Taking Incomplete Information Seriously: The Misunderstanding of John Harsanyi

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A Tripartite Distinction of Informational Assumptions in Economic Analysis

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   - No common knowledge assumptions at all

The Tripartite Distinction in Game Theory

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   - LEADING EXAMPLE: Almost all economic environments of interest?

A Pessimistic Assessment

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...we cannot avoid the assumption that all subjects under consideration are completely informed about the physical characteristics of the situation in which they operate

- Aumann (1987) wrote "The common knowledge assumption underlies all of game theory and much of economic theory. Whatever be the model under discussion ... the model itself must be assumed common knowledge; otherwise the model is insufficiently specified, and the analysis incoherent."

John Harsanyi 1967/68

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- we can incorporate any incomplete information without loss of generality!

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and analogously

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  - in economic model, it can encompass preferences, technology, etc...

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  - make those *implicit* common knowledge assumptions *explicit* and relax them
  - taking higher-order beliefs seriously

The Wilson Doctrine?

Wilson (1987):

> *Game theory....is deficient to the extent that it assumes other features to be common knowledge, such as one agent's probability assessment about another's preferences or information.*
>
> *I foresee the progress of game theory as depending on successive reductions in the base of common knowledge required to conduct useful analyses of practical problems. Only by repeated weakening of common knowledge assumptions will the theory approximate reality."*

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- key but subtle observation: relaxing common knowledge is equivalent to allowing richer type spaces

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- Strategic complementarities are important but what are the implications of multiple equilibria for empirical work, policy analysis or comparative statics more generally?
- CLAIM: It is important for lots of applied economic analysis to think about the implications of relaxing common knowledge assumptions in coordination games

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- If $0 < \theta_A < 1$ and $0 < \theta_B < 1$, then this game has multiple Nash equilibria

Global Games

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- Minor variant of Carlsson and van Damme (1993)

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  - for small noise, the risk dominant Nash equilibrium of the perfect information game is almost always played

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  - ....a contradiction

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- If $\sigma \approx 0$, then Ann always attaches probability $\approx \frac{1}{2}$ to $\theta_B \leq \theta_A$
- As $\sigma \rightarrow 0$, unique rationalizable outcome has each player invest if and only if $\theta_I \geq \frac{1}{2}$

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  - Can do lots of interesting comparative statics / policy analysis (and people have done....)
- Further extends to general supermodular games

Global Games Critique

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- In particular, we now have the unreasonable implicit and importnat common knowledge assumption about the distribution of signals conditional on the state....
- Let's go back to basics and examine our coordination game without making common knowledge assumptions.... or at least making fewer common knowledge assumptions....

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- Is a subset of our first universal type space

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- what can we say about strategic behavior?

Electronic Mail Game on Steroids

Rubinstein 89, Weinstein and Yildiz 07

Suppose that the state may be "good" with $(\theta_A, \theta_B) = \left(\frac{3}{4}, \frac{3}{4}\right)$:

	Invest	Not Invest
Invest	$\frac{3}{4}, \frac{3}{4}$	$-\frac{1}{4}, 0$
Not Invest	$0, -\frac{1}{4}$	$0, 0$

but Bob may have a dominant strategy to not invest, so the state is "bad", with $(\theta_A, \theta_B) = \left(\frac{3}{4}, -1\right)$:

	Invest	Not Invest
Invest	$\frac{3}{4}, -1$	$-\frac{1}{4}, 0$
Not Invest	$0, -2$	$0, 0$

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  - If Ann does not receive a first message, she thinks that the state is bad with probability $1 - \varepsilon$
  - If a player does not receive a confirmation of his/her message, he/she thinks that the other player did not receive his/her message with probability $1 - \varepsilon$

Infection Argument

- If Ann receives many confirmations, she is "close" (formally, in the product topology on the universal type space) to common knowledge that game is $(\theta_A, \theta_B) = \left(\frac{3}{4}, \frac{3}{4}\right)$

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- Weinstein Yildiz 07 show that this logic is completely general: (roughly) any action that is rationalizable in a perfect information game is uniquely rationalizable for a nearby type in the product topology

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- why is this useful (and feasible)?

Informationally Robust Analysis

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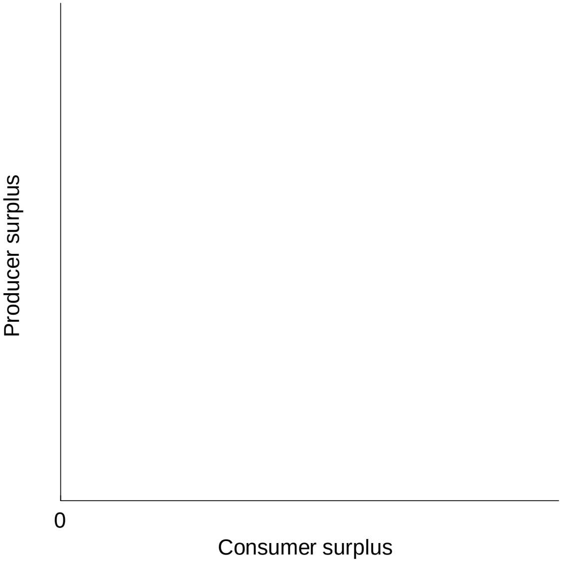
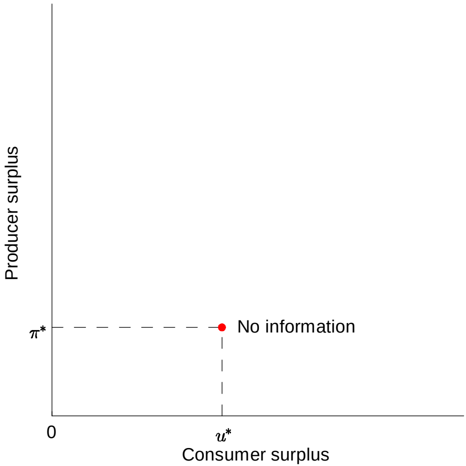
- Robust Prediction: What can we say about all (consumer surplus, producer surplus) pairs that can arise?

A Pictorial Characterization

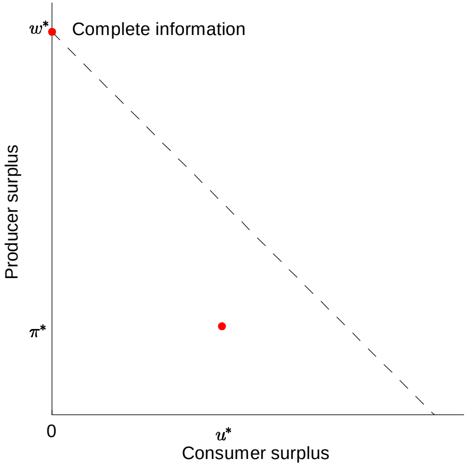


The Uniform Price Monopoly



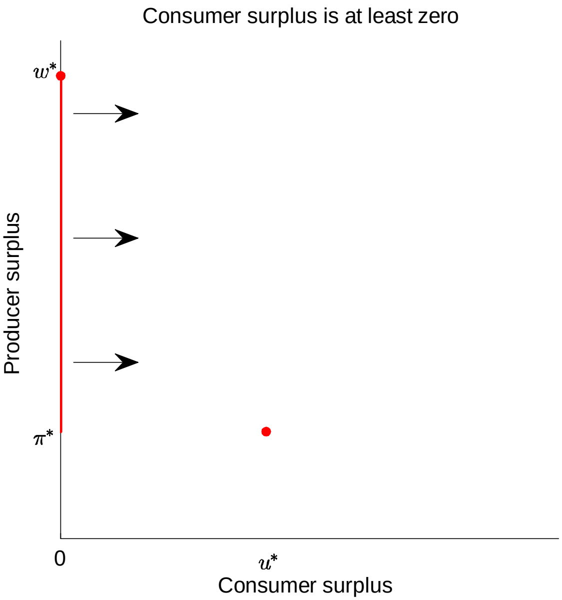
- producer charges (uniform) monopoly price
- consumers get positive consumer surplus, socially inefficient allocation

First Degree Price Discrimination: Perfect Discrimination

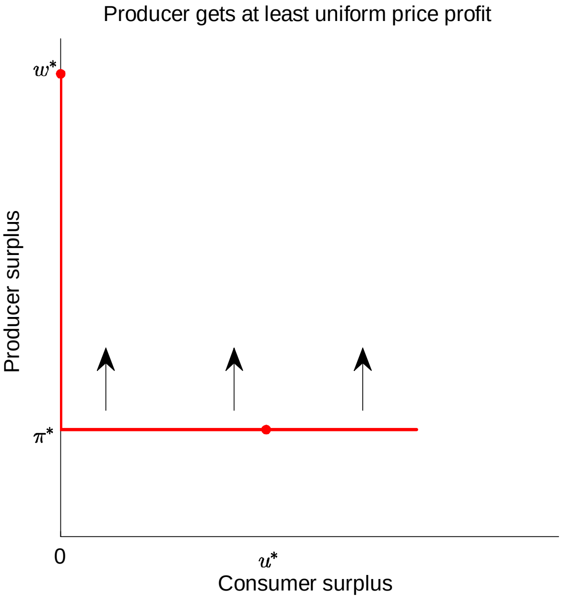


- producer extracts full surplus
- consumers get zero surplus, but socially efficient allocation

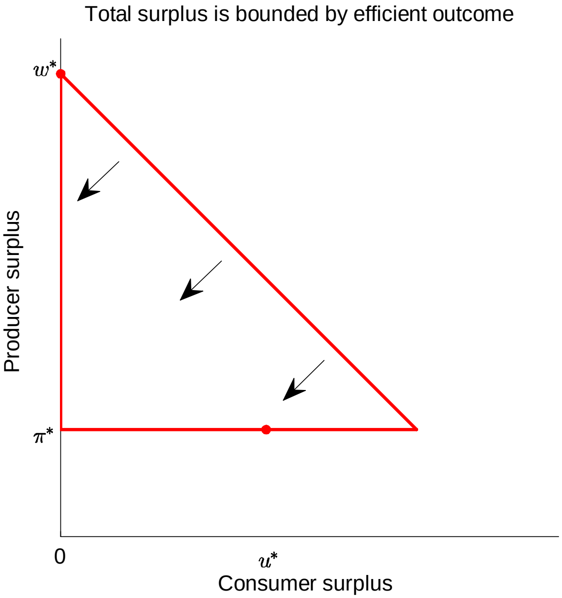
Welfare Bounds: Voluntary Participation



Welfare Bounds: Nonnegative Value of Information

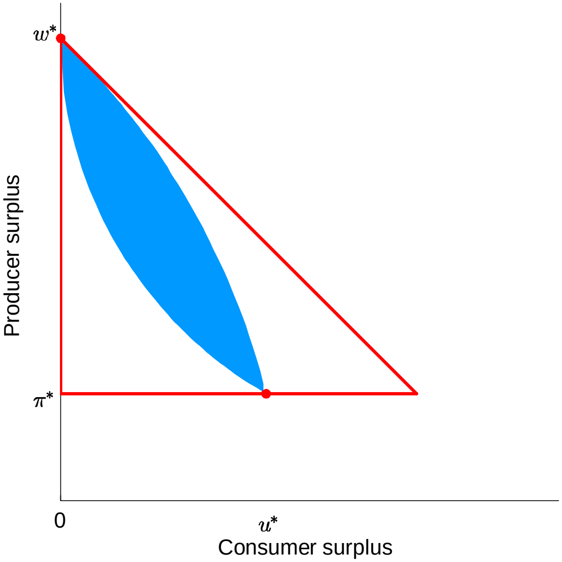


Welfare Bounds: Social Surplus

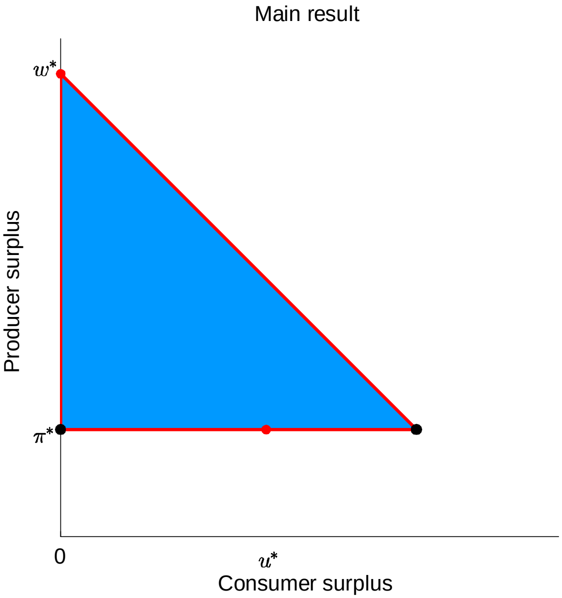


Welfare Bounds and Third Degree Price Discrimination

What is the feasible surplus set?

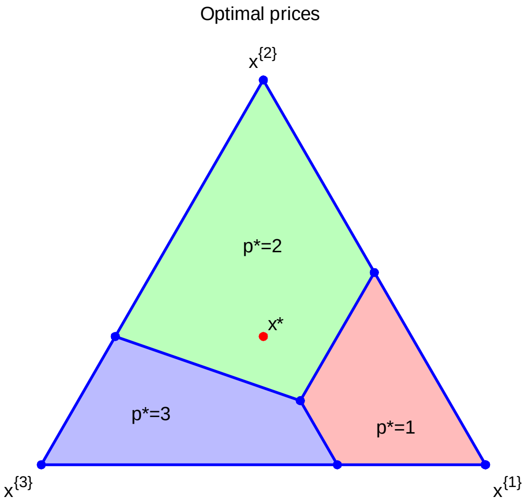


Main Result: No More Robust Predictions!



Example

- $\frac{1}{3}$ of consumers have valuation 1, $\frac{1}{3}$ have valuation 2 and $\frac{1}{3}$ have valuation 3
- optimal prices:



Splitting

- A segmentation of the three value uniform aggregate market:

	$v = 1$	$v = 2$	$v = 3$	weight
market 1	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$
market 2	0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$
market 3	0	1	0	$\frac{1}{6}$
total	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	

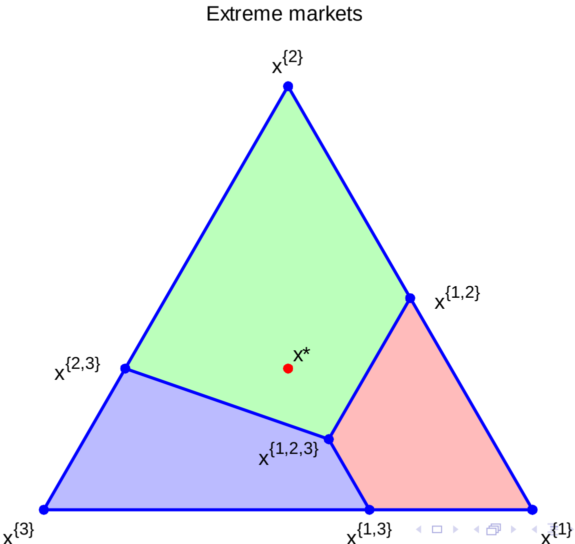
"Extremal Segmentation"

	$v = 1$	$v = 2$	$v = 3$	weight
$\{1, 2, 3\}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$
$\{2, 3\}$	0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$
$\{2\}$	0	1	0	$\frac{1}{6}$
total	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	

- price 2 is optimal in all markets
- in fact, seller is always indifferent between all prices in the support of the market
- this is always possible to do (this is the meat of our general argument)

Geometry of Extremal Markets

- extremal segment x^S: seller is indifferent between all prices in the support of S



Consumer Surplus Maximizing Segmentation

- *an* optimal policy: always charge lowest price in the support of every segment:

	$v = 1$	$v = 2$	$v = 3$	price	weight
$\{1, 2, 3\}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	1	$\frac{2}{3}$
$\{2, 3\}$	0	$\frac{1}{3}$	$\frac{2}{3}$	2	$\frac{1}{6}$
$\{2\}$	0	1	0	2	$\frac{1}{6}$
total	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		1

Social Surplus Minimizing Segmentation

- all incentive constraints in the support are binding
- another optimal policy: always charge highest price in each segment:

	$v = 1$	$v = 2$	$v = 3$	price	weight
$\{1, 2, 3\}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	3	$\frac{2}{3}$
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- In other settings, there are.... e.g., first price auction

Robust Identification

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Information Design

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- Compare Kamenica and Gentzkow (2011)

The Misunderstanding of John Harsanyi and Mechanism Design

Wilson (1987): (more complete quote)

> *Game theory has a great advantage in explicitly analyzing the consequences of trading rules that presumably are really common knowledge; it is deficient to the extent that it assumes other features to be common knowledge, such as one agent's probability assessment about another's preferences or information.*
>
> *I foresee the progress of game theory as depending on successive reductions in the base of common knowledge required to conduct useful analyses of practical problems. Only by repeated weakening of common knowledge assumptions will the theory approximate reality."*

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- Implementing on the universal type space is the same (modulo technicalities) as implementing on all types spaces

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  - Implemention of the efficient outcome in Bayes Nash equilibrium on universal type space may or may not be equivalent to dominant strategies implementation

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- Analogously, Bob's value is $v_B = \theta_B + \gamma\theta_A$

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2. Ann and Bob each have a signal that confounds a common value and private value component (cannot be distinguished)

Implicit Common Knowledge Assumptions and Interdependent Values

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- Whether this makes sense depends on the interpretation

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Should actually distinguish "higher order preference types", e.g.,

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Thus we are looking higher order preferences over acts

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- Higher order preference types correspond exactly to what would be learnt about players
- selling on the higher-order preference type space is complicated

Conclusion

- Incomplete information has not been fully incorporated into economic analysis
- Results are driven by implicit common knowledge whose role is sometimes not well understood
- But relaxing all common knowledge assumptions may be possible but unhelpful
- Focus on which are reasonable common knowledge assumptions and make them explicit