Limits of Price Discrimination with non-linear demand

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Price Discrimination

1. Segmenting Market ("Third Degree"): suppose that a seller can...
   - identify different types of consumers
   - offer them different prices for a good
   - can stop them re-trading
2. Screening Consumers (Second Degree): suppose that a seller can
   - offer non-linear price schedules (thus charging different prices for different units)
   - can stop them re-trading but does not know their utilities and must screen

Segmentation and Consumer Surplus with Single Unit Demand

- segmentation increases seller profits but has ambiguous effect on consumer surplus
  - may increase consumer surplus, most notably by selling to consumers who were otherwise excluded
  - may decrease consumer surplus by reducing consumer rents
  - classic literature: focusses on no exclusion case where it is harder to get increasing consumer surplus
  - our AER 15: consumer surplus can equal total possible surplus minus uniform monopoly profits (will review...)
- single unit demand same as linear utility with multi-unit demand
- rules out screening

Segmentation and Consumer Surplus with Non-Linear Demand

- same mechanism : reduce exclusion without losing consumer rents....
- new margin: "exclusion" is a matter of degree, i.e., rationing low valuation consumers
- revealing high value consumers reduces rent to 0 for those revealed high value types (infra-marginal effect) but reduces degree of exclusion / rationing of low value types (marginal effect)
- this trade-off continues to exist even under consumer optimal segmentation

This Talk

1. two type case: example and trade-off
2. convergence to simple unit demand
3. limits of price discrimination with non-linear demand

Two Type Model

- consumption q
- consumer of type θ gets concave utility

$$u_\theta(q)$$

- two types, low (L) and high (H)
- proportion x of low types
- constant marginal cost of production c

Optimal Contract

- efficient quantities q_L^* and q_H^* for low and high types
- optimal contract takes the form:
  - efficient quantity to high type ($q_H = q_H^*$)
  - perhaps distorted down quantity for low type $q_L \leq q_L^*$
  - no rent for low type (so $t_L = u_L(q_L)$)
  - rent $(\theta_H - \theta_L)\, u_H\,(q_L)$ for the high type (so $t_H = u_H(q_H) - cq_H - (\theta_H - \theta_L)\, u_H\,(q_L)$)
- $q_L(x)$ is an weakly increasing function of x
- must have exclusion at the bottom, i.e., $q_L(x) = 0$ for all $x \leq \underline{x}$.

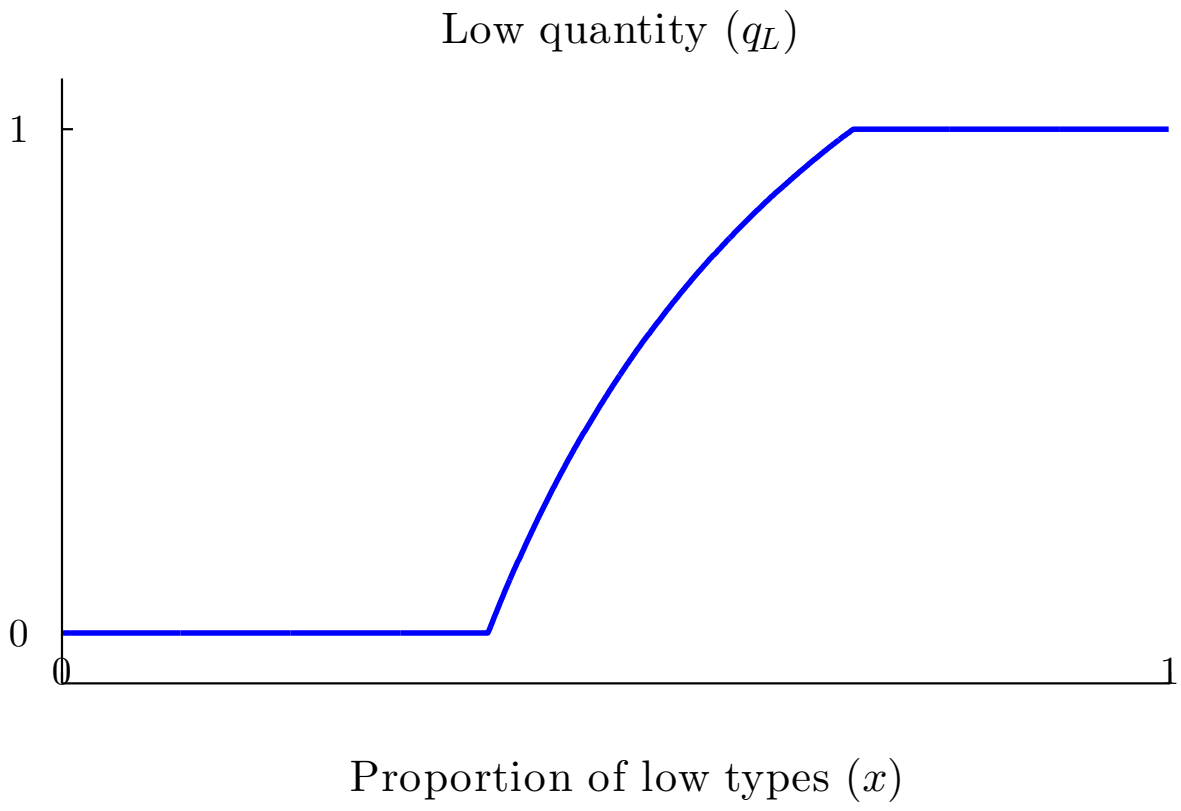
Example

- consumer of type θ get utility

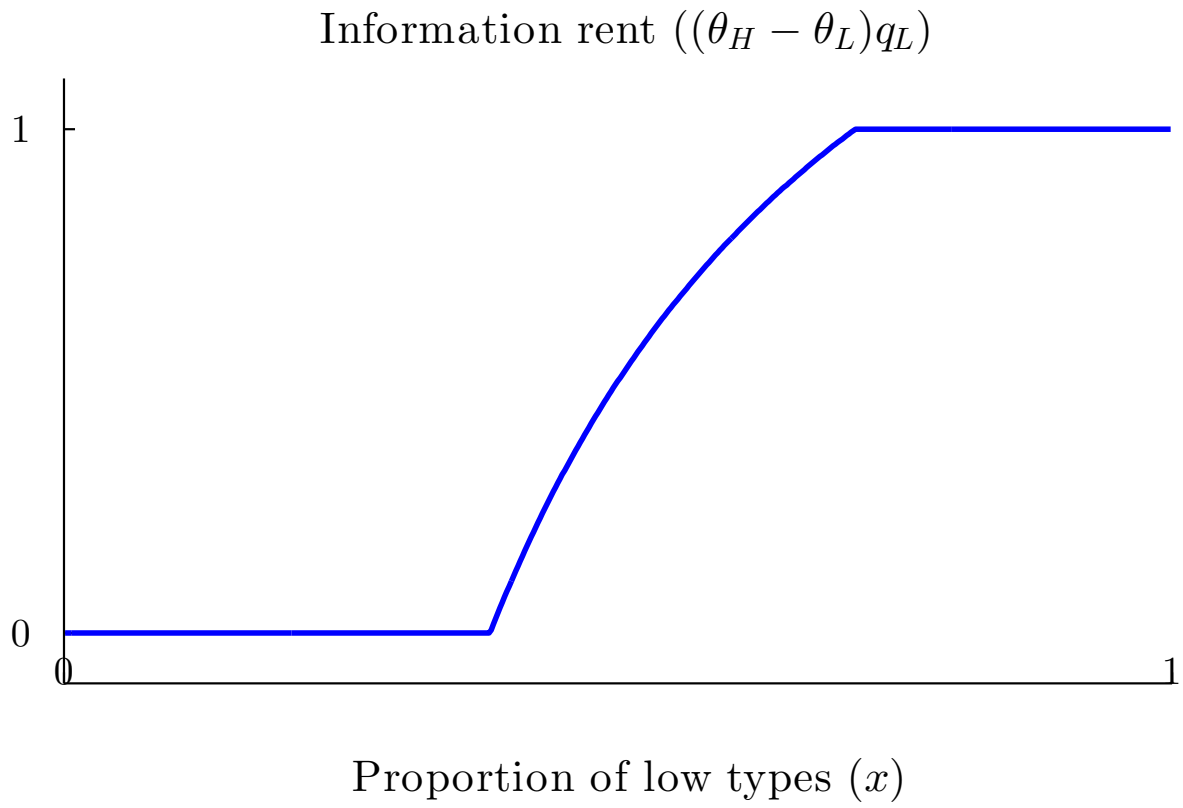
$$u_\theta(q) = min\{v_\theta, v_\theta q + \varepsilon q(1-q)\}$$

- where $v_L < v_H$ and $\varepsilon < v_L$
- single unit demand if $\varepsilon = 0$
- start with case $v_L = 1$, $v_H = 2$, $c = 0$ and $\varepsilon = 0.6$

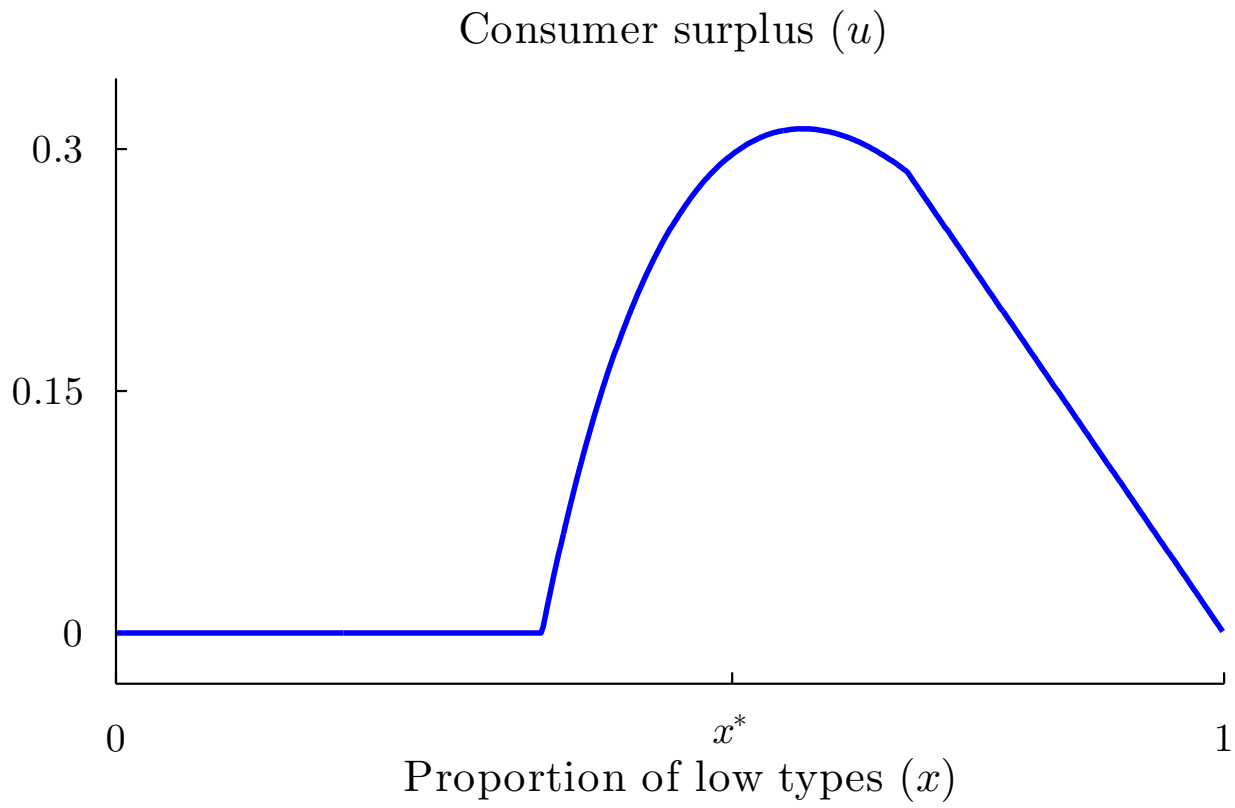
Quantity Distortion



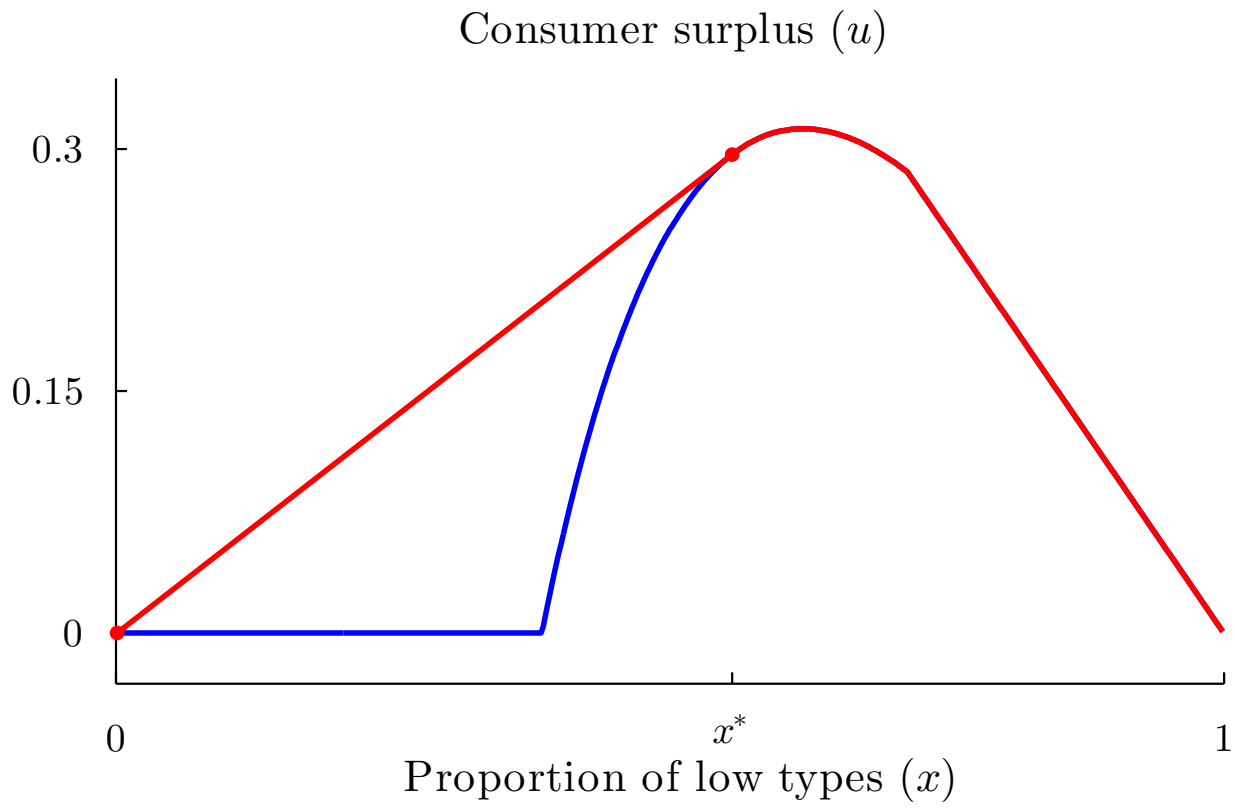
Rent for High Value Consumer



Ex Ante Consumer Surplus



Concavified Ex Ante Consumer Surplus



Optimal Segmentation

- optimal segmentation:
  - if $x \geq x^*$, do not segment. gains from marginal reduction in exclusion and not compensated by inframarginal loss of consumer rent
  - if $x \leq x^*$, segment into markets with all high value consumers are critical mixed market x^*
- Compare Kamenica-Gentzkow 11 “Bayesian Persuasion”

Optimal Segmentation

- by separating some high value consumers, we could increase the proportion of low value consumers in the residual market
- suppose we raised the proportion of low value consumers from x to $x + \varepsilon$
- to do this, we would have to reveal $\varepsilon/(x + \varepsilon)$ of the $1 - x$ high types to be high types
- there is an inframarginal cost of

$$(\varepsilon/(x + \varepsilon))(v_H - v_L)u(q_L(x))$$

- but there is a marginal benefit from reducing rationing of

$$(1 - x - \varepsilon)(v_H - v_L)(u(q_L(x + \varepsilon)) - u(q_L(x)))$$

- Thus incentive to segment at interior q_L if

$$(1 - x)u'(q_L)\frac{dq_L}{dx} \geq \frac{1}{x}u(q_L(x)$$

Approaching Linearity

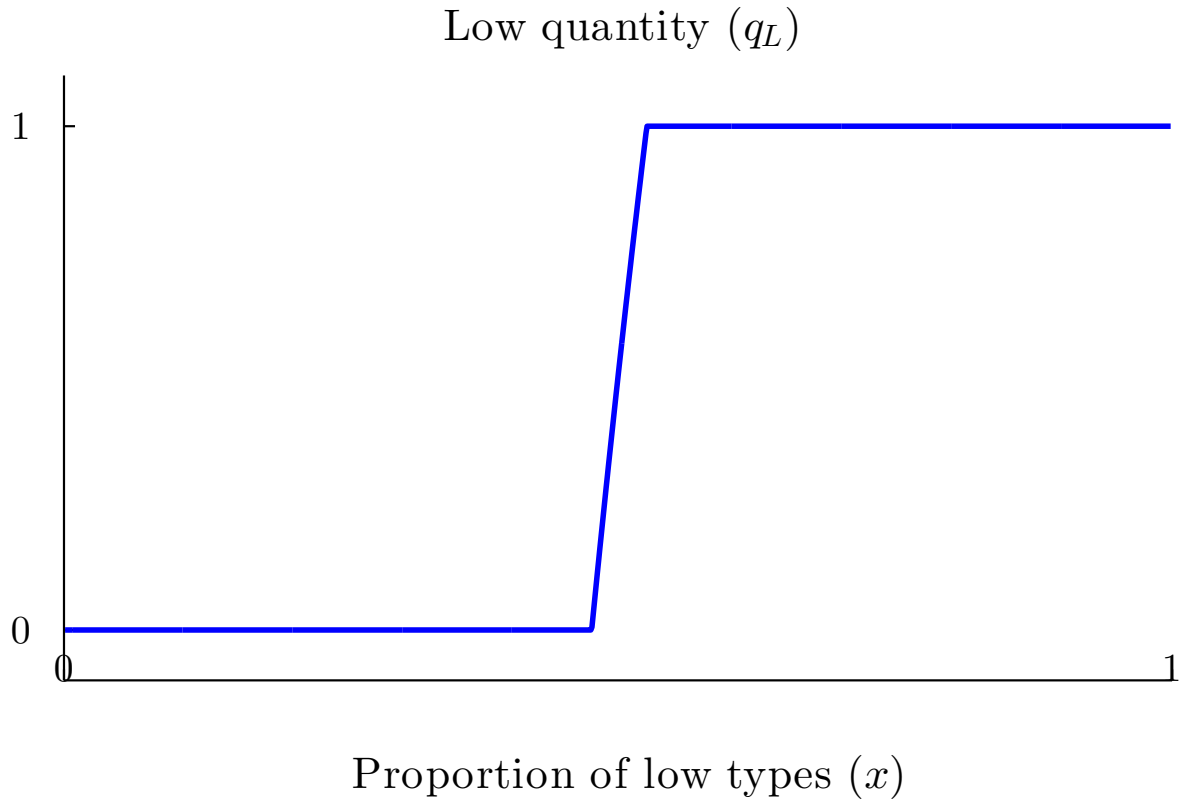
- our parameterized example was carefully chosen to converge nicely to single unit / linear demand: by separating some high value consumers, we could increase the proportion of low value consumers in the residual market:

$$u_\theta(q) = min\{v_\theta, v_\theta q + \varepsilon q(1-q)\}$$

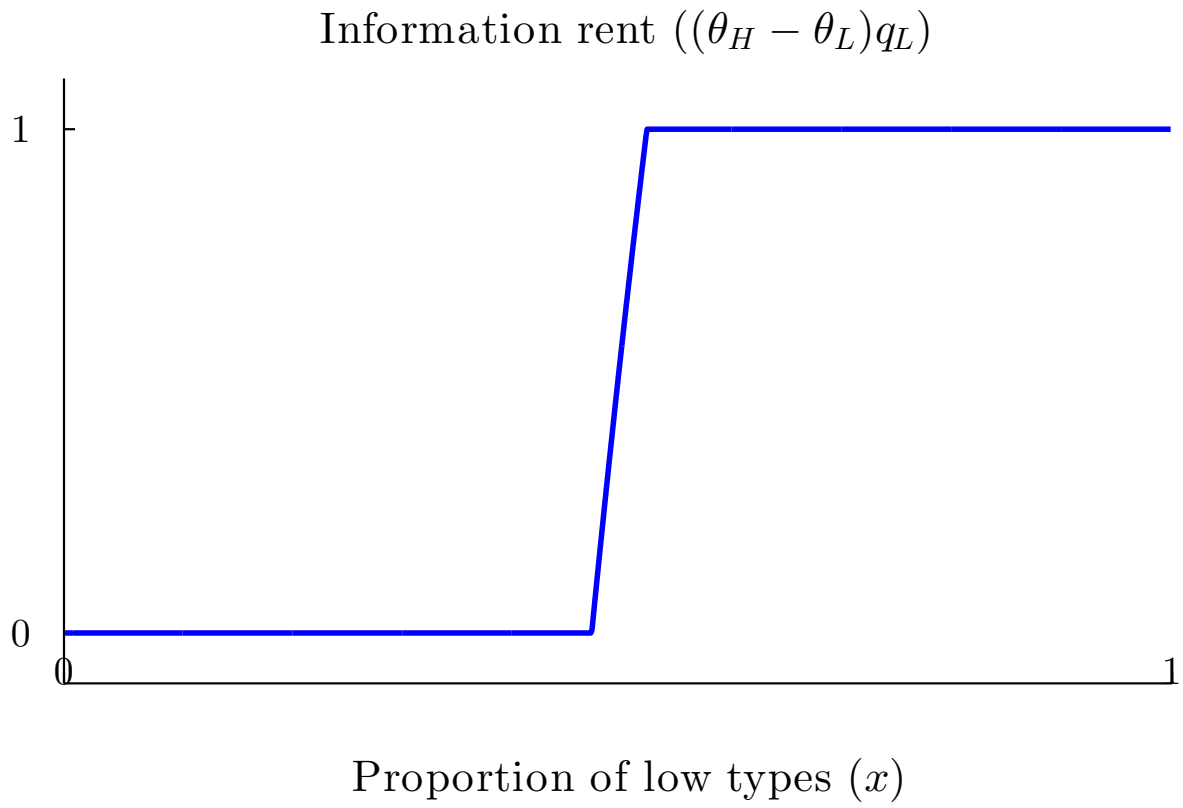
$$\lim_{\varepsilon \to 0} u_\theta(q) = min\{v_\theta, v_\theta q\}$$

- we can see same four pictures for $\varepsilon = 0.1$....

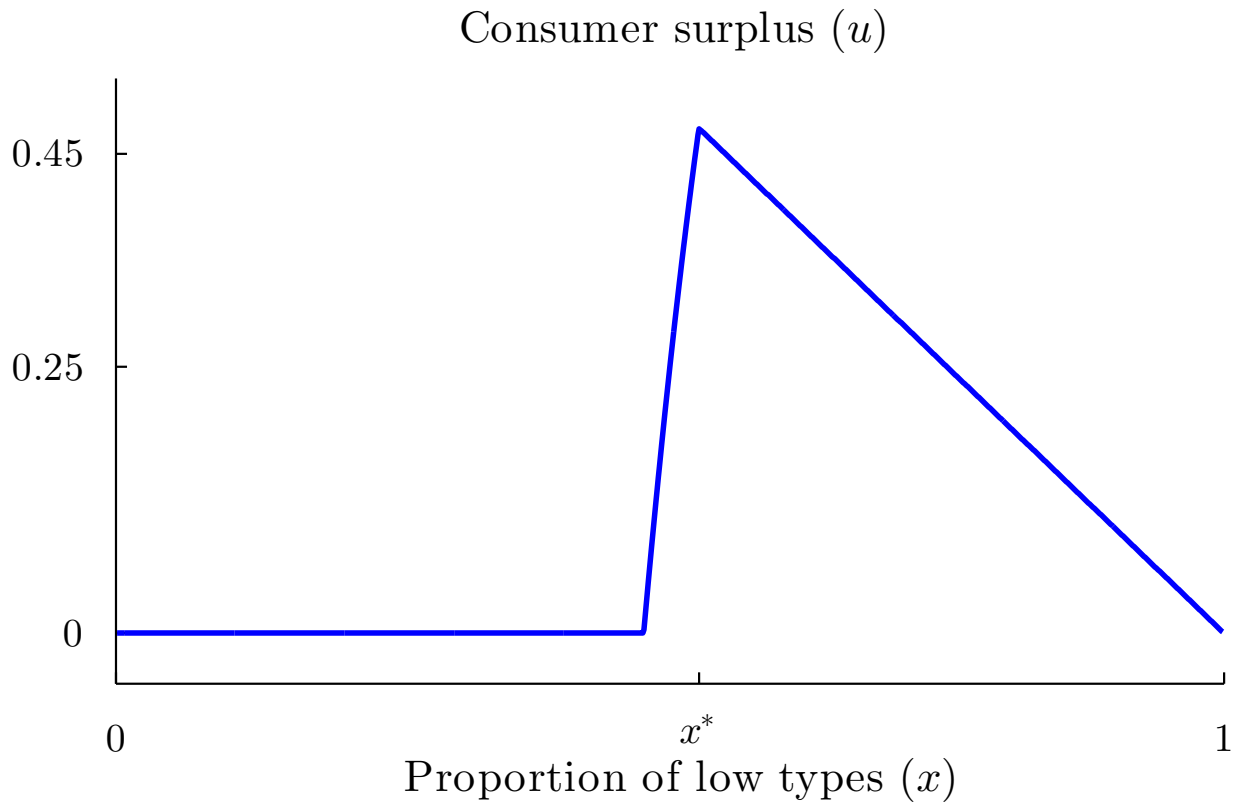
Quantity Distortion with almost linear demand



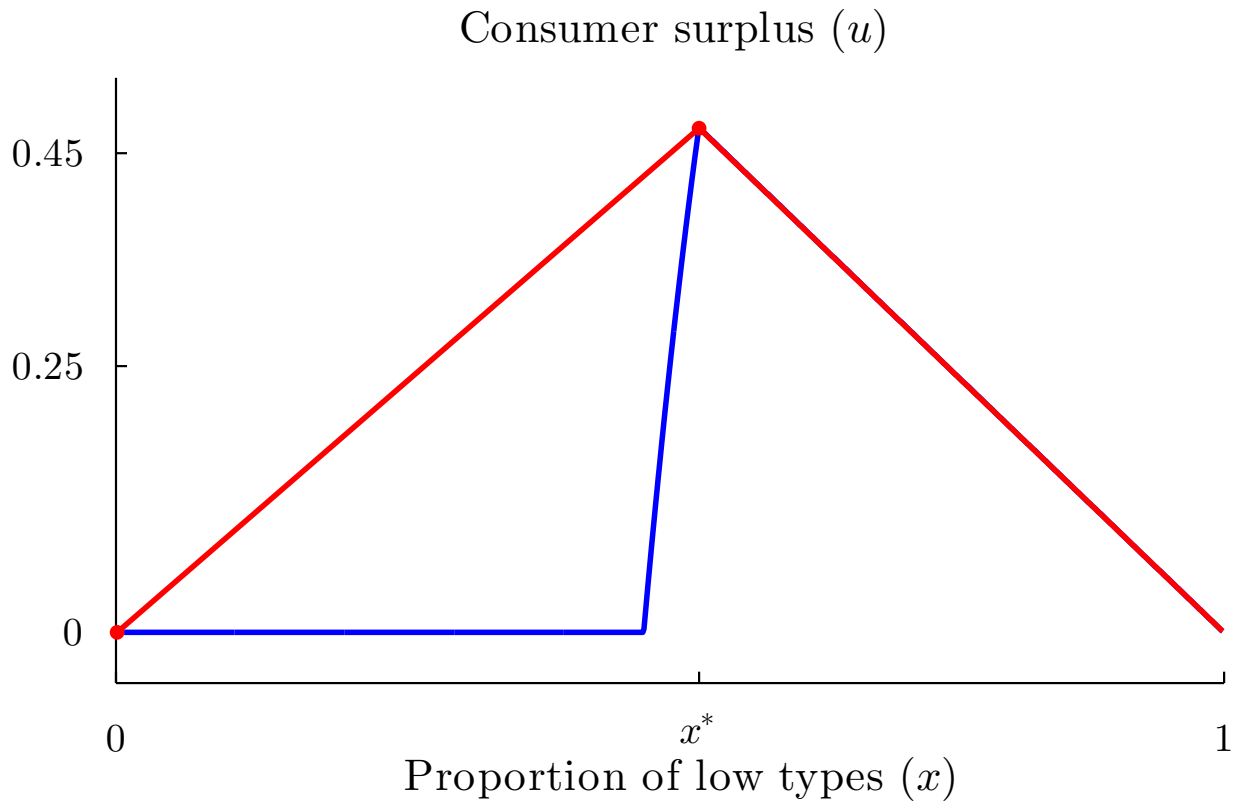
Rent for High Value Consumer with almost linear demand



Ex Ante Consumer Surplus with almost linear demand



Concavified Ex Ante Consumer Surplus with almost linear demand



Optimal Segmentation with almost linear demand

- optimal segmentation
- if $x \geq x^*$, do not segment. gains from marginal reduction in exclusion and not compensated by inframarginal loss of consumer rent
- if $x \leq x^*$, segment into markets with all high value consumers are critical mixed market x^*
- in the limit as $\varepsilon \to 0$, $x^* \simeq 1/2$ and monopolist sells about 1 to almost all consumers above x^*; and almost 0 to almost all consumers below x^*

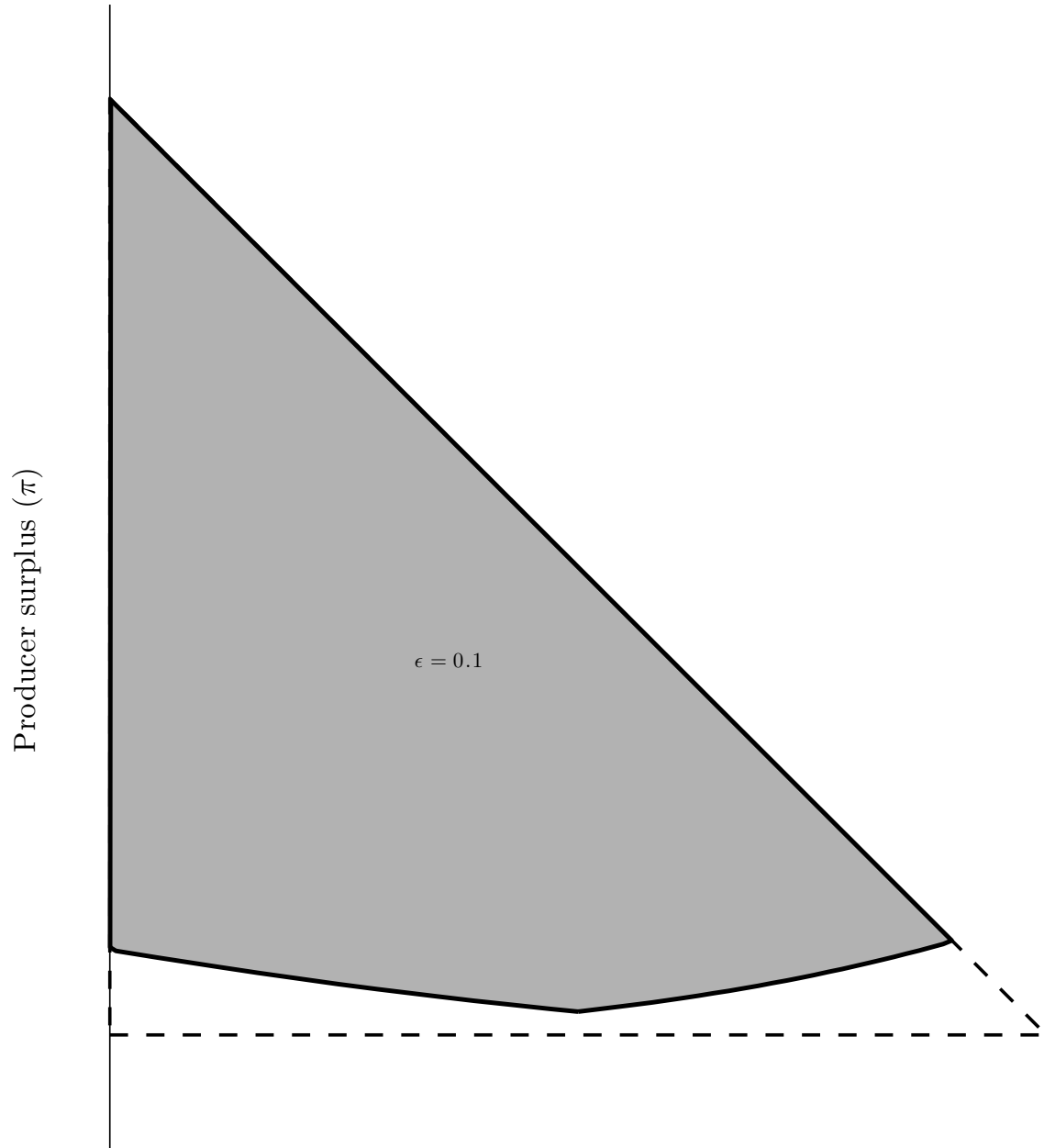
Optimal Segmentation with Single Unit Demand

- optimal segmentation
- if $x \geq 1/2$, sell to high and low consumers at low value
- if $x \leq 1/2$, segment into market with all high types and 50/50 market where low price is charged
- in this (special, linear) case, monopolist gains nothing from segmentation
- Bergemann, Brooks and Morris (2015) show that this observation generalizes to general demand curves to all surplus pairs

Limits to Price Discrimination with Single Unit Demand

- fix an arbitrary demand curve
- three bounds:
- consumer surplus is at least 0
- profits are at least uniform monopolist profits
- sum of consumer surplus and profits at most social surplus

Welfare Triangle Picture



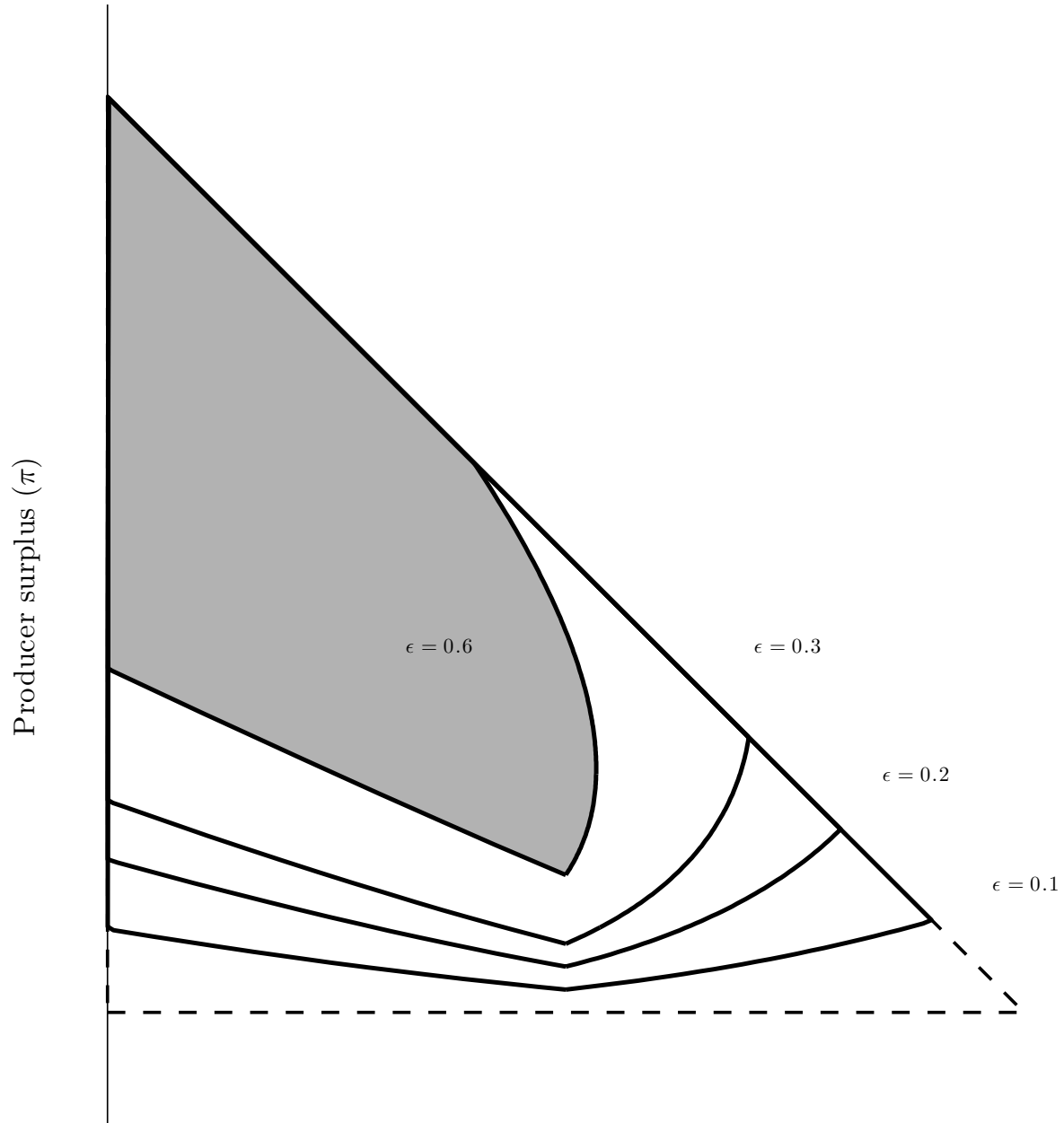
Simplest Argument for Consumer Surplus Result exactly generalizes this two type example

- fix an an arbitrary distribution over finite values (discrete demand curve)
- create a market containing all consumers with lowest value and critical proportion of all remaining consumers such that the monopolist is indifferent between charging the lowest value and the uniform monopoly price
- create a market containing all remaining consumers with the second lowest value and critical proportion of all remaining consumers such that the monopolist is indifferent between charging that second lowest value and the uniform monopoly price
- and so on....

Limits to Price Discrimination with Non-Linear Demand

- result does not extend beyond linear case, even when restricted two types.
- but we can look at the surplus triangle when we let $\varepsilon \to 0$ in example

Almost Welfare Triangle with almost linear demand



What Have We Learnt?

- there is an interesting unexplored interaction between screening (second degree) price discrimination and price discrimination by segmentation (thrid degree)
- we have focussed on what happens to consumer surplus
- as in single unit case, the economic trade-off is between increasing inclusion and maintaining consumer rents
- unlike in the single unit case, this trade-off remains non-trivial even under optimal segmentation
- illustrates tight connection to Kamenica/Gentzkow (2011) concavification; but Bayesian persuasion / information must be metaphorical in this context
- establishes continuity of BBM 2015 result

Paper

- Results about Nonlinear demand case appear in working paper versions of BBM 2015, e,g, Cowles Foundation Discussion Papers 1896, 1896R, 1896RR, 1896RRR and 1896 RRRR
- Most relevant is 1896RR
- May or may not expand into paper
- Slides available at http://www.princeton.edu/smorris/talks