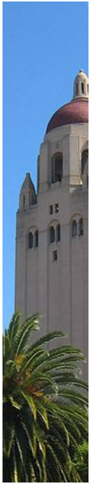


Artificial Intelligence and Economic Growth

P. Aghion, B. Jones, and C. Jones

in Agrawal et al *The Economics of Artificial Intelligence*, 2019

What are the implications of A.I. for economic growth?

- Build some growth models with A.I.
  - A.I. helps to make goods
  - A.I. helps to make ideas
- Implications
  - Long-run growth
  - Share of GDP paid to labor vs capital
  - Firms and organizations
- Singularity?

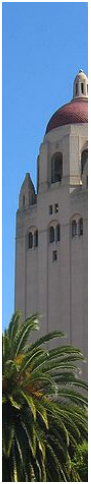
Two Main Themes

- A.I. modeled as a continuation of automation
  - Automation = replace labor in particular tasks with machines and algorithms
  - *Past:* textile looms, steam engines, electric power, computers
  - *Future:* driverless cars, paralegals, pathologists, maybe researchers, maybe everyone?

- A.I. may be limited by Baumol's cost disease
  - *Baumol:* growth constrained not by what we do well but rather by what is essential and yet hard to improve

Outline

- Basic model: automating tasks in production
- A.I. and the production of new ideas
- Singularity?
- Some facts



The Zeira 1998 Model

Simple Model of Automation (Zeira 1998)

- Production uses n tasks/goods:

$$Y = AX_1^{\alpha_1} X_2^{\alpha_2} \cdot \ldots \cdot X_n^{\alpha_n},$$

where $\sum_{i=1}^{n} \alpha_i = 1$ and

$$X_{it} = \begin{cases} L_{it} & \text{if not automated} \\ K_{it} & \text{if automated} \end{cases}$$

- Substituting gives

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

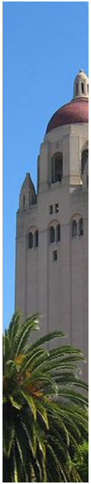
- Comments:
  - α reflects the *fraction* of tasks that are automated
  - Embed in neoclassical growth model $\Rightarrow$

$$g_y = \frac{g_A}{1-\alpha} \quad \text{where} \quad y_t \equiv Y_t / L_t$$

- Automation: $\uparrow \alpha$ raises both capital share and LR growth
  - Hard to reconcile with 20th century
  - Substantial automation but stable growth and capital shares

Subsequent Work

- Acemoglu and Restrepo (2017, 2018, 2019, 2020, 2021, ...)
  - Old tasks are gradually automated as new (labor) tasks are created
  - Fraction automated can then be steady
  - Rich framework, with endogenous innovation and automation, all cases worked out in great detail
- Peretto and Seater (2013), Hemous and Olson (2016), Agrawal, McHale, and Oettl (2017)



Automation and Baumol’s Cost Disease

Baumol's Cost Disease and the Kaldor Facts

- Baumol: Agriculture and manufacturing have rapid growth and declining shares of GDP
  - ... but also rising automation
- Aggregate capital share could reflect a balance
  - Rises within agriculture and manufacturing
  - But falls as these sectors decline
- Maybe this is a general feature of the economy!
  - First agriculture, then manufacturing, then services

AJJ Economic Environment

Final good $$Y_t = \left(\int_0^1 X_{it}^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}} \quad \text{where} \quad \sigma < 1$$

Tasks $$X_{it} = \begin{cases} K_{it} & \text{if automated} \quad i \in [0, \beta_t] \\ L_{it} & \text{if not automated} \quad i \in [\beta_t, 1] \end{cases}$$

Capital accumulation $$\dot{K}_t = I_t - \delta K_t$$

Resource constraint (K) $$\int_0^1 K_{it} di = K_t$$

Resource constraint (L) $$\int_0^1 L_{it} di = L$$

Resource constraint (Y) $$Y_t = Cons_t + I_t$$

Allocation $$I_t = \bar{s}_K Y_t$$

Automation and growth

- Combining equations

$$Y_t = \left[\beta_t \left(\frac{K_t}{\beta_t} \right)^{\frac{\sigma-1}{\sigma}} + (1-\beta_t) \left(\frac{L}{1-\beta_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

- How β interacts with K: two effects
  - β: what fraction of tasks have been automated
  - β: Dilution as $K/\beta \Rightarrow K$ spread over more tasks
- Same for labor: $L/(1-\beta_t)$ means given L concentrated on fewer tasks, raising "effective labor"

Rewriting in classic CES form

- Collecting the β terms into factor-augmenting form:

$$Y_t = F(B_t K_t, C_t L_t)$$

where

$$B_t = \left(\frac{1}{\beta_t}\right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad C_t = \left(\frac{1}{1-\beta_t}\right)^{\frac{1}{1-\sigma}}$$

- Effect of automation: $\uparrow \beta_t \Rightarrow \downarrow B_t$ and $\uparrow C_t$

Intuition: dilution effects just get magnified since $\sigma < 1$

Automation

- Suppose a constant fraction of non-automated tasks get automated every period:

$$\dot{\beta}_t = \theta(1 - \beta_t)$$

$$\Rightarrow \quad \beta_t \to 1$$

- What happens to $1 - \beta_t =: m_t$?

$$\frac{\dot{m}_t}{m_t} = -\theta$$

The fraction of labor-tasks falls at a constant exponential rate

Putting it all together

$$Y_t = F(B_t K_t, C_t L_t) \text{ where } B_t = \left(\frac{1}{\beta_t}\right)^{\frac{1}{1-\sigma}} \text{ and } C_t = \left(\frac{1}{1-\beta_t}\right)^{\frac{1}{1-\sigma}}$$

- $\beta_t \to 1 \Rightarrow B_t \to 1$
- But C_t grows at a constant exponential rate!

$$\frac{\dot{C}_t}{C_t} = -\frac{1}{1-\sigma}\frac{\dot{m}_t}{m_t} = \frac{\theta}{1-\sigma}$$

- When a constant fraction of remaining goods get automated and $\sigma < 1$, the automation model features an asymptotic BGP that satisfies Uzawa

Factor Shares of Income

- Ratio of capital share to labor share:

$$\frac{\alpha_{K_t}}{\alpha_{L_t}} = \left(\frac{\beta_t}{1-\beta_t}\right)^{1/\sigma} \left(\frac{K_t}{L_t}\right)^{\frac{\sigma-1}{\sigma}}$$

- Two offsetting effects ($\sigma < 1$):
  - $\uparrow \beta_t$ raises the capital share
  - $\uparrow K_t/L_t$ lowers the capital share

These balance and deliver constant factor shares in the limit

$$\alpha_{Kt} \equiv \frac{F_K K}{Y} = \beta_t^{\frac{1}{\sigma}} \left(\frac{K_t}{Y_t}\right)^{\frac{\sigma-1}{\sigma}} \to \left(\frac{\bar{s}_K}{g_Y+\delta}\right)^{\frac{\sigma-1}{\sigma}} < 1$$

Intuition for AJJ result

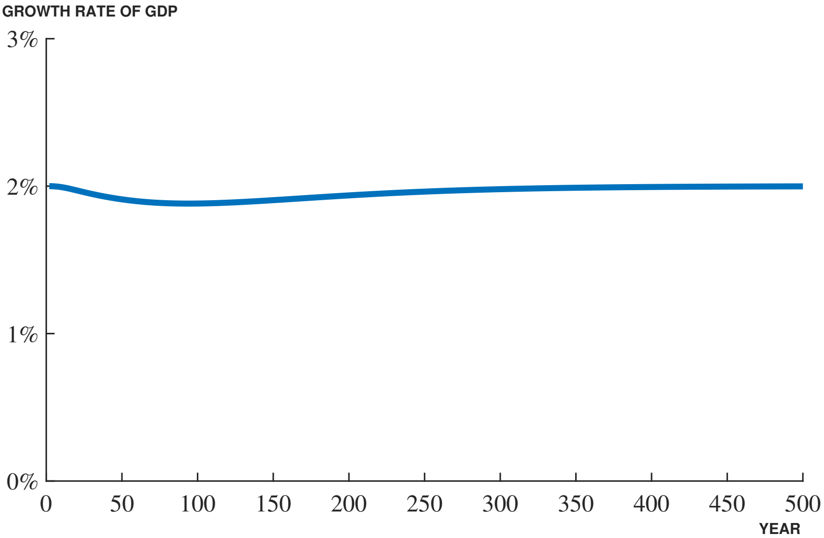
- Why does automation lead to balanced growth and satisfy Uzawa?
  - $\beta_t \to 1$ so the KATC piece "ends" eventually (all tasks automated)
  - Labor per task: $L/(1-\beta_t)$ rises exponentially over time!
  - Constant population, but concentrated on an exponentially shrinking set of goods $\Rightarrow$ exponential growth in "effective" labor

- Baumol logic
  - Agr/Mfg shrink as a share of the economy...
  - Labor still gets 2/3 of GDP! Vanishing share of tasks, but all else is cheap (Baumol)

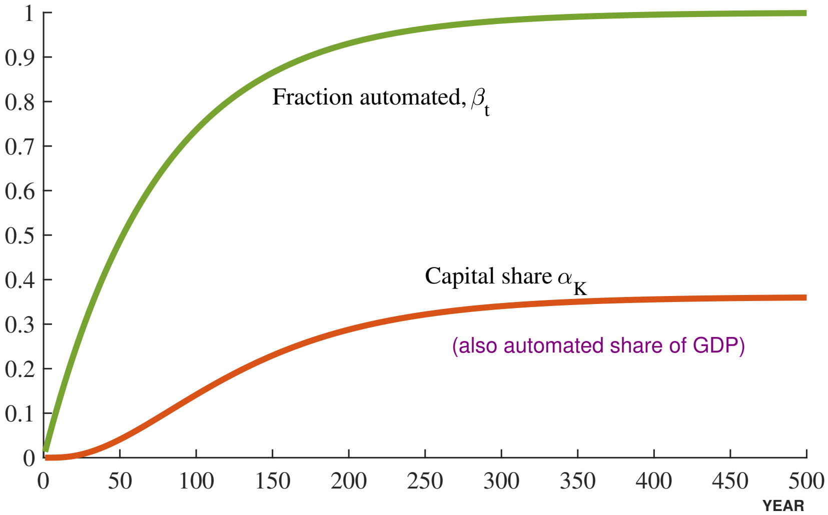
Interesting question: What fraction of tasks automated today? β_{2022}

(B. Jones and X. Liu 2022 on capital-embodied technical change)

Simulation: Automation and Asymptotic Balanced Growth



Simulation: Capital Share and Automation Fraction



Constant Factor Shares?

- Consider $g_A > 0$ — technical change beyond just automation
- Alternatively, factor shares can be constant if automation follows

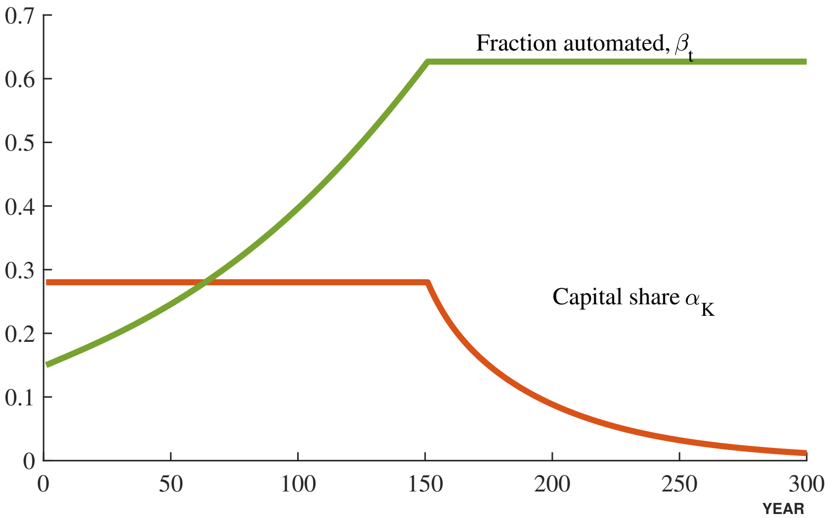
$$g_{\beta t} = (1 - \beta_t) \left(\frac{-\rho}{1 - \rho} \right) g_{kt},$$

- Knife-edge condition...
- Surprise: growth rates increase not decrease. Why? Requires

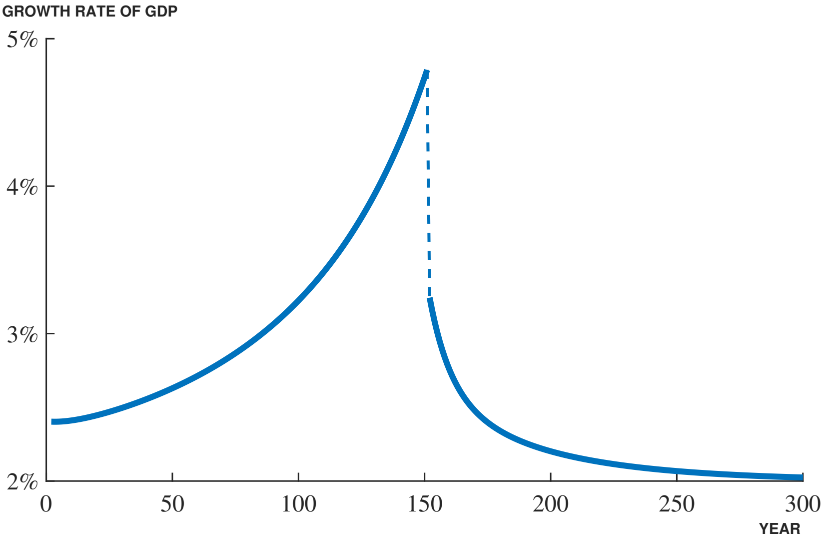
$$g_{Yt} = g_A + \beta_t g_{Kt}.$$

- $g_A = 0$ means zero growth. $g_A > 0$ means growth rises

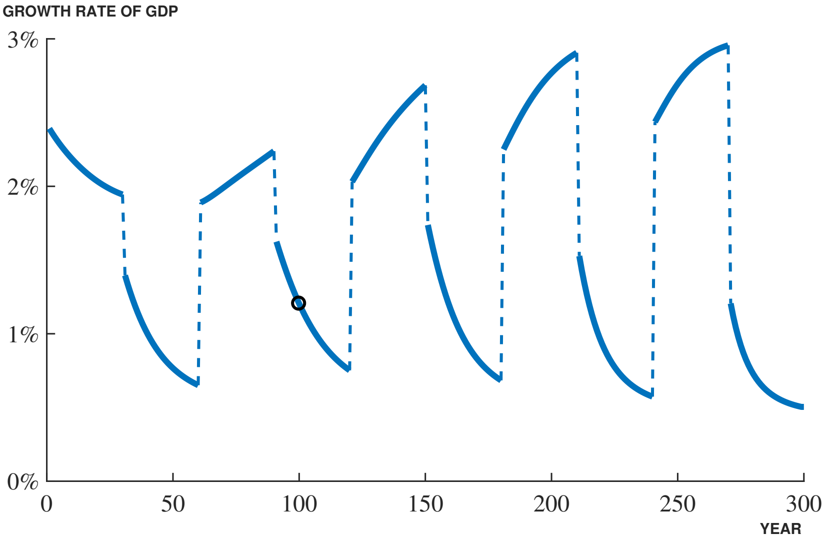
Simulation: Constant Capital Share



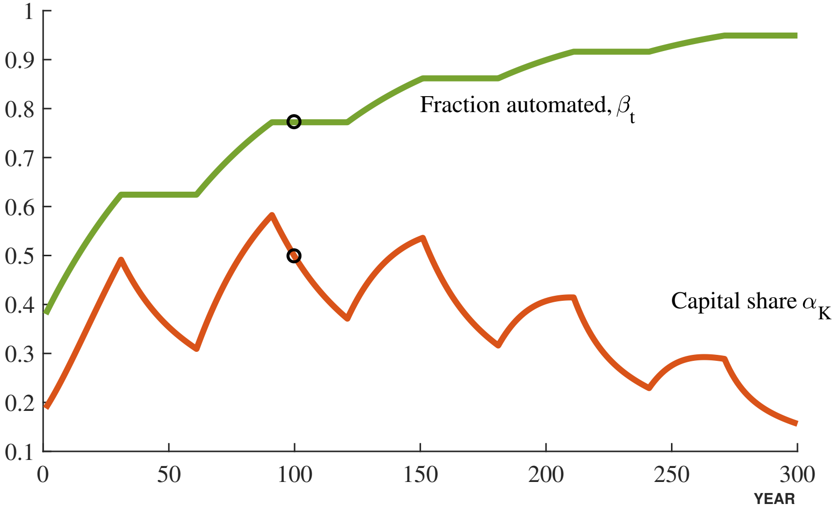
Simulation: Constant Capital Share

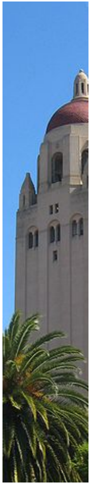


Simulation: Switching regimes...



Simulation: Switching regimes...





A.I. and Ideas

AI in the Ideas Production Function

- Let production of goods and services be $Y_t = A_t L_t$
- Let idea production be:

$$\dot{A}_t = A_t^{\phi} \left(\int_0^1 X_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \ \sigma < 1$$

- Assume fraction β_t of tasks are automated by date t. Then:

$$\dot{A}_t = A_t^{\phi} F(B_t K_t, C_t S_t)$$

where

$$B_t = \left(\frac{1}{\beta_t} \right)^{\frac{1}{1-\sigma}} \text{ and } C_t = \left(\frac{1}{1-\beta_t} \right)^{\frac{1}{1-\sigma}}$$

- This is like before...

AI in the Ideas Production Function

- Intuition: with $\sigma < 1$ the scarce factor comes to dominate

$$F(B_t K_t, C_t S_t) = C_t S_t \, F\left(\frac{B_t K_t}{C_t S_t}, 1\right) \to C_t S_t$$

- So, with continuous automation

$$\dot{A}_t \to A_t^{\phi} C_t S_t$$

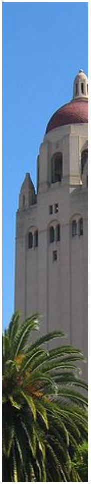
- And asymptotic balanced growth path becomes

$$g_A = \frac{g_C + g_S}{1 - \phi}$$

- We get a "boost" from continued automation (g_C)

Can automation replace population growth?

- Maybe! Suppose S is constant, $g_S = 0$
  - Intuition: Fixed S is spread among exponentially-declining measure of tasks
  - So researchers per task is growing exponentially!
- However
  - This setup takes automation as exogenous and at "just the right rate"
  - What if automation is endogenized?
  - Is population growth required to drive automation?
  - Could a smart/growing AI entirely replace humans?



Singularities

Singularities

- Now we become more radical and consider what happens when we go "all the way" and allow AI to take over all tasks.
- **Example 1:** Complete automation of goods and services production.

$$Y_t = A_t K_t$$

$\rightarrow$ Then growth rate can accelerate exponentially

$$g_Y = g_A + sA_t - \delta$$

we call this a "Type I" growth explosion

Singularities: Example 2

- Complete automation in ideas production function

$$\dot{A}_t = K_t A_t^{\phi}$$

- Intuitively, this idea production function acts like

$$\dot{A}_t = A_t^{1+\phi}$$

- Solution:

$$A_t = \left(\frac{1}{A_0^{-\phi} - \phi t} \right)^{1/\phi}$$

- Thus we can have a true **singularity** for $\phi > 0$. A_t exceeds any finite value before date $t^* = \frac{1}{\phi A_0^{\phi}}$.

Singularities: Example 3 – Incomplete Automation

- Cobb-Douglas, α and β are fraction automated, S constant

$$\dot{K}_t = \bar{s}L^{1-\alpha}A_t^{\sigma}K_t^{\alpha} - \delta K_t.$$

$$\dot{A}_t = K_t^{\beta}S^{\lambda}A_t^{\phi}$$

- Standard endogenous growth requires $\gamma = 1$:

$$\gamma := \frac{\sigma}{1-\alpha} \cdot \frac{\beta}{1-\phi}.$$

- If $\gamma > 1$, then growth explodes!
  - Can occur without full automation
  - Example: $\alpha = \beta = \phi = 1/2$ and $\sigma > 1/2$.

Objections to singularities

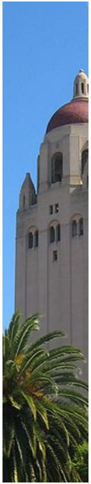
1. Automation limits (no $\beta_t \to 1$)
2. Search limits

$$\dot{A}_t = A_t^{1+\phi} \quad \text{or even} \quad A_t \leq \bar{A}$$

   but $\phi < 0$ (e.g., fishing out, burden of knowledge...)
3. Natural Laws

$$Y_t = \left(\int_0^1 (a_{it} Y_{it})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad \text{where } \sigma < 1$$

   now can have $a_{it} \to \infty$ for many tasks but no singularity
   - *Baumol theme:* growth determined not by what we are good at, but by what is essential yet hard to improve



Final Thoughts

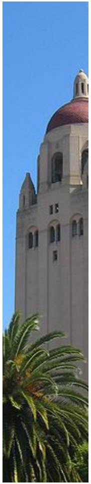
Conclusion: A.I. in the Production of Goods and Services

- Introduced Baumol's "cost disease" insight into Zeira's model of automation
  - Automation can act like labor augmenting technology (surprise!)
  - Can get balanced growth with a constant capital share well below 100%, even with nearly full automation

Conclusion: A.I. in the Ideas Production Function

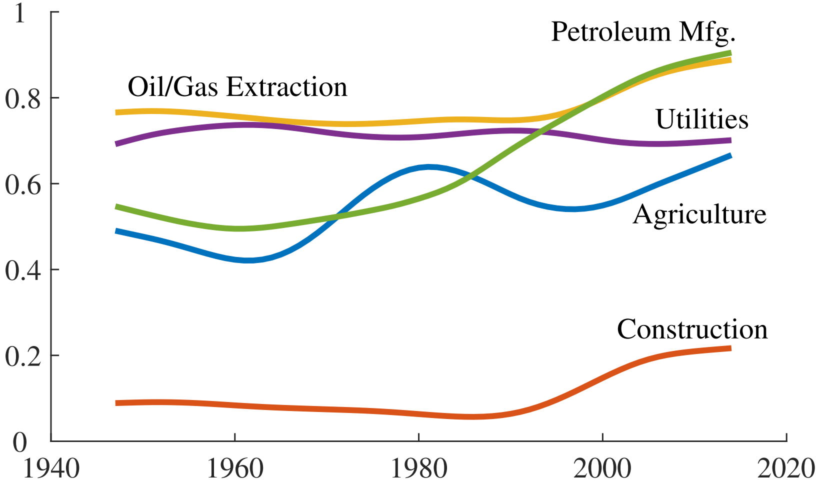
- Could A.I. obviate the role of population growth in generating exponential growth?
- Discussed possibility that A.I. could generate a singularity
  - Derived conditions under which the economy can achieve infinite income in finite time
- Discussed obstacles to such events
  - Automation limits, search limits, and/or natural laws (among others)

Extra Slides



Some Facts

Capital Shares in U.S. Industries



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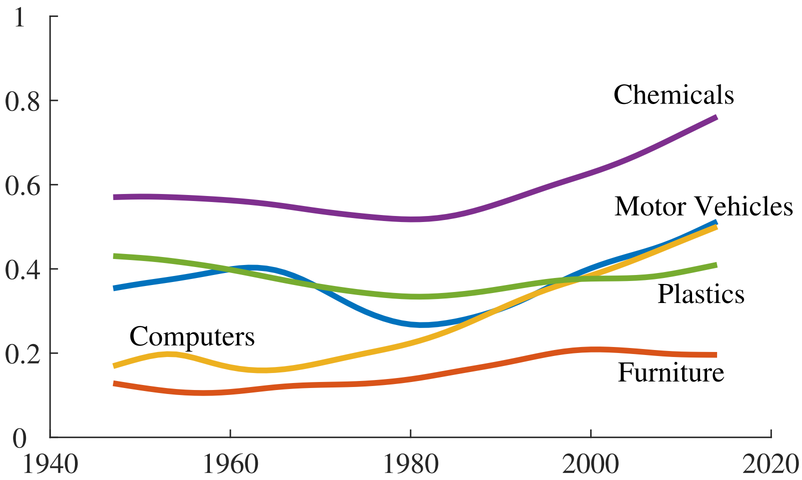

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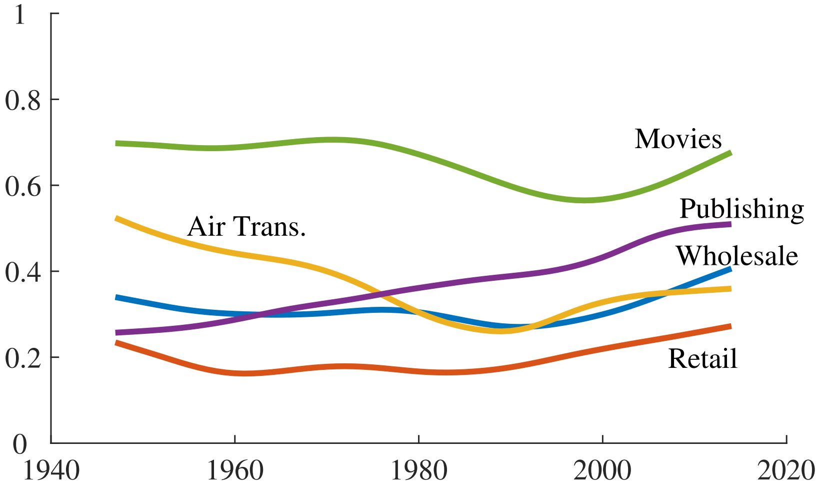

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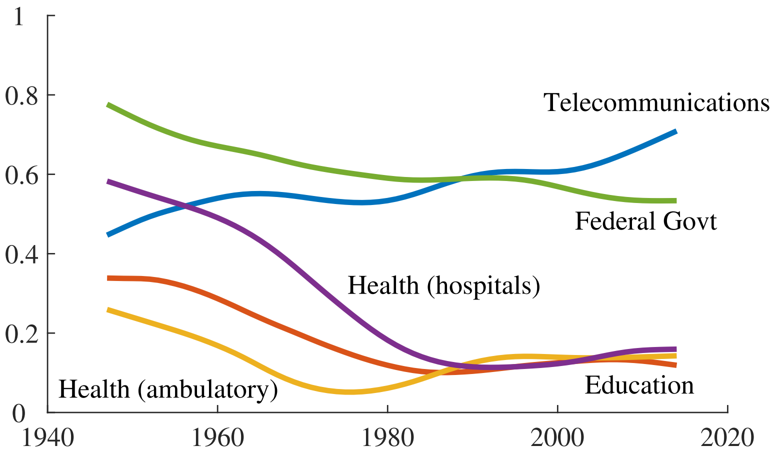
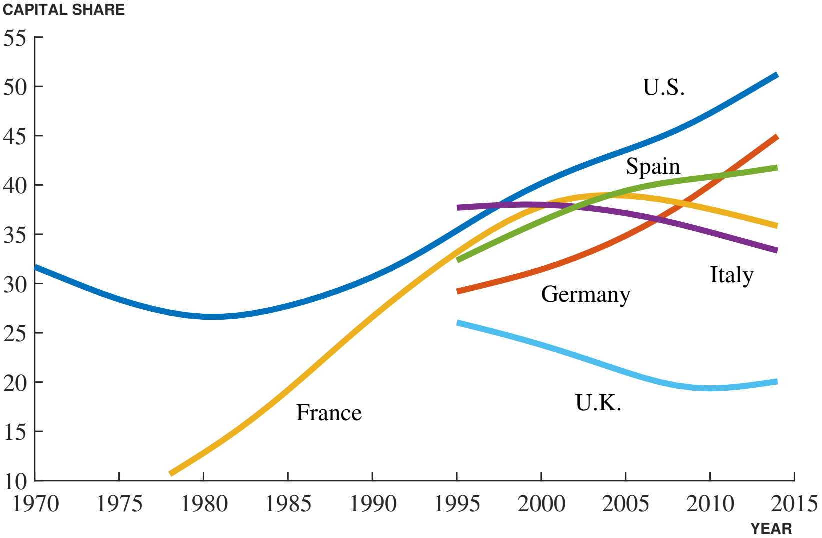
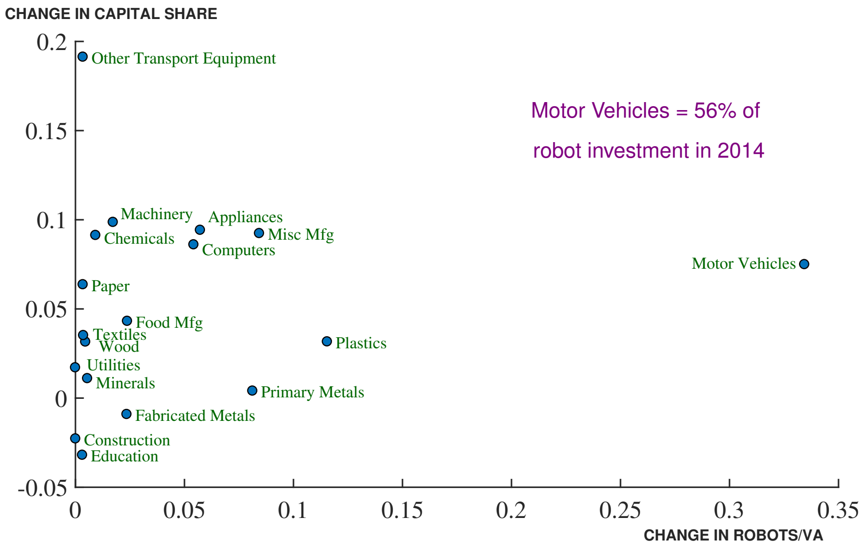


Capital Share of Income: Transportation Equipment



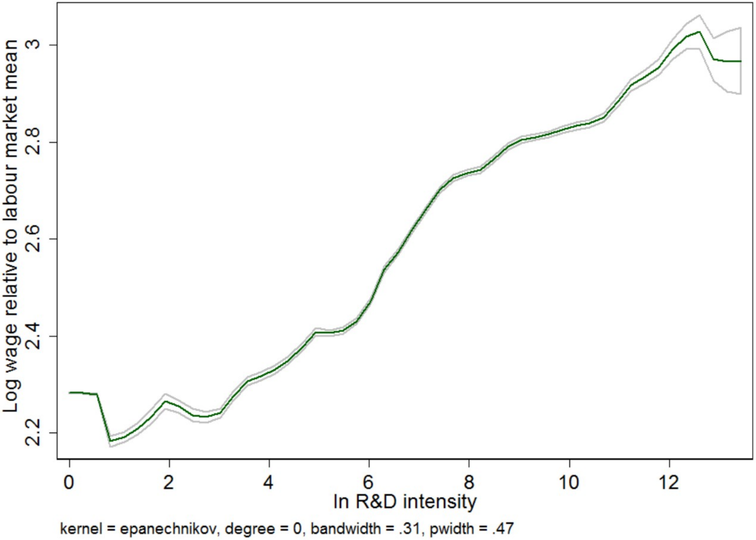
Adoption of Robots and Change in Capital Share



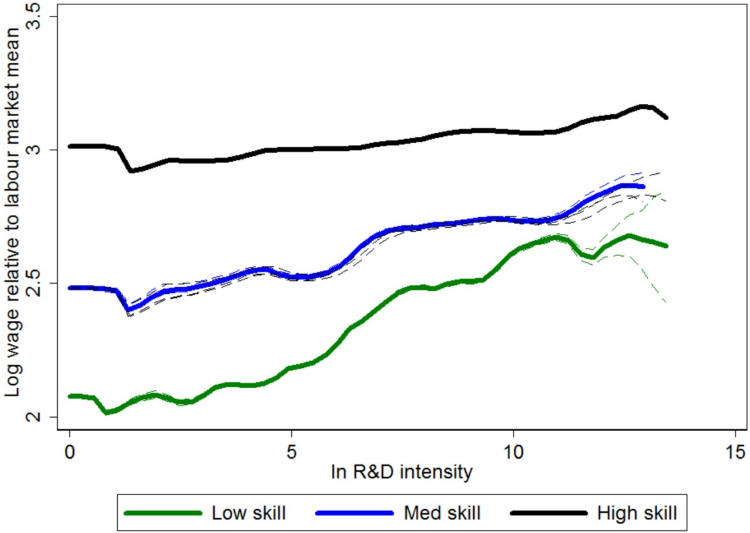
AI, Organizations, and Wage Inequality

- Usual story: robots replace low-skill labor, hence $\uparrow$ skill premium (e.g., Krusell et al. 2000)
- But solving future problems, incl. advancing AI, might be increasingly hard, suggesting $\uparrow$ complementarities across workers, $\uparrow$ teamwork, and changing firm boundaries (Garicano 2000, Jones 2009)
- Aghion et al. (2017) find evidence along these lines
  - outsouce higher fraction of low-skill workers
  - pay *increased* premium to low-skill workers kept

AI, Organizations, and Wage Inequality



AI, Skills, and Wage Inequality



Nonrivalry and the Economics of Data

Chad Jones and Christopher Tonetti

INET-IMF

Macroeconomics in the Age of AI

17 March 2020

Examples of Data

- Google, Facebook
- Amazon
- Tesla, Uber, Waymo
- Medical and genetic data
- Location history
- Speech records
- Physical action data

What is Data in this Paper?

- Data as a factor of production
- Data improves the quality of a product
  - We do not model data as helping a consumer or firm make a more informed decision (e.g., consumption, pricing)
- Data can be useful even if anonymous
- Other aspects of the economics of data are interesting (price discrimination, product specialization, etc.), but are purposely left out of the model

Canonical example: data as input into machine learning algorithm. E.g., medical detection algorithms, self-driving cars, voice recognition software.

Policies on Data Are Being Written Now

What policies governing data use maximize welfare?

- European General Data Protection Regulation (GDPR)
  - Privacy vs. social gain from sharing
  - “The protection of natural persons in relation to the processing of personal data is a fundamental right”
  - “The right. . . must be considered in relation to its function in society. . . ”
- The California Consumer Privacy Act of 2018 (start Jan 1 2020)
  - Allows consumers to opt out of having their data sold
- US Congress: COPRA, ACESS, etc.
- India’s Personal Data Protection bill

Data is Nonrival

- Growth literature: Ideas are nonrival
  - Unlike rival goods, ideas are infinitely usable
- Data is another nonrival good
  - Clearly not a blueprint / recipe $\Rightarrow$ different from ideas
  - Ideas are production functions, data is a factor of production
  - Multiple engineers/algorithms can use same data at same time (within and across firms)
- Nonrivalry implies increasing returns to scale: $Y = F(D, X)$
  - Constant returns to rival inputs: $F(D, \lambda X) = \lambda F(D, X)$
  - Increasing returns to data and rival inputs: $F(\lambda D, \lambda X) > \lambda F(D, X)$

Data Property Rights Matter

- Key point: allocations with different degrees of data use

  $\Rightarrow$ different output, welfare, etc.

- How do different property rights affect the use of data?
  - "Firms own data" versus "consumers own data"

- To illustrate, we assume (plausibly?) the Coase theorem fails
  - Consumers can't commit to selling data to just one firm
  - Firms can't commit to not using data they acquire
  - Useful for showing the role of data sharing

Data is Nonrival $\Rightarrow$ Interesting Questions

- Do markets produce the right amount of data?
- Why don't firms (always) sell their data?
- Who should own data as it's created?
- Implications of data nonrivalry for antitrust, economic growth, and comparative advantage across countries?

We develop a framework for thinking through these questions

Outline

- Economic environment
- Allocations:
  - Optimal allocation
  - Firms own data
  - Consumers own data
  - Extreme privacy protection: outlaw data sharing
- Theory results and a numerical example

Basic Setup

Overview

- Representative consumer with a love for variety
- Innovation $\Rightarrow$ endogenous measure of varieties
- Nonrivalry of data $\Rightarrow$ increasing returns to scale
- How is data produced?
  - Learning by doing: each unit consumed $\rightarrow$ 1 unit of data
  - Alternative: separate PF (Tesla vs Google self-driving car)
- Any data equally useful in all firms $\Rightarrow$ one sector of economy
- Data depreciates fully each period

The Economic Environment

Utility	$\int_0^\infty e^{-\rho t} L_t \, u(c_t) dt$
Flow Utility	$u(c_t) = \log c_t$
Consumption per person	$c_t = \left(\int_0^{N_t} c_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$ with $\sigma > 1$
Data production	$J_{it} = c_{it} L_t$
Variety resource constraint	$c_{it} = Y_{it} / L_t$
Firm production	$Y_{it} = D_{it}^{\eta} L_{it}, \quad \eta \in (0,1)$
Data used by firm i	$D_{it} \leq \alpha x_{it} J_{it} + (1-\alpha) B_t$ (nonrivalry)
Data of firm i used by others	$D_{sit} \leq \tilde{x}_{it} J_{it}$
Data bundle	$B_t = \left(N_t^{-\frac{1}{\epsilon}} \int_0^{N_t} D_{sit}^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}}$ with $\epsilon > 1$
Innovation (new varieties)	$\dot{N}_t = \frac{1}{\chi} \cdot L_{et}$
Labor resource constraint	$L_{et} + \int_0^{N_t} L_{it} \, di = L_t$
Population growth (exogenous)	$L_t = L_0 e^{g_L t}$
Creative destruction	$\delta(\tilde{x}_{it}) = \frac{\delta_0}{2} \tilde{x}_{it}^2$ (equilibrium)

The Economic Environment: Simple Privacy Costs

Utility	$\int_0^\infty e^{-\rho t} L_t \, u(c_t, x_{it}, \tilde{x}_{it}) dt$
Flow Utility	$u(c_t, x_{it}, \tilde{x}_{it}) = \log c_t - \frac{\kappa}{2} \frac{1}{N_t^2} \int_0^{N_t} x_{it}^2 \, di - \frac{\tilde{\kappa}}{2} \frac{1}{N_t} \int_0^{N_t} \tilde{x}_{it}^2 \, di$
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Creative destruction	$\delta(\tilde{x}_{it}) = \frac{\delta_0}{2} \tilde{x}_{it}^2$ (equilibrium)

The Planner Problem (using symmetry of firms)

$$\max_{\{L_{pt}, x_t, \tilde{x}_t\}} \int_0^\infty e^{-\tilde{\rho} t} L_0 \left(\log c_t - \frac{\kappa}{2} \frac{1}{N} x_t^2 - \frac{\tilde{\kappa}}{2} \tilde{x}_t^2 \right) dt, \quad \tilde{\rho} := \rho - g_L$$

subject to

$$
\begin{aligned}
c_t &= Y_t / L_t \\
Y_t &= N_t^{\frac{1}{\sigma-1}} D_{it}^{\eta} L_{pt} \\
D_{it} &= \alpha x_t Y_{it} + (1-\alpha) N_t \tilde{x}_t Y_{it} \\
Y_{it} &= D_{it}^{\eta} \cdot \frac{L_{pt}}{N_t} \\
\dot{N}_t &= \frac{1}{\chi} (L_t - L_{pt}) \\
L_t &= L_0 e^{g_L t}
\end{aligned}
$$

- More sharing ⇒ negative utility cost but more consumption
- Balance labor across production and entry/innovation

Scale Effect from Sharing Data

$$D_{it} = \alpha x_t J_{it} + (1-\alpha)\left(N_t^{-\frac{1}{\epsilon}} \int_0^{N_t} (\tilde{x}_t J_{it})^{\frac{\epsilon-1}{\epsilon}}\, di\right)^{\frac{\epsilon}{\epsilon-1}}$$

$$D_{it} = \alpha x_t Y_{it} + (1-\alpha) N_t \tilde{x}_t Y_{it}$$

$$= [\alpha x_t + (1-\alpha)\tilde{x}_t N_t] Y_{it}$$

- No sharing versus sharing:
  - No sharing: Only the αx_t term = no scale effect
  - Sharing: The $(1-\alpha)\tilde{x}_t N_t$ term = extra scale effect

  Source of Scale Effect: N_t scales with L_t

- Plugging into production function:

$$Y_{it} = \left([\alpha x_t + (1-\alpha)\tilde{x}_t N_t]^{\eta} L_{it}\right)^{\frac{1}{1-\eta}}$$

The Optimal Allocation on BGP (asymptotic)

$$\tilde{x}_{it} = \tilde{x}_{sp} = \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta}\right)^{1/2} \tag{1}$$

$$x_{it} = x_{sp} = \frac{\alpha}{1-\alpha}\frac{\tilde{\kappa}}{\kappa}\left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta}\right)^{1/2} \tag{2}$$

$$L_{it}^{sp} = \chi\rho \cdot \frac{\sigma-1}{1-\eta} := \nu_{sp} \tag{3}$$

$$N_t^{sp} = \frac{L_t}{\chi\left(g_L + \nu_{sp}\right)} := \psi_{sp} L_t \tag{4}$$

$$L_{pt}^{sp} = \nu_{sp}\psi_{sp}L_t \tag{5}$$

$$Y_t^{sp} = \left(\nu_{sp}(1-\alpha)^{\eta}\tilde{x}_{sp}^{\eta}\right)^{\frac{1}{1-\eta}} \left(\psi_{sp}L_t\right)^{\frac{1}{\sigma-1}+\frac{1}{1-\eta}} \tag{6}$$

$$c_t^{sp} = \frac{Y_t}{L_t} = \left(\nu_{sp}(1-\alpha)^{\eta}\tilde{x}_{sp}^{\eta}\right)^{\frac{1}{1-\eta}} \left(\psi_{sp}L_t\right)^{\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}} \tag{7}$$

$$g_c^{sp} = \left(\frac{1}{\sigma-1} + \frac{\eta}{1-\eta}\right) g_L \tag{8}$$

$$D_i^{sp} = \left((1-\alpha)\tilde{x}_{sp}\nu_{sp}\psi_{sp}L_t\right)^{\frac{1}{1-\eta}} \tag{9}$$

$$D^{sp} = ND_i = \left((1-\alpha)\tilde{x}_{sp}\nu_{sp}\right)^{\frac{1}{1-\eta}} \left(\psi_{sp}L_t\right)^{1+\frac{1}{1-\eta}} \tag{10}$$

$$Y_{it}^{sp} = \left(\nu_{sp}(1-\alpha)^{\eta}\tilde{x}_{sp}^{\eta}\right)^{\frac{1}{1-\eta}} \left(\psi_{sp}L_t\right)^{\frac{\eta}{1-\eta}} \tag{11}$$

The Optimal Allocation: GDP per person

$$c_t^{sp} = \frac{Y_t}{L_t} = \left(\nu_{sp}(1-\alpha)^{\eta}\tilde{x}_{sp}^{\eta}\right)^{\frac{1}{1-\eta}} \left(\psi_{sp}L_t\right)^{\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}}$$

$$g_c^{sp} = \left(\frac{1}{\sigma-1} + \frac{\eta}{1-\eta}\right) g_L$$

- Scale effect: $\underbrace{\frac{1}{\sigma-1}}_{\text{Love of Variety}} + \underbrace{\frac{\eta}{1-\eta}}_{\text{Data}}$
- More people make more data and all firms use all shared data

The Optimal Allocation: Data, Firm Size, Variety

$$\tilde{x}_{sp} = \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta} \right)^{1/2}$$

$$L_{it}^{sp} = \chi\rho \cdot \frac{\sigma - 1}{1-\eta} := \nu_{sp}$$

$$N_t^{sp} = \frac{L_t}{\chi g_L + \nu_{sp}} := \psi_{sp} L_t$$

- Data shared increasing in data production elasticity and decreasing in privacy cost
- Firm size constant on BGP. N has opposite comparative statics
- Higher entry cost, time preference, population growth, and elasticity of substitution raise firm size and reduce varieties
- Higher η raises firm size and reduces varieties: Entry does not create data

Firms Own Data

Firms Own Data: Consumer Problem

- Firms own data and choose one data policy $(x_{it}, \tilde{x}_{it})$ applied to all consumers

- Consumers just choose consumption:

$$
\begin{aligned}
U_0 &= \max_{\{c_{it}\}} \int_0^{\infty} e^{-\tilde{\rho} t} L_0 u(c_t, x_{it}, \tilde{x}_{it}) dt \\
\text{s.t.} \quad c_t &= \left(\int_0^{N_t} c_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \\
\dot{a}_t &= (r_t - g_L) a_t + w_t - \int_0^{N_t} p_{it} c_{it} di
\end{aligned}
$$

Firms own Data: Data Decisions

- Firms buy D_{bit} data from intermediary at given price p_b
- Firms sell D_{sit} data to intermediary at chosen price p_{si}
  - Perfect competition inconsistent with nonrival data!
  - Monopolistically competitive with own data
  - See the intermediary's downward-sloping demand curve and set price
- How much data to use / sell?
  - x_{it}: Use all of own data $\Rightarrow x_{it} = 1$
  - $\tilde{x}_{it}$: Trade off = selling data versus creative destruction

    $\delta(\tilde{x}_{it})$ = Poisson rate transferring ownership of variety

Firms own the Data: Incumbent Firm Problem

- Monopolistically competitive firm takes demand for variety as given (from FOC of consumer problem): $p_{it} = \left(\frac{c_t}{c_{it}}\right)^{\frac{1}{\sigma}} = \left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}}$

$$r_t V_{it} = \max_{L_{it}, D_{bit}, x_{it}, \tilde{x}_{it}} \left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}} Y_{it} - w_t L_{it} - p_{bt} D_{bit} + p_{st} \tilde{x}_{it} Y_{it} + \dot{V}_{it} - \delta(\tilde{x}_{it}) V_{it}$$

$$\text{s.t.} \quad Y_{it} = D_{it}^{\eta} L_{it}$$

$$D_{it} = \alpha x_{it} Y_{it} + (1-\alpha) D_{bit}$$

$$x_{it} \in [0,1], \; \tilde{x}_{it} \in [0,1]$$

$$p_{sit} = \lambda_{DI} N_t^{-\frac{1}{\epsilon}} \left(\frac{B_t}{\tilde{x}_{it} Y_{it}}\right)^{\frac{1}{\epsilon}}$$

- Data Intermediary $(p_{bt}, p_{st}, D_{bit})$ and Free Entry complete eqm.

Firms own the Data: Data Intermediary Problem

- A monopolist takes data purchase price as given and sees the downward sloping demand curve for data $p_{bt}(D_{bit})$:

$$\max_{p_{bt}, D_{sit}} \quad p_{bt} \int_0^{N_t} D_{bit}\, di - p_{st} \int_0^{N_t} D_{sit}\, di$$

s.t.

$$D_{bit} \leq B_t = \left(N_t^{-\frac{1}{\epsilon}} \int_0^{N_t} D_{sit}^{\frac{\epsilon-1}{\epsilon}}\, di \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$p_{bt} \leq p_{bt}^*$$

- Free entry at zero cost $\Rightarrow$ zero profits
- Problem incorporates data nonrivalry
  - Buys data once from each firm
  - But can sell the same bundle multiple times

Entry: Innovation Creates a New Variety

- χ units of labor needed to create an additional variety
- Free entry condition:

$$\chi w_t = V_{it} + \frac{\int_0^{N_t} \delta(\tilde{x}_{it}) V_{it} \, di}{\dot{N}_t}$$

- The value of a new variety and the per-entrant share of business stealing from creative destruction

Firms Own Data: A “No Trade” Law

- What if the government, in an attempt to protect consumers privacy, makes data sharing illegal?
- Government chooses
  - $x_{it} \in (0, 1]$
  - $\tilde{x}_{it} = 0$
- We call this the “Outlaw Sharing” allocation

Consumers Own Data

Consumers own Data: Consumer Problem

- Consumers own data, so now choose how much to sell $(x_{it}, \tilde{x}_{it})$:

$$U_0 = \max_{\{c_{it}, x_{it}, \tilde{x}_{it}\}} \int_0^\infty e^{-\tilde{\rho}t} L_0 u(c_t, x_{it}, \tilde{x}_{it}) dt$$

s.t. $$c_t = \left(\int_0^{N_t} c_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

$$\dot{a}_t = (r_t - g_L)a_t + w_t - \int_0^{N_t} p_{it} c_{it} di + \int_0^{N_t} x_{it} p_{st}^a c_{it} di + \int_0^{N_t} \tilde{x}_{it} p_{st}^b c_{it} di$$

- Firm problem similar to before, but now takes $x, \tilde{x}$ as given, can't sell data, and has to buy "own" data

Consumers own the Data: Incumbent Firm Problem

- Monopolistically competitive firm takes demand for variety as given (from FOC of consumer problem):
$q_{it} = \left(\frac{c_t}{c_{it}}\right)^{\frac{1}{\sigma}} = \left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}} = p_{it} - x_{it}p^a_{st} - \tilde{x}_{it}p^b_{st}$
- Firm buys data on its own variety (D_{ait}) and data on other firms varieties (D_{bit})

$$
\begin{aligned}
r_t V_{it} = \max_{L_{it}, D_{ait}, D_{bit}} & \left[\left(\frac{Y_t}{Y_{it}}\right)^{\frac{1}{\sigma}} + p^a_{st} x_{it} + p^b_{st} \tilde{x}_{it} \right] Y_{it} - w_t L_{it} \\
& - p_{at} D_{ait} - p_{bt} D_{bit} + \dot{V}_{it} - \delta(\tilde{x}_t) V_{it}
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad Y_{it} &= D^{\eta}_{it} L_{it} \\
D_{it} &= \alpha D_{ait} + (1-\alpha) D_{bit} \\
D_{ait} &\geq 0, \quad D_{bit} \geq 0
\end{aligned}
$$

Key Forces: Consumers vs. Firms vs. Outlaw Sharing

- Firms
  - use all data on own variety, ignoring consumer privacy
  - restrict data sharing because of creative destruction
- Consumers
  - respect their own privacy concerns
  - sell data broadly, ignoring creative destruction
- Outlaw sharing
  - maximizes privacy gains
  - missing scale effect reduces consumption

Results: Comparing Allocations

1. Planner Problem
2. Firms Own Data
3. Outlaw Data Sharing
4. Consumers Own Data

Key Allocations: $alloc \in \{sp, f, c, ns\}$

- Firm size: $L_i^{alloc} = L_{pt}/N_t = \nu_{alloc}$

$$\nu_{sp} := \chi\rho \cdot \frac{\sigma - 1}{1 - \eta}$$
$$\nu_{os} := \chi\rho \cdot \frac{\sigma - 1}{1 - \sigma\eta}$$
$$\nu_{c} := \chi g_L \cdot \frac{\rho + \delta(\tilde{x}_c)}{g_L + \delta(\tilde{x}_c)} \cdot \frac{\sigma - 1}{1 - \sigma\eta}$$
$$\nu_{f} := \chi g_L \cdot \frac{\rho + \delta(\tilde{x}_f)}{g_L + \delta(\tilde{x}_f)} \cdot \frac{\sigma - 1}{1 - \sigma\eta\frac{\epsilon - 1}{\epsilon}}$$

- Number of firms: $N_t^{alloc} = \psi_{alloc} L_t$

$$\psi_{alloc} := \frac{1}{\chi g_L + \nu_{alloc}}$$

Output

- For $alloc \in \{sp, c, f\}$:

$$Y_t^{alloc} = \left[\nu_{alloc}(1-\alpha)^{\eta}\tilde{x}_{alloc}^{\eta}\right]^{\frac{1}{1-\eta}} (\psi_{alloc}L_t)^{1+\frac{1}{\sigma-1}+\frac{\eta}{1-\eta}}$$

- For Outlaw Sharing:

$$Y_t^{os} = \left[\nu_{os}\alpha^{\eta}x_{os}^{\alpha}\right]^{\frac{1}{1-\eta}} (\psi_{os}L_t)^{1+\frac{1}{\sigma-1}}$$

- Two source of increasing returns to scale:
  - Standard variety effect: $\frac{\sigma}{\sigma-1}$
  - Data sharing: $\frac{\eta}{1-\eta}$
- Recall $\tilde{x}_t > 0$ from data sharing $\Rightarrow$ scale effect

Data Sharing

Own Firm Data	Sharing with Other Firms
$x_{sp} = \frac{\alpha}{1-\alpha} \frac{\tilde{\kappa}}{\kappa} \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta} \right)^{1/2}$	$\tilde{x}_{sp} = \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta} \right)^{1/2}$
$x_f = 1$	$\tilde{x}_f = \left(\frac{2\Gamma\rho}{(2-\Gamma)\delta_0} \right)^{1/2}, \Gamma := \frac{\eta(\sigma-1)}{\frac{\epsilon}{\epsilon-1} - \sigma\eta}$
$x_{os} \in (0, 1]$	$\tilde{x}_{os} = 0$
$x_c = \frac{\alpha}{1-\alpha} \frac{\tilde{\kappa}}{\kappa} \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta} \cdot \frac{\sigma-1}{\sigma} \right)^{1/2}$	$\tilde{x}_c = \left(\frac{1}{\tilde{\kappa}} \cdot \frac{\eta}{1-\eta} \cdot \frac{\sigma-1}{\sigma} \right)^{1/2}$

- Firms fear creative destruction and share less than planner (δ_0)
- Consumers share less than planner because of mark up
- No sharing law restricts data even more
- Firms use more own-variety data compared to consumer/planner

Numerical Example: Parameter Values

Description	Parameter	Value
Importance of data	η	0.06
Elasticity of substitution	σ	4
Weight on privacy	$\kappa = \tilde{\kappa}$	0.20
Population level	L_0	100
Population growth rate	g_L	0.02
Rate of time preference	ρ	0.025
Labor cost of entry	χ	0.01
Creative destruction	δ_0	0.4
Weight on own data	α	1/2
Elasticity of Substitution (data)	ϵ	50
Use of own data in NS	$\bar{x}$	1

Numerical Example: How large is η? (Approach 1 - Data Share)

- Share of GDP spent on data = $\frac{\eta}{1-\eta}\frac{\sigma-1}{\sigma}$
- Similar formula/quantity when consumers or firms own data
- Set $\sigma = 4$
- If data share of GDP is 5% $\Rightarrow \eta = 0.0625$
- If data share of GDP is 10% $\Rightarrow \eta = 0.12$
- Approach will be to explore $\eta \in \{0.03, 0.06, 0.12\}$

Numerical Example: Consumption Equivalent Welfare

$$U_{ss}^{alloc} = \frac{1}{\tilde{\rho}} \left(\log c_0^{alloc} - \frac{\tilde{\kappa}}{2} \tilde{x}_{alloc}^2 + \frac{g_c^{alloc}}{\tilde{\rho}} \right).$$

Let $U_{ss}^{alloc}(\lambda)$ denote steady-state welfare when we perturb the allocation of consumption by some proportion λ:

$$U_{ss}^{alloc}(\lambda) = \frac{1}{\tilde{\rho}} \left(\log(\lambda c_0^{alloc}) - \frac{\tilde{\kappa}}{2} \tilde{x}_{alloc}^2 + \frac{g_c^{alloc}}{\tilde{\rho}} \right).$$

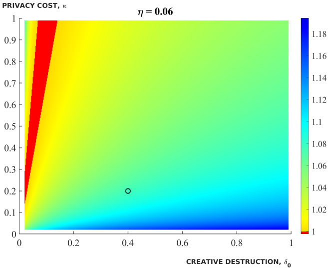
Define consumption equivalent welfare as λ^{alloc}:

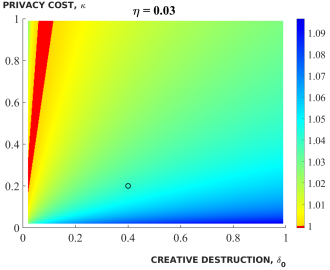
$U_{ss}^{sp}(\lambda^{alloc}) = U_{ss}^{alloc}(1)$ with

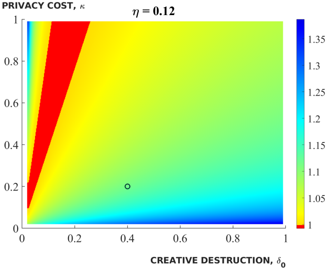
$$\log \lambda^{alloc} = \underbrace{\log c_0^{alloc} - \log c_0^{sp}}_{\text{Level term}} - \underbrace{\frac{\tilde{\kappa}}{2} \left(\tilde{x}_{alloc}^2 - \tilde{x}_{sp}^2 \right)}_{\text{Privacy term}} + \underbrace{\frac{g_c^{alloc} - g_c^{sp}}{\tilde{\rho}}}_{\text{Growth term}}$$

Note: The x_{it} terms drop out because scaled by $1/N$

Welfare Sensitivity Analysis (η, δ, κ): λ^c/λ^f







Allocations: Baseline

Allocation	Data Sharing “own” x	“others” $\tilde{x}$	Firm size ν	Variety $N/L = \psi$	Consu-mption c	Growth g	Creative Destruct. δ
Social Planner	0.66	0.66	1304	665	18.6	0.67%	0.0870
Consumers Own Data	0.59	0.59	1482	594	18.3	0.67%	0.0696
Firms Own Data	1	0.16	1838	491	16.0	0.67%	0.0052
Outlaw Sharing	1	0	2000	455	7.3	0.50%	0

- Firms overuse their own data and undershare with others
- Consumers share less data than planner, but not by much
- Growth rate scale effect is modest, level differences are large

Consumption Equivalent Welfare

Allocation	Welfare λ	$\log\lambda$	Level term	Privacy term	Growth term
Optimal Allocation	1	0	..	..	..
Consumers Own Data	0.9886	-0.0115	-0.0202	0.0087	0.0000
Firms Own Data	0.8917	-0.1146	-0.1555	0.0409	0.0000
Outlaw Sharing	0.3429	-1.0703	-0.9399	0.0435	-0.1739

- Outlaw sharing: particularly harmful law (66 percent worse!)
- Firms own data: substantially lower welfare (11 percent worse)
- Consumers own data: nearly optimal (1 or 2 percent worse)

Implications for IO

- Firms that use data might grow fast compared to those that don't
- Firms would like to merge into one single economy-wide firm
  - Implications for antitrust
  - Price/quantity behavior
- What are the costs of forced sharing?
  - Disincentive to collect/create data
  - Data as a barrier to entry
    (extension to quality ladder model)
  - Markets unraveling
- Targeted mandatory sharing?
  - E.g., airplane safety (after a crash)

Data versus Ideas: Excludability

- Maybe technologically easier to transmit data than ideas (usb key vs. education) . . .
- But data can be encrypted and monitored
- Data seems highly excludable
  - Idea: use machine learning to train self-driving car algorithm
  - ML needs lots of data. Each firm gathering own data

The Boundaries of Data Diffusion: Firms and Countries

- How does data diffuse across firms and countries?
  - Ideas eventually diffuse across firms or countries, so no country scale effect (e.g. HK vs China)
  - What about data?
- Scale effects and country size
  - Larger countries may have an important advantage as data grows in importance
- Scale effects and institutions
  - What if China mandates data sharing across Chinese firms and U.S. has no such policy
  - What if consumers in China have different privacy concerns than in the U.S. or Europe?

Conclusion

- Nonrival data ⇒ large social gain from sharing data
- If firms own data, they may:
  - privately use more data than consumers/planner would
  - sell less data across firms than consumers/planner would
- Nonrivalry ⇒ Laws that outlaw sharing could be very harmful
- Consumers owning data good at balancing privacy and sharing