Reward and Punishment in a Regime Change Game

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Question

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  - public meetings, boycotts, strikes, marches, demonstrations, sit-ins, freedom rides, street blockades, suicide bombings, assassination, hijacking, and guerrilla war

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  - public meetings, boycotts, strikes, marches, demonstrations, sit-ins, freedom rides, street blockades, suicide bombings, assassination, hijacking, and guerrilla war
- but what contentious performances can be elicited from supporters and how?
  - **pleasure in agency** (Wood 03): "the positive effect associated with self-determination, autonomy, self-esteem, efficacy, and pride that come from the successful assertion of intention"
  - "depends on the likelihood of success, which in turn depends on the number participating... yet the pleasure in agency is undiminished by the fact that one's own contribution to the likelihood of victory is vanishingly small"

Question

- **transformational leadership**: ability to create inspirational motivations via a variety of psychological mechanisms (Burns 78)
  - sociology: leaders raise participation by "identification, idealization, elevation of one or more values..." (Snow et al. 89)
  - politics: "people-oriented leaders are those who inspire people, give them a sense of identity..." (Goldstone 01)

Question

- **transformational leadership**: ability to create inspirational motivations via a variety of psychological mechanisms (Burns 78)
  - sociology: leaders raise participation by "identification, idealization, elevation of one or more values..." (Snow et al. 89)
  - politics: "people-oriented leaders are those who inspire people, give them a sense of identity..." (Goldstone 01)
- our question:
  - What happens when transformative leaders manipulate pleasure in agency in order to influence activists' choice among a repertoire of contentious performances?
- our answer (in a stylized model):
  - "the organization of the revolutionaries must consist first and foremost of people who make revolutionary activity their profession"...while others (workers) engage in various levels of contentious activities (Lenin 1902)

Key Tradeoffs

- if activists ("citizens") are heterogeneous, then there would be trade-offs in manipulating pleasure in agency ("benefits"); higher benefits from intermediate levels of contentious performances ("effort") will...
  - encourage those who would otherwise have chosen low effort to choose intermediate effort
  - discourage those who might otherwise have chosen high effort

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  - encourage those who would otherwise have chosen low effort to choose intermediate effort
  - discourage those who might otherwise have chosen high effort
- even if citizens are not heterogenous, at the margin where the revolution is occurring, the citizenry will have heterogeneous beliefs about the likelihood of success
  - if everyone thought revolution was going to succeed, everyone would choose maximum effort and the revolution would succeed
  - if everyone thought revolution was going to fail, everyone would choose minimum effort and the revolution would fail

Optimal Reward Schemes

- we will consider a situation where there is an upper bound on benefits: even charismatic and skillful "people-oriented" leaders have limited ability to incite pleasure in agency (intrinsic motivation) in citizens.
- the upper bound on benefits will imply an upper bound on effort (what would be chosen if you assigned probability 1 to success)

Optimal Reward Schemes

- we will consider a situation where there is an upper bound on benefits: even charismatic and skillful "people-oriented" leaders have limited ability to incite pleasure in agency (intrinsic motivation) in citizens.
- the upper bound on benefits will imply an upper bound on effort (what would be chosen if you assigned probability 1 to success)
- we will identify an optimal reward scheme (mapping from effort to benefits):
  - there will be a critical level of effort at which citizens will get the maximal benefit...
  - benefits will smoothly decline for lower efforts

Model of Coordination

- continuous action regime change game:
- citizens make a continuous effort decision
- revolution succeeds if aggregate effort exceeds a threshold
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- revolution succeeds if aggregate effort exceeds a threshold
- small uncertainty about threshold $\Rightarrow$ unique equilibrium ("global game")
- at critical threshold, there will be uniform distribution among citizens of the probability of success
- citizens trade off costs and success contingent reward scheme

....and Screening

- leader can pick optimal reward scheme
- screening problem because of heterogeneous beliefs about the probability of success

Methodological Contribution

- continuous action regime change game (Guimaraes-Morris 07 is unique applied precursor?)
- non-standard screening problem: role of maximum choice variable
- **global game with screening**

Model with Exogenous Benefits and Complete Information

- continuum of citizens with each choosing e_i.
- revolution succeeds if $\int e_i di \geq \theta$, where θ is "regime strength."
- uncontingent cost of effort $C(e)$.
- contingent benefit of effort $B(e)$.
- optimal effort correspondence:

$$e^*(p) = \arg\max_{e \geq 0} pB(e) - C(e).$$

- A correspondence is *weakly increasing* whenever:

$$p_2 > p_1 \;\Rightarrow\; e_2 \geq e_1,\; \forall e_i \in e^*(p_i),\; i = 1, 2.$$

Exogenous Benefits and Complete Information

- assume the optimal effort correspondence is increasing, so that $e^*(1) \geq e^*(0)$.
- maximum effort level is: $\bar{e} = sup(e^*(1))$.
- minimum effort level is: $\underline{e} = min(e^*(0))$.

Equilibrium:

1. $\theta > \bar{e}$: everyone puts in $e^*(0)$ and the regime survives.
2. $\theta \leq \underline{e}$: everyone puts in $e^*(1)$ and there is a regime change.
3. $\underline{e} < \theta \leq \bar{e}$:
    - everyone puts in $\underline{e}$ and the regime survives.
    - everyone puts in $\bar{e}$ and there is a regime change. (lots of other eq)

Exogenous Benefits and Incomplete Information

- citizens receive private signals: $x_i = \theta + \nu_i$.
- recall that each citizen's problem is:

$$\max_{e_i \geq 0} pr(success|x_i) \times B(e_i) - C(e_i).$$

- strategy: $s_i(x_i) : \mathbb{R} \to \mathbb{R}_+$.

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- A strategy profile $(s_i)_{i \in [0,1]}$ gives rise to aggregate efforts:

$$\hat{s}(\theta) = \int_{i=0}^{1} \int_{\nu_i=-\infty}^{\infty} s_i(\theta + \nu_i) f(\nu_i) d\nu_i \, di$$

Exogenous Benefits and Incomplete Information

- focusing on weakly decreasing strategies, $\hat{s}(\theta)$ is decreasing, and:
  - There is a unique θ^* such that $\hat{s}(\theta^*) = \theta^*$.
  - There is a regime change whenever $\theta < \theta^*$.

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- Thus,

$$s_i(x_i) = e^*(pr(success|x_i)) = e^*(pr(\theta < \theta^*|x_i)).$$

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$$s_i(x_i) = e^*(pr(success|x_i)) = e^*(pr(\theta < \theta^*|x_i)).$$

- Then, at the critical threshold θ^*, the aggregate effort is:

$$\theta^* = \hat{s}(\theta^*) = \int e^*(pr(\theta < \theta^*|x_i)) \, d\mu(pr(\theta < \theta^*|x_i)|\theta^*).$$

Exogenous Benefits and Incomplete Information

A Statistical Property

- Suppose θ is an unknown and uncertain random variable with a uniform distribution—improper distribution on $\mathbb{R}$ when relevant.
- Consider a signal $x = \theta + \nu$, where ν and θ are independent, and $\nu \sim f(\cdot)$.
- For a given threshold θ^*, what is $Pr(\theta < \theta^* | x)$?
- Because we have no prior information about x and θ, we can treat θ as a signal of x:

$$\theta = x - \nu.$$

- Thus,

$$Pr(\theta < \theta^* | x) = Pr(x - \nu < \theta^* | x) = Pr(x - \theta^* < \nu) = 1 - F(x - \theta^*).$$

A KEY Statistical Property

- **At the equilibrium critical threshold θ^*, beliefs about the likelihood of regime change is distributed uniformly.**

Let $H(\cdot|\theta^*)$ be the conditinal CDF of beliefs given $\theta = \theta^*$.

$$
\begin{aligned}
H(p|\theta^*) =& Pr(pr(\theta < \theta^*|x_i) \leq p|\theta^*) \\
=& Pr(1 - F(x_i - \theta^*) \leq p|\theta^*) \\
=& Pr(\theta^* + F^{-1}(1-p) \leq x_i|\theta^*) \\
=& 1 - F(\theta^* + F^{-1}(1-p) - \theta^*) \\
=& 1 - (1-p) = p.
\end{aligned}
$$

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Recall that:

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$$\theta^* = \int_{p=0}^{1} e^*(p) \; dp$$

The Road to Endogenous Benefits...

We assumed that $e^*(p)$ is a weakly increasing correspondence in order to characterize the equilibrium:

- There is a threshold θ^* such that there is a regime change if and only if $\theta \leq \theta^*$, where (with some abuse of notation) we have:

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When is $e^*(p)$ is a weakly increasing? Recall optimal effort correspondence:

$$e^*(p) = \arg\max_{e \geq 0} p\ B(e) - C(e).$$

- **If $C(e)$ is strictly increasing in e, then any selection from optimal effort correspondence is weakly increasing.**

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The Road to Endogenous Benefits...

Let $p_2 > p_1 > 0$, $e_i \in e^*(p_i) = \arg\max_{e \geq 0} p_i B(e) - C(e)$, $i \in \{1, 2\}$. Suppose $e_2 < e_1$.

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- Optimality of e_1 and e_2:

$$p_2\ B(e_2) - C(e_2) \geq p_2\ B(e_1) - C(e_1) \quad \Leftrightarrow \quad p_2\ [B(e_2) - B(e_1)] \geq C(e_2) - C(e_1$$

$$p_1\ B(e_1) - C(e_1) \geq p_1\ B(e_2) - C(e_2) \quad \Leftrightarrow \quad C(e_2) - C(e_1) \geq p_1\ [B(e_2) - B(e_1$$

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- $C(e)$ is strictly increasing. Thus, $e_2 < e_1$ implies $C(e_2) < C(e_1)$.

$$0 > C(e_2) - C(e_1) \geq p_1\ [B(e_2) - B(e_1)] \quad \Rightarrow \quad B(e_2) - B(e_1) < 0.$$

Endogenous Benefits

- Leader chooses success-contingent $B : \mathbb{R}_+ \to [0, M]$ to maximize the probability of regime change.
- Citizens observe $B(e)$ and their private signals and simultaneously decide how much effort to put in. Effort costs $C(e)$.
- Success or failure of the regime change attempt is realized.

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- Citizens observe $B(e)$ and their private signals and simultaneously decide how much effort to put in. Effort costs $C(e)$.
- Success or failure of the regime change attempt is realized.
- Assume $C(e)$ is strictly increasing and convex, with $C(0) = 0$.

$$\max_{B(\cdot)} \int_{p=0}^{1} e^*(p) dp$$

$$s.t.\ e^*(p) = \arg\max_{e \geq 0}\ p\ B(e) - C(e)$$

$$B(e) \in [0, M],$$

Recall Two Insights (from introduction)

- if supporters ("citizens") are heterogeneous, then there would be trade-offs in manipulating benefits; higher benefits from intermediate levels of effort will...
  - encourage those who would otherwise have chosen low effort to choose intermediate effort
  - discourage those who might otherwise have chosen high effort
- even if citizens are not heterogeneous, at the margin where the revolution is occuring, the citizenry will have heterogeneous beliefs about the likelihood of success
  - if everyone thought revolution was going to succeed, everyone would choose maximum effort and the revolution would succeed
  - if everyone thought revolution was going to fail, everyone would choose minimum effort and the revolution would fail

Screening Problem

- revelation principle / optimal screening argument:
- for each type $p \in [0,1]$, a "contract" is offered: $(e(p), B(p))$.

$$\max_{\{(e(p),B(p))\}} \int_{p=0}^{1} e(p)dp$$

$$\begin{aligned} s.t. \quad & pB(p) - C(e(p)) \geq 0, && \forall\ p \in [0,1] \\ & pB(p) - C(e(p)) \geq p\ B(p') - C(e(p')), && \forall\ p, p' \in [0,1] \\ & B(p) \in [0, M], && \forall\ p \in [0,1]. \end{aligned}$$

Main Result

RESULT: Suppose that C is strictly convex.

- the optimal $B(e)$ is continuous and weakly increasing. For some $0 < \widehat{e} < 1$, optimal B is strictly convex on $[0, \widehat{e}]$ and equal to M above $\widehat{e}$.
- The optimal $e(p)$ is continuous, strictly increasing for $p \in [0, 1/2]$, and equal to $\widehat{e}$ on $[1/2, 1]$.

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Why bunching?

- Suppose that $0 < B'(e) < C'(e)$ on an interval $[e_1, e_2]$; no one will choose effort in the interval $[e_1, e_2]$.

- Must have $B'(e) \geq C'(e)$ unless B' is zero.

Screening Problem: Incentive Compatibility

$$pB(p) - C(e(p)) \;\geq\; p\; B(p') - C(e(p'))$$

$$p'B(p') - C(e(p')) \;\geq\; p'\; B(p) - C(e(p))$$

$\Rightarrow \quad (p - p')\,(B(p) - B(p')) \geq 0.$

$\Rightarrow \quad B(p)$ is weakly increasing

$\Rightarrow \quad B(p)$ is almost everywhere differentiable.

$\Rightarrow \quad$ IC becomes $p\; B'(p) - h'(p) = 0$ (FOC) and $B'(p) \geq 0$ (SOC).

Screening Problem: Participation Constraint

- $pB(p) - C(e(p))$ is increasing in p, and hence:

$$0 \times B(0) - C(e(0)) = -C(e(0)) = -h(0) \geq 0 \Rightarrow pB(p) - C(e(p)) \geq$$

- $-C(e(0)) = -h(0) \geq 0$ implies $e(0) = h(0) = 0$.

Screening Problem

$$\max_{\{(e(p),B(p))\}} \int_{p=0}^{1} e(p)dp$$

$$\begin{aligned}
s.t. \quad & pB(p) - C(e(p)) \geq 0, && \forall\ p \in [0,1] \\
& pB(p) - C(e(p)) \geq p\ B(p') - C(e(p')), && \forall\ p, p' \in [0,1] \\
& B(p) \in [0, M], && \forall\ p \in [0,1].
\end{aligned}$$

$$\max_{\{(e(p),B(p))\}} \int_{p=0}^{1} e(p)dp$$

$$\begin{aligned}
s.t. \quad & pB'(p) - h'(p) = 0,\ B'(p) \geq 0,\ h(0) = 0 \\
& B(p) \in [0, M].
\end{aligned}$$

Screening Problem

- Recall that $h(p) = C(e(p))$, and hence $e(p) = C^{-1}(h(p))$. Letting $\Pi(\cdot) = C^{-1}(\cdot)$, we have:

$$\max_{\{(h(p),B(p))\}} \int_{p=0}^{1} \Pi(h(p))\, dp$$

$$s.t. \quad pB'(p) - h'(p) = 0,\ B'(p) \geq 0,\ h(0) = 0$$

$$B(p) \in [0, M].$$

- If $\Pi(h(p)) \neq h(p)$, we have a mechanism design without transfers: the "agent" gives up $h(p)$, the "principal" gets $\Pi(h(p))$.

Screening Problem

$$\max_{\{(h(p),B(p))\}} \int_{p=0}^{1} \Pi(h(p))\ dp$$

$$s.t. \quad pB'(p) - h'(p) = 0,\ B'(p) \geq 0,\ h(0) = 0$$

$$B(p) \in [0, M].$$

- Lemma: $C''(\cdot) > 0 \ \Rightarrow$ there is no jump in $B(p),\ h(p)$, or $e(p)$.
- The consequences of IC for $p \in (0,1]$:

$$p\ B'(p) - h'(p) = 0 \ \Leftrightarrow\ [p\ B'(e) - C'(e)]\ e'(p) = 0 \ \Rightarrow$$

1. $e'(p) = 0$ **OR**
2. $e'(p) > 0$ and $B'(e) > C'(e)$.

Screening Problem

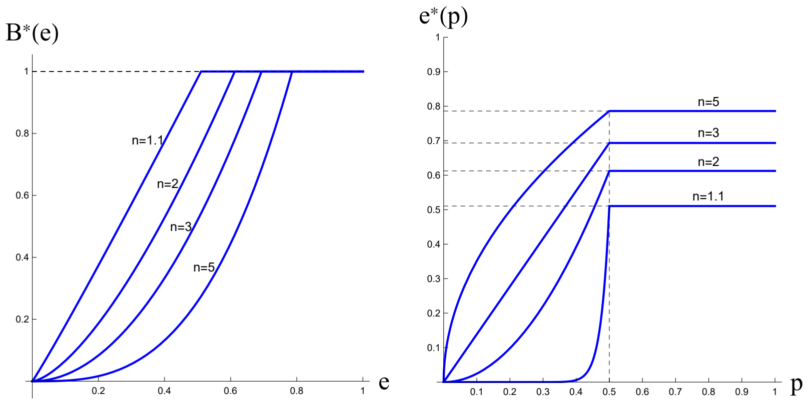

Figure: Optimal reward schedule, $B^*(e)$, and its induced effort schedule, $e^*(p)$, for $M = 1$ and cost functions of the form $C(e) = e^n$, $n > 1$. As the right panel illustrates, when the cost function approached the linear $C(e) = e$, effort schedule becomes binary as described in the text.

The Curious One Half

- Suppose $C(e) = e$, so that $h'(p) = C'(e)\ e'(p) = e'(p)$.
- Suppose types are distributed like $f(p)$.

$$\max_{\{(e(p),B(p))\}} \int_{p=0}^{1} e(p)\ f(p)\ dp$$

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$$e(p) \quad = \quad \int_{x=0}^{p} e'(x) dx.$$

The Curious One Half

$$\begin{aligned} e(p) &= \int_{x=0}^{p} e'(x)dx = \int_{x=0}^{p} xB'(x)dx = xB(x)\Big|_{x=0}^{p} - \int_{x=0}^{p} B(x)dx \\ &= pB(p) - \int_{x=0}^{p} B(x)dx. \end{aligned}$$

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&= pB(p) - \int_{x=0}^{p} B(x)dx.
\end{aligned}
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Moreover,

$$
\begin{aligned}
\int_{p=0}^{1} \left(\int_{x=0}^{p} B(x)dx \right) f(p)dp &= \left(\int_{x=0}^{p} B(x)dx \right) \; F(p) \; \Big|_{p=0}^{1} - \int_{p=0}^{1} B(p)F(p) \\
&= \int_{p=0}^{1} (1 - F(p)) \; B(p)dp.
\end{aligned}
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$$
\theta^* = \int_{p=0}^{1} e(p)f(p)dp = \int_{p=0}^{1}\left(p - \frac{1-F(p)}{f(p)}\right)\ B(p)\ f(p)dp.
$$

The Curious One Half

$$\max_{B(p)\in[0,M]} \int_{p=0}^{1} \left(p - \frac{1-F(p)}{f(p)} \right) B(p)\ f(p) dp.$$

$$B^*(p) = \begin{cases} 0 & ; p - \frac{1-F(p)}{f(p)} < 0 \\ \\ M & ; p - \frac{1-F(p)}{f(p)} > 0. \end{cases}$$

- Uniform Case: $p - \frac{1-F(p)}{f(p)} = p - \frac{1-p}{1} = 0 \ \Rightarrow \ p = 1/2.$

A Simpler Question: Optimal Contribution Restrictions

- Suppose $B(e)$ is exogenous, but the leader/manager can restrict efforts $e \geq 0$ to $\{e_1, \cdots, e_N\}$ for some $N \geq 1$.
- Whether and when he would restrict contributions? To how many and what contribution levels?

RESULT. Under some assumptions that ensures $e^*(p)$ is a strictly increasing function:

- If $B(e)$ is strictly concave, then the leader is strictly better off if he can restrict contributions to a single level.
- If $B(e)$ is linear, then he is indifferent.
- If $B(e)$ is strictly convex, then he is strictly better off not intervening.

Extensions and Open Questions

- designing costs instead of benefits
- a game between players choosing costs and benefits respectively
- more standard screening problems
- philanthropy and other applications

Conclusion

- **Substantive:**
  - subtle tradeoffs in design of optimal pleasure in agency
  - in support of the creation of vanguard
- **Methodology:**
  - global game of regime change with continuous actions...
  - ...with screening