Information Design, Informational Robustness and Non-Linear Pricing

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recent developments in information economics

1. information economy
   - information central to digital economy
2. second generation of information economics...
   - non parametric approach to modelling information
   - three related questions:
     1. Bayesian persuasion or **information design**: what information structure is optimal for an agent who can control it?
     2. **informationally robust predictions**: what bounds can the analyst put on outcomes without knowing the information structure?
     3. **revenue guarantees**: what mechanism provides the best guarantee of revenue (or some other objective), whatever the information structure?

my talk today

- illustrate the three questions and the relation to the information economy in a single economic setting: the non-linear pricing model of Mussa-Rosen (1978)
- in particular, I will.....
  1. sketch a slightly non-standard and informal treatment of the classic screening / mechanism design problem of Mussa-Rosen (1978)
  2. introduce the three questions by asking what happens if we vary the information structure - instead of or in addition to choosing the mechanism?
- talk is based on work with Dirk Bergemann (Yale University) and Tibor Heumann (Pontificia Universidad Catolica de Chile), as well as other co-authors and authors
- will post slides and with references...

setting 1

- continuum of buyers
  - the "value" of tth quantile buyer is $v = V(t)$, where increasing $V : [0, 1] \to [\underline{v}, \overline{v}]$
  - i.e., c.d.f. of values on $[\underline{v}, \overline{v}]$ is V^{-1}
- seller has fixed inventory of different quality goods
  - the "quality" of the tth quantile good is $q = Q(t)$, where increasing $Q : [0, 1] \to [\underline{q}, \overline{q}]$
  - i.e., c.d.f. of qualities on $[\underline{q}, \overline{q}]$ is Q^{-1}
- mass of buyers and goods both normalized to 1

setting 2

- utility of consumer of value v paying p for good of quality q is

$$v \cdot q - p$$

- can also
  1. endogenize inventory with convex cost (as in Mussa-Rosen 78)
  2. consider finite bidder auction with single unit demand
- we will first solve for the optimal selling mechanism assuming buyers know their values....
- ... and then discuss various scenarios where buyers' information is varied

modelling mechanism 1

- incentive compatibility implies that expected quality of the good must be an increasing function of buyer's expected value
- therefore, there must be assortative matching of expected qualities and expected values in any incentive compatible mechanism
- so all that matters is the distribution of expected qualities sold
- so a mechanism assigns expected quality $x = X(t)$ to the tth quantile buyer, where $X : [0,1] \to \left[\underline{q}, \overline{q}\right]$

modelling mechanism 2

- what mechanisms X can the seller choose among?
- seller can (i) pool qualities (offer lotteries) and (ii) exclude buyers (which we model as setting quality to 0)
- equivalently, the seller can induce a mean preserving contraction of the distribution of qualities (by pooling) and a first-order stochastic dominance shift downwards (by exclusion)
- equivalently, the seller can choose any X such that Q weakly majorizes X ($Q \succeq_w X$) i.e.,

$$\int_x^1 X(t)\,dt \leq \int_x^1 Q(t)\,dt$$

for all $x \in [0,1]$

information rent

- write $U(t)$ for the rent (utility) of quantile t buyer; by envelope theorem

$$U'(t) = V'(t)\,X(t)$$

- information rent by quantile is

$$U(t) = \int_{s=0}^{t} V'(s)\,X(s)\,ds$$

- total information rent is

$$U(V, X) = \int_{t=0}^{1} \left[\int_{s=0}^{t} V'(s)\,X(s)\,ds \right] dt$$

total surplus and profit

- total surplus is

$$S(V,X) = \int_{t=0}^{1} V(t)\,X(t)\,dt$$

- a famous profit formula:

$$\begin{aligned}\Pi(V,X) &= \overbrace{\int_{t=0}^{1} V(t)\,X(t)\,dt}^{\text{Surplus } S(V,X)} - \overbrace{\int_{t=0}^{1}\left(\int_{s=0}^{t} V'(s)\,X(s)\,ds\right)dt}^{\text{Information Rent } U(V,X)} \\ &= V(0)\,X(0) + \int_{t=0}^{1} V(t)\,(1-t)\,X'(t)\,dt\end{aligned}$$

mechanism design: regular case

- what is the optimal mechanism for the seller (maximizing profits), taking as given the distribution of values?
- choose $X \preceq_w Q$ to maximize $\Pi(V, X)$
- "regular" case: if virtual value $V(t) - V'(t)(1-t)$ is increasing (revenue $V(t)(1-t)$ is concave), then
  - optimal to exclude buyers with negative virtual values (marginal revenue), i.e., if

$$t \leq t_m = \underset{t \in [0,1]}{\arg\max} V(t)(1-t) -$$

  - allocate remaining inventory at higher quantiles without pooling

mechanism design: irregular case

- more generally,

$$\max_{X \prec Q} \Pi(V, X) = V(t_m)(1 - t_m) + \int_{t_m}^{1} \text{cav}[V(t)(1-t)]\, dQ(t)$$

- in the "irregular case" (where virtual value $V(t) - V'(t)(1-t)$ is not increasing), optimal to pool intervals whenever

$$\text{cav}[V(t)(1-t)] > V(t)(1-t)$$

- X is "monotone partitional", alternating pooling and full revelation
- Myerson (1981) ironing; also Kleiner et al (2022)

modelling information

- recall that the "value" of tth quantile buyer is $v = V(t)$, where $V : [0,1] \to [\underline{v}, \overline{v}]$
- an information structure for buyers will give rise to a distribution over expected values
- let expected value of the tth quantile buyer be $w = W(t)$, where $W : [0,1] \to [\underline{v}, \overline{v}]$
- expected values must be a mean preserving contraction of the (ex post) values
- equivalently, the seller can choose any W such that V majorizes W ($V \succeq W$) i.e.,

$$\int_x^1 W(t)\,dt \le \int_x^1 V(t)\,dt$$

for all $x \in [0,1]$, with equality if $x = 0$.

pure information design

- now suppose that the seller can choose an information structure W to maximize profits but the inventory must be sold efficiently (so $X = Q$).
- thus the seller chooses $W \preceq V$ to maximize $\Pi(W, Q)$
- "regular" case: if inverse hazard rate $(1-t)\,Q'(t)$ is increasing, optimal to fully reveal values
- if not, concavification argument gives optimal policy
- as in mechanism design problem, W is "monotone partitional", alternating pooling and full revelation
- under reasonable conditions, pooling at the top, separation at the bottom
- in particular, in second price auction, optimal to pool high valuation bidders, separate low valuation bidders

mechanism design and information design

- suppose seller can choose **both** mechanism **and** information (to maximize profits)
- thus the seller chooses $W \preceq V$ **and** $X \preceq_w Q$ to maximize $\Pi(W, X)$
- maximization subject to two majorization constraints
- optimal W has finite number of expected values in support, and mechanism has corresponding finite expected qualities
- intuition: if there was ever full revelation, pooling a small interval would give rise to third order decrease in total surplus, second order decrease in information rent
- under mild conditions, it is optimal to provide no information, sell the uniform lottery to all buyers and extract full surplus

digital markets

- suppose a seller must decide how many virtually differentiated variants of a good to sell and at what prices
- every variant is available to all buyers at common price (no personalized pricing)
- but seller can target consumers with a particular variant, i.e., make a (perhaps implicit) recommendation
- this gives implementation of previous direct mechanism
- but suggests that recommendation systems may not be optimal for vertically differentiated goods

varying information: more questions

1. **information design I**: what information structure maximizes profits (given efficient or optimal mechanism)?
2. **information design II**: what information structure maximizes information rent?
3. **revenue guarantee**: what information structure minimizes profits?
4. **robust predictions**: what bounds can one put on profits and information rent if you don't know the information structure

question 2: maximizing information rent

- what information structure maximizes information rent given that the seller will choose an optimal mechanism given the information structure?
- thus "the buyers" choose the chooses $W \preceq V$ to maximize $U(W, X)$ subject $X \preceq_w Q$ maximizing $\Pi(W, X)$
- compare Roessler and Szentes (2017) for the homogenous quality case
- continuous information structure is optimal
- equalize virtual values with generalized Pareto distribution but must also deter seller from exclusion

question 3: minimizing profits

- what information structure minimizes profits given that the seller will choose an optimal mechanism given the information structure?
- thus an "adversary" chooses $W \preceq V$ to minimize $\Pi(W, X)$ subject $X \preceq_w Q$ maximizing $\Pi(W, X)$
- or consider zero sum game where (i) seller chooses $X \preceq_w Q$ and (ii) adversary chooses $W \preceq V$ to maximize/minimize revenue respectively
- saddle point (X, W)
- compare Du (2018) for the homogenous quality case
- solution is revenue guarantee for the seller and X is the mechanism that attains it

question 4: robust predictions

- what bounds can the analyst put on profits and information rent if you don't know the information structure?
- to answer question, suppose a (metaphorical) informational designer maximizes a weighted sum of profits and information rent, anticipating that a seller would choose the profit maximizing mechanism
- thus the information designer chooses $W \preceq V$ to maximize $\lambda \Pi (W, X) + \mu U (W, X)$ (for positive and negative λ and μ) subject $X \preceq_w Q$ maximizing $\Pi (W, X)$
- finite support solution if weight on profits exceeds weight on information rent (for same reason as earlier)
- continuous solution otherwise
- see picture

conclusion

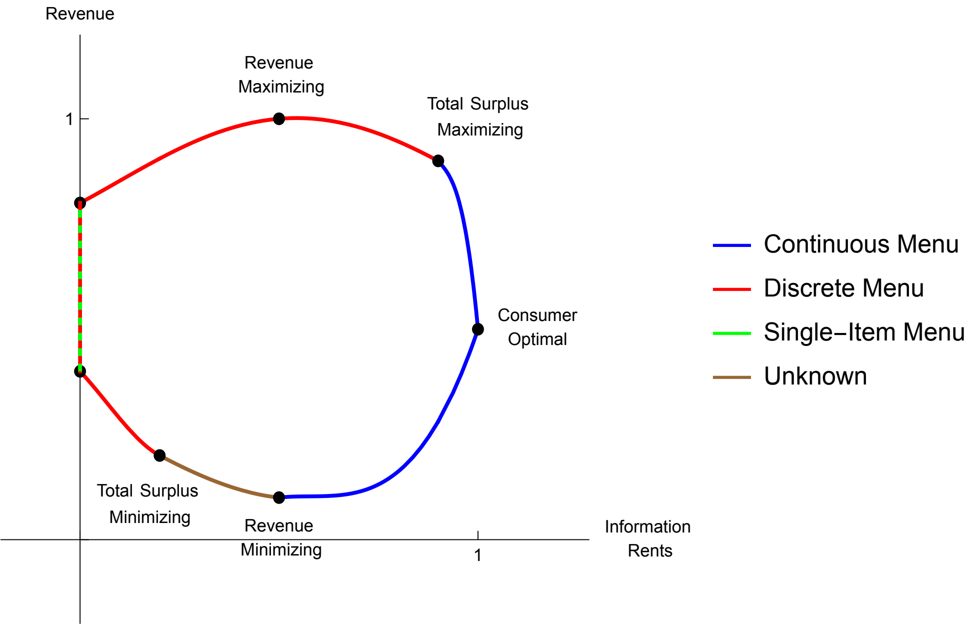
- you may have heard a lot about (and maybe written about) Bayesian persuasion and information design in recent years (also well represented at SAET Paris)
- to a significant extent, this comes from an internal dynamic in the theory community
- two external drivers of interest:
  - the information economy
  - the sensitivity of first generation information economics to parametric information structures
- the way forward is surely integration on second generation information economics and information economy applications

bergemann-heumann-morris papers

"Optimal Information Disclosure in Classical Auctions," also with Constantin Sorokin and Eyal Winter, *American Economic Review: Insights*

"Screening with Persuasion"

"The Consumer Optimal Information Structure in Optimal Auctions"


Revenue
1
Revenue
Maximizing
Total Surplus
Maximizing
Consumer
Optimal
Total Surplus
Minimizing
Revenue
Minimizing
1
Information
Rents
Continuous Menu
Discrete Menu
Single−Item Menu
Unknown