Erratum for Chattopadhyay and Duflo 2004

Esther Duflo and Rachael Meager

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A recent comment submitted to Econometrica by Mudit Kapoor and Arkodipta Sarkar concerns an error in Proposition 1 (i) in Chattopadhyay and Duflo 2004. The source of the contradiction is not an inconsistency in the model but a failure to understand that the model must be simplified in cases, and a lack of attention to the boundaries of the particular case examined in the proof. This note provides a correction and a full proof.

The proof in Chattopadhyay and Duflo is incomplete, and Proposition 1 (i) as stated is wrong because it fails to understand that the marginal male entrant of the particular case they examine lies on the boundary of the case. Kapoor and Sarkar offer an alternative condition to Proposition 1 (i), but their condition also fails to pay attention to the boundaries of the case, and is also incomplete. The correct condition for Proposition 1 (i) is derived in this note.

1 Statement of Revised Proposition 1 (i)

If $w_j < \mu'$, $M < \mu'$ and $\tilde{x_k} \geq m \geq \tilde{x_j}$, then no woman can run unopposed if

$$\min\left[\mu', \max\left[x_M, \alpha\frac{\delta_m + (1-\alpha)\mu' + \min[\bar{x_W}, \mu' - \delta_w]}{(2-\alpha)} + (1-\alpha)\mu'\right]\right] \leq 2m - \min[\bar{x_W}, \mu' - \delta_w].$$

2 Proof of Revised Proposition 1 (i)

We proceed by finding the most man-friendly woman (ie the highest w_j) who is willing to run unopposed, and then find the most woman-friendly man (ie the lowest w_k) who is willing to enter against this marginal woman. If this marginal man can win against this marginal woman, then no woman can run unopposed.

2.1 When does a woman "w_j" run uncontested?

A woman of type w_j runs uncontested iff

$$-|x_j - w_j| - \delta_w \geq -|\mu' - w_j|.$$

Where $x_j = \alpha w_j + (1-\alpha)\mu'$, reflecting policy capture by community elites who prefer policy outcome μ'. This condition can be simplified in two cases.

Case 1: $\mu' \geq w_j$. This implies $x_j \geq w_j$, which is why there are only two cases for this expression. In this case, the expression simplifies to $\mu' - x_j \geq \delta_w$. The highest possible value of x_j for which a woman runs uncontested is either $x_j = \mu' - \delta_w$ or the upper boundary of the set of x_j values to which this case applies. This upper bound is $x_W = \alpha W + (1-\alpha)\mu'$. Call this upper bound of the set $\bar{x_W}$. Therefore, the most man-friendly woman who will run is $\tilde{x_j} = \min[\bar{x_W}, \mu' - \delta_w]$.

Case 2: $\mu' \leq w_j$. This implies $x_j \leq w_j$. In this case, the expression simplifies to $x_j - \mu' \geq \delta_w$. In this case the highest possible x_j value for which a woman runs uncontested is $\tilde{x_j} = x_W$. This is not a very useful demarcation though, since the woman is more right-leaning than the elites in this case anyhow.

Case 2 is not relevant to the situation that Chattopadhyay and Duflo 2004 seeks to explain, so we will discard it. The remainder of the proof is completed assuming that $w_j < \mu'$. Note that in the algebra that follows, if the calculated value of the marginal female entrant lies below 0 then no female enters, and if the calculated value for the marginal male entrant lies above 1 then no male enters. In what follows we assume it is not the case.

2.2 When does a man "w_k" run against a woman "w_j"?

A man of type w_k runs against a woman of type w_j iff

$$-|x_k - w_k| - \delta_m \geq -|x_j - w_k|$$

and

$$|x_k - m| \leq |x_j - m|.$$

The first condition ensures it is worth his while to run and win; the second condition ensures he will win if he runs. Therefore, even the most man-friendly woman will face entry from a man iff there exists a w_k such that:

$$-|x_k - w_k| - \delta_m \geq -|\tilde{x_j} - w_k|$$

and

$$|x_k - m| \leq |\tilde{x_j} - m|.$$

That is, if there exists any man w_k for whom both of these conditions hold, then he wants to enter even against the most man-friendly woman, so no woman can run unopposed.

The first of the two conditions can be simplified in four cases as follows.

Case 1: $\mu' \geq w_k$ *and* $\tilde{x_j} \geq w_k$. This implies $\mu' \geq x_k \geq w_k$. The equation simplifies to $\tilde{x_j} - x_k \geq \delta_m$. This is certainly possible but as it requires $w_j \geq \mu'$ it is not relevant for Chattopadhyay and Duflo 2004.

Case 2: $\mu' \geq w_k$ and $\tilde{x_j} \leq w_k$. This implies $\mu' \geq x_k \geq w_k$. The equation simplifies to $2w_k - x_k - \tilde{x_j} \geq \delta_m$. This is certainly possible. In this case we must have $\tilde{w_j} \leq \mu'$ so this simplifies to

$$\begin{aligned} 2w_k - \alpha w_k - (1-\alpha)\mu' - \min[\bar{x_W}, \mu' - \delta_w] &\geq \delta_m \\ (2-\alpha)w_k &\geq \delta_m + (1-\alpha)\mu' + \min[\bar{x_W}, \mu' - \delta_w] \\ w_k &\geq \frac{\delta_m + (1-\alpha)\mu' + \min[\bar{x_W}, \mu' - \delta_w]}{(2-\alpha)} \end{aligned}$$

This provides an interior lower bound, that is, the most woman-friendly man who will run against a woman with $w_j < \mu'$. Denote the man for whom this holds with equality w_k^* and his implemented policy x_k^*. The lower boundary of the set of x_k to which this case applies is $x_M = \alpha M + (1-\alpha)\mu'$. So the most woman-friendly man who enters is $\tilde{x_k^*} = \max[x_k^*, x_M]$. But note that if this cutoff lies above μ', no men in this group run against the most man-friendly woman.

Case 3: $\mu' \leq w_k$ and $\tilde{x_j} \geq w_k$. This implies $\mu' \leq x_k \leq w_k$. This again requires $w_j \geq \mu'$ so it is not relevant.

*Case 4: $\mu' \leq w_k$ and $\tilde{x_j} \leq w_k$.*This implies $\mu' \leq x_k \leq w_k$. The entry condition simplifies to $x_k - \tilde{x_j} \geq \delta_m$. The smallest such x_k, which is the most woman-friendly man who would still run, is either $x_k = \tilde{x_j} + \delta_m$ or the lower bound of the set of x_k to which this case applies, which is $x_M = \mu'$ if $M \leq \mu'$, or $x_M = \alpha M + (1-\alpha)\mu'$ if $M \geq \mu'$. Call this lower bound of the set $\underline{x_M}$. Then the most woman-friendly man who runs produces the outcome $\tilde{x_k} = \max[\tilde{x_j} + \delta_m, \underline{x_M}]$.

Since we assumed $w_j < \mu'$, only cases 2 and 4 are possible. The original paper discards case 2 (without making it explicit), and instead works on case 4. As we will see, this is the source of the original problem: both case 2 and 4 should be examined for Proposition 1 (i). We proceed in case 4, and return to case 2 later.

The second condition for male entry requires that the man producing $\tilde{x_k}$ only runs against the woman producing $\tilde{x_j}$ if he can win. We are in a case where $\tilde{x_k} \geq \mu' \geq \tilde{x_j}$. Chattopadhyay and Duflo 2004 assume $\mu' \geq m$ so we know $\tilde{x_k} \geq m$, but we may have m larger or smaller than $\tilde{x_j}$. Suppose $m \geq \tilde{x_j}$. Then the man in question will always run against the woman iff

$$\tilde{x_k} - m \leq m - \tilde{x_j}$$

which "simplifies" to:

$$2m \geq \max[\min[\bar{x_W}, \mu' - \delta_w] + \delta_m, \underline{x_M}] + \min[\bar{x_W}, \mu' - \delta_w].$$

Now, we show that the supposed interior male cutoff $\tilde{x_k} = \tilde{x_j} + \delta_m$ for this case is not "interior" at all, and should be discarded. First, consider the highest possible value for the marginal woman, $\tilde{x_j} = \mu' - \delta_w$ (if she hits the boundary of her case, she lies below this value). Substituting this highest possible value into the expression for the interior marginal man $\tilde{x_k} = \tilde{x_j} + \delta_m$ produces the implication that $\tilde{x_k} = \mu' - \delta_w + \delta_w$. Since $\delta_w > \delta_m$ this implies that $\mu' \geq \tilde{x_k}$. Hence, this "interior" marginal man must lie below μ': but then he is not in the set of men to whom this case applies, the rules derived for case 4 do not apply to him. This tells us that any man with $w_k > \mu'$ enters against any woman with $w_j < \mu'$ for sure in the model: the marginal male entrant for case 4 is on the boundary, μ'.

What is the intuition for why the most woman-friendly man who runs is always found at or below μ' in the case where the most man-friendly woman is below μ'? This stems from the fact that $\delta_m < \delta_w$. If it is worth it for a woman to enter and win against the default outcome of μ', she must be very far from μ', because it costs her a lot to run. But since she is so far from μ', she must be relatively extreme (geared towards women) in her position, so therefore it is certainly worth it for some man just below μ' to enter against her, as long as he can win - both because he can get a final outcome very close to his actual preference, and because it costs him less to run in the first place.

We now derive the true, global cutoff condition for the case where $w_j < \mu'$. Since we know we hit the boundary of case 4 for the male entry condition, we now need to examine case 2 from the man's entry condition derived earlier, which encompasses men for whom $w_k < \mu'$. Notice that we must have $M \leq \mu'$ for this to be a relevant case. If we have this, then the true cutoff is the $\tilde{x_k^*}$ derived in case 2: the most woman-friendly man who runs is either M or $\frac{\delta_m + (1-\alpha)\mu' + \min[\bar{x_W}, \mu' - \delta_w]}{(2-\alpha)}$, whichever is larger. But this had better be below μ' or else we hit the upper bound of the case, and since we know from case 4 that the man at μ' enters, the cutoff becomes μ'. Therefore, the global cutoff condition should be:

$$\tilde{x_k} = \min\left[\mu', \max\left[x_M, \alpha\frac{\delta_m + (1-\alpha)\mu' + \min[\bar{x_W}, \mu' - \delta_w]}{(2-\alpha)} + (1-\alpha)\mu'\right]\right].$$

From this cutoff, one can proceed to derive the conditions under which this marginal male entrant can win, and therefore, the conditions under which no woman can run unopposed, as we did before. He can win if

$$|\tilde{x_k} - m| \leq |\tilde{x_j} - m|.$$

Notice that there are three possible cases here, even when we confine ourselves to the world in which $\tilde{w_j} \leq \mu'$. We know $x_k \geq \tilde{x_j}$ since we are in case 2. They might both be above m, both be below m, or we may have $\tilde{x_k} \geq m \geq \tilde{x_j}$. Simplifying in each case leads to a condition under which no woman can enter unopposed, for each case. The third case is the most interesting, so we will proceed with case 3. The condition becomes

$$\tilde{x_k} - m \leq m - \tilde{x_j}$$

which is fully written out as

$$\min\left[\mu', \max\left[x_M, \alpha\frac{\delta_m + (1-\alpha)\mu' + \min[\bar{x_W}, \mu' - \delta_w]}{(2-\alpha)} + (1-\alpha)\mu'\right]\right] \leq 2m - \min[\bar{x_W}, \mu' - \delta_w].$$

As mentioned previously, it is also necessary for the calculated value for the marginal female entrant to lie above zero, otherwise no woman will enter even if the above condition does not hold.