Wage Price Spirals

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When firms and workers disagree on the relative price of labor and goods, they try to outpace each other in setting nominal wages and prices, and inflation follows. This mechanism is at work in a standard new Keynesian model and the degree of disagreement is tied to the distance of aggregate output from its natural level. We look at how different shocks translate into different degrees of inflationary pressure on the good market and on the labor market side of the model. Depending on the relative force of these pressures, real wages can increase or fall. The direction in which the real wage moves is not indicative of how powerful the wage price spiral is. If the economy features a scarce non-labor input, inelastically supplied, with a relatively flexible price, episodes of excess demand are characterized by an initial spike in the input price, followed by persistent price inflation, and by a smaller but more persistent increase in wage inflation. The real wage falls early on and recovers later. In response to a supply shock optimal policy may involve choosing a positive output gap, if it helps relieve negative pressure on nominal wages.

1 Introduction

What is a wage price spiral? In this paper, we use the expression "wage price spiral" to describe a mechanism, present also in standard new Keynesian models, that amplifies the effects of a given inflationary shock.

The basic logic of the mechanism is that workers and firms disagree on the relative price of goods and labor, that is, on the real wage W/P. When firms adjust nominal prices they try to reach a certain ratio W/P, when workers negotiate nominal wages they try to reach a different, higher ratio. The outcome is nominal inflation in both prices and wages. This interpretation of the wage price spiral highlights the presence of a distributional conflict as a proximate cause of inflation.

After defining the wage price spiral in this way, we ask some positive and normative questions. Our analysis focuses on an economy in which there is a scarce non-labor input—which can capture an energy input, like oil or natural gas, but also other primary or intermediate goods that can cause occasional shortages, like lumber or microchips. The non-labor input is the crucial source of supply constraints and supply shocks, which seem important features to model the recent inflation experience in developed economies.

First, we ask whether the direction in which real wages move following a shock tells us something about the strength of the wage price spiral mechanism. We argue that that is not the case. The total force of the wage price spiral, that is, its power to translate a given shock into higher (price and wage) inflation, is different from its relative force on the price and on the wage sides. The direction of the real wage adjustment depends on the spiral's relative force, not on its total force.

Second, we ask whether the direction in which real wages move tells us something about the nature of the shock hitting the economy. In particular, we ask whether a pure aggregate demand shock can cause real wages to decline. We show that this depends on properties of the economy when the shock hits. If the economy is in a state in which the supply of the scarce non-labor input is inelastic and there is limited substitution for that input, then a demand shock can push prices up faster than wages and cause a real wage decline.

We show that the response of the economy to the demand shock described above is qualitatively similar to the response to a pure supply shock in which the input supply is temporarily reduced and the central bank fails to adjust output to its lower natural level. In both cases, there is excess demand in the economy, which translates into a tension between the level of the real wage to which firms and workers aspire, and thus into a wage price spiral.

We then show that these two shocks display a similar pattern of adjustment in prices. The adjustment takes place in three phases. First, there is a bout of high price inflation in the price of the inelastic non-labor inputs, followed by a gradual reduction in the price of these inputs. Second, there is a more persistent period of high good price inflation. Third, there is a smaller, but even more persistent increase in wage inflation. This pattern follows from our assumptions on the relative degree of price stickiness, with the input price being perfectly flexible, and with good prices being more flexible than wages. This pattern implies that at some point wage inflation crosses price inflation, so a period in which real wages fall is followed by a period in which they recover.

We then turn to normative questions and ask what is the optimal policy response to a supply shock coming from the scarce input. In particular, we ask two questions. First, could it be part of optimal policy to "run the economy hot"with have a positive output gap to attain high inflation? Second, could it be part of optimal policy to go further and have inflation in both prices and wages?

Our answer to the first question is affirmative: if the economy needs a lower real wage, it may be more efficient to reach the adjustment with the help of higher price inflation and moderate wage deflation, rather than though lower price inflation and deeper wage deflation. A positive output gap helps shift the adjustment in the direction of price inflation, so is socially beneficial in this manner.

The answer to the second question is also affirmative. We construct examples in which, at some point, along the adjustment path, the output gap is positive and price and wage inflation are both positive. The economic intuition is that this aspect of policy is a form of "forward guidance": by promising to heat up the economy in the future, we speed up the adjustment of the real wage today. Underlying this result is the assumption of forward-looking price- and wage-setting behavior and the commitment of policy. In contrast, when policy has full discretion the equilibrium outcome never features both price and wage inflation.

1.1 Related literature

Our paper makes contact and extends two related ideas: (i) the perspective that inflation is the result of a distributional conflict or disagreement; and (ii) the notion of wage-price spirals, related to the feedback between price and wage inflation and the resulting propagation shocks and persistence or inertia in inflation.

On the first point on distributional conflict origins of inflation an early relevant reference is Rowthorn (1977). This paper provides a model where each periods wages and prices are set in advance; wages are chosen first by workers, then prices are set by firms. Inflation is increasing in the conflict or "aspirational gap". Because of the structure of price and wage setting one requires this conflict anticipated by workers. an equilibrium with rational expectations and finite inflation is not possible in the presence of nonzero conflict. Our model features staggered wage and price setting.

The idea of the wage price spiral as an important element of inflation dynamics has a long history. Blanchard (1986) is a seminal paper on staggered price setting and money non-neutrality which focuses on the way in price setters and wage setters try to outdo each other. In that paper prices are set in even period and wages are set in odd periods, each for two periods a la Taylor; this generates oscillating dynamics for the real wage in response to a permanent money supply shock. Our paper instead builds on the (by now) canonical New Keynesian setting with sticky-price and sticky-wages of the Calvo variety, as developed by Erceg et al. (2000). We study supply and demand shocks under different policy responses.

One distinguishing feature of our approach is to separate the analysis of the price-setting component of the model from the general equilibrium determination of quantities. This approach allows us to identify the proximate forces that determine wage and price adjustment and to characterize the conditions that determine in what direction the real wage adjusts.

Our contribution also focuses on the role of non-labor inputs and to characterize the relative size and persistence of the adjustment of the input price, nominal prices, and wages. Our emphasis on the non-labor input connects our analysis to the analysis of oil shocks in Blanchard and Gali (2007a).[1] An important modeling difference is the assumptions on wage adjustment, we focus on nominal wage rigidities while they consider a form of real-wage rigidity.

On the normative side, our paper is connected to the welfare analysis of alternative policy rules in models where both prices and wages are rigid, going back to the original paper of Erceg et al. (2000) and to the real-rigidity model of Blanchard and Gali (2007b). The starting observation in the literature is that the presence of both price and wage rigidities breaks "divine coincidence" and introduces potentially interesting trade-offs in the response of monetary policy to supply shocks. We offer a complete characterization of optimal policy and explore conditions for the optimum to have a positive output gap in combination with high inflation, as well as cases where it is optimal to have both wage and price inflation.

2 Inflation Mechanics from Distributional Conflicts

Let us begin introducing a simple framework with price setting and wage setting. In the rest of the paper, we give micro foundations and develop a full general equilibrium model.

There is a large number of firms and a large number of workers (a continuum of both). Firms produce goods that are imperfect substitutes and enter symmetrically in the production of aggregate output. Similarly, workers' labor services are imperfect substitutes and enter symmetrically in the production function of each firm. Firms set the nominal price at which they are willing to sell their goods, workers set the nominal wage at which they are willing to sell their labor services. Both firms and workers are only allowed to re-set their price occasionally à la Calvo (1982). Time is continuous and at each point in time firms are selected randomly to re-set their price, with a Poisson intensity λ_p. The same happens for workers, with intensity λ_w. The assumption of staggered, monopolistic price setting for both wages and prices follows Blanchard (1986) and Erceg et al. (2000).

Assume that all workers who can re-set their wage at time t chose the same nominal wage (in logs)

$$w_t^* = p_t + g_t,$$

where p_t is the aggregate good price index. Similarly, assume all firms who can re-set choose

$$p_t^* = w_t - f_t,$$

where w_t is the wage index. For this section, we just take f_t and g_t as exogenously given variables. In the next sections, f_t and g_t will be derived from forward looking decisions in a general equilibrium model. For the sake of interpretation, think for now of myopic

[1] In turn, this connects us to the enormous literature on the effects of oil shocks, going back to Bruno and Sachs (1985).

workers and firms: workers re-set nominal wages to get their desired real wage g_t; firms re-set nominal prices to get their desired price markup over labor costs equal to $-f_t$, so f_t represents the firms' desired real wage.

Calvo assumptions give the differential equations:

$$\dot{w}_t = \lambda_w \left(w_t^* - w_t \right), \tag{1}$$
$$\dot{p}_t = \lambda_p \left(p_t^* - p_t \right). \tag{2}$$

Substituting for p_t^* and w_t^*, denoting price and wage inflation with π_t and π_t^w, and denoting the real wage with $\omega_t \equiv w_t - p_t$, these become

$$\pi_t^w = \lambda_w \left(g_t - \omega_t \right), \tag{3}$$
$$\pi_t = \lambda_p \left(\omega_t - f_t \right). \tag{4}$$

Combining them gives an ODE for the real wage

$$\dot{\omega}_t = \pi_t^w - \pi_t = \lambda_w \left(g_t - \omega_t \right) - \lambda_p \left(\omega_t - f_t \right), \tag{5}$$

which can be solved as

$$\omega_t = e^{-\nu t} \omega_0 + \nu \int_0^t e^{-\nu(t-s)} \left(\frac{\lambda_w}{\lambda_w + \lambda_p} g_s + \frac{\lambda_p}{\lambda_w + \lambda_p} f_s \right) ds,$$

where $\nu = \lambda_w + \lambda_p$. The real wage is a weighted average of its initial value and of workers' and firms' past desired real wages f_t and g_t.[2]

Let us use a bar to denote the following weighted average of past values

$$\bar{f}_t \equiv \nu \int_0^t e^{-\nu(t-s)} f_s ds,$$

and set the initial condition $\omega_0 = 0$, which is just a normalization if we interpret all variables in deviation from ω_0. Then the expression for the real wage can be written compactly as

$$\omega_t = \frac{\lambda_w}{\lambda_w + \lambda_p} \bar{g}_t + \frac{\lambda_p}{\lambda_w + \lambda_p} \bar{f}_t.$$

Substituting in equations (3) and (4) gives the following characterization.

Proposition 1. *Wage and price inflation can be written in terms of the average past distance between the real wage demands of workers and firms $\bar{g}_t - \bar{f}_t$ and of deviations of g_t and f_t from*

[2] As the sum of the weights is indeed

$$e^{-\nu t} + \nu \int_0^t e^{-\nu(t-s)} ds = 1.$$

their average past values:

$$\pi_t^w = \lambda_w (g_t - \bar{g}_t) + \frac{\lambda_w \lambda_p}{\lambda_w + \lambda_p} \left(\bar{g}_t - \bar{f}_t\right),$$
$$\pi_t = -\lambda_p \left(f_t - \bar{f}_t\right) + \frac{\lambda_w \lambda_p}{\lambda_w + \lambda_p} \left(\bar{g}_t - \bar{f}_t\right).$$

If $f_t = g_t$ for all t it is not possible for price and wage inflation to be both positive.

To interpret this result it is useful to start with simple examples with g_t and f_t constant over time: $g_t = g$, $f_t = f$ for all t. Consider the case of no disagreement: $f = g$. In this case, the real wage is $\omega_t = \left(1 - e^{-\nu t}\right) f$ and converges to f. Price and wage inflation converge to zero.

We can actually say more, in this case price and wage inflation are

$$\pi_t = -\lambda_p e^{-\nu t} f,$$

$$\pi_t^w = \lambda_w e^{-\nu t} f,$$

so depending on the sign of f, we can either have positive price inflation and negative wage inflation or the reverse. But we cannot have both price and wage inflation. In absence of disagreement, the real wage tends to adjust to its new equilibrium level with both nominal prices moving in the right direction, e.g., if W_t/P_t needs to fall, W_t will go down and P_t will go up. This is the first version of a more general result that will be derived in Section 6 (Proposition 5).

Conside next what happens when the real wage objectives of workers and firms are inconsistent: $g > f$. Suppose that the workers' desired real wage is in line with its initial value: $g = \omega_0 = 0$. At date 0, price inflation is positive and wage inflation is zero

$$\pi_0 = \lambda_p (\omega_0 - f) = -\lambda_p f > 0, \quad \pi_0^w = \lambda_w (g - \omega_0) = 0.$$

This pushes the real wage down, so, as time passes g and ω_t are no longer equal and workers start asking higher nominal wages because $g > \omega_t$. The initial inflationary pressure on the good market side spills over to the labor market side and we have

$$\pi_t = \lambda_p (\omega_t - f) > 0, \quad \pi_t^w = \lambda_w (g - \omega_t) > 0.$$

It is easy to show that both inflation rates remain positive for all t and that the real wage converges to a weighted average of firms' and workers' demands:

$$\lim_{t \to \infty} \omega_t = \bar{\omega} = \frac{\lambda_w}{\lambda_w + \lambda_p} g + \frac{\lambda_p}{\lambda_w + \lambda_p} f,$$

with $f < \bar{\omega} < g$. Price and wage inflation converge asymptotically to the same level

$$\pi = \pi^w = -\frac{\lambda_w \lambda_p}{\lambda_w + \lambda_p} (f - g) \,. \tag{6}$$

Disagreement on the real wage ends up yielding a stable real wage, in between the desired levels of the two sides. But it also produces constant inflation in the two nominal prices at the numerator and at the denominator. Both parties are unhappy about the real wage $\bar{\omega}$, so price and wage inflation pressures do not go away, they just become balanced on the two sides.

The long run inflation 7 has the following properties:

- it is proportional to the distance $f - g$;
- it is constant-returns-to-scale in λ_w and λ_p: if you double both frequencies of price adjustment inflation doubles;
- the two λs are complementary: if you increase λ_w the effect on inflation is larger if λ_p is larger (and vice versa);
- the role of λ_w and λ_p is entirely symmetric: if you swap one for the other long-run inflation is unchanged.

Let us turn now to a fully developed model. This will allow us to do two things: incorporate forward-looking elements in f_t and g_t, and trace back movements in f_t and g_t to underlying shocks in a general equilibrium model.

3 The Full Model

We now embed the derivations above in a relatively standard New Keynesian model. To capture supply shocks, an important ingredient we include is a scarce input that is used alongside labor for production. We assume this input has with a flexible price, and allow the firms' production function to have elasticity of substitution different from one.[3] One important example is energy but we interpret it more broadly to also capture shortages and bottlenecks in the supply of intermediates like microchips or lumber, which have appeared during the recent post-pandemic recovery.

[3]This is formally equivalent to having labor and capital, with capital renterd at a flexible price, although the interpretation is different. Erceg et al. (2000) have labor and capital, Blanchard and Gali (2007a) have an energy input.

3.1 Setup

The representative household has preferences

$$\int_0^\infty e^{-\rho t}\left(\frac{1}{1-\sigma}C_t^{1-\sigma}-\frac{\Phi_t}{1+\eta}\int_0^1 N_{jt}^{1+\eta}dj\right)dt,$$

where C_t is an aggregate of a continuum of varieties $C_t=\left(\int_0^1 C_{jt}^{1-1/\varepsilon_C}dj\right)^{\frac{1}{1-1/\varepsilon_C}}$, N_{jt} is the supply of specialized labor of type j and Φ_t is a labor supply shock. Each consumption variety j is supplied by a monopolistic firm with the production function

$$Y_{jt}=F\left(L_{jt},X_{jt}\right)\equiv\left(a_L L_{jt}^{\frac{\epsilon-1}{\epsilon}}+a_X X_{jt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}},$$

where L_{jt} is a labor aggregate made of all labor types $L_{jt}=\left(\int_0^1 L_{jkt}^{1-1/\varepsilon_L}dk\right)^{\frac{1}{1-1/\varepsilon_L}}$. Each labor type $k\in[0,1]$ is supplied by a monopolistic union that acts on behalf of the representative household. Integrating over firms the total employment of labor of type k is $N_{kt}=\int_0^1 L_{jkt}dj$. The representative household owns an exogenous endowment X_t of the input X and sells it to the monopolistic good producers on a competitive market at the price P_{Xt}.

Let us focus on characterizing the price and wage setting conditions of firms and unions. As in the simple model of Section 2 firms gets to reset with Poisson rate λ_p and unions get to reset with Poisson rate λ_w. Let P_t^* and W_t^* denote the price set by the firms and unions that can update at time t while P_t and W_t denote the price indexes for the good and labor aggregates.

Start with firms. The nominal marginal cost of producing good j is

$$\frac{W_t}{F_L\left(L_{jt},X_{jt}\right)}=\frac{W_t}{a_L Y_{jt}^{\frac{1}{\epsilon}}L_{jt}^{-\frac{1}{\epsilon}}},$$

or, in log-linear deviations from a steady state with constant quantities and prices

$$w_t-mpl_{jt} \tag{7}$$

where

$$mpl_{jt}=\frac{1}{\epsilon}\left(y_{jt}-l_{jt}\right)$$

is the marginal product of labor. The production function can be written in log-linear approximation

$$y_{jt}=s_L l_{jt}+s_X x_{jt}, \tag{8}$$

where s_L is the labor share and s_X is the share of input X in steady state, with $s_L+s_X=1$. All firms being price takers in the input market, they all hire labor and the input X using the same ratio L_{jt}/X_{jt} that satisfies the optimality condition

$$\frac{W_t}{P_{Xt}} = \frac{F_L\left(L_{jt}, X_{jt}\right)}{F_X\left(L_{jt}, X_{jt}\right)} = \frac{a_L}{a_X}\left(\frac{L_{jt}}{X_{jt}}\right)^{-\frac{1}{\epsilon}}.$$

Therefore, $l_{it} - x_{it} = l_t - x_t$ and the marginal product of labor is equalized across firms and satisfies

$$mpl_t = \frac{1}{\epsilon}\left(y_t - l_t\right) = \frac{s_X}{\epsilon}\left(x_t - l_t\right). \tag{9}$$

Also, using the optimality condition above, the price of the X input can then be written as

$$p_{Xt} = w_t - \frac{1}{\epsilon}\left(x_t - l_t\right).$$

3.2 Endogenous Price and Wage Resetting

Optimal price setting implies that firms set their price at time t as an average of future nominal marginal costs, conditional on not resetting. Since nominal marginal costs are given in (7), we get

$$p_t^* = \left(\rho + \lambda_p\right)\int_t^\infty e^{-\left(\rho+\lambda_p\right)(\tau-t)}\left(w_\tau - mpl_\tau\right)d\tau. \tag{10}$$

Similarly, we can derive the wage setting equation

$$w_t^* = \left(\rho + \lambda_w\right)\int_t^\infty e^{-\left(\rho+\lambda_w\right)(\tau-t)}\left(p_\tau + mrs_{\tau,t}\right)d\tau \tag{11}$$

where

$$mrs_{\tau,t} = \phi_\tau + \sigma y_\tau + \eta\left[n_\tau + \varepsilon_L\left(w_\tau - w_t^*\right)\right]$$

is the marginal rate of substitution between consumption and leisure at time τ for workers who reset their price at time $t \leq \tau$ (since their labor supply is $n_\tau + \varepsilon_L\left(w_\tau - w_t^*\right)$).[4]

The two equations capture the logic of the wage price spiral. Price setters aim to get a certain price-to-wage ratio in current and future periods, so they set their nominal price to catch up with current and anticipated future nominal wages. Symmetrically, wage setters aim to get a certain wage-to-price ratio and aim to catch up with current and future nominal prices.

[4] The model is consistent with the model of Section 2 if we define f_t as a forward looking expression capturing anticipated wage inflation and anticipated values of mpl

$$f_t = \left(\rho + \lambda_p\right)\int_t^\infty e^{-\left(\rho+\lambda_p\right)(\tau-t)}\left(\left(w_\tau - w_t\right) - mpl_\tau\right)d\tau,$$

and we define g_t in a symmetric manner.

The objectives of price setters and wage setters are captured, respectively, by mpl and mrs. In a flexible price economy, the two equations above would boil down to $p_t - w_t = mpl_t$ and $w_t - p_t = mrs_t$, which then requires necessarily $mpl_t = mrs_t$. In a flexible price economy the aspirations of firms and workers for the relative price of goods and labor must be consistent with each other. In a sticky price economy, instead, these aspirations can be inconsistent and, depending on shocks and on policy responses, the economy can feature $mpl_t \neq mrs_t$. When that happens, workers and firms try to set nominal prices to reach their desired relative price and, since these desired relative prices are inconsistent, the result is inflation, as in Section 2.

To go from equations (10) and (11) to wage and price inflation, let us combine them with the differential equations for p_t and w_t , (1) and (1). As shown in the appendix, this leads to the following expressions for price and wage inflation

$$\rho \pi_t = \Lambda_p \left(\omega_t - mpl_t \right) + \dot{\pi}_t, \tag{12}$$

$$\rho \pi_t^w = \Lambda_w \left(mrs_t - \omega_t \right) + \dot{\pi}_t^w, \tag{13}$$

where ω_t is the real wage and

$$mrs_t = \phi_t + \sigma y_t + \eta n_t \tag{14}$$

is the cross-sectional average of the marginal rate of substitution between consumption and leisure. The coefficients Λ_p and Λ_w reflect how fast prices and wages adjust in response to deviations of the real wage from, respectively, mpl_t and mrs_t, and their values are[5]

$$\Lambda_p \equiv \lambda_p \left(\rho + \lambda_p \right), \quad \Lambda_w = \lambda_w \frac{\rho + \lambda_w}{1 + \eta \varepsilon_L}.$$

Solving forward, these equations give price and wage inflation as functions of the path of mpl_t and mrs_t and of the real wage ω_t:

$$\pi_t = \Lambda_p \int_t^\infty e^{-\rho(s-t)} \left(\omega_s - mpl_s \right) ds, \tag{15}$$

$$\pi_t^w = \Lambda_w \int_t^\infty e^{-\rho(s-t)} \left(mrs_s - \omega_s \right) ds. \tag{16}$$

These equations show how the disagreement $mpl \neq mrs$ causes inflationary tensions. The first observation is that when $mpl \neq mrs$ there is no value of the real wage ω that can ensure that at the same time $\omega_t - mpl_t$ and $mrs_t - \omega_t$ are both zero. The second observation is that, due to price and wage stickiness the real wage ω_t will adjust gradually

[5] The presence of the term $1/\left(1 + \eta \varepsilon_L\right)$ in Λ_w reflects stragegic complementarity in wage setting. The model does not feature strategic complementarity in price setting, due to the assumption of constant returns to scale and a frictionless market for the X input, but it is easy to allow for strategic complementarity by introducing an additional firm-specific fixed factor.

through the equation

$$\dot{\omega}_t = \pi_t^w - \pi_t. \tag{17}$$

So there is a feedback between the tensions in $\omega_t - mpl_t$ and $mrs_t - \omega_t$ and the value of ω_t.

As an aside, notice that the simplest new Keynesian framework with flexible wages is a special case of our enviroment with $\lambda_w \to \infty$ and it also features a price wage spiral. With flexible wages the second equation becomes $\omega_t = mrs_t$, so price inflation is directly driven by the discrepancy $mrs_t - mpl_t$.

In the next section, we use the three conditions above to analyze the model predictions conditional on given paths of mpl_t and mrs_t, solving for the real wage ω_t. In the following sections, we go back to the full model and to the underlying shocks that determine mpl_t and mrs_t in general equilibrium.

4 Dynamics of Inflation and the Real Wage

We now characterize the equilibrium path of wages and prices for given paths of mpl_t and mrs_t. This allows us to formalize the idea that the direction in which the real wage moves is a symptom of relative forces on the demand and supply side of the labor market.

Combining equations (12) to (17) gives a second order ODE for the real wage ω_t

$$\ddot{\omega}_t = \rho\dot{\omega}_t + \left(\Lambda_p + \Lambda_w\right)\omega_t - \Lambda_p mpl_t - \Lambda_w mrs_t. \tag{18}$$

Notice that this equation plays a similar role as the ODE (5) in the simple analysis of Section 2. However, the explicit modeling of forward looking decisions and rational expectations implies that instead of a backward-looking first order ODE, we now have a second order ODE that must be solved selecting solutions that are stable in the long run.

The next proposition solves the ODE and provides an analytical characterization of ω_t.

Proposition 2. *The real wage satisfies the first order ODE*

$$\dot{\omega}_t = r_1\omega_t + \int_t^\infty e^{-r_2(s-t)}\left[\Lambda_p mpl_s + \Lambda_w mrs_s\right] ds, \tag{19}$$

where $r_1$and r_2 are the roots of the quadratic equation

$$r\left(r-\rho\right) = \Lambda_p + \Lambda_w,$$

and satisfy $r_1 < 0 < \rho < r_2$. Solving (19) gives the real wage as a function of $\{mpl_t\}_{t=0}^\infty$, $\{mrs_t\}_{t=0}^\infty$ and the initial condition ω_0

$$\omega_t = e^{r_1 t}\omega_0 + \int_0^\infty H_{t,s}\left(\Lambda_p mpl_s + \Lambda_w mrs_s\right) ds.$$

where $H_{t,s} = \left(\min\left\{e^{r_1(t-s)}, e^{r_2(t-s)}\right\} - e^{r_1 t - r_2 s}\right) / (r_2 - r_1)$.

The second term in (19) shows that real wage dynamics are driven by current and anticipated pressures on the two sides of the labor market: the real wage increases when either labor demand is high (high mpl) or labor supply is low (high mrs).

The first term in (19) shows that the real wage tends to mean revert, due to $r_1 < 0$. The reason is that a high real wage increases firms' marginal cost $\omega - mpl$, increasing price inflation, and, at the same time, reduces workers' marginal cost of labor supply $mrs - \omega$, reducing wage inflation. Both forces reduce the real wage.

The labor market pressures and the mean reversion force captured in (19) shape the real wage response to different shocks, as we can see in some simple examples that can be analyzed using a phase diagram.

4.1 Phase diagrams

Suppose the economy is in steady state and at date 0 we have a permanent reduction of mpl from zero to a constant value $\overline{mpl} < 0$, while stays mrs constant at zero.

The dynamics of ω are illustrated in the phase diagram of Figure 1. The stationary locus $\dot{\omega} = 0$ coincides with the x axis. The stationary locus $\ddot{\omega} = 0$ is downward sloping, from (18). They are both drawn in purple. The saddle path in blue comes from 19, which in this example becomes

$$\dot{\omega}_t = r_1 \omega_t + \frac{1}{r_2} \Lambda_p \overline{mpl}.$$

Setting $\dot{\omega} = 0$ and using $-r_1 r_2 = \Lambda_p + \Lambda_w$, the new steady state for ω is

$$\overline{\omega} = \frac{\Lambda_p}{\Lambda_p + \Lambda_w} \overline{mpl}. \tag{20}$$

Therefore, in this example the wage falls from $\omega_0 = 0$, along the saddle path, asymptoting at $\overline{\omega}$.

What is driving down real wages along the path towards $\overline{\omega}$?

To understand the intuition it is useful to go back to price and wage inflation, recalling equations (15) and (16) above. Firms face higher marginal costs due to the lower mpl. The anticipation of lower real wages in future periods partly dampens this force, because marginal costs are $\omega - mpl$. However, the net effect remains positive because real wages, as shown in Figure 1, are always higher than $\overline{\omega}$, which, in turn, is higher than $\overline{mpl}$ from

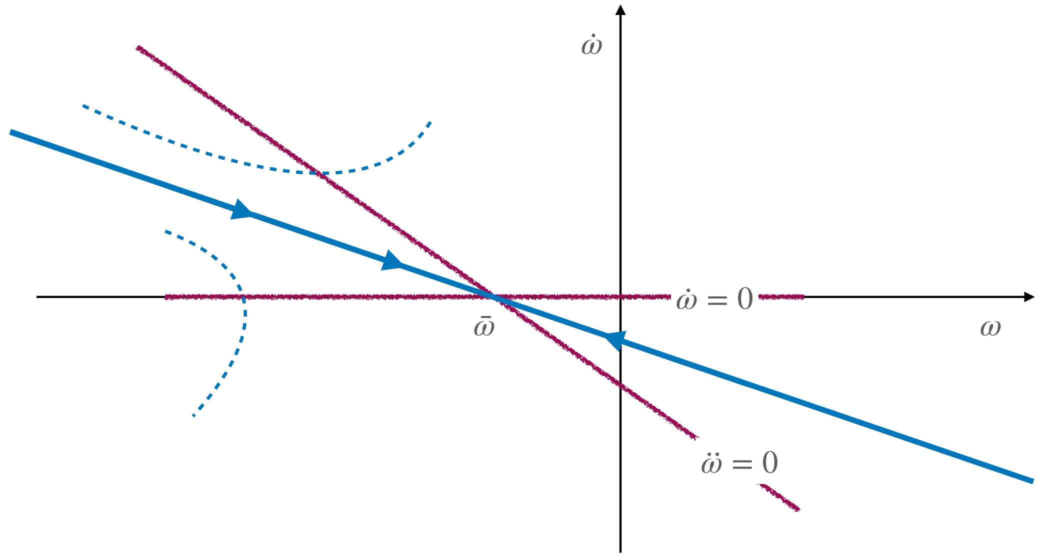

Figure 1: A permanent shock

(20). Therefore, the expression $\omega - mpl$ is positive at all dates and decreasing over time. Inflation is then always positive, decreases over time, and converges to

$$\lim_{t\to\infty} \pi_t = \frac{\Lambda_p}{\rho}\left(\overline{\omega} - \overline{mpl}\right) = -\frac{1}{\rho}\frac{\Lambda_p\Lambda_w}{\Lambda_p + \Lambda_w}\overline{mpl}. \tag{21}$$

Wage inflation is also positive, because workers try to make up for lower real wages in the future by trying to bid up their nominal wages. However, it can be shown that wage inflation is lower than price inflation. Wage inflation increases over time and converges to $\Lambda_w\overline{\omega}/\rho$, which is equal to the long run inflation rate (21).[6]

The interpretation is similar to Section 4. Due to the initial shift in mpl, firms and workers disagree on where the real wage should be: workers aim for $\omega = mrs = 0$, firms aim for $\omega = mpl < 0$. Price and wage inflation do not resolve this tension. The decline in the real wage reallocates the tension from the firms' side to the workers' side, until, asymptotically, the tension is balanced, wage and price inflation are equal, and the real wage converges to $\overline{\omega}$. The level $\overline{\omega}$ is a weighted average of the objectives of the two parties, with weights that depend on the speeds Λ_p and Λ_w at which price and wage

[6]Notice that if we make agents myopic by sending $\rho \to \infty$ and we set $\eta = 0$, we get an asymptotic real wage equal to

$$\lim_{\rho\to\infty} \frac{\Lambda_p}{\Lambda_p + \Lambda_w}\overline{mpl} = \frac{\lambda_p}{\lambda_p + \lambda_w}\overline{mpl}$$

and an asymptotic inflation rate equal to

$$\lim_{\rho\to\infty} -\frac{1}{\rho}\frac{\Lambda_p\Lambda_w}{\Lambda_p + \Lambda_w}\overline{mpl} = -\frac{\lambda_p\lambda_w}{\lambda_p + \lambda_w}\overline{mpl}$$

consistently with the expressions derived in Section 4.

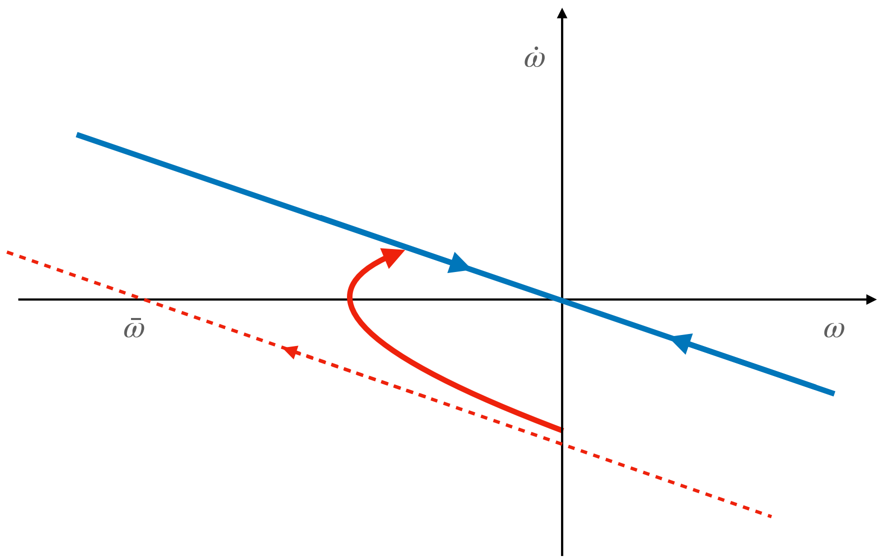

Figure 2: A transitory shock

inflation respond to deviations of mpl and mrs from ω.

An example featuring a permanent gap between mpl and mrs is useful but extreme. If calibrated with a realistically low value of ρ, such an example yields very large levels of wage and price inflation for a given shock $\overline{mpl}$. This is just a reflection of the fact that the long-run new Keynesian Phillips curve is very steep. Fortunately, a temporary change can also be analyzed using our phase diagram.

Consider an economy in steady state with all variables at 0. At $t = 0$, unexpectedly, firms realize that for a finite time interval $[0, T]$ they will face $\overline{mpl} < 0$. At T, mpl goes back to zero. The value of mrs remains at zero throughout.

The dynamics of ω following the shock are illustrated in Figure 2. First, the economy follows the red solid line, until that line meets the blue solid line at time T, then the economy follows the blue saddle path asymptoting back to the origin. The real wage first falls towards $\overline{\omega}$; at some point, before T, the real wage starts growing again, due to the increased strength of the mean-reverting force; finally, after the impulse to mpl is gone, the real wage converges back to zero.

The intuition for these dynamics is closely related to the case of a permanent shift. Figure 3 shows the responses of π, π_w and ω in a numerical example. In the interval $[0, T]$ real wages are below 0, but less so than productivity mpl. Therefore the marginal cost $\omega - mpl$ is positive in the interval $[0, T]$, driving up price inflation. Lower real wages are below the workers mrs, which is zero throughout, driving up wage inflation. The initial force on the price side is stronger, which is consistent with real wages falling. At some point, wage and price inflation cross and real wages start growing. This happens because as T approaches firms anticipate lower marginal costs, due to $\omega - mpl < 0$ after T. At the same time the force on the wage side remains positive throughout.

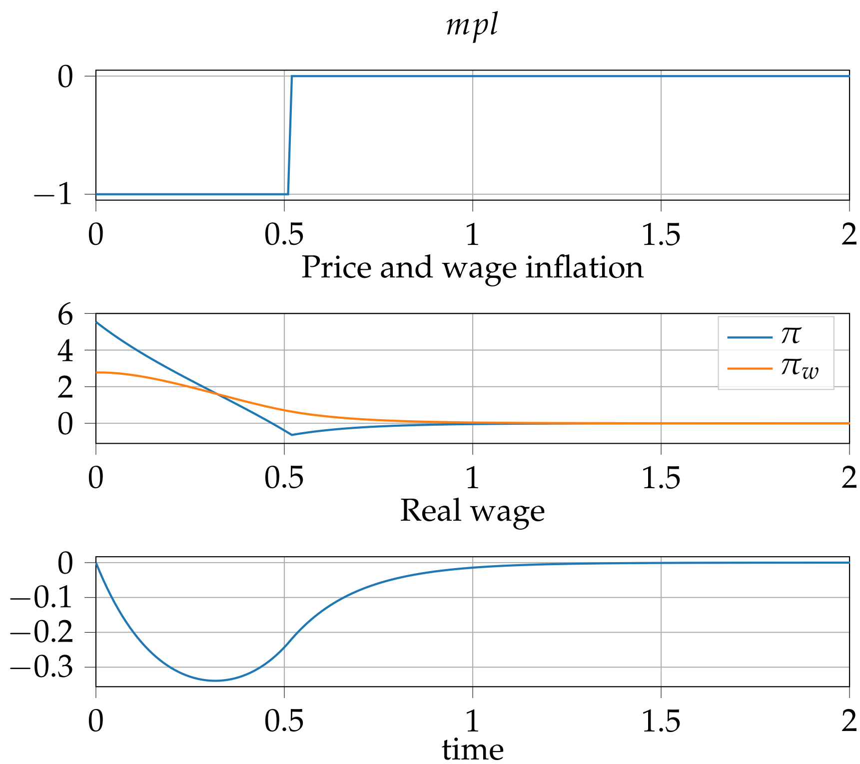

Figure 3: A temporary shift in mpl

As in the analysis of Section 2, the model cannot produce inertia in both price and wage inflation. After the shock disappears at date T, wage inflation remains positive but we have price deflation, as the real wage is low while the marginal product of labor is back at 0. This lack of inertia is the corollary of a more general result that we derive below, in Proposition 5.

Proposition 6 in the appendix provides formal derivations for a general class of experiments like the two just analyzed, in which only one side of the labor market is affected, that is, where only mpl or only mrs deviate from zero.

In most relevant cases, the underlying economic shocks change both mpl and mrs at the same time. In that case, the shape of the responses on the two sides can produce a variety of behaviors. In the next section, we focus on paths for mpl and mrs that decay exponentially over time.

5 Total and Relative Effects of the Wage Price Spiral

Building on the results of the previous section, we now turn to distinguishing the total effect of the wage price spiral mechanism on (wage and price) inflation, from its relative effect (wage vs price inflation).

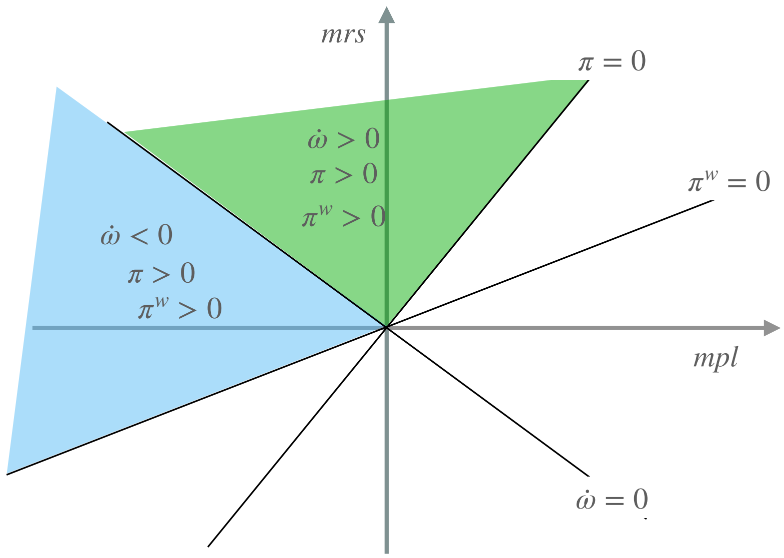

Figure 4: Regions for mpl_0 and mrs_0

Let's focus on shocks that produce exponentially decaying paths for mpl and mrs. The economy is in steady state before time 0. At time 0, there is an unexpected shock and from then on the paths of mpl and mrs are

$$mpl_t = mpl_0 e^{-\delta t}, \quad mrs_t = mrs_0 e^{-\delta t},$$

where δ is the speed at which the shock dies out.

Define the coefficients

$$\psi = \frac{r_2}{r_2 + \delta} \frac{-r_1}{-r_1 + \rho}, \quad \kappa = \frac{\Lambda_p}{\Lambda_p + \Lambda_w},$$

which are both in $(0, 1)$.

Proposition 3. *Given exponentially decaying paths for mpl and mrs, the effects on price and wage inflation at $t = 0$ are*

$$\dot\omega_0 > 0 \text{ iff } \Lambda_p \cdot mpl_0 + \Lambda_w \cdot mrs_0 > 0;$$
$$\pi_0 > 0 \text{ iff } mrs_0 > \frac{1 - \psi\kappa}{\psi(1-\kappa)} mpl_0;$$
$$\pi_0^w > 0 \text{ iff } mrs_0 > \frac{\psi\kappa}{1 - \psi(1-\kappa)} mpl_0;$$

Where $\frac{1-\psi\kappa}{\psi(1-\kappa)} > 1 > \frac{\psi\kappa}{1-\psi(1-\kappa)}$.

The proposition is illustrated in Figure 4, where we drop the time subscript 0 for readability. The green and blue regions are those in which the economy features both price and

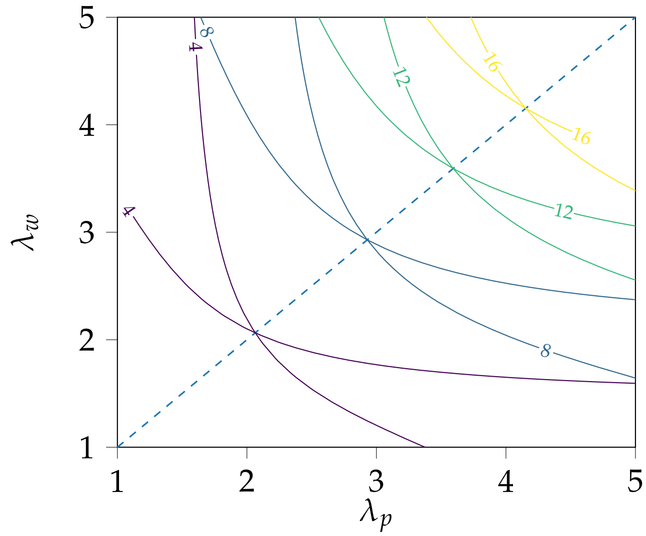

Figure 5: Price and wage inflation contours for different degrees of stickiness

wage inflation. Both $mrs_0 > 0$ and $mpl_0 < 0$ are inflationary forces, and we get inflation as long as one of them is present and strong enough. In particular, $mrs_0 > 0$ acts directly on workers' wage demands, $mpl_0 < 0$ acts directly on firms' price demands. Both also act indirectly. A higher mrs_0, by pushing up real wages tends to increase marginal costs and push up price inflation. A low mpl_0, by pushing real wages down, tends to increase wage demands and wage inflation. The fact that mrs acts directly on wages, while mpl acts directly on prices gives some intuition for why the slope of the $\pi = 0$ line is steeper than that of the $\pi^w = 0$ line. The difference between the green region and the blue region is that in the blue region the real wage declines at $t = 0$ while it increases in the green region. The reason for the difference is the relative strength of the pressure on price setters and wage setters.

Let us now do a different exercise: fix the size of the initial shocks $mrs_0 > 0$ and $mpl_0 < 0$ and change the economy's parameters to vary the degree by which the shocks get amplified. In particular, let us change the parameters λ_p and λ_w. As we increase the speed at which wages and prices are reset, the wage price spiral mechanism gets stronger. This is shown in Figure 5, where we plot level curves for π and π_w. The relatively steeper curves (in absolute value) correspond to π, the flatter ones to π_w. A higher frequency of price adjustment λ_p increases both π and π_w, but has a stronger effect on the former. The reverse holds for λ_w. For ease of illustration, we choose an economy with $\eta = 0$, hit by a symmetric shock $mrs_0 = -mpl_0$. This implies that $\lambda_p = \lambda_w$ implies $\Lambda_p = \Lambda_w$ and, from Proposition 3, it implies $\dot{\omega}_0 = 0$, which means that nominal price and wage inflation are equal, $\pi_0 = \pi_{w0}$. This is confirmed in the figure, where the contour levels corresponding to equal price and wage inflation meet on the 45 degree line.

Increasing either price or wage flexibility increases *both* price and wage inflation. This is what we call the total force of the wage price mechanism. At the same time, what happens to the real wage depends on the relative force on the two sides. Increasing λ_p tends to move us to the region below the 45 degree line, where real wages fall. Increasing λ_w has the opposite effect. This is what we mean by the relative power of the mechanism.

6 Demand and Supply Shocks

We now go back to the full model and trace back price and wage inflation to the general equilibrium effect of underlying shocks.

We focus on two shocks. First an expansionary demand shock, driven by easy monetary policy (easy fiscal policy would have similar implictions).

A common view is that excessive demand would work its way from a tight labor market, to higher wages, to higher prices. Following this intuition a pure demand shock should manifest itself in increasing real wages. We show that in our model general equilibrium forces are at work on both sides of the labor market and that the direction of adjustment of the real wage is in general ambiguous. This is especially true when the scarce, inelastic input X plays an important role.

Consider a monetary shock that leads to a temporary increase in employment $n_0 > 0$ on impact, the shock decays exponentially at rate δ, so

$$n_t > n_0 e^{-\delta t}.$$

The responses of mpl_t and mrs_t are easily derived from (9) and (14):

$$mpl_t = -\frac{s_X}{\epsilon} e^{-\delta t} n_0, \quad mrs_t = (\sigma s_L + \eta)\, e^{-\delta t} n_0.$$

Substituting in the conditions of Proposition 3 shows that price and wage inflation are both positive following the shock. What happens to the real wage, though, is in general ambiguous. The following is an immediate corollary of Proposition 3.

Proposition 4. *In response to a monetary shock that leads to a transitory increase in employment, real wages fall on impact if and only if*

$$\Lambda_p \frac{s_X}{\epsilon} > \Lambda_w (\sigma s_L + \eta)\,.$$

The left-hand side of the inequality captures the direct effect on price inflation. This term depends on the effect of higher employment on marginal costs and on stickiness in price setting, captured by Λ_p. The effect of employment on marginal costs is larger when the scarce input X is more important in the production of the final good (higher share s_X) and when the elasticity of substitution between labor and X is lower. The term on the right-hand side captures direct effects on wage inflation. This term depends on the effect on the marginal rate of substitution and on stickiness in wage setting, captured by Λ_w. The

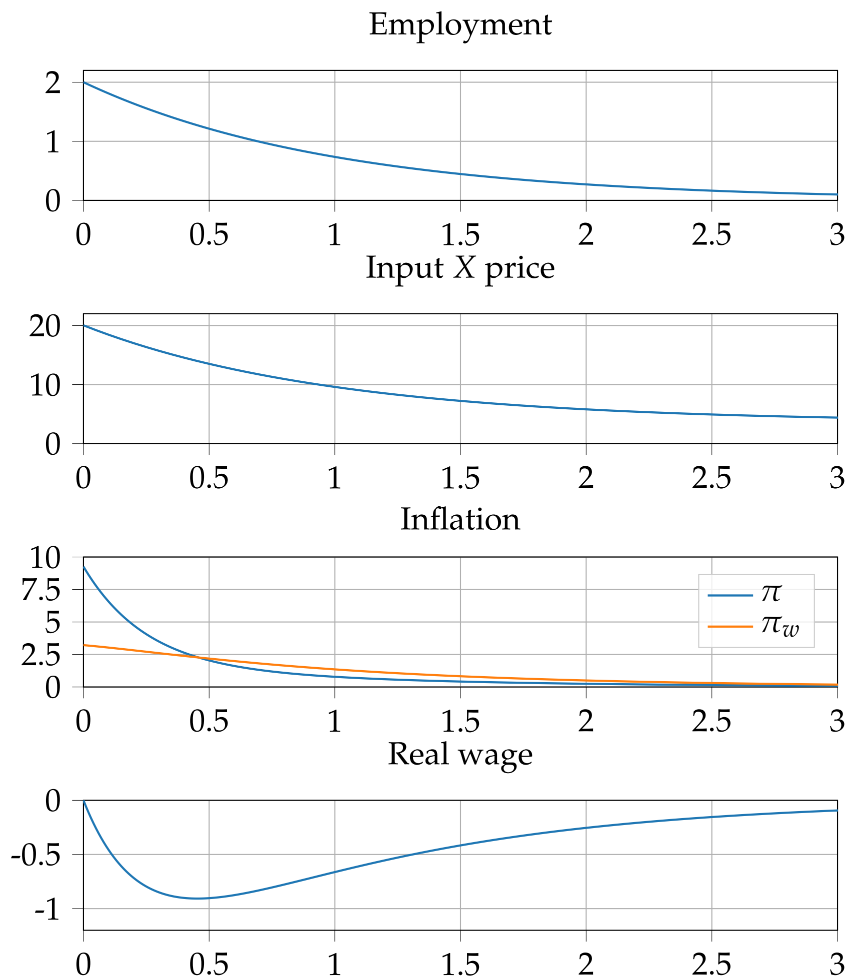

Figure 6: A supply-constrained demand shock

effect on the marginal rate of substitution, in turn, depends on an income effect, captured by the term σs_L, since s_L is the elasticity of output to the labor input, and on the inverse Frisch elasticity η.

Overall, if the effect on firms' marginal costs is relatively stronger than the effects on workers' marginal rate of subsitution and if prices are relatively more flexible than wages, we get a reduction in real wages.

In Figure 6 we plot the response to a temporary expansionary shock that increases n above its potential level by 2%, with a decay $\delta = 1$ in a simple numerical example.[7] The parameters used are in the Table 1.

The first panel shows the shock to n. The remaining panels show the responses of different prices.

[7]All plots show log deviations from steady state times 100, or, approximately, percentage deviations from steady state.

Preferences	$\sigma = 1$	$\eta = 0$	$\rho = 0.05$
Technology	$s_X = 0.1$	$\epsilon = 0.1$,	
Stickiness	$\lambda_p = 4$	$\lambda_w = 1$	

Table 1: Parameters

The input price is flexible, so it jumps on impact and then gradually goes back to its initial level, as the shock goes away. This is shown in the second panel of the figure. Notice that this panel shows the level of the input price, not its rate of inflation. Due to perfect flexibility P_X jumps by 20% at $t = 0$. This large increase is due to our assumption of a low elasticity of substitution between labor and the input X ($\epsilon = 0.1$), so when the employment is growing too fast relative to the supply of X, the price of X reacts strongly.

The effect of the increase in the input price is to increase firm's marginal costs. The impact effect on the nominal marginal cost $w_0 - mpl_0$ is 2%, as the input represents 10% of the cost in steady state ($s_X = 0.1$). This impulse translates into fast inflation on impact, due to our assumption of relatively flexible prices ($\lambda_p = 4$, i.e, prices reset every quarter). This is plotted in the third panel.

Wages respond because high employment translates into high real wage demands. In our simple model with $\eta = 0$, this is only due only to an income effect: as consumption grows, workers need higher wages to be induced to work. For illustration we have chosen parameters such that the impact effect on the nominal marginal cost of labor $p_0 + mrs_0$ is identical to the effect on the marginal cost of goods, both are 2%. However, wages are more sticky ($\lambda_w = 1$), so the effect on wage inflation is weaker. Wage inflation is also plotted in the third panel. The conditions for Proposition 4 are satisfied and the real wage falls on impact, as shown in the fourth panel.

To be clear, this is just a numerical example with numbers chosen for clarity of illustration. Nonetheless, there is clear qualitative feature that we want to highlight: the adjustment happens in three phases.

1. First, there is a bout of very fast inflation in the sector where the supply constraints are binding, here the market for input X.
2. Second, there is a phase in which price inflation is faster than wage inflation, as price setters react relatively quickly to the increase in input costs.
3. At some point (near $t = 0.5$ in our example) wage inflation crosses price inflation and we enter the third phase in which real wages recover. The input scarcity is going away, so the pressure on firms' marginal costs is weaker, while workers are still trying to catch up to the higher cost of living, given their real wage aspirations.

Consider now the same economy's response to a supply shock due to a temporary reduction in x. Suppose for now that the central bank responds in such a way as to keep

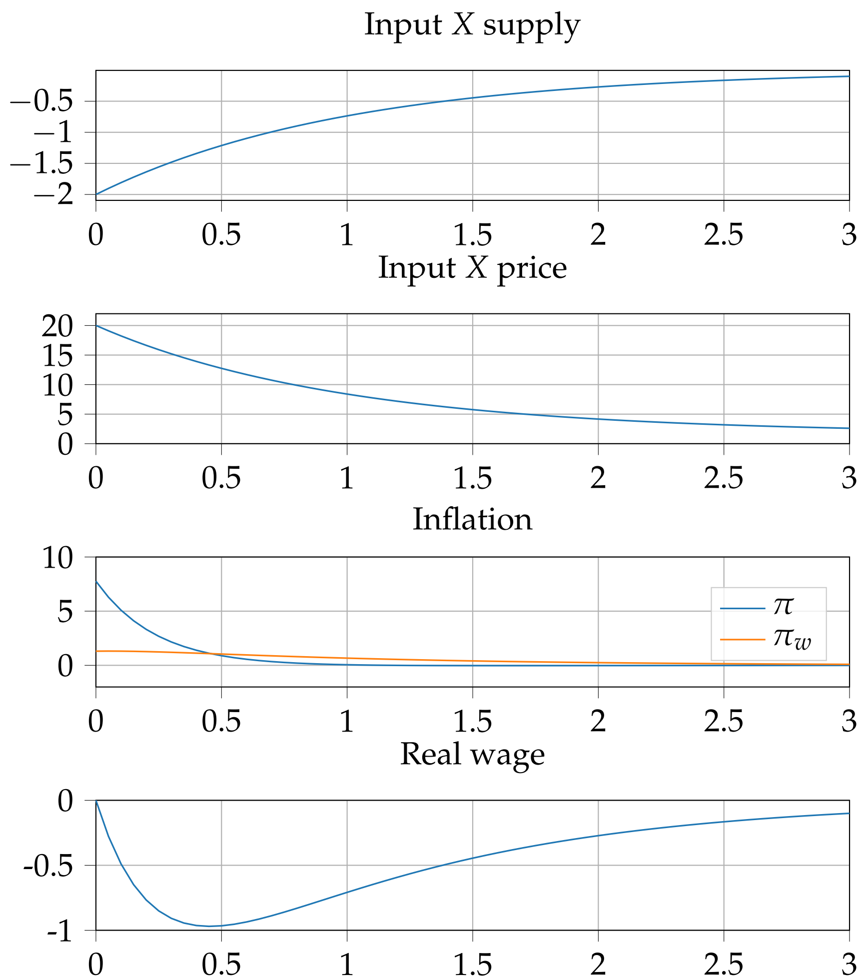

Figure 7: A supply shock

employment constant at $n_t = 0$. The responses of mpl and mrs are now

$$mpl_t = \frac{s_X}{\epsilon} e^{-\delta t} x_0 < 0, \quad mrs_t = \sigma s_X e^{-\delta t} x_0 < 0.$$

The main difference is that now the reduction in output reduces workers' mrs, via an income effect. This weakens real wage demands. However, given our parameter choices, the inflationary forces on the firms' side are still strong enough that we obtain positive wage and price inflation. In the representation of Figure 4 we are in the portion of the blue region that intersects with the lower left quadrant. From Proposition 3, we also know that $mpl_0 < 0$ and $mrs_0 < 0$ implies that the real wage falls on impact for any parameter configuration.

The responses are illustrated in Figure 7. For ease of comparison, we pick a negative shock to x_0 that produces the same increase in the input price as the positive n_0 shock in the demand shock exercise of Figure 6.

While nominal wages are growing less and the real wage drop is larger than in Figure 6, there is a common element to the demand and supply shocks just analyzed: the three-phase adjustment discussed above is qualitatively the same.

The response to the supply shock depend on how monetary policy adjusts. So far, we assumed a policy that keeps the employment path unchanged. However, the natural level of employment depends in general on x_t. In particular, keeping employment and output at their the natural level requires $mrs_t = mpl_t$, and so n_t^* can be derived from the condition

$$\sigma \left(s_N n_t^* + s_X x_t \right) + \eta n_t^* = \frac{s_X}{\epsilon} \left(x_t - n_t^* \right).$$

The responses of price and wage inflation when

$$n_t = n_t^* = \frac{\frac{1}{\epsilon} - \sigma}{\sigma \left(s_N + \frac{s_X}{\epsilon} \right) + \eta} s_X x_t$$

are plotted in Figure 8. Since our parametrization features a low degree of substitutability between labor and the input X, we have $\frac{1}{\epsilon} - \sigma > 0$ and a reduction in x_t lowers the natural level of employment, as shown in the first panel. The natural level of output $y_t^* = s_X x_t + s_N n_t^*$ is then lower for two reason, the direct effect of a lower x_t and for the lower level of natural employment. There is a clear difference in the inflation paths when quantities are at their natural levels: we see positive price inflation, but negative wage inflation. This goes on as long as the real wage falls, once the real wage starts growing again, the signs of price and wage inflation flip. In other words, real wage adjustments always take place with nominal prices and wages moving in opposite directions.

This is not just an outcome of our choice of parameters. When quantities are at their natural level we have $mrs_t = mpl_t$ and both are equal, by definition, to the natural real wage ω_t^*. The inflation equations then become

$$\pi_t = \Lambda_p \int_t^\infty e^{-\rho(s-t)} \left(\omega_s - \omega_s^* \right) ds,$$
$$\pi_t^w = \Lambda_w \int_t^\infty e^{-\rho(s-t)} \left(\omega_s^* - \omega_s \right) ds.$$

The following general result follows immediately.

Proposition 5. *If quantities are at their natural level, price and wage inflation π_t and π_t^w are either both zero or have opposite sign.*

This result can be visualized in the diagram of Figure 4, by noticing that the regions where π and π^w have the same sign are either entirely above or entirely below the 45 degree line, where $mrs = mpl$.

Comparing Figures 7 and 8 also shows that while employment falls more at the natural allocation, real wages fall less. This may seem surprising, but it is due to the fact the dynamics of the real wage are more strongly affected by mpl than by mrs, and mpl is higher

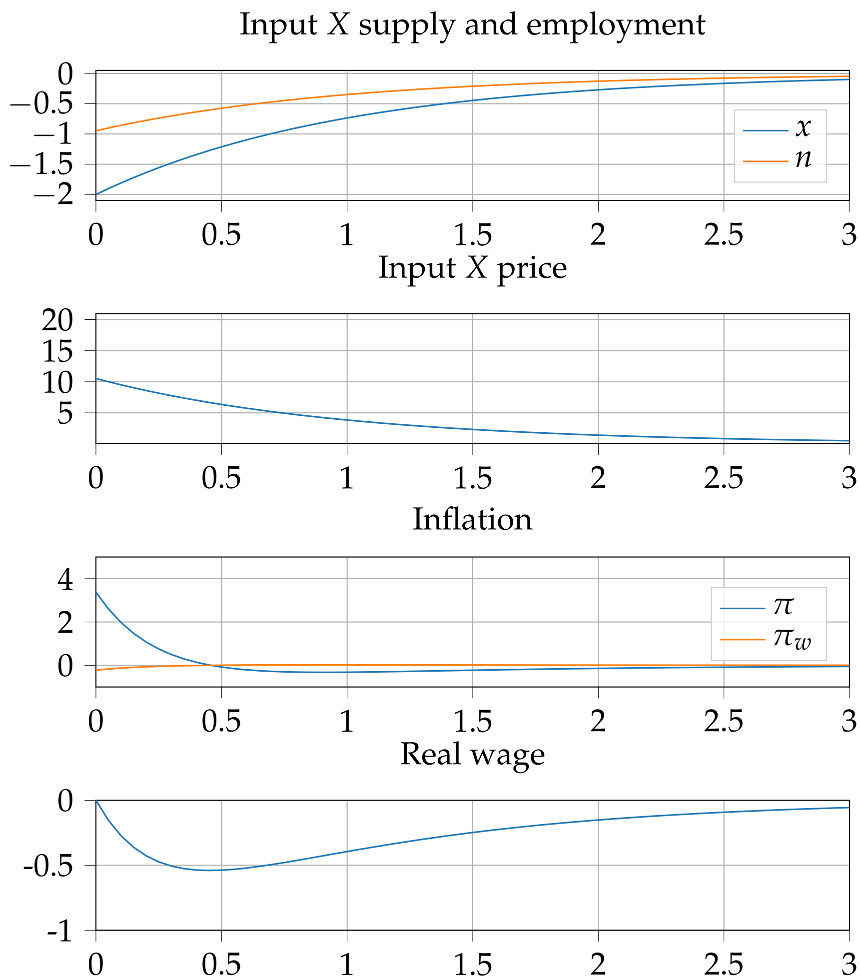

Figure 8: A supply shock with quantities on their natural path

along the path with lower employment. A different intuition for the same phenomenon is that lower employment reduces the pressure on the market for the scarce input, as seen in the second panel, weakening good inflation due to the high X price and increasing the real wage.

To summarize the findings of this section, there is a common adjustment pattern, illustrated in Figures 6, 7, that may be caused either by a positive demand shock or by an insufficient demand contraction in response to a negative supply shock. This adjustment pattern shows both price and wage inflation, with price inflation stronger early on and wage inflation catching up later. If the central bank keeps always the economy at its flexible price allocation this pattern is not present, as price and wage inflation have opposite signs.

However, as it's well known, an economy with both price and wage rigidities does not feature "divine coincidence," so a policy of keeping quantities at their flexible price levels is not necessarily optimal in our environment. In the next section, we turn to optimal policy.

7 Optimal Policy

In the previous section, we looked at economies in which the central bank unnecessarily stimulates the economy (demand shock) or in which the central bank responds weakly to a supply shock, so as to allow for both price and wage inflation (the supply shock with $n_t = 0$). The first example is a policy mistake, by construction. Of course, due to imperfect information and lags in the effects of monetary policy, similar mistakes can happen. However, in this section, we focus on the second shock, a supply shock, and ask what is the optimal response. Throughout, we assume monetary policy has perfect information on the underlying shocks and instantaneous control on the level of real activity.

The questions we address in this section are two: is it possible that following a supply shock the optimal response is to let the economy overheat, that is, to choose a positive output gap $y_t - y_t^* > 0$? Is it possible that the optimal response entails both positive price and wage inflation?

It is well known that divine coincidence fails in our environment. But that is really just a statement about feasibility: an outcome with no inflationary distortions, $\pi_t = \pi_t^w = 0$, and a zero output gap, $y_t = y_t^*$, are simply not feasible in our economy. The real wage needs to move in the flexible price equilibrium and that is incompatible with zero nominal inflation in p_t and w_t. Our contribution here is to characterize the signs of the deviations of π_t, π_t^w and $y_t - y_t^*$ from zero, under optimal policy.

In particular, Proposition 6 in the previous section tells us that if the central bank chooses $y_t = y_t^*$, then the signs of π_t and π_t^w will always be opposite. In other words, with a zero output gap the adjustment in the real wage never requires *both* price and wage inflation. Therefore, one could conjecture that generalized inflation, that is, inflation in both prices and wages is never optimal. However, a zero output gap is not necessarily optimal so that conjecture is not generally correct.

7.1 Optimal policy problem

Following standard steps, the objective function of the central bank can be derived as a quadratic approximation to the social welfare function:

$$\int_0^\infty e^{-\rho t} \frac{1}{2} \left[-\left(y_t - y_t^*\right)^2 - \Phi_p \pi_t^2 - \Phi_w \left(\pi_t^w\right)^2 \right] dt. \tag{22}$$

Deviations from first-best welfare come from two type of distortions: output deviations from its natural level, that is, from the level that equalizes the marginal benefit of producing goods with its marginal cost in terms of labor effort; and inflation in prices and wages that causes inefficient dispersion in relative prices of different varieties. The terms in (22) reflect these distortions. The value of the coefficients Φ_p and Φ_w depend on the model parameters and are derived and reported in the appendix.

The natural real wage following an X supply shock is

$$\omega_t^* = \frac{s_X}{\epsilon} \frac{\sigma + \eta + (\sigma - 1)\frac{s_X}{\epsilon}}{\sigma\left(s_L + \frac{s_X}{\epsilon}\right) + \eta} x_t.$$

We can then express mpl and mrs in terms of the natural real wage and deviations of employment from its natural path

$$mpl_t = \omega_t^* - \frac{s_X}{\epsilon}\left(n_t - n_t^*\right), \tag{23}$$

$$mrs_t = \omega_t^* + \left(\sigma s_L + \eta\right)\left(n_t - n_t^*\right). \tag{24}$$

The optimal policy problem is to maximize (22), subject to the constraints coming from price setting (12) and (13), condition

$$\dot{\omega}_t = \pi_t^w - \pi_t,$$

and the aggregate production function

$$y_t = s_L n_t + s_X x_t.$$

The optimality conditions that characterize an optimal policy are derived in the appendix.

7.2 Examples

We now consider examples that illustrate a variety of possible outcomes.

It helps the interpretation of the policy trade-offs to focus on the simple case of a permanent shock to x_t. With this shock, in all our examples, in the long run, the real wage is permanently lower and so are mpl and mrs, so that the economy eventually reaches a new steady state with zero inflation and zero output gap. To reach that new steady state requires ω_t to fall. This can be achieved by many combinations of price and wage inflation or deflation, as long as price inflation is larger than wage inflation. The question is what is the optimal way to get there.

Example 1: a symmetric case

Our first example is an economy with parameters that have the following properties:[8]

[8]The parameters are as follows:

$\sigma = 1$	$\eta = 0$	$\rho = 0.05$	
$s_X = 1/2$	$\epsilon = 1,$	$\varepsilon_C = 1.5$	$\varepsilon_L = 3$
$\lambda_p = 4$	$\lambda_w = 4$		

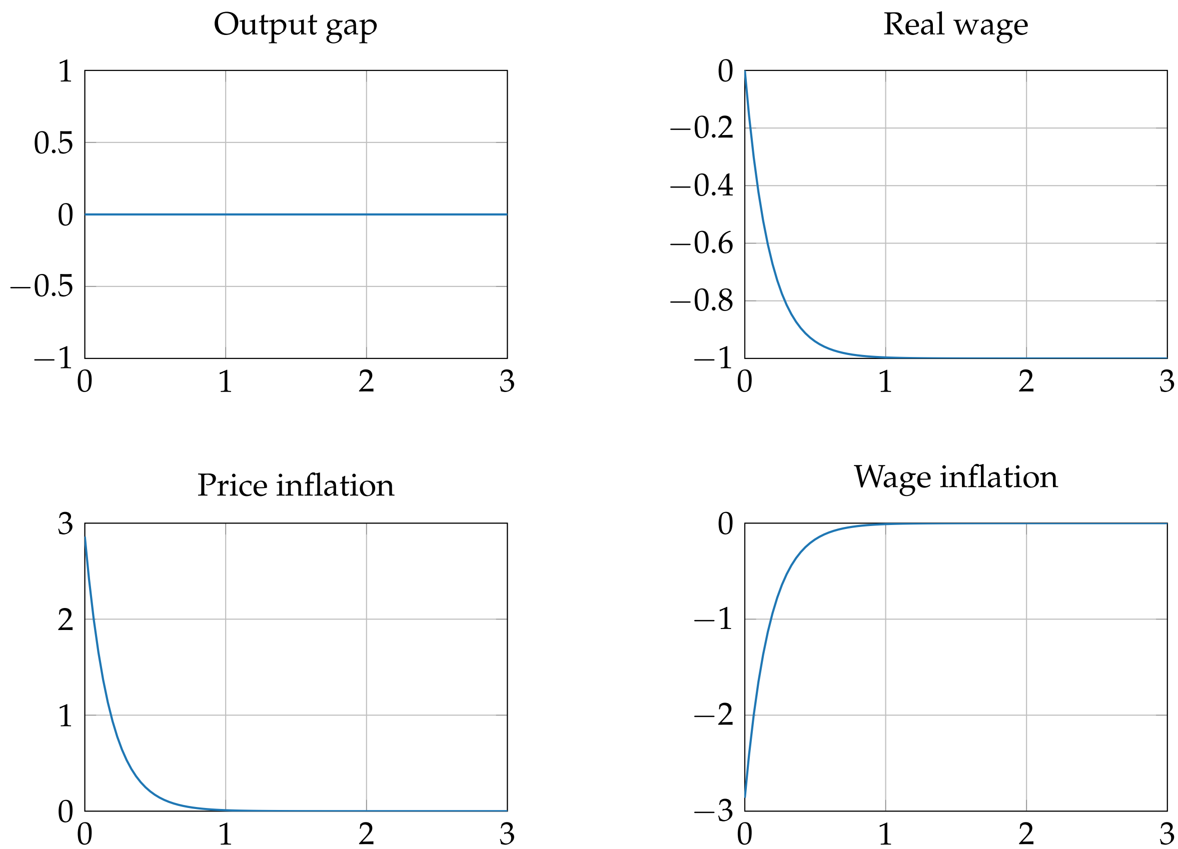

Figure 9: A symmetric example

- the welfare costs of wage and price inflation enter symmetrically the objective function, $\Phi_p = \Phi_w$;
- wages and prices are equally sticky, $\Lambda_p = \Lambda_w$;
- the output gap has symmetric effects on *mpl* and *mrs*.[9]

Figure 9 illustrates optimal policy outcomes in this example. Given the symmetry of the problem, the reduction in real wages is achieved by spreading the adjustment equally between nominal wage deflation and nominal price inflation. The output gap is kept exactly at zero. This example is clearly a knife edge case and relies on the symmetry of the parameters. As soon as we abandon this symmetry things get more interesting.

Example 2: a hot economy

In the second example, the parameters chosen imply that:[10]

[9] Given the expressions above this requires $\frac{s_X}{\epsilon} = \sigma s_L + \eta$.

[10] The parameters are as follows:

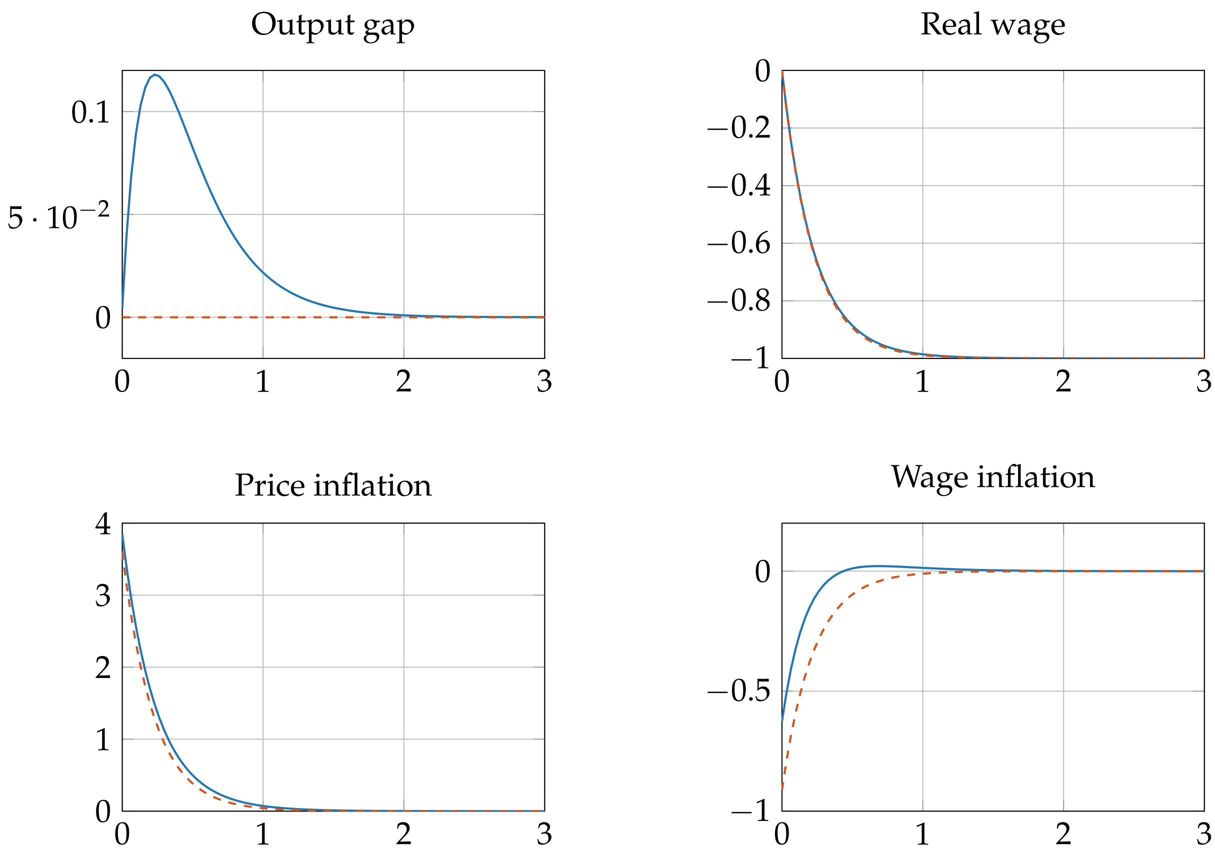

Figure 10: An optimal hot economy

- the welfare cost of wage inflation is larger than that of price inflation, $\Phi_p < \Phi_w$;
- wages are more sticky than prices, $\Lambda_p > \Lambda_w$.

We still have a set of parameters that imply roughly symmetric effects of the output gap on *mpl* and *mrs*, but the differences above are sufficient to obtain a quite different result. Figure 10 illustrates optimal policy outcomes in this case. For comparison, in the figure we also plot outcomes under a zero output gap policy (red dashed lines).

In this second example, it is optimal to have a positive output gap throughout the transition. To get some intuition for this result it is useful to recall from equations (12)-(13) and (23)-(24) that increasing the output gap has two direct effects. By decreasing *mpl* it leads to higher price inflation, by increasing *mrs* it leads to higher wage inflation. If we start at a zero-output-gap policy, with positive price inflation and negative wage inflation, the effect can be welfare improving because the welfare cost of price inflation is smaller than the welfare cost of wage deflation.

$\sigma = 1$	$\eta = 0$	$\rho = 0.05$	
$s_X = 0.1$	$\epsilon = 1,$	$\varepsilon_C = 1.5$	$\varepsilon_L = 4$
$\lambda_p = 4$	$\lambda_w = 2$		

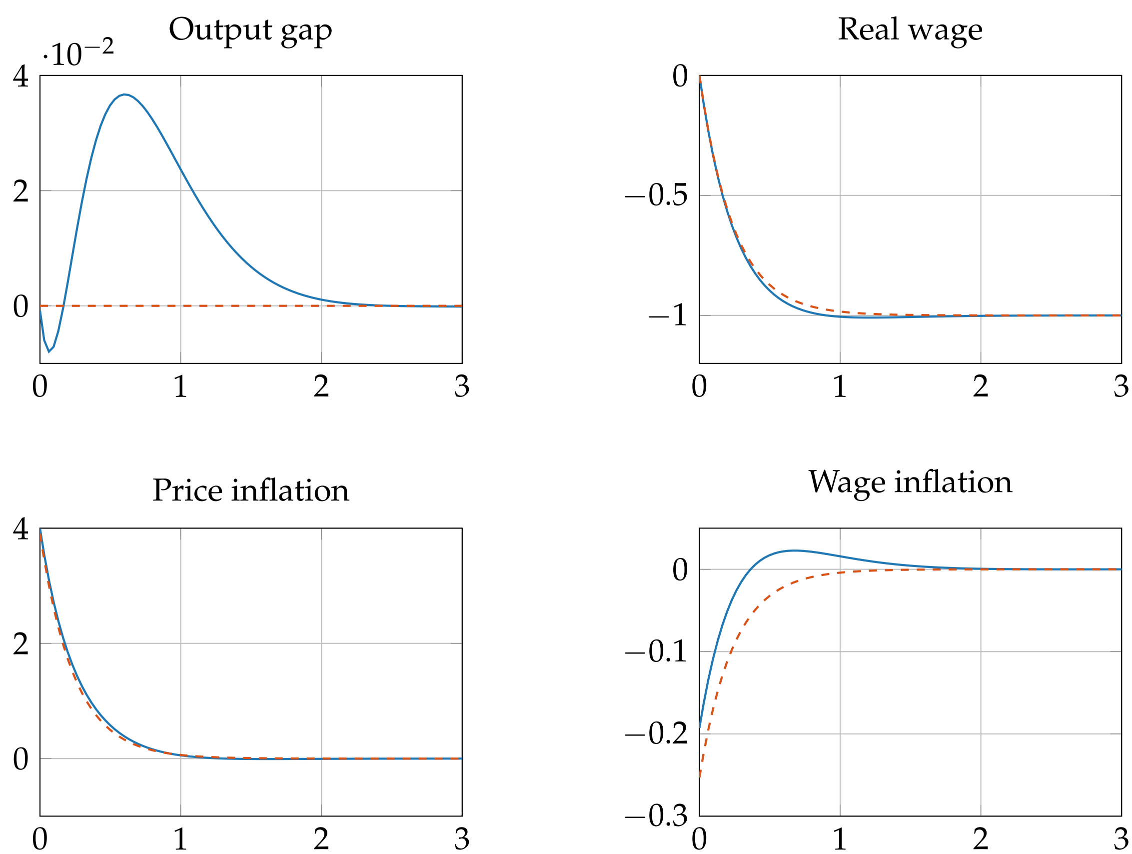

Figure 11: An example with generalized inflation and a hot economy

The role of $\Lambda_p > \Lambda_w$ is subtler and has to do with dynamics. With $\Lambda_p > \Lambda_w$ and $\xi_p \approx \xi_w$ a higher output gap also implies a faster declining real wage. Since a lower real wage in the future requires less adjustment, lowering the real wage today is welfare improving from a dynamic point of view. Therefore, a parametrization with $\Lambda_p > \Lambda_w$ makes it easier to obain examples with a welfare improving positive output gap.[11]

By choosing parameters that yield the opposite inequality, $\Phi_p > \Phi_w$, in the welfare coefficients it is possible to construct examples of the opposite: economies in which it is optimal to run a negative output gap in the transition.

Example 3: generalized inflation and a hot economy

Our third example is a variant on the second example, with an even larger welfare cost associated to wage dispersion (a larger Φ_w), a larger distance between price and wage stickiness, and with a smaller value of the elasticity of substitution between labor and the X input, ϵ, which implies that running a hot economy has larger benefits in terms

[11] The discussion of Figure X in the Appendix expands on this argument.

of lowering the real wage by having a larger effect on firms' marginal costs and thus on price inflation.[12]

The parametric choices above amplify the forces we saw in example 2 and they imply that there is an interval during the transition in which the optimal policy yields both a hot economy ($y_t > y_t^*$) and generalized price and wage inflation ($\pi_t > 0$ and $\pi_t^w > 0$).[13]

This result is surprising from a static point of view. Given the welfare function (22), at any point in time in which $y_t > y_t^*, \pi_t > 0$ and $\pi_t^w > 0$ it is welfare improving, from a static point of view, to reduce y_t, as it unambiguously lowers π_t and π_t^w and leads to an increase in the current payoff. However, from a dynamic perspectives there is an additional argument. Increasing y_t at time t has the effect of increasing π_s and π_s^w in all previous periods, due to the forward looking element in price setting. This entails welfare gains in early periods in the transition in which $\pi_s^w < 0$. Through this forward looking force a positive output gap later in the transition can be beneficial even if, at that point $\pi_t^w > 0$.

Now, while this example is theoretically interesting, it does have the flavor of a overly sophisticated form of forward guidance. Therefore, we do not think it provides a strong argument in favor of policies that deliver $y_t > y_t^*, \pi_t > 0$ and $\pi_t^w > 0$ at the same time. In the context of the present model, given the distortions it captures, it is hard to make a compelling practical case that the combination of a hot economy with positive wage and price inflation are a desirable outcome, even in response to a supply shock and even in presence of inelastic supply constraints.[14]

8 Conclusions

We explored the wage price spiral in a canonical model of price and wage setting.

Interpreting inflation as the outcome of inconsistent aspirations for the real wage (or other relative prices) opens the door to many theoretical and empirical questions. We are especially interested in extending our work to explore potential sources of inertia in the inflation process.

In the model analyzed here there is an instantaneous connection between the output gap and the real wage aspirations of workers' and firms. However, it is plausible that workers'

[12]The parameters are as follows:

$\sigma = 1$	$\eta = 0$	$\rho = 0.05$	
$s_X = 0.1$	$\epsilon = 0.1,$	$\varepsilon_C = 1.5$	$\varepsilon_L = 8$
$\lambda_p = 4$	$\lambda_w = 1$		

[13]Notice, that these qualitative features can actually be seen in example 2 too, but it is useful to choose an example where they are more clearly visible.

[14]This does not mean that such a case could not maybe be made in richer models, which capture, just to make an example, the benefits of labor reallocation. But that is clearly outside the scope of this paper.

real wage aspirations respond gradually to changes in labor market conditions. Similarly, changes in goods market conditions could affect slowly firms' expected profit margins. These are sources of inertia in inflation that come from agents' views on relative prices, and so are different from sources of inertia tied to future inflation expectations, on which most research has focused on. Even if inflation expectations are well anchored it is possible for inflation to persist if the disagreement between firms and workers is inertial. On the empirical front, while there is a large literature measuring inflation expectations, there has been limited effort so far at measuring workers' and firms' aspirations for real pay and for real profit margins.

Appendix

Derivation of equations (12) and (13)

To derive (12), take time derivatives on both sides of (10) to get

$$\dot{p}_t^* = -\left(\rho + \lambda_p\right)\left(w_t - mpl_t\right) + \left(\rho + \lambda_p\right) p_t^*.$$

Next, take time derivatives on both sides of (1). Substituting the expression just derived for $\dot{p}_t^*$ and adding and subtracting $\left(\rho + \lambda_p\right) p_t$ on the right-hand side yields

$$\begin{aligned}\dot{\pi}_t &= \lambda_p \left(-\left(\rho + \lambda_p\right)\left(w_t - p_t + mc_t\right) + \left(\rho + \lambda_p\right)\left(p_t^* - p_t\right) - \pi_t\right) = \\ &= -\lambda_p \left(\rho + \lambda_p\right)\left(w_t - p_t + mc_t\right) + \rho \pi_t,\end{aligned}$$

where the second equality uses $\lambda_p \left(p_t^* - p_t\right) = \pi_t$. Rearranging gives (12).

To derive (13), first rewrite (11), substituting for w_t^*, as

$$w_t^* = \frac{\rho + \lambda_w}{1 + \eta \varepsilon_L} \int_t^\infty e^{-(\rho + \lambda_w)(\tau - t)} \left(p_\tau + mrs_\tau + \eta \varepsilon_L w_\tau\right) d\tau.$$

Taking time derivatives on both sides gives

$$\dot{w}_t^* = -\frac{\rho + \lambda_w}{1 + \eta \varepsilon_L}\left(p_t + mrs_t + \eta \varepsilon_L w_t\right) + \left(\rho + \lambda_w\right) w_t^*.$$

Taking time derivatives on both sides of (1) and substituting for $\dot{w}_t^*$ yields

$$\begin{aligned}\dot{\pi}_t^w &= \lambda_w \left(-\frac{\rho + \lambda_w}{1 + \eta \varepsilon_L}\left(p_t + mrs_t + \eta \varepsilon_L w_t\right) + \left(\rho + \lambda_w\right) w_t + \left(\rho + \lambda_w\right)\left(w_t^* - w_t\right) - \pi_t^w\right) = \\ &= -\lambda_w \frac{\rho + \lambda_w}{1 + \eta \varepsilon_L}\left(p_t - w_t + mrs_t\right) + \rho \pi_t^w,\end{aligned}$$

which corresponds to (13).

Proof of Proposition 2

Consider the second order non-autonomous ODE

$$\ddot{\omega}_t - \rho\dot{\omega}_t - \Lambda\omega_t = f_t,$$

which is the general version of (18) with

$$\Lambda = \Lambda_p + \Lambda_w, \quad f_t = -\Lambda_p mpl_t - \Lambda_w mrs_t$$

With $\Lambda > 0$ there are two real eigenvalues r_1, r_2 that solve

$$r^2 - \rho r - \Lambda = 0,$$

or, equivalently, that satisfy $r_1 + r_2 = \rho$ and $r_1 r_2 = -\Lambda$. Then the ODE can be written as

$$(\partial - r_2)(\partial - r_1)\,\omega_t = f_t$$

where ∂ is the time-derivative operator. Notice that

$$(\partial - r_2)\int_t^\infty e^{-r_2(s-t)} f_s ds = -f_t$$

so we get

$$(\partial - r_1)\,\omega_t = \frac{1}{\partial - r_2} f_t = -\int_t^\infty e^{-r_2(s-t)} f_s ds,$$

which gives (19). Integrating backwards gives

$$\omega_t = e^{r_1 t}\omega_0 + \int_0^t e^{r_1(t-s)} \int_s^\infty e^{-r_2(\tau-s)} \left[\Lambda_p mpl_\tau + \Lambda_w mrs_\tau\right] d\tau ds,$$

and changing the order of integration gives

$$\begin{aligned}
\omega_t &= e^{r_1 t}\omega_0 + e^{r_1 t}\int_0^t e^{-r_2\tau}\left[\Lambda_p mpl_\tau + \Lambda_w mrs_\tau\right]\int_0^\tau e^{(r_2-r_1)s} ds d\tau + \\
&+ e^{r_1 t}\int_t^\infty e^{-r_2\tau}\left[\Lambda_p mpl_\tau + \Lambda_w mrs_\tau\right]\int_0^t e^{(r_2-r_1)s} ds d\tau = \\
&= e^{r_1 t}\omega_0 + \int_0^t \frac{e^{r_1(t-s)} - e^{r_1 t - r_2 s}}{r_2 - r_1}\left(\Lambda_p mpl_s + \Lambda_w mrs_s\right) ds + \\
&+ \int_t^\infty \frac{e^{r_2(t-s)} - e^{r_1 t - r_2 s}}{r_2 - r_1}\left(\Lambda_p mpl_s + \Lambda_w mrs_s\right) ds.
\end{aligned}$$

General Result for One-side Changes in mrs and mpl 3

The following result focuses on the effects of shocks that exclusively affect the labor demand side or the labor supply sidfe of the model, in the sense that they perturb mpl_t without affecting mrs_t, or, vice versa.

Proposition 6. *Suppose there is no change in $mrs_t = 0$ and the path for mpl_t is negative for all $t \in [0, \infty)$. Then the impact responses at $t = 0$ are*

$$\pi_0 > \pi_0^w > 0.$$

Suppose there is no change in $mpl_t = 0$ and the path for mrs_t is positive for all $t \in [0, \infty)$. Then the impact responses at $t = 0$ are

$$\pi_0^w > \pi_0 > 0.$$

Proof of Proposition 3

We first derive the real wage path using the expression in Proposition 2. Solving the integrals gives

$$\int_0^t e^{r_1(t-s)} \int_j^\infty e^{-r_2 j} e^{-\delta(s+j)} dj ds = \frac{1}{r_2 + \delta} \int_0^t e^{r_1(t-s) - \delta s} ds = \frac{1}{r_2 + \delta} \frac{e^{r_1 t} - e^{-\delta t}}{r_1 + \delta},$$

and then

$$\omega_t = \frac{1}{r_2 + \delta} \frac{e^{r_1 t} - e^{-\delta t}}{r_1 + \delta} \left(\Lambda_p mpl_0 + \Lambda_w mrs_0 \right).$$

To derive price inflation we solve the expression $\int_t^\infty e^{-\rho(\tau - t)} \omega_\tau d\tau$ in (15), using the following

$$\begin{aligned} &\int_t^\infty e^{-\rho(\tau-t)} \frac{1}{r_2 + \delta} \frac{e^{r_1 \tau} - e^{-\delta \tau}}{r_1 + \delta} d\tau = \\ &\int_0^\infty e^{-\rho j} \frac{1}{r_2 + \delta} \frac{e^{r_1(t+j)} - e^{-\delta(t+j)}}{r_1 + \delta} dj = \\ &\frac{1}{r_1 + \delta} \frac{1}{r_2 + \delta} \left(\frac{e^{r_1 t}}{-r_1 + \rho} - \frac{e^{-\delta t}}{\delta + \rho} \right). \end{aligned}$$

We then get that $\pi_t > 0$ if and only if

$$\frac{1}{r_1 + \delta} \frac{1}{r_2 + \delta} \left(\frac{e^{r_1 t}}{-r_1 + \rho} - \frac{e^{-\delta t}}{\delta + \rho} \right) \left[\Lambda_p mpl_0 + \Lambda_w mrs_0 \right] > \frac{e^{-\delta t}}{\rho + \delta} mpl_0,$$

which can be rewritten using $-r_1 r_2 = \Lambda_p + \Lambda_w$ (from the proof of Proposition (2)), to get

$$\frac{r_2}{r_2+\delta}\frac{-r_1}{r_1+\delta}\left(\frac{e^{r_1 t}}{-r_1+\rho} - \frac{e^{-\delta t}}{\delta+\rho}\right)\frac{\Lambda_p mpl_0 + \Lambda_w mrs_0}{\Lambda_p + \Lambda_w} > \frac{e^{-\delta t}}{\rho+\delta} mpl_0.$$

Setting $t = 0$ and rearranging gives the condition for $\pi_0 > 0$ in the statement of the proposition. Similar steps starting from equation (16), give the condition for $\pi_0^w > 0$.

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