A Proofs

A.1 Lemma 1

Proof of Lemma 1. ($\Rightarrow$) Suppose that (w^*, p, m, c^*, q^*) satisfies conditions (5)-(8). Let us start with the first-order condition associated with utility maximization abroad. Since we have normalized prices so that the marginal utility of income in Foreign is equal to one, the necessary first-order condition associated with (5) implies

$$u_i^{*\prime}(c_i^*) = p_i, \tag{27}$$

$$\int_i p_i c_i^* di = w^* L^*. \tag{28}$$

Turning to the necessary first-order condition associated with profit maximization abroad, condition (6), we get

$$p_i \leq w^* a_i^*, \text{ with equality if } q_i^* > 0. \tag{29}$$

Together with the definition of $m_i \equiv c_i - q_i$, the good market clearing condition (8) implies

$$c_i^* = q_i^* - m_i. \tag{30}$$

Combining conditions (27), (29), and (30) and using the convention $u_i^{*\prime}(-m_i) \equiv \infty$ if $m_i \geq 0$, we obtain equation (10). Similarly, we can rearrange equations (27) and (30) as

$$c_i^* = d_i^*(p_i), \tag{31}$$

$$q_i^* = c_i^* + m_i, \tag{32}$$

where $d_i^*(\cdot) \equiv u_i^{*\prime -1}(\cdot)$ denotes the foreign demand for good i. Equation (12) immediately derives from equations (29), (31), and (32). Equation (11) can then be obtained from equations (12) and (30). To conclude, note that equations (7) and (12) immediately imply equation (13), whereas equations (7) and (29) imply

$$\int_i p_i q_i^* di = w^* L^*.$$

Combining the previous expression with equations (10), (27), and (30), we obtain equation (14).

($\Leftarrow$) Now suppose that (w^*, p, m, c^*, q^*) satisfies equations (10)-(14). Equations (10) and (11) imply (27), whereas equations (10), (12), (13), and (14) imply equation (28). Since the foreign consumer's utility maximization problem is concave, the two first-order conditions (27) and (28) are sufficient for condition (5) to hold. Similarly, equations (10) and (12) imply condition (29). Since the foreign firm's profit maximization problem is concave, this first-order condition is sufficient for condition (6) to hold as well. Finally, equations (12) and (13) imply equation (7) and equations (11) and (12) imply equation (8). $\square$

A.2 Lemma 2

Proof of Lemma 2. ($\Rightarrow$) Suppose that $\left(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*}\right)$ solves Home's planning problem. By Definition 3, $\left(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*}\right)$ solves

$$\max_{w^*\geq 0, p\geq 0, c\geq 0, c^*, q\geq 0, q^*} \int u_i\left(c_i\right) di$$

subject to (5)-(9). By definition of $m \equiv c - q$, we know that $c_i = m_i + q_i$ for all i. By Lemma 1, we also know that $\left(w^*, p, c, c^*, q, q^*\right)$ satisfies conditions (5)-(8) if and only if equations (10)-(14) hold. The two previous observations imply that $\left(w^{0*}, m^0 = c^0 - q^0, q^0\right)$ solves

$$\max_{w^*\geq, m, q\geq 0} \int_i u_i(q_i + m_i) di \tag{P'}$$

subject to

$$\int_i a_i q_i di \leq L, \tag{33}$$

$$\int_i a_i^* q_i^* \left(m_i, w^*\right) di = L^*, \tag{34}$$

$$\int_i p_i(m_i, w^*) m_i di = 0. \tag{35}$$

The rest of the argument proceeds by contradiction. Suppose that $\left(w^{0*}, m^0, q^0\right)$ solves (P'), but does not solve (P). Then there must exist a solution $\left(w^{1*}, m^1, q^1\right)$ of (P) such that at least one of the two constraints (16) and (17) is slack. There are three possible cases. First, constraints (16) and (17) may be simultaneously slack. In this case, starting from m^1, one could strictly increase imports for a positive measure of goods by a small amount, while still satisfying (15)-(17). This would strictly increase utility and contradict the fact that $\left(w^{1*}, m^1, q^1\right)$ solves (P). Second, constraint (16) may be slack, whereas constraint (17) is binding. In this case, starting from w^{*1} and m^1, one could strictly increase imports for a positive measure of goods and decrease the foreign wage by a small amount such that (17) still binds. Since (15) is independent of w^* and m and (16) is slack to start with, (15)-(17) would still be satisfied. Since domestic utility is independent of w^*, this would again increase utility and contradict the fact that $\left(w^{1*}, m^1, q^1\right)$ solves (P). Third, constraint (17) may be slack, whereas constraint (16) is binding. In this case, starting from w^{*1} and m^1, one could strictly increase imports for a positive measure of goods and increase the foreign wage by a small amount such that (16) still binds. For the exact same reasons as in the previous case, this would again contradict the fact that $\left(w^{1*}, m^1, q^1\right)$ solves (P).

($\Leftarrow$) Suppose that $\left(w^{0*}, m^0, q^0\right)$ solves (P). From the first part of our proof we know that at any solution to (P), (16) and (17) must be binding. Thus $\left(m^0, q^0, w^{0*}\right)$ solves (P'). Now consider $\left(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*}\right)$ such that $p^0 = p\left(m^0, w^{0*}\right)$, $c^0 = m^0 + q^0$, $c^{0*} = c^*\left(m^0, w^{0*}\right)$, and $q^{0*} = q^*\left(m^0, w^{0*}\right)$. From Lemma 1, $\left(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*}\right)$ therefore also satisfies constraints (5)-(9). Furthermore, from the first part of our proof, any solution $\left(w^{1*}, p^1, c^1, c^{1*}, q^1, q^{1*}\right)$ to Home's planning problem must also solve (P') and, in turn, satisfy

$$\int_i u_i(q_i^1 + m_i^1)di = \int_i u_i(q_i^0 + m_i^0)di.$$

This implies that $\left(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*}\right)$ solves Home's planning problem. $\square$

A.3 Lemma 3

Proof of Lemma 3. ($\Rightarrow$) Suppose that $\left(m^0, q^0\right)$ solves (P_{w^*}). Let us first demonstrate that (P_{w^*}) is a concave maximization problem. Consider $f_i(m_i) \equiv p_i(m_i, w^{0*})m_i$. By equation (10), we know that

$$f_i(m_i) = \begin{cases} m_i w^* a_i^*, & \text{if } m_i > -d_i^*(w^* a_i^*), \\ m_i u_i^{*\prime}(-m_i), & \text{if } m_i \leq -d_i^*(w^* a_i^*). \end{cases}.$$

For $m_i > -d_i^*(w^* a_i^*)$, we have $f_i'(m_i) = w^* a_i^*$. For $m_i < -d_i^*(w^* a_i^*)$, $\sigma^* \geq 1$ implies $f_i'(m_i) = \left(1 - \frac{1}{\sigma^*}\right) \beta_i^* (-m_i)^{-\frac{1}{\sigma^*}} > 0$ and $f_i''(m_i) = \frac{1}{\sigma^*}\left(1 - \frac{1}{\sigma^*}\right) \beta_i^* (-m_i)^{-\frac{1}{\sigma^*}-1} > 0$. Since

$$\lim_{m_i \to^+ -d_i^*(w^* a_i^*)} f'(m_i) = w^* a_i^* > \lim_{m_i \to^- -d_i^*(w^* a_i^*)} \left(1 - \frac{1}{\sigma^*}\right) w^* a_i^*,$$

f_i is convex and increasing for all i.

Now consider $g_i(m_i) \equiv a_i^* q_i^*\left(m_i, w^{0*}\right)$. By equation (12), we know that

$$g_i(m_i) = \begin{cases} m_i a_i^* + a_i^* d_i^*(w^* a_i^*), & \text{if } m_i > -d_i^*(w^* a_i^*), \\ 0, & \text{if } m_i \leq -d_i^*(w^* a_i^*). \end{cases}$$

For $m_i > -d_i^*(w^* a_i^*)$, we have $g_i'(m_i) = a_i^*$. For $m_i < -d_i^*(w^* a_i^*)$, $g'(m_i) = 0$. Thus g_i is convex and increasing for all i.

Since u_i is strictly concave in (m_i, q_i), $a_i q_i$ is linear in q_i, and f_i and g_i are convex in m_i, the objective function is a concave functional, whereas the constraints are of the form $G(m, q) \leq 0$, with G a convex functional. Accordingly, Theorem 1, p. 217 in Luenberger (1969) implies the existence of $(\lambda, \lambda^*, \mu) \geq 0$ such that $\left(m^0, q^0\right)$ solves

$$\max_{m,q \geq 0} \mathcal{L}(m, q, \lambda, \lambda^*, \mu; w^*) \equiv \int_i u_i(q_i + m_i)\, di - \lambda \int_i a_i q_i di - \lambda^* \int_i a_i^* q_i^*(m_i, w^*)\, di - \mu \int_i p_i(m_i, w^*) m_i di.$$

and the three following conditions hold:

$$\begin{aligned}\lambda\left(L-\int_i a_i q_i^0 di\right) &= 0,\\ \lambda^*\left(L^*-\int_i a_i^* q_i^*\left(m_i^0,w^*\right)di\right) &= 0,\\ \mu\left(\int_i p_i(m_i^0,w^*)m_i^0 di\right) &= 0.\end{aligned}$$

Since $\left(m^0,q^0\right)$ satisfies constraints (15)-(17), we therefore have

$$\lambda\geq 0,\ \int_i a_i q_i^0 di\leq L,\text{ with complementary slackness,} \tag{36}$$

$$\lambda^*\geq 0,\ \int_i a_i^* q_i^*\left(m_i^0,w^*\right)di\leq L^*,\text{ with complementary slackness,} \tag{37}$$

$$\mu\geq 0,\ \int_i p_i(m_i,w^*)m_i^0 di\leq 0,\text{ with complementary slackness.} \tag{38}$$

To conclude, note that if $\left(m^0,q^0\right)$ solves $\max_{m,q\geq 0}\mathcal{L}\left(m,q,\lambda,\lambda^*,\mu;w^*\right)$, then for almost all i, $\left(m_i^0,q_i^0\right)$ must solve

$$\max_{m_i,q_i\geq 0}\mathcal{L}_i\left(m_i,q_i,\lambda,\lambda^*,\mu;w^*\right)\equiv u_i\left(q_i+m_i\right)-\lambda a_i q_i \\ -\lambda^* a_i^* q_i^*\left(m_i,w^*\right)-\mu p_i(m_i,w^*)m_i.$$

($\Leftarrow$) Now suppose that $\left(m_i^0,q_i^0\right)$ solves (P_i) for almost all i with λ,λ^*,μ such that conditions (36)-(38) hold. This implies

$$\left(m^0,q^0\right)\in\arg\max_{m,q\geq 0}\mathcal{L}\left(m,q,\lambda,\lambda^*,\mu;w^*\right).$$

Suppose first that all Lagrange multipliers are strictly positive: $\lambda>0$, $\lambda^*>0$, $\mu>0$, then conditions (36)-(38) imply

$$\begin{aligned}\int_i a_i q_i^0 di &= L,\\ \int_i a_i^* q_i^*\left(m_i^0,w^*\right)di &= L^*,\\ \int_i p_i(m_i^0,w^*)m_i^0 di &= 0.\end{aligned}$$

Thus Theorem 1, p. 220 in Luenberger (1969) immediately implies that $\left(m^0,q^0\right)$ is a solution to (P_{w^*}). Now suppose that at least one Lagrange multiplier is equal to zero. For expositional purposes suppose that $\lambda=0$, whereas $\lambda^*>0$ and $\mu>0$. In this case, we have

$$\left(m^0,q^0\right)\in\arg\max_{m,q\geq 0}\mathcal{L}\left(m,q,0,\lambda^*,\mu;w^*\right)$$

and

$$\begin{aligned}\int_i a_i^* q_i^*\left(m_i^0,w^*\right)di &= L^*,\\ \int_i p_i(m_i^0,w^*)m_i^0 di &= 0.\end{aligned}$$

Thus Theorem 1, p. 220 in Luenberger (1969) now implies that $\left(m^0, q^0\right)$ is a solution to

$$\max_{m,q\geq 0} \int_i u_i\left(q_i + m_i\right) di$$

subject to

$$\begin{aligned} \int_i a_i^* q_i^*\left(m_i, w^*\right) di &\leq L^*, \\ \int_i m_i p_i(m_i, w^*) di &\leq 0. \end{aligned}$$

Since $\int_i a_i q_i^0 di \leq L$ by condition (36), $\left(m^0, q^0\right)$ is therefore also a solution to (P_{w^*}). The other cases can be dealt with in a similar manner. $\square$

A.4 Proposition 1

Proof of Proposition 1. We first solve for the output level $q_i^0\left(m_i\right)$ that maximizes $\mathcal{L}_i\left(m_i, q_i, \lambda, \lambda^*, \mu; w^*\right)$, taking m_i as given. Since $\mathcal{L}_i\left(m_i, q_i, \lambda, \lambda^*, \mu; w^*\right)$ is strictly concave and differentiable in q_i, the optimal output level, $q_i^0\left(m_i\right)$, is given by the necessary and sufficient first-order condition,

$$u_i'\left(q_i^0\left(m_i\right) + m_i\right) \leq \lambda a_i, \text{ with equality if } q_i^0\left(m_i\right) > 0.$$

The previous condition can be rearranged in a more compact form as

$$q_i^0\left(m_i\right) = \max\left\{d_i\left(\lambda a_i\right) - m_i, 0\right\}. \tag{39}$$

Note that the domestic resource constraint (15) must be binding at any solution of (P_{w^*}). Otherwise the domestic government could strictly increase utility by increasing output. Thus λ must be strictly positive by Lemma 3, which implies that $q_i^0\left(m_i\right)$ is well-defined.

Let us now solve for the value of m_i that maximizes $\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right)$. The same arguments as in the proof of Lemma 3 imply that the previous Lagrangian is concave in m_i with a kink at $m_i = M_i^I \equiv -d_i^*(w^* a_i^*) < 0$, when Foreign starts producing good i; see equation (12). To study how $\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right)$ varies with m_i, we consider three regions separately: $m_i < M_i^I$, $M_i^I \leq m_i \leq M_i^{II}$, and $m_i > M_i^{II}$, where $M_i^{II} \equiv d_i\left(\lambda a_i\right) > 0$ is the import level at which Home stops producing good i; see equation (39).

First, suppose that $m_i < M_i^I$. In this region, equations (10), (12), and (39) imply

$$\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right) = u_i\left(d_i\left(\lambda a_i\right)\right) - \lambda a_i d_i\left(\lambda a_i\right) + \lambda a_i m_i - \mu m_i u_i^{*\prime}\left(-m_i\right).$$

CES utility further implies $u_i^{*\prime}\left(c_i^*\right) = \beta_i^*\left(c_i^*\right)^{-1/\sigma}$. Thus, $\mathcal{L}_i$ is strictly increasing if $m_i \in \left(-\infty, m_i^I\right)$ and strictly decreasing if $m_i \in \left(m_i^I, M_i^I\right)$, with $m_i^I \equiv -\left(\frac{\sigma^*}{\sigma^*-1}\frac{\lambda a_i}{\mu\beta_i^*}\right)^{-\sigma^*}$. Furthermore, by definition of $M_i^I \equiv -d_i^*(w^* a_i^*) = -(w^* a_i^*/\beta_i^*)^{-\sigma^*}$, the interval $\left(m_i^I, M_i^I\right)$ is non-empty if $\frac{a_i}{a_i^*} < A^I \equiv \frac{\sigma^*-1}{\sigma^*}\frac{\mu w^*}{\lambda}$.

When the previous inequality is satisfied, the concavity of $\mathcal{L}_i$ implies that Home exports m_i^I units of good i, whereas Foreign does not produce anything.

Second, suppose that $m_i \in \left[M_i^I, M_i^{II}\right]$. In this region, equations (10), (12), and (39) imply

$$\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right)$$
$$= u_i\left(d_i\left(\lambda a_i\right)\right) - \lambda a_i d_i\left(\lambda a_i\right) + \left(\lambda a_i - \left(\lambda^* + \mu w^*\right) a_i^*\right) m_i - \lambda^* a_i^* d_i^*\left(w^* a_i^*\right),$$

which is strictly decreasing in m_i if and only if $\frac{a_i}{a_i^*} < A^{II} \equiv \frac{\lambda^*+\mu w^*}{\lambda}$. When $\frac{a_i}{a_i^*} \in \left[A^I, A^{II}\right)$, the concavity of $\mathcal{L}_i$ implies that Home will export M_i^I units of good i. When $\frac{a_i}{a_i^*} = A^{II}$, the Lagrangian is flat between M_i^I and M_i^{II} so that any import level between M_i^I and M_i^{II} is optimal.

Finally, suppose that $M_i^{II} \leq m_i$. In this region, equations (10), (12), and (39) imply

$$\mathcal{L}_i\left(m_i, q_i^0\left(m_i\right), \lambda, \lambda^*, \mu; w^*\right) = u_i\left(m_i\right) - \left(\lambda^* + \mu w^*\right) a_i^* m_i - \lambda^* a_i^* d_i^*\left(w^* a_i^*\right),$$

which is strictly increasing if $m_i \in \left(M_i^{II}, m_i^{II}\right)$ and strictly decreasing if $m_i \in \left(m_i^{II}, \infty\right)$, with $m_i^{II} \equiv d_i\left(\left(\lambda^* + \mu w^*\right) a_i^*\right)$. Furthermore, by definition of $M_i^{II} \equiv d_i\left(\lambda a_i\right)$, $\left(M_i^{II}, m_i^{II}\right)$ is non-empty if $\frac{a_i}{a_i^*} > A^{II} \equiv \frac{\lambda^*+\mu w^*}{\lambda}$. When this inequality is satisfied, the concavity of $\mathcal{L}_i$ implies that Home will import m_i^{II} units of good i. Proposition 1 directly follows from the previous observations. $\square$

B Restricted Tax Instruments

The goal of this appendix is to characterize the solution to the domestic government's problem, as described in Definition 2, when ad-valorem trade taxes $t \equiv (t_i)$ are constrained to be zero for all exported goods. Our main finding is that optimal tariffs remain uniform under this restriction. The formal proof is an alternative to the one offered in Proposition 1 of Opp (2009).

Using the equilibrium conditions derived in Section 2.2, one can express the domestic government's problem in the absence of export taxes and subsidies as

$$\max_{t,\alpha,\alpha^*,c,c^*,\lambda,w,w^*} \int u_i(c_i) di$$

subject to foreign constraints

$$u_i^{*\prime}(c_i^*) = \min\left\{w^* a_i^*, w a_i\right\},$$

$$\int a_i^*\left(c_i^* \alpha_i^* + c_i \alpha_i\right) di = L^*,$$

$$\int \min\left\{w^* a_i^*, w a_i\right\} c_i^* di = w^* L^*,$$

$$\alpha_i^* = \begin{cases} = 0 & wa_i < w^* a_i^*, \\ \in [0,1] & wa_i = w^* a_i^*, \\ = 1 & wa_i > w^* a_i^*, \end{cases}$$

as well as local constraints

$$u_i'(c_i) = \lambda \min\left\{(1+t_i)\, w^* a_i^*, wa_i\right\},$$

$$\int a_i \left(c_i^*(1-\alpha_i^*) + c_i(1-\alpha_i)\right) di = L,$$

$$\alpha_i = \begin{cases} = 0 & wa_i < (1+t_i)\, w^* a_i^*, \\ \in [0,1] & wa_i = (1+t_i)\, w^* a_i^*, \\ = 1 & wa_i > (1+t_i)\, w^* a_i^*. \end{cases}$$

In the previous constraints, α_i and α_i^* measure the share of domestic and foreign consumption of good i, respectively, that is produced in Foreign. In a competitive equilibrium, any good that is produced by foreign firms for the foreign market must have a lower marginal cost than domestic firms: $\alpha_i^* > 0 \Rightarrow wa_i \geq w^* a_i^*$. Similarly, any good that is produced by foreign firms for the domestic market must have a lower marginal cost than domestic firms inclusive of the tariff: $\alpha_i > 0 \Rightarrow wa_i \geq (1+t_i) w^* a_i^*$. Note also that we normalize the Lagrange multiplier on the foreign budget constraint to one and omit the local budget constraint. This is implied by the other constraints, an expression of Walras' law.

For given (α^*, c^*, w, w^*), we focus on the inner problem:

$$\max_{t,\alpha,c,\lambda} \int u_i(c_i) di$$

subject to

$$\int a_i^* \left(c_i^* \alpha_i^* + c_i \alpha_i\right) di = L^*,$$

$$u_i'(c_i) = \lambda \min\left\{(1+t_i)\, w^* a_i^*, wa_i\right\},$$

$$\int a_i \left(c_i^*(1-\alpha_i^*) + c_i(1-\alpha_i)\right) di = L,$$

$$\alpha_i = \begin{cases} = 0 & wa_i < (1+t_i)\, w^* a_i^*, \\ \in [0,1] & wa_i = (1+t_i)\, w^* a_i^*, \\ = 1 & wa_i > (1+t_i)\, w^* a_i^*. \end{cases}$$

In order to characterize the solution of the inner problem, it is convenient to study first the relaxed version of the inner problem:

$$\max_{t,\alpha\in[0,1],c,\lambda} \int u_i(c_i) di$$

subject to

$$\int a_i^* \left(c_i^* \alpha_i^* + c_i \alpha_i\right) di = L^*,$$

$$\int a_i \left(c_i^* (1-\alpha_i^*) + c_i (1-\alpha_i)\right) di = L.$$

Let ν^* and ν, denote the Lagrange multipliers associated with the two previous constraints. The generalized Kuhn-Tucker conditions of the relaxed inner problem leads to

$$u_i'(c_i) = \alpha_i \nu^* a_i^* + (1-\alpha_i) \nu a_i,$$

as well as

$$\alpha_i = \begin{cases} 0 & \nu^* a_i^* > \nu a_i, \\ 1 & \nu^* a_i^* < \nu a_i. \end{cases}$$

Note that by the monotonicity of the ratio a_i^*/a_i, there exists a cutoff $\bar{i}$ such that $\nu^* a_{\bar{i}}^* = \nu a_{\bar{i}}$. Furthermore, it satisfies $\alpha_i = 0$ if $i < \bar{i}$ and $\alpha_i = 1$ if $i > \bar{i}$.

Now let us return to the original inner problem. Let us construct t and λ such that

$$\begin{aligned} 1 + t_i &= w\nu^*/w^*\nu, \\ \lambda &= w^*\nu/w. \end{aligned}$$

Given the previous values of t and λ, one can check that the solution of the relaxed problem, (α, c), also satisfies the two constraints that had been dropped

$$u_i'(c_i) = \lambda \min\left\{(1+t_i)\, w^* a_i^*, w a_i\right\},$$

$$\alpha_i = \begin{cases} = 0 & w a_i < (1+t_i)\, w^* a_i^*, \\ \in [0,1] & w a_i = (1+t_i)\, w^* a_i^*, \\ = 1 & w a_i > (1+t_i)\, w^* a_i^*. \end{cases}$$

To see this, first note that the two optimality conditions of the relaxed problem imply

$$u_i'(c_i) = \min\left\{\nu^* a_i^*, \nu a_i\right\}.$$

By construction of t and λ, we therefore have the second constraint directly follows from the monotonicity of a_i^*/a_i, the optimality condition for α, and the fact that $1 + t_i = w\nu^*/w^*\nu = w a_{\bar{i}}/w^* a_{\bar{i}}^*$.

Conversely, one can check that the two constraints that had been dropped from the inner problem can only be satisfied at a solution of the relaxed problem if t is uniform for all $i > \bar{i}$. To see this, consider $i_1 > i_2 \geq \bar{i}$. At a solution of the relaxed problem, $i_1 > i_2 \geq \bar{i}$ requires $\alpha_{i_1} = \alpha_{i_2} = 1$. Thus for the dropped constraint on α to be satisfied, t_{i_1} and t_{i_2} must be such that $w a_{i_1}/w^* a_{i_1}^* > 1 + t_{i_1}$ and $w a_{i_2}/w^* a_{i_2}^* > 1 + t_{i_2}$. In turn, for the dropped constraint on c to be satisfied, t_{i_1} and

t_{i_2} must be such that $1 + t_{i_1} = u'_{i_1}(c_{i_1})/\lambda a^*_{i_1}$ and $1 + t_{i_2} = u'_{i_2}(c_{i_2})/\lambda a^*_{i_2}$. At the solution of the relaxed problem, $i_1 > i_2 \geq \bar{i}$ further requires $u'_{i_1}(c_{i_1}) = \nu^* a^*_{i_1}$ and $u'_{i_2}(c_{i_2}) = \nu^* a^*_{i_2}$. Combining the two previous observations, we therefore get $1 + t_{i_1} = 1 + t_{i_2} = \nu^*/\lambda$, which establishes that for a solution of the relaxed problem to satisfy all the constraints of the inner problem, the tariff schedule on all imported goods, $i \geq \bar{i}$, must be uniform.

Since we have shown that starting from any solution of the relaxed inner problem, one can always construct t and λ such that all the constraints of the inner problem are satisfied, any solution of the inner problem must also be a solution of the relaxed problem. Furthermore, since the only solutions of the relaxed problem that satisfy all the constraints of the inner problem feature uniform import tariffs, any solution of the inner problem must feature uniform import tariffs. Finally, since any solution of the planning problem must also be a solution of the inner problem given (α^*, c^*, w, w^*), the previous observation implies that optimal tariffs must be uniform.

C Robustness

C.1 Armington Model

Consider a variation of the model presented in Section 2.1 with nested CES utility,

$$U = \sum_s \beta^s \ln \left[(c^s_h)^{(\sigma-1)/\sigma} + (c^s_f)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)},$$

where c^s_h and c^s_f denote the consumption of the domestic and foreign varieties of good s, $\sigma > 1$ denotes the elasticity of substitution between domestic and foreign varieties, and (β_s) are exogenous preference parameters such that $\sum_s \beta_s = 1$. All other assumptions are the same as in Section 2.1. When σ goes to infinity, the present model converges to the Cobb-Douglas version of the model presented in Section 2.1.[28]

In this environment, Home's planning problem can be expressed as

$$\max_{c^h, c^f, c^{h*} \geq 0} \sum_s \beta^s \ln \left[(c^s_h)^{(\sigma-1)/\sigma} + (c^s_f)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

subject to:

$$\begin{aligned}
\sum_s \left(w^* a^{s*} c^s_f - p^s(c^{s*}_h, w^*) c^{s*}_h \right) &\leq 0, \\
\sum_s a^s \left(c^s_h + c^{s*}_h \right) &\leq L, \\
\sum_s a^{s*} \left(c^s_f + \frac{(w^* a^{s*})^{-\sigma} \beta^{s*} w^* L^*}{(w^* a^{s*})^{1-\sigma} + \left(p^s(c^{s*}_h, w^*) \right)^{1-\sigma}} \right) &\leq L^*,
\end{aligned}$$

[28] In line with the analysis of Section 5.1, we assume a discrete number of sectors. All the results presented here extend to the case of a continuum of sectors in a straightforward manner.

where the world price of the domestic variety of good s, $p^s(c_h^{s*}, w^*)$, is implicitly defined as the solution of

$$p_h^s c_h^{s*}\left(\left(\frac{w^* a_i^{s*}}{p_h^s}\right)^{1-\sigma}+1\right)=\beta^{s*}w^*L^*, \tag{40}$$

with equation (40) deriving from the two following conditions

$$\begin{aligned} p_h^s &= \left(c_h^{s*}\right)^{\frac{-1}{\sigma}}\left(P^s\right)^{\frac{\sigma-1}{\sigma}}\left(\beta^{s*}w^*L^*\right)^{\frac{1}{\sigma}}, \\ \left(P^s\right)^{\frac{\sigma-1}{\sigma}} &= \left[\left(w^*a^{s*}\right)^{1-\sigma}+\left(p_h^s\right)^{1-\sigma}\right]^{\frac{-1}{\sigma}}. \end{aligned}$$

Now consider the following Lagrangian

$$\begin{aligned} \mathcal{L}\left(c_h, c_f, c_h^*, \lambda, \lambda^*, \mu; w^*\right) \equiv & \textstyle\sum_s \beta^s \ln\left[\left(c_h^s\right)^{(\sigma-1)/\sigma}+\left(c_f^s\right)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \\ & -\mu \textstyle\sum_s\left(w^*a^{s*}c_f^s - p^s(c_h^{s*}, w^*)c_h^{s*}\right)-\lambda\left(\sum_s a^s\left(c_h^s+c_h^{s*}\right)-L\right) \\ & -\lambda^*\left(\textstyle\sum_s a^{s*}\left(c_f^s+\frac{\left(w^*a^{s*}\right)^{-\sigma}\beta^{s*}w^*L^*}{\left(w^*a^{s*}\right)^{1-\sigma}+\left(p^s(c_h^{s*}, w^*)\right)^{1-\sigma}}\right)-L^*\right), \end{aligned}$$

with $\lambda, \lambda^*, \mu \geq 0$. Using the same arguments as in the baseline model, one can show that (c_h, c_f, c_h^*) solves Home's planning problem if and only if it maximizes $\mathcal{L}$. Furthermore, since this Lagrangian is separable across i, we can maximize $\mathcal{L}$ by maximizing the following Lagrangian sector-by-sector,

$$\mathcal{L}^s\left(c_h^s, c_f^s, c_h^{s*}, \lambda, \lambda^*, \mu; w^*\right) \equiv \mathcal{H}^s\left(c_h^s, c_f^s, \lambda, \lambda^*, \mu; w^*\right)+\mathcal{G}^s\left(c_h^{s*}, \lambda, \lambda^*, \mu; w^*\right)$$

where

$$\mathcal{H}^s\left(c_h^s, c_f^s\right) \equiv \beta^s \ln\left[\left(c_h^s\right)^{(\sigma-1)/\sigma}+\left(c_f^s\right)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}-\mu w^* a^{s*}c_f^s-\lambda a^s c_h^s-\lambda^* a^{s*}c_f^s$$

and

$$\mathcal{G}^s\left(c_h^{s*}\right) \equiv p^s(c_h^{s*}, w^*)c_h^{s*}-\lambda a^s c_h^{s*}-\lambda^*\frac{\left(w^*a^{s*}\right)^{-\sigma}\beta^{s*}w^*L^*}{\left(w^*a^{s*}\right)^{1-\sigma}+\left(p^s(c_h^{s*}, w^*)\right)^{1-\sigma}}.$$

The optimal values of c_h^s and c_f^s are given by the solution of the following system of equations,

$$\left(c_h^s\right)^{\frac{-1}{\sigma}}\left(\left(c_h^s\right)^{\frac{\sigma-1}{\sigma}}+\left(c_f^s\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1} = \lambda a^s, \tag{41}$$

$$\left(c_f^s\right)^{\frac{-1}{\sigma}}\left(\left(c_h^s\right)^{\frac{\sigma-1}{\sigma}}+\left(c_f^s\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1} = \left(\mu w^*+\lambda^*\right)a^{s*}. \tag{42}$$

Let $\tau_f^s \equiv \left(c_f^s\right)^{\frac{-1}{\sigma}}\left(\left(c_h^s\right)^{\frac{\sigma-1}{\sigma}}+\left(c_f^s\right)^{\frac{\sigma-1}{\sigma}}\right)^{-1}/w^*a^{s*}-1$ denote the wedge between the local price and the world price of the foreign variety of good s . In the decentralized equilibrium with trade taxes, τ_f^s corresponds to the optimal import tariff on foreign varieties. Equation (42) immediately implies that

$$\tau_f^s = \left(\mu + \frac{\lambda^*}{w^*}\right) - 1.$$

Thus import tariffs should be constant across sectors, as in Proposition 2.

The optimal value of c_h^{s*} is given by the solution of

$$\mu\left(p^s(c_h^{s*}, w^*) + c_h^{s*}\frac{dp^s(c_h^{s*}, w^*)}{dc_h^{s*}}\right) = \lambda a^s + \lambda^*(\sigma - 1)\frac{\left(w^* a^{s*}\right)^{1-\sigma}\beta^{s*}L^*\left(p^s(c_h^{s*}, w^*)\right)^{-\sigma}}{\left(\left(w^* a_i^*\right)^{1-\sigma} + \left(p^s(c_h^{s*}, w^*)\right)^{1-\sigma}\right)^2}\frac{dp^s(c_h^{s*}, w^*)}{dc_h^{s*}},$$

where equation (40) implies

$$\frac{dp^s(c_h^{s*}, w^*)}{dc_h^{s*}} = -\left(\frac{\left(p^s(c_h^{s*}, w^*)\right)^{\sigma-1}\left(w^* a^{s*}\right)^{1-\sigma} + 1}{\sigma\left(p^s(c_h^{s*}, w^*)\right)^{\sigma-1}\left(w^* a^{s*}\right)^{1-\sigma} + 1}\right)\frac{p^s(c_h^{s*}, w^*)}{c_h^{s*}}$$

Hence, we must have

$$p^s(c_h^{s*}, w^*) = \frac{\lambda a^s}{\left(\mu + \frac{\lambda^*}{w^*}\right)(\sigma - 1)}\left(\left(\frac{w^* a^{s*}}{p^s(c_h^{s*}, w^*)}\right)^{\sigma-1} + \sigma\right) \tag{43}$$

Let $\tau_h^s \equiv \lambda a_i / p^s(c_h^{s*}, w^*) - 1$ denote the wedge between the local price and the world price of the domestic variety of good s. In the decentralized equilibrium with trade taxes, τ_h^s corresponds to the optimal export subsidy on domestic varieties. One can then rearrange equation (43) as

$$\tau_h^s = \frac{\left(\mu + \frac{\lambda^*}{w^*}\right)}{\left(\frac{1}{(\sigma-1)}\left(\frac{w^* a^{s*}\left(1+\tau_h^s\right)}{\lambda a^s}\right)^{\sigma-1} + \frac{\sigma}{\sigma-1}\right)} - 1. \tag{44}$$

Totally differentiating equation (44) with respect to a^s/a^{s*}, one can check that τ_h^s is increasing in a^s/a^{s*}. Hence sectors with a weaker comparative advantage, i.e. a higher value of a^s/a^{s*}, should subsidized more (or taxed less), as in Proposition 2.

C.2 Trade Costs

Here we extend our model to incorporate exogenous iceberg trade costs, $\delta \geq 1$, such that if 1 unit of good i is shipped from one country to another, only a fraction $1/\delta$ arrives. We continue to define world prices, p_i, as those prevailing in Foreign and let

$$\phi(m_i) \equiv \begin{cases} \delta, & \text{if } m_i \geq 0, \\ 1/\delta, & \text{if } m_i < 0, \end{cases} \tag{45}$$

denote the gap between domestic and world prices in the absence of trade taxes.

As in our benchmark model, the domestic government's problem can be reformulated and transformed into many two-dimensional, unconstrained maximization problems using Lagrange multiplier methods. In the presence of trade costs, Home's objective is to find the solution $\left(m_i^0, q_i^0\right)$ of the good-specific Lagrangian,

$$\max_{m_i, q_i \geq 0} \mathcal{L}_i\left(m_i, q_i, \lambda, \lambda^*, \mu; w^*\right) \equiv u_i\left(q_i + m_i\right) - \lambda a_i q_i - \lambda^* a_i^* q_i^*\left(m_i, w^*\right) - \mu p_i(m_i, w^*)\phi\left(m_i\right) m_i,$$

where $p_i\left(m_i, w^*\right)$ and $q_i^*\left(m_i, w^*\right)$ are now given by

$$p_i\left(m_i, w^*\right) \equiv \min\left\{u_i^{*\prime}\left(-m_i \phi\left(m_i\right)\right), w^* a_i^*\right\}, \tag{46}$$

$$q_i^*\left(m_i, w^*\right) \equiv \max\{m_i \phi\left(m_i\right) + d_i^*(w^* a_i^*), 0\}. \tag{47}$$

Compared to the analysis of Section 3, if Home exports $-m_i > 0$ units abroad, then Foreign only consumes $-m_i/\delta$ units. Conversely, if Home imports $m_i > 0$ units from abroad, then Foreign must export $m_i \delta$ units. This explains why $\phi\left(m_i\right)$ appears in the two previous expressions.

The introduction of transportation costs leads to a new kink in the good-specific Lagrangian. In addition to the kink at $m_i = \delta M_i^I \equiv -\delta d_i^*(w^* a_i^*)$, there is now a kink at $m_i = 0$, reflecting the fact that some goods may no longer be traded at the solution of Home's planning problem. As before, since we are not looking for stationary points, this technicality does not complicate our problem. When maximizing the good-specific Lagrangian, we simply consider four regions in m_i space: $m_i < \delta M_i^I$, $\delta M_i^I \leq m_i < 0$, $0 \leq m_i < M_i^{II}$, and $m_i \geq M_i^{II}$.

As in Section 3, if Home's comparative advantage is sufficiently strong, $a_i/a_i^* \leq \frac{1}{\delta} A^I \equiv \frac{1}{\delta} \frac{\sigma^* - 1}{\sigma^*} \frac{\mu w^*}{\lambda}$, then optimal net imports are $m_i^0 = \delta^{1-\sigma^*} m_i^I \equiv -\left(\frac{\sigma^*}{\sigma^* - 1} \frac{\lambda a_i}{\mu \beta_i^*}\right)^{-\sigma^*} \delta^{1-\sigma^*}$. Similarly, if Foreign's comparative advantage is sufficiently strong, $a_i/a_i^* > \delta A^{II} \equiv \delta \frac{\lambda^* + \mu w^*}{\lambda}$, then optimal net imports are $m_i^0 = \delta^{-\sigma^*} m_i^{II} \equiv d_i\left(\left(\lambda^* + \mu w^*\right) \delta a_i^*\right)$. Relative to the benchmark model, there is now a range of goods for which comparative advantage is intermediate, $a_i/a_i^* \in \left(\frac{1}{\delta} A^{II}, \delta A^{II}\right)$, in which no international trade takes place. For given values of the foreign wage, w^*, and the Lagrange multipliers, λ, λ^*, μ, this region expands as trade costs become larger, i.e., as δ increases.

Building on the above observations, we obtain the following generalization of Proposition 1.

Proposition 4. *Optimal net imports are such that:* (a) $m_i^0 = \delta^{1-\sigma^*} m_i^I$, *if* $a_i/a_i^* \leq \frac{1}{\delta} A^I$; (b) $m_i^0 = \delta M_i^I$, *if* $a_i/a_i^* \in \left(\frac{1}{\delta} A^I, \frac{1}{\delta} A^{II}\right)$; (c) $m_i^0 \in \left[\delta M_i^I, 0\right]$ *if* $a_i/a_i^* = \frac{1}{\delta} A^{II}$; (d) $m_i^0 = 0$, *if* $a_i/a_i^* \in \left(\frac{1}{\delta} A^{II}, \delta A^{II}\right)$; (e) $m_i^0 \in \left[0, M_i^{II}\right]$ *if* $a_i/a_i^* = \delta A^{II}$; *and* (f) $m_i^0 = \delta^{-\sigma^*} m_i^{II}$, *if* $a_i/a_i^* > \delta A^{II}$.

Using Proposition 4, it is straightforward to show, as in Section 4.1, that wedges across traded goods are (weakly) increasing with Home's comparative advantage. Similarly, as in Section 4.2, one can show that any solution to Home's planning problem can be implemented using trade taxes and that the optimal taxes vary with comparative advantage as wedges do.