Crises: Equilibrium Shifts and Large Shocks

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- Widely credited with having shifted the Eurozone economy from a "bad equilibrium" (high sovereign debt spreads and growing fiscal deficits mutually reinforcing each other); to a "good equilibrium" (with low spreads and sustainable fiscal policy).

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e.g., sovereign debt markets, financial crises, revolutions

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- We explain when shifts must occur but allow for multiplicity and hysteresis in many scenarios

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- Key strategic implication:
  - with no or small shocks, can keep doing same thing as before because you may rationally be confident that others will do so
  - with large shocks,
    - not rational for marginal player to be confident of others' behavior; uniform rank beliefs select "risk dominant" equilibrium

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2. Don't always play risk dominant equilibrium. but switches only to risk dominant equilibrium

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- our main large shock result relies on fat tails (c.f., large normal prior, normal noise global game literature)

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- Static coordination game played repeatedly under evolving fundamentals and fat-tailed prior on common innovations
- Assume hysteresis: follow majority play from previous period if rationalizable, otherwise
- Majority behavior switches in response to either extreme enough level of fundamentals or a large shock

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- formally, payoff to not investing is 0 and payoff to investing is payoff to investing is $x + \alpha - 1$, where α is the proportion of other players investing

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    - a common shock η
    - an idiosyncratic shock ε_i

Maintained Assumptions about Shocks

1. **thick tailed common shocks**: η is distributed according to density g with thick (regularly varying) tails, i.e.,

$$\lim_{\lambda\to\infty} \frac{g(\lambda\eta)}{g(\lambda\eta')} \in (0,\infty) \text{ for all } \eta,\eta'$$

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2. **thinner tailed idiosyncratic shocks;** ε is distributed according to log-concave density f (i.e., $\log f$ is concave)

Rank belief: what probability does an agent assign to a representative agent having a lower return than his own?

$$R(z) \equiv \Pr(z_j \leq z | z_i = z) = \frac{\int F(\varepsilon) f(\varepsilon) g(z-\varepsilon) d\varepsilon}{\int f(\varepsilon) g(z-\varepsilon) d\varepsilon}$$

Equivalently, what is an agent's expectation of the proportion of other agents with lower returns?

- f is standard normal distribution $N(0, 1)$

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    - variance of η is unknown and distributed with inverse χ^2

Rank Beliefs in the Leading Example

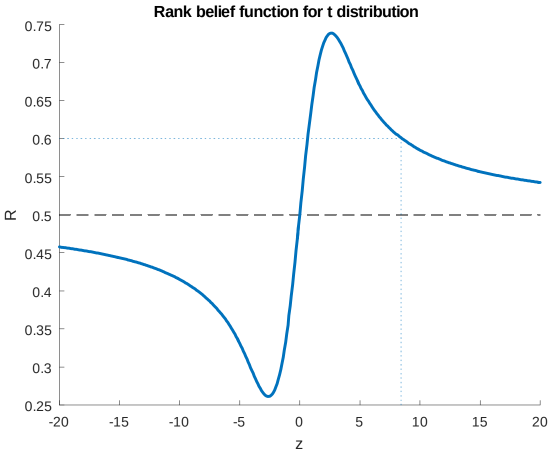

Figure: Rank belief function R.

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- **limit uniform rank beliefs**: $R(z) \to \frac{1}{2}$ as $z \to \infty$.

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- limit uniform rank beliefs as a primitive assumption?

Without Thick Tails

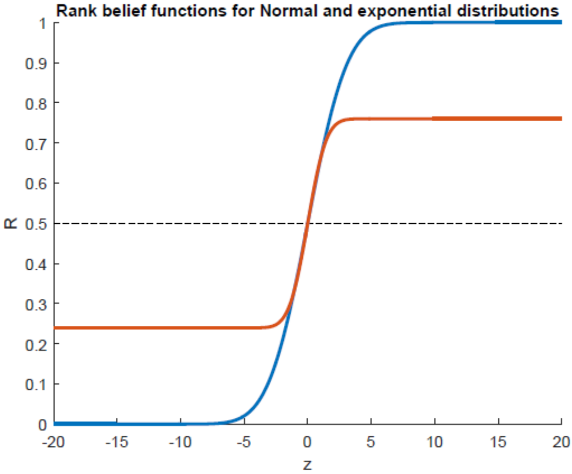

Figure: Rank belief function under normal idiosyncratic shocks and normal or exponential common shocks

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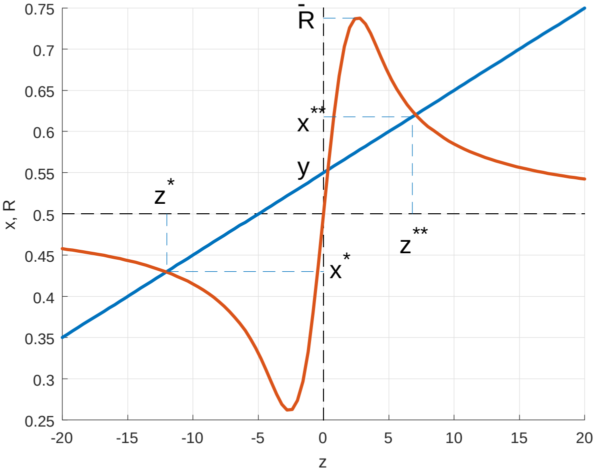
- also sufficient because log-concavity of f implies that when an agent has a high return, she has a higher (w.r.t. FOSD) belief about other player's return


R̄
x**
y
z*
x*
z**
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
x, R
-20
-15
-10
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0
5
10
15
20
z

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- PROOF: Let $\bar{z}$ be the largest shock at which not invest is rationalizable and suppose $\bar{z} > z^{**}$. The payoff to investing is at least

$$\overbrace{y + \sigma\bar{z}}^{\text{own return}} - \overbrace{R(\bar{z})}^{\text{proportion of others not investing}} > 0,$$

a contradiction.

Let $\overline{R}$ be the maximum possible rank belief:

$$\overline{R} = \max_{z \geq 0} R(z)$$

Proposition

Invest is uniquely rationalizable whenever $x > \overline{R}$

- equivalently, invest is uniquely rationalizable if $z > \frac{\overline{R}-y}{\sigma}$

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- for sufficiently high returns, it doesn't matter how you got there
- observe that $\frac{1}{2} < \overline{R} < 1$; thus this criterion is intermediate between risk dominance and dominant strategies

- For each $x \in \left(\frac{1}{2}, \overline{R}\right]$, define critical shock size $\overline{z}(x)$ to be the largest shock at which the rank belief is x:

$$\overline{z}(x) = \max R^{-1}(x)$$

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Invest is uniquely rationalizable if $x \in \left(\frac{1}{2}, \overline{R}\right]$ *and* $z > \overline{z}(x)$

Shock Sufficient Condition

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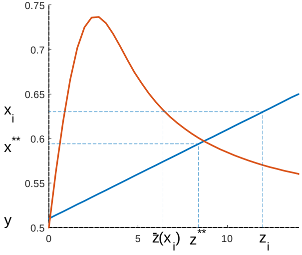
Invest is uniquely rationalizable if $x \in \left(\frac{1}{2}, \overline{R}\right]$ *and* $z > \overline{z}(x)$

- for intermediate returns, whether invest is uniquely rationalizable depends on whether there was a positive shock

- Invest will be uniquely rationalizable at fundamentals x_i if reached via a large shock (left panel) but not if reached by a small shock (right panel)



Ex Ante Level Sufficient Condition

- Let $\overline{y}$ be the critical level of fundamentals at which returns will exceed the rank belief whenever shocks are positive.

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- Formally, define $\overline{y}$ to be the largest y such that

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for some z.

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  for some z.
- For small σ, $\overline{y} \approx \overline{R}$

Proposition

Invest is uniquely rationalizable whenever $x > \frac{1}{2}$ and $y > \overline{y}$

- For small σ, sufficient conditions are also necessary....

Proposition

If R is single peaked and $y \leq \overline{R} - \sigma\overline{z}\left(\overline{R}\right) \leq \overline{y}$*, invest is uniquely rationalizable only if (i)* $x > \overline{R}$ *or (ii)* $x > \frac{1}{2}$ *and* $z > \overline{z}\left(x\right)$

- For small σ, sufficient conditions are also necessary....
- We get a partial converse under two additional restrictions:

Proposition

If R is single peaked and $y \leq \overline{R} - \sigma \overline{z}\left(\overline{R}\right) \leq \overline{y}$, invest is uniquely rationalizable only if (i) $x > \overline{R}$ or (ii) $x > \frac{1}{2}$ and $z > \overline{z}\left(x\right)$

- Call $\theta = y + \sigma\eta$ the *fundamental state*; fundamental state is the population mean return and also the median agent's return

Proposition

Invest is uniquely rationalizable for the majority if it is risk dominant ($\theta > \frac{1}{2}$) and, in addition, (i) the realized fundamentals are sufficiently high ($\theta > \overline{R}$), or (ii) the expected fundamentals were sufficiently high ($y > \overline{y}$), or (iii) the shock is sufficiently high $\eta > \overline{z}(\theta)$.

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    - for example, random walk $(y_{t+1} = \theta_t)$ or reversion to the mean $\left(y_{t+1} = \frac{1}{2} + \kappa\left(\theta_t - \frac{1}{2}\right)\right)$

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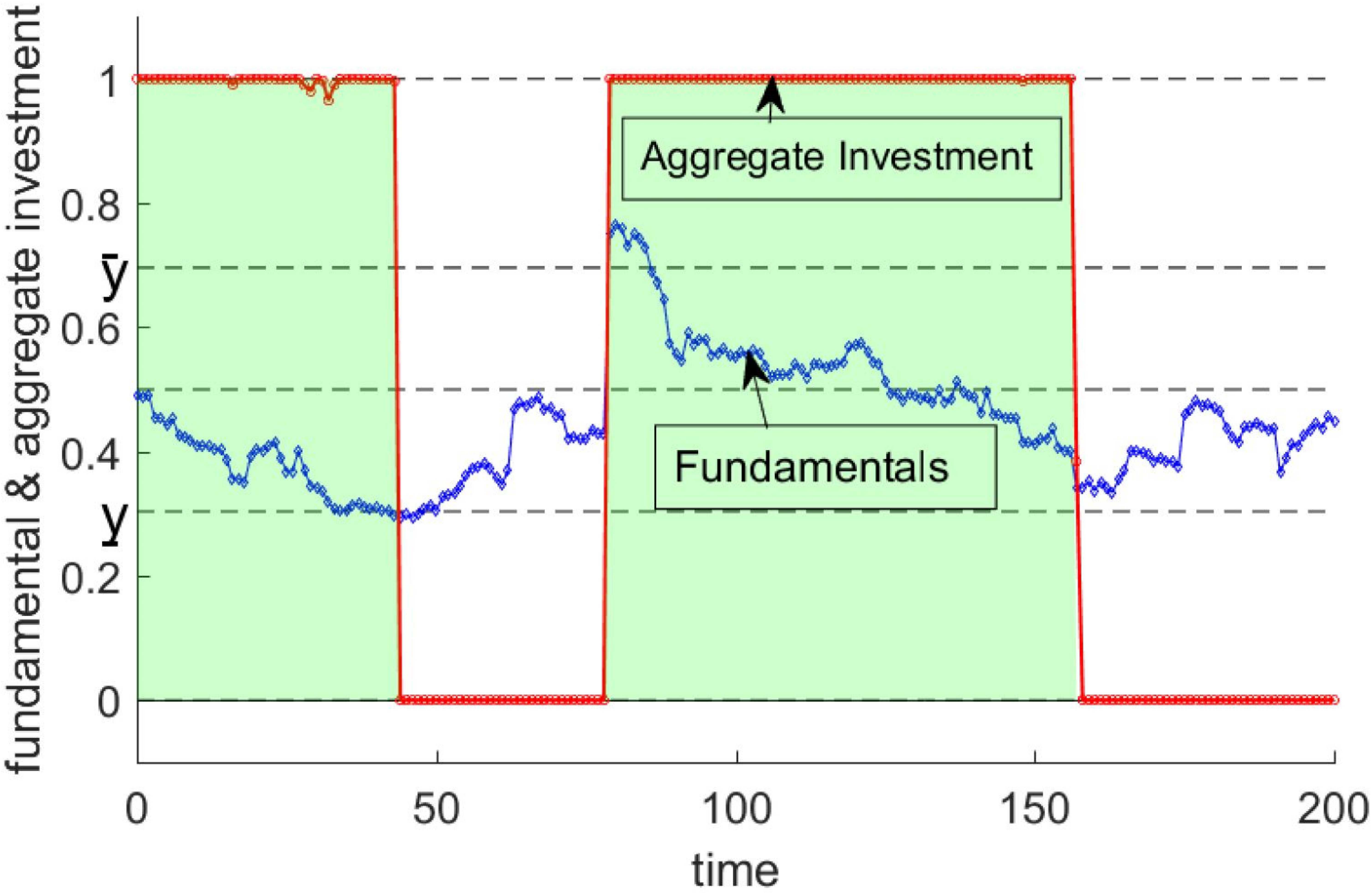
  - if not, do not invest whenever rationalizable



fundamental & aggregate investment
1
0.8
ȳ
0.6
0.4
y
0.2
0
0
50
100
150
200
time
Aggregate Investment
Fundamentals

Proposition

Shifts to majority investment will occur whenever invest is risk dominant ($\theta_t > \frac{1}{2}$) *and, in addition, (i) the realized fundamentals are sufficiently high* ($\theta_t > \overline{R}$)*, (ii) the expected fundamentals were sufficiently high* ($y_t > \overline{y}$) *or the shock was sufficiently high* $\eta_t > \overline{z}(\theta_t)$.

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  - slow news release good if you want to stay in current equilibrium (and vica versa)
  - simple mechanism that can be plugged into richer models

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- No role for shocks with monotone rank beliefs and $R_{\infty} = 1$ (e.g., normality)

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- SMALL SHOCKS PROPOSITION: Under multiplicity condition, there exists $\Delta > 0$ wuch that whenever $|x - y| \leq \Delta$, invest is uniquely rationalizable if and only if $y > \bar{y}$.

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- If a bad equilibrium is being played, and fundamentals are heading up, it is better to have fundamentals jump up (or good news released in chunks)

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    - Similarly, if going from uniqueness to multiplicity (c.f., global game arguments)
    - Feels like we go from multiplicity to uniqueness?