightforward to construct examples of strong absolute equilibrium bias. As a trivial example, take $F(Z, \theta) = 4Z^{1/2} + Z\theta - \theta^2/2$, which is concave in Z and θ, but not jointly concave in both for $Z > 1$. Consider a change from $\bar{Z} = 4$ to $\bar{Z} = 9$. It is easily verified that $\theta^*(\bar{Z} = 4) = 4$ while $\theta^*(\bar{Z} = 9) = 9$, so that $w_Z(\bar{Z} = 4, \theta^*(4)) = 5 < w_Z(\bar{Z} = 9, \theta^*(9)) = 9\frac{2}{3}$.

An immediate corollary of Theorem 4 is that strong absolute equilibrium bias is not possible in Economy D (where equilibria coincide with Pareto optima)[29]:

COROLLARY 1—No Strong Bias in Economy D: *Suppose that Θ is a convex subset of $\mathbb{R}^K$, F is twice continuously differentiable in (Z, θ), let the equilibrium technology at factor supplies $(\bar{Z}, \bar{L})$ be $\theta^*(\bar{Z}, \bar{L})$, and assume that $\partial\theta_j^*/\partial Z$ exists at $(\bar{Z}, \bar{L})$ for all $j = 1, \ldots, K$. Then there cannot be strong absolute equilibrium bias in Economy D.*

The proof follows because F is jointly concave in (Z, L, θ) in Economy D.

In Economy D, factor demand and technology choices, Z and θ, are made by the same agents. In contrast, in Economies C, M, and O, they are made by different agents. For example, in Economy M, final good producers choose input demands, while the technology monopolist chooses technology. This implies that we are at a maximum of F when we change only θ (and thus at a maximum of $F - w_Z Z$ for given w_Z), and also that we are at a maximum of $F - w_Z Z$ from final good producers' profit maximization. However, this does not guarantee that we are at a maximum of $F - w_Z Z$ in the entire (Z, θ) plane. In other words, the equilibrium may be a saddle point of $F - w_Z Z$ (or a quasi-saddle point of F) rather than a maximum.[30] When this is the case, a change in Z will induce θ to change in the direction of increasing both F and $F - w_Z Z$, and from the labor demand decisions of final good producers, this will increase the marginal product and price of factor Z.

Corollary 1 therefore shows that strong absolute bias is an *equilibrium* phenomenon and would not apply in Pareto optimal allocations. This is because when a social planner chooses both Z and θ, F (or $F - w_Z Z$) will be maximized in (Z, θ). Strong absolute bias is only possible when the equilibrium allocation corresponds to a quasi-saddle point of F, which in turn requires an interaction between the choices by different agents (e.g., final good producers and the technology monopolist). This discussion also clarifies the difference between the approach in this paper and the LeChatelier principle, which applies in partial equilibrium and implies that a firm's demand curve for a factor is always downward-sloping.

Finally, the following result further illustrates the importance of nonconvexities and characterizes what types of economies are likely to exhibit strong absolute equilibrium bias. Let $\mathcal{C}^2[B]$ denote the set of twice continuously differentiable functions over some compact set B and endow $\mathcal{C}^2$ with the standard

[29]Perhaps surprisingly, strong bias can arise in Economy D when Θ is nonconvex. See Acemoglu (2005) for an example.

[30]I refer to $(\bar{Z}, \theta^*)$ as a "quasi-saddle point" if it is a maximum of $F(Z, L, \theta)$ in θ and a maximum of $F(Z, L, \theta) - w_Z Z$ in Z, but there exists a direction in the (Z, θ) plane starting from $(\bar{Z}, \theta^*)$ along which both $F(Z, L, \theta)$ and $F(Z, L, \theta) - w_Z Z$ increase. Therefore, $(\bar{Z}, \theta^*)$ is a quasi-saddle point if $F(\bar{Z}, \theta)$ is maximized at $\theta = \theta^*$, but is not jointly concave in (Z, θ) at $(\bar{Z}, \theta^*)$.

"sup" metric. Let $\mathcal{C}^2_+[B] \subset \mathcal{C}^2[B]$ be the set of such functions that are strictly *convex* and let $\mathcal{C}^2_-[B] \subset \mathcal{C}^2[B]$ be the set of such functions that are strictly *concave* in each of their arguments (though not necessarily jointly so). Recall also that $F(Z, L, \theta) \equiv G(Z, L, \theta) - C(\theta)$. For simplicity, I assume $\Theta \subset \mathbb{R}$ (the generalization to $\Theta \subset \mathbb{R}^K$ is straightforward).

THEOREM 5—Conditions for Strong Bias: *Suppose that $\Theta \subset \mathbb{R}$ and $\mathcal{Z} \subset \mathbb{R}_+$ are compact, denote the equilibrium technology by θ^*, and for fixed $\bar{L} \in \mathcal{L}$, let $G(\bar{Z}, \bar{L}, \theta) \in \mathcal{C}^2_-[\mathcal{Z}\times\Theta]$. For each $C(\cdot) \in \mathcal{C}^2_+[\Theta]$, let $\mathcal{D}_C \subset \mathcal{C}^2_-[\Theta]$ be such that for all $G(\bar{Z}, \bar{L}, \theta) \in \mathcal{D}_C$ there is strong absolute equilibrium bias. Then we have*:

(i) *For each $C(\cdot) \in \mathcal{C}^2_+[\Theta]$, $\mathcal{D}_C$ is a nonempty open subset of $\mathcal{C}^2_-[\Theta]$.*

(ii) *Suppose that θ^* is an equilibrium technology for both $C_1(\cdot), C_2(\cdot) \in \mathcal{C}^2_+[\Theta]$ and that $\partial^2 C_1(\theta^*)/\partial\theta^2 < \partial^2 C_2(\theta^*)/\partial\theta^2$. Then $\mathcal{D}_{C_2} \subset \mathcal{D}_{C_1}$ (and $\mathcal{D}_{C_2} \neq \mathcal{D}_{C_1}$).*

PROOF: Because $C(\cdot) \in \mathcal{C}^2_+[\Theta]$, $0 < \partial^2 C(\theta^*)/\partial\theta^2 < \infty$, and because $G \in \mathcal{C}^2_-[\mathcal{Z}\times\Theta]$, $\partial^2 G/\partial Z^2 < 0$, and $\partial^2 G/\partial\theta^2 < 0$. From Theorem 4, only the values of the second derivatives evaluated at $(\bar{Z}, \bar{L}, \theta^*)$ determine whether there is strong bias. Because $\partial^2 C(\theta^*)/\partial\theta^2 > 0$, $\partial^2 G/\partial Z^2 < 0$, and $\partial^2 G/\partial\theta^2 < 0$, $\mathcal{D}_C$ includes all $G(\bar{Z}, \bar{L}, \theta)$ (for fixed $\bar{L} \in \mathcal{L}$) such that

$$\frac{\partial^2 G}{\partial Z^2} \times \left(\frac{\partial^2 G}{\partial\theta^2} - \frac{\partial^2 C}{\partial\theta^2}\right) < \left(\frac{\partial^2 G}{\partial Z\,\partial\theta}\right)^2$$

and is clearly a nonempty open subset of $\mathcal{C}^2_-[\Theta \times \mathcal{Z}]$, which proves part (i).

For part (ii), fix $G \in \mathcal{C}^2_-[\Theta \times \mathcal{Z}]$ and consider $C_1(\cdot), C_2(\cdot) \in \mathcal{C}^2_+[\Theta]$ with $\partial C_1(\theta^*)/\partial\theta = \partial C_2(\theta^*)/\partial\theta$ (so that θ^* is the equilibrium technology with both $C_1(\cdot)$ and $C_2(\cdot)$) and suppose that $\partial^2 C_1(\theta^*)/\partial\theta^2 < \partial^2 C_2(\theta^*)/\partial\theta^2$. Then $\partial^2 G/\partial Z^2 \times (\partial^2 G/\partial\theta^2 - \partial^2 C_2/\partial\theta^2) < (\partial^2 G/\partial Z\,\partial\theta)^2$ implies that

$$\frac{\partial^2 G}{\partial Z^2} \times \left(\frac{\partial^2 G}{\partial\theta^2} - \frac{\partial^2 C_1}{\partial\theta^2}\right) < \left(\frac{\partial^2 G}{\partial Z\,\partial\theta}\right)^2;$$

hence $\mathcal{D}_{C_2} \subset \mathcal{D}_{C_1}$. Next, take $G \in \mathcal{C}^2_-[\Theta \times \mathcal{Z}]$ such that

$$\frac{\partial^2 G}{\partial Z^2} \times \left(\frac{\partial^2 G}{\partial\theta^2} - \frac{\partial^2 C_2}{\partial\theta^2}\right) = \left(\frac{\partial^2 G}{\partial Z\,\partial\theta}\right)^2,$$

which is not in $\mathcal{D}_{C_2}$, but because

$$\frac{\partial^2 C_1(\theta^*)}{\partial\theta^2} < \frac{\partial^2 C_2(\theta^*)}{\partial\theta^2}, \quad \frac{\partial^2 G}{\partial Z^2} \times \left(\frac{\partial^2 G}{\partial\theta^2} - \frac{\partial^2 C_1}{\partial\theta^2}\right) < \left(\frac{\partial^2 G}{\partial Z\,\partial\theta}\right)^2,$$

$$G \in \mathcal{D}_{C_1}.$$

This establishes that $\mathcal{D}_{C_2} \neq \mathcal{D}_{C_1}$ and completes the proof. *Q.E.D.*

There are two important implications of this theorem. First, when the cost of generating new technologies is more "convex" in the sense that $\partial^2 C(\theta^*)/\partial\theta^2$ is higher, strong absolute equilibrium bias becomes less likely. This is intuitive, because a more convex C function corresponds to a more concave aggregate production possibilities set for the economy. The second and perhaps more important implication is that for any C function that is twice continuously differentiable over a compact set, there exists a nonempty open set of G functions that leads to strong absolute equilibrium bias. In other words, irrespective of the exact shape of the C function, strong bias is neither a pathological nor a nongeneric possibility.

Finally, I state a global version of Theorem 4[31]:

THEOREM 6—Nonconvexity and Global Strong Bias: *Suppose that Θ is a convex subset of $\mathbb{R}^K$ and that F is twice continuously differentiable in (Z,θ). Let $\bar{Z},\bar{Z}'\in\mathcal{Z}$, with $\bar{Z}'>\bar{Z}$, let $\bar{L}\in\mathcal{L}$, and let $\theta^*(\tilde{Z},\bar{L})$ be the equilibrium technology at factor supplies $(\tilde{Z},\bar{L})$, and assume that $\theta^*(\tilde{Z},\bar{L})$ is in the interior of Θ and that $\partial\theta_j^*/\partial Z$ exists at $(\tilde{Z},\bar{L})$ for all $j=1,\dots,K$ and all $\tilde{Z}\in[\bar{Z},\bar{Z}']$. Then there is* strong absolute equilibrium bias *at $(\{\bar{Z},\bar{Z}'\},\bar{L})$ if $F(Z,L,\theta)$'s Hessian, $\nabla^2 F_{(Z,\theta)(Z,\theta)}$, fails to be negative semidefinite at $(\tilde{Z},\bar{L},\theta^*(\tilde{Z},\bar{L}))$ for all $\tilde{Z}\in[\bar{Z},\bar{Z}']$.*

PROOF: The proof follows from the fundamental theorem of calculus and the proof of Theorem 4. Take $\bar{Z}$ and $\bar{Z}'>\bar{Z}$ in $\mathcal{Z}$ and fix $\bar{L}\in\mathcal{L}$. Then

$$w_Z(\bar{Z}',\bar{L},\theta^*(\bar{Z}',\bar{L}))-w_Z(\bar{Z},\bar{L},\theta^*(\bar{Z},\bar{L}))=\int_{\bar{Z}}^{\bar{Z}'}\frac{dw_Z(\bar{Z},\bar{L},\theta^*(Z,\bar{L}))}{dZ}\,dZ. \tag{43}$$

The hypotheses of the theorem, combined with the proof of Theorem 4, imply that $dw_Z(\bar{Z},\bar{L},\theta(Z,\bar{L}))/dZ>0$ for all $Z\in[\bar{Z},\bar{Z}']$, so (43) is positive, establishing the result. *Q.E.D.*

6. CONCLUSION

An investigation of the determinants of equilibrium bias is important both for a better understanding of the nature of technological change and for the

[31] At this point, let us return to Example 4 and to the question of why local weak bias did not lead to global weak bias there. One might have conjectured that an argument using the fundamental theorem of calculus similar to that in the proof of Theorem 6, in particular, equation (43), may work for weak bias as well as strong bias. To illustrate why this is not the case, take $\Theta\subset\mathbb{R}$, so that $dw_Z/dZ=\partial w_Z/\partial Z+(\partial w_Z/\partial\theta)\times(\partial\theta^*/\partial Z)$. Equation (43) and Theorem 6 apply to this entire expression, while weak bias concerns the second term, and it is not possible to apply the fundamental theorem of calculus just to this term.

study of the distributional implications of new technologies. In this paper, I analyzed the implications of changes in factor supplies on relative and absolute biases of technology and presented three main sets of results.

First, when the economy has two factors and two technologies, each of which augments one of the factors, equilibrium technology is relatively biased toward the factor that has become more abundant—in the sense that the induced change in technology will increase the relative price of the factor whose supply has increased. Moreover, when the elasticity of substitution between factors is sufficiently large, this induced bias can be strong enough to increase the relative price of the factor that becomes more abundant. These results concerning relative bias do not generalize to economies where the available menu of technologies includes non-factor-augmenting choices.

Second, I proposed the concept of absolute bias, which refers to how induced changes in technology affect the level of factor awards. Under fairly mild assumptions, changes in technology induced by small changes in factor supplies are always (absolutely) biased toward the factor that has become more abundant—in the sense that the induced change in technology increases the demand or the marginal product of the factor that has become more abundant.

Third, when the aggregate production possibilities set (inclusive of the technology costs) is nonconvex, there is strong absolute equilibrium bias—so that an increase in the supply of a factor induces a sufficiently large change in technology and raises the marginal product (price) of the factor that has become more abundant. Consequently, in such economies the endogenous-technology demand curves for factors are *upward-sloping*. The analysis established not only the possibility of strong bias, but also provided precise conditions for an economy to exhibit strong absolute equilibrium bias. In particular, economies without nonconvexities (or those where the equilibrium corresponds to a Pareto optimum) cannot exhibit strong bias. In contrast, in economies where factor demands and technology choices are made by different agents (e.g., technology producers versus final good producers), such nonconvexities are possible (in fact quite typical), and strong absolute bias and upward-sloping demand curves emerge in equilibrium.

To keep the exposition simple, the paper has made a number of simplifying assumptions. An obvious generalization is to introduce multiple goods rather than a single final good. This complicates the analysis, but the general insights do not appear to depend on the single good assumption. Another interesting direction for future research might be to integrate some of these results into growth models where there can be long-run growth due to technological change (see Acemoglu (1998, 2002, 2003b) or Jones (2005), for various growth models with relative equilibrium bias). It would also be interesting to investigate the implications of the results of absolute bias presented here for the theory of economic growth, for example, in the context of the effects of population growth on technological change, wages, and standards of living. Finally, the most important area for future research is an empirical investigation of whether the implications of these strong theorems actually hold in the data.

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APPENDIX

The following lemma is used in the proof of Theorem 4.[32] Recall that for a matrix (vector) v, v' denotes its transpose.

LEMMA 1: *Consider the* $n \times n$ *matrix*

$$B = \begin{pmatrix} Q & v \\ v' & b \end{pmatrix}, \tag{A1}$$

where Q *is an* $(n-1) \times (n-1)$ *symmetric negative definite matrix*, b *is a scalar, and* v *is an* $(n-1) \times 1$ *column vector. Then we have that* B *is negative semidefinite if and only if* $b - v'Q^{-1}v \leq 0$.

PROOF: $(\Longleftarrow)$ B is negative semidefinite if and only if

$$(x; y)'B(x; y) \leq 0,$$

where x is an arbitrary $(n-1) \times 1$ vector, y is a scalar, and $(x; y)$ is the $n \times 1$ column vector constructed by stacking x and y. Using the form of B in (A1), we have

$$(x; y)'B(x; y) = x'Qx + 2yx'v + by^2. \tag{A2}$$

When $y = 0$, the above expression is always nonpositive because Q is negative definite, so B is negative semidefinite as claimed.

Next consider the case where $y \neq 0$. In this case, let z be the $(n-1) \times 1$ vector constructed as $z = x/y$ and let us expand (A2) as

$$\begin{aligned}(x; y)'B(x; y) &= y^2(z'Qz + 2z'v + b) \\ &= y^2(z'Qz + 2z'v + v'Q^{-1}v) + y^2(b - v'Q^{-1}v).\end{aligned} \tag{A3}$$

Because Q is a real symmetric negative definite matrix, $-Q$ is a real symmetric and positive definite matrix, so there exists a nonsingular matrix M such that

[32] I thank Alp Simsek for help with the proof of this lemma. Alternatively, this lemma can be proved using the notion of Schur complements (e.g., Lay (1997, Chap. 2)).

$-Q = M'M$. Moreover, we also have that $-Q^{-1} = M^{-1}(M')^{-1} = M^{-1}(M^{-1})'$ (because $(M')^{-1} = (M^{-1})'$). Now, rewriting (A3) in terms of M, we have

$$\text{(A4)} \qquad (x;y)'B(x;y) \\ = -y^2(z'(-Q)z - 2z'v - v'Q^{-1}v) + y^2(b - v'Q^{-1}v) \\ = -y^2(z'(M'M)z - 2z'v + v'M^{-1}(M')^{-1}v) + y^2(b - v'Q^{-1}v).$$

Equation (A4) implies that B is negative semidefinite if and only if

$$\kappa \equiv y^2(z'(M'M)z - 2z'v + v'M^{-1}(M')^{-1}v) - y^2(b - v'Q^{-1}v) \geq 0.$$

Now, rearranging terms and with straightforward matrix manipulation, we have

$$\kappa \equiv y^2\big((Mz)'Mz - 2z'(M'(M')^{-1})v + ((M^{-1})'v)'(M^{-1})'v\big) \\ - y^2(b - v'Q^{-1}v) \\ \equiv y^2\big((Mz)'Mz - 2(Mz)'(M^{-1})'v + ((M^{-1})'v)'(M^{-1})'v\big) \\ - y^2(b - v'Q^{-1}v) \\ \equiv y^2\big[(Mz - (M^{-1})'v)'(Mz - (M^{-1})'v)\big] - y^2(b - v'Q^{-1}v).$$

Therefore, B is negative semidefinite if and only if

$$\text{(A5)} \qquad \kappa \equiv y^2\big[(Mz - (M^{-1})'v)'(Mz - (M^{-1})'v)\big] - y^2(b - v'Q^{-1}v) \geq 0.$$

Now suppose

$$b - v'Q^{-1}v \leq 0.$$

Then, from (A5), the first term of κ takes the form $y^2a'a$ for $a \equiv (Mz - (M^{-1})'v)'(Mz - (M^{-1})'v)$ and is always nonnegative for any z, so $\kappa \geq 0$, establishing that B is negative semidefinite.

($\Longrightarrow$) Conversely, suppose that B is negative semidefinite, which implies that $(x;y)'B(x;y) \leq 0$ for all $(x;y)$. To obtain a contradiction, suppose that

$$b - v'Q^{-1}v > 0.$$

Take $y \neq 0$, and in terms of (A5), set $z = M^{-1}(M')^{-1}v$, which yields $\kappa = -y^2(b - v'Q^{-1}v) < 0$ in (A5), contradicting the hypothesis that B is negative semidefinite (or that $(x;y)'B(x;y) \leq 0$ for all $(x;y)$), thus yielding a contradiction. *Q.E.D.*

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