The Life-cycle of Concentrated Industries*

Martin Beraja
MIT and NBER

Francisco Buera
University of St. Louis and NBER

June 15th, 2024

Abstract

Should governments promote competition in nascent industries or can they wait until they have concentrated? What determines the optimal mix of early and late interventions in the life-cycle of an industry? We build a model of the life-cycle of an oligopolistic industry: a version of Jovanovic and MacDonald (1994) with a finite number of firms. The equilibrium features a period of intense entry of small firms, followed by a shakeout and later industry concentration as some firms innovate and increase their scale and the majority exits. We analyze the second best problem of a government that can control the number of small firms by subdizing entry (or taxing exit). In an industry with large differences in scale between firms, a subsidy after the industry has concentrated suffices to implement the second best. Interventions early in the life-cycle are unnecessary. As scale differences shrink, the optimal subsidies become more uniform over the life-cycle. We apply these insights to empirically study digital and AI industries.

*Martin Beraja: maberaja@mit.edu. Francisco Buera: fjbuera@wustl.edu.

1 Introduction

Disruptive technologies often spur new industries. Many such industries have experienced a stark life-cycle. For example, car manufacturing saw an initial phase of intense firm entry early in the 20th century, followed by a shakeout and later industry concentration (Klepper and Simons, 2005). More recently, digital industries — spurred by computers, the Internet, and big data and AI — have rapidly concentrated as they matured.[1]

The fast concentration of digital industries has rekindled a debate about appropriate policy interventions to promote competition. Some interventions act on nascent industries *before* they become concentrated (ex-ante), such as laxer regulations on data privacy in digital industries (Goldfarb and Tucker, 2012), or subsidies to innovation (Bloom et al., 2019) and financing (Itskhoki and Moll, 2019) more generally. Other interventions come into play only *after* an industry has sufficiently concentrated (ex-post). For example, forcing leader firms in digital industries to share their data would lower barriers to entry (Abrahamson, 2014), as mandating access to essential infrastructure (Spulber and Yoo, 2007) or intellectual property (Tang, 2011) has achieved in industries of the past.

When should governments promote competition in a nascent industry? When can they wait until the industry has sufficiently concentrated? What determines the optimal mix of ex-ante and ex-post policy interventions? There is a dearth of results on optimal policy over the life-cycle of concentrated industries. The early literature focused instead on whether there is insufficient or excessive entry (Dixit and Stiglitz, 1977; Mankiw and Whinston, 1986; Reinganum, 1989; Aghion and Howitt, 1990). Recent work has shifted attention to measurement and quantification; for example, documenting evidence on rising concentration and market power (Philippon, 2019; De Loecker et al., 2020), and quantifying the effects of dynamic merger policies (Igami and Uetake, 2020; Mermelstein et al., 2020) or reducing markup distortions directly (Peters 2020; Boar and Midrigan 2019; Edmond et al. 2023).

[1] For example, personal computer operating systems (e.g., MS-DOS, Mac OS) arose in the early 1980s. Only a decade later Windows became the top system with about 90 percent market share (https://www.thestreet.com/technology/history-of-microsoft-15073246). The mid 1990s saw the rise of search engines (e.g., Yahoo!, AltaVista), with Google becoming the most popular engine in the early 2000s (Evans, 2008). Thousands of online marketplaces spawned in the late 1990s, and the industry experienced a sharp shakeout shortly after (Day et al., 2003).

In this paper, we build a model of the life-cycle of an oligopolistic industry: a version of Jovanovic and MacDonald (1994) with a finite number of firms. We characterize the equilibrium and the (constrained) optimal policy over the life-cycle of the industry, emphasizing the role of scale economies in determining both.[2] As an application, we empirically study digital and AI industries in the U.S. using a novel dataset from Venture Scanner.

In the model, a new technology gives birth to a new industry. Firms choose whether to enter the industry and use the technology to produce a horizontally differentiated product. This gives rise to *competition in the market*. Over time, firms receive random innovations which reduce their marginal cost and allow them to increase their scale. This makes firms vertically differentiated too. Firms can choose to exit at any point. Vertical differentiation and the possibility of exit give rise to *competition for the market*.

We solve for a (unique) equilibrium in Poisson mixed-strategies where firms choose an exit rate.[3] The equilibrium life-cycle of an industry features an initial growth in the number of small firms, followed by a shakeout as some of these firms randomly succeed and increase their scale, and the majority exits. In the long-run, an industry is characterized by a more concentrated market structure with only large firms remaining. A notable feature of the life-cycle in our model is the possibility of a gradual increase in the number of firms, with multiple periods of entry and exit, culminating in a sudden shakeout. These non-monotone dynamics are in sharp contrast with the monotonic life-cycle implied by the perfectly competitive version of our model (Jovanovic and MacDonald, 1994).[4] A non-monotonic life-cycle has been observed in many traditional industries — like car manufacturing (Klepper and Simons, 2005) — and digital industries more recently too.

The life-cycle of the industry is distinctly shaped by differences in scale be-

[2] Economies of scale have been key in driving the recent rise of US concentration (Covarrubias et al. 2020; Kwon et al. 2023) and superstar firms (Autor et al. 2020). Compared to traditional industries, sale economies seem to be particularly strong in many digital industries because of near-zero marginal costs (Goldfarb and Tucker, 2019) or intensive data use (Agrawal et al., 2019).

[3] A "War of Attrition" (Fudenberg and Tirole, 1986) leads to multiplicity of pure strategy equilibria. Our mixed-strategy equilibrium is unique under the refinement that large firms never exit.

[4] Under perfect competition, the life-cycle is *bang-bang:* firms do not exit when the price is high and then exit en masse once the price falls below a threshold. In our oligopolistic model, firms' incentives to enter may be especially strong right before the shakeout, resulting in a non-monotonic life-cycle. Such non-monotonicity can also arise from an *exogenous* industry-wide innovation opportunity late in the life-cycle, as in Jovanovic and MacDonald (1994).

tween large and small firms. At one extreme, consider a special case of our model where the relative scale between firms is arbitrarily large: firms have infinitely large marginal cost at entry and can only become productive after receiving the random innovation. This case captures an economy with strong scale economies for large firms relative to small ones. The model resembles those in the patent race literature (Reinganum, 1989), where firms enter the industry with the expectation of taking most of the market or exiting. The life-cycle features an initial outburst of firm entry, followed by a sharp shakeout and a few large firms remaining in the long-run. At the other extreme, suppose that differences in marginal costs (and thus scale) are negligible. The model is essentially static in this case. Firms only compete in the market as in models of imperfect competition based on horizontal differentiation (Benassy, 1996; Atkeson and Burstein, 2008). The initial and long-run industry concentration are similar.

We then turn to studying optimal policy. In principle, the government can implement a first best with a sufficiently rich set of instruments, including subsidizing production to correct markup distortions. These are unlikely to be available in practice due to political or informational constraints, which motivates us to study second best interventions.[5] In particular, we analyze the constrained Ramsey problem of a government that can only control the number of small firms in an industry. This is implemented via a time-varying subsidy to the fixed cost of production of small firms. The policy mimics proposed interventions to promote competition over an industry's life-cycle. Interventions after an industry has sufficiently concentrated (ex-post) are captured by subsidies late in the life-cycle, whereas interventions in a nascent industry (ex-ante) are captured by subsidies early on.

The relative scale between large and small firms in the industry crucially determines the optimal mix of early and late interventions in the life-cycle. When differences in scale are arbitrarily large, the government can wait to intervene until the industry has concentrated. Specifically, a subsidy to small firms after the industry has reached its long-run equilibrium suffices to implement the second best. Interventions earlier in the life-cycle are not necessary. The reason is that, in this limit case, small firms' entry and exit choices are *purely* driven by the option value of becoming larger and taking most of the market. Correcting profits

[5] For instance, the first best policy subsidizes firms which are already large and requires knowing firm-level elasticities (Edmond et al., 2023).

late in the life-cycle thus suffices to align private and social incentives earlier too. The Ramsey policy may be time-inconsistent when the required subsidies are too large. If the government cannot commit, the time-consistent policy must subsidize firms in a nascent industry as well, but the policy still remains heavily tilted towards subsidizing later in the life-cycle. Finally, as scale difference shrink, subsidies become more uniform over the life-cycle and eventually become identical in the (static) limit. These results are robust to a number of extensions to our baseline model; such as when small firms can choose their rate of innovation, there are innovation spillovers, or large firms can collude.

With our results in mind, the question of how to regulate an industry in practice can be understood as follows. Are firm choices mostly driven by dynamic (option value) considerations and competition for the market, or are static considerations and competition in the market important too? Our model points to differences in scale between large and small firms as a relevant moment for empirically diagnosing how close an industry is to each case.

We use this insight to empirically study modern digital and AI industries in the U.S. using a novel dataset from Venture Scanner. The dataset collects information on the universe of firms funded by venture capital — the primary funding source in these industries — and categorizes firms according to the technologies they produce or services they provide — such as "Deep and Machine Learning," "Consumer Payments," or "Short Term Rentals and Vacation Search." This is a key feature of this dataset, as it allows us to define an industry as a product market for a technology or service (a total of 155 industries). We find that digital and AI industries are still early on in their life-cycle, with the total number of active firms in almost all industries peaking in recent years. To benchmark digital and AI industries, we also digitized *The 100 Year Almanac* which collects information on automobile manufacturing firms in the U.S. We confirm the findings in Klepper and Simons (2005) in this data: the industry saw two decades of intense firm entry, followed by a shakeout and later concentration around WWII.

Regarding our moment of interest, we document that large firms (90th percentile of the size distribution) are roughly 40 times larger than small firms (10th percentile) in the median digital and AI industry (e.g., "Deep and Machine Learning Applications"). However, the distribution of relative scale (90th-10th percentile ratio) across industries is very skewed. More than 80 percent of industries have a relative scale larger than 35, with some industries like "Video Consumption Platforms" or "Short Term Rentals and Vacation Search" having large firms that are 120 times bigger than smaller ones. By comparison, the relative scale was 33 in the automobile industry at the peak of its life-cycle. Through the lens of our model, these findings suggest that most digital and AI industries have less of a need for interventions that promote competition in the present (nascent) stage than the automobile industry did at a similar point in its life-cycle. Instead, compared to the automobile industry, governments can intervene later in the life-cycle of these industries, waiting until they have sufficiently concentrated.

Beyond the recent literature on market power mentioned above (e.g., De Loecker et al., 2020 or Edmond et al., 2023), our paper also contributes to the literature studying innovation under imperfect competition. Aghion and Howitt (1990) spearheaded this literature, with Schumpeterian growth models having been used to analyze questions of competition and economic growth (Aghion et al., 2005; Liu et al., 2022; Olmstead-Rumsey, 2024) as well as in many other applications (see Aghion et al., 2014 for a survey). A common feature of this class of models is the absence of an industry life-cycle. There is typically one firm (the leader) which lies ahead of its competitor (the follower) — a perpetual duopoly.[6] In contrast, we model the life-cycle dynamics in an oligopolistic industry and analyze how the optimal policy varies over the life-cycle.

Our paper complements previous work that has studied the dynamic effects of policy interventions in different contexts. A literature on economic development has shown that interventions in nascent industries can act as a "big push" to resolve coordination problems (Murphy et al. 1989; Buera et al. 2021), make firms internalize dynamic learning spillovers (Melitz, 2005), or correct financial distortions (Itskhoki and Moll 2019). In advanced economies, front-loading carbon taxes may rapidly reduce environmental degradation (Hémous, 2016) and innovation subsidies can have a sluggish impact on aggregate productivity (Atkeson and Burstein, 2019). A separate literature in law and economics has studied the optimality of ex-post liability versus ex-ante regulation; as it applies to product safety, pollution, and other types of harm (Shavell, 2007; Glaeser and Shleifer, 2003; Posner, 2010).

[6] One exception is Impullitti and Licandro (2018). They build an endogenous growth model with oligopolistic industries that have a common number of homogeneous firms. Cavenaile et al. (2023) also develops an oligopolistic growth model, but features a competitive fringe of small firms together with a number of large firms engaged in Cournot competition.

Public health and environmental scientists advocate for ex-ante policy interventions based on the "precautionary principle" (Raffensperger and Tickner, 1999). This body of work emphasizes determinants of optimal policy which are not specific to — although can interact with — considerations of imperfect competition or the life-cycle of an industry.

2 Model

The model is a continous time analogue of the model of a life-cycle of a competitive industry in Jovanovic and MacDonald (1994), with one major difference. The industry has a finite number of strategic firms — an oligopoly.

The environment is as follows. Time is continuous and indexed by $t \geq 0$. The arrival of a radical new technology spurs a new industry, such as car manufacturing in the past or artificial intelligence (AI) more recently. Within the industry, there are $\underline{N}_t$ small firms producing with a high marginal cost technology and $\bar{N}_t$ large firms producing with a low marginal cost technology. The industry is characterized by state $\{\underline{N}, \bar{N}\}$.

2.1 Firms

Firms can freely enter and exit the industry at any point in time. Upon entry, firms produce using a basic technology with a fixed cost of production f and marginal cost $1/\underline{z}$. Over time, firms experience random Poisson innovations at rate λ. An innovation allows the firm to produce with a lower marginal cost technology ($1/\bar{z} < 1/\underline{z}$). We interpret λ as the rate at which a firm discovers new production processes and learns to produce at scale. We assume this innovation rate is exogenous for now (Section 5.2 relaxes this assumption). In the following, we will refer to high marginal cost firms as "small" and low marginal cost firms as "large."

Definition 1 (Firms' profits and values)**.** The function $\pi(\underline{N}, \bar{N}; z)$ is the flow profit of a firm with marginal cost $1/z$ in an industry with $\underline{N}$ small firms and $\bar{N}$ large firms. Accordingly, the value function $J(\underline{N}, \bar{N}; z)$ is the firm's expected present discounted value of profits.

For our theoretical results, we do not require specifying a particular microfoundation for the profit function — i.e. the cost structure, demand functions, nature of competition, whether collusion is allowed, etc.[7] We will only require that this function satisfies some natural regularity conditions in Assumption 1.

Assumption 1 (Profits)**.** *The profit function* $\pi\left(\underline{N},\bar{N};z\right)$ *is:*

(i) *decreasing in both* $\underline{N}$ *and* $\bar{N}$ *for any* z*,*

(ii) *increasing in* z *for any* $\underline{N}$ *and* $\bar{N}$*,*

(iii) *converges to minus the fixed cost of production* $-f$ *as* $z\to 0$ *and* $\bar{N}\to\infty$*, and*

(iv) *it is profitable for at least one firm to enter, i.e.,* $\pi\left(1,0;\underline{z}\right)+\lambda\pi\left(0,1;\bar{z}\right)/r>0$*.*

The following special case provides a particular microfoundation as an example. We will use this particular profit function in our numerical exercises.

Special case. Suppose that the cost of producing q units of a good is

$$\Gamma\left(q;z\right)=\frac{1}{z}q+f,$$

where z is the marginal cost and f is the fixed cost of production; and the inverse demand schedule to a firm i is

$$p_i=\frac{\sigma-1}{\sigma}\left[\sum_{j=1}^{\underline{N}_t+\bar{N}_t}\left(q_j\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}\frac{\sigma-1}{\sigma}-1}\left(q_i\right)^{-\frac{1}{\epsilon}},$$

where ϵ is the own price elasticity and σ governs cross-price elasticities, and $\epsilon>\sigma>0$. Moreover, suppose that firms compete in quantities a-la Cournot. In all, profits are given by $\pi\left(\underline{N},\bar{N};z\right)=p\left(\underline{N},\bar{N}\right)q\left(\underline{N},\bar{N};z\right)-\Gamma\left(q\left(\underline{N},\bar{N};z\right);z\right)$, where $p\left(\underline{N},\bar{N}\right)$ and $q\left(\underline{N},\bar{N};z\right)$ are the Cournot equilibrium price and quantity functions.

A firm's exit choice is a stopping time T. Large firms choose this stopping time to maximize the expected present discounted value of profits. Their value function is

$$J\left(\underline{N}_t,\bar{N}_t;\bar{z}\right)=\mathbb{E}_t\left[\max_T\int_t^{\min\{T,T_d\}}e^{-r(s-t)}\pi\left(\underline{N}_s,\bar{N}_s;\bar{z}\right)ds\right],\qquad(2.1)$$

[7] That said, our analysis abstracts for endogenous states at the firm level, and corresponding distribution of these individual states, which would imply dynamic pricing decisions.

where the expectation is taken over the industry state $\{\underline{N}_s, \bar{N}_s\}$, and $r > 0$ is the discount rate. Similarly, small firms choose T to maximize the expected present discounted value of profits. Their value function is

$$J\left(\underline{N}_t, \bar{N}_t; \underline{z}\right) = \mathbb{E}_t\left[\max_T \int_t^{\min\{T,S,T_d\}} e^{-r(s-t)}\pi\left(\underline{N}_s, \bar{N}_s; \underline{z}\right) ds + \mathbf{1}_{S<T} e^{-r(S-t)} J\left(\underline{N}_S, \bar{N}_S; \bar{z}\right)\right], \tag{2.2}$$

where the expectation is now also taken over the arrival time S at which the small firm becomes large (rate λ).

2.2 Households

The infinitely lived representative household has indirect utility function $U\left(\underline{N}, \bar{N}\right)$ in an industry state $\{\underline{N}, \bar{N}\}$. Their present discounted utility is

$$V\left(\underline{N}_t, \bar{N}_t\right) = \mathbb{E}_t\left[\int_t^{\infty} e^{-r(s-t)} U\left(\underline{N}_s, \bar{N}_s\right) ds\right], \tag{2.3}$$

where the expectation is taken over the industry state $\{\underline{N}_s, \bar{N}_s\}$, and $r > 0$ is the discount rate.

Again, for our theoretical results, we do not require a particular microfoundation for the indirect utility function. Below we provide a special case that we will use in our numerical examples.

Special case. The household has preferences

$$U = Q_t + X_t$$

over quantity Q_t of the good produced by the industry of interest and an outside good X_t.[8] The quantity Q_t is given by the CES aggregator across firm varieties i

$$Q_t = \left[\sum_{i=1}^{\underline{N}_t + \bar{N}_t} \left(q_{it}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}\frac{\sigma-1}{\sigma}}$$

[8] The outside good can be interpreted as consumption of goods produced by the rest of the economy, or as leisure.

with $\epsilon > \sigma > 0$. Households maximize flow utility subject to the budget constraint

$$\sum_{i=1}^{N_t+\bar{N}_t} p_{it}q_{it} + X_t = M + \Pi_t$$

given prices $\{p_{it}\}$, the price (normalized to 1) and endowment M of the outside good, and firm profits Π_t.

3 Equilibrium Industry Life-Cycle

We now characterize the equilibrium life-cycle of an industry. Section 3.1 provides a recursive characterization, starting from the long-run industry equilibrium (Section 3.1.1) and then moving backwards to characterize the full life-cycle (Sections 3.1.2 and 3.1.3). Lastly, Section 3.2 shows how differences in scale between firms affects the life-cycle of the industry.

3.1 Recursive Characterization

We now solve recursively for firms' values, and exit and entry policies in equilibrium. We will focus on equilibria where it is never optimal for large firms to exit. It is easy to accomodate cases where large firms exit. We refine the equilibrium to abstract from these cases, which add another source of inefficiency.

3.1.1 Long-run Equilibrium

Suppose that the industry has reached its long-run state $(0, \bar{N}_\infty)$ where there are only $\bar{N}_\infty$ large firms remaining. This state is absorbing and is always reached.[9] In what follows, we will refer to an industry in such long-run state as a *concentrated* industry.

Free exit implies that the laissez-faire equilibrium number of large firms $\bar{N}_\infty^{\mathrm{LF}}$ must satisfy

$$J\left(0, \bar{N}_\infty^{\mathrm{LF}}; \bar{z}\right) = \frac{\pi\left(0, \bar{N}_\infty^{\mathrm{LF}}; \bar{z}\right)}{r} \geq 0, \tag{3.1}$$

[9] The reason is that all small firms either exit or eventually become large at rate λ, and that we refine the equilibrium so that large firms never exit.

as otherwise at least one large firm would choose to exit. Moreover, free entry of small firms implies that $\bar{N}_\infty^{\text{LF}}$ must satisfy

$$J\left(1,\bar{N}_\infty^{\text{LF}};\underline{z}\right)=\frac{\pi\left(1,\bar{N}_\infty^{\text{LF}};\underline{z}\right)+\lambda\times J\left(0,\bar{N}_\infty^{\text{LF}}+1;\bar{z}\right)}{r+\lambda}<0 \tag{3.2}$$

$$J\left(1,\bar{N}_\infty^{\text{LF}}-1;\underline{z}\right)=\frac{\pi\left(1,\bar{N}_\infty^{\text{LF}}-1;\underline{z}\right)+\lambda\times J\left(0,\bar{N}_\infty^{\text{LF}};\bar{z}\right)}{r+\lambda}\geq 0. \tag{3.3}$$

The firm's values in (3.2) and (3.3) correspond to a small firm that is contemplating entering the industry when there are no other small firms. They reflect both the flow of profits $\pi(\cdot;\underline{z})$ while small, as well as the chance that the firm increases its scale, becoming a large firm with value $J(\cdot;\bar{z})$ at rate λ.

The equilibrium $\bar{N}_\infty^{\text{LF}}$ must satisfy the two conditions because, otherwise, an additional small firm would enter in the long-run if $J(1,\bar{N}_\infty;\underline{z})$ was positive, or the concentrated industry state could not be reached in equilibrium if $J(1,\bar{N}_\infty-1;\underline{z})$ was negative, as no small firm would choose to enter just before the industry concentrates.

Lastly, conditions (3.2) and (3.3) uniquely determine the equilibrium $\bar{N}_\infty^{\text{LF}}$ under Assumption 1, since it guarantees that $J(1,\bar{N};\underline{z})$ is strictly decreasing in $\bar{N}$ and that (3.1) is implied by (3.3). Furthermore, conditions (iii) and (iv) in (1) imply $1\leq\bar{N}_\infty^{\text{LF}}<\infty$. In all, the above characterization results in the following lemma.

Lemma 1 (Long-run equilibrium)**.** *The equilibrium number of large firms $1\leq\bar{N}_\infty^{LF}<\infty$ in a concentrated industry state $\left(0,\bar{N}_\infty^{LF}\right)$ is uniquely determined by (3.2) and (3.3).*

3.1.2 Equilibrium Life-Cycle

We now turn to industry states prior to long-run concentration, i.e., states $(\underline{N},\bar{N})$ with $\bar{N}<\bar{N}_\infty^{\text{LF}}$. For small firms, there is a strategic consideration: a firm could find it optimal to stay in the industry if some other firms would exit first.[10] We model a possible "war of attrition" (Fudenberg and Tirole, 1986; Takahashi, 2015) between firms as a mixed-strategy Poisson game. Formally, we let small firms choose an exit rate η.[11]

[10] There is no strategic consideration for large firms sincewe focus on equilibria where they never find it optimal to exit (our equilibrium refinement).

[11] This coordination issue is not necessary for our results on constrained inefficiency or optimal policy (Section 4).

The value of a small firm in state $(\underline{N}, \bar{N})$ is described by the Hamilton-Jacobi-Bellman (HJB) equation

$$
\begin{aligned}
rJ\left(\underline{N}, \bar{N}; \underline{z}\right) = & \pi\left(\underline{N}, \bar{N}; \underline{z}\right) + \lambda \times \left(J\left(\underline{N} - 1, \bar{N} + 1; \bar{z}\right) - J\left(\underline{N}, \bar{N}; \underline{z}\right)\right) \\
& + \lambda \times \left(\underline{N} - 1\right) \times \left(J\left(\underline{N} - 1, \bar{N} + 1; \underline{z}\right) - J\left(\underline{N}, \bar{N}; \underline{z}\right)\right) \\
& + \eta \times \left(0 - J\left(\underline{N}, \bar{N}; \underline{z}\right)\right) \\
& + \eta \times \left(\underline{N} - 1\right) \times \left(J\left(\underline{N} - 1, \bar{N}; \underline{z}\right) - J\left(\underline{N}, \bar{N}; \underline{z}\right)\right). \qquad (3.4)
\end{aligned}
$$

The first line shows the flow profits and the change in value when the firm innovates and increases its scale — rate λ. The second line shows the change in value when some other firm becomes large before the firm does — rate $\lambda \times (\underline{N} - 1)$. The third line shows the change in value when the firm exits — rate η. The last line shows the change in value when some other firm exits before the firm does — rate $\eta \times (\underline{N} - 1)$.

Consider the maximum number of small firms $\underline{N}^{\mathrm{LF}}(\bar{N})$ that an industry with $\bar{N}$ large firms can sustain in a laissez-faire equilibrium. That is, the maximum number beyond which small firms would choose to exit. This maximum $\underline{N}^{\mathrm{LF}}(\bar{N})$ must satisfy

$$
J\left(\underline{N}^{\mathrm{LF}}(\bar{N}), \bar{N}; \underline{z}\right) \leq 0 < J\left(\underline{N}^{\mathrm{LF}}(\bar{N}) - 1, \bar{N}; \underline{z}\right). \qquad (3.5)
$$

Suppose that the industry is in state $(\underline{N}, \bar{N})$ where there are more small firms than is sustainable $\underline{N} \geq \underline{N}^{\mathrm{LF}}(\bar{N})$. A mixed-strategy Poisson equilibrium requires that firms are indifferent between exiting or not. This implies the following exit policies. First, there are $\underline{N} - \underline{N}^{\mathrm{LF}}(\bar{N})$ small firms which exit immediately (their exit rate is $\eta = +\infty$) and obtain a zero value. Second, the remaining $\underline{N}^{\mathrm{LF}}(\bar{N})$ stay in the industry and exit at rate $\eta^{\mathrm{LF}}(\bar{N})$. This exit rate $\eta^{\mathrm{LF}}(\bar{N})$ ensures that stayers have zero value as well[12]

$$
J\left(\underline{N}^{\mathrm{LF}}(\bar{N}), \bar{N}; \underline{z}\right) = 0. \qquad (3.6)
$$

Alternatively, suppose that the industry is in state $(\underline{N}, \bar{N})$ where there are fewer small firms than is sustainable $\underline{N} < \underline{N}^{\mathrm{LF}}(\bar{N})$. Then, free entry implies that $\underline{N}^{\mathrm{LF}}(\bar{N}) -$

[12] The firms that exit immediately and those that stay are indifferent between choosing one option or the other. As such, while their identities will not pinned down in equilibrium, the numbers choosing each option are.

$\underline{N}$ small firms enter the industry immediately.

Finally, note that condition (3.5) uniquely determines $\underline{N}^{\text{LF}}(\bar{N})$ under Assumption 1, as it guarantees that $J(\underline{N}, \bar{N}; \underline{z})$ is strictly decreasing in $\bar{N}$. In all, this characterization results in the following lemma.

Lemma 2 (Equilibrium life-cycle)**.** *In an industry with $\bar{N}$ large firms, the equilibrium number of small firms $\underline{N}^{LF}(\bar{N})$ and the mixed-strategy Poisson exit rate $\eta^{LF}(\bar{N})$ are uniquely determined by conditions (3.5) and (3.6). The equilibrium features $\underline{N}^{LF}(\bar{N}) - \underline{N}$ firms entering immediately when there are few small firms in the industry $\underline{N} \leq \underline{N}^{LF}(\bar{N})$. Otherwise, a number $\underline{N} - \underline{N}^{LF}(\bar{N})$ of small firms exit immediately and the remaining ones exit at rate $\eta^{LF}(\bar{N})$.*

For completeness, the equilibrium value of a large firm is described by the HJB

$$\begin{aligned} rJ(\underline{N}, \bar{N}; \bar{z}) = &\pi(\underline{N}, \bar{N}; \bar{z}) + \lambda \times \underline{N} \times \left(J\left(\underline{N}^{\text{LF}}(\bar{N}+1), \bar{N}+1; \bar{z}\right) - J(\underline{N}, \bar{N}; \bar{z}) \right) \\ &+ \eta^{\text{LF}}(\bar{N}) \times \underline{N} \times \left(J\left(\underline{N}^{\text{LF}}(\bar{N}), \bar{N}; \bar{z}\right) - J(\underline{N}, \bar{N}; \bar{z}) \right), \end{aligned} \tag{3.7}$$

and the household's present discounted value utility in equation (2.3) is described recursively by the HJB

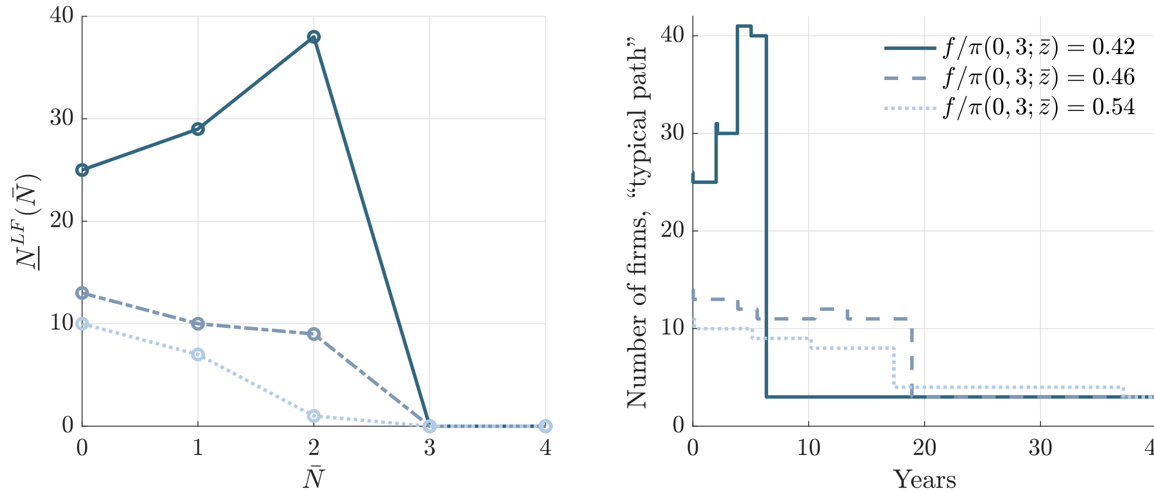
$$\begin{aligned} rV(\underline{N}, \bar{N}) = &U(\underline{N}, \bar{N}) + \lambda \times \underline{N} \times \left(V\left(\underline{N}^{\text{LF}}(\bar{N}+1), \bar{N}+1\right) - V(\underline{N}, \bar{N}) \right) \\ &+ \eta^{\text{LF}}(\bar{N}) \times \underline{N} \times \left(V\left(\underline{N}^{\text{LF}}(\bar{N}), \bar{N}\right) - V(\underline{N}, \bar{N}) \right). \end{aligned} \tag{3.8}$$

3.1.3 Entry, Shakeout, and Concentration: A Numerical Illustration

We now illustrate the results from the previous sections with a numerical example. We use the special case of our model described in Section 2. The parameters of the demand function are $\sigma = 2$ and $\epsilon = 8$, the arrival rate of innovations is $\lambda = 0.02$, the discount rate is $r = 0.03$, and the marginal cost parameters are $\bar{z}/\underline{z} = 1.3$ with $\bar{z}$ normalized to 1.

The left panel of Figure 1 illustrates the equilibrium number of small firms $\underline{N}^{\text{LF}}(\bar{N})$ as function of the number of large firms in the industry (Lemma 2). We consider three parameterizations that differ in their fixed cost of production f. For all parameterizations, the long-run number of firms $\bar{N}_\infty$ is 3 in the concentrated industry; we thus express the fixed cost relative to long-run profits $\pi(0, 3; \bar{z})$. When

Figure 1: Equilibrium industry life-cycle



the fixed cost is relatively large (dashed and dotted lines), the number of small firms $\underline{N}^{\text{LF}}(\bar{N})$ monotonically declines with the number of large firms $\bar{N}$ in the industry. As the fixed costs becomes smaller (solid line), $\underline{N}^{\text{LF}}(\bar{N})$ increases with $\bar{N}$ in states associated with a nascent industry with less than 2 small firms.

The right panel shows a "typical" realization of the equilibrium total number of firms $N_t = \underline{N}^{\text{LF}}(\bar{N}_t) + \bar{N}_t$ over an industry's life-cycle, where the time spend in each state is given by the expected time until the arrival of the next Poisson shock.[13] The equilibrium life-cycle consists of three phases. First, a nascent industry phase where small firms enter and begin producing. Second, a *shakeout* phase where firms find it optimal to exit. During this phase, a firm exits because other firms innovate and produce at scale before they do — rate $\lambda \times \left(\underline{N}^{\text{LF}}(\bar{N}) - 1\right)$ — or because the firm loses the war of attrition — rate $\eta^{\text{LF}}(\bar{N})$. In particular, shakeouts of multiple firms are triggered at times where a firm innovates. Finally, there is a concentrated industry phase where all remaining small firms have exited and only $\bar{N}_\infty$ large firms remain.

A notable feature of the industry life-cycle shown in Figure 1 is the possibility of a gradual growth in the number of firms, with multiple periods of entry and exit, culminating in a sudden shakeout. These dynamics are consistent with the

[13] For states $\bar{N} < \bar{N}_\infty^{\text{LF}}$, the length is given by $\left[\left(\eta^{\text{LF}}(\bar{N}) + \lambda\right)\underline{N}^{\text{LF}}(\bar{N})\right]^{-1}$ when the war of attrition is taken place and by $\left[\lambda\left(\underline{N}^{\text{LF}}(\bar{N}) - 1\right)\right]^{-1}$ after that.

life-cycle of new industries in the 20th century, e.g., car manufacturing (Klepper and Simons, 2005), and, more recently, digital industries — spurred by computers, the Internet, and big data and AI. However, such dynamics are in sharp contrast to those implied by the perfectly competitive version of our model (Jovanovic and MacDonald, 1994). In a competitive industry, the shakeout is unique and *bang-bang*: no firms exit when quantities are low (price is high) and then a mass of firms exit once quantities are beyond a threshold (price is low). The life-cycle is always monotonic. [14]

The non-monotonic life-cycle in an oligopolistic industry is explained by the fact that the incentives to enter are particularly strong right before the industry concentrates. For late entrants, the expected gain of scaling up right before the shakeout occurs soon after entry. Instead, early entrants need to wait longer for the shakeout to occur.

To see more clearly the forces driving non-monotonic life-cycles, consider an example where the industry concentrates with two firms in the long-run $\bar{N}_{\infty}^{LF} = 2$. The change in the value of delaying entry, from $\bar{N} = 0 \rightarrow \bar{N} = 1$, when competing with a common number of small firms $\underline{N}$, is given by

$$\begin{aligned} J\left(\underline{N}, 1; \underline{z}\right) - J\left(\underline{N}, 0; \underline{z}\right) &= \pi\left(\underline{N}, 1; \underline{z}\right) - \pi\left(\underline{N}, 0; \underline{z}\right) \\ &\quad + \frac{\lambda}{r + \delta + \lambda \underline{N}} \left[\pi\left(\underline{N}, 2; \bar{z}\right) - \pi\left(\underline{N}, 1; \bar{z}\right)\right] \\ &\quad + \underbrace{\frac{\lambda}{r + \delta + \lambda \underline{N}} \left[\pi\left(0, 2; \bar{z}\right) - \pi\left(\underline{N}, 2; \bar{z}\right)\right]}_{\text{benefits of entering closer to the shakeout} > 0}. \end{aligned} \tag{3.9}$$

The cost of delaying entry are straightforward. Late entrants face the competition of additional large firms, both when small and after scaling up. These are given by the first two terms in the right hand side of (3.9), $\pi\left(\underline{N}, 1; \underline{z}\right) - \pi\left(\underline{N}, 0; \underline{z}\right) < 0$ and $\pi\left(\underline{N}, 2; \bar{z}\right) - \pi\left(\underline{N}, 1; \bar{z}\right)$, respectively. The benefit from delaying entry is that the expected "business stealing" gains following the shakeout occur closer to the time of entry. These gains are given by the third term in the right hand side of (3.9), i.e., $\pi\left(0, 2; \bar{z}\right) - \pi\left(\underline{N}, 2; \bar{z}\right) > 0$. When the business stealing gains following the

[14] In Jovanovic and MacDonald (1994), non-monotonic entry dynamics are obtained by assuming that, later in the life-cycle of an industry, an exogenous industry-wide innovation opportunity arrives. This new opportunity spurs a protracted surge in entry.

shakeout are relatively large, there will be a burst of entry before the concentration of the industry, as illustrated by the solid lines in Figure 1.

The gains from delaying entry are larger the higher the rents $\pi\left(0, N_{\infty}^{\mathrm{LF}}; \bar{z}\right)/f$ are in the long-run, holding fixed the long-run state of the industry $\bar{N}_{\infty}^{\mathrm{LF}}$. The higher the long-run rents, the more entry will take place before concentration $\underline{N}^{\mathrm{LF}}\left(\bar{N}\right)$, and the higher will be the "business stealing" gains following the shakeout, $\bar{\pi}\left(0,2\right) - \bar{\pi}\left(\underline{N}^{\mathrm{LF}}\left(N_{\infty}^{\mathrm{LF}} - 1\right), 2\right)$. This is again illustrated by Figure 1. The case with lower fixed cost (solid lines) is associated with an increasing entry profile before concentration.

3.2 Scale Differences and the Equilibrium Life-Cycle

We now show that the industry life-cycle is distinctly shaped by differences in scale between large and small firms. We focus on such differences for two reasons. First, scale economies are a key driver of US concentration and markups (Covarrubias et al. 2020; Kwon et al. 2023; Autor et al. 2020), and are especially important in digital industries with near-zero marginal costs (Goldfarb and Tucker, 2019) or that use data intensively (Agrawal et al., 2019). Second, differences in scale crucially determine optimal policy over the life-cycle of the industry (Section 4).

Proposition 1 compares the life-cycle of an industry in two limit cases: an economy where there are arbitrarily large differences in scale *vis-a-vis* an economy where there are no differences. We assume throughout that profits are negative when the industry has a maximum of $\underline{N}^{\max}$ potential entrants at any point in time.[15]

Proposition 1 (Scale and equilibrium life-cycle)**.** *Let $\underline{N}^{max}$ be potential entrants in the industry. If the marginal cost of small firms relative to large firms is arbitrarily large*

[15] One interpretation is that these are the potential innovators or entrepreneurs that can create new products, or potential managers of the firm. Another aggregate constraint is that the resources used in production need to be feasible given the total resources available. This aggregate resource constraint also imposes an upper bound on the number of small firms. For example, in the special case of our model in Section 2, the constraint is $\underline{N} < \frac{M - \bar{N} \times \left(f + \frac{1}{\bar{z}} q(0, \bar{N}; \bar{z})\right)}{f}$ when $\underline{z} \to 0$. For our numerical exercises, we will assume that M is relatively large so that the binding constraint on the number of small firms is the number of potential entrants $\underline{N}^{\max}$ and not the upper bound imposed by the aggregate resource constraint.

($\bar{z}/\underline{z} \to \infty$ with $\underline{z} \to 0$), then

$$\underline{N}^{LF}(\bar{N}) = \begin{cases} \underline{N}^{max} & \text{for } \bar{N} < \bar{N}_{\infty}^{LF} \\ 0 & \text{otherwise.} \end{cases}$$

On the contrary, if there are no differences in marginal costs between firms ($\bar{z}/\underline{z} = 1$), then

$$\underline{N}^{LF}(\bar{N}) = \max\left\{\bar{N}_{\infty}^{LF} - \bar{N}, 0\right\}.$$

Proof. See Appendix A. □

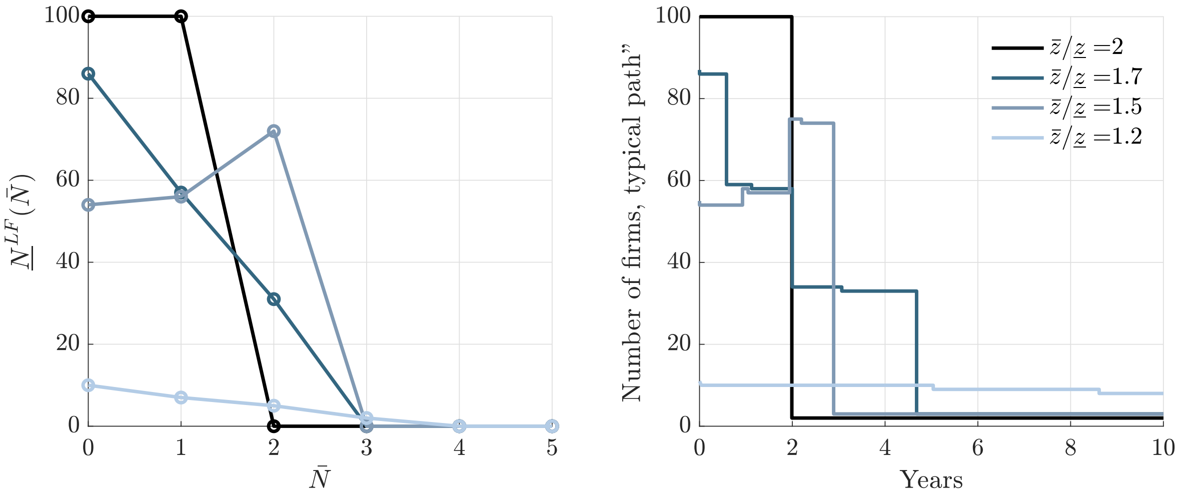
The industry features a sharp life-cycle when there are large differences in scale across firms ($\bar{z}/\underline{z} \to \infty$ with $\underline{z} \to 0$). The maximum number of small firms $\underline{N}^{\max}$ are present before the industry concentrates (i.e., when there are $\bar{N} < \bar{N}_{\infty}^{\mathrm{LF}}$ large firms). The shakeout occurs all at once, with all small firms exiting immediately as soon as $\bar{N} = \bar{N}_{\infty}^{\mathrm{LF}}$. Thus, this limit case describes winner-take-most industries where firms effectively *compete for the market*, similar to models in the patent race literature (Reinganum, 1989). On the contrary, there is no life-cycle when there are no differences in scale ($\bar{z}/\underline{z} = 1$). The total number of firms is equal to the long-run equilibrium $\bar{N}_{\infty}^{\mathrm{LF}}$ at all times. In this case, firms only *compete in the market*, as in models emphasizing horizontal differentiation (Benassy, 1996; Atkeson and Burstein, 2008).

Using the special case of our model in Section 2, Figure 2 illustrates the proposition and shows how the industry life-cycle varies for intermediate values of $\underline{z}$ (fixing $\bar{z} = 1$) away from the limit cases considered there. A difference in marginal costs of $\bar{z}/\underline{z} = 2$ (black solid line) already results in the sharp life-cycle dynamics associated with the limit case $\bar{z}/\underline{z} \to \infty$ with $\underline{z} \to 0$. As this difference shrinks, the life-cycle dynamics become more gradual; with the number of firms in a nascent industry and in the long-run being more similar.[16]

To provide intuition, it is useful to return to the value of a small firm described by the HJB equation (2.2). In the limit case where $\bar{z}/\underline{z} \to \infty$ with $\underline{z} \to 0$, the HJB

[16] For intermediate values of the marginal cost of small firms, $\bar{z}/\underline{z}$, industries tend to feature non-monotonic life-cycles. On one extreme, when the marginal cost is very high, the output of small firm is inconsequential for the outcome in the product market and the gain from delaying entry in (3.9) disappears, i.e., $\lim_{\underline{z}\to 0} \pi(0, 2; \bar{z}) - \pi(\underline{N}, 2; \bar{z}) = 0$. On the other extreme, the life-cycle is flat, as there is no scale advantage.

Figure 2: Scale and equilibrium life-cycle



equation becomes

$$\lim_{\underline{z}\to 0} J\left(\underline{N},\bar{N};\underline{z}\right) = \frac{1}{r+\lambda\underline{N}}\left[-f+\lambda\times J\left(\underline{N}^{\mathrm{LF}}\left(\bar{N}+1\right),\bar{N}+1;\bar{z}\right)\right.$$
$$\left.+\lambda\left(\underline{N}-1\right)\times\lim_{\underline{z}\to 0} J\left(\underline{N}^{\mathrm{LF}}\left(\bar{N}+1\right),\bar{N}+1;\underline{z}\right)\right]. \quad (3.10)$$

The industry state $\{\underline{N},\bar{N}\}$ does not affect a small firm's value through the flow profits. A small firm does not produce in this limit case and has negative profits due to fixed costs (Assumption 1). Thus, entry and exit decisions are purely driven by the option value of becoming a large firm. This value is either always positive, when there are few large firms $\bar{N}$ in the industry, or becomes negative thereafter. As such, the numerator in the expression above is either positive (for small $\bar{N}$) or negative: small firms either always find it profitable to enter or exit at all once.

On the contrary, if there are no differences in marginal costs between firms ($\bar{z}/\underline{z}=1$), then the model becomes static (as the Poisson innovations are irrelevant) and

$$J\left(\underline{N},\bar{N};1\right) = \frac{\pi\left(\underline{N},\bar{N};1\right)}{r} = \frac{\pi\left(\underline{N}+\bar{N},0;1\right)}{r}.$$

Firms either enter or not, and $\underline{N}^{\mathrm{LF}}\left(\bar{N}\right)+\bar{N}=\bar{N}_{\infty}^{\mathrm{LF}}$ at all times.

4 Optimal Policy

We now characterize the second best industry life-cycle and optimal policy. Section 4.1 states the Ramsey problem of a government that is constrained in its instruments, and characterizes the second best. Section 4.2 shows that the laissez-faire is generically constrained inefficient and discusses the sources of such inefficiency. Section 4.3 characterizes how differences in scale between firms affect the optimal policy over the life-cycle of an industry, both when the government can commit or not.

4.1 Constrained Ramsey problem

We consider a government that cannot directly tax or subsidize production. That is, it cannot directly address quantity distortions due to imperfect competition. Such interventions would implement a first best, but are unlikely to be feasible in practice due to political or informational constraints.[17] Instead, we analyze the constrained Ramsey problem of a government that only controls the number of small firms in an industry.[18] The second best can be implemented with a subsidy to the fixed cost of production of small firms. Depending on how the subisidy varies over the life-cycle of an industry, the optimal policy can resemble ex-ante or ex-post interventions to promote competition in practice. Subsidies late in the life-cycle mimic ex-post policies that intervene once an industry has sufficiently concentrated, whereas subsidies early in the life-cycle resemble ex-ante policies that intervene in nascent industries before they concentrate.

We assume for now that the government can commit to implementing the optimal subsidies to small firms. Section 4.3.1 discusses issues of time-consistency of this policy. Section 5.1 relaxes the government's problem allowing policies that affect the long-run profits of large firms too — for instance, a weaker antitrust enforcement of collusion; another type of ex-post intervention.

The following lemma states the government's HJB equation and characterizes

[17] For example, Edmond et al. (2023) show that implementing a first best in a model of monopolistic competition requires subsidizing large firms, and that the optimal policy requires knowledge of firm-level parameters.

[18] The goverment is subject to the same technological constraint than firms in equilibrium. That is, new firms have a high marginal cost (they are small), so the government cannot directly control the number of large, low marginal cost firms.

the second best industry life-cycle.

Lemma 3 (Second best life-cycle)**.** *Given indirect flow utility $U(\underline{N}, \bar{N})$, the government's HJB equation is*

$$rV(\bar{N}) = U\left(\underline{N}^{SB}(\bar{N}), \bar{N}\right) + \lambda \times \underline{N}^{SB}(\bar{N}) \times (V(\bar{N}+1) - V(\bar{N}))$$

where the optimal number of small firms $\underline{N}^{SB}(\bar{N})$ is such that the government scraps or creates firms until

$$U\left(\underline{N}^{SB}(\bar{N}), \bar{N}\right) - U\left(\underline{N}^{SB}(\bar{N}) - 1, \bar{N}\right) + \lambda \times (V(\bar{N}+1) - V(\bar{N})) \geq 0$$
$$U\left(\underline{N}^{SB}(\bar{N}) + 1, \bar{N}\right) - U\left(\underline{N}^{SB}(\bar{N}), \bar{N}\right) + \lambda \times (V(\bar{N}+1) - V(\bar{N})) < 0.$$

An additional small firm in the industry increases the expected present value of utility, as there is a higher chance that at least one of them innovates (rate λ) and increase their scale. However, the additional firm also affects the static flow utility $U(\underline{N}, \bar{N})$, lowering it when the extra fixed cost of production does not compensate the increase in consumer surplus. The optimal number of small firms $\underline{N}^{\text{SB}}(\bar{N})$ trades off these two forces.

4.2 Constrained Inefficiency

Consider the value of an additional firm for the government

$$\underbrace{U\left(\underline{N}^{\text{SB}}(\bar{N}), \bar{N}\right) - U\left(\underline{N}^{\text{SB}}(\bar{N}) - 1, \bar{N}\right)}_{\text{Static utility gain}} + \underbrace{\lambda \times (V(\bar{N}+1) - V(\bar{N}))}_{\text{Dynamic gain in utility}}$$

and compare it to the value of staying in the industry for a firm in equilibrium

$$\underbrace{\pi\left(\underline{N}^{\text{LF}}(\bar{N}), \bar{N}; \underline{z}\right)}_{\text{Static profits}} + \underbrace{\lambda \times J\left(\underline{N}^{\text{LF}}(\bar{N}+1), \bar{N}+1; \bar{z}\right)}_{\text{Dynamic gain in profits}}$$
$$+ \underbrace{\eta^{\text{LF}}(\bar{N}) \times \left(\underline{N}^{\text{LF}}(\bar{N}) - 1\right) \times J\left(\underline{N}^{\text{LF}}(\bar{N}), \bar{N}; \underline{z}\right)}_{\text{War of attrition}}.$$

There are three differences between the social and private incentives. Each is a source of inefficiency at the laissez-faire. The first source is static and is well known in the literature studying the optimal number of firms (or varieties) under imperfect competition (Dixit and Stiglitz, 1977; Mankiw and Whinston, 1986; Benassy, 1996). The government internalizes the static utility gained from an additional firm. Firms only internalize the profits they gain by staying in the industry, not the consumer surplus they generate. This pushes firms to exit excessively (or enter insufficiently) compared to optimal.

The second source of inefficiency is dynamic. But in a sense similar to the first and relates to forces present in Shumpeterian models of innovation (Aghion and Howitt, 1990). The government internalizes that an additional small firm increases the chances (by λ per unit of time) of at least one firm becoming a large firm, but destroys some surplus from existing firms. The firms, on the other hand, only internalize their own increase in the (expected) present discounted value of profits from becoming a large firm; not the surplus they destroy from other firms. This pushes firms to exit insufficiently (or enter excessively) compared to optimal.

The last source of inefficiency is the war of attrition. The firms do not coordinate their exit decisions in equilibrium, whereas the government does. As a result, they stay in the market with the expectation that other firms will exit before they do. This pushes firms to exit insufficiently compared to optimal.

4.3 Scale Differences and Optimal Policy over the Life-Cycle

We now show how the relative scale between large and small firms affects the optimal policy over the life-cycle of the industry. We begin by characterizing the second best life-cycle in the same two limit cases from Proposition 1, and then turn to optimal policy.

The second best life-cycle has identical dynamics than the equilibrium life-cycle in the limit cases, although the number of firms typically differs (Proposition 3 in the Appendix B). The second best features a sharp industry life-cycle when there are large differences in scale across firms ($\bar{z}/\underline{z} \to \infty$ with $\underline{z} \to 0$). The maximum number of small firms $\underline{N}^{\max}$ are present before the industry concentrates (i.e., for all $\bar{N} < N_{\infty}^{\text{SB}}$) and all small firms exit immediately when $\bar{N} = \bar{N}_{\infty}^{\text{SB}}$; where the long-run number of firms $\bar{N}_{\infty}^{\text{SB}}$ typically differs from the equilibrium $\bar{N}_{\infty}^{\text{LF}}$. The second best features no life-cycle dynamics when there are no differences in scale ($\bar{z}/\underline{z} = 1$). The total number of firms is equal to the long-run $\bar{N}_{\infty}^{\text{SB}}$, which again typically differs from the long-run equilibrium $\bar{N}_{\infty}^{\text{LF}}$.

Next, we turn to characterizing the optimal policy that implements the second best life-cycle. We are not interested on whether subsidizing or taxing small firms is optimal overall. This is ambigous for the reasons explained in Section 4.2, even in the limit cases considered in Propositions 1 and 3. Instead, our goal is to characterize the *timing* of the optimal policy over the life-cycle. When is it optimal to intervene in a nascent industry? When can the government wait until the industry has concentrated? What determines the optimal mix of early and late interventions?

With this in mind, we assume throughout that the long-run industry concentration is excessive at the laissez-faire ($\bar{N}_{\infty}^{\text{LF}} < \bar{N}_{\infty}^{\text{SB}}$). This is case of interest for regulations and policies aimed at promoting competition in practice.

Proposition 2 (Scale and optimal policy)**.** *Let $s(\bar{N})$ be the subsidy to the fixed cost of production of small firms in an industry with $\bar{N}$ large firms. If the marginal cost of small firms relative to large firms is arbitrarily large ($\bar{z}/\underline{z} \to \infty$ with $\underline{z} \to 0$), then the government can implement the second best by intervening only after the industry has concentrated in equilibrium. That is, the subsidy below suffices*

$$s(\bar{N}) = \begin{cases} 0 & \text{if } \bar{N} < \bar{N}_{\infty}^{LF} \\ > 0 & \text{if } \in \left[\bar{N}_{\infty}^{LF}, \bar{N}_{\infty}^{SB} - 1\right]. \end{cases}$$

In contrast, if there are no differences in marginal costs between firms ($\bar{z}/\underline{z} = 1$), then the government finds it optimal to intervene at all times:

$$s(\bar{N}) > 0 \iff \bar{N} < \bar{N}_{\infty}^{SB}.$$

Proof. See Appendix C. □

The first part of the proposition shows that policies that promote competition ex-post are sufficient when there are arbitrarily large differences in scale between small and large firms. A subsidy to small firms after the industry concentrates in the long-run in equilibrium suffices. There is no need for the government to intervene before then. In contrast, when there are arbitrary small differences in scale, the second part of the proposition shows that the government must also subsidize firms in a nascent industry before it becomes concentrated.

Using the special case of our model in Section 2, the left panel of Figure 3 illustrates the proposition and extends it for intermediate values of $\bar{z}/\underline{z}$ away from the limit cases considered there. While there are many subsidies that implement the second best life-cycle, we pick the lowest subsidies $\underline{s}(\bar{N})$ that make firms indifferent between staying or exiting; these minimize the fiscal cost of the intervention.[19] Finally, we consider values of $\bar{z}/\underline{z}$ that result in 3 firms in the long-run at the second-best and only 2 firms at the laissez-faire.

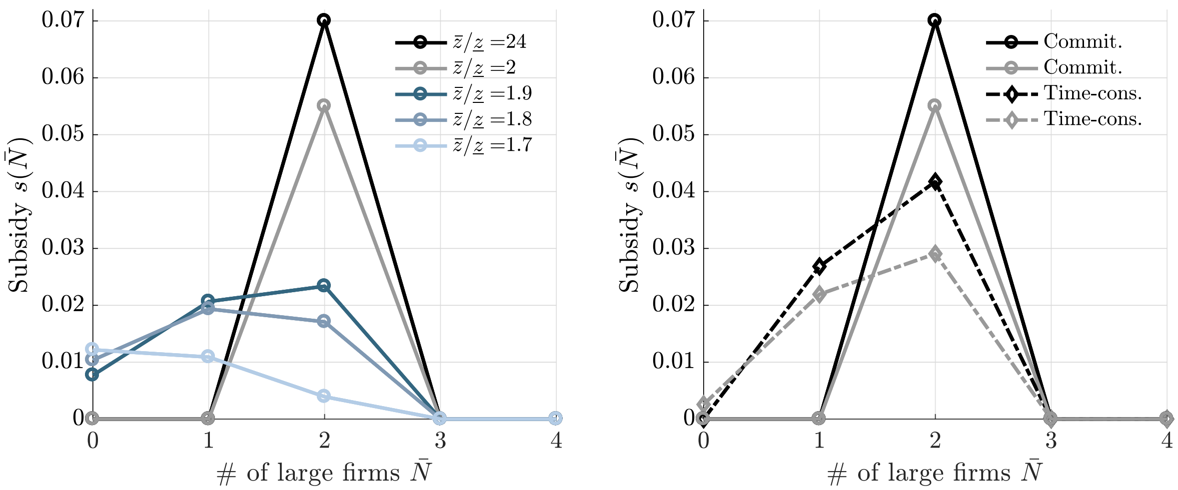
A relative marginal cost $\bar{z}/\underline{z}$ of 24 (black line) or 2 (gray line) already results in the sharp life-cycle at the second-best (and at the laissez-faire) associated with the limit case $\bar{z}/\underline{z} \to \infty$ with $\underline{z} \to 0$. Thus, the government can implement the second best without subsidizing intially ($s(0) = s(1) = 0$) and only subsidizing once the industry has concentrated in equilibrium with 2 firms in the long-run ($s(2) > 0$). As $\bar{z}/\underline{z}$ falls (blue lines), the subsidies begin to flatten out over the life-cycle and even become frontloaded for $\bar{z}/\underline{z} = 1.7$. Figure 6 in Appendix D shows similar patterns for even smaller values of $\bar{z}/\underline{z}$. The second best has more firms in the long-run in these cases (up to 7), and the subsidies are eventually flat when $\bar{z}/\underline{z} = 1$. In all, when scale differences are large, the optimal policy is heavily tilted towards interventions late in the life-cycle after an industry has become concentrated. As $\bar{z}/\underline{z}$ falls and scale differences shrink, early and late subsidies become more similar and eventually become identical. The optimal policy subsidizes firms much more uniformly over the life-cycle.

The first part of proposition follows from the fact that, in the limit when $\bar{z}/\underline{z} \to \infty$ with $\underline{z} \to 0$, the entry and exit decisions of small firms are purely driven by the option value of becoming a large firm later on. Thus, subsidies that affect profits later in the life-cycle of an industry suffice to align private and social incentives to enter or exit the industry earlier in the life-cycle too, implementing the second best.

To make the above intuition concrete, it helps to go over some of the steps of the proof in an example. Suppose that the second best features just one more firm in the long-run compared to the laissez faire, i.e., $\bar{N}_\infty^{\mathrm{SB}} = \bar{N}_\infty^{\mathrm{LF}} + 1$. Adapting equation (3.10), the value of a small firm in the state with $\bar{N} = \bar{N}_\infty^{\mathrm{LF}}$ under the optimal

[19] Section 4.3.1 discusses the question of time-consistency.

Figure 3: Scale and optimal policy



subsidies is

$$\lim_{\underline{z}\to 0} J^{\mathrm{SB}}\left(\underline{N}^{\max}, \bar{N}_\infty^{\mathrm{LF}}; \underline{z}\right) = \frac{s\left(\bar{N}_\infty^{\mathrm{LF}}\right) - f + \lambda \times J^{\mathrm{SB}}\left(0, \bar{N}^{\mathrm{SB}}; \bar{z}\right)}{r + \lambda \underline{N}^{\max}}$$

in the limit case where $\bar{z}/\underline{z} \to \infty$ with $\underline{z} \to 0$. A subsidy $s\left(\bar{N}_\infty^{\mathrm{LF}}\right)$ large enough that small firms find it optimal to enter — i.e., such that $\lim_{\underline{z}\to 0} J^{\mathrm{SB}}\left(\underline{N}^{\max}, \bar{N}_\infty^{\mathrm{LF}}; \underline{z}\right) \geq 0$ — implements the second best $\underline{N}^{\mathrm{SB}}\left(\bar{N}_\infty^{\mathrm{LF}}\right) = \underline{N}^{\max}$, whereas they would have exited $\underline{N}^{\mathrm{LF}}\left(\bar{N}_\infty^{\mathrm{LF}}\right) = 0$ at the laissez faire (Proposition 1).

For earlier states in the life-cycle with $\bar{N} < \bar{N}_\infty^{\mathrm{LF}}$, remember that firms already found it optimal to enter at the laissez-faire and the maximum number of small firms was present (Proposition 1). However, subsidizing small firms in the state $\bar{N} = \bar{N}_\infty^{\mathrm{LF}}$ affects firm values in earlier states $\bar{N} < \bar{N}_\infty^{\mathrm{LF}}$ too, potentially lowering them as more firms are present later in the life-cyle. The optimal subsidy ensures that firm values $\lim_{\underline{z}\to 0} J^{\mathrm{SB}}\left(\underline{N}^{\max}, \bar{N}; \underline{z}\right)$ remain positive in these earlier states. In particular, the subsidy may have to be larger than the lower bound subsidy which is just as large to make firms indifferent $\lim_{\underline{z}\to 0} J^{\mathrm{SB}}\left(\underline{N}^{\max}, \bar{N}_\infty^{\mathrm{LF}}; \underline{z}\right) = 0$, raising the question of time-consistency (Section 4.3.1).

The intuition for the second part of the proposition is more straightforward. The model becomes static when there are no differences in scale ($\bar{z}/\underline{z} = 1$). There is no life-cycle and the number of firms is identical to the long-run at all times, both at the laissez-faire and second best. Thus, the government finds it optimal to subsidize at all times too when the number of firms is less than the second best $\bar{N} < \bar{N}^{\text{SB}}_{\infty}$.

4.3.1 Time-Consistency

In the limit with arbitrary large differences in scale ($\bar{z}/\underline{z} \to +\infty$ with $\underline{z} \to 0$), the proof of Proposition 2 shows that the optimal subsidies after the industry has concentrated in equilibrium — i.e., states with $\bar{N} \in \left[\bar{N}^{\text{LF}}_{\infty}, \bar{N}^{\text{SB}}_{\infty} - 1\right]$ — may be larger than the lower bound $\underline{s}\left(\bar{N}\right)$ at which firms are indifferent between staying or exiting in such states. The reason is that subsidies at the lower bound may not be large enough to ensure that firms enter in industry states prior to concentration too — i.e., in states with $\bar{N} < \bar{N}^{\text{LF}}_{\infty}$.

However, the constrained Ramsey policy is not time consistent *if* the subsidies need to be larger than the lower bound $\underline{s}\left(\bar{N}\right)$. The government would find it optimal to promise to subsidize above the lower bound after the industry had concentrated in equilibrium, but would later "renege" on these promised subsidies. Instead, the government would subsidize at the lower bound $\underline{s}\left(\bar{N}\right)$ because it implements the second best life-cycle for all $\bar{N} \in \left[\bar{N}^{\text{LF}}_{\infty}, \bar{N}^{\text{SB}}_{\infty} - 1\right]$ at a lower fiscal cost.[20]

Suppose that the required subsidies are indeed larger than $\underline{s}\left(\bar{N}\right)$ after the industry has concentrated and the government cannot commit to such large subsidies. To implement the second best life-cycle, a time-consistent policy now needs to subsidize small firms both *before* and after the industry has concentrated in equilibrium. The subsidies are set at the lower bound after the industry has concentrated, and they are positive for states prior to concentration with $\bar{N} < \bar{N}^{\text{LF}}_{\infty}$. The subsidies in states prior to industry concentration need to be large enough to ensure that the second best (maximum) number of firms $\underline{N}^{\max}$ enter the industry in these states. The following corollary to Proposition 3 summarizes this discussion.

Corollary 1 (Time-consistency)**.** *Suppose that the subsidies $\underline{s}\left(\bar{N}\right)$ that make firms indifferent between staying or exiting in states with $\bar{N} \in \left[\bar{N}^{LF}_{\infty}, \bar{N}^{SB}_{\infty} - 1\right]$ are an optimal policy in Proposition 3 in the limit case where $\bar{z}/\underline{z} \to +\infty$ with $\underline{z} \to 0$. Then, such optimal policy*

[20] This logic also implies that subsidies to *large* firms after an industry concentrates are never time-consistent. This is one reason why we rule out such subsidies to large firms in our constrained Ramsey problem (Section 4.1).

is time-consistent. Otherwise, the time-consistent policy is such that

$$s\left(\bar{N}\right)=\begin{cases}>0 & \textit{if } \bar{N}<\bar{N}_{\infty}^{LF}\\ \underline{s}\left(\bar{N}\right) & \textit{if } \in\left[\bar{N}_{\infty}^{LF},\bar{N}_{\infty}^{SB}-1\right].\end{cases}$$

The right panel of Figure 3 illustrates the corollary for the two cases where the relative marginal cost $\bar{z}/\underline{z}$ is 24 and 2. For such large differences in scale, the optimal subsidies under commitment (black and gray solid lines) are already those associated with the limit case $\bar{z}/\underline{z}\rightarrow\infty$ with $\underline{z}\rightarrow 0$. Given our parameterization, the subsidies turn out to be time-inconsistent. The time-consistent policy (dashed lines) requires that the government subsidizes not only once the industry concentrates at the laissez-faire ($s(2)>0$) but also earlier in the life-cycle ($s(1)>s(0)>0$). That said, the time-consistent policy retains qualitatively similar features to the policy under commitment in this numerical example; both policies are tilted towards subsidizing later in the life-cycle.

5 Extensions

We next discuss three extensions of our baseline model. First, we consider the case in which the large firms collude and choose quantities to maximize their joint surplus. This extension highlights the role of antitrust policies, an important set of policies that intervene after the industry has sufficiently concentrated (ex-post). Second, we endogenize the arrival rate of innovations λ by letting small firms invest by paying a convex cost. Finally, we allow for innovation spillovers where the arrival rate of an innovation λ depends on the number of large firms.

While each extension enriches the analysis of the life-cycle of concentrated industries, and affects optimal policy, the main lesson from Proposition 3 remains valid. When there are arbitrarily large differences in scale between firms, the government can implement the second best by subsidizing small firms only after the industry has concentrated. Interventions early on in the life-cycle are not necessary. As scale differences fall, intervening both before and after an industry concentrates is optimal.

5.1 Collusion and Antitrust Policies

In the benchmark numerical examples, we consider cases in which the profit function $\pi(\underline{N}, \bar{N}; z)$ is the outcome of a static Cournot Nash equilibrium. Instead, we now explore examples where the profit function is the outcome of Nash equilibria in which large firms form a cartel and collude. In particular, we assume that large firms jointly choose the quantities they supply and products they operate to maximize their joint profits, taking as given the quantities supplied by small firms.[21] Each large firm receives an equal share of the joint profits.

The resulting profit function $\pi^{\text{Cartel}}(\underline{N}, \bar{N}; z)$ still satisfies Assumption 1. Therefore, Propositions 1 and 2 are valid as well. But how are the equilibrium life-cycle and the optimal policy affected by collusion among large firms? Naturally, for each value of the aggregate state $(\underline{N}, \bar{N})$, the profits of large firms $\pi^{\text{Cartel}}(\underline{N}, \bar{N}; \bar{z})$ are higher compared to our benchmark, fueling the incentives of small firms to enter. This results in more large firms in the long-run equilibrium than in our benchmark $N_{\infty}^{\text{Cartel}} \geq N_{\infty}^{\text{LF}}$, and more entry through the life-cycle. For the parametrization we consider in Figure 1, the cartel chooses to operate fewer products, although there are more firms that innovate. Some of the products that can be produced with a low marginal cost $\bar{z}$ are not supplied. As a consequence, the constrained planner chooses to limit entry. The constrained planner only values the innovation of the products that would be active in the long-run.

An effective antitrust policy is the additional policy implication in this case. This policy consist in breaking up the cartel and implementing the static Nash equilibrium that was the feature of our benchmark analysis. Importantly, when there are arbitrarily large differences in scale, it is enough to implement the antitrust policy after the industry has concentrated, which reinforces the conclusions of the benchmark analysis.

5.2 Endogenous Rate of Innovation

In this extension, we allow small firms to choose the arrival rate of an innovation λ. In particular, we assume that firms incur a cost $c(\lambda)$ to innovate at the rate λ, with $c(0) = 0$, $c'(\lambda) > 0$ and $c''(\lambda) > 0$. This version of the model thus

[21] To save on fixed costs, the $\bar{N}$ large firms may choose to operate a number of products that is strictly smaller than $\bar{N}$.

features an intensive margin of innovation in addition to the extensive margin in the benchmark model. The optimal policy results in Section 4.3 are largely robust to allowing for innovation along the intensive margin, although there are some interesting interactions between the extensive and intensive margins over the life-cycle of an industry.

The optimal innovation rate of small firms $\lambda\left(\underline{N}, \bar{N}\right)$ in state $\left(\underline{N}, \bar{N}\right)$ satisfies the first order condition

$$J\left(\underline{N}^{LF}\left(\bar{N}+1\right), \bar{N}+1; \bar{z}\right) - J\left(\underline{N}, \bar{N}; \underline{z}\right) = c'\left(\lambda\left(\underline{N}, \bar{N}\right)\right) \tag{5.1}$$

where the value $J\left(\underline{N}, \bar{N}; \underline{z}\right)$ is now calculated net of innovation costs.[22] As in Proposition 1, the equilibrium features a sharp life-cycle when there are arbitrarily large differences in scale between firms. The maximum number of small firms $\underline{N}^{\max}$ are present before the industry concentrates (i.e., when there are $\bar{N} < \bar{N}_{\infty}^{\text{LF}}$ large firms). The shakeout occurrs all at once, with all small firms exiting immediately as soon as $\bar{N} = \bar{N}_{\infty}^{\text{LF}}$. The intensive margin of innovation exhibits a more gradual life-cycle. The endogenous rate of innovation $\lambda\left(\underline{N}, \bar{N}\right)$ is largest early in the life-cycle, as the marginal gains from a successful innovation are the largest. As the industry is closer to the concentration in the long-run, the marginal gains from a succesful innovation diminish, leading to a lower individual optimal arrival rate of an innovation.

For intermediate cases, the equilibrium arrival rate of innovations could be increasing or decreasing over the life-cycle. For the marginal entrant, equation (5.1) simplifies to

$$J\left(\underline{N}^{\text{LF}}\left(\bar{N}+1\right), \bar{N}+1; \bar{z}\right) = c'\left(\lambda\left(\underline{N}^{\text{LF}}\left(\bar{N}\right), \bar{N}\right)\right).$$

[22] In particular, the value of a small firm in state $\left(\underline{N}, \bar{N}\right)$ is described by the Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned} rJ\left(\underline{N}, \bar{N}; \underline{z}\right) = \max_{\lambda} \{ & \pi\left(\underline{N}, \bar{N}; \underline{z}\right) - c\left(\lambda\right) + \lambda \times \left(J\left(\underline{N}-1, \bar{N}+1; \bar{z}\right) - J\left(\underline{N}, \bar{N}; \underline{z}\right)\right) \\ & + \lambda_{-1} \times \left(\underline{N}-1\right) \times \left(J\left(\underline{N}-1, \bar{N}+1; \underline{z}\right) - J\left(\underline{N}, \bar{N}; \underline{z}\right)\right) \\ & + \eta \times \left(0 - J\left(\underline{N}, \bar{N}; \underline{z}\right)\right) \\ & + \eta \times \left(\underline{N}-1\right) \times \left(J\left(\underline{N}-1, \bar{N}; \underline{z}\right) - J\left(\underline{N}, \bar{N}; \underline{z}\right)\right) \}. \end{aligned} \tag{5.2}$$

where λ_{-1} is the innovation rate of the $\underline{N}-1$ competing small firms.

Thus, as long as the equilibrium value of a large firms decreases with the number of competing large firms along the industry equilibrium life-cycle, the intensive margin of innovation is decreasing for the marginal entrant. In contrast, for non-monotonic life-cycles in which entry is maximal before the concentration of the industry, the value of a large firm can increase over the life-cycle.. In these cases, the extensive and intensive margin of innovation can both increase over the life-cycle. Figures 7 and 8 in Appendix D illustrate the life-cycle of these two margins for alternative values of the scale parameter and the elasticity of the cost function $c(\lambda)$. Importantly, the implications discussed earlier for the life-cycle of entry and optimal policy are largely robust to the inclusion of an intensive margin of innovation.

Lastly, we find that having an intensive margin of innovation can result in a distribution of long-run industry states, in contrast with our benchmark model where the long-run industry state was unique. In particular, for the case of a relative concentrated industry with few entrants, we can construct life-cycle equilibria that feature a (unique) distribution over two long-run industry states: (i) a highly-concentrated long-run equilibrium with a single small firm which chooses never to innovate, and (ii) a long-run equilibrium with two large firms. In this example, there are initialy two firms entering the industry, investing in innovation, $\lambda^{LF}(2,0) > 0$, an exiting at a positive rate, $\eta^{LF}(0) > 0$. If exit occurs before the arrival of a succesful innovation, the industry has a (small) monopolist who chooses not to innovate, $\lambda^{LF}(1,0) = 0$. On the contrary, if an innovation occurs first, the industry eventually converges to a duopoly with two large firms.[23]

This example illustrates that there are subtle interactions between entry, exit and the intensive margin. Current entry is complementary with the intensive margin of innovation, as $\lambda^{LF}(2,0) > \lambda^{LF}(1,0) = 0$, but future entry lowers the incentives to innovate. In particular, the monopolist would choose to innovate if there would be no entry in the second state, i.e., $\underline{N}(1) = 0$.

[23] The example assumes that parameters of the demand function are $\sigma = 2$ and $\varepsilon = 8$, that the cost function is quadratic $c(\lambda) = 33 \cdot \lambda^2$, the discount rate is $r = 0.03$, the marginal cost parameters are $\bar{z}/\underline{z} = 1.3$ with $\bar{z}$ normalized to 1, fixed cost $f = 0.09$, and that there is an additional fixed cost of 0.04 for unproductive firms in state $(2,0)$ to guarantee that exactly two firms enters when $\bar{N} = 0$, i.e., $\underline{N}(0) = 2$.

5.3 Innovation Spillovers

In the benchmark analysis we abstracted from knowledge spillovers, an important theme in the discussion of the development and diffusion of new technologies and the growth of new industries. A simple way to incorporate these considerations is to assume that the arrival rate of an innovation is a function of the number of firms that have already innovated $\lambda\left(\bar{N}\right)$. This captures the idea that it is easier innovate after others have "walked the path."

Propositions 1 and 2 still go through. For each value of $\bar{N}$, the entry decision is still *bang-bang* and the intuition in equation (3.10) applies. When there are arbitrarily large differences in scale between firms, the government can implement the second best by subsidizing small firms only after the industry has concentrated. However, the time-consistent subsidies that implement the second best can be different in this case. For instance, if the arrival rate of innovations is particularly low initially, $\lambda\left(0\right) \ll \lambda\left(1\right)$, then the time-consistent subsidies would be larger in the initial period than in subsequent ones.

6 Evidence from Digital and AI Industries

Having shown our main results, the question of whether early or late interventions to promote competition in an industry are optimal can now be understood as follows. Are firm entry and exit choices mostly driven by the option value of taking over the market after the industry shakeout and concentration, or is competition in the market in nascent industries an important consideration too? From a measurement perspective, our results show that differences in scale between large and small firms are a key moment for empirically diagnosing how close an industry is to each case.

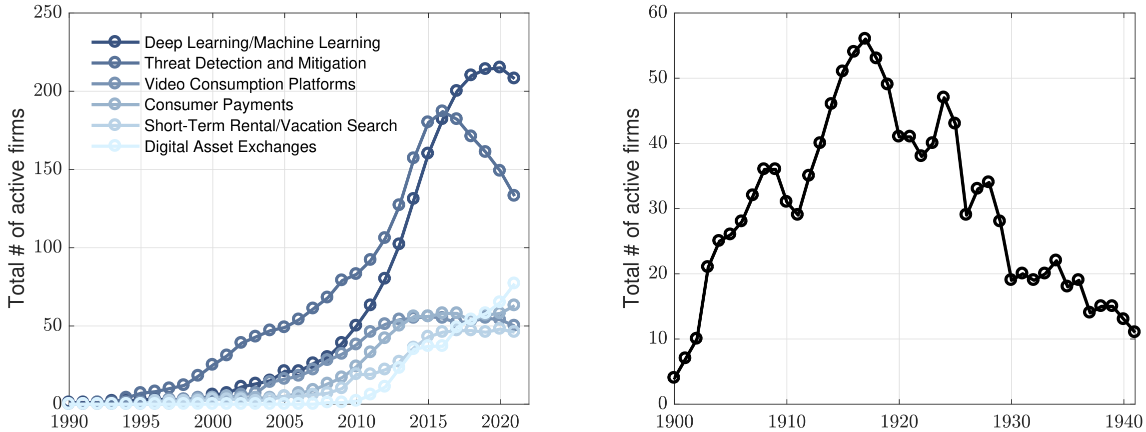
Next, we use this measurement insight to analyze digital and AI industries in the U.S. We focus on these industries for two reasons. First, digital and AI industries have been the target of much scrutiny by policymakers, and new regulations are already being passed (such as the *Digital Markets Act* in Europe). Second, it is still early enough that our results can inform policymakers in practice: many digital and AI industries are nascent and are far from fully concentrating. Our goal here is not to provide a full quantitative analysis, though. Our model is arguably too stylized and we lack basic information on many important parameters for a credible quantification exercise, such as demand elasticities or fixed costs of production. Instead, our goal is to provide a sense of magnitudes and variation in the key moment highlighted by our theory. How important are the differences in scale between large and small firms? Is there much variation across industries in this moment or should all be regulated more or less similarly?

We use a novel dataset from Venture Scanner that collects information on the universe of firms that have ever been funded by venture capital — the primary funding source in digital and AI industries. Venture Scanner uses a propietary algorithm to categorize firms according to the technologies they produce or services they provide. This as an important feature of this dataset: it allows for defining product markets for a technology or service. There are 17 broad technology or service categories; such as "Artificial Intelligence," "Financial," "Real Estate," or "Security." Each is divided into narrower subcategories; like "Deep and Machine Learning," "Consumer Payments," "Short Term Rentals and Vacation Search," or "Threat Detection and Compliance." We define an industry as a technology or service subcategory, which results in a total of 155 industries. Finally, the dataset includes information on a firm's starting year, whether the firm is still active in a given year, and which employment interval it belongs to: 1 to 10 employees, 11 to 50, 51 to 100, 101 to 250, and 251 to 10,000. We measure a firm's size as the mean of their employment interval.

To benchmark digital and AI industries, we compare them to the automobile industry in the U.S — a traditional industry which has been studied at length and has already experienced a full life-cycle (Klepper, 2002). The data comes from digitizing *The 100 Year Almanac* which collects information on automobile manufacturing firms. Important for our purposes, the *Almanac* collects the number of units sold for each firm and year. We count a firm as being active in any given year when it sold at least one unit. We measure the size of the firm as the number of units sold.

The left panel of Figure 4 shows the total number of active firms in selected industries in the Venture Scanner dataset since 1990. We document that these industries are still early on in their life-cycle with the total number of firms peaking in recent years. The same is true for almost all other digital and AI industries in our dataset: they are nascent industries which are far from concentrating. The

Figure 4: The life-cycle across industries



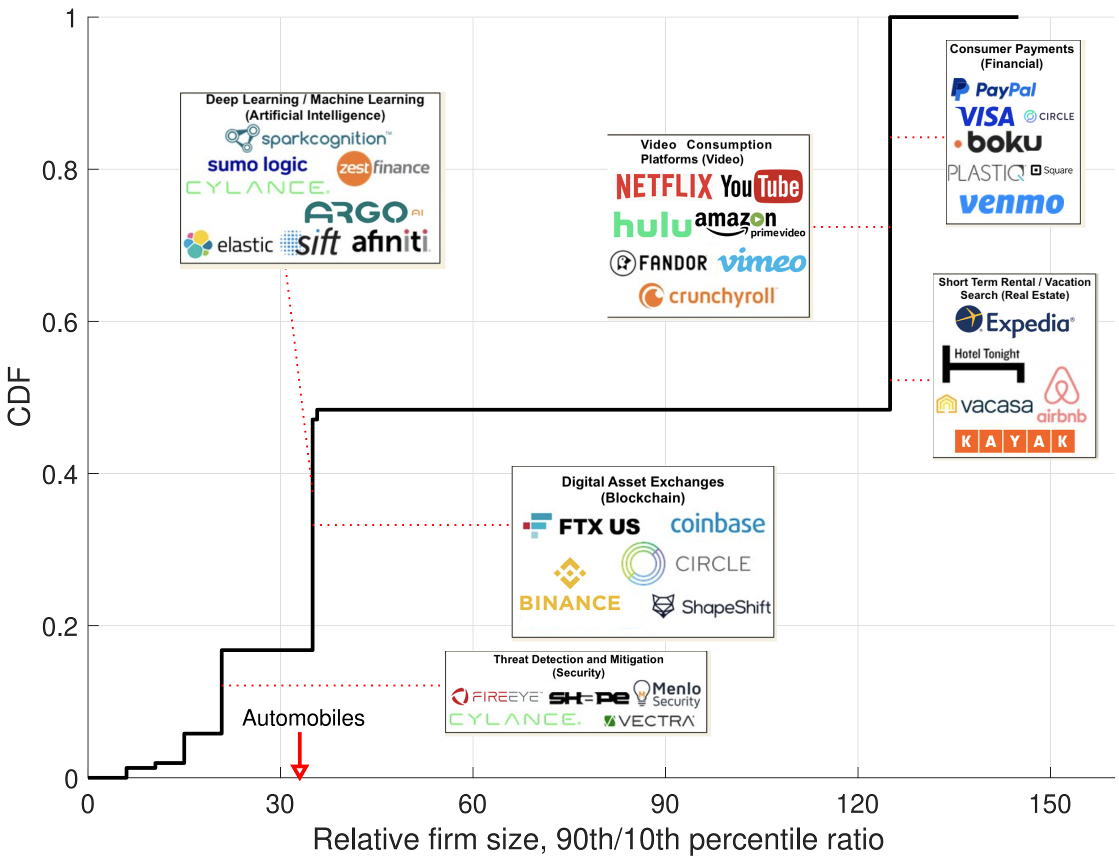
right panel of the figure shows the total number of active firms in the automobile industry from 1900 until 1941 (when the U.S. entered WWII). We confirm the findings in Klepper and Simons (2005) in our data: the industry experienced about two decades of intense entry, followed by a shakeout and later concentration.[24]

Next, we turn to documenting the differences in scale between "large" and "small" firms in each industry. We associate large firms with those above the 90th percentile in an industry's size distribution, and small firms to those below the 10th percentile. The relative scale between large and small firms — our moment of interest — is thus the 90th to 10th percentile ratio. We measure this ratio for each industry in the Venture Scanner dataset, and then compute its empirical cumulative distribution (CDF) across industries.

Figure 5 shows our findings. Some industries — like "Video Consumption Platforms" or "Short Term Rental / Vacation Search" — have particularly large differences in scale. Large firms in these industries are 120 times larger than small firms. In other industries, scale differences are much smaller, such as in the "Threat Detection and Mitigation" industry. Overall, the median digital and AI industry (e.g., "Deep Learning and Machine Learning") has a relative scale of about 40, but the distribution is very skewed: more than 80 percent of industries have a rela-

[24] Klepper and Simons (2005) put together information from the several data sources; the main one being *Thomas' Register of American Manufacturers*. Despite the similarities in the life-cycle, the peak number of firms in their data is larger than in ours (about 275 versus 55). This means that *The 100 Year Almanac* is missing many relatively small firms. However, *Thomas' Register* does not have information on firms' output, which is crucial for our purposes.

Figure 5: Relative scale across industries



tive scale larger than 35. As a comparison, we also compute the relative scale in the automobile industry at the peak of the life-cycle in Figure 4.[25] We find that the relative scale was 33 at the time. Thus, most digital and AI industries have larger differences in scale than the automobile industry did at a similar point in its life-cycle.

Through the lens of our model, these findings imply that most digital and AI industries have less of a need for interventions that promote competition in the present (early) stage in their life-cycle than the automobile industry did at the same stage. Instead, relative to the automobile industry, governments can wait longer and intervene later in the life-cycle after these industries have sufficiently

[25] Size is measured in terms of employment in the Venture Scanner industries, whereas it is measured in terms of output in the automobile industry. Under constant returns to scale in production, output and employment are proportional to each other, thus making the two relative scale measures comparable.

concentrated.

References

ABRAHAMSON, Z. (2014): "Essential data," *Yale LJ*, 124, 867.

AGHION, P., U. AKCIGIT, AND P. HOWITT (2014): "What do we learn from Schumpeterian growth theory?" in *Handbook of economic growth*, Elsevier, vol. 2, 515–563.

AGHION, P., N. BLOOM, R. BLUNDELL, R. GRIFFITH, AND P. HOWITT (2005): "Competition and innovation: An inverted-U relationship," *The quarterly journal of economics*, 120, 701–728.

AGHION, P. AND P. HOWITT (1990): "A model of growth through creative destruction," .

AGRAWAL, A., J. GANS, AND A. GOLDFARB (2019): *The economics of artificial intelligence: an agenda*, University of Chicago Press.

ATKESON, A. AND A. BURSTEIN (2008): "Pricing-to-market, trade costs, and international relative prices," *American Economic Review*, 98, 1998–2031.

——— (2019): "Aggregate implications of innovation policy," *Journal of Political Economy*, 127, 2625–2683.

AUTOR, D., D. DORN, L. F. KATZ, C. PATTERSON, AND J. VAN REENEN (2020): "The fall of the labor share and the rise of superstar firms," *The Quarterly Journal of Economics*, 135, 645–709.

BENASSY, J.-P. (1996): "Taste for variety and optimum production patterns in monopolistic competition," *Economics Letters*, 52, 41–47.

BLOOM, N., J. VAN REENEN, AND H. WILLIAMS (2019): "A toolkit of policies to promote innovation," *Journal of economic perspectives*, 33, 163–184.

BOAR, C. AND V. MIDRIGAN (2019): "Markups and inequality," Tech. rep., National Bureau of Economic Research.

BUERA, F. J., H. HOPENHAYN, Y. SHIN, AND N. TRACHTER (2021): "Big push in distorted economies," Tech. rep., National Bureau of Economic Research.

CAVENAILE, L., M. A. CELIK, AND X. TIAN (2023): "Are Markups Too High? Competition, Strategic Innovation, and Industry Dynamics," *Working Paper*.

COVARRUBIAS, M., G. GUTIÉRREZ, AND T. PHILIPPON (2020): "From Good to Bad Concentration? US Industries over the past 30 years," *NBER Macroeconomics Annual*, 34, 1–46.

DAY, G. S., A. J. FEIN, AND G. RUPPERSBERGER (2003): "Shakeouts in digital markets: Lessons from B2B exchanges," *California Management Review*, 45, 131–150.

DE LOECKER, J., J. EECKHOUT, AND G. UNGER (2020): "The rise of market power and the macroeconomic implications," *The Quarterly Journal of Economics*, 135, 561–644.

DIXIT, A. K. AND J. E. STIGLITZ (1977): "Monopolistic competition and optimum product diversity," *The American economic review*, 67, 297–308.

EDMOND, C., V. MIDRIGAN, AND D. Y. XU (2023): "How costly are markups?" *Journal of Political Economy*, 131, 000–000.

EVANS, D. S. (2008): "Competition and regulatory policy for multi-sided platforms with applications to the web economy," *Concurrences*, 57–62.

FUDENBERG, D. AND J. TIROLE (1986): "A theory of exit in duopoly," *Econometrica: Journal of the Econometric Society*, 943–960.

GLAESER, E. L. AND A. SHLEIFER (2003): "The rise of the regulatory state," *Journal of economic literature*, 41, 401–425.

GOLDFARB, A. AND C. TUCKER (2012): "Privacy and innovation," *Innovation policy and the economy*, 12, 65–90.

——— (2019): "Digital economics," *Journal of economic literature*, 57, 3–43.

HÉMOUS, D. (2016): "The dynamic impact of unilateral environmental policies," *Journal of International Economics*, 103, 80–95.

IGAMI, M. AND K. UETAKE (2020): "Mergers, innovation, and entry-exit dynamics: Consolidation of the hard disk drive industry, 1996–2016," *The Review of Economic Studies*, 87, 2672–2702.

IMPULLITTI, G. AND O. LICANDRO (2018): "Trade, firm selection and innovation: The competition channel," *The Economic Journal*, 128, 189–229.

ITSKHOKI, O. AND B. MOLL (2019): "Optimal Development Policies With Financial Frictions," *Econometrica*, 87, 139–173.

JOVANOVIC, B. AND G. M. MACDONALD (1994): "The life cycle of a competitive industry," *Journal of Political Economy*, 102, 322–347.

KLEPPER, S. (2002): "The capabilities of new firms and the evolution of the US automobile industry," *Industrial and corporate change*, 11, 645–666.

KLEPPER, S. AND K. L. SIMONS (2005): "Industry shakeouts and technological change," *International Journal of Industrial Organization*, 23, 23–43.

KWON, S. Y., Y. MA, AND K. ZIMMERMANN (2023): "100 years of rising corporate concentration," *University of Chicago, Becker Friedman Institute for Economics Working Paper*.

LIU, E., A. MIAN, AND A. SUFI (2022): "Low interest rates, market power, and productivity growth," *Econometrica*, 90, 193–221.

MANKIW, N. G. AND M. D. WHINSTON (1986): "Free entry and social inefficiency," *The RAND Journal of Economics*, 48–58.

MELITZ, M. J. (2005): "When and how should infant industries be protected?" *Journal of International Economics*, 66, 177–196.

MERMELSTEIN, B., V. NOCKE, M. A. SATTERTHWAITE, AND M. D. WHINSTON (2020): "Internal versus external growth in industries with scale economies: A computational model of optimal merger policy," *Journal of Political Economy*, 128, 301–341.

MURPHY, K. M., A. SHLEIFER, AND R. W. VISHNY (1989): "Industrialization and the big push," *Journal of political economy*, 97, 1003–1026.

OLMSTEAD-RUMSEY, J. (2024): "Market Concentration and the Productivity Slowdown," *Mimeo*.

PETERS, M. (2020): "Heterogeneous markups, growth, and endogenous misallocation," *Econometrica*, 88, 2037–2073.

PHILIPPON, T. (2019): *The great reversal: How America gave up on free markets*, Harvard University Press.

POSNER, R. A. (2010): "Regulation (agencies) versus litigation (courts): an analytical framework," in *Regulation vs. litigation: Perspectives from economics and law*, University of Chicago Press, 11–26.

RAFFENSPERGER, C. AND J. TICKNER (1999): *Protecting Public Health and the Environment: Implementing The Precautionary Principle*, Island Press.

REINGANUM, J. F. (1989): "The timing of innovation: Research, development, and diffusion," *Handbook of industrial organization*, 1, 849–908.

SHAVELL, S. (2007): "Liability for accidents," *Handbook of law and economics*, 1, 139–182.

SPULBER, D. F. AND C. S. YOO (2007): "Mandating access to telecom and the internet: the hidden side of Trinko," *Colum. L. Rev.*, 107, 1822.

TAKAHASHI, Y. (2015): "Estimating a war of attrition: The case of the us movie theater industry," *American Economic Review*, 105, 2204–2241.

TANG, G. (2011): "Intel and the x86 architecture: A legal perspective," *Harvard Journal of Law and Technology*.

Online Appendix for:

"The Life-Cycle of Concentrated Industries"

This online appendix contains the proofs and derivations of all theoretical results for the article.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded letters (e.g., "A." or "B.") refer to the main article.

A Proof of Proposition 1

Economy where $\underline{z}/\underline{z} \to +\infty$ with $\underline{z} \to 0$. The profits of smalls firms are $\pi\left(\underline{N}, \bar{N}; \underline{z}\right) = -f$ when $\underline{z}/\underline{z} \to +\infty$ with $\underline{z} \to 0$. We first show that there are no Poisson mixed strategies in this case. Small firms either always find it optimal to enter or exit. The proof is by contradiction.

Suppose that a mixed strategy is optimal. This requires that, when other firms are exiting at rate $\eta^{\mathrm{LF}}\left(\bar{N}\right)$ in industry state $\pi\left(\underline{N}, \bar{N}; \underline{z}\right)$, a small firms is indifferent between exiting or staying $J\left(\underline{N}, \bar{N}; \underline{z}\right) = 0$. This is equivalent to

$$-f + \lambda \times J\left(\underline{N}^{\mathrm{LF}}\left(\bar{N}+1\right), \bar{N}+1; \bar{z}\right) + \eta^{\mathrm{LF}}\left(\bar{N}\right) \times \left(\underline{N}-1\right) \times J\left(\underline{N}^{\mathrm{LF}}\left(\bar{N}\right), \bar{N}; \underline{z}\right) = 0, \tag{A.1}$$

where

$$J\left(\underline{N}^{\mathrm{LF}}\left(\bar{N}\right), \bar{N}; \underline{z}\right) = -f + \lambda \times J\left(\underline{N}^{\mathrm{LF}}\left(\bar{N}+1\right), \bar{N}+1; \bar{z}\right) > 0$$

and we have already used the fact that $J\left(\underline{N}^{\mathrm{LF}}\left(\bar{N}+1\right), \bar{N}+1; \underline{z}\right) = 0$ in equilibrium. Combining the two conditions abovem, we can re-write (A.1) as

$$\left(1 + \eta^{\mathrm{LF}}\left(\bar{N}\right) \times \left(\underline{N}-1\right)\right) \times \left(-f + \lambda \times J\left(\underline{N}^{\mathrm{LF}}\left(\bar{N}+1\right), \bar{N}+1; \bar{z}\right)\right) = 0.$$

But this condition cannot hold for any mixed-strategy $\eta^{\mathrm{LF}}\left(\bar{N}\right) \geq 0$ generically — i.e., except in a knife-edge case $-f + \lambda \times J\left(\underline{N}^{\mathrm{LF}}\left(\bar{N}+1\right), \bar{N}+1; \bar{z}\right) = 0$. So we have arrived at the desired contradiction.

We next show that all potential entrants $\underline{N}^{\max}$ find it optimal to enter the industry and never exit before the industry concentrates. We begin from the industry state with $\bar{N} = \bar{N}_{\infty}^{\mathrm{LF}} - 1$. The value of a small firm is given by the HJB equation

$$J\left(\underline{N}, \bar{N}_{\infty}^{\mathrm{LF}} - 1; \underline{z}\right) = \frac{-f + \lambda \times J\left(0, \bar{N}_{\infty}^{\mathrm{LF}}; \bar{z}\right)}{r + \lambda \underline{N}}.$$

Condition (3.3) determining the long-run $\bar{N}_{\infty}^{\mathrm{LF}}$ requires that $J\left(1, \bar{N}_{\infty}^{\mathrm{LF}} - 1, \underline{z}\right) \geq 0$, which is true if and only if $-f + \lambda \times J\left(0, \bar{N}_{\infty}^{\mathrm{LF}}; \bar{z}\right) \geq 0$. This immediately implies that $J\left(\underline{N}, \bar{N}_{\infty}^{\mathrm{LF}} - 1; \underline{z}\right) \geq 0$ for any $\underline{N} \geq 1$. Therefore, firms always have incentives to enter and $\underline{N}^{\mathrm{LF}}\left(\bar{N}_{\infty}^{\mathrm{LF}} - 1\right) = \underline{N}^{\max}$ in equilibrium.

Next, consider the industry state just prior with $\bar{N} = \bar{N}_{\infty}^{\mathrm{LF}} - 2$. The HJB equation is

$$J\left(\underline{N},\bar{N}_{\infty}^{\mathrm{LF}}-2;\underline{z}\right)=\frac{-f+\lambda\times J\left(\underline{N}^{\max},\bar{N}_{\infty}^{\mathrm{LF}}-1;\bar{z}\right)+\lambda\times\left(\underline{N}-1\right)\times J\left(\underline{N}^{\max},\bar{N}_{\infty}^{\mathrm{LF}}-1;\underline{z}\right)}{r+\lambda\underline{N}}.$$

Again, we have that $J\left(\underline{N},\bar{N}_{\infty}^{\mathrm{LF}}-2;\underline{z}\right)\geq 0$ for any $\underline{N}$ and so $\underline{N}^{\mathrm{LF}}\left(\bar{N}_{\infty}^{\mathrm{LF}}-2\right)=\underline{N}^{\max}$ in equilibrium. The reason is that $J\left(\underline{N}^{\max},\bar{N}_{\infty}^{\mathrm{LF}}-1;\underline{z}\right)\geq 0$ and $-f+\lambda\times J\left(\underline{N}^{\max},\bar{N}_{\infty}^{\mathrm{LF}}-1;\bar{z}\right)$ is

$$\begin{aligned}
&=-f+\lambda\times\frac{\pi\left(\underline{N}^{\max},\bar{N}_{\infty}^{\mathrm{LF}}-1;\bar{z}\right)+\lambda\underline{N}^{\max}\times J\left(0,\bar{N}_{\infty}^{\mathrm{LF}};\bar{z}\right)}{r+\lambda\underline{N}^{\max}}\\
&=-f+\lambda\times J\left(0,\bar{N}_{\infty}^{\mathrm{LF}};\bar{z}\right)+\lambda\times\frac{\pi\left(\underline{N}^{\max},\bar{N}_{\infty}^{\mathrm{LF}}-1;\bar{z}\right)-r\times J\left(0,\bar{N}_{\infty}^{\mathrm{LF}};\bar{z}\right)}{r+\lambda\underline{N}^{\max}}\\
&=\underbrace{-f+\lambda\times J\left(0,\bar{N}_{\infty}^{\mathrm{LF}};\bar{z}\right)}_{\geq 0}+\lambda\times\underbrace{\frac{\pi\left(\underline{N}^{\max},\bar{N}_{\infty}^{\mathrm{LF}}-1;\bar{z}\right)-\pi\left(0,\bar{N}_{\infty}^{\mathrm{LF}};\bar{z}\right)}{r+\lambda\underline{N}^{\max}}}_{\geq 0}\geq 0,
\end{aligned}$$

where the last inequality follows from the fact (i) that we have shown above that $-f+\lambda\times J\left(0,\bar{N}_{\infty}^{\mathrm{LF}};\bar{z}\right)\geq 0$, and (ii) that $\pi\left(\underline{N}^{\max},\bar{N}_{\infty}^{\mathrm{LF}}-1;\bar{z}\right)-\pi\left(0,\bar{N}_{\infty}^{\mathrm{LF}};\bar{z}\right)\geq 0$ due to Assumption 1 and that the profits of large firms are independent of the number of small firms in this limit case when $\underline{z}/\underline{z}\to+\infty$ with $\underline{z}\to 0$ (since small firms do not produce).

The recursion above can be repeated n times for each $\bar{N}=\bar{N}_{\infty}^{\mathrm{LF}}-n$ until reaching the initial industry state $\bar{N}=0$. This shows that $\underline{N}^{\mathrm{LF}}\left(\bar{N}\right)=\underline{N}^{\max}$ in equilibrium for any $\underline{N}<\bar{N}_{\infty}^{\mathrm{LF}}$, which completes the first part of the proof.

Economy where $\bar{z}/\underline{z}=1$. The profits are all firms are identical and only the total number of firms matters in this case. That is, $\pi\left(\underline{N},\bar{N};\bar{z}\right)=\pi\left(\underline{N},\bar{N};\underline{z}\right)=\pi\left(0,\underline{N}+\bar{N};\underline{z}\right)$. Without loss of generality, we can assume that $\lambda=0$ in this case too, since all firms are identical. This immediately implies that the industry is always at its long-run equilibrium. The total number of firms is $\underline{N}+\bar{N}=\bar{N}_{\infty}^{\mathrm{LF}}$ determined by the free exit and entry conditions (3.1) and (3.2).

B Scale and Second Best Industry Life-Cycle

The following proposition characterizes the second best life-cycle in the same two limit cases from Proposition 1.

Proposition 3 (Scale and second best life-cycle)**.** *Suppose that the marginal cost of small firms relative to large firms is arbitrarily large ($\bar{z}/\underline{z} \to +\infty$ with $\underline{z} \to 0$).*

1. The second best number of large firms in the long-run $\bar{N}_\infty^{SB}$ typically differs from the laissez-faire $\bar{N}_\infty^{LF}$.

2. As in the laissez-faire, the second best industry life-cycle also features the maximum number of small firms $\underline{N}^{\max}$ present before the industry concentrates (i.e., for all $\bar{N} < N_\infty^{SB}$), and all small firms exiting immediately when $\bar{N} = \bar{N}_\infty^{SB}$.

On the contrary, suppose that there are no differences in marginal cost between firms ($\bar{z}/\underline{z} = 1$). The industry features no life-cycle at the second best. The total number of firms is equal to the long-run $\bar{N}_\infty^{SB}$ that maximizes flow household utility $U(\cdot)$ at all times, which typically differs from the long-run equilibrium $\bar{N}_\infty^{LF}$.

The proof is similar to that of Proposition 1 in Appendix A.

Economy where $\underline{z}/\underline{z} \to +\infty$ with $\underline{z} \to 0$. The flow utility satisfies $U(\underline{N}, \bar{N}) = U(0, \bar{N}) - f\underline{N}$ in this case. The long-run number of firms satisfies

$$V\left(1, \bar{N}_\infty^{\text{SB}} - 1\right) - V\left(0, \bar{N}_\infty^{\text{SB}} - 1\right) \geq 0 \iff -f + \lambda \times \frac{U\left(0, \bar{N}_\infty^{\text{SB}}\right) - U\left(0, \bar{N}_\infty^{\text{SB}} - 1\right)}{r} \geq 0 \tag{B.1}$$

Next, consider the industry state with $\bar{N} = \bar{N}_\infty^{\text{SB}} - 1$. Condition (B.1) immediately implies that

$$V\left(\underline{N}, \bar{N}_\infty^{\text{SB}} - 1\right) - V\left(\underline{N} - 1, \bar{N}_\infty^{\text{SB}} - 1\right) \propto -f + \lambda \times \frac{U\left(0, \bar{N}_\infty^{\text{SB}}\right) - U\left(0, \bar{N}_\infty^{\text{SB}} - 1\right)}{r} \geq 0$$

for any $\underline{N}$. Therefore, the second best number of small firms is the maximum $\underline{N}^{\text{SB}}\left(\bar{N}_\infty^{\text{SB}} - 1\right) = \underline{N}^{\max}$.

Consider now the industry state just prior with $\bar{N} = \bar{N}_\infty^{\text{SB}} - 2$. We have that

$V\left(\underline{N}, \bar{N}_{\infty}^{\mathrm{SB}}-2\right) - V\left(\underline{N}-1, \bar{N}_{\infty}^{\mathrm{SB}}-2\right)$ is

$$
\begin{aligned}
&\propto \lambda \times \left(V\left(\underline{N}^{\max}, \bar{N}_{\infty}^{\mathrm{SB}}-1\right) - \frac{U\left(0, \bar{N}_{\infty}^{\mathrm{SB}}-2\right)}{r} \right) - f \\
&= \underbrace{-f + \lambda \times \frac{U\left(0, \bar{N}_{\infty}^{\mathrm{SB}}-1\right) - U\left(0, \bar{N}_{\infty}^{\mathrm{SB}}-2\right)}{r}}_{\geq 0} \\
&\quad + \frac{\lambda \underline{N}^{\max}}{r + \lambda \underline{N}^{\max}} \times \left(\underbrace{-f + \lambda \times \frac{U\left(0, \bar{N}_{\infty}^{\mathrm{SB}}\right) - U\left(0, \bar{N}_{\infty}^{\mathrm{SB}}-1\right)}{r}}_{\geq 0} \right) \geq 0,
\end{aligned}
$$

where both terms are positive by Condition (B.1). Therefore, the second best is again $\underline{N}^{\mathrm{SB}}\left(\bar{N}_{\infty}^{\mathrm{SB}}-2\right) = \underline{N}^{\max}$.

The recursion above can be repeated n times for each $\bar{N} = \bar{N}_{\infty}^{\mathrm{SB}} - n$ until reaching the initial industry state $\bar{N} = 0$. This shows that $\underline{N}^{\mathrm{SB}}\left(\bar{N}\right) = \underline{N}^{\max}$ in equilibrium for any $\underline{N} < \bar{N}_{\infty}^{\mathrm{SB}}$, which completes the first part of the proof.

Economy where $\bar{z}/\underline{z} = 1$. The flow utility from any firm is identical and only the total number of firms matters in this case. That is, $U\left(\underline{N}, \bar{N}\right) = U\left(0, \underline{N} + \bar{N}\right)$. Without loss of generality, we can assume that $\lambda = 0$ in this case too, since all firms are identical. This immediately implies that the total number of firms is constant at the second best, and given by $\bar{N}_{\infty}^{\mathrm{SB}}$.

C Proof of Proposition 2

Economy where $\underline{z}/\underline{z} \to +\infty$ with $\underline{z} \to 0$. Suppose that the industry has reached the long-run equilibrium with $\bar{N}_{\infty}^{\mathrm{LF}}$ large firms. Without government intervention, all small firms would exit then. But now the government can subsidize f for small firms to implement the second best. Trivially, the optimal subsidies are zero for all $\bar{N} \geq \bar{N}_{\infty}^{\mathrm{SB}}$, since there are no small firms at the second best (Proposition 3).

We want to show that subsidies $s^{\mathrm{SB}}\left(\bar{N}\right)$ that lower the fixed cost of small firms to $f - s^{\mathrm{SB}}\left(\bar{N}\right)$ for all $\bar{N} \in \left[\bar{N}_{\infty}^{\mathrm{LF}}, \bar{N}_{\infty}^{\mathrm{SB}} - 1\right]$ and are zero otherwise suffice to implement the second best. That is, the subsidies need to guarantee that the maximum number of small firms $\underline{N}^{\max}$ enter the industry for all $\bar{N} < \bar{N}_{\infty}^{\mathrm{SB}}$.

Starting from states $\bar{N} \in \left[\bar{N}_{\infty}^{\text{LF}}, \bar{N}_{\infty}^{\text{SB}} - 1\right]$, this requires subsidies large enough that

$$\lim_{\underline{z} \to 0} J^{\text{SB}}\left(\underline{N}^{\max}, \bar{N}; \underline{z}\right) = \frac{1}{r + \lambda \underline{N}^{\max}} \left[s^{\text{SB}}\left(\bar{N}\right) - f + \lambda \underline{N}^{\max} \times J^{\text{SB}}\left(\underline{N}^{\max}, \bar{N}+1; \bar{z}\right) \right.$$
$$\left. + \lambda\left(\underline{N}^{\max} - 1\right) \times \lim_{\underline{z} \to 0} J^{\text{SB}}\left(\underline{N}^{\max}, \bar{N}+1; \underline{z}\right) \right] \geq 0, \tag{C.1}$$

where $J^{\text{SB}}\left(\cdot\right)$ refers to firm values under the optimal subsidies. When the condition (C.1) holds with equality, this defines a lower bound $\underline{s}\left(\bar{N}\right)$ for the optimal subsidy in an industry with $\bar{N}$ large firms.

Next, consider industry states with $\bar{N} < \bar{N}_{\infty}^{\text{LF}}$. Remember that, at the laissez-faire, the value of small firms $J\left(\underline{N}^{\max}, \underline{N}; \underline{z}\right)$ is positive for $\bar{N} < \bar{N}_{\infty}^{\text{LF}}$ and the maximum number of small firms is present (Proposition 1). However, these firm values change once the government subsidizes small firms at later industry states with $\bar{N} \in \left[\bar{N}_{\infty}^{\text{LF}}, \bar{N}_{\infty}^{\text{SB}} - 1\right]$. The are two possible cases. Suppose first that $\lim_{\underline{z} \to 0} J^{\text{SB}}\left(\underline{N}^{\max}, \bar{N}; \underline{z}\right)$ remains positive for all $\bar{N} < \bar{N}_{\infty}^{\text{LF}}$ when the government sets subsidies equal to their lower bound $\underline{s}\left(\bar{N}\right)$ for all $\bar{N} \in \left[\bar{N}_{\infty}^{\text{LF}}, \bar{N}_{\infty}^{\text{SB}} - 1\right]$ and sets zero subsidies otherwise.[26] In this case, the lower bound subsidies for states with $\bar{N} \in \left[\bar{N}_{\infty}^{\text{LF}}, \bar{N}_{\infty}^{\text{SB}} - 1\right]$ not only make the maximum number $\underline{N}^{\max}$ of small firms enter in these states, but also in all states with $\bar{N} < \bar{N}_{\infty}^{\text{LF}}$. Therefore, the second best life-cycle in Proposition 3 is implemented with subsidies $s^{\text{SB}}\left(\bar{N}\right) = \underline{s}\left(\bar{N}\right)$ for all $\bar{N} \in \left[\bar{N}_{\infty}^{\text{LF}}, \bar{N}_{\infty}^{\text{SB}} - 1\right]$ and zero otherwise.

On the contrary, suppose that $\lim_{\underline{z} \to 0} J^{\text{SB}}\left(\underline{N}^{\max}, \bar{N}; \underline{z}\right)$ turns negative for some $\bar{N} < \bar{N}_{\infty}^{\text{LF}}$ under the lower bound subsidies. The government now needs to subsidize above the lower bound for industry states with $\bar{N} \in \left[\bar{N}_{\infty}^{\text{LF}}, \bar{N}_{\infty}^{\text{SB}} - 1\right]$ in order to make the maximum number $\underline{N}^{\max}$ of small firms enter in states with $\bar{N} < \bar{N}_{\infty}^{\text{LF}}$. Therefore, the second best life-cycle in Proposition 3 is implemented with subsidies $s^{\text{SB}}\left(\bar{N}\right) > \underline{s}\left(\bar{N}\right)$ for all $\bar{N} \in \left[\bar{N}_{\infty}^{\text{LF}}, \bar{N}_{\infty}^{\text{SB}} - 1\right]$ and zero otherwise; where the lowest required subsidies $s^{\text{SB}}\left(\bar{N}\right)$ are such that $\lim_{\underline{z} \to 0} J^{\text{SB}}\left(\underline{N}^{\max}, \bar{N}; \underline{z}\right) \geq 0$ for all $\bar{N} < \bar{N}_{\infty}^{\text{LF}}$, with equality for at least some $\bar{N}$.

[26] This is the case when the lower bound subsidies are large enough to compensate firms for the fact that future profits are lower for $\bar{N} \geq \bar{N}_{\infty}^{\text{LF}}$ — as there are more large firms in the industry — or when future profits do not fall so much that firms are discouraged to enter until reaching the maximum $\underline{N}^{\max}$.

Economy where $\bar{z}/\underline{z} = 1$. The proof is straightfoward. Let the subsidy $s^{\text{SB}}(\bar{N})$ be such that $\pi(0, \bar{N}; \bar{z}) + s^{\text{SB}}(\bar{N}) = U(0, \bar{N})$ for any $\bar{N}$. This subsidy aligns the private and social incentives to enter the industry, and implements the second best number of total firms $\bar{N}_{\infty}^{\text{SB}}$ at all times.

D Additional Figures

Figure 6: Scale and optimal policy

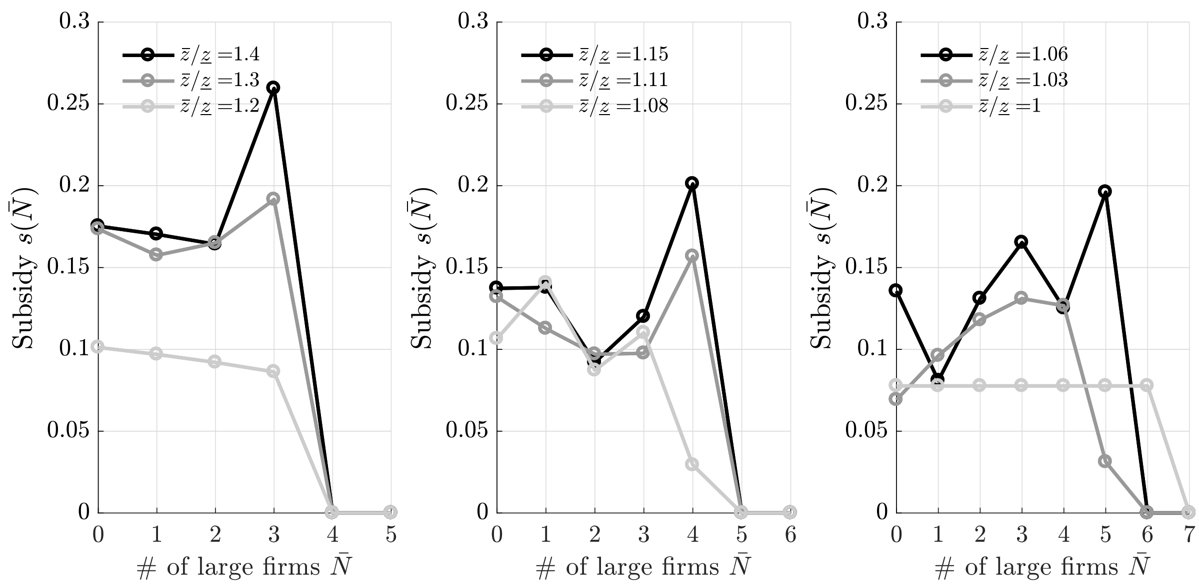
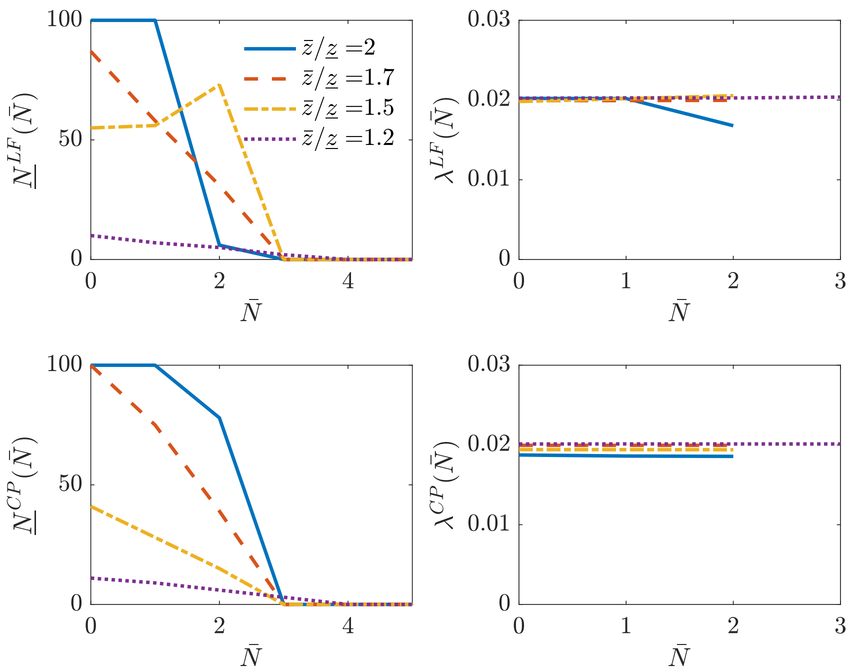
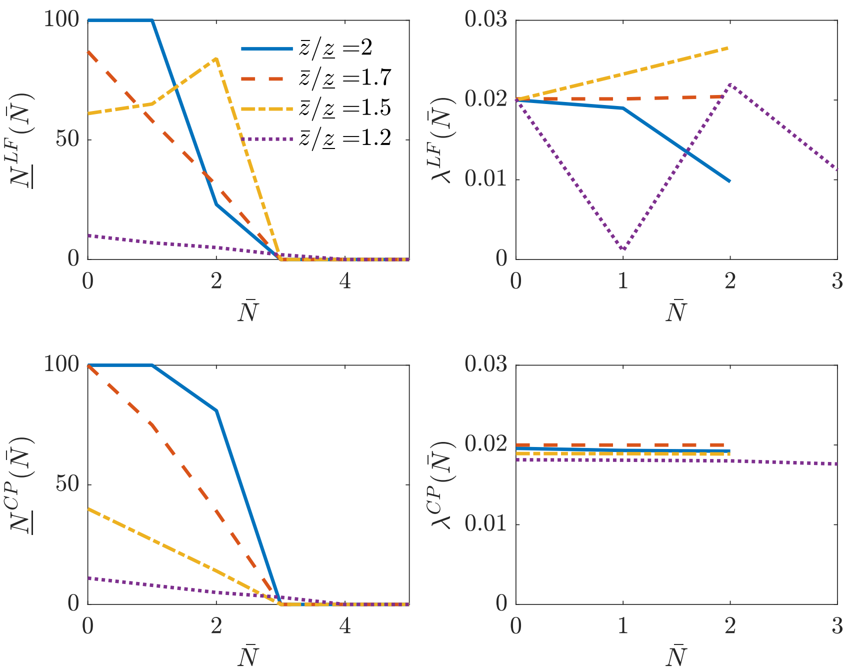


Figure 7: Life-Cycle with Intensive Margin of Innovation, Inelastic Case



Note: The simulations were done with the parameter values used in Figure 2 and a quadratic innovation cost function, $c(\lambda) = c_0\lambda^2$, where the constant term c_0 was calibrated so that $\lambda^{LF}(0) = 0.02$. In addition, the fixed cost for small firms $\underline{z}$ was adjusted downwards so that the profits net of the innovation costs were the same in the initial state to those of the model with exogenous λ, i.e., $f - c_0 0.02^2$.

Figure 8: Life-Cycle with Intensive Margin of Innovation, Elastic Case



Note: The simulations were done with the parameter values used in Figure 7 and a higher elasticity of the cost function, $c(\lambda) = c_0 \lambda^{1.1}$. Again, the constant term c_0 was calibrated so that $\lambda^{LF}(0) = 0.02$.