

FROM HYPERINFLATION TO STABLE PRICES: ARGENTINA'S EVIDENCE ON MENU COST MODELS*

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In this article, we analyze how inflation affects firms' price-setting behavior. For a class of menu cost models, we derive several predictions about how price-setting changes with inflation at very high and at near-zero inflation rates. Then, we present evidence supporting these predictions using product-level data underlying Argentina's consumer price index from 1988 to 1997—a unique experience where monthly inflation ranged from almost 200% to less than zero. For low inflation rates, we find that (i) the frequency and absolute size of price changes as well as the dispersion of relative prices do not change with inflation, (ii) the frequency and size of price increases and decreases are symmetric around zero inflation, and (iii) aggregate inflation changes are mostly driven by changes in the frequency of price increases and decreases, as opposed to the size of price changes. For high inflation rates, we find that (iv) the elasticity of the frequency of price changes with respect to inflation is close to two-thirds, (v) the frequency of price changes across different products becomes similar, and (vi) the elasticity of the dispersion of relative prices with respect to inflation is one-third. Our findings confirm and extend available evidence for countries that experienced either very high or near-zero inflation. We conclude by showing that a hyperinflation of 500% a year is associated with a cost of approximately 8.5% of aggregate output a year as a result of inefficient price dispersion alone. *JEL* Codes: E31, E50.

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I. INTRODUCTION

Infrequent nominal price adjustments are at the center of much of the literature studying positive and normative implications of inflation. In this article, we study how price-setting behavior changes with inflation, both theoretically and empirically. We consider menu cost models with idiosyncratic firm-level shocks, that is, models where monopolistic firms set prices subject to a fixed cost of adjustment and are hit by shocks to their real marginal cost. We derive sharp predictions concerning how changes in steady-state inflation affect price-setting behavior at near-zero and very high inflation rates. Then we confront these predictions with their empirical counterparts using product-level data underlying Argentina's consumer price index from 1988 to 1997. Argentina's experience provides a unique opportunity to analyze price-setting behavior through the lens of menu cost models because it encompasses several years of price stability (and even deflation) as well as years of sustained very high inflation.

We find evidence supporting many predictions of menu cost models. Our empirical findings involve confronting a number of variables (summarizing price-setting behavior) across time periods with different inflation rates. Our theoretical results involve comparative statics about how these same variables change across steady states with different inflation rates. However, we show that when idiosyncratic shocks are persistent and discount rates are low, these comparative statics depend on the ratio of inflation to the variance of firm-level idiosyncratic shocks. Therefore, they are comparable to our empirical findings under the assumption that the variance of idiosyncratic shocks remains approximately constant across time periods and that current inflation approximates expected future inflation.¹ The first set of results concerns low-inflation economies. Theoretically and empirically, we show that in a neighborhood of zero inflation (i) the frequency of price changes is unresponsive to inflation; (ii) the dispersion of relative prices is unresponsive to inflation; (iii) the frequency of price increases is equal to the frequency of price decreases; (iv) conditional on a price change, the size of price increases is equal to the size of price decreases; and (v) inflation changes mostly (to be precise, 90%) due to the changes in the difference between the frequency

1. Later we discuss how we empirically implement measures of expected future inflation.

of price increases and of price decreases—as opposed to changes in the size of price increases and decreases. The second set of results concerns high-inflation economies. We find that (vi) the frequency of price adjustment becomes the same for all products/firms, (vii) the elasticity of the average frequency of price changes with respect to inflation converges to two-thirds, (viii) the elasticity of the dispersion of relative prices converges to one-third, (ix) the elasticity of the average size of price changes with respect to inflation converges to one-third, and (x) the frequency of price decreases converges to zero.

We believe these results are interesting because they underlie the welfare costs of inflation in menu cost models (as well as other models of price stickiness) and because they test this class of models. First, the menu cost paid when changing prices is a direct welfare cost of inflation because these resources are wasted. Second, the “extra” price dispersion created by nominal variation in prices is another avenue for inefficiency in menu cost models as well as in other models with sticky prices. For example, in models with an exogenous frequency of price adjustment, as described in [Woodford \(2003, ch. 6\)](#).² We find that the direct cost of more frequent price changes is unlikely to be significant for inflation rates below 5% a year (see [Figure V](#) later). Moreover, the costs of inflation arising from inefficient price dispersion is also likely to be negligible for inflation rates below 10% a year ([Figure X](#) later). Therefore, findings (i) and (ii) imply that the welfare cost of increasing the rate of inflation is negligible around zero inflation. For higher rates of inflation, the welfare costs of inflation increase as the frequency of price changes and the standard deviation of relative prices increase. We estimate that the welfare costs of the additional relative price dispersion caused by inflation are approximately 1.5% of GDP for an inflation rate of 100% a year and approximately 8.5% of GDP for inflation rates of 500% a year.

We begin by presenting a model of monopolistic firms that are hit by idiosyncratic shocks to their real marginal cost and face

2. See [Bénabou \(1992\)](#) and [Burstein and Hellwig \(2008\)](#) for earlier and recent examples of analysis that takes both effects of inflation into account, the former using heterogeneous consumers that search for products and homogeneous firms, and the latter using differentiated products on the demand side and heterogeneity in the firm’s cost. Recently, and most related to our article, [Nakamura et al. \(2018\)](#) reexamine these issues for the United States using new micro data including the higher inflation period for the United States as well as calibrated menu cost models.

a fixed cost of changing prices. Similar models have been introduced by Barro (1972), Bertola and Caballero (1990), Danziger (1999), Golosov and Lucas (2007), and Gertler and Leahy (2008). Then, in Sections II.B and II.C, we derive our comparative static results about how inflation affects price-setting behavior and illustrate them with a numerical example based on the model in Golosov and Lucas (2007), so that one can evaluate how the local theoretical results apply to a large range of quantitatively relevant parameter values. The results for low inflation are new: in particular, the prediction that most of the changes in inflation (90%) are accounted for by changes in the difference between the frequency of price increases and decreases.³ Furthermore, a new insight of this work is that under some simplifying assumptions, price-setting behavior depends only on the ratio between inflation and the variance of the idiosyncratic shocks. In this sense, high-inflation economies are equivalent to economies in which firms do not face idiosyncratic shocks for nominal price adjustment decisions. Hence, we can apply benchmark results on the effect of inflation on price-setting behavior in the deterministic case (Sheshinski and Weiss 1977; and Bénabou and Konieczny 1994, which consider models without idiosyncratic shocks) to the case in which firms face persistent idiosyncratic shocks and there is high inflation.

We continue with our main empirical findings in Section IV. Hereinafter, we discuss the most notable ones as well as their relationship to previous literature. We first describe our data set in Section IV.A. We use the micro-data underlying the construction of the Argentine consumer price index from 1988 to 1997 for 506 goods covering 84% of expenditures. The unique feature of these data is the range of inflation during this time period. The inflation rate was almost 5,000% during 1989 and 1,500% during 1990. After the stabilization plan of 1991, there was a quick disinflation episode and after 1992 there was virtual price stability with some deflationary periods.

Using these data, we estimate the frequency of price changes, the average size of price changes, and the dispersion of relative prices for different time periods. We find that the frequency of price changes as a function of the rate of inflation is flat at low

3. Alvarez et al. (2011) and Alvarez and Lippi (2014) show that the frequency of price changes and the dispersion of relative prices are insensitive to inflation at low inflation rates under a more restrictive set of assumptions.

inflation levels and has a constant elasticity for high inflation levels (a novel empirical finding). We estimate this elasticity to be between one-half and two-thirds, which is close to our theoretical prediction of two-thirds.⁴ There is a large literature that estimates the frequency of price changes. Various papers do this for different countries, different time periods, different rates of inflation, and different sets of goods.⁵ A feature of our data set (and a success of the theory) is that our estimates of the frequency of price changes for each level of inflation in the Argentine data are similar to the estimates of the other studies with the same rate of inflation. This is illustrated in [Figure VI](#) later. Further details on the samples and inflation ranges considered in other studies in the literature are provided in [Online Appendix G](#).

Then, we find that even though the frequency of price changes is unresponsive to inflation when inflation is low, the difference between the frequency of price increases and the frequency of price decreases is an increasing function of inflation. This is consistent with previous evidence (see, for example, [Nakamura and Steinsson 2008](#) for the United States; [Gagnon 2009](#) for Mexico; [Berardi, Gautier, and Le Bihan 2015](#) for France; and [Cavallo 2015](#) for cross-country evidence). Our contribution is to provide a new theoretical interpretation of this fact through the lens of menu cost models. We show that under some assumptions, when inflation is low, this fact is consistent with 90% of the changes in inflation resulting from the extensive margin of price adjustments, that is, changes in the difference between the frequency of price increases and decreases. Furthermore, we document that the cross-good dispersion of the frequency of price changes falls with inflation when inflation is high. As is well known in the literature on low-inflation economies, there is large variation in the frequency of price changes across firms selling different goods.

4. To accommodate the range of estimates in the data, our simple model could be extended, allowing firms to freely change prices at random times. [Nakamura and Steinsson \(2010\)](#) consider a version of this model (also see [Dotsey and King 2005](#); [Caballero and Engel 2007](#); and [Alvarez, Le Bihan, and Lippi 2016](#)).

5. For example, for low-inflation economies [Álvarez et al. \(2006\)](#) for the Euro area; [Bils and Klenow \(2004\)](#), [Klenow and Kryvtsov \(2008\)](#), and [Nakamura and Steinsson \(2008\)](#) for the United States. For moderate levels of inflation, [Gagnon \(2009\)](#) for Mexico, [Barros et al. \(2009\)](#) for Brazil, and [Wulfsberg \(2016\)](#) for Norway. [Baharad and Eden \(2004\)](#) and [Lach and Tsiddon \(1992\)](#) for Israel, as well as [Konieczny and Skrzypacz \(2005\)](#) for Poland, look at high-inflation economies, but not as high as our Argentine sample.

The new finding is that as inflation rises, the frequency of price changes becomes similar across different products, reflecting that the importance of idiosyncratic differences across products disappears as a motive for changing prices when inflation is high.

Next we are concerned with the dispersion of relative prices and size of price changes. Despite being of theoretical and practical importance because of its welfare implications, this is the first article to look at the relationship between inflation and the dispersion of relative prices across stores, selling the same good, for a wide range of inflation rates.⁶ We document that the dispersion of relative prices across stores is insensitive to inflation for low inflation rates and it becomes an increasing function of inflation for high inflation rates. In fact, for very high inflation rates, the empirical elasticity of the dispersion of relative prices with respect to inflation is close to the theoretical upper bound of one-third. The average size of price changes, conditional on a price change taking place, exhibits a similar pattern. It is insensitive to inflation at low inflation rates and then increases with inflation as inflation becomes higher.

We conclude by showing that the welfare costs of inflation due to inefficient price dispersion are only relevant for very high inflation rates. In particular, we find that the cost of the additional price dispersion resulting from a hyperinflation of 500% a year is approximately 8.5% of aggregate output a year.

Some of our results for the frequency and size of price changes have recently been documented for the United States as well by [Nakamura et al. \(2018\)](#). In particular, using newly found post 1977 BLS micro-data on U.S. consumer prices, they find that in calibrated models the frequency of price changes increases with inflation and that the size of price changes is

6. Many papers look at the dispersion of inflation rates across goods, which is a different concept. [Reinsdorf \(1994\)](#), [Sheremirov \(2015\)](#), and [Perez and Drenik \(2015\)](#) are some exceptions. However, due to the low variation of inflation in their samples, it is hard to make inferences about the elasticity of interest in those studies. In [Reinsdorf \(1994\)](#), inflation varies between 3.7% and 10.4% in the United States in the early 1980s and has conflicting results depending on the measure of inflation. [Nakamura et al. \(2018\)](#) have a better sample, and their results are consistent with ours. [Sheremirov \(2015\)](#) looks at U.S. data for 2001–2011 when the variation in the rate of inflation is small. [Perez and Drenik \(2015\)](#) find a positive correlation between inflation and the dispersion of relative prices for inflation rates ranging from 10% to 20% a year, which they attribute to other confounding factors.

insensitive to inflation for inflation rates under 14% a year. While they study the consequences of these findings for the welfare cost of inflation in calibrated New Keynesian models, our article differs in that we are interested in providing a more general theoretical characterization within menu cost models of these as well as other statistics for both very low and very high inflation. In addition, on the empirical side we are able to study very high and very low inflation during this period, since the first two years of the sample had annual inflation rates in the thousands, and the last few years essentially enjoyed price stability, with a disinflation period in between (see [Figure IV](#) and the figures in the background discussion in [Online Appendix H](#)).

Finally, several online appendixes provide supplementary material. [Online Appendix A](#) contains the proofs of the propositions in [Section II.A](#). Some of these proofs are of independent interest because they fully characterize the solution of the menu cost model with a closed-form solution. [Online Appendix BA](#) contains an analytical characterization of our version of the [Golosov and Lucas \(2007\)](#) model. [Online Appendix C](#) describes some details of our data. Both in the main body and in [Online Appendix D](#), we perform extensive robustness checks to evaluate the sensitivity of our estimates. Our findings are robust to different treatment of sales, product substitutions, and missing values in the estimation of the frequency of price changes and with respect to the level of aggregation of price changes. They are also robust to using contemporaneous inflation or an estimate of expected future inflation for the relevant time frame, and to excluding observations corresponding to periods with inflation above some threshold for which we have reasons to believe that discrete sampling might bias the estimates. [Online Appendix H](#) contains a short description of the history of economic policy and inflation in Argentina for the years before and during our sample.

II. COMPARATIVE STATIC PROPERTIES OF MENU COST MODELS

In this section, we study how inflation affects price-setting behavior in menu cost models. We show several theoretical predictions for a stylized menu cost model where competitive monopolistic firms face a fixed cost of changing their nominal price in the presence of idiosyncratic real marginal-cost shocks and common constant inflation.

In [Section II.A](#), we write down a simple setup and obtain the main analytical results. In [Section II.B](#) and [Section II.C](#), we obtain the results for low and high inflation, respectively; explain the nature of the assumptions needed for the results; highlight which form of these results are already present in the literature; and discuss pros and cons of applying these comparative statics results to time series data. [Section II.D](#) decomposes changes in the rate of inflation into those arising from changes in the frequency of price changes and those arising from changes in the size of price changes. In [Section III](#), we present Kehoe and Midrigan's (2015) version of Golosov and Lucas's (2007) model and illustrate the theoretical results in this section for both low and high inflation. This model relaxes some assumptions and we parameterize the model in an empirically reasonable fashion to show that the theoretical results of [Section II.A](#) are quantitatively applicable to typical economies.

In [Online Appendix A](#), we prove the theoretical propositions in this section in a setup with a more general profit function and a less restrictive process for the shocks, but that retains certain symmetry properties.

II.A. The Basic Menu Cost Model

We study the problem of a monopolist adjusting the nominal price of its product in an environment with inflation, idiosyncratic real marginal-cost shocks, and a fixed cost (the menu cost) of changing nominal prices. We think of this problem as a simplified version of the problem in [Golosov and Lucas \(2007\)](#), and as an almost identical problem to the one considered in [Barro \(1972\)](#).

We assume that the instantaneous profit of the monopolist depends on its price relative to the economy- or industry-wide average price and on an idiosyncratic shock. We let $F(p, z) = \zeta(z) - B(p - z)^2$ be the real value of the profit per period as a function of p , the log of the nominal price charged by the firm relative to the log of the economy-wide price, and z , an idiosyncratic shifter of the profit function. We let z be the static profit-maximizing real price. Thus, our quadratic specification can be interpreted as an approximation of a general function F around the static profit-maximizing price. In this case, $\zeta(z)$ is the static maximized profit, and $B > 0$ is half the second derivative of F around that price. We assume that the economy-wide price grows at a constant inflation rate π , so that when the firm does not change its nominal price its relative log-price decreases, that is, $dp = -\pi dt$. We also allow

the menu cost to depend on z . In this case, we write $C_t = c \zeta(z_t)$, where $c \geq 0$ is a constant, so $c = 0$ represents the frictionless problem. We assume that $\{z_t\}$ is a diffusion with law of motion $dz = \sigma dW$, where $\{W(t)\}$ is a standard Brownian motion with $W(t) - W(0) \sim N(0, t)$. We use $r \geq 0$ for the real discount rate of profits and adjustment costs. We let $\{\tau_i\}$ be the stopping times at which prices are adjusted and $\{\Delta p(\tau_i)\}$ the corresponding price changes, so that the problem of the firm can be written as

$$(1) \quad V(p, z) = \max_{\{\tau_i, \Delta p_i\}_{i=0}^{\infty}} \mathbb{E} \left[\int_0^{\infty} e^{-rt} F(p(t), z(t)) dt - \sum_{i=0}^{\infty} e^{-r\tau_i} c \zeta(z(\tau_i)) \mid z(0) = z \right],$$

with $p(t) = p(0) + \sum_{i=0}^{\tau_i < t} \Delta p(\tau_i) - \pi t$ and $z(t) = \sigma W(t)$ for all $t \geq 0$, and the initial state is given by $p(0) = p$ and $z(0) = z$.

Given the simple form of the period return function and the law of motion of the state, the optimal policy that solves this problem can be described by three numbers that control the difference between p and z , namely:

$$\Psi(\pi, \sigma^2) = \left[\underline{\psi}(\pi, \sigma^2), \bar{\psi}(\pi, \sigma^2), \hat{\psi}(\pi, \sigma^2) \right],$$

where we include the parameters π and σ^2 as explicit arguments of the decision rules to conduct comparative statics. The numbers $\underline{\psi}(\pi, \sigma^2)$ and $\bar{\psi}(\pi, \sigma^2)$ define the inaction set as follows:

$$\mathcal{I}(\pi, \sigma^2) = \left\{ (p, z) \in \mathbb{R} \times \mathbb{Z} : \underline{\psi}(\pi, \sigma^2) + z \leq p \leq \bar{\psi}(\pi, \sigma^2) + z \right\}.$$

If the firm’s relative price is within the inaction set, that is, if $(p, z) \in \mathcal{I}$, then it is optimal not to change prices. Outside the interior of the inaction set the firm will adjust prices so that its relative price just after adjustment is given by $p = \hat{\psi}(\pi, \sigma^2) + z$. Since $\{z(t)\}$ has continuous paths, all adjustments will occur at the boundary of the inaction set (given additional regularity conditions). For instance, a firm with a relative price p and an idiosyncratic shock z such that the relative price hits the lower boundary of the inaction set—that is, such that $p = \underline{\psi}(\pi, \sigma^2) + z$ —will raise its price by $\Delta p = \hat{\psi}(\pi, \sigma^2) - \underline{\psi}(\pi, \sigma^2) > 0$. Likewise, when hitting the upper boundary, it will change its price (decrease it) by $\Delta p = \hat{\psi}(\pi, \sigma^2) - \bar{\psi}(\pi, \sigma^2) < 0$.

Using the optimal decision rules, we can compute the density of the invariant distribution of the state, $g(p, z; \pi, \sigma^2)$, as well as the expected time between adjustments $T(p, z; \pi, \sigma^2)$ starting from the state (p, z) . Note that using $g(\cdot)$ we can readily find the distribution of relative prices in the economy (or industry) and we can compute the expected time elapsed between consecutive adjustments under the invariant distribution, and its reciprocal, the expected number of adjustments per unit of time, which we denote by $\lambda_a(\pi, \sigma^2)$.

We denote by $\lambda_a^+(\pi, \sigma^2)$ and $\lambda_a^-(\pi, \sigma^2)$ the frequencies of price increases and decreases, respectively. Furthermore, we let $\Delta_p^+(\pi, \sigma^2)$ be the expected size of price changes, conditional on having an increase, and $\Delta_p^-(\pi, \sigma^2)$ the corresponding expected size of price changes, conditional on having a decrease. Formally $\Delta_p^+(\pi, \sigma^2) = \int_Z [\hat{\psi}(z) - \underline{\psi}(z)] \frac{g(\psi(z), z)}{\int_Z g(\psi(z'), z') dz'} dz$ where we omit (π, σ^2) as arguments of g , $\hat{\psi}$, and $\underline{\psi}$ to simplify notation. $\Delta_p^-(\pi, \sigma^2)$ is defined analogously.

II.B. Comparative Statics with Low Inflation

In this section, we show that when inflation is zero and firms face idiosyncratic profit shocks, changes in the inflation rate do not have a first-order effect on the frequency of price changes or on the distribution of relative prices. The intuition for this result is that at zero inflation, price changes are triggered by idiosyncratic shocks and small variations in inflation have only a second-order effect. Moreover, we show that there is a type of symmetry in this case: the frequency of price increases and decreases and the size of price increases and decreases are the same.

We let $h(\hat{p}; \pi, \sigma^2) = \int g(\hat{p}, z; \pi, \sigma^2) dz$ be the invariant distribution of log relative prices $\hat{p} = p - \bar{p}$, for an economy, or industry, with (π, σ^2) , when it exists. Using h we can compute several statistics of interest, such as $\bar{\sigma}(\pi, \sigma^2)$, the standard deviation of relative prices. As in the case of the frequency of price changes, we include (π, σ^2) explicitly as arguments of this statistic.

PROPOSITION 1.

- (i) *If the frequency of price changes $\lambda_a(\pi, \sigma^2)$ is differentiable at $\pi = 0$, then the frequency of price changes is insensitive to inflation,*

$$\frac{\partial}{\partial \pi} \lambda_a(0, \sigma^2) = 0.$$

- (ii) *If the density of the invariant distribution $h(\hat{p}; \pi, \sigma^2)$ is differentiable at $\pi = 0$, then the dispersion of relative prices under the invariant distribution is insensitive to inflation,*

$$\frac{\partial}{\partial \pi} \bar{\sigma}(0, \sigma^2) = 0.$$

- (iii) *The frequencies of price changes and the size of price adjustment are symmetric at $\pi = 0$ in the sense that*

$$\lambda_a^+(0, \sigma^2) = \lambda_a^-(0, \sigma^2), \quad \frac{\partial \lambda_a^+(0, \sigma^2)}{\partial \pi} = -\frac{\partial \lambda_a^-(0, \sigma^2)}{\partial \pi} \text{ and}$$

$$\Delta_p^+(0, \sigma^2) = \Delta_p^-(0, \sigma^2), \quad \frac{\partial \Delta_p^+(0, \sigma^2)}{\partial \pi} = -\frac{\partial \Delta_p^-(0, \sigma^2)}{\partial \pi}$$

where λ_a^+ is the frequency of price increases, Δ_p^+ is the average size of price increases, and λ_a^- , Δ_p^- are the analogous concepts for price decreases.

The proof, for a more general case, is in [Online Appendix A](#). The main idea is to use the symmetry of the objective function $F(p, z)$ with respect to (p, z) to show the results. Indeed, [Online Appendix A](#) extends the model to one with a general F as well as a law of motion for z given by $dz = a(z)dt + b(z)\sigma dW$ where both $F(\cdot, \cdot)$ and $a(\cdot)$, $b(\cdot)$ satisfy certain symmetry properties. The proof shows that the expected number of adjustments is symmetric around zero inflation, that is, $\lambda_a(\pi, \sigma^2) = \lambda_a(-\pi, \sigma^2)$ for all π . Given the symmetry of the profit function we view this property as quite intuitive: a 1% inflation should give rise to as many price changes as a 1% deflation. Symmetry implies that if λ_a is differentiable then $\frac{\partial}{\partial \pi} \lambda_a(\pi, \sigma^2) = -\frac{\partial}{\partial \pi} \lambda_a(-\pi, \sigma^2)$, which establishes the first result.

Analogously, for the distribution of relative prices, the main idea is to show that the marginal distribution of relative prices is symmetric in the sense that $h(\hat{p}; \pi, \sigma^2) = h(-\hat{p}; -\pi, \sigma^2)$ for all \hat{p}, π , that is, the probability of high relative prices with positive inflation is the same as that of low relative prices with deflation. As these symmetric functions are locally unchanged with respect to π when $\pi = 0$, inflation has no first-order effect on the second moment of the distribution of relative prices at $\pi = 0$. Similarly, the symmetry of the frequency and of the average size of price

increases and decreases also follow from the symmetry assumptions.

The assumption of differentiability of λ_α and $\bar{\sigma}$ with respect to π is not merely a technical condition. The function $\lambda_\alpha(\pi, \sigma^2)$ could have a local minimum at $\pi = 0$ without being smooth. This is indeed the case for $\sigma^2 = 0$ to which we turn in the next subsection.⁷ Likewise, the differentiability of $h(\hat{p}; \pi, \sigma^2)$ at $\pi = 0$ requires $\sigma > 0$. In Sheshinski and Weiss's (1977) model, that is, when $\sigma = 0$, the distribution h is degenerate, uniform at $\pi \neq 0$, but nondifferentiable at $\pi = 0$.

1. Remarks and Relation to the Literature. We find [Proposition 1](#)'s theoretical predictions interesting because they extend an important result on the welfare cost of inflation from sticky-price models with exogenous price changes (e.g., the Calvo model) to menu cost models with endogenous frequency of price changes. The result is that in cashless economies with low inflation, there is no first-order welfare effect of inflation, that is, the welfare cost of inflation can be approximated by a “purely quadratic” function of inflation.

Inflation imposes welfare costs through two channels in cashless economies. First, the “extra” price dispersion created by inflation is an avenue for inefficiency in models with sticky prices, because it creates “wedges” between the marginal rates of substitution in consumption and the marginal rates of transformation in production. See, for example, [Woodford \(2003, ch. 6\)](#), and references therein for the analysis of this effect. Part (ii) of [Proposition 1](#) extends this result to the menu cost model. Second, a higher endogenous frequency of price adjustments because of inflation is an obvious source of welfare losses when these adjustments are costly. Part (i) of [Proposition 1](#) establishes that this second channel is also negligible for low inflation rates.⁸

7. We conjecture, but have not proved at this level of generality, that as long as $\sigma^2 > 0$, the problem is regular enough to become smooth, that is, the idiosyncratic shocks will dominate the effect of inflation. For several examples one can either compute all the required functions or show that they are smooth, given the elliptical nature of the different ODEs involved. Based on this logic, as well as on computations of different models, we believe that the length of the interval for inflations around zero for which $\lambda_\alpha(\cdot, \sigma^2)$ is approximately flat is increasing in the value of σ^2 .

8. There are other publications that take more than one effect into account. [Burstein and Hellwig \(2008\)](#) compute numerical examples in a model closer to

The results of [Proposition 1](#) should apply to a wider class of models as long as one essential assumption is maintained: the symmetry of the profit function around the profit-maximizing price. For example, a version of [Proposition 1](#) applies to models with both observations and menu costs ([Alvarez et al. 2011](#)), to models that have multiproduct firms ([Alvarez and Lippi 2014](#)), and to models that combine menu costs and Calvo-type adjustments ([Nakamura and Steinsson 2010](#); [Alvarez, Le Bihan, and Lippi 2016](#)). In [Section III](#), we solve numerically for Golosov and Lucas's (2007) version of the model (which does not satisfy the aforementioned symmetry properties). We show that for empirically reasonable parameter values, the functions λ_a and $\bar{\sigma}$ are approximately flat for a wide value of inflation rates around zero.

We do not know of other theoretical results analyzing the sensitivity of $\lambda_a(\pi, \sigma^2)$ and $\bar{\sigma}(\pi, \sigma^2)$ to inflation around $\pi = 0$ in this setup. However, there is a closely related model that contains a complete analytical characterization by [Danziger \(1999\)](#). In fact, we can show that for a small cost of changing prices, [Proposition 1](#) holds in Danziger's characterization.

II.C. Comparative Statics with High Inflation

Now we turn to the analysis of price-setting behavior for large values of inflation. In highly inflationary environments, the main reason for firms to change nominal prices is to keep their relative price in a target zone as the aggregate price level grows. Idiosyncratic shocks in the high-inflation case become less important and therefore the analysis of the deterministic case is instructive. This leads us to proceed in two steps. First we derive comparative statics results in the deterministic case—that is, when $\sigma^2 = 0$. This is a version of the problem studied by [Sheshinski and Weiss \(1977\)](#). Then we study the conditions under which these comparative statics are the same as for the case of $\sigma^2 > 0$ and very large π .

[Sheshinski and Weiss \(1977\)](#) study a menu cost model similar to the deterministic case in our basic setup. The firm's problem is to decide when to change prices and by how much when aggregate prices grow at the rate π . In [Sheshinski and Weiss's \(1977\)](#) model, the time elapsed between adjustments is simply a constant, which we denote by $\mathcal{T}(\pi)$. [Sheshinski and Weiss \(1977\)](#) find sufficient

ours, which also includes the traditional money demand cost. [Bénabou \(1992\)](#) uses a different framework, with heterogeneous consumers that search for products and homogeneous firms subject to menu costs but constant production costs.

conditions so that the time between adjustments decreases with the inflation rate (see their Proposition 2), and several authors have further refined the characterization by concentrating on the case where the fixed cost c is small. Let $p^* = \arg \max_p F(p, 0)$ be the log price of the static monopolist maximization profit, where $z = 0$ is a normalization of the shifter parameter, which stays constant. In the deterministic setup the optimal policy for $\pi > 0$ is to let the log price reach a value s , at which time it adjusts to S , where $s < p^* < S$. The time between adjustments is then $T(\pi) = \frac{S-s}{\pi}$. Furthermore, we highlight another implication obtained in Sheshinski and Weiss's (1977) model: the setup with $\sigma^2 = 0$. The distribution of the log relative price is uniform in the interval $[s, S]$. Thus, the standard deviation of the log relative prices in this economy, denoted by $\bar{\sigma}$, is given by $\bar{\sigma} = \sqrt{\frac{1}{12}}(S - s)$. As established in Proposition 1 in Sheshinski and Weiss (1977), the range of prices $S - s$ is increasing in the inflation rate π . Obviously the elasticities of λ_a and of $\bar{\sigma}$ with respect to π are related because $S - s = \pi T$ and $\lambda_a = \frac{1}{T}$.

LEMMA 1. Assume that $\sigma^2 = 0$ and $\pi > 0$. Then it follows immediately that $\lambda_a^-(\pi, \sigma^2) = 0$ and that $\Delta_p^+(\pi, \sigma^2) = S - s$. Furthermore assume that $F(\cdot, 0)$ is three times differentiable, then

$$(2a) \quad \lim_{c \rightarrow 0} \frac{\partial \lambda_a}{\partial \pi} \frac{\pi}{\lambda_a} = \frac{2}{3} \text{ and}$$

$$(2b) \quad \lim_{c \rightarrow 0} \frac{\partial \bar{\sigma}}{\partial \pi} \frac{\pi}{\bar{\sigma}} = \frac{1}{3}$$

Proof. See Online Appendix AB. □

The lemma establishes that in the deterministic case when menu costs c are small, there are no price decreases, and the magnitude of price increases, $S - s$, increases with inflation at a rate of one-third. Also, as inflation increases, the time between consecutive price changes shrinks and the frequency of price adjustment increases with an elasticity of two-thirds.

Next, Lemma 2 analyzes the conditions under which the limiting values of the elasticities in Lemma 1 for the Sheshinski and Weiss (1977) model are the same as for the case with idiosyncratic costs, $\sigma > 0$, and very large π . Lemma 2 establishes that when the idiosyncratic shocks z are persistent and interest rates and menu

costs are small, the frequency of price adjustment is homogeneous of degree one in (π, σ^2) so that it can be written as a function of the ratio $\frac{\sigma^2}{\pi}$.

For the next results we write the frequency of price adjustment as a function of the rate of inflation, π , the variance of the idiosyncratic shock, σ^2 , the discount factor, r , and the inverse of the menu cost, $\frac{1}{c}$, that is, $\lambda_a(\pi, \sigma^2, r, \frac{1}{c})$. We also write the policy rules as functions of the parameters for each z ; $\Psi(\pi, \sigma^2, r, \frac{1}{c}; z) = \left[\underline{\psi}(z; \pi, \sigma^2, r, \frac{1}{c}), \bar{\psi}(z; \pi, \sigma^2, r, \frac{1}{c}), \hat{\psi}(z; \pi, \sigma^2, r, \frac{1}{c}) \right]$ and the expected price change functions as $\Delta_p^+(\pi, \sigma^2, r, \frac{1}{c})$ and $\Delta_p^-(\pi, \sigma^2, r, \frac{1}{c})$.

LEMMA 2. The function $\lambda_a(\pi, \sigma^2, r, \frac{1}{c})$ is homogeneous of degree one and the policy functions $\Psi(\pi, \sigma^2, r, \frac{1}{c})$ are homogeneous of degree zero in all the parameters. Therefore,

$$(3a) \quad \lim_{\pi \rightarrow \infty} \left[\lim_{c \downarrow 0, r \downarrow 0} \frac{\partial \lambda_a(\pi, \sigma^2, r, \frac{1}{c})}{\partial \pi} \frac{\pi}{\lambda_a(\pi, \sigma^2, r, \frac{1}{c})} \right]_{\sigma > 0} \\ = \lim_{\sigma \rightarrow 0} \left[\lim_{c \downarrow 0, r \downarrow 0} \frac{\partial \lambda_a(\pi, \sigma^2, r, \frac{1}{c})}{\partial \pi} \frac{\pi}{\lambda_a(\pi, \sigma^2, r, \frac{1}{c})} \right]_{\pi > 0}.$$

Also,

$$(3b) \quad \frac{\lim_{\pi \rightarrow \infty} \left[\lim_{c \downarrow 0, r \downarrow 0} \Psi(\pi, \sigma^2, r, \frac{1}{c}; z) \right]_{\sigma > 0}}{\lim_{\sigma \rightarrow 0} \left[\lim_{c \downarrow 0, r \downarrow 0} \Psi(\pi, \sigma^2, r, \frac{1}{c}; z) \right]_{\pi > 0}} = \mathbb{1} \quad \text{for all } z,$$

and

$$(3c) \quad \frac{\lim_{\pi \rightarrow \infty} \left[\lim_{c \downarrow 0, r \downarrow 0} \Delta_p^+(\pi, \sigma^2, r, \frac{1}{c}) \right]_{\sigma > 0}}{\lim_{\sigma \rightarrow 0} \left[\lim_{c \downarrow 0, r \downarrow 0} \Delta_p^+(\pi, \sigma^2, r, \frac{1}{c}) \right]_{\pi > 0}} = 1.$$

Proof. See [Online Appendix AC](#). □

The intuition underlying [Lemma 2's](#) proof is that multiplying r , π , σ^2 , and the profit function $F(\cdot)$ by a constant $k > 0$ is akin to changing the units in which we measure time. Moreover, the objective function on the right-hand side of [equation \(1\)](#) is homogeneous of degree one in $F(\cdot)$ and c and hence the policy function is the same whether we multiply $F(\cdot)$ by k or divide c by it. Thus λ_a is homogeneous of degree one in $(\pi, \sigma^2, r, \frac{1}{c})$. Likewise, $\lambda_a(\pi, \sigma^2)$ is homogeneous of degree one in (π, σ^2) when menu costs are

small, $c \downarrow 0$, and the interest rate is very low, $r \downarrow 0$. The interpretation of r going to zero is that instead of maximizing the expected discounted profit, the firm is maximizing the expected average profit, a case frequently analyzed in stopping-time problems (see, for example, Maurice 1981; Andrew and Zervos 2006), which we study in detail in Online Appendix AD.

Lemma 2 extends the result on the elasticity of the frequency of price adjustment with respect to inflation of equation (2a) in Lemma 1 to the case with $\sigma > 0$ and with an arbitrarily large π . This lemma requires that the shifter z has only permanent shocks. In a model with permanent shocks, there is no invariant distribution of z , and hence no invariant distribution of relative prices. However, in the version of Golosov and Lucas’s (2007) model considered in Section III, the invariant distribution is well defined. Indeed, the elasticity of the standard deviation of relative prices conditional on z with respect to inflation converges to one-third, as in Sheshinski-Weiss. Furthermore, as discussed in Section III, for large inflation rates, this implies that the elasticity of the unconditional standard deviation of relative prices with respect to inflation has an upper bound of one-third.

Using Lemma 1 and Lemma 2, we obtain the following result for the high-inflation case:

PROPOSITION 2. Assume that F is three times differentiable. Consider two firms with $\sigma_1, \sigma_2 > 0$. Then,

$$(4a) \quad \lim_{\pi \rightarrow \infty} \left[\frac{\lim_{c \downarrow 0, r \downarrow 0} \lambda_a(\pi, \sigma_1^2)}{\lim_{c \downarrow 0, r \downarrow 0} \lambda_a(\pi, \sigma_2^2)} \right] = 1$$

$$(4b) \quad \lim_{\pi \rightarrow \infty} \left[\lim_{c \downarrow 0, r \downarrow 0} \frac{\partial \lambda_a(\pi, \sigma_i^2)}{\partial \pi} \frac{\pi}{\lambda_a(\pi, \sigma_i^2)} \right] = \frac{2}{3} \text{ for } i = 1, 2$$

$$(4c) \quad \lim_{\pi \rightarrow \infty} \left[\lim_{c \downarrow 0, r \downarrow 0} \frac{\partial \Delta_p^+(\pi, \sigma_i^2)}{\partial \pi} \frac{\pi}{\Delta_p^+(\pi, \sigma_i^2)} \right] = \frac{1}{3} \text{ for } i = 1, 2.$$

Proposition 2 contains strong predictions about the limiting behavior of the frequency of price adjustment as inflation becomes large. The first part is a direct consequence of Lemma 2. It implies

that if we think that different industries have systematically different idiosyncratic shocks, we would expect the variance of these shocks to differ across industries and, hence, the frequency of price adjustment to be different across industries when inflation is low. Equation (4a) implies that differences in the frequency of price adjustment observed with low inflation should wash away as inflation becomes large. This is illustrated in the numerical example in the next section (see Figure II) and verified in the data (see Section IV.D). The intuition is that when inflation is low, the main driver of idiosyncratic nominal price changes are idiosyncratic shocks, and when inflation is high, the main driver of price changes is the growth of aggregate prices. The second part of Proposition 2 is a sharp prediction about the rate at which firms change the frequency of price adjustment when inflation grows. It states that this elasticity should be two-thirds in the limit when $\pi \rightarrow \infty$. Finally, equation (4c) states that the elasticity of price increases with respect to inflation converges to one-third as $\pi \rightarrow \infty$. It follows from the fact that Δ_p^+ is $S - s = \frac{\pi}{\lambda_q}$ when $\sigma = 0$.

The results of Proposition 2 apply to a wider set of models such as those mentioned in the comments to Proposition 1.

II.D. Decomposition of Changes in the Rate of Inflation

This section shows how steady-state changes in the rate of inflation can be decomposed into changes in the extensive and in the intensive margins of price adjustment, that is, changes in inflation accounted for by the frequency of price changes and changes in inflation accounted for by the size of price changes conditional on a price change taking place. Our main theoretical result is that for low inflation, the extensive margin accounts for 90% of changes in inflation while for high inflation it accounts for two-thirds of inflation changes.

As a matter of accounting, we can decompose the inflation rate as the difference between the product of the frequency of price increases times the average size of price increases and the product of the frequency of price decreases times the average size of price decreases. Formally:

$$\pi = \lambda_a^+ \Delta_p^+ - \lambda_a^- \Delta_p^-.$$

Totally differentiating the previous expression with respect to the inflation rate, and using the optimal decision rules, we can decompose the changes in the inflation rate into those due to changes

in the frequency, denoted by δ , and those due to changes in the average size, denoted by $1 - \delta$:

$$1 = \underbrace{\frac{\partial \lambda_a^+}{\partial \pi} \Delta_p^+ - \frac{\partial \lambda_a^-}{\partial \pi} \Delta_p^-}_{\text{Extensive Margin } \delta(\pi)} + \underbrace{\frac{\partial \Delta_p^+}{\partial \pi} \lambda_a^+ - \frac{\partial \Delta_p^-}{\partial \pi} \lambda_a^-}_{\text{Intensive Margin } 1-\delta(\pi)}.$$

We derive the decomposition of inflation for $\pi = 0$ and for $\pi \rightarrow \infty$ with a quadratic profit function,⁹ no discounting, and where z represents the (log of the) product cost and follows a driftless continuous time random walk.

PROPOSITION 3. Assume that $\sigma > 0$ and $F(p - \bar{p}, z) = -B(p - z)^2$, where $B > 0$ is a positive constant. Then,

$$\lim_{r \downarrow 0} \delta(0; r) = \frac{9}{10} \text{ and } \lim_{\pi \rightarrow \infty} \lim_{r \downarrow 0} \delta(\pi; r) = \frac{2}{3}.$$

Proof. See [Online Appendix AD](#). □

Notably, while [Proposition 3](#) states that 90% of changes in inflation around zero inflation are accounted for by the extensive margin of price adjustment, [Proposition 1](#) states that the frequency of price changes is insensitive to inflation. To gain insights into the interplay between the two propositions observe that for zero inflation the frequency of price increases and decreases are the same, that is $\lambda_a^+ = \lambda_a^-$, and also that the sizes are the same, that is $\Delta_p^+ = \Delta_p^-$. [Proposition 1](#) implies that $\frac{\partial \lambda_a^+}{\partial \pi} = -\frac{\partial \lambda_a^-}{\partial \pi}$ so the extensive margin at zero is $\delta(0) = 2\Delta_p^+ \frac{\partial \lambda_a^+}{\partial \pi}$. Since inflation introduces a negative trend in relative prices, it induces them to hit more often the lower limit of the inaction set, prompting more price increases and fewer price decreases. The characterization of optimal policies in the proof of [Proposition 3](#) shows that these changes in the frequency of price increases and decreases account for 90% of changes in the rate of inflation at $\pi = 0$. A similar argument holds for the decomposition of the change of inflation in a mild deflation.

9. Equivalently, we can write the result for a small fixed cost c , so that prices are close to the profit-maximizing value, and thus a second-order expansion of the profit function is accurate.

The key technical insight in the proof of [Proposition 3](#) is that $\lim_{r \rightarrow 0} rV(x, r)$ is finite and independent of x , where $V(x, r)$ is the value function and where $x \equiv p - z$ is the markup. This allows us to obtain an analytical solution of the value function and to characterize optimal policies.

Finally, the following corollary to [Proposition 3](#) presents a sharp prediction about how changes in inflation affect the frequency of price increases and decreases when inflation is low.

COROLLARY 1. Around $\pi = 0$, the difference between the frequency of price increases and decreases rises with inflation. Formally,

$$\frac{\partial(\lambda_a^+ - \lambda_a^-)}{\partial\pi}\Big|_{\pi=0} = \frac{\delta(0)}{\Delta_p^+|_{\pi=0}} = \frac{\frac{9}{10}}{\Delta_p^+|_{\pi=0}} > 0.$$

Taken together, the results at low inflation in [Proposition 3](#) and its corollary imply that inflation rises when inflation is low mostly because the frequency of price increases rises and that of price decreases falls (the extensive margin), as opposed to the size of price increases rising and that of price decreases falling (the intensive margin).

1. General Remarks. We conclude this section with a few remarks on the applicability of these comparative static results to the time series variation in our data set. The propositions in this section were obtained under the assumption that inflation is to remain constant at the rate π , and that the frequency of price changes is computed under the invariant distribution. Thus, strictly speaking, our propositions are not predictions for time series variation but comparative static results.

We give three comments in this regard. First, this should be less of a concern for very high inflation, since the model becomes close to static, that is, firms change prices very often and thus the adjustment to the invariant distribution happens very fast. Second, when we analyze the Argentinean data, we correlate the current frequency of price changes with an average of the current and future inflation rates. We experiment with different definitions of these averages and find that the estimates of the elasticities in the first two propositions of this section are not sensitive to this choice. Moreover, with Argentina's experience in mind, [Beraja \(2013\)](#) studies the transitional dynamics in a menu cost

model where agents anticipate a disinflation in the future and performs the same comparative statics with artificial data generated from such model. He finds that the theoretical results in this section are robust to conducting the analysis in a nonstationary economy during a disinflation process calibrated to the Argentine economy. Third, in [Section III](#) we numerically solve a standard version of the menu cost model for reasonable parameter values for menu cost c and discount rate r , which are positive but small, and for finite but large inflation rates π , of the order that are observed in Argentina. We find that the propositions in this section (which use limit values for c , r , and π) accurately predict the behavior of the statistics of interest computed in the calibrated model.

III. ILLUSTRATING THE THEORY WITH GOLOSOV AND LUCAS'S (2007) MODEL

In this section, we specify a version of the firm's problem studied in [Section II.A](#) to illustrate the theory. We characterize the solution of the model analytically and numerically and show how changes in the rate of inflation affect optimal pricing rules, the frequency of price changes, and the size of price adjustments. The example also verifies the robustness of the analytical predictions obtained so far. In [Section II.A](#), we obtained sharp analytical results under a variety of simplifying assumptions such as limit values of parameters (e.g., vanishing menu cost c and or discount rate r), or the shape of profit functions F . Also, our analytical results were obtained at two extreme values of inflation. In this section, we check the robustness of the simplifying assumptions by computing a version of the model away from the limit cases, considering values of inflation in the range observed in Argentina.

The example is a version of the [Goloso and Lucas \(2007\)](#) model, identical to the one in [Kehoe and Midrigan \(2015\)](#).¹⁰ Specifically, we assume a constant elasticity of demand, a constant returns to scale production technology, idiosyncratic shocks to marginal cost that are permanent, an exponentially distributed product life, and a cost of changing prices that is proportional to current profits (but independent of the size of the price change).

10. We zero out the transitory shock that gives rise to sales in [Kehoe and Midrigan \(2015\)](#) and write the model in continuous time.

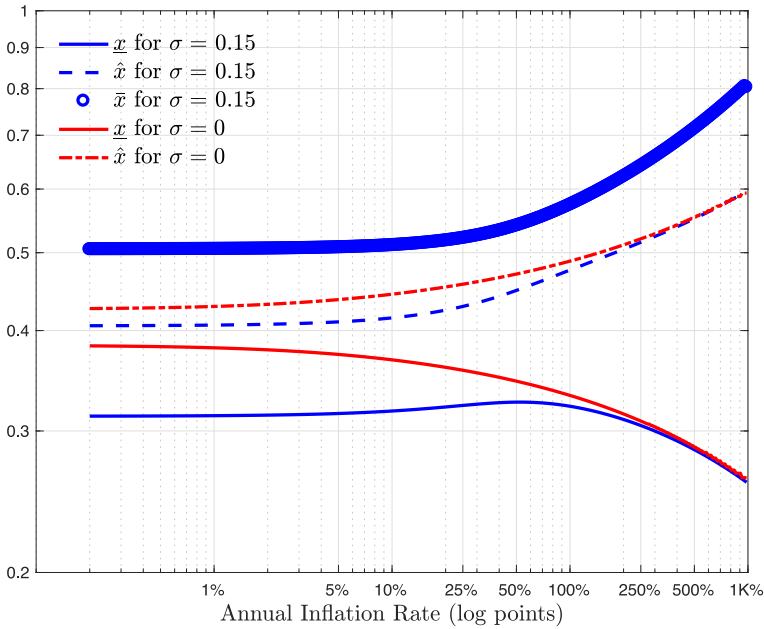


FIGURE I

Optimal Thresholds for Different Inflation Rates

Online Appendix B presents a more detailed description of the setup.

Furthermore, in Online Appendix BA, we present several propositions with an analytical characterization of the solution of this model. A novel contribution of this article is to derive a system of three equations in three unknowns for optimal pricing rules, as well as the explicit solution to the value function. We also derive an explicit solution for the expected number of adjustments per unit of time λ_a , and we characterize the density g of the invariant distribution of (p, z) . We believe these derivations could be useful for researchers interested in menu cost models more generally.

The remainder of this section contains several figures that describe numerically how changes in the rate of inflation affect the optimal pricing rules, the frequency of price changes, and the size of price adjustments. Again, see Online Appendix B for details on the calibration of the model underlying these figures.

Figure I illustrates how the optimal pricing policies vary with inflation for two cases, $\sigma = 0.15$ and $\sigma = 0$. The dashed center

lines are the optimal return markup and the outer lines are the boundaries of the inaction set in each case. With no inflation and $\sigma > 0$, the markup will drift away from the starting optimal markup, driven by the idiosyncratic shock. The firm will keep its nominal price fixed as long as it does not hit the boundaries. Once the markup hits either boundary, \underline{x} or \bar{x} , the firm resets the price and the markup returns to \hat{x} . The light gray (in color version, online, red) lines depict the optimal thresholds for Sheshinski and Weiss's (1977) case with $\sigma = 0$. Markups always fall when there are no idiosyncratic shocks after the firm resets its nominal price. Hence, the upper limit of the inaction set becomes irrelevant. The firm resets its nominal price to $\hat{x} + z$ when the markup hits the lower bound \underline{x} and waits for it to fall again.

Figure I shows several properties of the menu cost models we study.¹¹ At very low inflation rates, and when $\sigma > 0$, the thresholds are symmetric, that is, the distance between \underline{x} and \hat{x} is the same as the distance between \bar{x} and \hat{x} . This symmetry implies that the size of price increases is equal to the size of price decreases, $\Delta_p^+(0, \sigma) = \Delta_p^-(0, \sigma)$, and that the frequency of price increases is equal to the frequency of price decreases, $\lambda_a^+(0, \sigma) = \lambda_a^-(0, \sigma)$. These illustrate the results obtained in part (iii) of Proposition 1. At very high inflation rates, the models with $\sigma > 0$ and with $\sigma = 0$ are equivalent in the sense that the critical values \underline{x} and \hat{x} in Golosov and Lucas's (2007) model converge to the Ss bands in Sheshinski and Weiss's (1977) model as established in equation (3b) in Lemma 2. As a result, the magnitude of price changes in the two models is the same as in equation (3c) in Lemma 2, $\Delta_p^+(\pi, 0) = S - s = \hat{x} - \underline{x}$. For rates of inflation above 250% a year, Figure I also shows that the elasticity of $\Delta_p^+(\pi, 0)$ with respect to inflation is close to one-third—equation (4c) in Proposition 2.

Figure II, Panel A (color version online) displays the frequency of price increases λ_a^+ , together with the frequency of all adjustments λ_a , for two values of the cost volatility σ . There are several interesting observations about this figure. First, the frequency λ_a is insensitive to inflation in the neighborhood of zero inflation as established in part (i) of Proposition 1. Second, the length of the inflation interval around $\pi = 0$ where λ_a is

11. This example does not exactly satisfy all the assumptions of the model in Section II.A since the profit function F derived from a constant elasticity demand is not symmetric. Yet for small cost c , the terms in the quadratic expansion, which are symmetric by construction, should provide an accurate approximation.

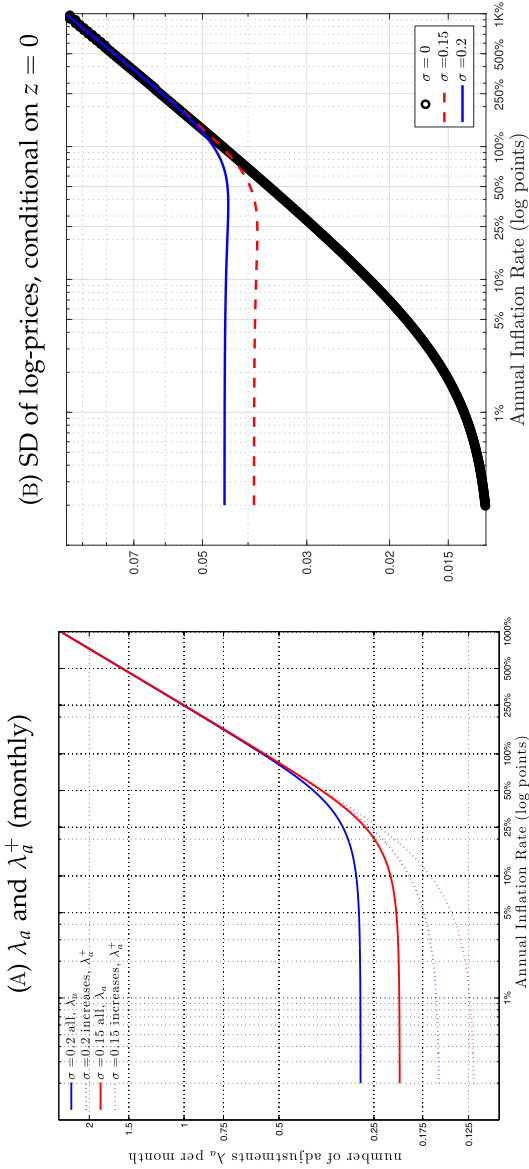


FIGURE II
Frequency of Price Changes and Standard Deviation of log-prices

approximately constant increases with σ —(see the discussion in note 7). Third, the last part of [Proposition 1](#) predicts that the frequency of price increases and price decreases is the same when $\pi = 0$. The figure shows that for low inflation the frequency of price increases is about half of the frequency of price changes, indicating that half the price changes are increases and half are decreases. Fourth, for values of inflation above 250% a year, the frequency of price changes λ_a for different values of σ are approximately the same, consistent with the limiting results in [equation \(4a\)](#) of [Proposition 2](#). Fifth, since the graph is in log scale, it is clear that the common slope is approximately constant for high inflation, and close to two-thirds as established in [equation \(4b\)](#) in [Proposition 2](#). Finally, as inflation becomes large all price adjustments are price increases—as can be seen from the fact that λ_a^+ converges to λ_a for each value of σ .

[Figure II](#), Panel B displays the standard deviation of log prices, conditional on $z = 0$. It shows that the elasticity of the standard deviation of relative prices conditional on z with respect to inflation is approximately zero for $\pi = 0$ and $\sigma > 0$ (as in [Proposition 1](#)) and it is approximately one-third for large π (as in [Lemma 1](#)). Moreover, for the case when $\sigma > 0$, Panel B shows that the standard deviation of relative prices converges to the case with $\sigma = 0$ and large enough π . Thus, the elasticity with respect to inflation of the standard deviation of relative prices, conditional on $z = 0$, converges to one-third as in Sheshinski and Weiss. Even though the figure only shows this property conditional on $z = 0$, this also holds for all z . This can be seen by using the characterization of the invariant distribution of relative prices in [Proposition 6](#) in the [Online Appendix](#) and taking limits as $\pi \rightarrow \infty$.

Next we analyze the unconditional dispersion of relative prices. The standard deviation of relative prices conditional on z mostly captures the price dispersion coming from asynchronous price adjustments to inflation. The only other remaining source of dispersion in log prices is due to firm idiosyncratic shocks z . To see this, it is helpful to decompose the unconditional variance of relative prices $\bar{\sigma}^2(p; \pi, \cdot)$ for a given inflation rate π as follows:

$$(5) \quad \bar{\sigma}^2(p; \pi, \cdot) = \mathbb{E}[\text{Var}(p|z; \pi, \cdot)] + \text{Var}[\mathbb{E}(p|z; \pi, \cdot)].$$

Here, the first term, $\mathbb{E}[\text{Var}(p|z; \pi, \cdot)]$ corresponds to [Figure II](#), Panel B because, as we show in [Online Appendix BA](#), the log-price distribution $g(\cdot, z)$ has the same shape for any value

of z . As for the term $\text{Var} [\mathbb{E}(p|z; \pi, \cdot)]$, this source of dispersion is mostly exogenous because the variance of the average price is equal to the cross-sectional dispersion of z for all values of π when menu costs are zero.

Taken together, the foregoing discussion implies that relative price dispersion is insensitive to inflation when inflation is low because it reflects idiosyncratic firm shocks. However, for large enough inflation rates, relative price dispersion has an elasticity with respect to inflation that has an upper bound of one-third because the conditional variance for a given z has an elasticity of exactly one-third and the second term in [equation \(5\)](#) is insensitive to inflation. To illustrate this, [Figure XII in Online Appendix BB](#) plots the unconditional standard deviation of log-prices for different values of volatility of the idiosyncratic shocks and average product life. We observe that the elasticity increases when the second term in [equation \(5\)](#) becomes smaller (for example, when the average product life decreases). Yet it takes much higher inflation rates than the ones observed in the peak months in Argentina for the elasticity to reach the upper bound of one-third for high inflation rates.

IV. ARGENTINA'S EVIDENCE ON MENU COST MODELS OF PRICE DYNAMICS

In [Section II](#) we uncovered several properties of menu cost models that can be contrasted with data. The presentation of the empirical results in this section is organized around those predictions. As a reminder, these are:

- (i) The elasticity of the frequency of price changes λ with respect to changes in the rate of inflation is zero at low inflation rates, and it approximates two-thirds as inflation becomes very large.¹²
- (ii) The dispersion of the frequency of price changes across goods decreases with inflation. It is zero when inflation goes to infinity and the model converges to the [Sheshinski and Weiss \(1977\)](#) model with no idiosyncratic shocks.

12. This result is robust to extending the menu cost model to one where the menu cost is zero at some random times, so that they combine menu costs and Calvo-type price adjustments. See, for example [Nakamura and Steinsson \(2010\)](#) and [Alvarez, Le Bihan, and Lippi \(2016\)](#). In such random-menu-cost models, as $\pi \rightarrow \infty$, the frequency also converges to two-thirds.

- (iii) Intensive and extensive margins of price increases and decreases.
 - (a) The frequency of price increases and of price decreases are similar at low inflation rates.
 - (b) The size of price increases and price decreases are similar at low inflation rates.
 - (c) At low inflation rates, as inflation grows, the frequency of price changes remains constant while the frequency of price increases rises and the frequency of price decreases falls.
 - (d) For high inflation rates, the frequency of price increases converges to λ and the frequency of price decreases converges to zero.
 - (e) The size of price changes is an increasing function of the inflation rate.
- (iv) The elasticity of the dispersion of prices across stores with respect to inflation is zero for low inflation rates and it is bounded by one-third when inflation becomes very large.

Next, we look at each of these predictions in the Argentinean data.

IV.A. Description of the Data Set

Our data set contains 8,618,345 price quotes underlying the consumer price index for the Buenos Aires metropolitan area in the period December 1988 to September 1997. Each price quote represents an item, that is, a good or service of a determined brand sold in a specific outlet in a specific period of time. Goods¹³ and outlets are chosen to be representative of consumer expenditure in the 1986 consumer expenditure survey.¹⁴ Price quotes are for 506 goods that account for about 84% of household expenditures.

Goods are divided into two groups: homogeneous and differentiated goods. Differentiated goods represent 50.5% of the expenditure in our sample while homogeneous goods account for the remaining 49.5%.¹⁵ Prices are collected every two weeks for all homogeneous goods and for those differentiated goods sold in

13. To simplify the exposition, when it is clear, we use goods to refer to either goods or services.

14. Encuesta Nacional de Gasto de los Hogares.

15. Examples of homogeneous goods are barley bread, chicken, and lettuce. Examples of differentiated goods are moccasin shoes, utilities, tourism, and professional services.

supermarket chains. They are gathered every month for the rest of the differentiated goods. The data set contains 233 prices collected every two weeks and 302 prices collected every month. Twenty-nine of each of these goods are gathered both monthly and twice a month.¹⁶

An important feature of the data set is the rich cross section of outlets where prices are recorded at each point in time. Over the whole sample there are 11,659 outlets, roughly around 3,200 outlets per month for homogeneous goods and about the same number for differentiated goods. On average, across the nine years, there are 166 outlets per good (81 outlets per product collected monthly and 265 per good collected bimonthly). [Online Appendix C](#) contains further information on data collection and on the classification of goods.

We exclude from the sample price quotes for baskets of goods, rents, and fuel prices. Baskets correspond to around 9.91% of total expenditure and are excluded because their prices are gathered for any good in a basket, that is, if one good is not available, it is substituted by another in the basket. Examples are medicines and cigarettes. Rents are sampled monthly for a fixed set of representative properties. Reported prices represent the average of the sampled properties and include what is paid on that month, as opposed to what is paid for a new contract. Rents represent 2.33% of household expenditure. Fuel prices account for 4% of total expenditure, and we exclude them because they were gathered in a separate database that we do not have access to.

The data set has some missing observations and flags for stock-outs, price substitutions, and sales. We treat stock-outs (10.5% of observations) and price quotes with no recorded information (2.25% of observations) as missing observations. The statistical agency substitutes the price quote of an item for a similar item, typically when the good is either discontinued by the producer or not sold any longer by an outlet. Using this definition, across the nine years of our data set, we have an average of 2.39% of price quotes that have been substituted. The data set contains an indicator of whether an item was on sale. Around 5% of items have a sale flag. This is small compared with the 11% frequency

16. The outlets are divided into 20 waves, corresponding to the 20 working days of the month. Each outlet is visited roughly on the same working day every 10 working days in the case of homogeneous goods and differentiated goods gathered at supermarkets. The data set includes the particular day when each price is gathered.

of sales reported by [Klenow and Kryvtsov \(2008\)](#) for the United States. Seventy percent of the sales correspond to homogeneous items (this is similar to [Klenow and Kryvtsov 2008](#). They report that sales are more frequent for food items). The time series data for the number of outlets per good and for the frequencies of missing observations, substitutions, and sales are depicted in [Figure III](#) (color version online).

IV.B. *Estimating the Frequency of Price Changes*

We extend the methodology of [Klenow and Kryvtsov \(2008\)](#) for estimating the frequency of price changes to the case of time-varying frequencies of price changes.¹⁷ We assume a constant probability of a price change per unit of time (a month for differentiated goods and two weeks for homogeneous goods) so that the arrival rate of a price change follows a Poisson process. In this case, the maximum likelihood estimator of the frequency of price changes is

$$\lambda_t = -\ln(1 - \text{fraction of outlets that changed price between } t \text{ and } t - 1).$$

(6)

The fraction of outlets that changed their price between periods can be calculated for individual goods or for the aggregate by pooling the data for all outlets and all goods together. In this computation, we drop observations with missing price quotes. This simple estimator just counts the fraction of price changes in a period of time, and transforms it into a per unit of time rate, λ . We refer to λ as the “instantaneous” frequency of price changes.

Later, we perform robustness checks by using different methods of aggregation across goods, by considering different treatments for sales, substitutions, and missing observations, and by dropping the assumption that price changes follow a Poisson process.

[Figure IV](#) plots the monthly time series of the simple pooled estimator of λ and the expected inflation rate. It assumes that all homogeneous and all differentiated goods have the same frequency of price changes and estimates this aggregate frequency by using the simple pooled estimator for the homogeneous and for the

17. Using the same methodology makes our study comparable to most of the papers in the literature.

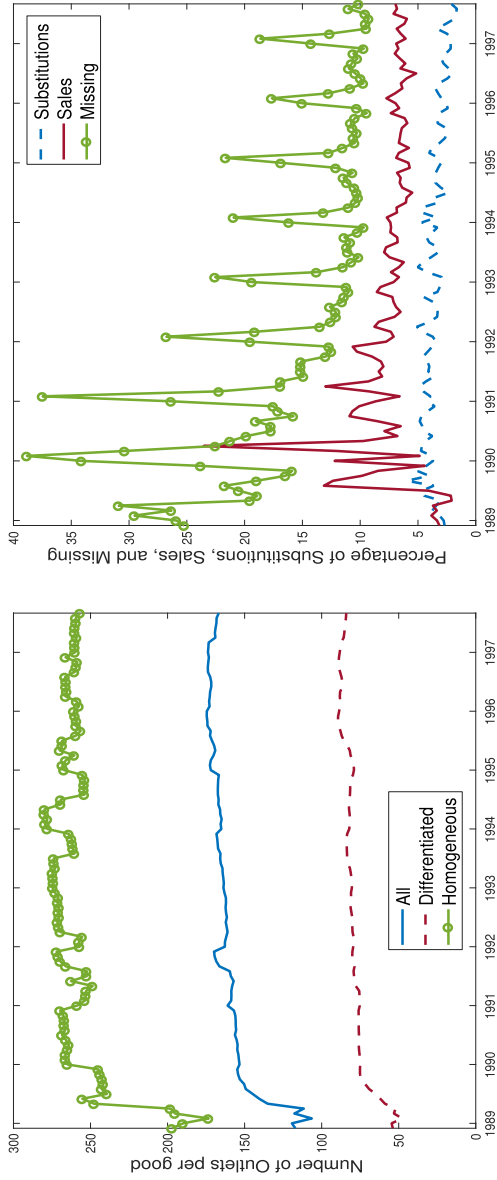


FIGURE III

Number of Outlets per Good, and Frequencies of Substitution and Sales

For the homogeneous goods during a month, we count a sale or substitution if there was one such event in any of the two 15-day subperiods. Missing includes stock-outs.

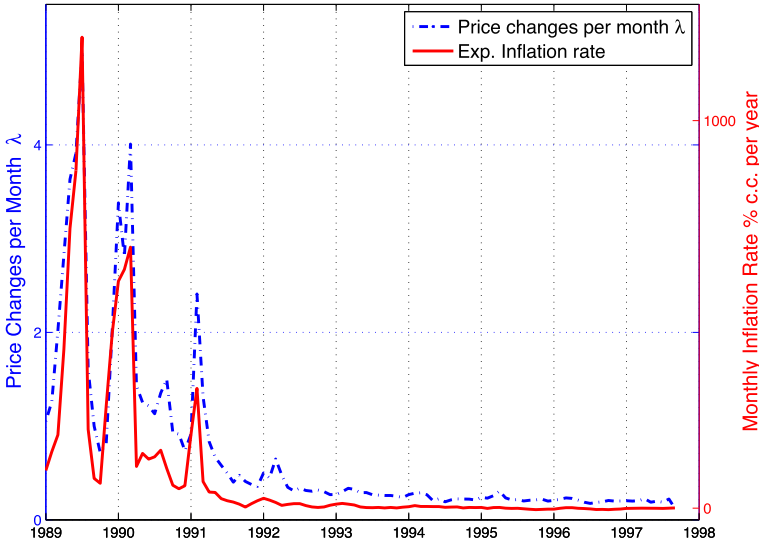


FIGURE IV

Estimated Frequency of Price Changes λ and Expected Inflation

The λ shown is the expenditure-share-weighted average of the homogeneous and differentiated goods' simple estimator $\hat{\lambda} = -\log(1 - f_t)$, where f_t is the fraction of outlets that changed price in period t . Inflation is the average of the log-difference of monthly prices multiplied by 1,200 and weighted by expenditure shares. Expected inflation is the average inflation rate $\frac{1}{\lambda_t}$ periods ahead.

differentiated goods. The biweekly estimates of the homogeneous goods are aggregated to a monthly frequency,¹⁸ and the plot shows the weighted average of these two estimators, using the share of household expenditures as weights. Finally, the expected inflation is computed as the average inflation rate $\frac{1}{\lambda_t}$ periods ahead. We observe that the two variables are correlated. For instance, during the mid-1989 hyperinflation, the implied expected duration of a price spell is close to one week; after 1993, the implied expected duration is close to half a year.

IV.C. The Frequency of Price Changes and Inflation

In this section, we report how the estimated frequency of price changes varies with inflation. We find that, as predicted by

18. The monthly frequency is the sum of the biweekly frequencies of each month.

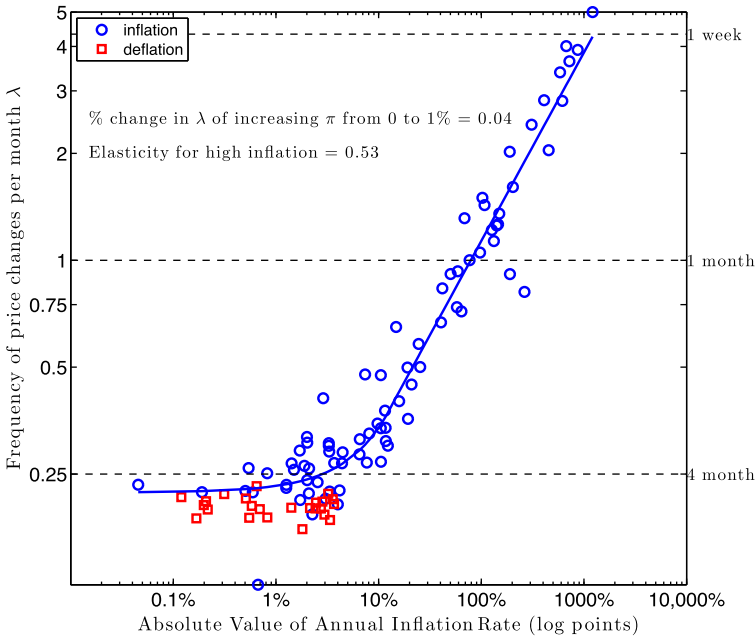


FIGURE V

The Frequency of Price Changes (λ) and Expected Inflation

The λ shown is the expenditure-share-weighted average of the homogeneous and differentiated goods' simple estimator $\hat{\lambda} = -\log(1 - f_t)$, where f_t is the fraction of outlets that changed price in period t . Inflation is the average of the log difference of monthly prices weighted by expenditure shares. Expected inflation is the average inflation rate $\frac{1}{\lambda_t}$ periods ahead. The fitted line is $\log \lambda = a + \epsilon \min \{ \pi - \pi^c, 0 \} + \nu (\min \{ \pi - \pi^c, 0 \})^2 + \gamma \max \{ \log \pi - \log \pi^c, 0 \}$. The squares represent negative expected inflation rates and the circles positive ones.

the menu cost model, the frequency of price changes is insensitive to inflation when inflation is low. Moreover, for high inflation rates, we find that the elasticity of the frequency of price changes with respect to inflation is between one-half and the theoretical two-thirds.

Figure V plots the frequency of price changes against the rate of inflation using a log scale for both variables.¹⁹ On the right axis we indicate the implied instantaneous duration, that is, $\frac{1}{\lambda}$. In interpreting this figure, as well as the other estimates presented

19. See Section II for caveats on these results and for the interpretation of contemporaneous correlations.

below, it is worth noting that $\frac{1}{\lambda_t}$ is the expected duration of prices at time t if λ_t would remain constant in the future and provided that the probability of a price change is the same within the smallest period of observation (one month for differentiated goods and two weeks for homogeneous goods).

Motivated by the theoretical considerations in [Section II](#), as well as the patterns evidenced in [Figure V](#), we fit (estimated via nonlinear least squares) the following statistical model to the data:

$$(7) \quad \log \lambda = a + \epsilon \min \{ \pi - \pi^c, 0 \} + \nu (\min \{ \pi - \pi^c, 0 \})^2 \\ + \gamma \max \{ \log \pi - \log \pi^c, 0 \}.$$

This model assumes that $\log \lambda$ is a quadratic function of inflation for inflation rates below the critical value, π^c and that $\log \lambda$ is a linear function of $\log \pi$ for inflation rates above π^c . In [Figure V](#) we observe that λ is insensitive to inflation at low inflation rates. Increasing inflation from 0% to 1% a year increases the frequency of price changes by only 0.04%. Moreover, the behavior of λ is symmetric around zero. The frequency of price changes starts to rise for inflation rates under 5% a year. For high inflation rates, the elasticity of λ with respect to inflation is captured by the parameter γ . We estimate γ to be at least one-half but smaller than the theoretical limit of two-thirds. Also, as predicted by the menu cost model, as inflation rises this elasticity becomes constant—that is, the linear fit for $\log \lambda$ as a function of $\log \pi$ works well for high inflation rates. In this estimation, the critical value π^c in the statistical model, which has no theoretical interpretation, is of 14% a year.²⁰ The expected duration of price spells for zero inflation is 4.5 months, which is consistent with international evidence, as the next section shows.

The strong results in [Figure V](#) are surprising as the model applies to steady states in which the inflation rate has been at the same level for a long time. [Section IV.C.2](#) presents two sets of robustness checks along these lines. First, we replicated [Figure V](#) using current inflation as well as different measures of expected

20. The comparative static of the models discussed in [Section II](#) does not imply a kink like the one in [equation \(7\)](#), we merely use this specification because it is a low-dimensional representation of interesting patterns in the data that provides a good fit and has properties at the extreme values that are consistent with our interpretation of the theory.

inflation instead of inflation. The results are similar to those in [Figure V](#) but somewhat weaker. Second, we redo the analysis on data simulated from our model in [Section III](#), but in a nonstationary economy where agents anticipate a disinflation like the one that occurred in Argentina following the exchange rate peg. Again, our results are similar to conducting the analysis in a stationary economy with a fixed inflation rate.

1. International Evidence on the Frequency of Price Changes and Inflation. The previous section shows that certain aspects of price-setting behavior in Argentina are consistent with the predictions of menu cost models. In particular, the elasticity of the frequency of price changes is close to zero at low inflation rates and close to two-thirds for high inflation rates. Here we show that Argentina's inflationary experience is of special interest because it both spans and extends previous findings in the literature.

There are several studies that estimate the frequency of price changes for countries experiencing different inflation rates. [Figure VI](#) provides a visual summary of these studies and compares them to ours by adding the international evidence to [Figure V](#).

First, observe how the wide range of inflation rates covered by our sample makes this article unique: none of the other papers covers inflation rates ranging from a mild deflation to 7.2 million percent a year (annualized rate of inflation in July 1989). This is what allows us to estimate the elasticity of the frequency of price changes with respect to inflation both at low and high inflation rates. In the other samples it is hard to test these hypotheses because of their limited inflation range. Second, we note how the patterns of the data for each country are consistent with the two predictions of the menu cost model. Third, we note that in most cases, the level of the estimated frequency of price changes is similar to Argentina's. The similarity between our results and the existing literature is remarkable since the other studies involve different economies, different goods, and different time periods. It is a strong indicator that our results are of general interest, as the theory suggests, and are not a special feature of Argentina.²¹

21. Table XV in [Online Appendix G](#) provides a succinct comparison of the data sets used in these studies and of the inflationary environment in place in each case. The table shows that in addition to covering a wider range of inflation rates our data set is special due to its broad coverage that includes more than 500 goods representing 85% of Argentina's consumption expenditures.

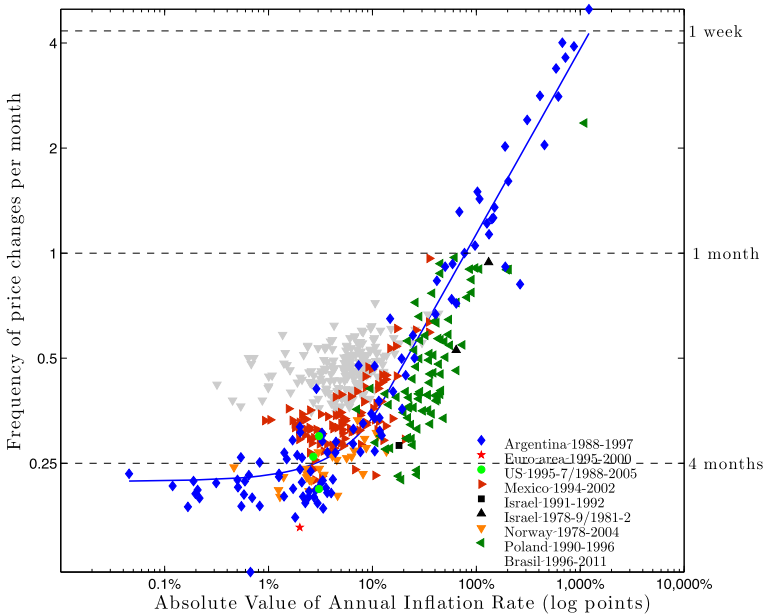


FIGURE VI

The Frequency of Price Changes (λ) and Expected Inflation: International Evidence

A color version of this figure is available online. Price changes per month for Argentina are the simple pooled estimator of λ . For the other cases we plot $-\log(1 - f)$, where f is the reported frequency of price changes in each study. The (λ, π) pairs for Argentina, Mexico, and Brazil are estimated once a month and for the other countries once a year. Expected inflation is the average inflation $\frac{1}{\lambda}$ months ahead. Data for the Euro area is from [Álvarez et al. \(2006\)](#), for the United States from [Bils and Klenow \(2004\)](#), [Klenow and Kryvtsov \(2008\)](#), and [Nakamura and Steinsson \(2008\)](#), for Mexico from [Gagnon \(2009\)](#), for Israel from [Baharad and Eden \(2004\)](#) and [Lach and Tsiddon \(1992\)](#), for Poland from [Konieczny and Skrzypacz \(2005\)](#), for Brazil [Barros et al. \(2009\)](#), and for Norway from [Wulfsberg \(2016\)](#). Logarithmic scales for both axes.

The studies included in the figure are all the ones we could find covering a wide inflation range. For the low inflation range we included studies for the United States by [Bils and Klenow \(2004\)](#), [Klenow and Kryvtsov \(2008\)](#), and [Nakamura and Steinsson \(2008\)](#), and for the Euro area by [Álvarez et al. \(2006\)](#).²² Our estimates of the frequency of price changes are consistent with all

22. Recently [Nakamura et al. \(2018\)](#) have extended the U.S. data to cover an earlier period, which includes inflation rates going up to 14% a year.

of them.²³ We have three data points for Israel corresponding to an inflation rate of 16% a year between 1991 and 1992 (Baharad and Edén 2004), 64% a year between 1978 and 1979, and 120% a year between 1981 and 1982 (Lach and Tsiddon 1992). The frequency of price changes for these three points is well aligned with the Argentine data. The same is true for the Norwegian data (Wulfsberg 2016) that ranges from 0.5% to 14% a year. For Poland, Mexico, and Brazil, we were able to obtain monthly data for a wide range of inflation rates. The Polish sample ranges from 18% to 249% a year (Konieczny and Skrzypacz 2005) and the Mexican one ranges from 3.5% to 45% a year (Gagnon 2009). In both cases, the observations are aligned with the Argentina sample. The Brazilian data (Barros et al. 2009) yields an elasticity of the frequency of price changes at high inflation that is consistent with ours. However, it yields a higher level of the frequency of price changes than ours and other studies.

2. *Robustness.* Next we conduct a number of robustness exercises to evaluate the sensitivity of the main results regarding the frequency of price changes. The first set of exercises deals with recurrent issues when analyzing micro-price data sets, such as missing observations and price changes due to substitutions or sales, as well as issues of aggregation across products. Second, we address biases resulting from discrete sampling. Third, we present results using different measures of expected inflation. Finally, we address the possibility that the theoretical propositions that hold in the steady state are a poor description of the Argentine experience in the high-inflation period leading to the stabilization plan in 1991 where agents are likely to have anticipated the strong disinflation that followed.

The conclusions are twofold. First, at low inflation rates the empirical findings of this section go through intact. Second, at high inflation, we observe some quantitative but not qualitative differences. Most notably, depending on the estimator used to aggregate the data, the elasticity of the frequency of price changes can range from approximately one-half to the theoretical two-thirds.

i. Missing Data, Substitutions, Sales, and Aggregation. Table I reports the sensitivity of the estimates of the elasticity γ , the

23. There are other studies for low-inflation countries, especially for the Euro area, but since they mostly yield estimates similar to those of Álvarez et al. (2006) we do not report them (see Álvarez et al. 2006 and Klenow and Malin 2011 for references to these studies).

TABLE I
THE FREQUENCY OF PRICE ADJUSTMENT AND INFLATION: ROBUSTNESS CHECKS

Aggregation	Elasticity γ			Semi-elasticity at zero $\Delta\% \lambda$			Expected duration at $\pi = 0$ (months)		
	Diff.	Hom.	Agg.	Diff.	Hom.	Agg.	Diff.	Hom.	Agg.
Panel A: Simple estimator (no information from missing price quotes)									
Pooled	0.51	0.5	0.53	0.08	-0.01	0.04	9.1	2.9	4.5
Weighted average	0.52	0.48	0.52	0.07	-0.01	0.04	8.2	2.8	4.4
Median	0.64	0.64	0.68	0.1	0.01	0.05	15	7	9.3
Weighted Median	0.65	0.64	0.68	0.09	0	0.04	12.2	5.5	7.8
Pooled (excluding sales)	0.5	0.47	0.52	0.08	0.01	0.05	10	3.8	5.7
Panel B: All price quotes									
Pooled	0.51	0.5	0.52	0.08	0	0.04	8.8	3	5.1
Weighted average	0.52	0.45	0.49	-0.05	-0.01	0.04	8.9	2.9	4.6
Median	0.62	0.58	0.65	0.09	-0.1	0.02	15.3	4.4	10.2
Weighted median	0.62	0.65	0.65	0.09	0.01	0.04	12.9	5.4	7.5
Panel C: Excluding substitution quotes									
Pooled	0.55	0.5	0.52	0.09	-0.01	0.03	7	3.2	6.2
Weighted average	0.52	0.45	0.51	0.06	-0.01	0.03	10.7	3	6.7
Median	0.66	0.65	0.68	0.13	0.01	0.02	18.8	9	12.4
Weighted median	0.66	0.62	0.66	0.07	-0.03	0.02	16.5	6.1	10.4
Panel D: Excluding substitution spells									
Pooled	0.52	0.5	0.52	0.07	0	0.05	10.7	3	5.5
Weighted average	0.53	0.44	0.49	-0.1	-0.01	0.04	8.6	2.9	4.8
Median	0.62	0.64	0.64	0.09	0.02	-0.02	18.4	8.3	10.9
Weighted median	0.63	0.6	0.66	0.09	-0.07	-0.05	15.4	4.7	8.2
Panel E: Excluding substitution and sales quotes									
Pooled	0.5	0.47	0.52	0.08	0.03	0.05	9.9	2.6	6.7

Notes. Diff. denotes differentiated goods, which are sampled once a month. Hom. denotes homogeneous goods, which are sampled twice a month. Agg. denotes the weighted average of the differentiated and homogeneous goods, with weights given by the expenditure shares and where the homogeneous goods have been aggregated to monthly frequencies. For each case we use NLLS to fit: $\log \lambda_t = a + \epsilon \min \{\pi_t - \pi^c, 0\} + \nu (\min \{\pi_t - \pi^c, 0\})^2 + \gamma \max \{\log \pi_t - \log \pi^c, 0\} + \omega_t$. The semi-elasticity at zero $\Delta\% \lambda$ is the percentage change in λ when inflation goes from 0% to 1%. Panel A estimates λ with the simple estimator in equation (6) discarding information from missing prices, Panel B is the full information maximum likelihood estimator described in Online Appendix DA, Panel C replaces price quotes with a product substitution by missing data, Panel D replaces price spells ending in a product substitution by missing data, and Panel E replaces sales quotes by the previous price and product substitutions by a missing quote.

semi-elasticity $\Delta\% \lambda$ and the duration at low inflation obtained with the simple estimator to different treatments of missing data, sales, product substitution, and broad aggregation levels. In Online Appendix DA we describe in detail the methodologies and the definitions of all estimators in Table I.

We report estimates of the three parameters of equation (7) for the sample of differentiated goods (sampled monthly), for the sample of homogeneous goods (sampled twice a month) and for the aggregate. The latter is obtained by averaging the estimated λ s with their expenditure shares after converting the biweekly estimates to monthly ones.

The first and second columns show the elasticities at high and low inflation. The third block of columns shows the implied duration of price spells when inflation is low (below the threshold) under the assumption that the frequency of price adjustment is constant.

The first row in [Table I](#) corresponds to the pooled simple estimator reported in [Figure V](#). The estimates of the elasticity of the frequency of price adjustment with respect to inflation, γ , are very similar for the λ s in the two samples and for the aggregate λ . The estimates for the semi-elasticity and expected duration at low inflation are markedly higher for differentiated goods in comparison to homogeneous goods. The other lines in the table provide estimates of the three parameters of interest for different aggregation methods and for the different treatments of missing observations, product substitutions, and sales. The values for the elasticity γ across all these estimation techniques ranges from approximately one-half to two-thirds. The variation in $\Delta\% \lambda$ estimates is much smaller across methodologies. Both differences in the estimators result from alternative aggregation methodologies and not from the treatment of sales, substitutions, and missing values. For instance, the elasticity at high inflation when using the simple estimator with pooled data climbs from 0.53 to 0.68 when using the median estimate across industries.

The treatment of sales and substitutions does seem to have an effect on the estimates of the expected duration of price spells when inflation is low, as in other papers in the literature (see [Klenow and Malin 2011](#)). For example, durations increase from 4.5 months to 5.7 months when sales price quotes are replaced by the price quote of the previous regular price. In [Klenow and Kryvtsov \(2008\)](#) durations go from 2.2 months to 2.8 after the sales treatment and in [Nakamura and Steinsson \(2008\)](#) they go from 4.2 to 3.2 months. Time series for frequency of substitution, sales, and missing values in the sample can be seen in [Figure III](#).

What accounts for the differences in the estimated implied duration at low inflation between the sample of differentiated and homogeneous goods? Expected durations are much higher for differentiated goods than for homogeneous goods. In principle, we believe that this discrepancy can be attributed to two features: an intrinsic difference between the type of goods or due to the fact that the prices of homogeneous goods are sampled bimonthly and prices for differentiated goods once a month.

TABLE II
DISTRIBUTION OF FITTED COEFFICIENTS AT THE FIVE-DIGIT LEVEL

	Elasticity γ		Semi-elasticity at zero $\Delta\% \lambda$		Duration at $\pi = 0$	
	Diff.	Hom.	Diff.	Hom.	Diff.	Hom.
Mean	0.58	0.56	0.03	0.01	18.5	7.5
Median	0.58	0.55	0.03	0	15	6.4
Perc 75	0.7	0.6	0.1	0.02	25.6	9.3
Perc 25	0.48	0.5	-0.03	-0.02	6.9	4.7
Std. dev.	0.14	0.08	0.1	0.07	14.2	5.2

Notes. Diff. denotes differentiated goods. Hom. denotes homogeneous goods. For each five-digit industry we use NLLS to fit: $\log \lambda_t = a + \epsilon \min \{\pi_t - \pi^c, 0\} + \nu (\min \{\pi_t - \pi^c, 0\})^2 + \gamma \max \{\log \pi_t - \log \pi^c, 0\} + \omega t$. The semi-elasticity at zero $\Delta\% \lambda$ is the percentage change in λ when inflation goes from 0 to 1%. λ is estimated with the simple estimator in [equation \(6\)](#).

Finally, we explore the robustness of the parameter estimates for the elasticity of the frequency of price changes with respect to inflation at high and low inflation rates by fitting [equation \(7\)](#) for each of the five-digit industries, using the simple estimator of λ .²⁴

[Table II](#) presents statistics describing the distribution of the coefficient estimates derived from [equation \(7\)](#) across five-digit industries. The elasticity estimates confirm our previous findings: (i) the elasticity of the frequency of price changes at high inflation, γ , varies between one-half and two-thirds; and (ii) the semi-elasticity $\Delta\% \lambda$ is approximately zero regardless of the industry. Consistent with the results in [Table III](#) in the next section, there is large variation in the expected duration at low inflation, particularly so for differentiated goods.

ii. Sampling Periodicity. So far we have been using the estimator of the theoretical frequency λ_a that has been proposed in the literature, $\hat{\lambda}_t = -\ln(1 - f_t)$. If price changes follow a Poisson process, this is the maximum likelihood estimator of λ_a . However, since we only observe frequency of price changes f_t at discrete times, a well known bias may arise if prices change more than once within the time interval and these changes are not independent. In particular, we would expect the bias to become larger as inflation increases and prices change more frequently.

In this section, we consider an alternative estimator $\hat{\lambda}_t^{SW} = f_t$ and compare it to $\hat{\lambda}_t$. In the Sheshinski-Weiss model with no

24. We performed the same exercise at a six-digit level obtaining qualitatively similar results. See [Table V](#) for disaggregation levels.

TABLE III
CROSS INDUSTRY DISPERSION OF DURATION $\frac{1}{\lambda}$

Annual inflation range (%)	Median duration	75-25 pct difference	90-10 pct difference
Homogeneous goods			
<10	6.3	6.4	14.0
[10, 100]	1.7	3.0	6.4
[100, 500]	0.6	0.7	1.9
≥ 500	0.2	0.3	0.6
Differentiated goods			
[0, 10]	9.9	12.0	27.0
[10, 100]	2.9	3.8	6.9
[100, 500]	0.8	0.7	1.6
≥ 500	0.3	0.2	0.4

Notes. Duration is in months and calculated as $\frac{1}{\lambda}$ for each five digit industry. The cross-industry statistics, for example, 75-25 pct, are computed by pooling all λ s corresponding to inflation rates in the interval.

idiosyncratic shocks (or in the limit as inflation becomes very large compared to the volatility of the shocks), this is a maximum likelihood estimator of λ_α .

In the left panel of [Figure VII](#) we present the results of Monte Carlo simulations using data generated by the model in [Section II](#). We sample observations every two weeks and calculate both estimators of λ_α for different inflation rates. The true frequency of price adjustment, λ_α , is represented by the medium gray (red) line, the frequency, f , by the light gray (green) line and the simple estimator $\hat{\lambda}_t$ by the dark gray (blue) line (color version online). The figure points to the existence of an upward bias in $\hat{\lambda}_t$ for high inflation rates, and as such, in the elasticity of the frequency of price adjustments to inflation.

To reduce the incidence of such bias in our empirical estimates, we proceed by reestimating the elasticity of the frequency of price changes to inflation by excluding observations corresponding to inflation above some threshold. In the right panel of [Figure VII](#), we show this for a threshold inflation of 200%. This threshold is where our Monte Carlo estimates show that the bias starts becoming more pronounced. The estimated elasticities are 0.63 and 0.48, respectively, much in line with our benchmark estimates.²⁵

25. Excluding observations corresponding to inflation below 50% and above 200% or below 50% and above 100% results in estimated elasticities of 0.64 and 0.76 when using $\hat{\lambda}$; when using f instead, these are 0.44 and 0.59.

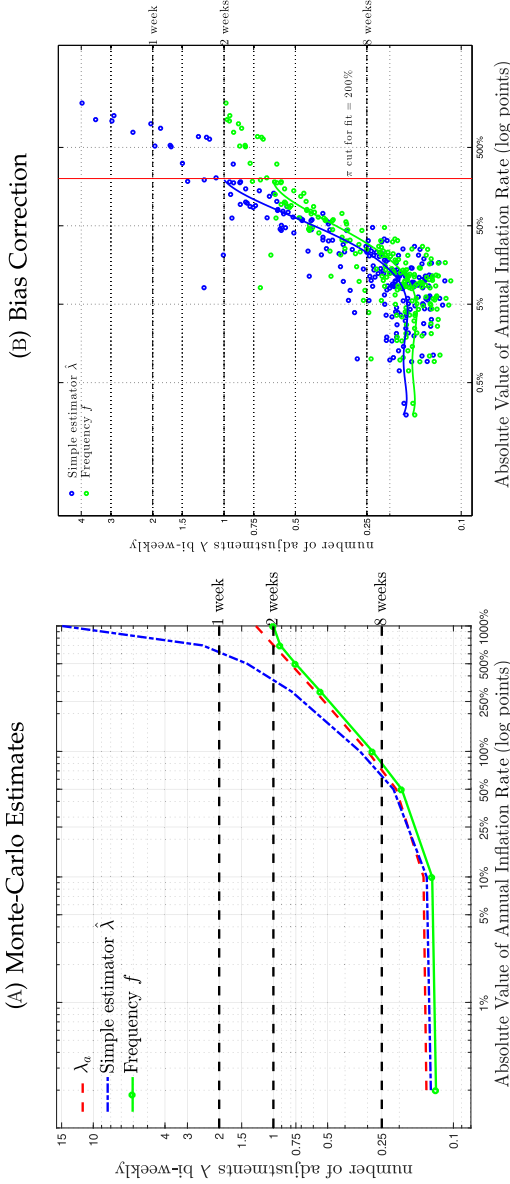


FIGURE VII

Sensitivity to Sampling Periodicity.

A color version of this figure is available online. Panel A shows the theoretical λ_a and estimators $\hat{\lambda}$, f when we simulate the model in Section II and sample observations twice a month. Panel B shows both estimators for goods sampled twice a month in our data (i.e., homogeneous goods) and the fit of a polynomial of order 5 when we drop observations associated with inflation above 200%.

iii. *Expected Inflation.* Next we check the robustness of our results to measuring expected inflation differently. So far, we have used the average realized inflation for the expected duration of the price set in period t , $\frac{1}{\lambda_t}$. We now consider the average of the actual inflation rate of the following k_t months, where $k_t = \left\lceil \frac{n}{\lambda_t} \right\rceil$ and the operator $\lceil \cdot \rceil$ refers to the integer part of a number. Formally, this is $\pi_t^e = \frac{1}{k_t} \sum_{s=t}^{t+k_t} \pi_s$. We refer to n as the forward-looking factor. Thus, as inflation falls (and implied durations rise) in our sample, agents put an increasing weight on future inflation. When $n = 0$ expected and actual inflation are the same.

Table XIII in [Online Appendix DG](#) shows that the results presented here are not very sensitive to estimating [equation \(7\)](#) using different forward-looking factors in [equation \(59\)](#).

iv. *Expected Disinflation.* Motivated by Argentina's history in the years prior to the exchange rate peg of the 1990s, it seems reasonable to believe that forward-looking agents anticipating lower future money growth rates and inflation would have altered their pricing behavior before the exchange rate peg was actually in place. This could cast doubts on our interpretation of the evidence on menu cost models by studying the Argentinean economy during this exact period, since our theoretical results are derived for stationary economies with constant money growth rates.

[Beraja \(2013\)](#) studies this issue. He conducts the same comparative statics analysis from this section on data simulated from the model in [Section III](#) during a disinflation process calibrated to the Argentine economy. He finds that our theoretical results are robust to studying a nonstationary economy where agents anticipate a disinflation in the future.

IV.D. *Inflation and the Dispersion of the Frequency of Price Changes*

This section reports how the dispersion of the frequency of price changes varies as inflation grows. [Proposition 2](#) states that under certain conditions, the firm's pricing behavior when inflation is high is independent of the variance of the idiosyncratic shocks. This implies that as inflation becomes higher, it swamps the effect of idiosyncratic differences across firms that result in differences in the frequency with which they change prices. [Figure II](#) illustrates this point in the numerical example of our version of the [Golosov and Lucas \(2007\)](#) model in [Section III](#). We

use this result assuming that firms in an industry have the same parameters, but that the parameters differ across industries.

In [Table III](#) we estimate λ for each narrowly defined industry (at a five-digit level of aggregation),²⁶ calculate the implied average duration $\frac{1}{\lambda}$ and present two measures of the dispersion of λ 's across such industries: the 75-25 and 90-10 percentile differences. We observe a significant decline in dispersion as inflation rises both across homogeneous and differentiated good industries. For example, for homogeneous goods, the 90-10 percentile difference in λ s when inflation is above 500% a year is about 23 times smaller than the percentile difference at single-digit inflation.

IV.E. Inflation and the Intensive and Extensive Margins of Price Adjustments

In this section, we confront theoretical predictions about the behavior of the intensive and extensive margins²⁷ of price changes with the data.

We first look at the predictions of the theory ([Propositions 1 and 3](#) and [Corollary 1](#)) with respect to the frequency of price changes (the extensive margin of price adjustments) for near-zero inflation rates. According to theory, for near-zero inflation, the frequency of price increases and of price decreases is the same, the frequency of price changes is insensitive to inflation, and the difference between the frequency of price increases and decreases rises with inflation. [Figure II](#) illustrates some of these properties of the menu cost model in the numerical example in [Section III](#).

[Figure VIII](#) (color version online) takes these predictions to the data for our two groups of goods. The red crosses plot the frequency of price changes λ against inflation and the blue circles represent the difference between the frequency of price increases and that of price decreases, $\lambda^+ - \lambda^-$. The range of inflation in the figure was chosen by picking the lowest, negative rate of inflation (excluding outliers) and its positive opposite. The quadratic function fitting the red crosses reflects both the insensitivity of the frequency of price changes to inflation as well as the symmetry between the frequency of price increases and price decreases—that is, quadratic functions have zero derivative and are symmetric around zero. This fact is particularly evident for homogeneous

26. Examples of five-digit aggregation are citrus fruits, soaps, and detergents. See [Table V](#) in [Online Appendix C](#).

27. See [Section II.D](#) for a definition of these margins.

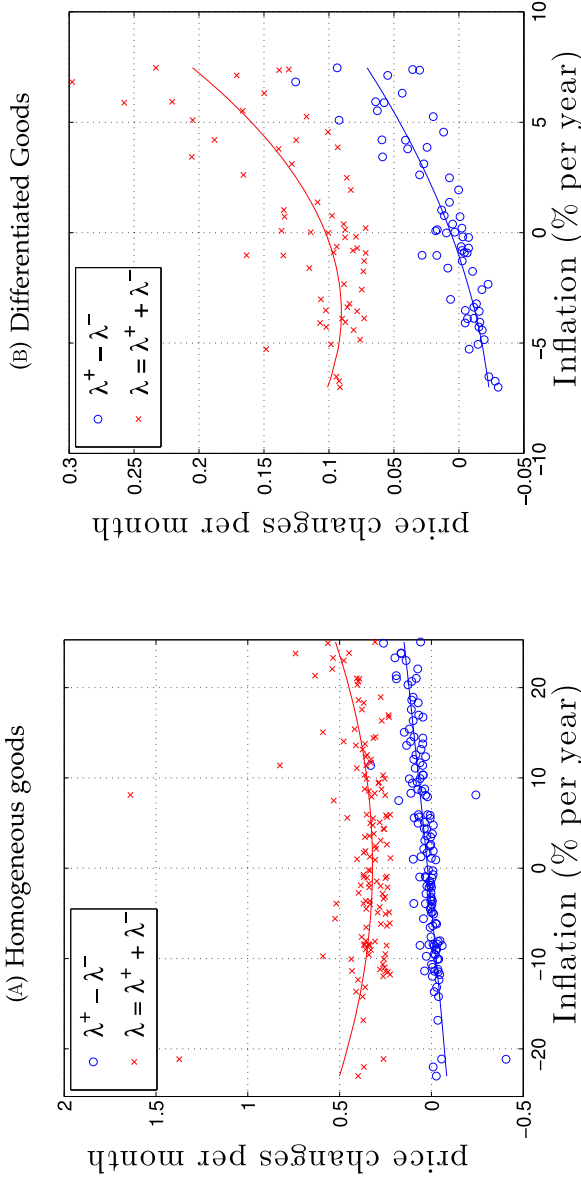


FIGURE VIII

Decomposition of Inflation for Low Inflation Rates

A color version of this figure is available online. λ is the frequency of price changes per month. λ^+ (λ^-) is the frequency of price increases (decreases) per month. Inflation is the annualized log difference of the average price between two consecutive periods. The inflation range is chosen by picking the one-percentile inflation (minimum inflation rate removing outliers) and its positive opposite. Lines are least squares second-degree polynomials.

goods (Panel A), but somewhat less apparent for differentiated goods (Panel B).²⁸ Furthermore, the two panels in Figure VIII show that the prediction that the derivative of $\lambda^+ - \lambda^-$ with respect to inflation is positive for low rates of inflation seems to be consistent with the data.

We conclude the analysis of the extensive margin of price changes at low inflation with a variance decomposition of inflation. Proposition 3 states that in near-zero inflation, most of the changes in inflation result from the extensive margin (90% to be precise). Then, in our data, we compute the extensive margin contribution as follows. Remember that inflation can be decomposed as:

$$\pi = (\lambda^+ - \lambda^-)\Delta_p^+ + (\Delta_p^+ - \Delta_p^-)\lambda^-,$$

so that its variance can be written as

$$\text{var}(\pi) = \text{cov}[(\lambda^+ - \lambda^-)\Delta_p^+, \pi] + \text{cov}[(\Delta_p^+ - \Delta_p^-)\lambda^-, \pi].$$

Therefore, the extensive margin contribution is simply $\frac{\text{Cov}[(\lambda^+ - \lambda^-)\Delta_p^+, \pi]}{\text{Var}(\pi)}$. Because at $\pi = 0$ the frequency of price increases and decreases are identical, this calculation approximates the theoretical extensive margin contribution in Proposition 3.²⁹ We find that for both homogeneous and heterogeneous goods, the contribution of the extensive margin to total inflation variance is between 80% and 90%, depending on whether we define “near-zero inflation” as inflation belonging to a large range (e.g., between 20% and -20%) or small (e.g., between 5% and -5%).

28. Figure IX, Panel A shows the same fact by plotting the frequency of price increases (green circles) and the frequency of price decreases (red squares) against the absolute value of inflation. The axes in Figure VIII are not on a log scale, unlike Figure V and Figure IX, thus ensuring that the insensitivity of the frequency of price adjustment to inflation is not an artifact of the scale.

29. This variance decomposition, guided by Proposition 3, differs from Klenow and Kryvtsov (2008), who do not distinguish between the frequency of price increases and decreases. Because the frequency of price changes is unresponsive to inflation for low inflation rates Klenow and Kryvtsov (2008), as well as Gagnon (2009), conclude that the variance of inflation at low inflation rates is mostly explained by the intensive margin, whereas we conclude that it is explained by the extensive margin. In our case, the latter is the change in the difference between the frequency of price increases and that of price decreases captured by $\frac{\text{Cov}[(\lambda^+ - \lambda^-)\Delta_p^+, \pi]}{\text{Var}(\pi)}$.

Next, we look at extensive and intensive margin predictions for high inflation rates. We present results for the homogeneous goods alone. The results for differentiated goods are similar. As a reminder, our theoretical propositions showed that at high inflation rates, the frequency of price increases λ^+ should converge to λ and the frequency of price decreases λ^- should converge to zero (Lemma 1 and Lemma 2). Figure IX, Panel A shows that this is indeed the case in the Argentine data. Furthermore, Panel B shows the empirical behavior of the intensive margin. We find that for low inflation rates, the size of price changes is insensitive to inflation and that the size of price increases and decreases is the same (approximately 10%).³⁰ This is consistent with the last part of Proposition 1. As inflation rises, the size of price increases and decreases rises, with the magnitude of price increases becoming larger than that of price decreases. This is consistent with the properties of our numerical example, shown in Figure I and Figure IX.³¹

As with the results for the frequency of price changes in the previous section, one concern is that the average size of price increases in Figure IX, Panel B are biased at very high inflation rates because of the aforementioned issues with time aggregation. Thus, we repeat the analysis of Figure VII, Panel B. We calculate the size of price increases for different inflation rates using data generated by the model in Section III and sample observations every two weeks. Figure XIII in Online Appendix E compares this to the theoretical average size of price increases in the model. As opposed to what we found in Figure VII, there is almost no bias in the average size of price increases because of the sampling periodicity.

IV.F. Inflation and the Dispersion of Relative Prices

In this section, we document the empirical sensitivity of the cross-sectional price dispersion to the inflation rate at very low and very high values of inflation. In Section II and in the example in Section III, we analyzed how inflation affects the dispersion of relative prices in menu cost models. We showed that the

30. Gagnon (2009) finds similar patterns in Mexico. For low inflation, the frequency and the absolute size of price changes are unresponsive to inflation, while the share of price increases (decreases) rises (falls) with inflation

31. Note that Figure IX, Panel B is the analog to Figure XI in Online Appendix BB, which we computed with the numerical example described in Section III. The theoretical and the empirical figures are indeed qualitatively very similar.

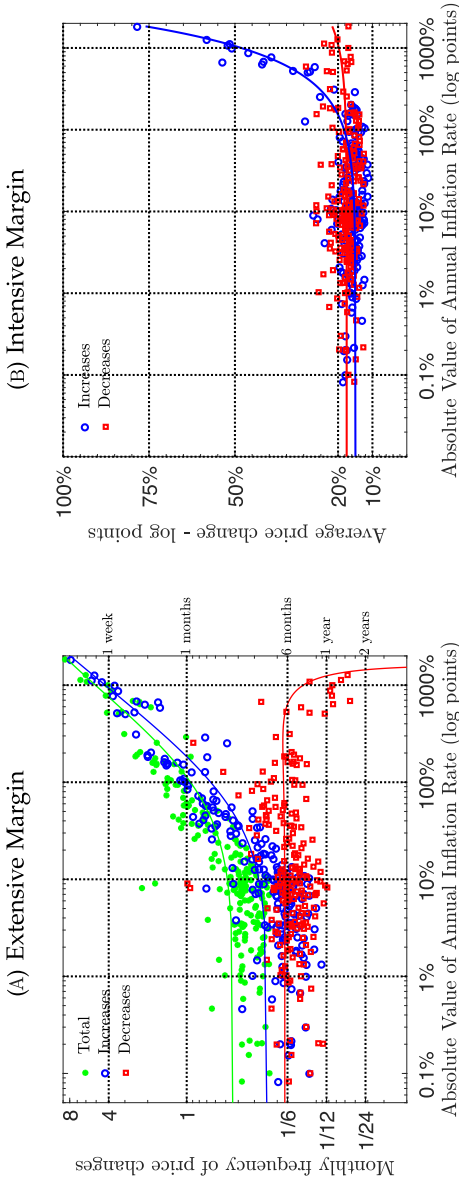


FIGURE IX

Intensive and Extensive Margins of Price Adjustments for Homogeneous Goods

A color version of this figure is available online. In Panel A, the frequency of price increases and decreases is calculated as $-\log(1 - f)$, where f is the fraction of outlets increasing or decreasing price on a given date. In Panel B, the average price change is the log difference in prices, conditional on a price change taking place, averaged with expenditure weights over all homogeneous and differentiated goods, on a given date. Both panels use data on homogeneous goods alone. Lines are least squares second-degree polynomials.

dispersion of relative prices is insensitive to changes in inflation when inflation is low, whereas it increases with inflation when inflation is high. In the limit, as the rate of inflation relative to the variance of idiosyncratic shocks becomes infinite, $\frac{\pi}{\sigma} \rightarrow \infty$, the elasticity of the standard deviation of relative prices with respect to inflation is bounded by one-third. In this section, we contrast these predictions with our data set and find that the empirical elasticities are remarkably close to the ones predicted by the theory.

This aspect of the data is of independent interest because at the core of the welfare costs of inflation in sticky-price models is that higher inflation introduces relative price dispersion that decouples marginal rates of substitution from marginal rate of transformation. In our model, this is captured by the effect of inflation on the standard deviation of relative prices.

Our strategy is to estimate the cross-sectional dispersion of prices in each period and correlate it with inflation. The assumption behind it is that the cross-sectional dispersion is changing through time only because of the time series variation in inflation. We think that in our case this is a reasonable assumption due to the very large changes in inflation in relatively short periods of time.³² We measure the dispersion of relative prices through the residual variance in a regression of prices at each time, store, and good on a rich set of fixed effects.

To estimate the effect of inflation on the distribution of relative prices, ideally we need to compare identical goods or, at least, control for factors that affect individual price levels (for example, quality or store characteristics). We proxy for these factors by controlling for goods, stores, and nonsubstitution spells, which we define next.

The Argentine statistical agency (INDEC) fixes the exact characteristics of a good in each store during what we call a non-substitution spell (see [Online Appendix C](#)). In particular, when an INDEC enumerator first goes to a store, she fixes all the characteristics of a good and records them. For instance, suppose we are talking at the most disaggregated level of goods defined as “carbonated drink of top brand in a small bottle”—this will be i

32. As pointed out by [Nakamura et al. \(2018\)](#) this may be a more pressing issue for their U.S. data which has much smaller changes in inflation over a longer time period.

in the notation below.³³ The first time the enumerator goes to the store s , she fixes the brand and the exact package of the good i , based on the information given by the manager of store s on which brand and package is sold most often in that particular store. We refer to this particular brand-package as $j(i, s, t)$.³⁴ From then on, when the enumerator visits store s she keeps measuring the price of that particular brand and package for the good i “carbonated drink of top brand in small bottle.” The first time the particular brand and package is no longer available in that store, the enumerator records this as a substitution. Subsequently the enumerator fixes a new brand and package for that store, again using the information from the store manager. We refer to the times t between these two events as a nonsubstitution spell, that is, a period where the variable $j(i, s, t)$ takes the same value. After a substitution takes place for the same (s, i) then $j(i, s, t)$ increases by 1. Thus, for each good-store combination (i, s) , the variable $j(i, s, t)$ takes positive integer values. Although we do not have access to the agency record of the description of the good in each substitution spell, we have access to the indicator of the times at which substitutions have taken place, and hence we can compute $j(i, s, t)$ for each time t , good i , and store s .

We consider five cases for the specification of fixed effects where we progressively include more dummies. In each case, we estimate a weighted regression for prices of goods at each store in each time period, using the CPI weights. Then we compute the residual variance for each time across the goods and stores and convert it into a standard deviation. We plot this standard deviation against the expected inflation for each date—which correspond to a two-week period. We use the homogeneous goods because for these goods there are more outlets per good and because of the higher frequency of these goods, which is required to have a large number of good \times stores with more than one non-substitution spell. The data set has about 5.5 million price quotes (combination of times, goods, and stores with valid prices for homogeneous goods). In case 1 we have 222 time dummies, one for each two-week period. In case 2 we have about 5,000 separate dummies for times, goods, and stores. In case 3 we have about 75,000 dummies for time and for goods \times store combinations. In

33. To be concrete, we have 233 different i 's for homogeneous goods.

34. To be concrete, this can be regular Coca-Cola in a particular type of plastic bottle.

TABLE IV
REGRESSIONS USED TO COMPUTE RESIDUAL VARIANCE OF PRICE LEVELS

Models <i>i</i> : indicate dummies	# of dummies	Adj R^2	Elast at $\pi = 100\%$	Elast at $\pi = 500\%$	Elast at $\pi = 700\%$
1: time	212	0.751	0.03	0.21	0.31
2: time + good + store	4,978	0.982	0.06	0.26	0.34
3: time + good \times store	74,755	0.987	0.14	0.35	0.37
4: time + good \times store \times non-subst-spell	153,896	0.989	0.16	0.37	0.38
5: time \times store + time \times good + good \times store \times non-subst-spell	464,505	0.996	0.13	0.30	0.28

Notes. 5,497,452 price observations are used in each regression for 233 goods with prices collected twice a month over 212 periods.

case 4 we have dummies for time and for each nonsubstitution spell in each of the store \times good combinations, which requires us to estimate 155,000 parameters. For many good \times store combinations there is only one nonsubstitution spell during the time spanned by our data set, while for some there are dozens. Finally, in case 5 we have dummies for combinations of time \times store, dummies for combinations of time \times good, and separate dummies for nonsubstitution spells of each store \times good combination. In this case we have about 470,000 dummies. Table IV summarizes this information. On conceptual grounds, our preferred specification is number 4: time dummies and nonsubstitution-spell dummies for each good and store. Yet we are mindful that we are borderline in terms of the degrees of freedom left to be able to accurately estimate the residual variance in each two-week period.

Figure X displays the standard deviation of the residuals for each case, plotted against the expected inflation at each point of time. For each case we also include a fitted polynomial regression. It is clear that for low inflation the cross-sectional standard deviation of relative prices varies very little, just as the theory predicts. Instead, at very high inflation, the elasticity of the cross-sectional dispersion of relative price with respect to inflation is about one-third. In particular, the last three columns of Table IV display the elasticities of the fitted polynomial regressions evaluated at annual continually compounded inflation rates of 100%, 500%, and 700% for each case. We view the values of the elasticity of the fitted line at both low and high inflation as consistent with the theory. The elasticity is zero at low inflation rates and

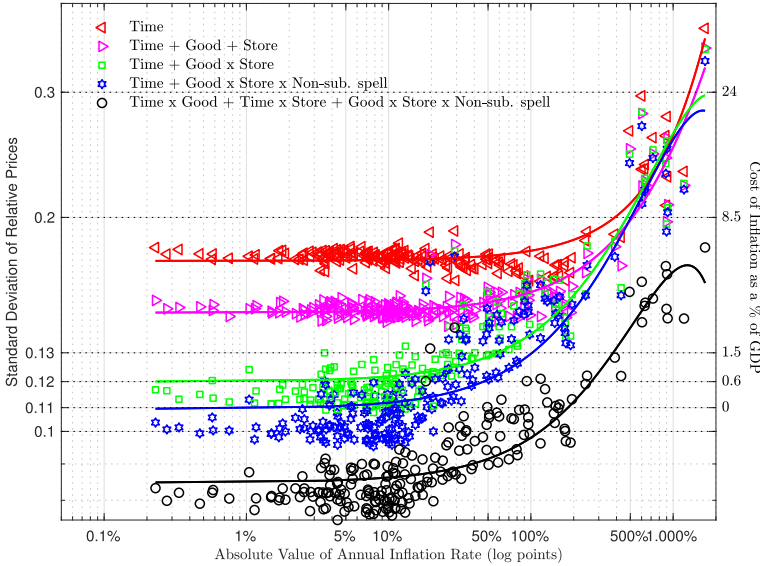


FIGURE X

Cross-Sectional Standard Deviation of Prices and Costs of Price Dispersion versus Inflation

A color version of this figure is available online. Each point is an inflation–standard deviation pair. The residual standard deviation is derived from each regression in Table IV for the sample of goods with two visits a month. The lines are OLS fitted values for a second-order polynomial in levels. The right axis shows the costs of inflation captured by equation (8) with $\eta = 6$ for $\pi = 0$ ($\sigma = 0.108$), $\pi = 50$ ($\sigma = 0.1175$), $\pi = 100$ ($\sigma = 0.129$), and $\pi = 500$ ($\sigma = 0.2$), measured as a % of GDP.

approaches the upper bound of one-third for sufficiently large inflation rates, as discussed in Section II.B and Section III.³⁵

Finally, we perform two types of exercises to evaluate the robustness of these results. As for the frequency and size of price changes, the first concern is that the standard deviation of relative prices is biased at very high inflation rates because of issues with

35. These results are sensitive to the method employed to fit the relation between the standard deviation of relative prices and expected inflation. In Figure X we fit a second-degree polynomial in levels. Fitting an equation similar to the one in Figure V, for example, the elasticity of the standard deviation with respect to inflation for low inflation is still zero, but for high inflation it is lower.

time aggregation.³⁶ Thus we repeat the Monte Carlo analysis of previous sections. Using data generated by the model in [Section III](#), we sample observations every two weeks and calculate the standard deviation of log prices for different inflation rates. Figure XIV in [Online Appendix E](#) compares it to the theoretical standard deviation of log prices. We observe that for inflation rates higher than 500%, some bias exists. However, in [Table IV](#), we found that the estimated elasticity remains close to the theoretical upper bound of one-third even at 500% inflation rates.

The second concern is that of sample selection. Because our sample is an unbalanced panel, it might be the case that the goods or stores for which we have price quotes at different rates of inflation have different variance of relative prices. To account for this we reproduced [Figure X](#) and [Table IV](#) for a sample of store-good pairs with 190 (out of 212) nonmissing observations. The sample is reduced by 4 million observations, so it has approximately 1.5 million observations. The results are very similar to those from the full sample and can be found in [Online Appendix F](#).

1. *The Cost of Inflation.* Taken together, the foregoing results imply that the welfare cost of inflation due to the inefficient dispersion in relative prices—as emphasized in [Woodford \(2003, ch. 6\)](#)—is likely to be relevant only for high rates of inflation, as evidenced by the insensitivity of relative price dispersion to inflation for inflation rates below ten percent per year.

Using a second-order approximation to the expression for the decrease in output due to price dispersion, one obtains the following expression for the cost of inflation (expressed as a fraction of output lost per period):

$$(8) \quad \text{cost}(\pi) = \frac{\eta}{2} \left(\bar{\sigma}^2(\pi) - \bar{\sigma}^2(0) \right),$$

where $\bar{\sigma}^2(\pi) - \bar{\sigma}^2(0)$ is the change in the variance of relative prices between an annual inflation rate of zero and π , and η is the

36. Prices of the same good are gathered from different stores over a rolling window of two weeks, as opposed to being measured simultaneously across all stores on the same day. Thus, if the prices gathered late in the two-week period are systematically higher than those gathered early on because almost all prices do change due to inflation, we obtain an upward-biased estimate of the cross-sectional standard deviation of prices at a point in time.

elasticity of substitution between the different goods. Note that this is a “typical” Harberger’s triangle formula: proportional to half of the elasticity and to the (average) square of the tax wedge. Furthermore, it is worth emphasizing that this is only the part of the cost that corresponds to the distortion due to extra price dispersion: the total cost also includes the average menu cost spent per year at the inflation rate π .

Equation (8) and Figure X indicate that the cost of inefficient price dispersion due to inflation is highly nonlinear. The right axes of Figure X shows the costs of inefficient price dispersion for the benchmark case—regression 4 in Table IV—for different (log) annual inflation rates with $\eta = 6$. We take the standard deviation of relative prices with no inflation to be the level of the fitted line close to zero. For an inflation rate of $\pi = 50\%$ a year the cost of inflation is approximately 0.6% of GDP, and for $\pi = 100\%$ it is approximately 1.5% of GDP. Thus, for inflation rates below 100% a year the costs of inflation arising from inefficient price dispersion are relatively moderate. For higher rates of inflation, these costs rise quickly. An inflation of $\pi = 500\%$ a year, for example, results in a cost of approximately 8.5% of aggregate output per year due to the additional price dispersion alone.

V. CONCLUSIONS

After deriving several predictions of menu cost models of nominal price setting at very high and near-zero inflation rates, we empirically analyzed how inflation affects price-setting behavior by using a novel micro-data set underlying Argentina’s consumer price index. Argentina’s experience is unique because it encompasses periods of very high and near-zero inflation, thus allowing us to test sharp predictions of menu cost models in these extreme scenarios.

We found that when inflation is low, the frequency of price changes, the dispersion of relative prices, and the absolute size of price changes are insensitive to inflation. Furthermore, we showed, both theoretically and empirically, that the difference between the frequency of price increases and decreases rises with inflation when inflation is low. These findings are consistent with predictions of menu cost models at low inflation, where idiosyncratic firm shocks swamp inflation as a motive for changing prices.

At high inflation, we found that inflation swamps idiosyncratic shocks as a driver of price changes. The frequency of price

changes across different products becomes similar, and the frequency of price changes, the dispersion of relative prices, and the average size of price changes all rise with inflation with elasticities that are quantitatively in line with Sheshinski and Weiss's (1977) menu cost model with no idiosyncratic shocks.

Furthermore, we confirmed and extended available evidence for the relationship between the frequency of price changes and inflation for countries that experienced either very high or low inflation. Despite large structural differences between these countries, we view these findings as reflecting common, robust economic mechanisms captured by menu cost models driving price changes and inflation.

Finally, we showed that the cost of inflation resulting from inefficient price dispersion is likely to be quantitatively large only for very high inflation rates.

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SUPPLEMENTARY MATERIAL

An [Online Appendix](#) for this article can be found at *The Quarterly Journal of Economics* online. Data and code replicating tables and figures in this article can be found in [Alvarez et al. \(2018\)](#), in the Harvard Dataverse, [doi:10.7910/DVN/C8ZOAS](https://doi.org/10.7910/DVN/C8ZOAS).

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From Hyperinflation to Stable Prices: Argentina's Evidence on Menu Cost Models

**SUPPLEMENTARY MATERIAL
FOR ONLINE PUBLICATION**

A PROOFS OF COMPARATIVE STATIC RESULTS

We prove propositions 1 and 2 in a more general setup than the one in Section II. We assume that the instantaneous profit of the monopolist $F(p, z)$ is strictly concave in its first argument and that $\{z_t\}$ is a diffusion with coefficients $a(\cdot)$ and $b(\cdot)$:

$$dz = a(z) dt + b(z) \sigma dW \quad (9)$$

where $\{W(t)\}$ is a standard Brownian Motion so $W(t) - W(0) \sim N(0, t)$. We keep the parameter σ separately from $b(\cdot)$ so that when $\sigma = 0$ the problem is deterministic.

AA Proof of Proposition 1

In order to prove this result we assume that the idiosyncratic shock z has strictly positive volatility and we make some mild symmetry assumptions on the firm's problem, i.e., that the process for the shocks is symmetric around zero, and that the profit function is symmetric in the log of the static profit maximizing relative price as well as in its shifter.

More precisely we define symmetry as follows. Assume that $z \in Z = [-\bar{z}, \bar{z}]$ for some strictly positive \bar{z} . Let the profit maximizing relative price given z be $p^*(z) \equiv \arg \max_x F(x, z)$. We say that $a(\cdot), b(\cdot), \zeta(z)$ and $F(\cdot)$ are *symmetric* if

$$a(z) = -a(-z) \leq 0 \text{ and } b(z) = b(-z) > 0 \text{ for all } z \in [0, \bar{z}] , \quad (10)$$

$$p^*(z) = -p^*(-z) \geq 0 \text{ for all } z \in [0, \bar{z}] \quad (11)$$

$$F(\hat{p} + p^*(z), z) = F(-\hat{p} + p^*(-z), -z) + f(z) \text{ for all } z \in [0, \bar{z}] \text{ and } \hat{p} \geq 0 , \quad (12)$$

$$\zeta(z) = \zeta(-z) > 0 \text{ for all } z \in [0, \bar{z}] , \quad (13)$$

for some function $f(z)$ and with the normalization $p^*(0) = 0$.

Let $\mu(z)$ be the density of the invariant distribution of z , when it exists. Equation (10) implies that the invariant distribution μ as well as the transition densities of the exogenous process $\{z_t\}$ are symmetric around $z = 0$. Equations (11)-(12) state that the profit function is symmetric around the (log) maximizing price and its cost shifter. Thus, if the price is \hat{p} higher than the optimal for a firm with z , profits deviate from its optimal value by the same amount as with prices \hat{p} lower than the optimal when the shifter is $-z$. The function f allows to have an effect of the shifter z on the level of the profits that is independent of the price. The quadratic case with $a(z) = 0$ and $b(z) = 1$ for all Z in Section II. satisfies these symmetry conditions.

Under these symmetry assumptions, first we establish that the value function, the optimal adjustment function and the inaction sets are all symmetric. We show that:

$$\begin{aligned} V(\hat{p} + p^*(z), z; \pi, \sigma^2) &= V(-\hat{p} + p^*(-z), -z; -\pi, \sigma^2) + v(z) , \\ \hat{\psi}(z; -\pi, \sigma^2) &= -\hat{\psi}(-z; \pi, \sigma^2), \text{ and} \\ (\hat{p} + p^*(z), z) \in \mathcal{I}(\pi, \sigma^2) &\implies (-\hat{p} + p^*(-z), -z) \in \mathcal{I}(-\pi, \sigma^2) \end{aligned}$$

for all $z \in [0, \bar{z}]$, $\hat{p} \geq 0$ and $\pi \in (-\bar{\pi}, \bar{\pi})$. The symmetry of these three objects can be established using a guess and verify argument in the Bellman equation. This argument has two parts, one deals with the instantaneous return and the second with the probabilities of different paths of z 's. First we show that the instantaneous return satisfy the analogous property that the required symmetry property for the value function stated above. We note that for $t \leq \tau$, where we let 0 be the time where prices were last set, we have:

$$\begin{aligned} F(p(t) - \bar{p}(t), z(t)) &= F(p(0) - \bar{p}(0) - \pi t - p^*(z(t)) + p^*(z(t)), z(t)) \\ &= F(-p(0) + \bar{p}(0) + \pi t - p^*(-z(t)) + p^*(-z(t)), -z(t)) + f(z(t)) \\ &= F(-p(0) + \bar{p}(0) + \pi t + p^*(z(t)) + p^*(-z(t)), -z(t)) + f(z(t)), \end{aligned}$$

where the second equality holds by symmetry of $F(\cdot)$ setting $\hat{p}(t) = p(0) - \bar{p}(0) - p^*(z(t)) - \pi t$. Thus fixing the path of $\{z(t)\}$ for $0 \leq t \leq \tau$, starting with $p(0) - \bar{p}(0)$ and $z(0)$ and having inflation π , gives the same profits, assuming symmetry of $F(\cdot)$, than starting with $-p(0) + \bar{p}(0)$ and having inflation $-\pi$ and $-z(0)$. Finally the probability of the path $\{z(t)\}$ for $t \in [0, \tau]$ conditional on $z(0)$, given the symmetry of $a(\cdot)$ and $b(\cdot)$ is the same as the one for the path $\{-z(t)\}$ conditional on $-z(0)$. From here one obtains that the inaction set is symmetric. Likewise, from this property it is easy to see that the optimal adjustment is also symmetric. If with inflation π a firm adjust with current shock z setting $p = \bar{p} + \hat{p}(z; \pi, \sigma^2)$, then with inflation $-\pi$ and current shock $-z$ it will adjust to $p = \bar{p} + \hat{p}(-z, -\pi) = \bar{p} - \hat{p}(z, -\pi)$. To see this, let $t = 0$ be a date where an adjustment take place, let $p(0)$ the price right after the adjustment, and let τ the stopping time until the next adjustment. The value of $p(0)$ maximizes

$$\begin{aligned} p(0) &= \arg \max_{\bar{p}} \mathbb{E} \left[\int_0^\tau e^{-rt} F(\bar{p} - \bar{p}(0) - \pi t, z(t)) | z(0) \right] \\ &= \arg \max_{\bar{p}} \mathbb{E} \left[\int_0^\tau e^{-rt} F(\bar{p} - \bar{p}(0) - \pi t - p^*(z(t)) + p^*(z(t)), z(t)) | z(0) \right] \\ &= \arg \max_{\bar{p}} \mathbb{E} \left[\int_0^{\tau'} e^{-rt} F(-\bar{p} + \bar{p}(0) + \pi t - p^*(-z(t)) + p^*(-z(t)), -z(t)) | -z(0) \right] \\ &= \arg \max_{\bar{p}} \mathbb{E} \left[\int_0^{\tau'} e^{-rt} F(-\bar{p} + \bar{p}(0) + \pi t, -z(t)) | -z(0) \right]. \end{aligned}$$

where τ' is the stopping time obtained from τ but defined flipping the sign of the z 's.

Given the symmetry of the inaction set and optimal adjustment it is relatively straightforward to establish the symmetry of the expected time to adjustment \mathcal{T} and of the invariant density g . With the \mathcal{T} and g symmetric, it is immediate to establish that λ_a is symmetric. Finally, if λ_a is differentiable, then $\frac{\partial}{\partial \pi} \lambda_a(\pi, \sigma^2) = -\frac{\partial}{\partial \pi} \lambda_a(-\pi, \sigma^2)$, which establish part (i) of the proposition.

Now we show that inflation has no first order effect of $\bar{\sigma}$, i.e., we establish part (ii). For that we first use that the symmetry of the decision rules and of the invariant distribution of the shocks (which follows from the symmetry of a and b) implies that $h(\hat{p}, \pi) = h(-\hat{p}, -\pi)$ where for simplicity we suppress σ^2 as an argument. Differentiating this expression with respect to π and evaluating at $\pi = 0$ we obtain: $h_\pi(\hat{p}, 0) = -h_\pi(-\hat{p}, 0)$. Let $f(\hat{p}, \pi)$ be any symmetric differentiable function in the sense that $f(\hat{p}) = f(-\hat{p})$. Then writing the expected value of f as $\mathbb{E}[f|\pi] = \int_{-\infty}^0 f(\hat{p})h(\hat{p}, \pi)d\hat{p} + \int_0^\infty f(\hat{p})h(\hat{p}, \pi)d\hat{p}$ and differentiating both terms with respect to π

and evaluating it at $\pi = 0$, using the implications for symmetry for the derivatives of h and the symmetry of f we have $\frac{\partial}{\partial \pi} \mathbb{E}[f|0] = 0$. Applying this to $f(\hat{p}) = \hat{p}^2$ we obtain that inflation does not have a first order effect on the second non-centered moment of the relative prices. Finally, to examine the effect of inflation on the variance of the relative prices, we need to examine the effect of inflation on the square of the average relative price,

$$\frac{\partial}{\partial \pi} \left[\int_{-\infty}^{\infty} \hat{p} h(\hat{p}, \pi) d\hat{p} \right]^2 \Big|_{\pi=0} = 2 \left[\int_{-\infty}^{\infty} \hat{p} h(\hat{p}, 0) d\hat{p} \right] \left[\int_{-\infty}^{\infty} \hat{p} \frac{\partial}{\partial \pi} h(\hat{p}, 0) d\hat{p} \right] = 0,$$

since by symmetry of $h(\cdot, 0)$ around $\hat{p} = 0$ we have $\int_{-\infty}^{\infty} \hat{p} h(\hat{p}, 0) d\hat{p} = 0$. Then, we have shown that inflation has no first order effect on the variance of relative prices around $\pi = 0$.

Finally, observe that by symmetry we have that for any inflation rate $\lambda_a^+(\pi) = \lambda_a^-(\pi)$ and $\Delta_p^+(\pi) = \Delta_p^-(\pi)$. Differentiating with respect to inflation yields $\frac{\partial \lambda_a^+(\pi)}{\partial \pi} = -\frac{\partial \lambda_a^-(\pi)}{\partial \pi}$ and $\frac{\partial \Delta_p^+(\pi)}{\partial \pi} = -\frac{\partial \Delta_p^-(\pi)}{\partial \pi}$. Evaluating it at $\pi = 0$ gives the result. Q.E.D.

AB Proof of Lemma 1

Benabou and Konieczny (1994) compute the value of following an sS policy assuming that the period return function $F(p, 0)$ is cubic in terms of deviations from the profit maximizing price, i.e. $p - p^*$. This allows for explicit computation of the value of the policy and to obtain the first order conditions at s and S for any value of π . Adding equations (8) and (14) in **Benabou and Konieczny (1994)** we get the expression $S - s = 2\delta + \frac{2}{3} \left(\frac{r}{\pi} - a \right) \delta^2$ for $\delta = \left(\frac{3}{2} \frac{c\pi}{F''} \right)^{\frac{1}{3}}$ and $a = -\frac{F'''}{2F''}$. Thus,

$$S - s = 2 \left(-\frac{3}{2} \frac{c}{F''} \right)^{\frac{1}{3}} \pi^{\frac{1}{3}} + \frac{2}{3} \left(-\frac{3}{2} \frac{c}{F''} \right)^{\frac{2}{3}} r \pi^{-\frac{1}{3}} - \frac{2}{3} \left(-\frac{3}{2} \frac{c}{F''} \right)^{\frac{2}{3}} a \pi^{\frac{2}{3}}$$

and

$$\frac{d(S-s)}{d\pi} \frac{\pi}{S-s} = \omega_1 \frac{1}{3} + \omega_2 \frac{-1}{3} + \omega_3 \frac{2}{3},$$

where $\omega_1 = 2 \left(-\frac{3}{2} \frac{c}{F''} \right)^{\frac{1}{3}} \pi^{\frac{1}{3}} / (S-s)$, $\omega_2 = \frac{2}{3} \left(-\frac{3}{2} \frac{c}{F''} \right)^{\frac{2}{3}} r \pi^{-\frac{1}{3}} / (S-s)$, and $\omega_3 = -\frac{2}{3} \left(-\frac{3}{2} \frac{c}{F''} \right)^{\frac{2}{3}} a \pi^{\frac{2}{3}} / (S-s)$. Using L'hospital's rule $\lim_{c \rightarrow 0} \omega_1 = 1$, and $\lim_{c \rightarrow 0} \omega_2 = \lim_{c \rightarrow 0} \omega_3 = 0$, so that $\frac{d(S-s)}{d\pi} \frac{\pi}{S-s} = 1/3$.

The same argument for $\mathcal{T} = (S-s)/\pi$ yields

$$\mathcal{T} = 2 \left(-\frac{3}{2} \frac{c}{F''} \right)^{\frac{1}{3}} \pi^{-\frac{2}{3}} + \frac{2}{3} \left(-\frac{3}{2} \frac{c}{F''} \right)^{\frac{2}{3}} r \pi^{-\frac{4}{3}} - \frac{2}{3} \left(-\frac{3}{2} \frac{c}{F''} \right)^{\frac{2}{3}} a \pi^{-\frac{1}{3}}$$

As $\lambda_a = \mathcal{T}^{-1}$, taking limits as $c \rightarrow 0$ yields $\lim_{c \rightarrow 0} \frac{d\lambda_a}{d\pi} \frac{\lambda_a}{\pi} = \frac{2}{3}$. QED.

AC Proof of Lemma 2

First notice that if in the problem described in (1) we multiply r, π, σ^2 and the functions $a(\cdot), F(\cdot)$ by a constant $k > 0$, we are just changing the units at which we measure time. Moreover, the objective function in the right hand side of (1) is homogeneous of degree one in $F(\cdot)$ and c and, hence, the policy function is the same whether we multiply $F(\cdot)$ by k or divide c by it. Thus, if the function $a(\cdot)$ equals zero, λ_a is homogeneous of degree one

in $(\pi, \sigma^2, r, \frac{1}{c})$. Using this homogeneity:

$$\frac{1}{\lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})} \frac{\partial \lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})}{\partial \pi} = \frac{\pi}{\lambda_a(\pi, \sigma^2, r, \frac{1}{c})} \frac{\partial \lambda_a(\pi, \sigma^2, r, \frac{1}{c})}{\partial \pi}$$

Taking limits for $\sigma > 0$:

$$\begin{aligned} \lim_{c \downarrow 0, r \downarrow 0} \frac{1}{\lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})} \frac{\partial \lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})}{\partial \pi} &= \frac{1}{\lambda_a(1, \frac{\sigma^2}{\pi}, 0, \infty)} \frac{\partial \lambda_a(1, \frac{\sigma^2}{\pi}, 0, \infty)}{\partial \pi} \\ &= \lim_{c \downarrow 0, r \downarrow 0} \frac{\pi}{\lambda_a(\pi, \sigma^2, r, \frac{1}{c})} \frac{\partial \lambda_a(\pi, \sigma^2, r, \frac{1}{c})}{\partial \pi} = \frac{\pi}{\lambda_a(\pi, \sigma^2, 0, \infty)} \frac{\partial \lambda_a(\pi, \sigma^2, 0, \infty)}{\partial \pi} \end{aligned}$$

Thus

$$\lim_{\pi \rightarrow \infty} \left[\lim_{c \downarrow 0, r \downarrow 0} \frac{1}{\lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})} \frac{\partial \lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})}{\partial \pi} \right]_{\sigma^2 > 0} = \lim_{\sigma \rightarrow 0} \left[\lim_{c \downarrow 0, r \downarrow 0} \frac{1}{\lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})} \frac{\partial \lambda_a(1, \frac{\sigma^2}{\pi}, \frac{r}{\pi}, \frac{1}{\pi c})}{\partial \pi} \right]_{\pi > 0}.$$

For the same reason all the elements of Ψ for a given z are homogeneous of degree 0 so, (3b) follows.

Finally we establish (3c). Using the same notation we index the invariant density as follows $g(p - \bar{p}, z; \pi, \sigma^2, r, 1/c)$. Note that since $a(z) = 0$, scaling the four parameters scales the units of time of z . Moreover, scaling the four parameters does not change Ψ . Thus for each $(p - \bar{p}, z)$ the invariant density is homogeneous of degree zero in $(\pi, \sigma^2, r, 1/c)$. The result follows from the definition of Δ_p^+ and Δ_p^- in terms of g and Ψ . QED.

AD Proof of Proposition 3

Let $x = p - z$ be the log of the real gross mark-up, which we refer to as the net markup. The firm's optimal pricing policy, in this case, can be characterized in terms of three constants $X \equiv (\underline{x}, \bar{x}, \hat{x})$. The policy function takes the simple form $\Psi(z) = X + z$. We can write the inaction set in terms of the net markups as $\mathcal{I} = \{x : \underline{x} < x < \bar{x}\}$. It is optimal to keep the price unchanged when the net markup x is in the interval (\underline{x}, \bar{x}) . When prices are not changed, the real markup evolves according to $dx = -\pi dt + \sigma dW$. When the real markup hits either of the two thresholds, prices are adjusted so that the real markup is \hat{x} and thus the optimal return relative price is $\hat{\psi}(z) = \hat{x} + z$. Price increases are equal to $\Delta_p^+ = \hat{x} - \underline{x}$ and price decreases are equal to $\Delta_p^- = \bar{x} - \hat{x}$. In the case where $\sigma = 0$ and $\pi > 0$, we obtain a version of Sheshinski and Weiss's (1977) model, and the optimal policy can be characterized simply by two thresholds $\underline{x} < \hat{x}$.

We present a series of lemmas that yield the proof of Proposition 3. First, Lemma 3 simplifies the Hamilton-Jacobi-Bellman for the undiscounted case. It shows that $\lim_{r \rightarrow 0} rV(x, r)$ in problem 1 is a constant. Second, Lemma 4 represents the value function as a power series and finds its coefficients as functions of π and σ . Lemma 5 is an analytical solution for the zero inflation case that characterizes $(\underline{x}, \bar{x}, \hat{x})$, the size of prize changes and the frequency of price increases and of price decreases. Lemma 6 characterizes the derivatives of these elements with respect to inflation at $\pi = 0$. Finally, Lemma 7 is a complete analytical characterization of Sheshinski and Weiss's (1977) case.

LEMMA 3. The limit as $r \downarrow 0$, of the value function and of the thresholds $\underline{x}, \hat{x}, \bar{x}$ can be obtained by solving for a

constant A and a function $v : [\underline{x}, \bar{x}] \rightarrow \mathbb{R}$ which satisfy:

$$A = Bx^2 - \pi v'(x) + \frac{\sigma^2}{2} v''(x) \text{ for all } x \in [\underline{x}, \bar{x}] \quad (14)$$

and the boundary conditions:

$$v(\bar{x}) = v(x^*) + c,$$

$$v(\underline{x}) = v(x^*) + c$$

$$v'(\bar{x}) = 0,$$

$$v'(\underline{x}) = 0,$$

$$v'(\hat{x}) = 0.$$

Remark: $v(\hat{x})$ can be normalized to zero, and A has the interpretation of the expected profits per unit of time net of the expected cost of changing prices.

Proof. The solution of the firm's problem (1) can be characterized by the equation

$$rV(x_0, r) = F(x) - \pi V'(x, r) + \frac{\sigma^2}{2} V''(x, r) \text{ for all } x \in [\underline{x}, \bar{x}] \quad (15)$$

and the following boundary conditions: two value matching conditions $V(\hat{x}, r) - V(\bar{x}, r) = V(\hat{x}, r) - V(\underline{x}, r) = c$ and the smooth pasting and optimal return conditions $V'(\hat{x}, r) = V'(\bar{x}, r) = V'(\underline{x}, r) = 0$.

Let $v(x) = \lim_{r \rightarrow 0} V(x, r)$ for $x \in [\underline{x}, \bar{x}]$ and $v'(x) = \lim_{r \rightarrow 0} V'(x, r)$ for $x \in [\underline{x}, \bar{x}]$.

We show that when $r \rightarrow 0$ the left hand side of (15) is a constant—i.e., $\lim_{r \rightarrow 0} rV(x, r) = A$ —so it becomes (14). Write $V(x, r)$ as $V(x, r) = V_1(x, r) + V_2(x, r)$, where the first term is the present value of expected profits and the second is the present value of the expected adjustment costs. Multiplying the former by r we get

$$rV_1(x_0, r) = \lim_{T \rightarrow \infty} E \left[\int_0^T re^{-rt} F(x(t)) dt \mid x = x_0 \right]$$

for some profit function $F(x(t))$. Observe that rV_1 is a weighted average of $F(x(t))$ with positive weights, re^{-rt} that satisfy $\int_0^T re^{-rt} dt = 1$. As $r \rightarrow 0$, the terms re^{-rt} become a constant, which has to be $1/T$ to still integrate to 1. Then,

$$\lim_{r \rightarrow 0} E \left[\lim_{T \rightarrow \infty} \int_0^T re^{-rt} F(x(t)) dt \mid x = x_0 \right] = \lim_{T \rightarrow \infty} \int_0^\infty \frac{1}{T} E [F(x(t)) \mid x = x_0] dt.$$

If the path of $x(t)$ that solves the firm's problem (1) is ergodic then the average $E [F(x(t)) \mid x = x_0]$ is independent of the state so

$$\lim_{T \rightarrow \infty} \int_0^\infty \frac{1}{T} E [F(x(t)) \mid x = x_0] dt \rightarrow E [F(x)] \text{ for all } x(0) \in [\underline{x}, \bar{x}].$$

Now write the second part of the value function in problem (1), the expected costs of price adjustments,

as $V_2(x_0, r) = \lim_{T \rightarrow \infty} E \left[\sum_{i=1}^{N(T)} e^{-r\tau_i} c \mid x = x(0) \right]$, where τ_i is the time of each price adjustment and $N(T) = \max\{i : \tau_i \leq T\}$ is the number of price adjustments before T . Letting τ_N be the time of the N^{th} adjustment, we can write $V_2(x, r)$ as $V_2(x_0, r) = E \left[\sum_{i=1}^N e^{-r\tau_i} c \mid x = x_0 \right] + E \left[e^{-r\tau_N} \sum_{i=N+1}^{\infty} e^{-r\tau_i} c \mid x = x_0 \right]$. Multiplying both terms by r , noticing that immediately after an adjustment x reverts to the reset value \hat{x} , adding and subtracting $V(\hat{x}, r)$ on the left side of the equality, and collecting terms yields

$$rV_2(x_0, r) = \frac{r}{1 - E[e^{-r\tau_N} \mid x = x_0]} E \left[\sum_{i=1}^N e^{-r\tau_i} c \mid x = x(0) \right] - \frac{r}{1 - E[e^{-r\tau_N} \mid x = x_0]} [V_2(x_0, r) - V_2(\hat{x}, r)].$$

Taking limits as $r \rightarrow 0$, we get $\lim_{r \rightarrow 0} \sum_{i=1}^N e^{-r\tau_i} = N$ and $\lim_{r \rightarrow 0} \frac{r}{1 - E[e^{-r\tau_N} \mid x = x_0]} = \frac{1}{E[\tau_N \mid x = x_0]}$ so that

$$\lim_{r \rightarrow 0} rV_2(x_0, r) = c \frac{N}{E[\tau_N \mid x = x_0]} - \frac{1}{E[\tau_N \mid x = x_0]} [V_2(x_0, r) - V_2(\hat{x}, r)]$$

For $N \rightarrow \infty$, the first term in the right hand side converges to λ_a by the strong law of large numbers of renewal theory, and the second term vanishes since $|V_2(x_0, r) - V_2(\hat{x}, r)| \leq 1$ for all $r > 0$, so that

$$\lim_{r \rightarrow 0} rV_2(x_0, r) = \lambda_a \text{ for all } x_0$$

Equation (14) together with its boundary conditions is a functional equation to find a constant A , the values $(\underline{x}, \hat{x}, \bar{x})$ and a twice differentiable function $v : [\underline{x}, \bar{x}] \rightarrow \mathbb{R}$. Moreover, A is the expected profit net of the expected cost of adjustment—i.e., $A = E[F(x)] - \lambda_a c$.

In the case with $\sigma = 0$, the integral $\lim_{T \rightarrow \infty} \int_0^{\infty} \frac{1}{T} F(x(t)) dt$ converges to a value that is independent of $x(0)$ for the path of $x(t)$ that solves the firm's problem and a similar argument applies. Analogously for the expected costs of price adjustment V_2 .

■

LEMMA 4. Given \underline{x} and \bar{x} , the function v described in **Lemma 3** is the power series

$$v(x) = \sum_{i=1}^{\infty} \alpha_i x^i. \quad (16)$$

In the case with $\pi = 0$ and $\sigma > 0$ the coefficients are

$$\alpha_2 = \frac{A}{\sigma^2}, \alpha_4 = -\frac{1}{6} \frac{B}{\sigma^2} \text{ and } \alpha_i = 0 \text{ for all } i \neq 2, 4$$

In the case with $\pi > 0$ and $\sigma = 0$, the coefficients are

$$\alpha_1 = -\frac{A}{\pi}, \alpha_3 = \frac{1}{3} \frac{B}{\pi} \text{ and } \alpha_i = 0 \text{ for } i = 2 \text{ and for } i \geq 4.$$

Proof. We look for the coefficients of (16) that solve (14) for all $x \in [\underline{x}, \bar{x}]$. Substituting for v' and for v'' in (14) and matching coefficients yields the results. When $\sigma > 0$, the conditions for matching coefficients are

$$\begin{aligned} A &= -\alpha_1\pi + \alpha_2\sigma^2 \\ \alpha_3 &= \frac{1}{3} \left(\frac{2\pi}{\sigma^2} \right) \alpha_2 \\ \alpha_4 &= -\frac{1}{6} \frac{B}{\sigma^2} + \frac{1}{12} \left(\frac{2\pi}{\sigma^2} \right)^2 \alpha_2 \\ \alpha_i &= \left[-2 \frac{B}{\sigma^2} + \left(\frac{2\pi}{\sigma^2} \right)^2 \alpha_2 \right] \frac{2}{i!} \left(\frac{2\pi}{\sigma^2} \right)^{i-4} \quad \text{for } i \geq 5. \end{aligned}$$

For $\pi = 0$ we have that $\alpha_i = 0$ for all i except for α_2 and α_4 . For $i = 3$ and for $i \geq 5$ this follows directly from the conditions above. $\alpha_1 = 0$ is a condition for the symmetry of v when $\pi = 0$. ■

Finding $v(x)$ then requires to solve a two dimensional problem in α_1 and α_2 when $\sigma > 0$ (or in α_1 and α_3 when $\sigma = 0$) using the boundary conditions in Lemma 3.

LEMMA 5. The solution for the thresholds when $\pi = 0$ is $\hat{x} = 0$, $\bar{x} = -\underline{x} = \left(6c \frac{\sigma^2}{B}\right)^{\frac{1}{2}}$ and the constant $A = \left(\frac{2}{3}Bc\sigma^2\right)^{\frac{1}{2}}$.

Proof. We will use the normalization $v(\hat{x}) = 0$, the smooth pasting condition $v'(\bar{x}) = 0$ and the value matching conditions $v(\bar{x}) = v(\underline{x}) = c$ together with Lemma 4. From Lemma 4 we know that for the case of $\pi = 0$ the only terms in (16) are the powers 2 and 4 of x with $A = \alpha_2\sigma^2$ and $\alpha_4 = -\frac{1}{6} \frac{B}{\sigma^2}$. This implies that for $v'(\bar{x}) = 0$ we need $A = B/3\bar{x}^2$, which, in turn, implies that $v(\bar{x}) = 1/6 \frac{B}{\sigma^2} \bar{x}^4 = c$ from where we obtain the expressions for \bar{x} and A . $\hat{x} = 0$ then follows from $v(\hat{x}) = 0$. Later we use the fact the $\alpha_2 = \frac{1}{3} B \frac{\bar{x}^2}{\sigma^2}$ ■

LEMMA 6. The derivatives of the thresholds are for $\pi = 0$ are given by:

$$\frac{\partial \bar{x}}{\partial \pi} = \frac{\partial \underline{x}}{\partial \pi} = \frac{2}{15} \frac{1}{\lambda_a(0)} \quad \text{and} \quad \frac{\partial \hat{x}}{\partial \pi} = \frac{21}{90} \frac{1}{\lambda_a(0)}$$

COROLLARY 2. The derivative of the size of price changes with respect to inflation for $\pi = 0$ is

$$\frac{\partial \Delta_p^+(0)}{\partial \pi} = -\frac{\partial \Delta_p^-(0)}{\partial \pi} = 0.1 \frac{1}{\lambda_a(0)}$$

Proof. We solve first for the derivatives of α_1 and α_2 with respect to π at $\pi = 0$.

Derivative of α_2 : From the representation of $v(x; \pi)$ in (16) we have that $\frac{\partial^2}{(\partial x)^2} v(0; \pi) = 2\alpha_2$. The symmetry of the value function implies that $v(x, \pi) = v(-x, -\pi)$, which implies $\frac{\partial^3}{(\partial x)^2 \partial \pi} v(x, \pi) = -\frac{\partial^3}{(\partial x)^2 \partial \pi} v(-x, -\pi)$ for all $x \in [\underline{x}, \bar{x}]$ and for all π . Hence, $\frac{\partial^3}{(\partial x)^2 \partial \pi} v(0, 0) = -\frac{\partial^3}{(\partial x)^2 \partial \pi} v(0, 0) = 0$. Finally, $\frac{\partial^3}{(\partial x)^2 \partial \pi} v(0, 0) = 2 \frac{\partial \alpha_2}{\partial \pi} = 0$.

Derivative of α_1 : From the representation of $v(x, \pi)$ in (16), using $\frac{\partial \alpha_2}{\partial \pi} = 0$ at $\pi = 0$, we have that

$$\frac{\partial v(x, 0)}{\partial \pi} = \frac{\partial \alpha_1(0)}{\partial \pi} x + \frac{2\alpha_2}{3\sigma^2} x^3 - \frac{1}{15} \frac{B}{\sigma^4} x^5$$

Differentiating value matching, $v(\hat{x}(\pi), \pi) + c = v(\bar{x}(\pi), \pi)$ with respect to inflation and using smooth pasting we get $\frac{\partial v(\hat{x}, \pi)}{\partial \pi} = \frac{\partial v(\bar{x}, \pi)}{\partial \pi}$. Evaluating at $\pi = 0$, $\frac{\partial v(\bar{x}, 0)}{\partial \pi} = \frac{\partial v(0, 0)}{\partial \pi}$, which is equal to zero since by symmetry $\frac{\partial v(0, 0)}{\partial \pi} = 0$. Therefore, evaluating the first equation at \bar{x} and dividing by \bar{x} we get

$$0 = \frac{\partial \alpha_1(0)}{\partial \pi} + \frac{2}{3\sigma^2} \alpha_2 \bar{x}^2 - \frac{1}{15} \frac{B}{\sigma^4} \bar{x}^4$$

On the other hand, smooth pasting evaluated at $\pi = 0$, yields $\frac{\partial v(\bar{x}, 0)}{\partial x} = 2\alpha_2 \bar{x} - 4\frac{1}{6} \frac{B}{\sigma^2} \bar{x}^3 = 0$, implying $\alpha_2 \bar{x}^2 = \frac{1}{3} \frac{B}{\sigma^2} \bar{x}^4$. Substituting above implies

$$0 = \frac{\partial \alpha_1(0)}{\partial \pi} + \frac{2}{3\sigma^2} \frac{1}{3} \frac{B}{\sigma^2} \bar{x}^4 - \frac{1}{15} \frac{B}{\sigma^4} \bar{x}^4$$

and we get

$$\frac{\partial \alpha_1(0)}{\partial \pi} = -\frac{7}{45} B \frac{\bar{x}^4}{\sigma^4}.$$

Now we are ready to take the derivative of the thresholds X with respect to π at $\pi = 0$.

Derivative of \hat{x} . Using the implicit function theorem on the smooth pasting condition $\frac{\partial v(\hat{x}, 0)}{\partial x} = 0$ and recalling that at $\pi = 0$ $\hat{x} = 0$ we get

$$\frac{\partial \hat{x}(0)}{\partial \pi} = -\frac{\frac{\partial^2 v(0, 0)}{\partial x \partial \pi}}{\frac{\partial^2 v(0, 0)}{(\partial x)^2}} = -\frac{\frac{\partial \alpha_1(0)}{\partial \pi}}{2\alpha_2(0)} = \frac{7}{45} B \frac{\bar{x}^4}{\sigma^4} = \frac{21}{90} \frac{\bar{x}^2}{\sigma^2}$$

Derivative of \bar{x} and of \underline{x} . Using the implicit function theorem on the smooth pasting condition $\frac{\partial v(\bar{x}, 0)}{\partial x} = 0$ we get

$$\frac{\partial \bar{x}(0)}{\partial \pi} = -\frac{\frac{\partial^2 v(\bar{x}, 0)}{\partial x \partial \pi}}{\frac{\partial^2 v(\bar{x}, 0)}{(\partial x)^2}}$$

Using the expression of α_2 for $\pi = 0$ in the proof of Lemma 5 and the result for the derivative of α_1 , the numerator is

$$\begin{aligned} \frac{\partial^2 v(\bar{x}, 0)}{\partial x \partial \pi} &= \frac{\partial \alpha_1(0)}{\partial \pi} + \frac{2\alpha_2}{3\sigma^2} \bar{x}^2 - \frac{1}{15} B \frac{\bar{x}^4}{\sigma^4} \\ &= -\frac{7}{45} B \frac{\bar{x}^4}{\sigma^4} + \frac{1}{3} B \frac{\bar{x}^4}{\sigma^4} = \frac{8}{45} B \frac{\bar{x}^4}{\sigma^4} \end{aligned}$$

Using (16) with the coefficients in Lemma 4 evaluated at $\pi = 0$ as well as the expression for α_2 for $\pi = 0$, the

denominator is

$$\begin{aligned}\frac{\partial^2 v(\bar{x}, 0)}{(\partial x)^2} &= 2\alpha_2 - 2B \frac{\bar{x}^2}{\sigma^2} \\ &= \frac{2}{3}B \frac{\bar{x}^2}{\sigma^2} - 2B \frac{\bar{x}^2}{\sigma^2} = -\frac{4}{3}B \frac{\bar{x}^2}{\sigma^2}.\end{aligned}$$

Symmetry implies that $\frac{\partial \bar{x}(0)}{\partial \pi} = \frac{\partial \underline{x}(0)}{\partial \pi}$. Therefore,

$$\frac{\partial \bar{x}(0)}{\partial \pi} = \frac{\partial \underline{x}(0)}{\partial \pi} = \frac{2}{15} \frac{\bar{x}^2}{\sigma^2}$$

Size of price changes. Recall that the size of price increases and of price decreases is given by

$$\Delta_p^+(\pi) = \hat{x}(\pi) - \underline{x}(\pi) \text{ and } \Delta_p^-(\pi) = \bar{x}(\pi) - \hat{x}(\pi)$$

Using the previous results $\frac{\partial \Delta_p^+(0)}{\partial \pi} = \frac{\partial \hat{x}(0)}{\partial \pi} - \frac{\partial \underline{x}(0)}{\partial \pi} = \frac{21}{90} \frac{\bar{x}^2}{\sigma^2} - \frac{2}{15} \frac{\bar{x}^2}{\sigma^2} = 0.1 \frac{\bar{x}^2}{\sigma^2}$. We also know that $\lambda_a(\pi)$ for $\pi = 0$ is $\lambda_a(0) = \frac{\sigma^2}{\bar{x}^2}$. Hence,

$$\frac{\partial \Delta_p^+(0)}{\partial \pi} = -\frac{\partial \Delta_p^-(0)}{\partial \pi} = 0.1 \frac{1}{\lambda_a(0)}$$

■

LEMMA 7. Assume that $\sigma = 0$ and $\pi > 0$ or, equivalently, that $\pi/\sigma \rightarrow \infty$ for $\sigma > 0$. Then,

$$\hat{x}(\pi) = -\underline{x}(\pi) = \frac{1}{2} \Delta_p^+(\pi) = \left(\frac{3c}{4B} \pi \right)^{1/3}$$

$$\lambda_a = \lambda_a^+ = \frac{\pi}{\Delta_p^+(\pi)} = \frac{1}{2} \left(\frac{3c}{4B} \right)^{-1/3} \pi^{2/3}$$

Proof. Consider the case in which $\sigma = 0$. This is equivalent to taking the limit for $\pi \rightarrow \infty$ with $\sigma > 0$ —see [Lemma 2](#) or [Lemma 4](#). The state moves deterministically from \hat{x} to \underline{x} at speed π . We find closed form solutions for these thresholds. From [Lemma 4](#) we know that for $\sigma = 0$ the value function is $v(x) = \alpha_1 \pi + \alpha_3 x^3$ with $\alpha_1 = -A/\pi$ and $\alpha_3 = \frac{B}{3\pi}$. We use smooth pasting at \hat{x} and \underline{x} , together with value matching to solve for α_1 , \hat{x} and \underline{x} . Smooth pasting, $v'(\hat{x}) = 0$ and $v'(\underline{x}) = 0$ implies that $\hat{x} = -\underline{x} = \sqrt{\frac{-\alpha_1 \pi}{B}}$. Value matching, $v(\hat{x}) - v(\underline{x}) = c$, then implies $\alpha_1 = -\left(c \frac{3}{4}\right)^{2/3} \left(\frac{B}{\pi}\right)^{1/3}$. The results of the lemma follow. ■

B ILLUSTRATING THE THEORY WITH GOLOSOV AND LUCAS'S (2007) MODEL

In this section we specify a version of the firm's problem studied in [Section II.A](#) to illustrate the theory. We characterize the solution of the model analytically and numerically and show how changes in the rate of inflation

affect optimal pricing rules, the frequency of price changes, and the size of price adjustments.

We compute this example to verify the robustness of the analytical predictions obtained so far. In [Section II.A](#) we obtained sharp analytical results under a variety of simplifying assumption such as limit values of parameters (e.g., vanishing menu cost c and or discount rate r), or the shape of profit functions (e.g., symmetry of F). Also, our analytical results were obtained at two extreme values of inflation. In this section, we check the robustness of the simplifying assumptions by computing a version of the model away from the limit cases, considering values of inflation in the range observed in Argentina.

The example assumes a constant elasticity of demand, a constant returns to scale production technology, idiosyncratic shocks to marginal cost that are permanent, an exponentially distributed product life and a cost of changing prices that is proportional to current profits (but independent of the size of the price change). This version of the [Golosov and Lucas \(2007\)](#) model is identical to the one in [Kehoe and Midrigan \(2015\)](#).³⁷

We assume that period profits are given by a demand with constant elasticity η and with a constant return to scale technology with marginal cost given by e^z , so that

$$F(p, z) = e^{-\eta p} (e^p - e^z) ,$$

where $p - z$ is the log of the gross markup (the net markup).

The shocks on the log of the cost are permanent in the sense that

$$dz = \mu_z dt + \sigma dW - z dN.$$

where N is the counter of a Poisson process with constant arrival rate per unit of time ρ . We interpret this as products dying with Poisson arrival rate $\rho > 0$ per unit of time, at which time they are replaced by a new one which starts with $z = 0$ and must set its initial price. A positive value of μ_z can be interpreted as a vintage effect, i.e., the technology for new products grows at rate μ_z . The disappearance of products assures the existence of an ergodic distribution of relative prices, a realistic device given the rate at which product are substituted in most datasets.

The menu cost, $\zeta(z)$, is assumed to be $\zeta(z) = c F(p^*(z), z)$ for some constant $c > 0$, so that it is proportional to the maximized profit. Here, $p^*(z) = z + m$ where $m = \log(\frac{\eta}{\eta-1})$ is the log of the gross optimal static markup. Thus $F(p^*(z), z) = e^{(1-\eta)z} \left(\frac{\eta-1}{\eta}\right)^\eta \frac{1}{\eta-1}$. Note that $F(p^*(z), z)$ is decreasing and strictly convex on z for $\eta > 1$.

We will assume that $\eta > 1$ so that the static monopolist problem has a solution, and that

$$r + \rho \geq (1 - \eta) \left[\mu_z + (1 - \eta) \frac{\sigma^2}{2} \right] \quad (17)$$

so that the profits of the problem with zero fixed cost, $c = 0$ are finite. Since for $\eta > 1$, period profits are decreasing and convex on z , and hence discounted expected profits are finite if the discount rate $r + \rho$ is high enough, or if the cost increases at a high enough rate μ_z , or if $\sigma^2/2$ is low enough.³⁸

³⁷We zero out the transitory shock that gives rise to sales in [Kehoe and Midrigan \(2015\)](#) and write the model in continuous time.

³⁸In this case the expected discounted value of profits, starting with z_0 is given by $\mathbb{E}_0 \int_0^\infty e^{-(r+\rho)t} F(p^*(z_t), z_t) dt =$

The firm's optimal pricing policy for the $\sigma > 0$ case can be characterized in terms of three constants $X \equiv (\underline{x}, \bar{x}, \hat{x})$. This is due to the combination of assumptions of constant elasticity of demand, constant returns to scale and permanent shocks to cost while the product lasts. In this example, the policy function takes the simple form $\Psi(z) = X + z$. Letting $x = p - z$ be the log of the real gross mark-up, which we refer to as the net markup, we write the inaction set as $\mathcal{I} = \{x : \underline{x} < x < \bar{x}\}$. It is optimal to keep the price unchanged when the net markup x is in the interval (\underline{x}, \bar{x}) . When prices are not changed, the real markup evolves according to $dx = -(\mu_z + \pi)dt + \sigma dW$. When the real markup hits either of the two thresholds, prices are adjusted so that the real markup is \hat{x} and thus the optimal return relative price is $\hat{\psi}(z) = \hat{x} + z$. For the case where $\sigma = 0$ and $\pi + \mu_z > 0$, we obtain a version of [Sheshinski and Weiss's \(1977\)](#) model, and the optimal policy can be characterized simply by two thresholds $s \equiv \underline{x} < \hat{x} \equiv S$.³⁹

In [online appendix BA](#), we present several propositions with an analytical characterization of the solution of this model. For the numerical examples in [Section III](#), we use those propositions together with the following calibration. We follow [Kehoe and Midrigan \(2015\)](#) and use $\eta = 3$, which implies a very large markup, but it is roughly inline with marketing/IO estimates of demand elasticities. We set $\rho = 0.1$ so products have a lifetime of 10 years, and $r = 0.06$ so yearly interest rates are 6%. We let $c = 0.002$ so that adjustment cost is 20 basis point of yearly frictionless profits. We let $\mu_z = 0.02$, i.e., a 2 percent per year increase in cost (or a 2% increase in vintage productivity). We consider three values for $\sigma \in \{0, 0.15, 0.20\}$, the first corresponds to [Sheshinski and Weiss's \(1977\)](#) model, and the others are 15% and 20% standard deviation in the change in marginal cost, at annual rates. The values of $c / (\eta(\eta - 1))$ and $\sigma = 0.15$ were jointly chosen so that at zero inflation the model matches both the average number of price changes $\lambda_a = 2.7$ and the average size of price changes of $\Delta_p = 0.10$, roughly the values corresponding to zero inflation in our data set.⁴⁰

BA Analytical Characterization of the Model in [Section B](#)

In what follows we use $x = p - z$ for the log of the real gross mark-up. In [Proposition 4](#) we show that when $\sigma > 0$, the inaction set is given by $\mathcal{I} = \{(p, z) : \underline{x} + z < p < \bar{x} + z\}$ and that the optimal return point is given by $\hat{\psi}(z) = \hat{x} + z$ for three constants $X \equiv (\underline{x}, \hat{x}, \bar{x})$. This is due to the combination of assumptions of constant elasticity of demand, constant returns to scale and permanent shocks to cost while the product last. This means that it is optimal to keep the price unchanged when the real markup x is in the interval (\underline{x}, \bar{x}) . When prices are not changed, the real markup evolves according to $dx = -(\mu_z + \pi)dt + \sigma dW$. When the real markup hits either of the two thresholds, prices are adjusted so that the real markup is \hat{x} . [Proposition 4](#) derives a system of three equations in three unknowns for X , as well as the explicit solution to the value function, as function of the parameters $\Theta \equiv (\pi, \mu_z, \sigma^2, \rho, r, \eta, c)$. [Proposition 5](#) derives an explicit solution for the expected number of adjustment per unit of time λ_a given a policy X and parameters $(\pi, \mu_z, \sigma^2, \rho)$. [Proposition 6](#) characterizes the

$$\frac{(\eta-1)^{\eta-1} e^{(1-\eta)\gamma_0}}{\eta^\eta} \int_0^\infty e^{[-(r+\rho)+(1-\eta)\mu_z+(1-\eta)^2 \frac{\sigma^2}{2}]t} dt.$$

³⁹ In this case $\lambda_a = \rho / \left[1 - \exp\left(-\frac{\rho}{\pi + \mu_z}(\hat{x} - \underline{x})\right)\right] = (\pi + \mu_z) / (\hat{x} - \underline{x}) + o(\rho / (\pi + \mu_z))$, so that it coincides with the expression used for $\sigma = 0$ if ρ is small relative to π .

⁴⁰ Using results in [Alvarez, Lippi, and Paciello \(2011\)](#), when z is a random walk and the fixed cost is proportional to profits, there are mappings from observations to parameters $\lambda_a = \left[\frac{\sigma^2 (1/2) \eta (\eta-1)}{c}\right]^{\frac{1}{2}}$ and $\Delta_p = \left[\sigma^2 \frac{c}{(1/2) \eta (\eta-1)}\right]^{\frac{1}{4}} \sqrt{\frac{2}{3.14159\dots}}$

density g for invariant distribution of (p, z) implied by the policy X and the parameters $(\pi, \mu_z, \sigma^2, \rho, \eta)$.

For future reference we define \hat{c} implicitly $\zeta(z) = \hat{c} e^{z(1-\eta)}$

PROPOSITION 4. Assume that $\sigma > 0, c > 0$ and that (17) holds. The inaction set is given by $\mathcal{I} = \{(p, z) : \underline{x} + z < p < \bar{x} + z\}$. The optimal return point is given by $\hat{\psi}(z) = \hat{x} + z$. The value function in the range of inaction and the constants $X \equiv (\underline{x}, \hat{x}, \bar{x})$ with $\underline{x} < \hat{x} < \bar{x}$ solve

$$V(p, z) = e^{z(1-\eta)} V(p - z, 0) \equiv e^{z(1-\eta)} v(p - z) \quad (18)$$

$$v(x) = a_1 e^{x(1-\eta)} + a_2 e^{-x\eta} + \sum_{i=1}^2 A_i e^{v_i x} \quad (19)$$

where the coefficients a_i, v_i are given by

$$\begin{aligned} 0 &= -b_0 + b_1 v_i + b_2 (v_i)^2 \\ a_1 &= \frac{1}{b_0 - (1-\eta) b_1 - (1-\eta)^2 b_2} \text{ and } a_2 = -\frac{1}{b_0 + \eta b_1 - (\eta)^2 b_2} \text{ where} \\ b_0 &= r + \rho - \mu_z(1-\eta) - (1-\eta)^2 \frac{\sigma^2}{2}, \quad b_1 = -\left[\mu_z + \pi + 2(1-\eta) \frac{\sigma^2}{2} \right], \quad b_2 = \frac{\sigma^2}{2}. \end{aligned}$$

and where the five values A_1, A_2, X solve the following five equations:

$$\begin{aligned} \hat{c} - a_1 \left(e^{\hat{x}(1-\eta)} - e^{\bar{x}(1-\eta)} \right) - a_2 \left(e^{-\hat{x}\eta} - e^{-\bar{x}\eta} \right) &= \sum_{i=1}^2 A_i \left(e^{v_i \hat{x}} - e^{v_i \bar{x}} \right), \\ \hat{c} - a_1 \left(e^{\hat{x}(1-\eta)} - e^{\underline{x}(1-\eta)} \right) - a_2 \left(e^{-\hat{x}\eta} - e^{-\underline{x}\eta} \right) &= \sum_{i=1}^2 A_i \left(e^{v_i \hat{x}} - e^{v_i \underline{x}} \right), \\ 0 &= a_1 (1-\eta) e^{\hat{x}(1-\eta)} - a_2 \eta e^{-\hat{x}\eta} + \sum_{i=1}^2 A_i v_i e^{v_i \hat{x}}, \\ 0 &= a_1 (1-\eta) e^{\bar{x}(1-\eta)} - a_2 \eta e^{-\bar{x}\eta} + \sum_{i=1}^2 A_i v_i e^{v_i \bar{x}}, \\ 0 &= a_1 (1-\eta) e^{\underline{x}(1-\eta)} - a_2 \eta e^{-\underline{x}\eta} + \sum_{i=1}^2 A_i v_i e^{v_i \underline{x}}. \end{aligned}$$

The first two equations are linear in (A_1, A_2) , given X .

Proof. To simplify the notation we evaluate all the expressions when $\bar{p} = 0$, so the relative price and the nominal price coincide.

The Bellman equation in the inaction region $(p, z) \in \mathcal{I}$ is

$$(r + \rho)V(p, z) = e^{-\eta p} (e^p - e^z) - \pi V_p(p, z) + V_z(p, z)\mu_z + V_{zz}(p, z)\frac{\sigma^2}{2}$$

for all $p \in [\underline{p}(z), \bar{p}(z)]$. The boundary conditions are given by first order conditions for the optimal return point:

$$V_p(\hat{\psi}(z), z) = 0 \quad (20)$$

the value matching conditions, stating that the value at each of the two boundaries is the same as the value at the optimal price after paying the cost:

$$V(\underline{p}(z), z) = V(\hat{\psi}(z), z) - \zeta(z), \quad V(\bar{p}(z), z) = V(\hat{\psi}(z), z) - \zeta(z). \quad (21)$$

and the smooth pasting conditions, stating that the value function should have the same slope at the boundary than the value function in the control region (which is flat), so:

$$V_p(\underline{p}(z), z) = 0, \quad V_p(\bar{p}(z), z) = 0, \quad (22)$$

Under this conditions, the value function and optimal policies are homogeneous in the sense that:

$$V(p, z) = e^{z(1-\eta)} V(p-z, 0) \equiv e^{z(1-\eta)} v(p-z) \quad (23)$$

$$\underline{p}(z) = \underline{z} + z, \quad \bar{p}(z) = \bar{x} + z, \quad \text{and} \quad \hat{\psi}(z) = \hat{x} + z, \quad (24)$$

where \underline{z} , \bar{x} and \hat{x} are three constant to be determined.

Using the homogeneity of the value function in (23) we can compute the derivatives

$$V_p(p, z) = e^{z(1-\eta)} v'(p-z)$$

$$V_z(p, z) = (1-\eta)e^{z(1-\eta)} v(p-z) - e^{z(1-\eta)} v'(p-z)$$

$$V_{zz}(p, z) = (1-\eta)^2 e^{z(1-\eta)} v(p-z) - 2(1-\eta)e^{z(1-\eta)} v'(p-z) + e^{z(1-\eta)} v''(p-z)$$

Replacing this derivatives in the Bellman equation for the inaction region we get

$$\begin{aligned} (r + \rho)v(p-z) &= e^{-(p-z)\eta} (e^{p-z} - 1) - \pi v'(p-z) + [(1-\eta)v(p-z) - v'(p-z)]\mu_z \\ &+ [(1-\eta)^2 v(p-z) - 2(1-\eta)v'(p-z) + v''(p-z)]\frac{\sigma^2}{2} \end{aligned}$$

or

$$\begin{aligned} \left[r + \rho - \mu_z(1-\eta) - (1-\eta)^2 \frac{\sigma^2}{2} \right] v(p-z) &= e^{(p-z)(1-\eta)} - e^{-\eta(p-z)} \\ - v'(p-z) [\mu_z + \pi + 2(1-\eta) \frac{\sigma^2}{2}] &+ v''(p-z) \frac{\sigma^2}{2} \end{aligned}$$

We write $x = p - z$ be the log of the gross markup, or the net markup. Consider the free boundary ODE:

$$b_0 v(x) = e^{x(1-\eta)} - e^{-\eta x} + b_1 v'(x) + b_2 v''(x) \text{ for all } x \in [\underline{x}, \bar{x}]$$

$$v(\underline{x}) = v(\hat{x}) - \hat{c}, \quad v(\bar{x}) = v(\hat{x}) - \hat{c},$$

$$v'(\underline{x}) = 0, \quad v'(\bar{x}) = 0, \quad v'(\hat{x}) = 0,$$

where

$$\begin{aligned} b_0 &= \left[r + \rho - \mu_z(1 - \eta) - (1 - \eta)^2 \frac{\sigma^2}{2} \right], \\ b_1 &= - \left[\mu_z + \pi + 2(1 - \eta) \frac{\sigma^2}{2} \right], \\ b_2 &= \frac{\sigma^2}{2}. \end{aligned}$$

The value function is given by the sum of the particular solution and the solution of the homogeneous equation:

$$v(x) = a_1 e^{x(1-\eta)} + a_2 e^{-x\eta} + \sum_{i=1}^2 A_i e^{v_i x}$$

where v_i are the roots of the quadratic equation

$$0 = -b_0 + b_1 v_i + b_2 (v_i)^2$$

and where the coefficients for the particular solution are

$$\begin{aligned} a_1 &= \frac{1}{b_0 - (1 - \eta) b_1 - (1 - \eta)^2 b_2} \\ a_2 &= - \frac{1}{b_0 + \eta b_1 - (\eta)^2 b_2}, \end{aligned}$$

since

$$\begin{aligned} b_0 a_1 e^{x(1-\eta)} &= e^{x(1-\eta)} + a_1 (1 - \eta) e^{x(1-\eta)} b_1 + a_1 (1 - \eta)^2 e^{x(1-\eta)} b_2 \\ b_0 a_2 e^{-x\eta} &= -e^{-x\eta} - a_2 \eta e^{-x\eta} b_1 + a_2 (\eta)^2 e^{-x\eta} b_2. \end{aligned}$$

The five constants A_1, A_2 and $X \equiv (\bar{x}, \underline{x}, \hat{x})$ are chosen to satisfies the 2 value matching conditions (21), the two smooth pasting conditions (22) and the optimal return point (20). It is actually more convenient to solve the value function in two steps. First to solve for the constants $A_i(X)$ for $i = 1, 2$ using the two value matching conditions. Mathematically, the advantage of this intermediate step is that, given X , the equations for the A_1, A_2 are linear. Conceptually, the advantage is that the solution represent the value of the policy described by the triplet $X = (\bar{x}, \underline{x}, \hat{x})$. Then we solve for $(\bar{x}, \underline{x}, \hat{x})$ using the conditions for the optimality of the thresholds, namely the two smooth pasting (22) and the f.o.c. for the return point (20).

Solving A_1, A_2 for a given policy X amount to solve the following linear system:

$$\begin{aligned} \hat{c} - a_1 \left(e^{\hat{x}(1-\eta)} - e^{\bar{x}(1-\eta)} \right) - a_2 \left(e^{-\hat{x}\eta} - e^{-\bar{x}\eta} \right) &= \sum_{i=1}^2 A_i \left(e^{v_i \hat{x}} - e^{v_i \bar{x}} \right) \\ \hat{c} - a_1 \left(e^{\hat{x}(1-\eta)} - e^{\underline{x}(1-\eta)} \right) - a_2 \left(e^{-\hat{x}\eta} - e^{-\underline{x}\eta} \right) &= \sum_{i=1}^2 A_i \left(e^{v_i \hat{x}} - e^{v_i \underline{x}} \right) \end{aligned}$$

Given $A_1(X), A_2(X)$ we need to solve the following three equations:

$$\begin{aligned} 0 &= a_1 (1 - \eta) e^{\hat{x}(1-\eta)} - a_2 \eta e^{-\hat{x}\eta} + \sum_{i=1}^2 A_i(X) v_i e^{v_i \hat{x}}, \\ 0 &= a_1 (1 - \eta) e^{\bar{x}(1-\eta)} - a_2 \eta e^{-\bar{x}\eta} + \sum_{i=1}^2 A_i(X) v_i e^{v_i \bar{x}}, \\ 0 &= a_1 (1 - \eta) e^{\underline{x}(1-\eta)} - a_2 \eta e^{-\underline{x}\eta} + \sum_{i=1}^2 A_i(X) v_i e^{v_i \underline{x}}. \end{aligned}$$

■

The expected number of adjustments per unit of time is given in the next proposition:

PROPOSITION 5. Given a policy described by $X = (\underline{x}, \hat{x}, \bar{x})$ the expected number of adjustment per unit of time λ_a and the expected number of price increases λ_a^+ are given by

$$\begin{aligned} \lambda_a &= 1 / \left[\frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i \hat{x}} \right] \\ \lambda_a^+ &= 1 / \left[\frac{1}{\rho} + \sum_{i=1}^2 B_{l,i} e^{q_i \hat{x}} \right] \end{aligned}$$

where q_i are the roots of $\rho = -(\pi + \mu_z)q_i + \frac{\sigma^2}{2}(q_i)^2$ and where B_i and $B_{l,i}, B_{H,i}$ solve the following system of linear equations:

$$\begin{aligned} 0 &= \frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i \underline{x}} = \frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i \bar{x}}, \\ \frac{1}{\rho} &= -B_{h,1} e^{q_1 \hat{x}} - B_{h,2} e^{q_2 \hat{x}}, \quad 0 = B_{h,1} (e^{q_1 \hat{x}} - e^{q_1 \bar{x}}) + B_{h,2} (e^{q_2 \hat{x}} - e^{q_2 \bar{x}}), \\ -\frac{1}{\rho} &= B_{l,1} e^{q_1 \underline{x}} + B_{l,2} e^{q_2 \underline{x}}, \quad 0 = B_{l,1} q_1 e^{q_1 \hat{x}} + B_{l,2} q_2 e^{q_2 \hat{x}} - B_{h,1} q_1 e^{q_1 \hat{x}} - B_{h,2} q_2 e^{q_2 \hat{x}}. \end{aligned}$$

Proof. The expected time until the next adjustment solves the following Kolmogorov equation:

$$\rho \mathcal{T}(p, z) = 1 - \pi \mathcal{T}_p(p, z) + \mathcal{T}_z(p, z) \mu_z + \mathcal{T}_{zz}(p, z) \frac{\sigma^2}{2}$$

for all p such that $\underline{p}(z) < p < \bar{p}(z)$, and all z . The boundary conditions are that time reaches zero when it hits the barriers:

$$\mathcal{T}(\bar{p}(z), z) = \mathcal{T}(\underline{p}(z), z) = 0.$$

Given the homogeneity of the policies we look for a function satisfying

$$\mathcal{T}(p, z) = T(p - z)$$

Given the form of the expected time we have:

$$\mathcal{T}_p(p, z) = T'(p - z), \quad \mathcal{T}_z(p, z) = -T'(p - z) \text{ and } \mathcal{T}_{zz}(p, z) = T''(p - z),$$

so the Kolmogorov equation becomes:

$$\rho T(x) = 1 - (\pi + \mu_z)T'(x) + T''(x)\frac{\sigma^2}{2} \text{ for all } x \in (\underline{x}, \bar{x}).$$

The solution of this equation, given \underline{x}, \bar{x} is:

$$T(x) = \frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i x} \text{ for all } x \in (\underline{x}, \bar{x})$$

where q_i are roots of

$$\rho = -(\pi + \mu_z)q_i + \frac{\sigma^2}{2}(q_i)^2, \quad (25)$$

and where the B_1, B_2 are chosen so that the expected time is zero at the boundaries:

$$0 = \frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i \underline{x}} \quad (26)$$

$$0 = \frac{1}{\rho} + \sum_{i=1}^2 B_i e^{q_i \bar{x}} \quad (27)$$

Given the solution of this two linear equations $B_1(\underline{x}, \bar{x}), B_2(\underline{x}, \bar{x})$ the expected number of adjustments per unit of time λ_a is given by

$$\lambda_a = \frac{1}{T(\hat{x})} = \frac{1}{\frac{1}{\rho} + \sum_{i=1}^2 B_i(\underline{x}, \bar{x}) e^{q_i \hat{x}'}}$$

Finally, we derive the expression for the frequency of price increases. The time until the next price increase is the first time until x hits \underline{x} or the product dies while $\underline{x} < x < \hat{x}$. If x hits \bar{x} , or the product dies exogenously while $\bar{x} > x > \hat{x}$, then x then is returned to \hat{x} . Thus the expected time until the next increase in price solves the following Kolmogorov equation:

$$\rho \mathcal{T}(p, z) = \begin{cases} 1 - \pi \mathcal{T}_p(p, z) + \mathcal{T}_z(p, z)\mu_z + \mathcal{T}_{zz}(p, z)\frac{\sigma^2}{2} & \text{if } p < z + \hat{x} \\ 1 + \rho \mathcal{T}(z + \hat{x}, z) - \pi \mathcal{T}_p(p, z) + \mathcal{T}_z(p, z)\mu_z + \mathcal{T}_{zz}(p, z)\frac{\sigma^2}{2} & \text{if } p > z + \hat{x} \end{cases}$$

for all p such that $\underline{p}(z) < p < \bar{p}(z)$, and all z . The boundary conditions are that time reaches zero when it hits the barriers:

$$\mathcal{T}(\bar{x} + z, z) = \mathcal{T}(\hat{x} + z, z) \text{ and } \mathcal{T}(\underline{x} + z, z) = 0.$$

We look for a solution that is continuous and once differentiable at $(p, z) = (\hat{x} + z, z)$, and otherwise twice continuously differentiable. To do so we let $\mathcal{T}(p, z) = T_h(x)$ for $x \in [\hat{x}, \bar{x}]$ and $\mathcal{T}(p, z) = T_l(x)$ for $x \in [\underline{x}, \hat{x}]$ and

$$\rho T_l(x) = 1 - (\pi + \mu_z)T_l'(x) + T_l''(x)\frac{\sigma^2}{2}$$

$$\begin{aligned}
\rho T_h(x) &= 1 + \rho T_h(\hat{x}) - (\mu_z + \pi) T_h'(x) + T_h''(x) \frac{\sigma^2}{2} \\
T_l(\hat{x}) &= T_h(\hat{x}), T_l'(\hat{x}) = T_h'(\hat{x}) \\
T_h(\hat{x}) &= T_h(\bar{x}), T_l(\underline{x}) = 0.
\end{aligned}$$

The solution for T_j for $j = h, l$ are:

$$T_l(x) = \frac{1}{\rho} + \sum_{i=1}^2 B_{l,i} e^{q_i x} \quad \text{and} \quad T_h(x) = \frac{1}{\rho} + \sum_{i=1}^2 B_{h,i} e^{q_i x} + \left(\frac{1}{\rho} + \sum_{i=1}^2 B_{l,i} e^{q_i \hat{x}} \right)$$

The four boundary conditions become the following four linear equations of the constants B 's:

$$\begin{aligned}
\sum_{i=1}^2 B_{l,i} e^{q_i \hat{x}} &= \sum_{i=1}^2 B_{h,i} e^{q_i \hat{x}} + \sum_{i=1}^2 B_{l,i} e^{q_i \hat{x}} + \frac{1}{\rho} \\
\sum_{i=1}^2 B_{l,i} q_i e^{q_i \hat{x}} &= \sum_{i=1}^2 B_{h,i} q_i e^{q_i \hat{x}} \\
\sum_{i=1}^2 B_{l,i} e^{q_i \bar{x}} &= 0 \\
\sum_{i=1}^2 B_{h,i} e^{q_i \bar{x}} &= \sum_{i=1}^2 B_{h,i} e^{q_i \bar{x}}
\end{aligned}$$

Hence, the frequency of price increases λ_a^+ is given by

$$\lambda_a^+ = 1/T_l(\hat{x}) = 1 / \left(\frac{1}{\rho} + \sum_{i=1}^2 B_{l,i} e^{q_i \hat{x}} \right). \blacksquare$$

Now we turn to the density of the invariant distribution

PROPOSITION 6. Given a policy described by $X = (\underline{x}, \hat{x}, \bar{x})$ the density of the invariant distribution $g(p, z)$ is given by

$$g(p, z) = \begin{cases} e^{\phi_1 z} \left[U_1^+ e^{\xi_1(p-z)} + U_2^+ \right] & \text{if } p - z \in (\hat{x}, \bar{x}), z > 0 \\ e^{\phi_1 z} \left[L_1^+ e^{\xi_1(p-z)} + L_2^+ \right] & \text{if } p - z \in [\underline{x}, \hat{x}], z > 0 \\ e^{\phi_2 z} \left[U_1^- e^{\xi_2(p-z)} + L_2^- \right] & \text{if } p - z \in (\hat{x}, \bar{x}), z < 0 \\ e^{\phi_2 z} \left[L_1^- e^{\xi_2(p-z)} + L_2^- \right] & \text{if } p - z \in [\underline{x}, \hat{x}], z < 0 \\ 0 & \text{otherwise} \end{cases} \quad (28)$$

where $\{\phi_1, \phi_2, \xi_1, \xi_2\}$ are given by

$$\begin{aligned}
\rho &= -\mu_z \phi_j + \frac{\sigma^2}{2} \phi_j^2 \quad \text{for each of the roots } j = 1, 2 \quad \text{and} \\
\xi_j &= -\frac{\pi + \mu_z - 2\phi_j \frac{\sigma^2}{2}}{\sigma^2/2}
\end{aligned}$$

and where the coefficients $\{U_i^+, L_i^+, U_i^-, L_i^-\}_{i=1,2}$ solve 8 linear equations:

$$\begin{aligned}
0 &= U_1^+ e^{\xi_1 \bar{x}} + U_2^+ = L_1^+ e^{\xi_1 \bar{x}} + L_2^+ \\
0 &= U_1^- e^{\xi_2 \bar{x}} + U_2^- = L_1^- e^{\xi_2 \bar{x}} + L_2^- \\
\frac{\phi_1 \phi_2}{\phi_1 - \phi_2} &= \frac{L_1^+}{\xi_1} [e^{\xi_1 \hat{x}} - e^{\xi_1 \bar{x}}] + L_2^+ [\hat{x} - \bar{x}] + \frac{U_1^+}{\xi_1} [e^{\xi_1 \bar{x}} - e^{\xi_1 \hat{x}}] + U_2^+ [\bar{x} - \hat{x}] \\
\frac{\phi_1 \phi_2}{\phi_1 - \phi_2} &= \frac{L_1^-}{\xi_2} [e^{\xi_2 \hat{x}} - e^{\xi_2 \bar{x}}] + L_2^- [\hat{x} - \bar{x}] + \frac{U_1^-}{\xi_2} [e^{\xi_2 \bar{x}} - e^{\xi_2 \hat{x}}] + U_2^- [\bar{x} - \hat{x}] \\
U_1^+ e^{\xi_1 \hat{x}} + U_2^+ &= L_1^+ e^{\xi_1 \hat{x}} + L_2^+ \\
U_1^- e^{\xi_2 \hat{x}} + U_2^- &= L_1^- e^{\xi_2 \hat{x}} + L_2^- .
\end{aligned}$$

Proof. The density of the invariant distribution for (p, z) solves the forward Kolmogorov p.d.e.:

$$\rho g(p, z) = \pi g_p(p, z) - \mu_z g_z(p, z) + g_{zz}(p, z) \frac{\sigma^2}{2} \quad (29)$$

for all $(p, z) \neq (\hat{x} + z, z) = (\hat{\psi}(z), z)$ and all $p : \underline{p}(z) = \underline{x} + z \leq p \leq \bar{x} + z = \bar{p}(z)$ and all z . The pde does not apply at the optimal return point, since local consideration cannot determine g there. The other boundary conditions are zero density at the lower and upper boundaries of adjustments, and that g integrates to one:

$$\begin{aligned}
g(\underline{x} + z, z) &= g(\bar{x} + z, z) = 0 \text{ for all } z \\
1 &= \int_{-\infty}^{\infty} \int_{\underline{x}+z}^{\bar{x}+z} g(p, z) dp dz
\end{aligned}$$

We will show that g can be computed dividing the state space in four regions given by whether (p, z) is such that $z > 0$ and $z < 0$ and given z whether $p \in [\underline{x} + z, \hat{x} + z]$ and $p \in [\hat{x} + z, \bar{x} + z]$.

As a preliminary step we solve for the marginal on z of the invariant distribution, which we denote by \tilde{g} . This is the invariant distribution of the process $\{z\}$ which with intensity ρ is re-started at zero and otherwise follows $dz = \mu_z dt + \sigma dW$. It can be shown that \tilde{g} is given by

$$\tilde{g}(z) \equiv \int_{\underline{x}+z}^{\bar{x}+z} g(p, z) dp = \begin{cases} \frac{\phi_1 \phi_2}{\phi_1 - \phi_2} e^{\phi_2 z} & \text{if } z < 0 \\ \frac{\phi_1 \phi_2}{\phi_1 - \phi_2} e^{\phi_1 z} & \text{if } z > 0, \end{cases} \quad (30)$$

where $\phi_1 < 0 < \phi_2$ are the two real roots of the characteristic equation

$$\rho = -\mu_z \phi + \frac{\sigma^2}{2} \phi^2. \quad (31)$$

We conjecture that g can be written as follows:

$$g(p, z) = \begin{cases} e^{\phi_1 z} k(p - z) & \text{if } z < 0 \\ e^{\phi_2 z} k(p - z) & \text{if } z > 0, \end{cases} \quad (32)$$

In this case we compute the derivatives as:

$$\begin{aligned} g_p(p, z) &= e^{\phi z} k'(p - z), \\ g_z(p, z) &= e^{\phi z} \phi k(p - z) - e^{\phi z} k'(p - z), \\ g_{zz}(p, z) &= e^{\phi z} \phi^2 k(p - z) - 2e^{\phi z} \phi k'(p - z) + e^{\phi z} k''(p - z). \end{aligned}$$

where $\phi = \phi_1$ for $z < 0$ and $\phi = \phi_2$ for $z > 0$. The p.d.e. then becomes:

$$\rho k(p - z) = \pi k'(p - z) - \mu_z [\phi k(p - z) - k'(p - z)] + \left[\phi^2 k(p - z) - 2\phi k'(p - z) + k''(p - z) \right] \frac{\sigma^2}{2}$$

for $p - z \neq \hat{x}$ or

$$\left[\rho + \phi \mu_z - \phi^2 \frac{\sigma^2}{2} \right] k(p - z) = \left(\pi + \mu_z - 2\phi \frac{\sigma^2}{2} \right) k'(p - z) + k''(p - z) \frac{\sigma^2}{2}$$

Thus

$$k(p - z) = \begin{cases} U_1 e^{\tilde{\zeta}_{1j}(p-z)} + U_2 e^{\tilde{\zeta}_{2j}(p-z)} & \text{if } p - z \in (\hat{x}, \bar{x}] \\ L_1 e^{\tilde{\zeta}_{1j}(p-z)} + L_2 e^{\tilde{\zeta}_{2j}(p-z)} & \text{if } p - z \in [\underline{x}, \hat{x}] \end{cases}$$

where $\tilde{\zeta}_{1j}, \tilde{\zeta}_{2j}$ solves the quadratic equation:

$$\left[\rho + \phi_j \mu_z - \phi_j^2 \frac{\sigma^2}{2} \right] = \left(\pi + \mu_z - 2\phi_j \frac{\sigma^2}{2} \right) \tilde{\zeta} + \frac{\sigma^2}{2} \tilde{\zeta}^2 \quad (33)$$

for each $j = 1, 2$ corresponding to $\phi = \phi_1$ and $\phi = \phi_2$, i.e., the positive and negative values of z . We note that, by definition of ϕ in (31), the left hand side of (33) equal zero, and hence one of the two roots is always equal to zero. Thus we label $\tilde{\zeta}_{2j} = 0$ for $j = 1, 2$. The remaining root equals:

$$\tilde{\zeta}_{1j} = -\frac{\pi + \mu_z - 2\phi_j \frac{\sigma^2}{2}}{\sigma^2/2} \quad \text{and} \quad \tilde{\zeta}_{2j} = 0 \quad \text{for } j = 1, 2. \quad (34)$$

We integrate $g(p, z)$ over p and equate it to $\tilde{g}(z)$ to obtain a condition for coefficients C . First we consider the case of $z > 0$:

$$\begin{aligned} \tilde{g}(z) &= \int_{\underline{x}+z}^{\hat{x}+z} e^{\phi_1 z} \sum_{i=1}^2 L_i^+ e^{\tilde{\zeta}_{i1}(p-z)} dp + \int_{\hat{x}+z}^{\bar{x}+z} e^{\phi_1 z} \sum_{i=1}^2 U_i^+ e^{\tilde{\zeta}_{i1}(p-z)} dp \\ &= e^{\phi_1 z} \sum_{i=1}^2 \frac{L_i^+ e^{-\tilde{\zeta}_{i1} z}}{\tilde{\zeta}_{i1}} \left[e^{\tilde{\zeta}_{i1}(\bar{x}+z)} - e^{\tilde{\zeta}_{i1}(\hat{x}+z)} \right] + e^{\phi_1 z} \sum_{i=1}^2 \frac{U_i^+ e^{-\tilde{\zeta}_{i1} z}}{\tilde{\zeta}_{i1}} \left[e^{\tilde{\zeta}_{i1}(\hat{x}+z)} - e^{\tilde{\zeta}_{i1}(\underline{x}+z)} \right] \\ &= e^{\phi_1 z} \left(\sum_{i=1}^2 \frac{L_i^+}{\tilde{\zeta}_{i1}} \left[e^{\tilde{\zeta}_{i1} \bar{x}} - e^{\tilde{\zeta}_{i1} \hat{x}} \right] + \sum_{i=1}^2 \frac{U_i^+}{\tilde{\zeta}_{i1}} \left[e^{\tilde{\zeta}_{i1} \hat{x}} - e^{\tilde{\zeta}_{i1} \underline{x}} \right] \right) \\ &= e^{\phi_1 z} \left(\frac{L_1^+}{\tilde{\zeta}_{11}} \left[e^{\tilde{\zeta}_{11} \hat{x}} - e^{\tilde{\zeta}_{11} \underline{x}} \right] + L_2^+ [\hat{x} - \underline{x}] + \frac{U_1^+}{\tilde{\zeta}_{11}} \left[e^{\tilde{\zeta}_{11} \bar{x}} - e^{\tilde{\zeta}_{11} \hat{x}} \right] + U_2^+ [\bar{x} - \hat{x}] \right) \end{aligned}$$

where the last line uses that $\xi_{2,1} = 0$. The analogous expression for $z < 0$ is

$$\tilde{g}(z) = e^{\phi_2 z} \left(\frac{L_1^-}{\xi_{12}} \left[e^{\xi_{12}\hat{x}} - e^{\xi_{12}\underline{x}} \right] + L_2^- [\hat{x} - \underline{x}] + \frac{U_1^-}{\xi_{12}} \left[e^{\xi_{12}\bar{x}} - e^{\xi_{12}\hat{x}} \right] + U_2^- [\bar{x} - \hat{x}] \right)$$

The value of the density at the boundary of the range of inaction is given by

$$g(\bar{x} + z, z) = \begin{cases} e^{\phi_1 z} \sum_{i=1}^2 U_i^+ e^{\xi_{i1}\bar{x}} & \text{for } z > 0 \\ e^{\phi_2 z} \sum_{i=1}^2 U_i^+ e^{\xi_{i2}\bar{x}} & \text{for } z < 0 \end{cases}$$

$$g(\underline{x} + z, z) = \begin{cases} e^{\phi_1 z} \sum_{i=1}^2 L_i^+ e^{\xi_{i1}\underline{x}} & \text{for } z > 0 \\ e^{\phi_2 z} \sum_{i=1}^2 L_i^+ e^{\xi_{i2}\underline{x}} & \text{for } z < 0 \end{cases}$$

If the density g at $(p, z) = (\hat{\psi}(z), z) = (z + \hat{x}, z)$ is continuous on p for a given z we have:

$$g(\hat{x}, z) = \begin{cases} e^{\phi_1 z} \left[\sum_{i=1}^2 U_i^+ e^{\xi_{i1}\hat{x}} \right] = e^{\phi_1 z} \left[\sum_{i=1}^2 L_i^+ e^{\xi_{i1}\hat{x}} \right] & \text{if } z > 0, \\ e^{\phi_2 z} \left[\sum_{i=1}^2 U_i^- e^{\xi_{i2}\hat{x}} \right] = e^{\phi_2 z} \left[\sum_{i=1}^2 L_i^- e^{\xi_{i2}\hat{x}} \right] & \text{if } z < 0. \end{cases} \quad (35)$$

We summarize the results for the invariant density g here

$$g(p, z) = \begin{cases} e^{\phi_1 z} \left[U_1^+ e^{\xi_1(p-z)} + U_2^+ \right] & \text{if } p - z \in (\hat{x}, \bar{x}], z > 0 \\ e^{\phi_1 z} \left[L_1^+ e^{\xi_1(p-z)} + L_2^+ \right] & \text{if } p - z \in [\underline{x}, \hat{x}], z > 0 \\ e^{\phi_2 z} \left[U_1^- e^{\xi_2(p-z)} + L_2^- \right] & \text{if } p - z \in (\hat{x}, \bar{x}], z < 0 \\ e^{\phi_2 z} \left[L_1^- e^{\xi_2(p-z)} + L_2^- \right] & \text{if } p - z \in [\underline{x}, \hat{x}], z < 0 \end{cases} \quad (36)$$

where $\{\phi_1, \phi_2\}$ are the two roots of the quadratic (31) and where the use $\xi_1 \equiv \xi_{11}, \xi_2 \equiv \xi_{12}$ are given by the non-zero roots (34). The 8 values for $\{U_i^+, L_i^+, U_i^-, L_i^-\}_{i=1,2}$ solve two system of 4 linear equations, one for $\{U_i^+, L_i^+\}_{i=1,2}$ and one for $\{U_i^-, L_i^-\}_{i=1,2}$. The upper and lower boundary of the range of inaction has zero density for both positive and negative values of z :

$$0 = U_1^+ e^{\xi_1 \bar{x}} + U_2^+ = L_1^+ e^{\xi_1 \bar{x}} + L_2^+ \quad (37)$$

$$0 = U_1^- e^{\xi_2 \bar{x}} + U_2^- = L_1^- e^{\xi_2 \bar{x}} + L_2^- \quad (38)$$

The marginal distribution of the z computed using g coincides with \tilde{g} for positive and negative values of z :

$$\frac{\phi_1 \phi_2}{\phi_1 - \phi_2} = \frac{L_1^+}{\xi_1} \left[e^{\xi_1 \hat{x}} - e^{\xi_1 \underline{x}} \right] + L_2^+ [\hat{x} - \underline{x}] + \frac{U_1^+}{\xi_1} \left[e^{\xi_1 \bar{x}} - e^{\xi_1 \hat{x}} \right] + U_2^+ [\bar{x} - \hat{x}] \quad (39)$$

$$\frac{\phi_1 \phi_2}{\phi_1 - \phi_2} = \frac{L_1^-}{\xi_2} \left[e^{\xi_2 \hat{x}} - e^{\xi_2 \underline{x}} \right] + L_2^- [\hat{x} - \underline{x}] + \frac{U_1^-}{\xi_2} \left[e^{\xi_2 \bar{x}} - e^{\xi_2 \hat{x}} \right] + U_2^- [\bar{x} - \hat{x}] \quad (40)$$

The density is continuous at $(p, z) = (\hat{\psi}(z), z) = (z + \hat{x}, z)$. Thus

$$U_1^+ e^{\xi_1 \hat{x}} + U_2^+ = L_1^+ e^{\xi_1 \hat{x}} + L_2^+ \quad (41)$$

$$U_1^- e^{\xi_2 \hat{x}} + U_2^- = L_1^- e^{\xi_2 \hat{x}} + L_2^-. \quad (42)$$

■

We are now ready to characterize the marginal density of prices, denoted by h .

$$h(p) \equiv \int_{-\infty}^{\infty} g(p, z) dz = \int_{p-\bar{x}}^{p-\underline{x}} g(p, z) dz = \int_{p-\bar{x}}^{p-\hat{x}} g(p, z) dz + \int_{p-\hat{x}}^{p-\underline{x}} g(p, z) dz \quad (43)$$

$$h(p) = \begin{cases} \int_{p-\bar{x}}^{p-\hat{x}} e^{\phi_1 z} [U_1^+ e^{\xi_1(p-z)} + U_2^+] dz + \int_{p-\hat{x}}^{p-\underline{x}} e^{\phi_1 z} [L_1^+ e^{\xi_1(p-z)} + L_2^+] dz & \text{if } p > \bar{x} \\ \int_{p-\bar{x}}^0 e^{\phi_2 z} [U_1^- e^{\xi_2(p-z)} + U_2^-] dz + \int_0^{p-\hat{x}} e^{\phi_1 z} [U_1^+ e^{\xi_1(p-z)} + U_2^+] dz \\ + \int_{p-\hat{x}}^{p-\underline{x}} e^{\phi_1 z} [L_1^+ e^{\xi_1(p-z)} + L_2^+] dz & \text{if } p \in [\hat{x}, \bar{x}] \\ \int_{p-\bar{x}}^{p-\hat{x}} e^{\phi_2 z} [U_1^- e^{\xi_2(p-z)} + U_2^-] dz + \int_{p-\hat{x}}^0 e^{\phi_2 z} [L_1^- e^{\xi_2(p-z)} + L_2^-] dz \\ + \int_0^{p-\underline{x}} e^{\phi_1 z} [L_1^+ e^{\xi_1(p-z)} + L_2^+] dz & \text{if } p \in [\underline{x}, \hat{x}] \\ \int_{p-\bar{x}}^{p-\hat{x}} e^{\phi_2 z} [U_1^- e^{\xi_2(p-z)} + U_2^-] dz + \int_{p-\hat{x}}^{p-\underline{x}} e^{\phi_2 z} [L_1^- e^{\xi_2(p-z)} + L_2^-] dz & \text{if } p < \underline{x} \end{cases} \quad (44)$$

Using that

$$\begin{aligned} \int_a^b e^{\phi z} [D_1 e^{\xi(p-z)} + D_2] dz &= D_1 e^{\xi p} \int_a^b e^{(\phi-\xi)z} dz + D_2 \int_a^b e^{\phi z} dz \\ &= D_1 \frac{e^{\xi p + (\phi-\xi)b} - e^{\xi p + (\phi-\xi)a}}{\phi - \xi} + D_2 \frac{e^{\phi b} - e^{\phi a}}{\phi} \end{aligned}$$

we can solve the integrals for each of the branches of h where $D_i \in \{U_i^+, U_i^-, L_i^+, L_i^-\}$ for $i = 1, 2$ and $(\phi, \xi) \in \{\phi_1, \xi_1, \phi_2, \xi_2\}$ and a and b take different values accordingly.

For completeness, we give the expression for the case with $\sigma = 0$, a version of [Sheshinski and Weiss \(1977\)](#).

PROPOSITION 7. Assume that $\sigma = 0, c > 0, \pi + \mu_z > 0$ and (17) holds. The inaction set is given by $\mathcal{I} = \{(p, z) : \underline{x} + z < p < \bar{x} + z\}$. The optimal return point is given by $\hat{\psi}(z) = \hat{x} + z$. The value function in the range of inaction and the constants $X \equiv (x, \hat{x})$ solve

$$\begin{aligned} V(p, z) &= e^{z(1-\eta)} V(p-z, 0) \equiv e^{z(1-\eta)} v(p-z) \\ v(x) &= a_1 e^{x(1-\eta)} + a_2 e^{-x\eta} + A e^{\nu x} \end{aligned}$$

where the coefficients a_i, v are given by

$$\begin{aligned} v &= \frac{b_0}{b_1}, \quad a_1 = \frac{1}{b_0 - (1 - \eta) b_1} \quad \text{and} \quad a_2 = -\frac{1}{b_0 + \eta b_1} \quad \text{where} \\ b_0 &= r + \rho - \mu_z(1 - \eta), \quad b_1 = -[\mu_z + \pi]. \end{aligned}$$

and where the three values $A, X \equiv (\underline{x}, \hat{x})$ solve the following three equations:

$$\begin{aligned} \hat{c} &= -a_1 \left(e^{\hat{x}(1-\eta)} - e^{\underline{x}(1-\eta)} \right) - a_2 \left(e^{-\hat{x}\eta} - e^{-\underline{x}\eta} \right) = A \left(e^{v\hat{x}} - e^{v\underline{x}} \right), \\ 0 &= a_1 (1 - \eta) e^{\hat{x}(1-\eta)} - a_2 \eta e^{-\hat{x}\eta} + A v e^{v\hat{x}}, \\ 0 &= a_1 (1 - \eta) e^{\underline{x}(1-\eta)} - a_2 \eta e^{-\underline{x}\eta} + A v e^{v\underline{x}}. \end{aligned}$$

Furthermore:

$$\lambda_a = \frac{\rho}{1 - \exp\left(-\frac{\rho}{\pi + \mu_z}(\hat{x} - \underline{x})\right)}$$

Proof. The expressions for the value function are obtained by setting $\sigma = 0$ and imposing the sS policy between the bands \underline{x}, \hat{x} . This problem is identical to the one in [Sheshinski and Weiss \(1977\)](#), where the discount rate is $r + \rho$. For the frequency of price adjustment we need to include the death and replacement of the products. For this we let \mathcal{T} the expected time until an adjustment:

$$\begin{aligned} \rho \mathcal{T}(p, z) &= 1 + \mathcal{T}_z(p, z) \mu_z - \mathcal{T}_p(p, z) \pi \\ \rho T(x) &= 1 - T'(x) (\mu_z + \pi). \end{aligned}$$

with boundary conditions $T(\underline{x}) = 0$. So the solution is $T(x) = 1/\rho + B \exp\left(-\frac{\rho}{\pi + \mu_z} x\right)$ with $B = -\exp\left(\frac{\rho}{\pi + \mu_z} \underline{x}\right) / \rho$ so

$$T(x) = \frac{1}{\rho} \left[1 - \exp\left(-\frac{\rho}{\pi + \mu_z} (x - \underline{x})\right) \right]$$

and hence

$$1/\lambda_a = T(\hat{x}) = \frac{1}{\rho} \left[1 - \exp\left(-\frac{\rho}{\pi + \mu_z} (\hat{x} - \underline{x})\right) \right] \quad \blacksquare$$

BB Some graphical illustrations of the model in Section B

Figure XI plots the size of the “regular” price increases and “regular” price decreases, for different inflation rates. Imitating the empirical literature, we define as regular price changes those not triggered by the jump shock that reset the value of z to zero. The figure shows that for low inflation rates the size of price increases and of price decreases is very similar as predicted by part (iii) of [Proposition 1](#). The figure also that the slope of the size of price changes with respect to inflation is very small, consistent with the prediction that $\frac{\partial \Delta_p^+(0, \sigma^2)}{\partial \pi} + \frac{\partial \Delta_p^-(0, \sigma^2)}{\partial \pi} = 0$. As

inflation rises, the size of both price increases and decreases becomes larger. Panel (b) of Figure IX in Section IV. is the empirical counterpart of Figure XI.

Figure XI: Average size of price increases Δ_p^+ and decreases Δ_p^-

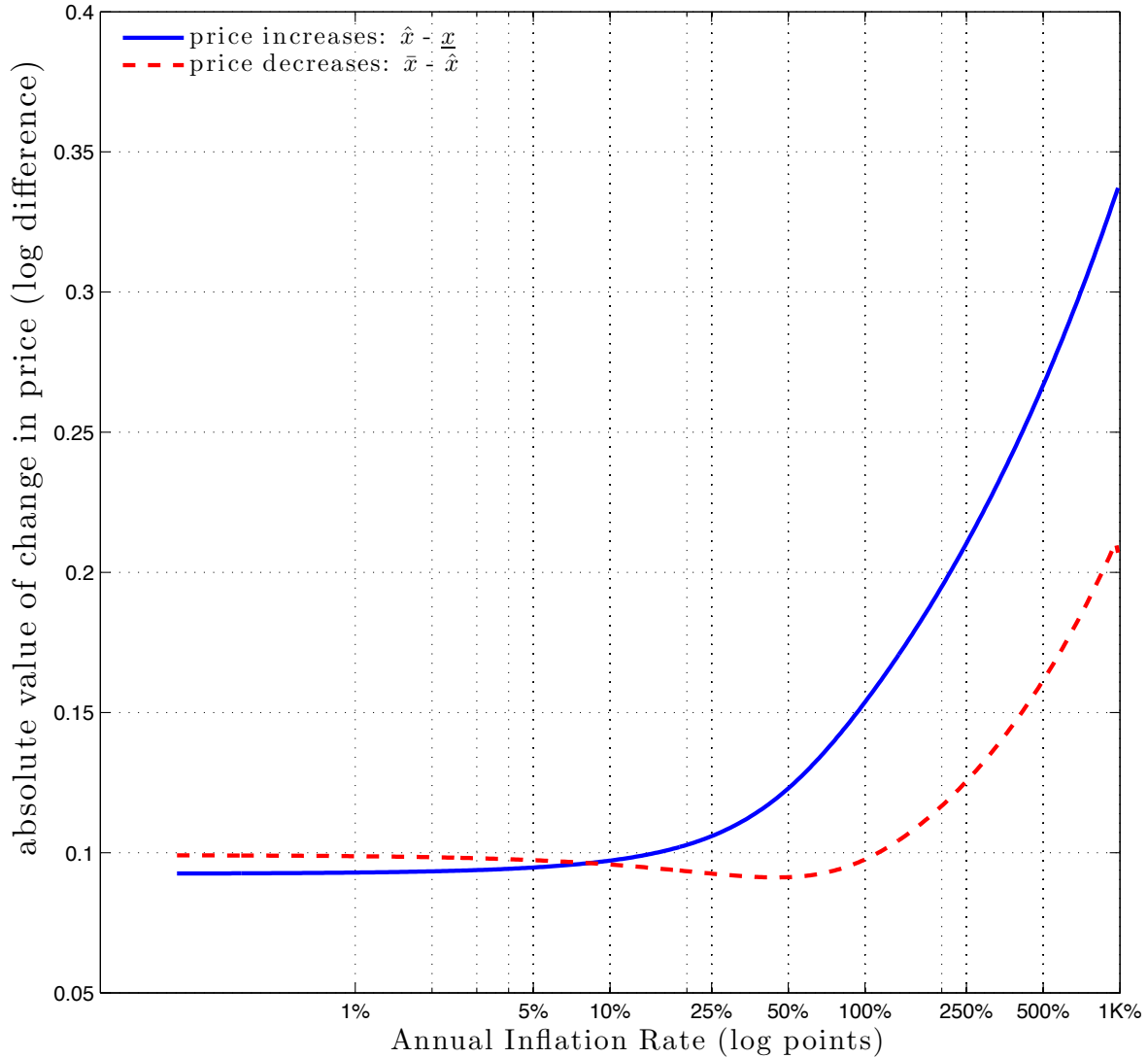
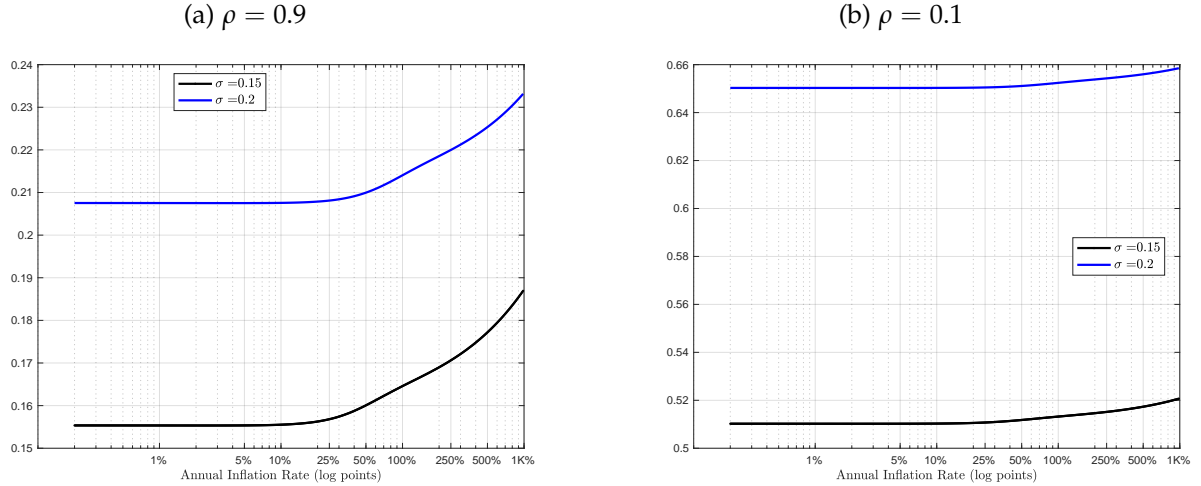


Figure XII plot the unconditional standard deviation of relative prices as a function of inflation. For low values of inflation the unconditional standard deviation is insensitive to inflation as predicted by the theory. For large enough inflation rates the width of the inaction range should swamp the effect the variation of z on σ . In our numerical examples, however, it takes inflation rates even much higher than the ones observed in the peak months in Argentina for this to happen. Nevertheless, the elasticity with respect to inflation at high inflation increases when we decrease the average product life $1/\rho$ (and thus the unconditional variance at low inflation), getting closer to the theoretical upper bound of $1/3$.

Figure XII: Unconditional standard deviation of log relative prices $\bar{\sigma}$



C CLASSIFICATION OF GOODS AND SERVICES AND DATA COLLECTION

Goods/services in the dataset are classified according to the MERCOSUR Harmonized Index of Consumer Price (HICP) classification. The HICP uses the first four digit levels of the Classification of Individual Consumption According to Purpose (COICOP) of the United Nations plus three digit levels based on the CPI of the MERCOSUR countries. The *goods/services* in the database are the seven digit level of the HICP classification; six digit level groups are called *products*; five digit level groups are called *sub-classes*; four digit level categories are called *classes*; three digit level categories are called *groups* and two digit level groups are called *divisions*. Table V shows two examples of this classification.

Table V: Example of the Harmonized Index of Consumer Price Classification

Classification	Example 1	Example 2
Division	Food and Beverages	Household equipment and maintenance
Group	Food	Household maintenance
Class	Fruits	Cleaning tools and products
Sub-Class	Fresh Fruits	Cleaning products
Product	Citric Fruits	Soaps and detergents
Good	Lemons	Liquid soap

For most cases, the brand chosen for the product is the one most widely sold by the outlet, or the one that occupy more space in the stands, if applicable (hence brands can change from month to month or from two-weeks to two-weeks). For some cases, the brand is part of the attributes, the product is defined as one from a “top brand”.

The weight of each good in the CPI is obtained from the 1986 National Expenditure Survey (Encuesta Nacional de Gasto de los Hogares). Weights are computed as the proportion of the households expenditure on each good over the total expenditure of the households. We re-normalize these weights to reflect the fact that

our working dataset represents only 84% of expenditure. The weight of a particular good is proportional to the importance of its expenditure with respect to the total expenditure without taking into account the percentage of households buying it.

Table VI shows the top 20 goods, in terms of the importance of their weights in our sample. As it can be seen from the table, most goods whose prices are gathered twice a month are represented by food and beverages while goods whose prices are gathered monthly include services, apparel and other miscellaneous goods and services.

Table VI: Goods ordered by weight whose prices are gathered once and twice per month

Differentiated)	Weight (%)	Homogeneous	Weight (%)
Lunch	2.02	Whole chicken	1.51
Lunch in the workplace	1.67	Wine	1.49
Car	1.52	French bread (less than 12 pieces)	1.38
Housemaid	1.48	Fresh whole milk	1.31
Monthly union membership	1.22	Blade steaks	0.96
Snack	0.91	Standing rump	0.93
Medical consultation	0.83	Eggs	0.87
Gas bottle	0.58	Short ribs (Roast prime ribs)	0.85
Laides hairdresser	0.56	Striploin steaks	0.78
Labor for construction	0.54	Apple	0.73
Adult cloth slippers	0.50	Oil	0.72
Color TV	0.50	Rump steaks	0.72
Funeral expenses	0.49	Potatos	0.71
Men's dress shirt	0.49	Soda (coke)	0.70
Dry cleaning and ironing	0.48	Cheese (quartirollo type)	0.70
Sports club fee	0.47	Tomatoes	0.63
Movie ticket	0.47	Minced meat	0.60
Men's denim pants	0.45	Sugar (white)	0.59
Disposable diapers	0.43	Coffee (in package)	0.57
Shampoo	0.40	Yerba mate	0.55

Table VII shows the weight structure in our database classified by divisions. The table shows goods in terms of their weight with respect to the total weight in the sample (Total column), and with respect to the total weight of their belonging category (one or two visit goods) Food and non-alcoholic beverages represent almost 43% of the total weight in the sample and 82% of the total weight of goods whose prices are gathered twice a month. On the other hand, weights of one visit goods are less concentrated. Almost 12% of the total weight in the sample corresponds to furniture and household items and around 9% correspond to apparel. These percentages are around 24% and 19%, respectively, when computing percentages over the total weight in the one visit goods category.

CA Instructions to CPI's Pollsters

Pollsters record item's prices. Remember that an item is a good/service of a determined brand sold in a specific outlet in a specific period of time. Prices are transactional, meaning that the pollster should be able to buy the product in the outlet. Goods are defined by their attributes. For the majority of the goods, the brand is not an attribute. The brand of a specific item is determined the first time the pollster visits an outlet. The brand is the most sold/displayed by the outlet. Once the item is completely defined, the pollster collects the price of that item next time she visits the same outlet. After the first visit, in the following visits, the pollsters arrive to each

Table VII: Weights for Divisions of the Harmonized Index of Consumer Price

Divisions	Weight		Weight	
	Total	One visit	Total	Two visits
Food and non-alcoholic beverages	4.94	9.91	42.67	82.39
Apparel	9.24	18.53		
Conservation and repair of housing plus gas in bottle	0.99	1.99		
Furniture and household items	11.86	23.79		
Medical products, appliances and equipment plus external medical services	3.39	6.79		
Transportation	2.60	5.22		
Recreation and culture	4.50	9.02		
Education	2.42	4.85		
Miscellaneous goods and services (Toiletries, haircut, etc.)	5.07	10.16		
Jewelry, clocks, watches plus other personal belongings	4.86	9.75		
Alcoholic beverages			3.94	7.62
Non-durable household goods			2.40	4.64
Other items and personal care products			2.77	5.36

outlet with a form that includes all items for which prices are to be collected.

For example, assume the good is *soda-cola top brand* and the attributes are *package: plastic bottle and weight: 1.5 liters*. The first time the pollster goes to, say, outlet *A* she ask for the *cola top brand* most sold in that outlet. Assume that *Coca-Cola* is the most sold soda in outlet *A*. Then the item is completely defined: *Coca-Cola* in plastic bottle of 1.5 liters in outlet *A*. Next time the pollster goes to outlet *A* she records the price of that item.

All prices are in Argentine pesos. Our dataset does not contain flags for indexed prices. In traditional outlets pollsters ask for the price of an item even when, for example it is displayed in a blackboard, because the good has to be available in order to record its price. In supermarket chains the price recorded is collected from the shelf/counter display area.

There are a number of special situations to be taken into account:

1. *Substitutions*: every time there is a change in the attributes of the good the pollster has to replace that particular good for another one. In this case, the pollster mark the price collected with a flag indicating a substitution has occurred. The goods that are substituted should be similar in terms of the type of brand and or quality.
2. *Stockouts*: every time the pollster could not buy the item, either because the good is out of stock in the outlet or because the same good or a similar one is not sold by outlet at the time the price has to be collected, she has to mark the item with a flag of stockout and she has to assign a missing price for that item. Stock-outs include what we label "pure stock-outs", the case where the outlet has depleted the stock of the good, including end of seasonal goods/services. Stock outs also include the case where the outlet no longer carries the same good/service and it does not offers a similar good/service of comparable quality/brand. Examples of stock-outs include many fruits and vegetables not available off-season, as well as clothing such as winter coats and sweaters during summer.
3. *Sales*: every time the pollster observes a good with a sale flag in an outlet, she has to mark the price of

that particular item with a sale flag.

All pollsters were supervised at least once a month. Supervisors visited some of the outlets that had been visited earlier the same day by the pollster and collected a sample of the prices that the pollster should have collected. Then, at the National Statistic Institute, another supervisor compared both forms.

D ROBUSTNESS

DA Missing Data, Substitutions, Sales and Aggregation

Here we describe in detail the methodologies and the definitions of all estimators in [Table I](#) in [Section IV.C.2](#).

We start by describing the assumptions used to estimate the probability of a price change when we observe missing prices. If between two observed prices there are some missing prices we use the following assumption. If the two observed prices are exactly equal we assume there has been *no* changes in prices in any times between these dates. This is the same assumption as in [Klenow and Kryvtsov \(2008\)](#). Instead, if the two observed prices are different we assume there has been *at least one change* in prices in between. The first assumption allow us to complete the missing prices in between two observed prices that are equal. From here on, assume that the missing prices in such string of prices have been replaced.

We will refer from now on to the sequence of prices between two different observed prices as a *spell of constant prices*, or for short a *spell of prices*. Without any missing prices, a spell of constant prices is just a sequence of repeated prices ending with a different price. Notice that the last (observed) price in a spell of constant prices is the first price of the next spell.

Next we describe the possible patterns of prices, and its implications for the estimation of the probability of a price change. After following the procedure described above, all spell of prices and missing observations have only two possible patterns. The first pattern is a spell of prices ending with a price change, but with no missing observations. We consider the second pattern in the following section, where we deal with the effect of missing prices. Consider the following example for a spell for an outlet i that contains no missing prices nor substitutions:

Table VIII: Example of a spell of constant prices without missing prices

p_t	10	10	10	10	10	15
	t	$t + 1$	$t + 2$	$t + 3$	$t + 4$	$t + 5$
	$\widehat{\lambda}_{t+1}$		$\widehat{\lambda}_{t+2}$		$\widehat{\lambda}_{t+3}$	
I_t	y	0	0	0	0	1
γ_t	y	1	1	1	1	1

The braces on top of the values of λ are meant to remind the reader that λ_t refers to the constant probability of change in prices between $t - 1$ and t . The indicator I_{it} adopts the value one if, in outlet i , the price in period t is different from the price at period $t - 1$, and zero otherwise, except for the first temporal period of the first

string of prices where it is missing⁴¹. In this example we have exactly no changes for the first four periods and at least one change in the next period. The probability of observing this completed spell of constant prices is thus:⁴²

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \times e^{-\lambda_{t+3}} \times e^{-\lambda_{t+4}} \times (1 - e^{-\lambda_{t+5}}) \quad (45)$$

It follows that in this simple case, assuming all the outlets selling the same good have the same λ_j , the likelihood function of prices observed for product j is

$$L_j = \prod_{i \in O_j} \left[e^{-\lambda_{j,t}} \right]^{(1-I_{it})} \times \left[1 - e^{-\lambda_{j,t-\tau}} \right]^{I_{it}} \quad (46)$$

The maximum likelihood estimator of the arrival rate of price changes for product j between times t and $t + 1$ in the simple case without missing prices and without substitutions is

$$\lambda_{j,t+1} = \log \left(\frac{\sum_{i \in O_j} 1}{\sum_{i \in O_j} (1 - I_{it})} \right) = -\log(1 - f_{jt}) \quad (47)$$

where we let O_j denote the set of the outlets i of the product j and f_{jt} is the fraction of outlets of good j that changed prices in period t . In words, $\lambda_{j,t+1}$ is the log of the ratio of the number of outlets to the number of outlets that have not changed the price between t and $t + 1$. Thus $\lambda_{j,t+1}$ ranges between zero, if no outlets have change prices, and infinite if all outlets have changed prices. The probability of at least one change in prices in period t for product j is $1 - e^{-\lambda_{jt}} = f_{jt}$.

DB Incorporating Information on Missing Prices

Now we consider the case where there are some missing prices before the price change, but we postpone the discussion of the effect of price substitutions.

In general, a spell of constant prices is a sequence of $n + 1$ prices that starts with an observed price p_t , possibly followed by a series of prices all equal to p_t , then followed, possibly, by a series of missing prices, that finally ends with an observed price at p_{t+n} that differs from the value of the initial price p_t . Notice that while we also refer to this sequence of prices as a spell of constant prices, it can include more than one price change if there were missing observations, a topic to which we return in detail below.

To deal with missing prices, the interesting patterns for a spell of constant prices are those which end with a price change, but that contain some missing price(s) just before the end of the spell. For example, consider a spell of prices for an outlet, i , in which there are exactly no changes in the first four periods and at least one change sometime during the next three periods.

In **Table IX** an x means that the variable is not defined for that case, the m denotes a missing/imputed price and the y denotes that for indicator I and counter γ the first observation in the spell of prices is missing because it depends on the prices in period $t - 1$ which are not in the information of the table. The probability of

⁴¹We also include the indicator γ_t , which we explain below, for completeness.

⁴²We assume that the number of price changes between $t - 1$ and t follows a homogeneous Poisson process with arrival rate λ_t per unit of time. The probability of k occurrences is $e^{-\lambda} \lambda^k / k!$ and the waiting time between occurrences follows an exponential distribution.

Table IX: Example of a spell of constant prices with observed and missing prices

p_t	10	10	10	10	10	m	m	15
t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+7$
	$\widehat{\lambda}_{t+1}$	$\widehat{\lambda}_{t+2}$	$\widehat{\lambda}_{t+3}$	$\widehat{\lambda}_{t+4}$	$\widehat{\lambda}_{t+5}$	$\widehat{\lambda}_{t+6}$	$\widehat{\lambda}_{t+7}$	
I_t	y	0	0	0	0	x	x	1
γ_t	y	1	1	1	1	x	x	3
χ_t	0	0	0	0	0	1	1	0

observing this spell is:

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \times e^{-\lambda_{t+3}} \times e^{-\lambda_{t+4}} \times \left(1 - e^{-\lambda_{t+5}} \times e^{-\lambda_{t+6}} \times e^{-\lambda_{t+7}}\right) \quad (48)$$

The first four products are the probability of no change during the first four periods, and the last term is the probability of at least one change during the last three periods. The second term is the complement of the probability of no change in prices during the last three periods.

The likelihood of the sample of T periods (with $T + 1$ prices) of all the outlets for the good j —denoted by the set O_j —is the product over all outlets i of the product of all spells for outlet i of the probability (48). To write the likelihood we define an indicator, χ_{it} , and a counter γ_{it} . The indicator χ_{it} adopts the value one if a price is missing for outlet i in period t , and zero otherwise. The value of γ_t counts the number of periods between two non-missing prices. The counter γ_{it} is [Klenow and Kryvtsov \(2008\)](#) duration clock. Then the likelihood function of the prices observed for product j is:

$$L_j = \prod_{i \in O_j} \prod_{t=1}^T \left[e^{-\lambda_{j,t} \gamma_{it}} \right]^{(1-I_{it})(1-\chi_{it})} \times \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} \right]^{I_{it}(1-\chi_{it})} \quad (49)$$

Since the λ 's are the probability of a price change and they are indexed at the end of a period, the first temporal observation of prices at $t = 0$ does not enter the likelihood. The log likelihood is:

$$\ell_j = \sum_{i \in O_j} \left(\sum_{t=1}^T (1 - \chi_{it})(1 - I_{it}) \times (-\lambda_{j,t} \gamma_{it}) + \sum_{t=1}^T (1 - \chi_{it}) I_{it} \ln \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} \right] \right) \quad (50)$$

To compute the contribution to the likelihood of a given value of $\lambda_{j,t}$ for $t = 1, \dots, T$ we find convenient to introduce two extra counters: κ_{it} and η_{it} for any period t in which prices are missing/imputed. The variable κ_{it} counts the number of periods since the beginning of a string of missing/imputed prices. The variable η_{it} counts the number of periods of missing/imputed prices until the next price is observed. For example, consider the string of prices in [Table X](#).

[Table X](#) shows an example of a spell of constant prices for a given variety and a given outlet. In (51) we highlight the contribution to the log-likelihood of the value of λ_t for a given outlet i :

Table X: Example of a spell of constant prices, w/ missing prices and counters

p	10	10	m	m	m	m	m	m	15
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	
	λ_{t+1}	λ_{t+2}	λ_{t+3}	λ_{t+4}	λ_{t+5}	λ_{t+6}	λ_{t+7}		
I	y	0	x	x	x	x	x	x	1
χ	0	0	1	1	1	1	1	1	0
γ	y	1	x	x	x	x	x	x	6
κ	x	x	1	2	3	4	5	x	x
η	x	x	5	4	3	2	1	x	x

$$\begin{aligned} \ell_j = & \dots + (1 - \chi_{it})(1 - I_{it}) \times (-\lambda_{j,t}\gamma_{it}) + (1 - \chi_{it})I_{it} \ln \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} \right] \\ & + \chi_{it} \ln \left[1 - e^{-(\sum_{\tau=0}^{\kappa_{it}-1} \lambda_{j,t-\tau} + \sum_{\tau=1}^{\eta_{it}} \lambda_{j,t+\tau})} \right] + \dots . \end{aligned} \quad (51)$$

The first two terms have the contribution to the likelihood of $\lambda_{j,t}$, if the price at time t is not missing. The first case corresponds to a price at the beginning of the spell of constant prices. The second to a case where the price is the last one of the spell, and uses γ_{it} to be able to write the corresponding probability. The third term, correspond to the contribution of $\lambda_{j,t}$, if the price at time t is missing, and uses κ_{it} , and η_{it} to write the corresponding probability.

Using (51) it is easy to write the FOC with respect to $\lambda_{j,t}$ of the sample as:

$$\begin{aligned} \frac{\partial \ell_j}{\partial \lambda_{j,t}} = & \sum_{i \in O_j} (1 - \chi_{it})(1 - I_{it}) \times (-\gamma_{it}) + \sum_{i \in O_j} (1 - \chi_{it}) I_{it} \frac{1}{e^{(\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau})} - 1} \\ & + \sum_{i \in O_j} \chi_{it} \frac{1}{e^{(\sum_{\tau=0}^{\kappa_{it}-1} \lambda_{j,t-\tau} + \sum_{\tau=1}^{\eta_{it}} \lambda_{j,t+\tau})} - 1} = 0 , \end{aligned} \quad (52)$$

for $t = 1, \dots, T$.

Roughly speaking the estimator for $\lambda_{j,t}$ computes the ratio of the number of outlets i that a time t have change the prices with those that have not change prices or that have missing prices. This approximation is exact if no outlet has a missing/imputed price at period t or before. In this case $\chi_{is} = 0$ for $s = 1, 2, \dots, t$ for all $i \in O_j$, then (52) becomes

$$\sum_{i \in O_j} (1 - I_{it}) \times \gamma_{it} = \sum_{i \in O_j} \left[\frac{I_{it}}{e^{\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} - 1} \right] ,$$

which, if we make $\lambda_{j,t} = \lambda_{j,t-1} = \dots = \lambda_{j,t-\gamma_{it}+1}$ is the same expression than in [Klenow and Kryvtsov \(2008\)](#). In the case of no missing observations, and where all the λ 's are assumed to be the same, this maximum likelihood estimator coincide with the simple estimator introduced in (47).

Our data contains a flag indicating whether an item was on sale. We consider a procedure that disregards the changes in prices that occur during a sale. The idea behind this procedure is that sales are anomalies for the point of view of some models of price adjustment, and hence they are not counted as price changes. To explain this assumption we write an hypothetical example:

Table XI: Example of a spell of constant prices removing sales

p_t	10	10	10	m	8	15
	t	$t + 1$	$t + 2$	$t + 3$	$t + 4$	$t + 5$
	$\widehat{\lambda}_{t+1}$	$\widehat{\lambda}_{t+2}$	$\widehat{\lambda}_{t+3}$	$\widehat{\lambda}_{t+4}$	$\widehat{\lambda}_{t+5}$	
I_t	y	0	0	0	0	1
a_t	y	0	0	0	1	0

In **Table XI** the indicator a_t takes the value of one if the good is on sale on period t and if the price at the time t is smaller than the previous recorded price. In this case, the price in periods $t + 3$ and $t + 4$ is changed to 10, the value of the previous recorded price. The general principle is to consider a string of recorded prices, possible missing values, and a price that has a sale flag, and replace the price of the string of missing values and the period with a sale flag for the previous recorded price. In other words, we replace the price at the period with a sale flag for the previous recorded price and then using our first assumption on missing prices we complete the missing price in between two prices that are equal. When the sales are disregarded the number and duration of price spells can change. Once this procedure is implemented, the likelihood is the one presented above using the modified price series -indeed in **Table XI** the indicator I_t is the one that corresponds to the modified price string. We refer to the corresponding estimates as those that *excludes sale quotes*. This is a procedure used by many, e.g., [Klenow and Kryvtsov \(2008\)](#). By construction with this method the estimates for λ will be smaller.

DD Incorporating Missing Prices and Price Substitutions

In this section we discuss different assumptions on the treatment of missing data and good substitutions that allow us to construct four estimators of the frequency of price changes that we report later on.

We use the indicators $\tilde{e}_{it}, e_{it}^*, \bar{e}_{it}$ to consider two different assumptions on how to treat a price spell that ends in some missing values or price substitutions. In particular, consider the case of a substitution of a product or a missing price. As explained above, our data set contains the information of whether the characteristic of the product sold at the outlet has changed and was subsequently substituted by a similar product. We also have information on whether the price is missing (mostly due to a stock-out). To be concrete, consider the following example of a spell of constant prices in **Table XII**. In this table, and $s_{it} = 1$ denotes the period where a price substitution has occurred. Thus, the example has a spell of 9 prices, with two periods (3 prices) with no change in prices, then 5 periods with missing prices, and finally in the last period there is an observed price that correspond to a substitution of the good.

Table XII: Example: spell of constant prices w/ counters for substitutions

p	10	10	10	m	m	m	m	15
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$
	λ_{t+1}	λ_{t+2}	λ_{t+3}	λ_{t+4}	λ_{t+5}	λ_{t+6}	λ_{t+7}	
I	y	0	x	x	x	x	x	1
χ	0	0	0	1	1	1	1	0
s	y	0	0	0	0	0	0	1
\bar{e}	y	0	0	0	0	0	0	0
e^*	y	1	1	1	1	1	1	1
\bar{e}	y	0	0	1	1	1	1	1

As in the previous examples, for the first observation an y indicates that the value of the indicator cannot be decided based on the information in the table.

The issue is the interpretation of when and whether there has been a price change in the previous price spell. One interpretation is that there has been a price change somewhere between periods $t+2$ and $t+7$. A different interpretation is that, because the price spell ends with the substitution of the good, the price has not changed. The idea for this interpretation is that if the good would have not changed, the price could have been constant beyond $t+7$. The next three cases explain how to implement the first interpretation, and two ways to implement the second one. The last two cases present two simple estimators, one that treats quotes with substitutions as regular price changes, and one that exclude them.

1. We disregard the information of the substitution of a good, and proceed as we have done so far: including all price quotes as if the good have been not changed. In the previous example, it consists on assuming that the price has changed between periods $t+2$ and $t+7$. In this case we say that the probability of observing the spell in the table is given by:

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \times \left(1 - e^{-\lambda_{t+3} - \lambda_{t+4} - \lambda_{t+5} - \lambda_{t+6} - \lambda_{t+7}}\right) \quad (53)$$

In this case we set $\tilde{e}_{it} = 0$ for all periods, since we don't want to exclude any part of this price spell. We refer to these assumptions as *including all price quotes*.

2. We follow [Klenow and Kryvtsov \(2008\)](#) and others and exclude completely any spell of prices that ends with a substitution of a product. We implement this by defining the indicator $e_{it}^* = 1$ for any price corresponding to this spell, i.e., we exclude all the observations –with the exception of the first price, which is relevant for the previous string. In this case we have no associated probability for this spell. The idea behind this treatment is that if there would have been no substitution of the good the price could have stayed constant even beyond $t+7$. We refer to these assumptions as *excluding substitution spells* for short.
3. We introduce a new way to handle this information based on the following underlying assumption: if a spell of constant prices ends up in a substitution we interpret that the product has changed, and hence we cannot infer from the observed price whether the price has changed or not, as in the previous case.

Yet, unlike the previous case, we do not discard the information at the beginning of the string, where the product was the same and its price was not changing. In this case the associated probability for this spell is:

$$P = e^{-\lambda_{t+1}} \times e^{-\lambda_{t+2}} \quad (54)$$

In this case we use the indicator $\bar{e}_{it} = 0$ for the first two observations, among which we know that there was no price change, and $\bar{e}_{it} = 1$ for the rest of the observations where we can't conclude if there was a price change for the *same* product. We refer to these assumptions as *excluding substitution quotes* for short.

4. We present an alternative estimator to the maximum likelihood, which has the advantage of being simpler to describe and understand. This estimator imitates the one for the case where there are no missing price quotes in (47), and simply excludes the values of the missing. In this case, this estimator is:

$$\lambda_{j,t+1} = \log \left(\frac{\sum_{i \in O_j} (1 - \chi_t) (1 - \chi_{t+1})}{\sum_{i \in O_j} (1 - \chi_t) (1 - \chi_{t+1}) (1 - I_{it})} \right) \quad (55)$$

In words, $\lambda_{j,t+1}$ is a non-linear transformation of the probability of the change of prices. This probability is estimated as the ratio of the outlets that have changed prices over all the outlets, including only those price quotes that are simultaneously not missing at t and $t + 1$. While this estimator is simpler than the maximum likelihood, it does not use all the information of the missing values efficiently. We refer to this estimator as the *simple* estimator.

Finally, to completely state the notation for the likelihood function, we use the indicator ζ to deal with missing prices at the beginning of the sample. In particular, if for an outlet i the sample starts with $n + 1$ missing prices, we exclude these observations from the likelihood since we cannot determine the previous price. We do this by setting the indicator $\zeta_{it} = 1$ for $t = 0, 1, \dots, n$. Thus, depending of the assumption, the exclusion indicator e takes the values given by \tilde{e} , \bar{e} or e^* , besides the value of 1 for all the missing observations at the beginning of the sample. The log likelihood function is then:

$$\ell_j = \sum_{i \in O_j} \sum_{t=1}^T (1 - e_{it}) \left((1 - \chi_{it})(1 - I_{it}) (-\lambda_{j,t} \gamma_{it}) + (1 - \chi_{it}) I_{it} \ln \left[1 - e^{-\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}} \right] \right) \quad (56)$$

The first order condition for $\lambda_{j,t}$, using the counters κ and η is:

$$\begin{aligned} \frac{\partial \ell_j}{\partial \lambda_{j,t}} &= \sum_{i \in O_j} (1 - \chi_{it}) (1 - e_{it}) (1 - I_{it}) \times (-\gamma_{it}) + \\ &\sum_{i \in O_j} (1 - \chi_{it}) (1 - e_{it}) I_{it} \frac{1}{e^{\left(\sum_{\tau=0}^{\gamma_{it}-1} \lambda_{j,t-\tau}\right)} - 1} \\ &+ \sum_{i \in O_j} \chi_{it} (1 - e_{it}) \frac{1}{e^{\left(\sum_{\tau=0}^{\kappa_{it}-1} \lambda_{j,t-\tau} + \sum_{\tau=1}^{\eta_{it}} \lambda_{j,t+\tau}\right)} - 1} = 0, \end{aligned} \quad (57)$$

for $t = 1, \dots, T$.

DE Computation of estimates and standard errors of λ

We compute the maximum likelihood estimator by using an iterative procedure. In this discussion we fix a good or item. We denote the iterations by superindex j . The initial guess for λ_t^0 is the log of the ratio of the outlets that change the price between $t - 1$ and t . Then we solve for λ_t^{j+1} in (57) for the foc of λ_t taking as given the values of λ_{t-i}^j and $\lambda^{j,t+i}$ for all $i \neq 0$. Notice that this equation has a unique solution. Also notice that the solution can be ∞ , for instance if all prices change, or zero.

We have not derived expression for standard error for the different estimator of λ . Nevertheless to give an idea of the precision of our estimator, we note that for the simple pooled estimator, assuming that missing values are independent, we can use the expression for the standard error of a binomial distributed variable for the probability of a price change π , while using the estimate $\hat{\pi}_t = \text{number of outlets with a price change} / N_t$ where N_t is the number of outlet with a price quote between t and t_1 . In this case $se(\hat{\pi}) = \sqrt{\hat{\pi}(1 - \hat{\pi})/N}$. Using the delta method and $\lambda(p) = -\log(1 - p)$ then we have:

$$se(\hat{\lambda}) = \frac{\sqrt{\exp(\hat{\lambda}) - 1}}{\sqrt{N}} \quad \text{and} \quad se(\log \hat{\lambda}) = \frac{1}{\hat{\lambda}} \frac{\sqrt{\exp(\hat{\lambda}) - 1}}{\sqrt{N}}. \quad (58)$$

To give an idea of the magnitudes for our estimated parameters we use some round numbers for both homogeneous and differentiated goods. For the case where we pool *all* the differentiated goods we can take $N_d = 230 \times 80 \times 0.80 = 14720 \approx \# \text{ diff. goods} \times \text{avg. \# outlets diff. goods} \times \text{fraction of non-missing quotes}$. Pooling *all* the homogeneous goods we $N_h = 60000 = 300 \times 250 \times 0.80 \approx \# \text{ homog. goods} \times \text{avg. \# outlets homog. goods} \times \text{fraction of non-missing quotes}$. So for high value of λ such as for $\log \hat{\lambda} = \log 5$, we have that the standard errors are $se(\log \hat{\lambda}_d) = 0.02$ and $se(\log \hat{\lambda}_h) = 0.010$. Instead for low values of λ such as for $\log \hat{\lambda} = \log 1/5$, we have that the standard errors are $se(\log \hat{\lambda}_d) = 0.019$ and $se(\log \hat{\lambda}_h) = 0.009$. Instead if we estimate λ at the level of each good, the standard errors should be larger by a factor $\sqrt{230} \approx 15$ and $\sqrt{330} \approx 17$ for differentiated and homogeneous goods respectively.

DF Aggregation: Weighted Average, Median, Weighted Median and Pooled Maximum Likelihood

In this section we deal with the issue of aggregation. So far we have described how to estimate the frequency of price adjustment for each good category separately.

Remember that those goods that fall in the homogeneous goods category are sampled bi-monthly and so will be our estimates. In this way, the first step in order to aggregate all categories is to convert them into monthly estimates, which is done simply by adding the two estimates in any given month (this results from the exponential assumption for our likelihood function).

Next, we compute three aggregated estimations. First, we calculate the weighted average of all monthly estimates (both differentiated and homogeneous goods), where the weights are the corresponding expenditure shares of each good category.

$$\lambda_t = \sum_{i=1}^N \omega_i \lambda_{it}$$

For future reference, we will call this *weighted average* or simply *WA*, followed by the specific treatment of

missing, sales and substitutions. For example, *Weighted Average Excluding Sales*.

The other two estimates are the median and weighted median of all monthly estimates (both differentiated and homogeneous goods). The aggregated median estimation (*median* for short) consists in taking the median λ of all products at each time period. The aggregate weighted median estimation is computed by sorting, at each time period, the expenditure weights of each product by the value of their associated λ from lowest to highest. Then we compute the accumulated sum of the weights until reaching 0.5. The aggregate weighted median of the frequency of price adjustment is the associated λ of the product whose weight makes the accumulated sum equal to or greater than 0.5. We refer to this estimate as *weighted median*.

Finally we consider a last aggregated estimation. As mentioned, the estimates for the frequency of price changes presented allow the value of λ to depend on the time period and the good. We now consider an estimate based on the assumption that the frequency price changes is common for all goods, but that it can change between time periods. This simply puts together the outlets for all goods in our sample. Thus, the log likelihood is:

$$\ell(\lambda_1, \dots, \lambda_T) = \sum_{j=1}^N \ell_j(\lambda_1, \dots, \lambda_T)$$

where the $\ell_j(\lambda_1, \dots, \lambda_T)$ corresponding to the log likelihood for each assumption about missing prices and or price substitutions as it has been introduced in the previous sections, and where N is the number of goods in our sample. We refer to this estimator as the *pooled maximum likelihood* or for short PML. Likewise, when we assume that all goods have the same frequency of price changes but use the simple estimator for λ , we refer to it as *simple pooled estimator*.

DG Expected Inflation

The robustness check in this section pertains to the measure of expected inflation as opposed to our estimates of the frequencies of price adjustment. Theoretical models of price setting behavior, such as the menu cost model presented in [Section II.](#), predict that firms decision to change prices, and the magnitude of such change, depend on *expected* inflation between adjustments. Our measure of expected inflation so far was the average realized inflation for the expected duration of the price set in period t , $1/\lambda_t$.

We now consider a more flexible form for expected inflation as an average of the actual inflation rate of the following k_t months, where $k_t = [n/\lambda_t]$ and $[x]$ is the integer part of x ; that is

$$\pi_t^e = \frac{1}{k_t} \sum_{s=t}^{t+k_t} \pi_s \quad (59)$$

We refer to n as the *forward looking factor*. Thus, as inflation falls (and implied durations rise) in our sample agents put an increasing weight on future inflation. When $n = 0$ expected and actual inflation are the same.

[Table XIII](#) shows that the results presented in [Section IV.C](#) are not very sensitive to estimating (7) using different forward looking factors in (59). The first row of the table shows the results when we use the actual rate of inflation⁴³. The following four rows shows that as expectations are more forward looking the threshold

⁴³ We use the inflation rate estimated by INDEC for the whole CPI instead of the inflation rate in our sample because we need forward looking values of inflation at the end of the sample that otherwise we cannot obtain.

Table XIII: Elasticities and implied duration for different estimates of expected inflation.

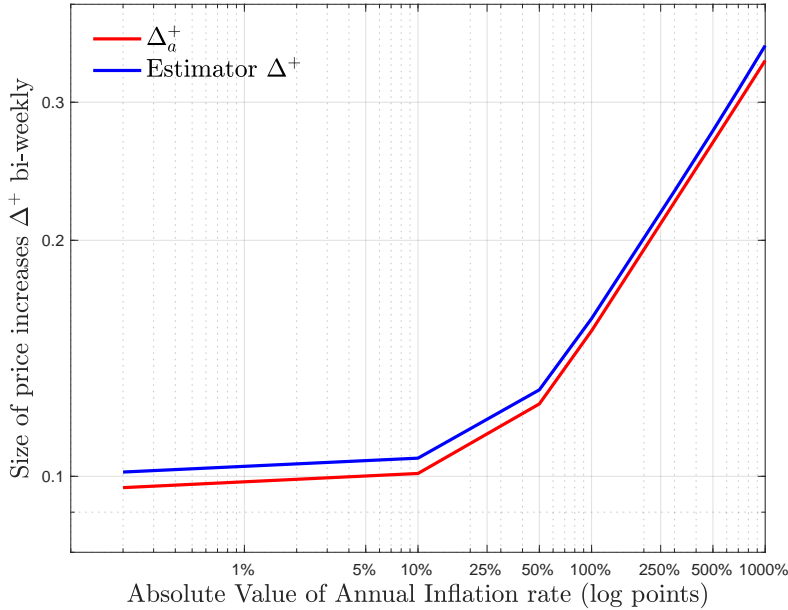
Forward looking factor	Elasticity γ	Semi-elasticity at zero $\Delta\% \lambda$	Duration at $\pi = 0$	R^2
$n = 0$	0.57	0.01	4.4	0.959
$n = 0.5$	0.54	0.03	4.4	0.949
$n = 1$	0.53	0.04	4.5	0.953
$n = 1.5$	0.48	0.04	4.6	0.946
$n = 2$	0.46	0.04	4.7	0.944

inflation and the elasticity fall slightly and the implied duration at the threshold remains constant. The R^2 of the regression is maximized for $n = 1$, so that the best fit is when we assume agents set expected inflation equal to the average of the perfect foresight future inflation rates for the expected duration of the prices. All the estimates of the elasticity γ are consistent with the theoretical prediction in Section II.

E MONTE CARLO

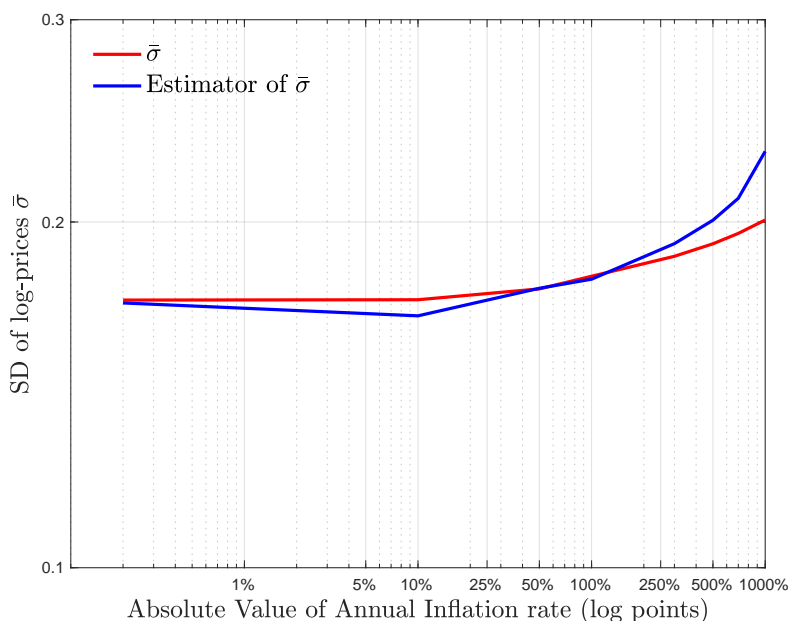
Figure XIII and Figure XIV show the Monte Carlo simulations to evaluate whether time-aggregation is biasing our estimates of the average size of price changes and the standard deviation of relative prices. See the text for a description of the simulation and results.

Figure XIII: Size of Price Increases: Sensitivity to Sampling Periodicity



Note: Figure shows the theoretical Δ_a^+ and the estimator Δ^+ when we simulate the model in Section II. and sample observations twice a month.

Figure XIV: Standard deviation of relative prices:
Sensitivity to Sampling Periodicity



Note: Figure shows the theoretical and the estimator of the standard deviation of log-prices when we simulate the model in Section II. and sample observations twice a month.

F BALANCED SAMPLE

In this section we explore the sensitivity of the relationship of the price dispersion and inflation displayed in Section IV.F due to sample selection. In particular, we analyzed whether the increase in price dispersion displayed in Table IV.F and Figure X are due to a compositional effect of the dispersion of relative prices across the good and or stores for which we have price quotes at different inflation rates. For this purpose we select a sample of good and stores that is more balanced than our benchmark case. In particular, we select the good \times store combinations for which we have 190 non-missing observations out of the 212 two-weeks periods. We select 190 as a compromise to have enough observations to compute the relevant fixed-effects and to obtain a balanced sample. We end up with about 1.6 million of price observations instead of 5.5 millions for the complete sample. We refer to this as the “balanced sample”.

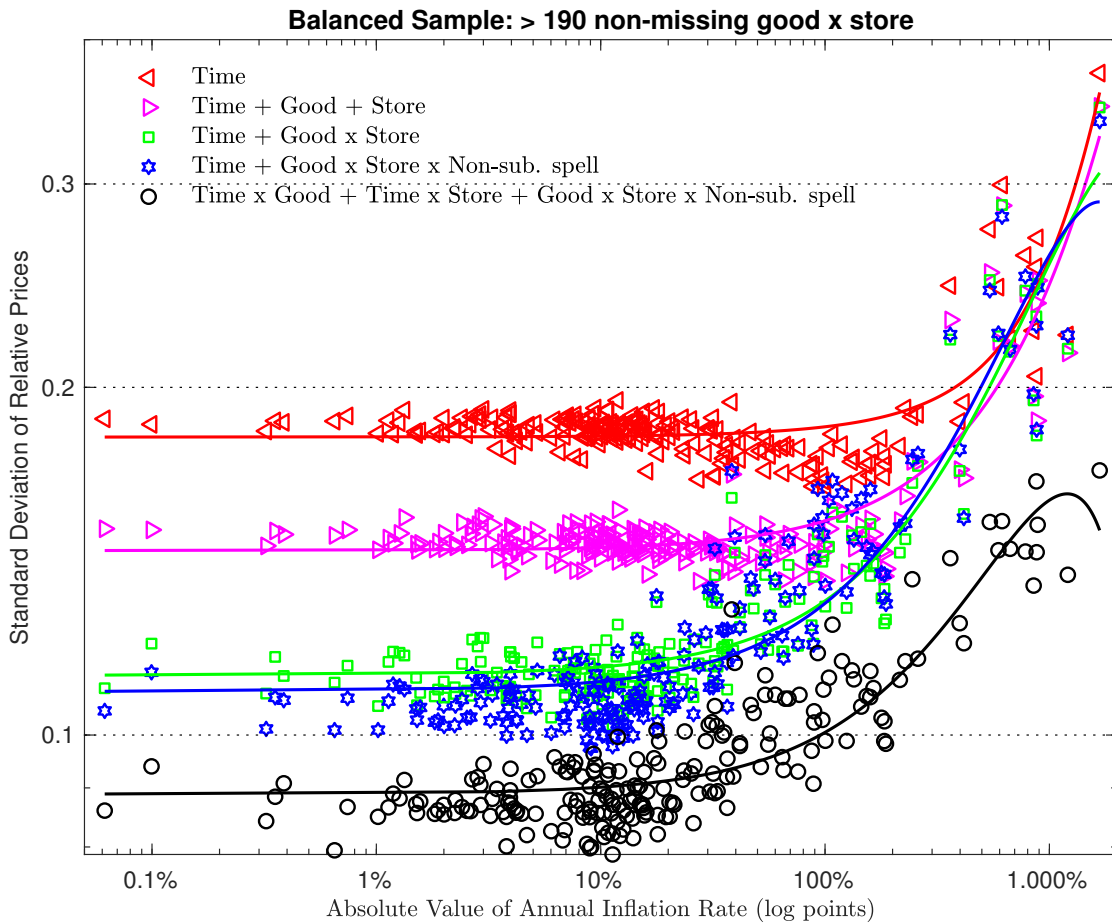
The results of the “balanced sample” are analyzed in Table XIV and displayed in Figure XV. As can be seen, the results of the balanced sample are very similar to the results of the full sample, from which we conclude that sample selections does not substantially biased our estimate of the relationship between inflation and price dispersion.

Table XIV: Regressions used to compute residual variance of price levels: balanced sample

Models i : indicate dummies	# of dummies	Adj- R^2	Slope at $\pi =$ 100%	Slope at $\pi =$ 500%	Slope at $\pi =$ 700%
1: time	212	0.739	0.02	0.18	0.24
2: time + good + store	1091	0.981	0.06	0.26	0.35
3: time + good \times store	8613	0.984	0.13	0.37	0.40
4: time + good \times store \times non-sub-spell	32220	0.987	0.16	0.38	0.38
5: time \times store + time \times good + good \times store \times non-sub-spell	169131	0.995	0.11	0.26	0.24

Price observations used in each regression 1613623 for homogenous goods, in 212 two-week periods. We have kept the good \times store combinations which have non-missing price quotes for at least 190 out of the 212 two week periods.

Figure XV: Cross sectional standard deviation of prices vs. inflation



G COMPARISON TO OTHER STUDIES

Table XV provides a succinct view of the samples used in other studies of price dynamics under high inflation. It shows the countries analyzed, the product coverage of each sample, and the range of inflation rates in each

study. The most salient features of this paper are the broader coverage of our sample and the larger variation in inflation rates.

Table XV: Comparison with other Studies in Countries with High Inflation

Country	Authors	Sample product coverage	Observ. per month	Sample	Inflation (%. a.r.)	Monthly freq. (%)
Argentina	This paper	506 goods/services, representing 84% of Argentinian consumption expenditures	81,305 on average	1988-1997	$0 - 7.2 \times 10^6$	16-99
Brazil	Barros et al. (2009)	70% of Brazilian consumption expenditures	98,194 on average	1997-2010	2-13	39-50
Israel	Llach and Tsiddon (1992)	26 food products (mostly meat and alcoholic beverages)	250	1978-1979	64	41
Israel	Llach and Tsiddon (1992), Eden (2001),	26 food products (mostly meat and alcoholic beverages)	530	1981-1982	118	61
Israel	Baharada and Eden (2004)	up to 390 narrowly-defined products from the Israeli CPI	2800	1991-1992	16	24
Mexico	Gagnon (2007)	227 product categories, representing 54.1 percent of Mexican consumption expenditures	31,500	1994-2002	3.5-45	27-45
Poland	Konieczny and Skrzypacz (2005)	52 goods, including 37 grocery items, and 3 services	up to 2400	1990-1996	18-249	59-30

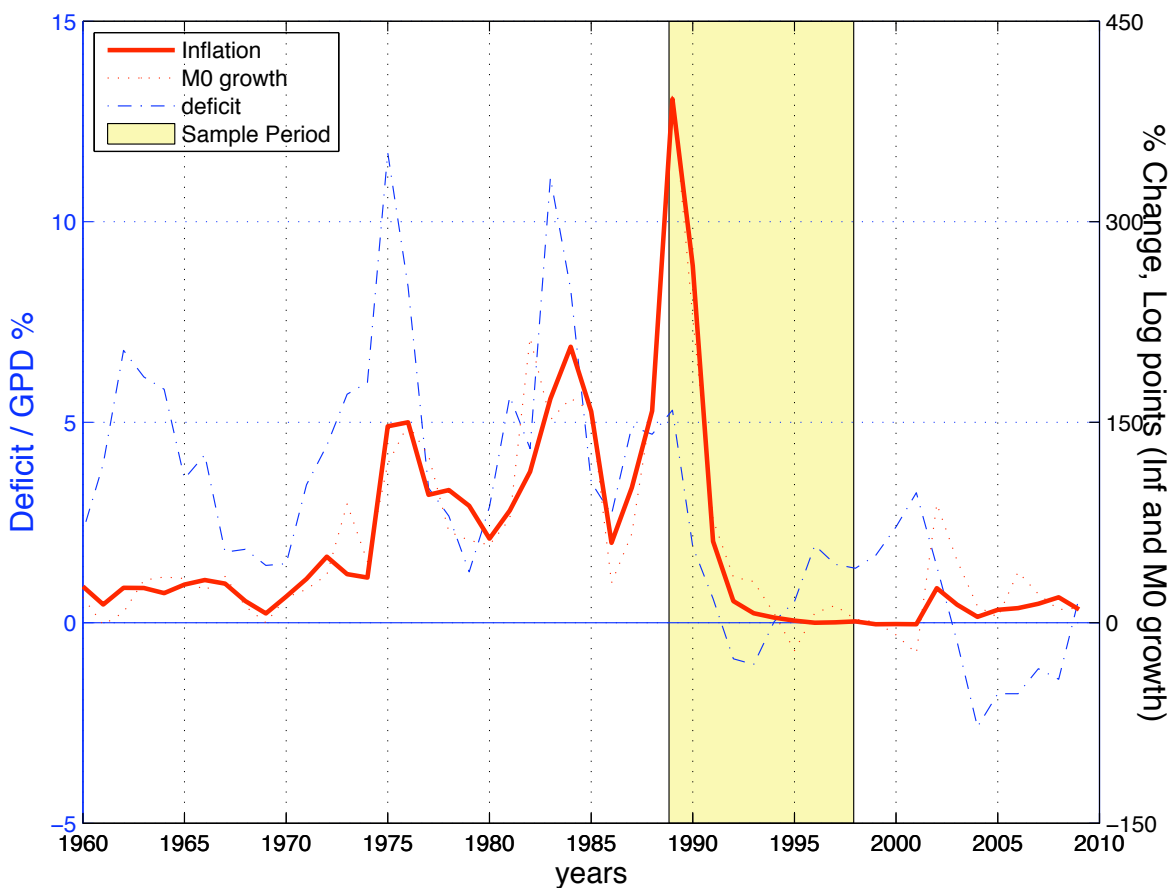
H BACKGROUND ON INFLATION AND ECONOMIC POLICY

Here we give a brief chronology of economic policy to help readers understand the economic environment in our sample period, which goes from December 1988 to September 1997. The beginning of our sample coincides with decades of high inflation culminating in two years of extremely high inflation (typically referred as two short hyperinflations) followed by a successful stabilization plan, based on a currency board, started in April of 1991 which brought price stability in about a year, and stable prices until at least three more years after the end of our sample.

The years before the introduction of the currency board witnessed several unsuccessful stabilization plans, whose duration become shorter and shorter, and that culminated in the two short hyperinflations, all of these during a period of political turmoil. Several sources describe the inflation experience of Argentina since the 1970, such as [Kiguel \(1991\)](#) and [Alvarez and Zeldes \(2005\)](#) for the period before 1991 and [Cavallo and Cottani \(1997\)](#) for descriptions right after 1991. For a more comprehensive study see [Buera and Nicolini \(2010\)](#).

Argentina had a very high average inflation rate since the beginning of the 1970s. Institutionally, the Central Bank has been part of the executive branch with no independent powers, and typically has been one of the most important sources of finance for a chronic fiscal deficit. Figure XVI plots inflation, money growth and deficits between 1960 and 2010 with our sample period highlighted in yellow. The deficit as a percentage of GDP was on average well above 5% from 1975 to 1990, see Figure XVI. At the beginning of the 1980s fiscal deficit and its financing by the Central Bank were large even for Argentinean standards. These years coincide with a bout of high inflation that started in the second half of the last military government, 1980 to 1982, and continued during the first two years of the newly elected administration of Dr. Alfonsín, 1983 to 1984.

Figure XVI: Inflation, Money Growth and Deficits



In June of 1985 there was a serious attempt to control inflation by a new economic team which implemented what it is referred to as the Austral stabilization plan (the name comes from the introduction of the "Austral" currency in place of the "Argentine Peso"). The core of this stabilization plan was to fix the exchange rate, to control the fiscal deficit and its financing from the Central Bank, and to introduce price and wage controls. While the Austral plan had some initial success, reducing the monthly inflation rate from 30% in June of 1985 to 3.1% in August of 1985, by mid 1986 the exchange rate was allowed to depreciate every month and inflation reached about 5% per month. By July of 1987 the monthly inflation rate was already above 10%. The same economic team started what is referred to as the "Primavera" stabilization plan in October 1988, when the inflation rate was

again around 30% per month, at a time when the Alfonsín administration was becoming politically weak. The primavera plan was a new short lived exchange rate based stabilization plan that was abandoned in February of 1989.⁴⁴ Our data set starts right after the beginning of the Primavera stabilization plan, in December of 1988.

After the collapse of the “Plan Primavera” the economy lost its nominal anchor and a perverse monetary regime was in place. Legal reserve requirements for banks were practically 100% and the Central Bank paid interest on reserves (most of the monetary base) printing money. Thus a self fulfilling mechanism for inflation was in place. High inflationary expectations, led to high nominal deposit rates, which turned into high rates of money creation that validated the inflationary expectations.

Figure XVII displays the yearly percent continuously compounded inflation rate and interest rate for the first years of our sample, together with references to some of the main changes in economic policy during the period. Observe how interest rates and inflation skyrocketed after the plan Primavera’s crawling peg was abandoned. In May 1989 a presidential election took place where the opposition candidate, Dr. Menem, was elected. The finance minister and the central bank president that carried the Primavera plan resigned in April 1989. Thereafter, Dr. Alfonsín’s administration had two different finance ministers and two different central bank presidents, in the midst of a very weak political position and rampant uncertainty about the policies to be followed by the next administration. During the campaign for the presidential election Dr. Menem proposed economic policies that can be safely characterized as populist, with a strong backing from labor unions. Indeed, the core of his proposed economic policy was to decree a very large generalized wage increase, “el salarizado”.⁴⁵ In July 1989 the elected president, Dr Menem, took office, several months before the stipulated transition date, in the midst of uncontrolled looting, riots and extreme social tension. The inflation rate at this time was the highest ever recorded in Argentina, almost 200% per month—2.3% per day.

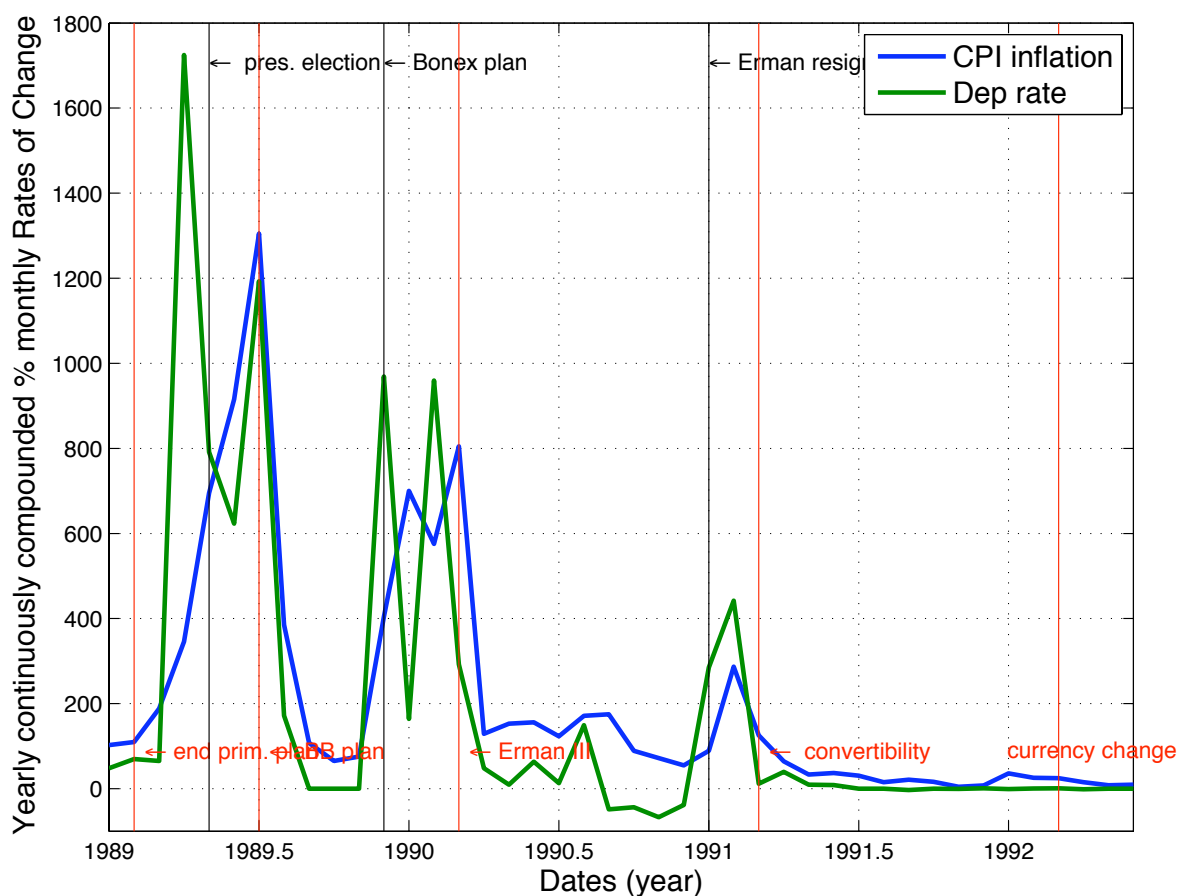
The beginning of the Menem administration started with a large devaluation of the Argentine currency, in what is known as the BB stabilization plan, for the name of the company Bunge and Born, where the two first secretaries of the treasury came from. Indeed these appointments made by the Menem administration were a surprise to most observers, given the promises made in the campaign. The announcement of tight control of the fiscal deficit, and the management of the exchange rate of this plan were also surprising for most observers. During this time inflation transitorily fell.

In December 1989, amid large losses in the value of the Argentine peso, a new finance minister was appointed, Dr. Erman Gonzalez, who started yet a new “stabilization plan” (referred to as Plan Bonex). The core of this plan was a big compulsory open market operation by which the central bank exchanged all time deposits in the Banking system (mostly peso denominated time deposits with maturities of less than a month) for 10 year US Dollar denominated government bonds (Bonex 1989). This big open market operation changed the monetary regime and allowed the Central Bank to regain control of the money supply, as the government no longer had to pay interest on money (reserves on time deposits) by printing money. During Dr. Gonzalez tenure there were several fiscal measures aimed at controlling the fiscal deficit. In March 1990 a renewed version of the stabilization plan was launched, with a stricter control of the money supply and of the fiscal deficit. The actual

⁴⁴The peg started at about 12 units of Argentine currency (“the Austral”) per US dollar. To put it in perspective, at the beginning of the Austral plan, the peg was 0.8 units of Argentine currency per US dollar.

⁴⁵The slogan to summarize his proposed economic policy was “el salarizado”, i.e., “the huge wage increase” in Spanish.

Figure XVII: Inflation and Depreciation Rate during HyperInflations



percentage inflation rates during 1989 and 1990 were 4924% and 1342% respectively!

In January of 1991, Dr. Gonzalez resigned and Dr. Cavallo was appointed as finance minister. During the first two months of his tenure there was a large devaluation of the currency and a large increase in the prices of government owned public utility firms. In April 1st of 1991 there was a regime shift that lasted until 2001. The new regime was a currency board that fixed the exchange rate and enacted the independence of the Central Bank, first by means of presidential decrees, and then by laws approved by congress. At this time the argentinean currency, the Austral, was pegged to the US dollar at 10,000 units per USD.⁴⁶

On January 1st 1992 there was a currency reform that introduced a new currency (the Peso Argentino) to replace the Austral, chopping four zeros of the latter (so that one peso was pegged to one dollar, and to 10,000 australes).

There are a host of changes that were introduced at this time, both in term of deregulation and in terms of fiscal arrangements (broadening of the value added tax's base, sale of state owned firms, etc.), which in the first years reduced the size of the fiscal deficit. There was also a renewed access to the international bond markets, and a constant increase in public debt. There were also an acceleration of the trade liberalization that started in

⁴⁶To have an idea of the average inflation rate until 1991, notice that the exchange rate when the Austral was introduced in June of 1985 was 0.8 austral per US dollars.

the mid 80s and a liberalization of all price and wage controls. During the years covered in our sample, GDP grew substantially, despite the short and sharp recession during 1995, typically associated with the balance of payment crisis in Mexico. The exchange rate remained fixed until January of 2002, where the exchange rate was depreciated in the midst of a banking run that started in the last quarter of 2001, a recession that started at least a year prior, and the simultaneous default of the public debt.