Selling Impressions: Efficiency vs Competition

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  - Or something in between?

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  - with no information: inefficiency but no information rent - revenue is common ex ante expected value

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- this is our main theoretical result and first main contribution

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- Advertisers values' might well be correlated, but will be independent as long as advertiser / viewer variation is "horizontal", i.e., attributes and preferences are symmetric across viewers and bidders

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    - A model of the market for impressions with two sided information
    - We describe when this model reduces statistically and strategically to our first model and result

Hour Long Talk

1. Main Result
2. Market for Impressions
    1. Institutional Details
    2. Stylized Model of Market for Impressions with Two-Sided Information
    3. Statistical and Strategic Analysis

Part I: Main Result

Setting for Main Result

- N bidders

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- if F is a mean preserving spread of G we write $F \prec G$

Revenue

- second-order statistic $w_{(2)}$ of N symmetrically and independently distributed random variables is

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- non-linear problem in optimization variable G
- neither convex nor concave program

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- quantile distribution S is independent of the underlying distribution F or G
- just as quantile of any random variable is uniformly distributed, the quantile of second-order statistic of N random variables is distributed according to S for every distribution

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- objective is linear in G^{-1}

Optimal Information Structure

Proposition (Optimal Information Structure)

Suppose that F is absolutely continuous, then the unique optimal symmetric information structure is given by:

$$s(v_i) = \begin{cases} v_j & \text{if } q_i(v_i) \leq q^* \\ \mathbb{E}[v_j \mid F(v_j) \geq q] & \text{if } q_i(v_i) \geq q^* \end{cases}$$

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- with change of variables, "upper censorship"

Competition through Information

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- information rent of the winning bidder is depressed considerably with a corresponding gain in the revenue for the seller

Intuitive Proof Step 1: Integrate by Parts

- if $\overline{v} = G^{-1}(1)$ is the upper bound on expected value, by integration by parts, revenue is:

$$\int_0^1 S'(q)G^{-1}(q)dq = \overline{v} - \int_0^1 S(q)dG^{-1}(q)$$

so we have minimization problem

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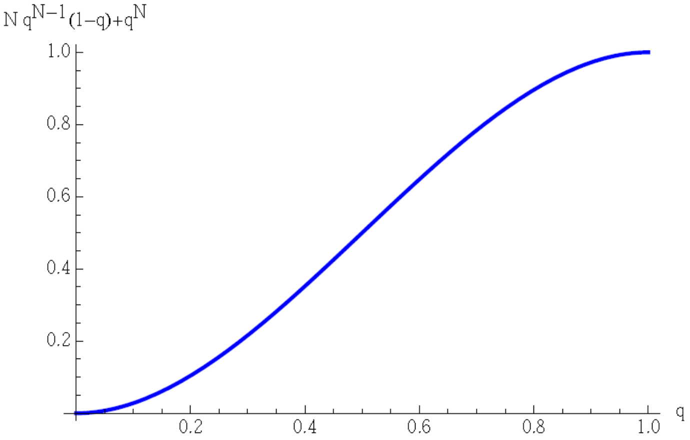
- HINT: if $\overline{v} = 1$, G^{-1} is itself a distribution function.

Step 2: Convexification of Second Order Statistic



- graph of $S(q)$ for $N = 3$

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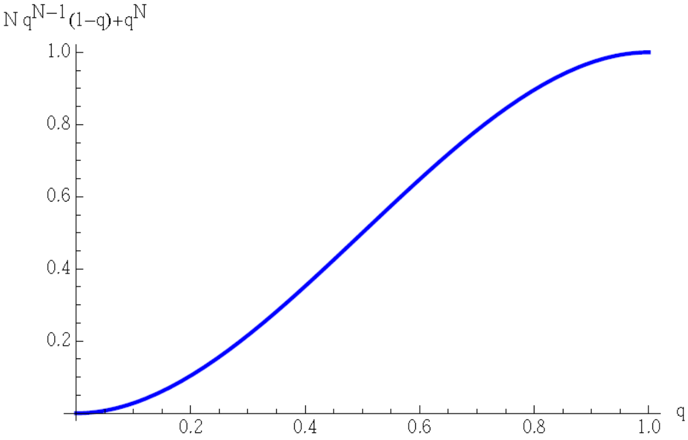
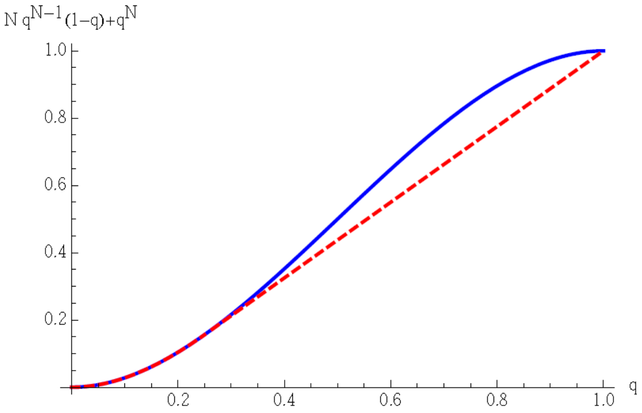


- graph of $S(q)$ for $N = 3$
- unique inflection point for all N

Convex Hull of Quantile Function



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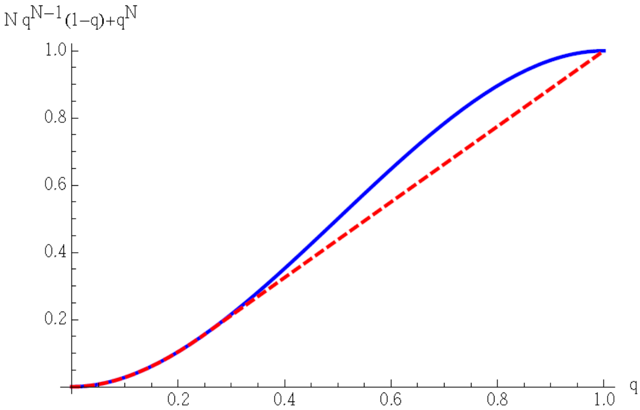
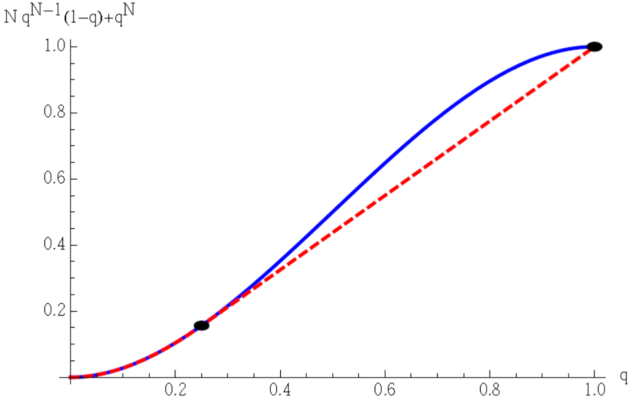


- find largest convex function below the original one
- problem reduces to finding q such that:

$$S(q) + S'(q)(1-q) = S(1) = 1$$

End Points of Affine Segment



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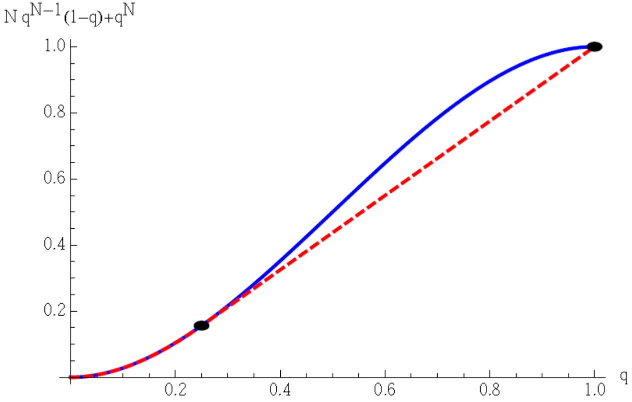
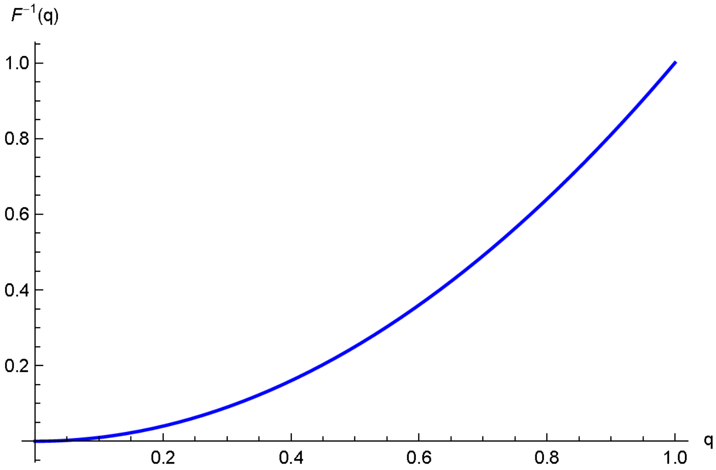


- we take the mass of F^{-1} to the extremes of the affine segment
- the mass at each extreme must keep the expected mean of quantile constant

Step 3: Back to Value Distribution



- map back to value distribution of bidder i

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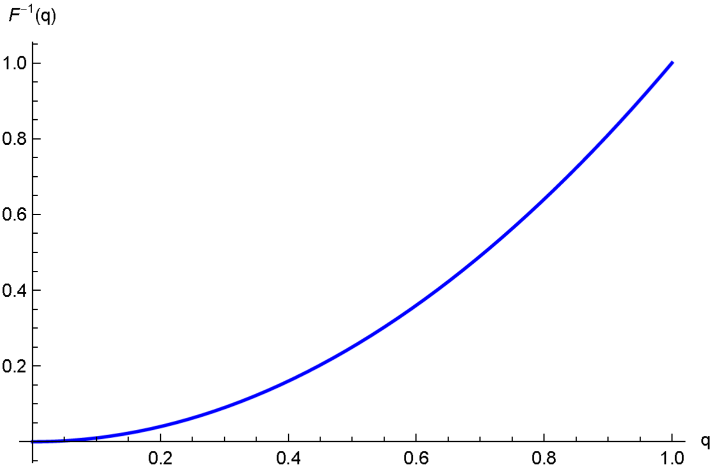
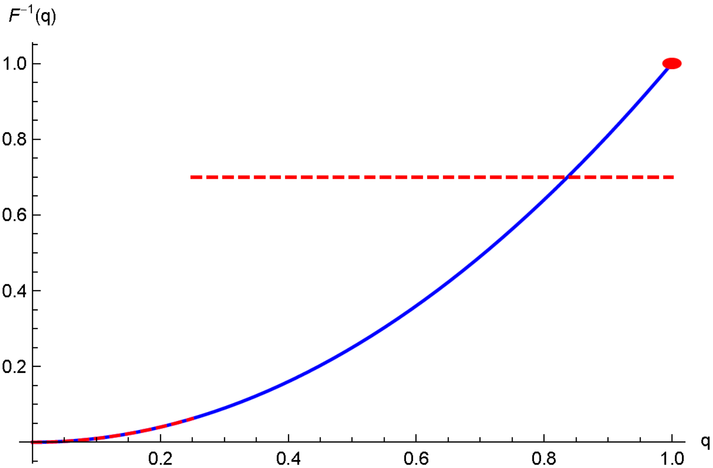


- map back to value distribution of bidder i
- we draw the quantile function for $F(v) = \sqrt{v}$

From Quantile to Convexified Quantile



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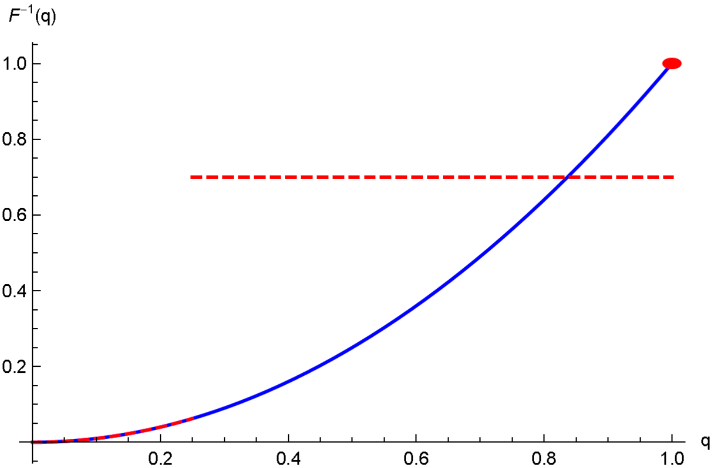

- the mass is moved to the end points
- while keeping expectation of quantile constant

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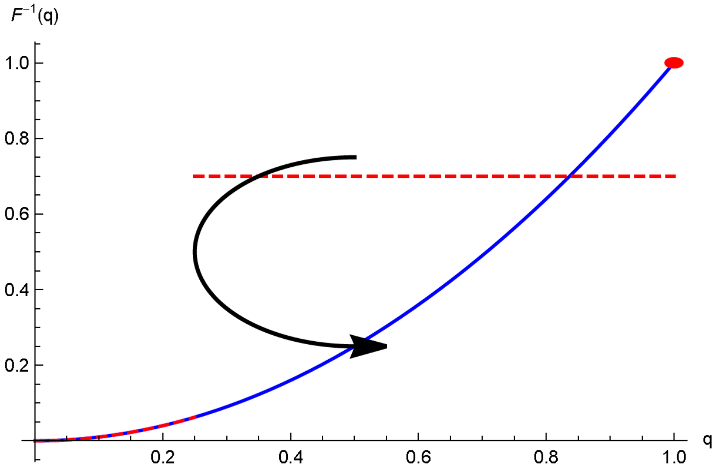

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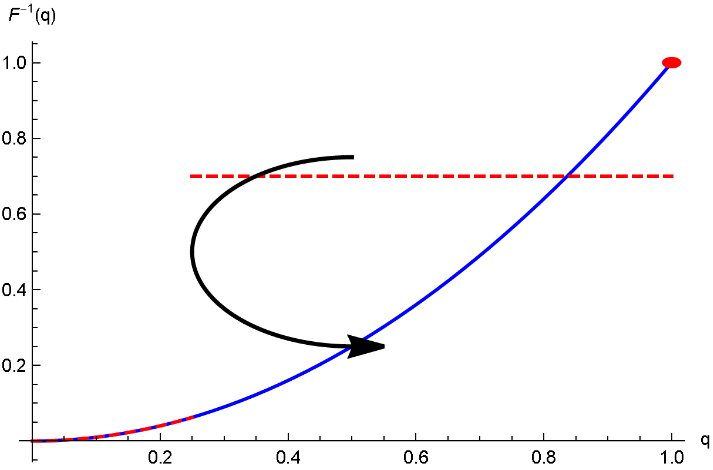
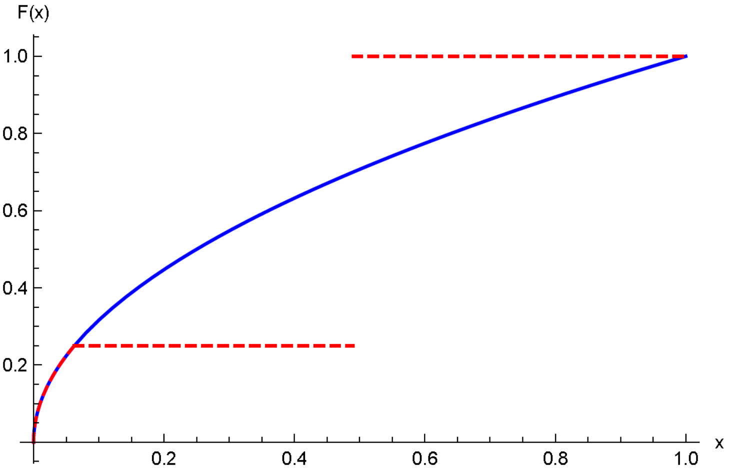


- we have been working with the quantile function
- to recover the distribution we rotate

From Convex Distribution to Information Structure



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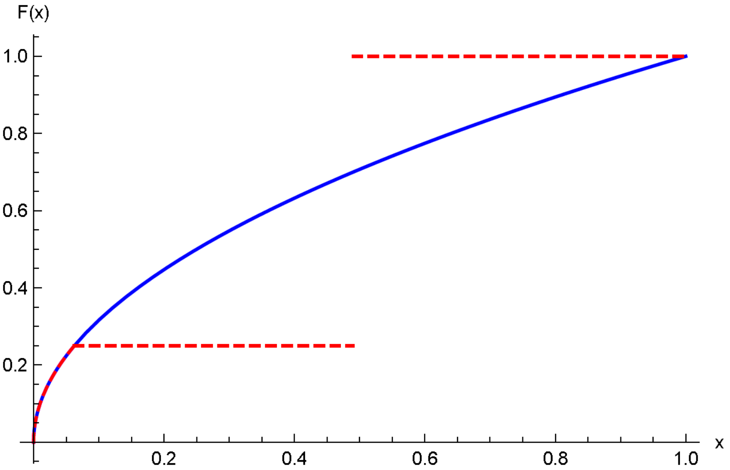

- we now have the distribution Q
- there is one step in distribution of expected value

Verification

- this is an example of a problem of characterizing extreme points of monotone functions subject to majorization constraints (Kleiner et al. 2021)

Proposition (Kleiner et al. Proposition 2)

Let G^{-1} be such that for some countable collection of intervals $\{[\underline{x}_i, \bar{x}_i) \mid i \in I\}$,

$$G^{-1}(q) = \begin{cases} F^{-1}(q) & q \notin \cup_{i \in I}[\underline{x}_i, \bar{x}_i) \\ \frac{\int_{\underline{x}_i}^{\bar{x}_i} F^{-1}(t)dt}{\bar{x}_i - \underline{x}_i} & q \in [\underline{x}_i, \bar{x}_i) \end{cases}$$

If $\operatorname{conv} S$ is affine on $[x_i, x_i)$ for each $i \in I$ and if $\operatorname{conv} S = S$ otherwise, then G solves the maximization problem. Moreover, if F is strictly increasing the converse holds.

What is the Critical Quantile?

Proposition (Critical Quantile)

The quantile $q^*(N) \in [0,1)$ *that determines the optimal information structure is* 0 *if* $N = 2$*, is increasing in* N *and approaches* 1 *as* $N \to \infty$*; for* $N \geq 3$*, it is implicitly defined as the solution of:*

$$S'(q)(1-q) = 1 - S(q)$$

- this is an Nth degree polynomial in q

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$$\overbrace{\varepsilon S'(q)}^{\text{prob low payment}} \times \overbrace{\overline{v}-\underline{v}}^{\text{decrease in payment}}$$

Critical Quantiles

N	$q^* (N)$
2	0
3	0.25
4	0.46
5	0.58
10	0.81
100	0.98

Part II: Market for Impressions

Market for Impressions: Qualitative Features

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- value of the match or impression between advertiser and viewer is jointly determined by these different sources of private information

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    - extension: motivates modifications of our main result

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- There is an induced distribution F over value v_i

Statistical Assumptions

- An advertiser's preference tells them nothing about their or others' valuation of the object (without knowing the attribute)

$$(x, v_1, ..., v_N) \text{ and } (y, v_1, ..., v_N)$$

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2. The publisher commits to submitting advertiser optimal bid as a function of his reported preference and the publisher's signal
3. Preferences and attributes are realized, preferences are reported to the advertiser, signals and bids are realized and the impression is allocated to the highest bidder at the second highest price

Auto-Bidding

Proposition (Truthful Reporting)

Advertisers have an incentive to truthfully report their preferences in the auto-bidding mechanism.

- Corollary: With those commitment powers, publisher's problem reduces to our main result

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- we can show that manual bidding sometimes creates an incentive for advertisers to misreport their preferences.... in order to attain information about their high values
- however, there is an information structure where seller pools high and low values and reveals values in between, which attains close to first best

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5. As mentioned, manual bidding or other mechanisms (hard)
6. Relaxing statistical assumptions, allow vertical differentiation of bidders and viewers (hard)

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- Bergemann, Brooks and Morris (2017): seller can do even better if he can reveal information to buyers about other buyers' valuations

Literature II: Mechanics of Bidding in the Market for Impressions

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- publishers can and do control the amount of information reflected in bids by limiting bidding language, releasing information
- a key incentive to pool premium impressions while allowing information about non-premium impressions

Fixed N doesn't seem very empirically relevant/interpretable. We solve two interesting cases with a large number of bidders....

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2. power law tails
   - gain from pooling is strictly positive even as $N \to \infty$ and $q^* \to 1$

Large Markets

- large number of (possible) bidders is arguably the prevailing condition in digital advertising how does information respond to random participation of bidders
- revenue performance of auction with optimal information structure when the actual number of participating bidders grows large.

Random Number of Bidder

- with probability p, valuation is equal zero
- with probability $1 - p$,valuation is distributed with F
- limit as $N \to \infty$ and $p \to 1$ while expected number of bidders with positive values constant at:

$$\lambda \triangleq N(1 - p)$$

- critical number ρ of expected bidders

$$\rho \triangleq N(1 - q^*) \tag{1}$$

- as $N \to \infty$, (1) converges in terms of ρ:

$$\rho^2 e^{-\rho} = 1 - e^{-\rho} - \rho e^{-\rho} \Leftrightarrow \rho \approx 1.793$$

Equilibrium Information

Proposition

As $N \to \infty$, $p \to 1$, the optimal information structure is:

1. *If $\lambda \leq \rho$, then bidders observe binary signals and expected value is either 0 or $\mathbb{E}[v_i]\,\lambda/\rho$.*
2. *If $\lambda > \rho$, bidder v_i with $F(v_i) \leq (\lambda - \rho)/\lambda$ learns value, and bidder $v_i \in [F^{-1}((\lambda - \rho)/\lambda), 1]$ is bundled.*

- bundle zero values with positive values ("broad search")
- increase number of bidders even at cost of decreasing expected valuations
- with sufficently many bidders, we have pooling of high-valuation bidders

Large Number of Bidders with Heavy Tails

- Arnosti, Beck and Milgrom (2016) argued heavy tails distribution prevail in digital advertising.
- F has regularly varying tails with index α, if

$$\lim_{t \to \infty} \frac{1 - F(kt)}{1 - F(t)} = k^{\alpha},$$

- we assume $\alpha < 0$, and with $\alpha < -1$, we guarantee finite mean
- for example Pareto distribution satisfies this assumption

Revenue Comparison with Heavy Tails

- expected revenue in second price auction with complete disclosure of information, R_c :

$$R_c \triangleq \mathbb{E}[v_{(2)}].$$

- compare revenue of optimal information structure, R with revenue of complete disclosure, R_c for large N

Proposition (Revenue Ratio with Many Bidders)

As $N \to \infty$, there exists $z \in (1, \infty)$ s.th.:

$$\lim_{N \to \infty} \frac{R}{R_c} = z.$$

Furthermore, in the limit $\alpha \to -1$, $z \to \infty$.

Revenue Gains

- gains from optimal information structure do not vanish
- when the distribution has fat tails, or $\alpha < 0$

$$\mathbb{E}[v_{(1)}] - \mathbb{E}[v_{(2)}] \to \infty, \text{ as } N \to \infty.$$

- optimal information structure thickens the market at the tail of the distribution
- thus provide a revenue improvement even as the numbers of bidders becomes arbitrarily large

Honesty and Obedience

Eliciting Advertisers' Preferences

- examine advertisers' incentives to truthfully report their preferences
- a reporting strategy for bidder i is denoted by:

$$\widetilde{y}_i : \{-1, 1\}^J \to \Delta\{-1, 1\}^J.$$

- given reported preferences, the seller discloses to the bidder a signal $s(\widetilde{v}_i)$, where

$$\widetilde{v}_i \triangleq u\left(\frac{1}{\sqrt{J}} \sum_{j=1}^{J} \widetilde{y}_{ij}(y_{ij}) x_j\right)$$

- since preferences and attributes are symmetrically distributed, a sufficient statistic for the bidder's strategy is the fraction of preferences truthfully reported:

$$t_i \triangleq \sum_{j=1}^{J} \frac{\widetilde{y_i y_i}}{J}$$

Critical Reporting Strategies

- with preferences and attributes symmetrically distributed, a sufficient statistic is:

$$t_i \triangleq \sum_{j=1}^{J} \frac{\widetilde{y}_i y_i}{J}$$

- in other words, for any reporting strategy $\widetilde{y}_i, \widetilde{y}'_i$ satisfying $t_i = t'_i$, the induced distribution of expected valuations will be the same: $\widehat{G}_i = \widehat{G}'_i$
- if $t = 1$ then preferences have been correctly reported; if $t = 0$ then half of all preference components have been misreported; if $t = -1$ then every preference component has been incorrectly reported

Honesty and Informativeness

- following lemma establishes that the only relevant incentive constraints are those induced by reporting the exact opposite preference

Lemma (Informativeness of Signals)

Let s be the optimal information structure. For every $t \in [0, 1)$, the generated signal is less informative than the signal generated when reporting truthfully. For every $t \in (-1, 0]$, the generated signal is less informative than the signal generated when reporting the exact opposite preference (i.e., $t = -1$).

Truthful Reporting Under Auto Bidding

- informative lemma helps to establish:

Proposition (Honesty)

Under auto bidding and the optimal information structure, it is a dominant strategy for the advertiser to report his preference truthfully.

- misreporting leads to automated bids different from the expected value given limited information
- truthtelling guarantees that bid always equals expected value

Manual Bidding

- truthtelling is not an equilibrium for every $N,\ u$
- there is a class of information structures balancing revenue-maximization and incentive compatibility with large N
- consider the two-sided pooling structure:

$$s(v_i) = \begin{cases} \mathbb{E}[v_j \mid F(v_j) \leq 1-q] & \text{if } F(v_j) \leq 1-q^* \\ v_j & \text{if } 1-q^* \leq F(v_j) \leq q^* \\ \mathbb{E}[v_j \mid F(v_j) \geq q] & \text{if } F(v_j) \geq q^* \end{cases}$$

- above information structure adds pooling at the bottom to pooling at the top

Truthful Reporting Under Manual Bidding

Proposition (Honesty and Obedience)

Under manual bidding, it is a dominant strategy for the advertiser to report his preference truthfully in the two-sided pooling structure.

Proposition (Approximate Optimality)

Under the two-sided pooling information structure the revenue converges to the one under the optimal information structure when the number of bidders grows large:

$$\lim_{N \to \infty} (\mathbb{E}[w_{(2)}] - R) = 0.$$

- revenue under two-sided pooling is given by $w_{(2)}$

Discussion and Conclusion

- correlated values and adverse selection
- vertical differentation of attributes
- auction format
- reserve price and optimal auction
- asymmetric information structure