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**INFORMATION TRANSMISSION AND
INCENTIVES IN MARKETS**

Matti Suominen

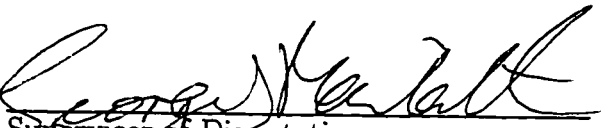
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of the Requirements for the Degree of Doctor of Philosophy

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Supervisor of Dissertation


Graduate Group Chairperson

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To my parents

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All remaining errors are my own.

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August 1997

ABSTRACT

INFORMATION TRANSMISSION AND INCENTIVES IN MARKETS

MATTI SUOMINEN

George Mailath

This dissertation comprises of three independent essays studying information transmission and incentives in financial and non financial markets. The first essay develops a structural model of stock markets that accounts for several stylized facts on the relationship between stock returns and trading volume. In this model the rate of public and private information arrival are probabilistic, the latter depending on the informed trader's ability and effort. The ability is uncertain and stochastically changing over time. Time series properties of the model include contemporaneous correlation between volatility and volume, both unconditionally and conditional on the current information set, and autocorrelation in volatility. When short sales constraints are included, there is a positive correlation between stock returns and volume. Secondly, the essay studies traders' incentives under delegated portfolio management.

The second essay studies firms incentives in credit markets over industry evolution. As industry evolves, the number of producers first increases and later rapidly falls. Sometimes even seventy percent of the prevailing firms exit in such a shakeout. Other stylized facts are that the price of the output initially falls

and the quantity produced by each producer increases, both at a decreasing rate, and stabilize after the shakeout. Even though the number of firms is increasing in growing markets, these markets are also characterized by higher exit rates than the mature markets. In this essay I develop a model of reputation formation in credit markets which, when applied to growing markets, gives rise to these phenomena.

The third essay (joint work with Illtae Ahn) studies community enforcement in a private information, random matching setting, where buyers privately “network” for information and sellers have a short term incentive to supply low quality. We show that high quality can be sold in a sequential equilibrium when there are M sellers and buyers, even when each buyer periodically observes only $N^*(M)$ other buyers’ and sellers’ games where $0 < \lim_{M \rightarrow \infty} N^*/M < \infty$. We show that when such networking is costly and M is large, low quality is supplied with positive probability in any Nash equilibrium. For this case, we characterize conditions for a sequential equilibrium in which both high and low quality are supplied.

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Chapter 1

Trading Volume and Information Revelation in Stock Markets

1.1 Introduction

In this paper I develop a structural model of stock markets that accounts for several major stylized facts on the relationship between stock returns and trading volume: (1) positive contemporaneous correlation between volume and absolute returns, (2) positive correlation between volume and stock returns itself and (3) serially correlated volatility. In addition, I study the trader's incentives under delegated portfolio management.

I assume a single large trader, representing a leading brokerage firm, who is engaged in information acquisition and asset management. I call him the portfolio manager. I assume that his access to private information depends on his ability and effort. Ability is unobservable to other market participants and changes stochastically over time, capturing effects such as changes in the firm's employees, their

ability to process private information and changes in the availability of private information. The investors and the market maker estimate the portfolio manager's ability. Portfolio manager's reputation is their estimate of his current ability. I assume a market mechanism similar to Kyle (1985) and Admati and Pfleiderer (1988), with the exceptions that the arrival of public information about the assets underlying value is probabilistic and the quantity traded by the informed trader, the informed portfolio manager, becomes public information at the end of each day.¹

In equilibrium, the amount of funds delegated to the portfolio manager increases in his reputation. Because of concern about his reputation, the portfolio manager trades only in the event that he receives private information. His trades then reveal information to the other market participants, resulting in a positive correlation between volatility and volume. The expected price volatility depends on the true ability of the informed trader and inherits any autocorrelation in the process determining ability. I assume that the true ability of the portfolio manager changes according to a two state Markov process and show that in this case the correlation between volatility and volume is positive both unconditionally and conditional on the public information set. I solve for the autocorrelation function for volatility and demonstrate that it is positive and geometrically decaying, very much like the autocorrelation functions of individual stocks and stock indices documented by the empirical research. I also show that the conditional volatility is autocorrelated, mean reverting and that it may be positively (or negatively) correlated with the expected trading volume. When short sales constraints are included there is

¹The assumption that trades of the informed trader become public information at the end of the day is consistent with the structure of markets in some countries, e.g. Finland, where a list of security transactions between brokers is publicly distributed at the end of each day. We could replace this assumption with a less stringent assumption that only the market maker observes the trades between different traders. This would better reflect the structure of NYSE, where a specialist observes all trades between different brokers with some delay.

a positive correlation between stock returns and trading volume.² Finally, I show that endogenous information acquisition can amplify the changes in volatility and trading volume.

The empirical regularities that I study are documented in Karpoff (1987) and Gallant et. al. (1992). The latter, however, conclude that “there seems to be no model with dynamically optimizing, heterogenous agents that could jointly account for the major stylized facts” listed in the first paragraph of this paper. The positive autocorrelation in volatility for lags as long as twenty days make it unlikely that this phenomenon could be explained merely by the dissemination of single pieces of public or private information. Several theoretical articles have recently emerged to address this discrepancy. Foster and Viswanathan (1995), for instance, build a speculative trading model that can account for several of these stylized facts, but make an assumption of changing expected volatility of the underlying asset returns to obtain their results. Harris and Raviv (1993), on the other hand, build a model based on differences in opinion that can account for the positive volume-absolute price change correlation and the positive autocorrelation in volume. One drawback of their model, however, is that it concerns solely the arrival of public information, whereas in reality it is often difficult to relate the changes in volatility and volume to the arrival of any public information. Other articles that have addressed the issue are at least: Brock and LeBaron (1996), de Fontnouvelle (1996) and Shalen (1993). First two of these are adaptive beliefs models where agents follow naive, ad hock updating rules when assessing the market informativeness and the profitability of information acquisition. These models are able to produce roughly similar

²Diamond and Verrecchia (1987) and Karpoff (1987) have earlier suggested short sales constraints as a plausible explanation for the return-volume correlation. This explanation is particularly appealing since this correlation is not present in futures markets where the costs of short selling are small.

unconditional volume-volatility correlations and autocorrelations as observed in the empirical data. Shalen (1993), on the other hand, is a two period model which argues that dispersion of beliefs can be positively related to both volume-volatility correlation and autocorrelation in volatility.

Another, more empirically oriented approach to explaining these phenomena is the “Mixture of Distribution Hypothesis”, extended by Andersen (1996). In these models the dynamic features are governed by information flow, modelled as a stochastic volatility process. Other closely related articles include Benabou and Laroque (1992), Bollerslev, Chou and Kroner (1992), Blume, Easley and O’hara (1994), Brown, Harlow and Starks (1996), Campbell, Grossman and Wang (1993), Chevalier and Ellison (1996), Diamond and Verrecchia (1987), Dow and Gorton (1995), Grundy and McNichols (1989) and Wang (1994).

The rest of the paper is organized as follows: Section two describes the model. Section three describes an equilibrium where the portfolio manager trades only when he receives private information. Section four studies dynamic properties of this equilibrium. Section five extends the model to include short sales constraints and allows the portfolio manager to invest (in terms of effort) in information acquisition. Section six concludes the paper.

1.2 Model Description

There are T periods $t \in \{1, 2, \dots, T\}$, a safe asset and a single risky asset. There is a continuum $[0, I]$ of small investors, each endowed with unit wealth, who can invest directly to the markets or invest through the portfolio manager. There are two other kinds of market participants, whose behavior I take as exogenously determined.

They are the market maker, whose behavior is characterized by the zero profit constraint, and the noise traders. Similarly as in Kyle (1985) and Admati and Pfleiderer (1988), in each period the different traders submit “market orders” to the market maker.

The value of the risky asset in period T is $F_T = \bar{F} + \sum_{t=1}^T \delta_t$, where \bar{F} is a constant larger than T and δ_t a random innovation realized in the beginning of period t . I assume that in every period

$$\delta_t = \begin{cases} 1 & \text{with pr. } k \\ 0 & \text{with pr. } (1 - 2k) \\ -1 & \text{with pr. } k \end{cases} ,$$

where $k < 1/2$. As in Easley and O'hara (1992), the assumption that k is less than half reflects “event uncertainty”. The return on the safe asset is normalized to zero.

With probability ρ there is a public announcement, a_t , in period t that reveals δ_t . With probability $(1 - \rho)$ there is no announcement. In addition there is an annual report in period T that reveals all δ_t 's. Implicit in this structure is that the asset returns over the entire time interval $[1, T]$ is exogenously given.

There is a single portfolio manager (a leading brokerage firm) who may have access to private information. He observes a signal $z_t \in \{\delta_t, 0\}$ in the beginning of each period. The probability that $z_t = \delta_t$ depends on his ability (in a later section also his effort). When $z_t = 0$ the portfolio manager can not tell apart the events that $\delta_t = 0$ and $z_t = 0$ and that $\delta_t \neq 0$ but $z_t = 0$. The portfolio manager can be two types, good and bad, and his type changes over time in a way that investors

can not perfectly observe.^{3,4} I assume that

a) when the manager is good, $\Pr\{z_t = \delta_t\} = \bar{\alpha} < 1$, and

b) when the manager is bad $\Pr\{z_t = \delta_t\} = \underline{\alpha}$, where $0 < \underline{\alpha} < \bar{\alpha}$.

Let $s_t \in \{g, b\}$ denote the portfolio manager's type in period t ; Here g stands for good and b for bad portfolio manager. The portfolio manager starts as good with probability $\varepsilon/(\varepsilon + \mu)$. In each period, good managers become bad with probability μ and bad managers good with probability ε . Both μ and ε are less than half. Investors can not observe the type of the portfolio manager and must base their decisions solely on the portfolio managers reputation, which is based on the portfolio manager's past actions and all other publicly available information. The portfolio manager is not endowed with any wealth nor does he have the ability to save.⁵ The portfolio manager's commission is an exogenously determined fraction c of the assets under management. To keep the calculations simple there is no discounting by any of the traders and all agents are risk neutral. The noise trading, u_t , is independent across periods and in each period it is drawn from a uniform distribution on the

³One interpretation is that the portfolio manager is a firm owned by it's employees and its employees change over time in a way that investors can not perfectly observe. Similar idea of firm reputation is developed in Tirole (1996). In Tirole's model there are infinitely many employees, however, so the average quality of a worker is always the same. My assumption of a single but changing type corresponds to a model with finitely many employees, since in both cases the average quality of the worker is changing over time. This makes my equilibrium history dependent in a very natural way. Similar assumption of changing types, in their model honest and dishonest, is made in Benabou and Laroque (1992). See also Holmstrom (1982) and Cole, Dow and English (1995).

⁴For tractability I have assumed that the portfolio manager's signals are perfect when $z_t \neq 0$. I believe, however, that the model can be extended to the more realistic case of imperfect signals as long as the probability of an error in the bad portfolio manager's signal is larger than that in the good portfolio manager's signal. Because of tractability I have also assumed that the signal reveals only δ_t , not for instance $\sum_{\tau=1}^t \delta_\tau$. Again, it seems possible to extend the model to incorporate this, perhaps more natural signal.

⁵The idea is that the portfolio manager's own trading is not directly observable and it is very small relative to the trading on the assets that he manages.

interval $[-m, m]$.

There is a competitive market maker who is willing to be the opposing party to the market demand at a price p_t that equals the expected value of the asset conditional on his information set. That is, if we denote by v the net order flow that is presented to the market maker at some time in period t and by Ψ the public information set that is available to the market maker at that point in time, then the market clearing price is given by

$$P_t = E(\bar{F} + \sum_{\tau=0}^T \delta_\tau \mid v, \Psi).$$

In every period the timing of events is as follows:

- (a) Portfolio manager's type is determined.
- (b) Investors allocate their investments between the different assets. Portfolio manager privately observes z_t .
- (c) Traders submit 'market orders' to the market maker and an opening price p_t^o is determined, trades are executed.
- (d) A public signal a_t that fully reveals δ_t is published with probability ρ . Portfolio manager's previous trade, y_t , becomes public information.
- (e) Markets are open for a second time and trading with market maker is possible at the closing price p_t^{cl} . The portfolio manager sells his position to the market maker and pays his investors $x_t(1 - c) + y_t [p_t^{cl} - p_t^o]$.⁶

In period T , there is an additional stage 6 in which all δ_τ 's for $\tau \leq T$ are

⁶We are ignoring the possibility that this payment is negative. We may, for instance, assume that investors do not have a limited liability and that negative payments are possible.

announced.

The assumption that the portfolio manager sells his position to the market maker at stage five is innocent since the portfolio manager's trade has already become public information. This trading round is not informative about the asset's value and its trading volume is unimportant for our results. Indeed, the results would be similar if this trading round would include some noise trading, or there was no trading at all at price p^{cl} .

I therefore assume a setting where the informed trader, the professional portfolio manager, receives a private signal in the beginning of the day on the daily innovation on the value of the security. This innovation may become public information either through a direct disclosure of information, the public announcement, or it may be revealed through the trading of the informed portfolio manager. In the equilibrium that I construct, the public signal will provide a yardstick to evaluate the trading performance of the portfolio manager and prevent him from *churning*, i.e., trading even in the event that he has no private information.⁷ In reality, an alternative yardstick could be the observable actions of other, competing portfolio managers.

Let Ψ_t denote the public information set available at the end of the period t . Let $R_t(\Psi_{t-1})$ denote the public belief (at the beginning of the period t) that the portfolio manager is good in period t . Let α_t denote the associated probability that the portfolio manager observes δ_t , i.e., let

$$\alpha_t = \bar{\alpha}R_t + (1 - R_t)\underline{\alpha}.$$

Note that $\alpha_t = R_t$ in the extreme case where $\bar{\alpha} = 1$ and $\underline{\alpha} = 0$. When there is no

⁷I will use this terminology throughout this paper when referring to the portfolio managers trading in the event that $z_t \neq 0$.

risk of confusion, I will drop the time subscripts from R_t and α_t . Throughout the paper I assume that the amount of potential investors, I , is larger than m/c .

1.3 Equilibrium

In this section I show that when the probability of public information arrival, ρ , is large and the probability of nonzero innovation on asset returns, k , is small, there exists sequential equilibria where the portfolio manager follows a linear trading strategy in the opening round of trade: $y_t = y^* z_t$, where y^* is a positive constant. He therefore trades only in the event that z_t is different from zero. His trades then reveal his private information, which results in early revelation of information and increases the price volatility prior to period T .

There exists several sequential equilibria with linear trading strategies. We focus our attention to the equilibrium in which both the periodic returns to investors and the amount of investments to the portfolio manager are maximized.⁸ In this equilibrium the value of the portfolio manager's future payoff increases in his reputation and reputational concerns prevent him from churning: i.e., trading even in the event that he has no information. Investors use Bayes rule to update the probability of a good portfolio manager and invest through the portfolio manager whenever their expected payoff exceeds zero. The investments to the portfolio manager, $x_t(R_t)$, are strictly increasing in R_t and the portfolio manager follows the following simple linear trading rule: $y_t(z_t) = m(1 - k\alpha_t) z_t$.

To develop the argument it is instructive to first look at the market maker's

⁸It is straightforward to show that when ρ is large this trading strategy maximizes the periodic trading profits even within a larger class of feasible trading strategies: symmetric trading strategies where the portfolio manager churns with probability ℓ .

problem assuming that the portfolio manager follows a linear trading rule: $y_t(z_t) = y^*(R_t)z_t$, where $y^* \neq 0$. Let us introduce the following notation: Let $\hat{\delta}_t = E[\delta_t | \Psi_\tau]$. Since the public information set at the end of the period τ , Ψ_τ , includes both the public announcement, a_τ , and the portfolio manager's trade, y_τ , these two variables should be used in calculating $\hat{\delta}_t$. We have: $\hat{\delta}_\tau = 1$ if $a_\tau = 1$ or $y_\tau = y^*$ and there is no a_τ ; $\hat{\delta}_\tau = -1$ if $a_\tau = -1$ or $y_\tau = -y^*$ and there is no a_τ ; $\hat{\delta}_\tau = 0$ otherwise.

Given this notation and given α_t , the pricing rule for the market maker can be written as:

$$P_t^o = \bar{F} + \sum_{\tau=0}^{t-1} \hat{\delta}_\tau + E(\delta_t | v_t, \alpha_t)$$

$$= \bar{F} + \sum_{\tau=0}^{t-1} \hat{\delta}_\tau + \begin{cases} 1 & \text{if } v_t > m \\ \frac{k\alpha}{1-k\alpha} & \text{if } m - y^* \leq v_t \leq m \\ 0 & \text{if } y^* - m \leq v_t \leq m - y^* \\ \frac{-k\alpha}{1-k\alpha} & \text{if } -m \leq v_t \leq y^* - m \\ -1 & \text{if } v_t < -m \end{cases}$$

and

$$P_t^{cl} = \bar{F} + \sum_{\tau=0}^t \hat{\delta}_\tau.$$

Given the portfolio manager's trading strategy and the structure of the game, the market maker can infer the portfolio manager's private information from his trades when setting the closing price p^{cl} . When setting the opening price, however, the market maker can not perfectly infer the value of δ_t from v_t when $v_t \in [-m, m]$, and must calculate the expected value of δ_t using the Bayes rule.⁹

⁹For instance in the region $m - y^* \leq v_t \leq m$ there are two possible events: either $z_t = 1$ and

Having looked at the market maker's problem we can already state the following result:

Lemma 1: The trading profits in the equilibrium where the portfolio manager follows the trading rule $y_t(z_t) = m(1 - k\alpha_t)z_t$, are larger than the trading profits in any equilibrium with trading strategies linear in z_t .

Proof. Taking as given the market maker's pricing rule, Ψ_{t-1} and $z_t = 1$, the expected periodic trading profits can be written as

$$\pi^e = E \left[y_t (p_t^{cl} - p_t^o) \mid p_t^{cl} = \sum_t^{t-1} \hat{\delta}_\tau + 1 \right]. \quad (1.1)$$

For $y_t \leq y^*$ we have

$$\begin{aligned} \pi^e &= y \left[1 - \frac{k\alpha}{1 - k\alpha} \right] \int_{m-y-y^*}^{m-y} (1/2m) du + y \int_{y^*-y-m}^{m-y-y^*} (1/2m) du \\ &\quad + y \left[1 + \frac{k\alpha}{1 - k\alpha} \right] \int_{-m}^{y^*-y-m} (1/2m) du \\ &= \frac{1}{2m} \left(y \left[1 - \frac{k\alpha}{1 - k\alpha} \right] y^* + 2y [m - y^*] + y \left[1 + \frac{k\alpha}{1 - k\alpha} \right] (y^* - y) \right) \\ &= y - \frac{y^2}{2(1 - k\alpha)m}, \end{aligned} \quad (1.2)$$

$u_t = v_t - y^*$ or $z_t = 0$ and $u_t = v_t$. The p.d.f. $g(v \mid z_t = 1) = g(v \mid z_t = 0) = 1/2m$, so Bayes rule implies

$$\Pr\{z_t = 1 \mid v_t\} = \frac{k\alpha \frac{1}{2m}}{k\alpha \frac{1}{2m} + (1 - 2k\alpha) \frac{1}{2m}} = \frac{k\alpha}{1 - k\alpha}.$$

It is straightforward to check that the expression for the expected trading profit is identical in the event that $z_t = -1$ as a function of $-y$ for $y \geq -y^*$. In any linear equilibrium the expected trading profits are then $k2\alpha_t$ times the expected trading profits given by equation (1.2) with y^* replacing y . Maximizing this with respect to y^* gives the result. ■

The expected trading profits from following this trading strategy are

$$\pi^e(R_t) = 2k\alpha_t \left[m(1 - k\alpha) - \frac{[m(1 - k\alpha)]^2}{2(1 - k\alpha)m} \right] = m(k\alpha_t - k^2\alpha_t^2). \quad (1.3)$$

Before we go on to prove that there exists a sequential equilibrium in which the portfolio manager follows this trading strategy, we must specify how the beliefs are updated in a linear equilibrium. Since we are interested in sequential equilibria, we must specify these beliefs for all possible events on and off the equilibrium path. The beliefs over different types of portfolio managers at the beginning of period $t+1$ are obtained using the Bayes rule whenever possible. At off-the-equilibrium paths this is not possible, however. There are a number of ways to specify the beliefs at off-the-equilibrium paths. Here I have assumed that what appears to be churning is taken as evidence of a bad portfolio manager. These investor beliefs can easily be shown to be sequentially rational. Furthermore, in a two period version of the model it is easy to show that since $\underline{\alpha} < \bar{\alpha}$, the bad portfolio manager does indeed have a

higher incentive to churn.¹⁰ More precisely, if \hat{R}_t denotes the posterior probability of a good portfolio manager given Ψ_t , then

$$\hat{R}_t = \begin{cases} \frac{\bar{\alpha}R_t}{\alpha_t} & \text{if } \hat{\delta}_t \neq 0 \text{ and } \text{sign}[\hat{\delta}_t] = \text{sign}[y_t] \\ R_t & \text{if } a_t = y_t = 0 \\ \frac{(1-\bar{\alpha})R_t}{1-\alpha_t} & \text{if } \bar{\delta}_t \neq y_t = 0 \\ \frac{((1-2k)+2k(1-\bar{\alpha}))R_t}{(1-2k)+2k(1-\alpha_t)} & \text{if } y_t = 0 \text{ and no } a_t \\ 0 & \text{if } \hat{\delta}_t \neq 0 \text{ and } \text{sign}[\hat{\delta}_t] \neq \text{sign}[y_t] \end{cases} \quad (1.4)$$

In particular, $\hat{R}_t = 0$ when the portfolio manager is caught churning. Given the transition probabilities μ and ε for good and bad portfolio managers, we have:

$$\begin{aligned} R_{t+1} &= \hat{R}_t(1 - \mu) + (1 - \hat{R}_t)\varepsilon \\ &= \hat{R}_t(1 - \varepsilon - \mu) + \varepsilon. \end{aligned}$$

For a complete description of the strategies and beliefs we must still specify the portfolio manager's beliefs: Let his beliefs be that others update α_t as specified and play according to their equilibrium strategies. We can now state the first proposition.

Proposition 1: There exists $\underline{k} > 0$ and $\bar{\rho} < 1$ such that for all k and ρ that satisfy $k \leq \underline{k}$ and $\rho \geq \bar{\rho}$, the above strategies form a sequential equilibrium of

¹⁰If the no churning constraint fails for the good portfolio manager it always fails also for the bad portfolio manager. The converse, however, is not true. Indeed, there exists sequential equilibria where only the bad portfolio managers churn with positive probability.

the game. In this equilibrium the investments to the portfolio manager, $x(R_t)$, are larger than the investments in any other linear equilibrium. Furthermore, $x(R_t)$ is strictly increasing in R_t .

The formal proof of the proposition is given in the appendix. The intuition is as follows: Because the portfolio manager's commission depends only on the amount of assets under his management he is indifferent between trading strategies that result in the same level of reputation. Among actions that result in the same level of reputation he may as well follow the trading strategy that, in equilibrium, gives the largest expected profits to his investors. Effectively, the portfolio manager then chooses y_t among three alternatives: $y_t \in \{y^*, -y^*, 0\}$. Given that the expected trading profits (1.3) increase in R_t , it is easy to show that the portfolio manager's value function is increasing in R_t . Given the updating of investors' beliefs, it is also straightforward to show that the portfolio manager wants to trade according to this strategy when $z_t \neq 0$. The problem, however, is to prevent churning. It is here, where the conditions on k and ρ are needed. High ρ and small k reduce the probability that churning leads to higher reputation and increase the probability of the converse. Since the portfolio manager's value function increases in his reputation, when ρ is high enough and k low enough, the portfolio manager prefers not to churn.

The condition for no churning can be relaxed in many other ways besides raising ρ and reducing k : First, if there existed a honest portfolio management firm who never churns, the rational dishonest portfolio management firms would have an incentive to pretend they are honest, since by not churning more can be extracted from the noise traders. Similar reputation effect could prevail in a framework where T period intervals, such as above, are repeated infinitely often. As a third way of

relaxing the incentives to churn, consider a framework where the portfolio manager is given an opportunity to periodically (at stage 5) reveal his information, $z_t \neq 0$, in the form of an investment report and assume that these reports are verifiable. In such a framework, there exists an equilibrium with identical trading rule as above, in which the portfolio manager periodically reveals all non zero z_t 's to the public, whenever *either* ρ or k is small enough.^{11,12}

Interestingly, the equilibrium changes somewhat when we include a profit based commission for the portfolio manager (the main time series properties remain similar, however). It turns out that the profit function (1.2) is concave for $y \leq y^*$ and $y > y^*$ and continuous at $y_t = y^*$. The derivative, however, is not continuous at y^* , but rather there is an upward jump at that point. This implies that the equilibrium with profit based commissions is one in mixed strategies. Furthermore, it can be shown that such equilibrium results in smaller or equal trading profits as compared to those given by equation (1.3.). It seems that with a commission based solely on the assets under management, the portfolio manager can commit to less aggressive trading strategy than in the presence of profit based commission and may therefore obtain higher expected trading profits.

It is instructive to compare this equilibrium to one that would prevail when there is a single informed trader who is trading for his own account and whose trading is not observable. This is the framework in Kyle (1985). In his model the informed trader trades relatively small quantities on information in order to hide his information and benefit from it over several periods. As compared with his model,

¹¹I have assumed that no effort by the portfolio manager is necessary to observe z_t . The same equilibrium, however, would hold even if we included such cost of effort, as long as this cost was small enough.

¹²Fischer and Verrecchia (1996) discuss evidence that fund managers are eager to publicly announce their trades after taking their positions. The framework of this paper could easily be altered to justify that kind of behavior.

here the fact that the portfolio manager's trades are publicly observable result in early revelation of information and an increased volatility in asset returns prior to T (we will soon study the relationship between information revelation and volatility).

1.4 Time Series Properties

Let us now look at some of the dynamic properties of this equilibrium.¹³ To begin with, let us calculate the expressions for expected price variance and the expected trading volume, conditional on Ψ_{t-1} . Let us measure these from the closing prices, for instance $\sigma_t^2(\Psi_{t-1}) = \text{var}[P_t^c - P_{t-1}^c \mid \Psi_{t-1}]$, and use the associated trading volume. I will drop the c superscript from P_t^c so that P_t always refers to closing price unless otherwise mentioned. The expressions for price variability and trading volume are somewhat different when we measure the price variability from opening prices and use the corresponding trading volume. As you recall, there is no asymmetric information in closing prices, whereas in opening prices there is.

As Admati and Pfleiderer (1988) argue a proper measure for the volume of trade can be obtained as follows. Let $d^+ = \max(d, 0)$, $d^- = \max(-d, 0)$. The volume of trade at stage two (including trades that are crossed out between traders) is $\omega_t^o = \max(u_t^+ + y_t^+, u_t^- + y_t^-)$. Similarly, the volume of trade at stage five is $\omega_t^c = |y_t|$. Our measure volume, ω_t , is the sum of these two volumes. Our results are similar if we measure the volume only at stage two. Regarding variance, note that $P_t - P_{t-1} = \hat{\delta}_t \equiv E[\delta_t \mid \Psi_t]$. The martingale property of prices, formally shown in proposition two below, implies that $E[P_t - P_{t-1} \mid \Psi_{t-1}] = 0$. Given this and the fact that $\hat{\delta}_t$ is either zero, one or minus one, we obtain that $\text{var}[P_t - P_{t-1} \mid \Psi_{t-1}] = E[\hat{\delta}_t^2 \mid \Psi_{t-1}]$.

¹³When doing so we are restricting our attention to the cases where ρ is high and k is small. See the discussion at the end of the previous section for motivation.

Straightforward calculations give:

$$\begin{aligned}\sigma_t^2 &= \text{var}[P_t - P_{t-1} \mid \Psi_{t-1}] = 2k(\rho + (1 - \rho)\alpha_t), \\ \omega_t^c &= E[\omega_t \mid \Psi_{t-1}] = \frac{m}{2} + k\alpha_t \left[3y^*(R_t) + \frac{y^*(R_t)^2}{2m} \right].\end{aligned}$$

ω_t is different from v_t for two reasons: First because we now include the cases where the portfolio manager's and noise trader's orders cancel and secondly we take into account the unwinding of the portfolio manager's position in stage five at in each period (for instance the selling of y^* in the event that $y_t = y^*$).

We summarize some of the resulting dynamic properties of the model in the next two propositions. The following two propositions after that characterize the dynamics of σ_t and ω_t^c , i.e., the dynamics of conditional volatility and volume.

Proposition 2: The asset prices are a martingale with respect to Ψ_{t-1} .

Proof.

$$\begin{aligned}E[P_t^c \mid \Psi_{t-1}] &= \bar{F} + \sum_{\tau=0}^{t-1} \hat{\delta}_\tau + E[E[\delta_t \mid \Psi_t] \mid \Psi_{t-1}] \\ &= \bar{F} + \sum_{\tau=0}^{t-1} \hat{\delta}_\tau = P_{t-1}^c \blacksquare\end{aligned}$$

Proposition 3: Let $0 < s < T - t$. The following results hold:

$$\text{cov} \left[(P_t - P_{t-1})^2, \omega_t \mid \Psi_{t-1} \right] > 0,$$

$$\text{cov} [(P_t - P_{t-1})^2, \omega_t] > 0,$$

$$\begin{aligned} \text{cov} [(P_{t+s} - P_{t+s-1})^2, (P_t - P_{t-1})^2] = \\ \left[\frac{4\varepsilon\mu k^2(1-\rho)^2[\bar{\alpha} - \underline{\alpha}]^2}{(\varepsilon + \mu)^2} \right] (1 - \varepsilon - \mu)^s > 0. \end{aligned}$$

The proofs of this and all remaining propositions are given in the appendix. So the volatility and trading volume are positively correlated both unconditionally and conditional on Ψ . The autocorrelation function for volatility is positive and geometrically decaying, very much like the autocorrelation functions of stocks and stock indices documented by the empirical research (see e.g. Brock et. al., 1996).

Equally interesting as to show that our model can match these stylized facts, is to characterize the dynamics of conditional volatility and volume. The next two propositions show that the conditional volatility is autocorrelated and mean reverting (as the literature on GARCH assumes). The proposition after that characterizes the relationship between conditional volatility and expected trading volume. Before proceeding, let us introduce the following notation: Let $E_t(\cdot)$ denote the expectation operator conditioned on the public information set Ψ_{t-1} . We can now state the following propositions:

Proposition 4: $\text{cov}[\sigma_{t+s}^2(\Psi_{t+s-1}), \sigma_t^2(\Psi_{t-1})] > 0.$

Proposition 5: σ_t^2 is mean reverting. In particular, $E_t[\sigma_{t+1}^2] < \sigma_t^2$ when $\sigma_t^2 > \bar{\sigma}^2$ and $E_t[\sigma_{t+1}^2] > \sigma_t^2$ for $\sigma_t^2 < \bar{\sigma}^2$, where

$$\bar{\sigma}^2 = 2k \left[\rho + (1 - \rho) \left(\underline{\alpha} + \frac{\varepsilon(\bar{\alpha} - \underline{\alpha})}{(\varepsilon + \mu)} \right) \right].$$

Proposition 6: When $k \leq \frac{8 - \sqrt{43}}{3} \approx 0.48$, the conditional volatility, σ_t^2 , and the conditional expected trading volume, ω_t^e , are positively correlated random variables.

What is also interesting is that under extreme conditions, very high k and $\underline{\alpha}$, also the opposite to proposition 6 may hold, that is, that these two random variables are actually negatively correlated. This possibility is due to the fact that as α_t increases the size of the portfolio manager's trade is reduced. It is interesting that this can occur even in a model where the amount of noise trading is exogenously given.

The positive correlation between volume and volatility conditional on the current information set arises from the fact that the portfolio manager trades only when he receives private information and the fact that his trades carry information and affect prices. This effect is so strong that there is a positive correlation between volume and volatility even when we are not conditioning on the public information set. It is not obvious that this should be the case because good ability is correlated with a high reputation and high reputation, on the other hand, leads the portfolio manager to reduce the size of his trade (as we argued the expected trading volume may actually be negatively correlated with the expected volatility).

The evolution of the true volatility depends on the evolution of the portfolio

manager's ability. When the portfolio manager is good he discovers more information and prices are more volatile. When the portfolio manager's true ability changes according to a two state Markov process, the autocorrelation function is geometrically decaying. This happens because the longer the time period between return observations, the larger is the probability of switches in the portfolio manager's type.

The evolution of conditional volatility and volume are characterized by the evolution of R_t . Besides being dictated by the process determining the true ability and by chance, the evolution of R_t has the following characteristic: In periods of high volatility in asset returns, $\hat{\delta}_t \neq 0$, agents reputations change because these events separate "informed" portfolio managers from the "uninformed" and allow inference on the type of the portfolio manager. In contrast when the volatility of asset returns is low, $\hat{\delta}_t = 0$,

$$R_{t+1} = R_t(1 - \mu - \varepsilon) + \varepsilon,$$

and reputation drifts down towards its long run average, $\varepsilon/(\mu + \varepsilon)$, or below.¹⁴

An additional observation about the stock return - trading volume relation is made in Jones et. al. (1994). They show that size of transactions has no information value beyond the number of transactions. Since the above model is a simplified model with only one informed trader, discussing this observation in the

¹⁴If we use the above results and derive a dynamic evolution equation for conditional volatility this looks somewhat similar to a GARCH (1,1) process: $\sigma_t^2 = \varphi\sigma_{t-1}^2 + \psi r_{t-1}^2$. In our formulation σ_{t-1}^2 and r_{t-1}^2 are not sufficient statistics for predicting the conditional volatility σ_t^2 , however, and also y_{t-1}^2 is needed. In addition, these variables do not enter the evolution equation for volatility in a simple linear form as in the GARCH model. As Anderssen (1995) pointed out, the first of these observations suggests a reason for why empirical models predicting volatility may gain by incorporating volume of trade (proxy for y_t) in the evolution equation for volatility, as is done in Anderssen (1995) and Lamoreux and Lastrapes (1991). We should note, however, that when ρ is small, little information may be lost by omitting y_{t-1} from the right hand side. See Bollerslev, Chou and Kroner for a survey on the work related to GARCH.

context of this model is only indicative. Nevertheless, if we assume that the noise trading comes from a single noise trader, then in line with the findings of Jones et. al. (1994), the model predicts that the number of transactions contains all the relevant information regarding the price change that is imbedded in the volume of trade: i.e., are there two transactions or only one. In this model, the size of the orders is determined by the size of the noise trading and has no information value beyond the number of transactions, which reflects the arrival of new information.

1.5 Extending the Basic Model

1.5.1 Introducing Short Sales Constraints

In this section we repeat the above analysis but include constraints on short selling. To do this in the simplest possible way, let us assume that there is some exogenously determined and independent state variable f_t , which determines whether the portfolio manager can sell stock short in period t and that the realization of f_t is public information.¹⁵ Let us assume that $f_t = 1$ in the presence of short sales constraints and that $\text{prob}[f_t = 1] = f > 0$. In the appendix it is argued that for a slightly different trading rule:

$$y_t = \begin{cases} y^* z_t & \text{when } f_t = 0 \\ \hat{y} & \text{when } f_t = 1 \text{ and } z_t = 1 \\ 0 & \text{otherwise} \end{cases} ,$$

¹⁵This kind of probabilistic short sales constraint could arise for instance if the portfolio manager could sell short some stocks, but not the others. In such a case, if the periodic innovations are stock specific, whether the short sales constraint is binding or not depends on which stock the innovation concerns. Alternatively we may think that the portfolio manager holds a imperfectly diversified portfolio and can not sell stocks short at all. In such case he can react to a negative innovation only if he happens to hold the relevant stock.

the propositions one through three continue to hold. In addition we have:

Proposition 7:

$$\text{cov}((P_t - P_{t-1}), \omega_t) > 0.$$

The intuition behind this result is clear: When $f_t = 1$, the portfolio manager is trading only in the event that $z_t = 1$ and, as before, his trades carry information. As for all other propositions, the formal proof is given in the appendix.

1.5.2 Endogenous Information Acquisition

In this section we show that endogenous information acquisition can amplify the changes in volatility and trading volume. There are at least two possible ways to model endogenous information acquisition: either the investments on information acquisition are observable to the investors, for instance the act of hiring more security analysts, or they are not, for instance the amount of effort that is put into the information acquisition. We shall concentrate on the second possibility.¹⁶

Let us now assume that the bad portfolio manager may also observe δ_t with probability $\bar{\alpha}$, instead of $\underline{\alpha}$, if he invests $e > 0$. Let the strategies and the beliefs for the investors and portfolio managers be similar to those in section three, except for the bad portfolio manager who now searches information with probability $q^*(R_t) \in [0, 1]$. Let

$$\alpha_t = \bar{\alpha}R_t + (1 - R_t) [\bar{\alpha}q^*(R_t) + \underline{\alpha}(1 - q^*(R_t))].$$

¹⁶Let us first briefly discuss the possibility of observable investments. Suppose that both types of managers may hire more security analysts but only the good managers can pick out good security analysts. Other things equal, the incentives of both types of firms to invest would then seem to increase in R_t simply because investors then place higher probability for the event that good security analysts are hired.

Lemma 1: In any equilibrium with the above strategies $q_t^* < 1$.

Proof. Assume that $q_t^* = 1$. Then in equilibrium both types of portfolio managers invest and reputation in period $t + 1$ is independent of the performance in period t . In this case it is optimal for the bad manager not to invest and to set $q_t = 0$. A contradiction. ■

Proposition 8: Let $T = 2$. When ρ is large and k is small, there exists $q_t^*(R) \in [0, 1)$ for $t \leq T$ such that the above strategies form a sequential equilibrium of the game. In this equilibrium $q^*(R_t) = 0$ when R_t is close enough to either zero or one.

The reasoning for why the portfolio manager follows the trading strategy $y_t = y^* z_t$ is similar to before. Intuition behind the result concerning $q^*(R_t)$, on the other hand, is as follows: When the portfolio manager's reputation is close to zero or one all successes (failures) are attributed to good luck (bad luck) and are not taken as evidence of the portfolio manager's type. In this case the portfolio manager has no incentives to invest in information acquisition. This is an example of a well known property of Bayes rule: the signals, the successes or failures of the portfolio manager, affect the posterior probability most when there is uncertainty over the agent's type.

The above proposition suggests that endogenous information acquisition will amplify the changes in volatility and trading volume provided that ε is small relative to μ , so that managers are typically bad. As the proposition shows, when ε is small

and reputation is at or below its long run average $\varepsilon/(\varepsilon + \mu)$ the portfolio manager does not have any incentive to invest in information acquisition and q^* is equal to zero. If, however, the portfolio manager succeeds to develop a higher reputation, so that there becomes more uncertainty over his type, it may then pay for him to invest in information acquisition. So when R_t is high, there is both a high probability that the portfolio manager is good, and if he is bad, there is a high probability that he invests in information acquisition. Similarly, however, if the portfolio manager's reputation ever becomes very high, the manager will start to reduce the level of his investment.

How would the results on the asset market dynamics extend to the case where the number of portfolio managers is large. If we interpret the changes in α as changes in the availability of private information, then nothing changes. How about if we insist on the interpretation of α as ability and assume that the abilities are independent across portfolio managers. Even then, however, the results may extend even to the case where the number of portfolio managers is very large. With several portfolio managers, as with a single portfolio manager, the initial volatility in asset returns, $\delta_\tau \neq 1$, separates good portfolio managers from bad, but it may also lead to reallocation of assets to the successful portfolio managers, whose reputations have increased. If there are always enough successful portfolio managers, whose incentives to search for information are increased, the expected price volatility may always go up following a period of volatility in asset returns. When $\hat{\delta}_t = 0$, on the other hand, the reputations of all portfolio managers move towards (or below) the long run averages of the good portfolio managers. This suggests that as long as there is event uncertainty, i.e., $k < 1/2$, most of our results on asset market dynamics, namely the volatility and volume correlation, their positive autocorrelation and

mean reversion can be maintained even with a large number of informed portfolio managers.

1.6 Conclusion

This paper developed a simple model of stock market microstructure that can account for the major stylized facts on the stock return and trading volume dynamics. I assumed a single informed trader, called portfolio manager, who is engaged in information acquisition and asset management. His access to private information depends on his ability and effort. The ability is unobservable to other market participants and changes stochastically over time, capturing effects such as changes in the firm's employees, their ability to process private information and changes in the availability of private information. The investors and the market maker estimate the portfolio manager's ability. The portfolio manager's reputation is their estimate of his current ability. The market mechanism is similar to Kyle (1985) and Admati and Pfleiderer (1988), with the exceptions that there is probabilistic public information arrival on the underlying asset returns and the quantity traded by the informed trader, the informed portfolio manager, becomes public information at the end of each day (see footnote one for motivation).

In equilibrium, because of concern about his reputation, the informed portfolio manager trades only in the event that he receives private information. His trades then reveal information to the other market participants, resulting in a positive correlation between volatility and volume. The expected price volatility depends on the true ability of the informed trader and inherits any autocorrelation in the process determining this ability. I assumed that the true ability of the portfolio manager

changes according to a two state Markov process and showed that in this case the correlation between volatility and volume is positive both conditional on the public information set and not. I solved for the autocorrelation function for volatility and verified that it is positive and geometrically decaying, very much like the autocorrelation functions of individual stocks and stock indices documented by the empirical research. I showed that the conditional volatility is autocorrelated, mean reverting and that it may be positively (or negatively) correlated with the expected trading volume. When short sales constraints are included there was a positive correlation between stock returns and trading volume. Finally, I showed that endogenous information acquisition can amplify the changes in volatility and trading volume.

Chapter 2

Reputation, Credit Markets and Industry Evolution

2.1 Introduction

It is well established that in the presence of limited liability and moral hazard, fierce competition and zero profits cannot prevail in mature markets, where demand and technology no longer improve; see for instance Klein and Leffler (1981) on quality provision, and Stiglitz and Weiss (1981) on the incentives for taking risks. The reason is that with limited possibilities to punish the entrepreneurs for defection, entrepreneurs must be bribed in terms of positive mark-ups to act prudently. The threat of losing these mark-ups gives the firms an incentive to refrain from opportunistic behavior. What is perhaps less well understood is that such phenomena need not characterize evolving markets, where demand and technology improve rapidly. In such markets, producers have an incentive to behave prudently even in the absence of any mark-ups just in the hope of a better future, i.e., the

positive mark-ups of mature markets. These observations provide an alternative explanation for the stylized facts of industry evolution, as documented by Gort and Klepper (1982).

Stylized facts of an evolving industry are that first the number of producers increases and later it rapidly falls. The price of the product falls and the quantity produced by each producer increases, both at a decreasing rate, and stabilize after the shakeout. Despite the net entry into the young, growing markets, these markets are also characterized by higher exit rates than the mature markets. In the existing literature, e.g., Gort and Klepper (1982), Klepper and Graddy (1990) and Jovanovic and MacDonald (1994), these phenomena are often related to the arrival of product innovations. Jovanovic and MacDonald (1994) argue that in the case of the American tire industry the shakeout occurred in response to the emergence of a major invention. Gort and Klepper (1982), on the other hand, provide evidence from a number of industries, and find that typically the shakeout occurs at the end of a period of rapid technological improvement. According to the data regarding prices and quantities presented in their paper and recollected in Hopenhayn (1993), the shakeout seems also to be associated with a large drop in the growth rate of demand. As one would expect, these two events seem interrelated.

Klepper and Graddy (1990) further discuss the evidence in Gort and Klepper (1982), and present a model where there is much entry and exit in young markets, and where over time only the most efficient firms survive. Here the shakeout is caused by the gradual exit of the least efficient firms. One drawback of their model is that the entry rate is exogenously determined by an assumption that there is only a small number of new potential entrants in every period. Other related articles are at least Klepper and Miller (1995), Klepper (1996) and Horvath, Schivardi and

Woywode (1996).

In this paper we study the financial aspects of industry evolution and identify a phenomenon related to the reputation formation in credit markets that gives rise to similar dynamics in industry evolution. Our explanation of industry dynamics is therefore radically different from those in the existing literature: here the shakeout is explained by a change in the severity of the borrowers moral hazard problem in response to an exogenous change in the growth rate of demand or the rate of technological improvement.¹

We analyze a model where the entrepreneurs must borrow a fixed amount from the financial markets in every period in order to set up their production facilities. We assume that entrants who have successfully obtained credit have access to two possible production technologies: a safe and a risky technology. Because of the convexity in their payoffs due to their limited liability, the entrepreneurs have a short term incentive to choose the risky technology. We study the borrowers' incentives to choose the safe technology, and analyze the resulting market equilibrium as the function of the market size.

The model has infinitely many potential entrepreneurs. Because of fixed costs in production and a moral hazard problem, in equilibrium only finitely many firms obtain credit and enter, and the competition is imperfect.² As one would expect there are more firms in the larger markets, and the competition is therefore more

¹Hopenhayn (1993) also considers a situation where the shakeout occurs in response to an exogenous change in the growth rate of demand or the rate of technological improvement. In his model there are several types of firms, characterized by their cost functions, and the types of the firms change stochastically over time. When the growth stops there is a gradual shakeout because the less efficient firms are slowly driven out of the markets by efficient producers whose proportion slowly increases (for each cohort of producers the proportion of efficient firms increases over time). His model of industry evolution is closely related to the models by Gort and Klepper (1990) and Klepper (1996).

²In a later section we consider a model where there is technological improvement over time and a continuum of firms enter the markets in every period as long as the technology improves.

intense. Nevertheless, in contrast to the free entry model of Cournot competition by Novshek (1980), here the firms in larger markets earn larger profits. The reason is that in larger markets the firms are also producing larger quantities and have a greater potential gains from taking risks related to their costs of production. To prevent these firms from taking risks, the equilibrium profit stream in large markets must be larger than in small markets.

We then look at what happens in growing markets. We assume a setting where the demand for output grows rapidly until, at some random point in time, the growth rate of demand slows and the market matures. In this case, fixed costs and the moral hazard problem reduce the entry into the small markets. As markets grow more entrants obtain credit and enter, and competition increases resulting in a gradually declining price. In contrast to the mature markets where demand no longer increases, here the firms do not necessarily earn strictly positive profits. The firms are willing to behave prudently in growing markets merely because they hope to be among the producers who make it to the large and highly profitable (stationary) markets in the future. Since larger stationary markets imply larger profits for incumbent producers, the continuation value for keeping a good credit rating is typically much higher in growing markets than in stationary markets of the same size. As a result more firms can be issued credit. This effect is intensified by the fact that when the number of firms increases in such small and growing markets, all firms produce less and the potential gains from taking risks become even smaller; allowing for an even larger number of firms to enter. As the market then suddenly matures and the growth stops, there is shakeout caused by an increase in the severity of the moral hazard problem that results from a decrease in the value of the expected future returns for existing firms.

There is some evidence that at least some of the shakeouts have been financially driven: e.g., the US ski resort industry. Until the late 1980's the demand for ski resorts' services grew rapidly and the number of firms in the industry increased.

(Now the) growth in the industry has been flat for 10 years... Over the past 10 years, the number of US ski resorts has dropped from 709 to 519 as intense competition has led to the sale or closure of smaller resorts that lacked the finances to undertake popular improvements, such as faster ski lifts (Financial Times, February 5th, 1997).

One additional stylized fact was that in addition to a higher entry rate, the growing markets are also characterized by a relatively high exit rate. We extend our model to account for this observation by assuming that there are two types of entrepreneurs: ones that can avoid taking risks and ones that cannot. This extension of the model can account not only for the observed dynamics in the number of firms, prices and quantities but also the dynamics in entry and exit rates. It can also explain why the latest cohorts of entrants are more likely to exit than firms who have entered before them. Especially the structure of this extended model is closely related to Diamond (1989).

2.2 The Basic Model

In this section we study a model of industry evolution that has the following characteristics: By assumption the firms must borrow funding for a machine from the

capital markets in every period in order to produce. There is a possible conflict of interest between borrowers and lenders: because of limited liability the entrepreneurs may have a short term incentive to produce with risky technology that is dominated (in expectation) by the safe technology. We investigate how entry to the markets is affected by the firms incentives to choose the safe technology and study the resulting industry equilibrium. Because of fixed costs and the above mentioned moral hazard problem, our equilibrium is characterized by finitely many active producers and imperfect competition.

We assume infinitely many agents of two types: Lenders and potential entrepreneurs. The entrepreneurs live infinitely long whereas the lenders only for a period. We make this assumption, along with Diamond (1989), so that the only intertemporal linkage is that for the borrowers. To produce an entrepreneur must borrow F units of money from the capital markets to buy a machine or, alternatively, to cover their fixed costs. Because F is larger than zero only finitely many firms can ever obtain credit and enter. By assumption, the entrepreneurs can not save and so they must borrow F units of money in every period in which they wish to produce. As in Diamond (1989) we assume that the lender and the borrower can ex ante commit to use a costly monitoring technology that destroys the current periods profits in certain pre specified states of the world.

Once in possession of a machine the entrepreneurs have access to two different production technologies: a safe and a risky technology. With the safe technology the costs of production are:

$$C(q_t) = F + cq_t,$$

and with the risky technology these costs are

$$C(q_t) = \begin{cases} F + \bar{c}q_t & \text{with pr. } \gamma \\ F + \underline{c}q_t & \text{with pr. } (1 - \gamma) \end{cases},$$

where $\bar{c} > c > \underline{c} = 0$.

We assume that the entrepreneurs have no wealth and they have limited liability so that their periodic payoffs are never less than zero. We will confine most of our analysis to the case where γ , \bar{c} and F are such that in equilibrium only the firms who are expected to produce with riskless technology are issued credit. We assume that the firms' outputs and profits are not observable to the lenders (although this is not crucial) and that entering firms compete in quantities. We assume that the entrepreneurs discount future with a constant discount factor δ and that the values of their outside opportunities are zero.

As mentioned the lenders, in contrast to borrowers, live only for one period. They are risk neutral and, for simplicity, they do not discount their payoffs within the period that they live. They are endowed with F units of wealth in the beginning of the period and they derive utility from their end of the period wealth. Along with Diamond (1989) we assume that the only statistics that lenders observe from the past are the number of times that a borrower has and has not paid his debt. In each period the loan markets work as follows: The potential entrepreneurs $i \in \{1, 2, \dots\}$ are organized into a queue. The potential entrepreneurs approach lenders sequentially one at a time and pick two lenders $j \in \{1, 2, \dots\}$ randomly and offer them interest rates $R_{i,j} \in [0, \infty]$ to borrow F . Once an entrepreneur obtains F units of money from a lender he and the lender exit the loan markets and the next potential entrepreneur enters. If both banks refuse to lend to an entrepreneur at

the interest rates that he offered, the entrepreneur must exit the loan markets. The loan markets for a given period stop once an entrepreneur with good credit history is declined credit twice.

The lenders $j \in \{1, 2, \dots\}$ make their lending decisions sequentially, knowing the actions of all other lenders before them. The ordering of entrepreneurs as they apply for lending is important in determining the identities of entrepreneurs who obtain credit and it is not necessarily the same as the order in which they are indexed. We will assume throughout the paper that the incumbent entrepreneurs are always given priority in obtaining lending, that is, they are always ahead of the other entrepreneurs in the queue. The ordering of incumbents in this queue, on the other hand, is important as it determines the exit rule in the event that the aggregate number of firms in the current period is smaller than in the previous period. We look at two possible rules of organizing incumbents: one where their ordering is determined randomly in the beginning of each period and one where their ordering corresponds to the entrepreneurs' initial dates of entry. These two ways to order the entrepreneurs correspond to exit rules "Equal probability of exit" and "First in last out," which are specified more carefully below. The ordering of potential entrepreneurs who were not active in the previous period is assumed to be random.³

Our assumption of non-observable profits guarantees that the optimal contract between bank and the entrepreneur is a loan contract. Note however that on the equilibrium path, the firms' profits are non stochastic so the payoffs to differ-

³By giving priority to former borrowers the lenders increase the incentives of the current entrepreneurs to behave prudently and thereby the amount of lending that can be done in any given period. This assumption is also the most natural one if there are several types of entrepreneurs and those with a proven track record are perceived somehow less risky by the lenders than those with no track record. We will analyze such model in section five.

ent contracts, equity and debt could be made identical. Nevertheless the choice of contract, equity or debt, could affect the firms' incentives off-the-equilibrium path. Note also that in this setting the lenders will behave competitively since each borrower can ask credit from two lenders and the number of lenders is infinite, whereas only finitely many firms can ever obtain credit, meaning that the probability that two entrepreneurs approach any particular lender in any given period is zero. We have here been very explicit about the organization of the loan markets. It should be mentioned, however, that this is only one possible organization of loan markets where the lenders behave competitively, the number of entrepreneurs who obtain credit is non random and lending takes place until there are no lenders who would like to lend to potential entrepreneurs at any positive interest rate.

We now look how the demand for the industry output is determined. It is assumed that in every period the demand is exogenously given by:

$$Q_t = G_t(\alpha - \beta P_t), \quad (2.1)$$

where Q_t is the quantity, P_t the price, α and β positive constants and G_t a (possibly) time dependent parameter that reflects the size of the markets. Equation (2.1) can be interpreted as there being G_t customers at time t , each of whom has demand $(\alpha - \beta P_t)$. In terms of the inverse demand function, this can be written as:

$$P_t = a - bQ_t/G_t,$$

where $\alpha = a/b$, and $\beta = 1/b$.

We want to analyze the industry evolution in this kind of setting under the assumption that the demand for industry output, G_t , grows in every period at some positive rate $g - 1 \geq 0$, until the growth of demand stops at some random point in time. For simplicity we take the rate of growth, $g - 1$, and the stopping time of growth as exogenously determined. For simplicity, we will also assume that there is a possibly very large bound on G_t , so that $G_t \leq \bar{G}$, where $\bar{G} = g^T G_0$.

Let us denote the state of the economy as (g_t, G_t) , where $g_t \in \{g, s\}$ is the current growth state, $g > s = 1$, and G_t the size of the markets in period t . We assume that the state of the economy evolves as follows: If the state of the economy is currently (g, G_t) the next periods state is (s, G_t) with probability ρ and $(g, \min\{gG_t, \bar{G}\})$ with probability $(1 - \rho)$. When the current state of the economy is (s, G_t) , the next period's state is (s, G_t) for certain. For $t < T$, irrespective of the number of periods that the industry demand has grown, the growth stops with probability ρ in the next period and when it stops it never resumes again. Note also that the current periods growth is realized in the beginning of the period in the sense that the demand for that period already reflects the entire growth for that period. Note also that the growth state matters only when $G_t < \bar{G}$.

2.3 Equilibrium

We characterize a market equilibrium, where incumbent firms use the safe technology and no additional firm can obtain sufficient capital to enter. Throughout this section we assume that in product markets the firms compete in quantities and in product markets the equilibrium is the one shot Nash equilibrium. Because of this and our assumption that the firms have similar costs, the equilibrium in the

product market is always symmetric with respect to incumbent firms. The decision over which of all the potential entrepreneurs produce in the market in any particular period is made by the capital markets by allocating their supply of funds to a subset of potential producers. In the equilibrium that we consider the lenders lend F only to $n(g_t, G_t)$ entrepreneurs who have never failed in their debt and, when determining the identity of those entrepreneurs, they give priority to entrepreneurs who were active in the previous period. Their strategy is to commit to the use of the costly monitoring technology whenever the interest payment by the entrepreneur is below the scheduled payment. We make the assumption about giving priority to old entrepreneurs because it seems to reflect the reality and it is likely to be the one that maximizes the amount of funds lent to the industry (subject to the perceived exit rule in shakeout and the assumption that the firms compete in quantities).⁴ In equilibrium the market structure depends only on the state of the economy (g_t, G_t) .

2.3.1 Stationary Markets

To characterize the market evolution we must first characterize the market structure in all states (s, G_t) for $G_t \leq \bar{G}$, that is in all the stationary markets, and then by backward induction figure out the market structure in growing markets, states (g, G_t) . Throughout the paper we assume that the cost parameters \bar{c} and F are large enough so that in the equilibrium where the entrepreneurs use the safe technology the number of firms is larger than in the equilibrium where the firms use the risky technology. We will later provide conditions on \bar{c} , F and γ , the probability of high marginal costs, that guarantee that this is indeed the case. In any state, in addition

⁴Note that if we did allow for collusion, the profits would be higher and the maximum number of firms might be higher as well. However with competitive loan markets and many potential entrants such collusion could be very difficult to sustain.

to market clearing conditions in product markets, the following three conditions must hold for an equilibrium in which the entrepreneurs produce with the riskless technology: (1) the incumbent firms must make non-negative profits and (2) they must have an incentive to use the safe technology and (3) no additional lending to new entrants is profitable.

Let us now derive the equilibrium conditions for such an equilibrium for an arbitrary stationary market (s, G_t) . Denote by $\pi(n, G_t)$ the equilibrium profit, and by $q(n, G_t)$ the equilibrium output of a single firm who engages in a quantity competition with $n-1$ other firms, all producing with safe technology. It is straightforward to show that:

$$\pi(n, G_t) = \frac{G_t(a-c)^2}{b(n+1)^2} - F, \quad (2.2)$$

and

$$q(n, G) = \frac{G_t(a-c)}{b(n+1)}.$$

To calculate the equilibrium conditions we must also calculate the profits of the firm when optimally deviating to the risky technology, $\pi^d(n, G, \bar{c})$ for $\bar{c} \in \{0, \bar{c}\}$, when the other $(n-1)$ producers produce $q(n, G)$. When the marginal cost \bar{c} is below the price so that the firms produces a strictly positive quantity even in the event that $\bar{c} = \bar{c}$ it is easy to show that:

$$\pi^d(n, G_t, \bar{c}) = \frac{G_t(2a + (n-1)c - (n+1)\bar{c})^2}{4b(n+1)^2} - F$$

When \bar{c} is higher than the market price that would prevail if the deviating firm did not produce at all, then $\pi^d(n, G_t, \bar{c}) = 0$.

One of our assumptions was that the entrepreneur has a limited liability. This means that if the entrepreneur obtains credit with an interest rate R the entrepreneurs payoff is $\max\{\pi - R, 0\}$. Since $\pi^d(n, G_t, 0) > \pi(n, G_t) \geq R$, when $R > \pi^d(n, G_t, \bar{c})$ it may be in the interest of the entrepreneur to choose the risky technology although $E\pi^d(n, G, \bar{c}) < \pi(n, G_t)$. This is because of the convexity that limited liability introduces to his payoff.

We are focusing on the equilibrium in which the firms use the riskless technology and in which the lenders do not discount future within a period. In this equilibrium $R = 0$, as the lenders are behaving competitively, and entry to the markets brings profits down so that in equilibrium $\pi(n, G_t) > 0 > \pi^d(n, G_t, \bar{c})$. As mentioned, we will discuss later the optimality of the equilibrium with riskless technology.

Denote by $V^S(n, G)$ the value of the future payoff stream for an entrepreneur in stationary markets of size G , assuming that n producers, including the entrepreneur, are active in every period. Abstracting from the integer problems, the three equilibrium conditions in state (s, G) can then be written as:

$$(1 - \gamma)\pi^d(n, G, 0) \leq \frac{(1 - \delta + \gamma\delta)\pi(n, G)}{(1 - \delta)} = (1 - \delta + \gamma\delta)V^S(n, G), \quad (2.3)$$

$$\pi(n, G) \geq 0, \quad (2.4)$$

and either

$$(1 - \gamma)\pi^d(n, G, 0) = (1 - \delta + \gamma\delta)V^S(n, G_t) \quad \text{or} \quad (2.5)$$

$$\pi(n, G) = 0$$

By looking at equation (2.3) we note that (because $\pi^d(n, G, 0) > \pi \geq 0$) equation (2.4) is never binding in the equilibrium, and by equation (2.5) we conclude that equation (2.3) holds as an equality in mature markets.⁵ The equality in (2.3) then determines $n(s, G)$ uniquely, where $n(s, G)$ is the maximum number of firms that can obtain credit repeatedly in a stationary market of size G . Substituting for $\pi(n, G)$ and $\pi^d(n, G, 0)$ and rearranging we obtain:

$$\left[\frac{G_t(a-c)^2}{b(n+1)^2} \left[\frac{1-\delta+\gamma\delta}{(1-\delta)(1-\gamma)} \right] - \frac{G_t(2a+(n-1)c)^2}{4b(n+1)^2} \right] = \frac{\gamma F}{(1-\delta)(1-\gamma)}$$

or

$$\frac{G_t}{b(n+1)^2} \left[\frac{\gamma(a-c)^2}{(1-\delta)(1-\gamma)} - (a-c)c(n+1) - \frac{(n+1)^2c^2}{4} \right] = \frac{\gamma F}{(1-\delta)(1-\gamma)} \quad (2.7)$$

This gives us a second order equation for $n(s, G)$ which has the unique solution:

⁵If we require that n is an integer the condition for equality is replaced by:

$$(1 - \gamma)\pi^d(n + 1, G) > \frac{(1-\delta+\gamma\delta)\pi(n+1, G)}{(1-\delta)} = V^S(n + 1, G_t) \quad \text{or} \quad (2.6)$$

$$\pi(n + 1, G) < 0$$

$$n(s, G) = \frac{2c(a-c)}{\xi} \left(\sqrt{1 + \frac{\gamma\xi}{(1-\delta)(1-\gamma)c^2}} - 1 \right) - 1 \quad (2.8)$$

where

$$\xi = \frac{4b\gamma F}{(1-\delta)(1-\gamma)G} + c^2$$

Equation (2.8) shows that both $n(s, G_t)$ and $G_t/(n+1)^2$ are increasing in G_t . In fact, it is easy to see from equation (2.7) or (2.8) that for a very large G , $n(s, G)$ approaches some constant. We have that:

$$n(s, G) \leq \bar{n}^s = \frac{2(a-c)}{c} \left(\sqrt{1 + \frac{\gamma}{(1-\delta)(1-\gamma)}} - 1 \right) - 1$$

and $n(s, G)$ approaches \bar{n}^s as $G \rightarrow \infty$. Given that $G_t/(n+1)^2$ is strictly increasing in G_t we have also the result that $\pi(s, G)$ is strictly increasing in G . Let us summarize these important results:

$$n(s, gG) > n(s, G), \quad \forall g > 1$$

and

$$\pi(s, gG) > \pi(s, G), \quad \forall g > 1.$$

In the equilibrium above the incentives to cheat are bigger in larger markets. Similarly then, so must be the value of behaving prudently, or in other words, the discounted stream of equilibrium profits. The result that (2.3) never binds is similar to the result in Klein and Leffler (1980) and extends the result of Stiglitz and Weiss (1981) to a repeated game setting: in stationary markets the sellers must receive a positive stream of profits to prevent cheating from taking place. What we have also shown is that with quantity competition this stream of profits is larger in larger markets.

Now there are two questions that we have omitted so far and which we can now answer. First question is a condition on F , γ and \bar{c} that guarantees that the equilibrium with safe technology is robust to the threat of fly-by-night entry from firms producing with the risky technology. For this it is sufficient that

$$E\pi^d(n(s, G), G, \bar{c}) \leq 0$$

When this condition holds there is no room for any firm producing with risky technology to enter, even if the lender could somehow persuade the entrant to distribute its entire profits as interest to the lender. Second question is what is a sufficient condition on F , γ and \bar{c} so that the number of firms in the equilibrium in which the firms use the safe technology is larger than the number of firms in the equilibrium with risky technology. The precise conditions for this are given in the appendix but similarly as the first condition, this condition is satisfied as long as γ , F and \bar{c} are large enough. Throughout the analysis we assume this to be the case.

2.3.2 Growing Markets

Having solved what happens in the mature markets in an equilibrium where the firms use the riskless technology we can look what happens in the growing markets. We show that the growth opportunities relax the incentive compatibility constraint allowing for a larger number of firms to operate in growing markets than in stationary markets of the same size. We show that the zero profit constraint can be binding in growing markets whereas in static markets it could not. To get some intuition let us look more closely at the incentive compatibility constraint for the growing markets.

The situation is now more complicated than in the stationary markets. Several things may happen even if the entrepreneur pays back his loan at the end of the period. First there is uncertainty over the next period's growth state. Second, for any given growth state if the equilibrium number of producers is smaller in that state than what it is today, there may be uncertainty over whether any particular entrepreneur is issued credit in the next period or not. That is, there may be some credit rationing among incumbent producers in some states of the world. To calculate the value function at any growth state we must specify the rule that determines who gets credit and who does not in the event that some firms must exit. Given that all entrepreneurs are identical, there are several ways to define consistent decision rules that determine which of the firms exit and which remain. We look at two possible decision rules for exit: (1) "Equal probability of exit for all incumbents" and (2) "First in last out." These definitions are self explanatory. The latter is well motivated in a setting, such as the one that we analyze in section 5, where there are several types of entrepreneurs and a long good credit history increases the probability of a good entrepreneur. For the second exit rule to be well specified we

assume that even if there are several entrants in any given period they enter in some order so that there are no ties.⁶ In both cases, once the entrepreneur is declined credit we assume that he is not issued credit in any of the future periods.⁷

Let us denote by $\psi_j(n, m_i, G_t)$ the probability that entrepreneur i who was issued credit in the previous state (g, G_t) , gets credit in state $(g_j, \min\{g_j G_t, \bar{G}\})$ given that $n - 1$ other entrepreneurs were given credit in previous period and m_i of them have entered the markets before i . That is, $\psi_j(n, m_i, G_t)$ is entrepreneur i 's probability of survival when the next period's growth state is $j \in \{s, g\}$. When the exit rule is "Equal probability of exit for all incumbents" $\psi_j(n, m_i, G_t)$ does not depend on m_i and it is:

$$\psi_j(n, G_t) = \min \left\{ 1, \frac{n(g_j, \min\{G_t g_j, \bar{G}\})}{n(g, G_t)} \right\}.$$

As profits, this probability is adversely related to the current number of producers, but now only in the first order. Similarly when the exit rule is "Last in first out." In that case $\psi_j(n, m_i, G_t)$ does not depend on n and is simply:

$$\psi_j(m_i, G_t) = \begin{cases} 1 & \text{if } n(g_j, \min\{G_t g_j, \bar{G}\}) \geq m_i \\ 0 & \text{if } n(g_j, \min\{G_t g_j, \bar{G}\}) < m_i \end{cases}.$$

Let us first assume that the exit rule is "Equal probability of exit." As before, let us denote by $V^g(G_t)$ the value of the future payoff stream to an incumbent

⁶In a later part of the paper we will assume that in the case of ties the decision rule is "Equal probability of exit". Formally the exit rule is determined by determined how incumbents are ordered in queue as they enter the credit markets.

⁷Since all entrepreneurs are identical and there are infinitely many entrepreneurs we can assume this kind of behavior. Irrespective of that, this would be the case in the equilibria that we consider even if the firm that exited with a good credit history was the next one to enter if there was further entry. This is because once the number of firms declines in the equilibria that we consider, i.e., there is a shakeout, the number of firms never increases again.

entrepreneur in state (g, G_t) . Remember that the value of the outside opportunities to all entrepreneurs are zero. Now, as before, the incentive compatibility and zero profit conditions in state (g, G_t) can be written as:

$$(1 - \gamma)\pi^d(n, G_t, 0) - \pi(n, G_t) \leq \quad (2.9)$$

$$\gamma\delta \left[\rho\psi_s(n, G_t)V^s(G_t) + (1 - \rho)\psi_g(n, G_t)V^g(\min\{G_t g_j, \bar{G}\}) \right],$$

$$\pi(n, G_t) \geq 0. \quad (2.10)$$

In addition, in equilibrium one of the equations holds as an equality.

Look at the state (g, G_t) such that $gG_t = \bar{G}$. We know by our previous results concerning stationary markets that in this case $\psi_g \geq \psi_s$ because more producers can stay in the market if there is growth and the state is (g, \bar{G}) than if it is (s, G_t) . We also know that when there are exactly $n(s, G_t)$ producers in markets $\psi_g = \psi_s = 1$ for all incumbents and since $V^s(\bar{G}) > V^s(G_t)$ neither the incentive compatibility nor the zero profit constraint hold as an equality. Therefore the equilibrium number of firms, $n(g, G_t)$, must be strictly larger than $n(s, G_t)$. But this means that $\psi_s < 1$ and that there is exit when the growth stops at G_t . More generally the same result, namely that $\psi_s < 1$ for any $G_t < \bar{G}$, follows if the following two conditions hold: $V^g(gG_t) > V^s(G_t)$ and $n^g(gG_t) > n^s(G_t)$.

We can solve for the optimal amount of firms in growing markets by backward induction. Having solved for $n(g, gG_t)$ we can work back equations (2.9) and (2.10) recursively to determine $n(g, G_t)$. Because of the structure of the model, however, it is not possible to obtain a closed form representation for $n(g, G_t)$, and therefore the profits or the magnitude of the shakeout, except for extreme values of g and

δ . The following proposition characterizes the number of firms in growing markets when g and δ are large.

Proposition 1: When the exit rule is “Equal probability of exit,” there exists \bar{g} such that for all $g > \bar{g}$, there exists $\bar{\delta}(g)$ such that for all $\delta \geq \bar{\delta}$ and for all $G_t < g^T G_0$:

1. $\pi(g, G_t) = 0$
2. $\psi_g(g, G_t) = 1$
3. $\psi_s(g, G_t) < 1$

That is, when g is larger than some boundary value and δ is sufficiently large, the profits in growing markets are always zero. The number of firms is constantly increasing and there is a shakeout once the growth of the demand stops.

The proof of the proposition is given in the appendix. The intuition is that when g and δ are sufficiently large the value of behaving prudently is very large and the incentive compatibility constraint is not binding even when there is the maximum number of firms operating in markets and the profits are zero. When the zero profit constraint holds in the growing markets we can easily characterize the evolution in prices, quantities produced by all firms and the number of firms in growing markets. We have:

$$n(g, G_t) = \sqrt{\frac{G_t(a-c)^2}{Fb}} - 1$$

As the markets grow, the number of producers increases. This increases competition, increasing the production by each producer:

$$q(g, G_t) = \sqrt{\frac{G_t F}{b}},$$

and resulting in a geometrically decreasing market price:

$$p(g, G_t) = \frac{a(b^2 - 1) + c}{b^2} + \sqrt{\frac{F}{b^3 G_t}}.$$

When the growth stops and the markets mature, the number of firms decreases to the point where the incentive compatibility constraint holds. That is, there is a shakeout as the growth stops. At this point the market price increases to some extent because of a decrease in competition. After that the number of producers, prices and quantities stabilize.⁸

We now look at the second possible exit rule: “Last in first out.” Since the exit rule now depends on m_i , the order in which the entrepreneur i entered the markets, the value function representing the value of a player i ’s expected payoff stream can also depend on m_i in growing markets. So let us denote by $V^g(G_t, m)$ the value of the remaining payoff stream in state (g, G_t) to the firm who was (is) the m ’th firm among incumbents to enter the markets. As before the equilibrium conditions are that

$$(1 - \gamma)\pi^d(n, G_t, 0) - \pi(n, G_t) \leq \tag{2.11}$$

⁸Note that given that the zero profit constraint is binding there can be no strategic behavior by the firms to deter entry. This is no longer true when we allow for heterogenous firms.

$$\gamma\delta \left[\rho\psi_s(m, G_t)V^s(G_t) + (1 - \rho)\psi_g(m, G_t)V^g(\min\{G_t g_j, \bar{G}\}, m) \right],$$

$$\pi(n, G_t) \geq 0. \tag{2.12}$$

for all $m \leq n$, and either

$$(1 - \gamma)\pi^d(n, G) - \pi(n, G) =$$

$$\gamma\delta\rho\psi_s(n, G_t)V^s(G_t) + (1 - \rho)\psi_g(n, G_t)V^g(\min\{G_t g_j, \bar{G}\}, n),$$

or

$$\pi(n, G) = 0$$

where we have made use of the fact that $\psi_g(m, G_t)$ and $V^g(m, G_t)$ are decreasing in m . In this case, we can show that a somewhat similar result holds:

Proposition 2: Assume that the exit rule is “First in last out.” There exists \bar{g} such that for all $g > \bar{g}$, there exists $\bar{\delta}(g)$ such that for all $\delta \geq \bar{\delta}$ and for all $G_t < g^T G_0$ the number of firms is given by $n(g, G_t) = \min \left\{ n_s(\bar{G}), \sqrt{\frac{G_t(a-c)^2}{Fb}} - 1 \right\}$.

That is, when g is positive and δ large enough up to $n_s(\bar{G})$ firms are willing to enter the markets even if they were to earn zero profits during the growth stage of the industry. The intuition is the same: there are large positive profits to be made by the firms if they make it to the large and very profitable markets. With this exit rule there can not, however, be more than $n_s(\bar{G})$ entrants because even

if we were to reach the state (s, \bar{G}) no additional firms would obtain credit in the future. Proof for the proposition is given in the appendix.

An example will help to fix the ideas:

Example:

A numerical example assuming "Equal probability of exit"

G_t	1	2	4	8	16	32	64	128	256	512	1024
$n(g, G_t)$	1.0	1.9	3.0	4.7	7.1	10.4	15.1	21.8	31.2	44.5	20.5
$n(s, G_t)$	1.0	1.7	2.8	4.2	6.1	8.5	11.2	14.1	16.9	19.0	20.5
$q(g, G_t)$	4.5	6.3	8.9	12.6	17.9	25.3	35.8	50.6	71.6	101	429
$q(s, G_t)$	4.6	6.6	9.5	13.8	20.3	30.5	47.1	76.1	129	230	429
$p(g, G_t)$	0.55	0.42	0.32	0.26	0.21	0.18	0.16	0.14	0.13	0.12	0.14
$p(s, G_t)$	0.56	0.43	0.34	0.27	0.23	0.20	0.17	0.16	0.15	0.14	0.14
$\pi(s, G_t)$	0.1	0.2	0.2	0.4	0.6	0.9	1.5	2.5	4.5	8.4	16.0
shakeout	5%	6%	7%	10%	13%	19%	26%	35%	46%	57%	-

$$a = 1, b = 0.1, c = 0.1, \bar{c} = 2, F = 2, g = 2, \rho = 0.1, \delta = 0.9, \gamma = 0.3, \bar{G} = 1024$$

The market size is initially small, demand given by $P_0 = 1 - 0.1Q_0$, but grows at a rate $g - 1$, i.e., 100% per period. For these parameters the positive profit constraint is always binding in growing markets. The number of firms increases as the market size increases and there is a shakeout once the growth stops. In this example the shakeout is larger if the growth stops at a later point in time.

2.4 A Model with Technological Innovations

In this section we look at a model where there are technological innovations until, at some random point in time the technological possibilities become fully exhausted. There are two types of technological innovations: first there are innovations that increase the demand by increasing the consumers' valuation for the product. Second there are technological innovations that merely decrease the costs of production of an existing product. Empirical evidence cited in Gort and Klepper (1982) suggests that both kinds of innovations occur in the young and growing markets, suggesting that they are interrelated. They discuss evidence that the periods of heavy innovation activity are associated with both increasing demand and a continuously declining price. In this section we show that similar results to the previous section can be obtained in competitive markets if the growth of demand is driven by technological innovations that simultaneously reduce the costs of production and increase the optimal plant size.⁹

We assume a similar setting to the previous section in that there are infinitely many of two types of agents: entrepreneurs and lenders. The entrepreneurs are infinitely lived whereas the lenders live only for one period. The entrepreneurs choose between two technologies, riskless and risky, parametrized by a risk parameter θ . As before, the lenders' strategy is to give priority to previous borrowers who have successfully paid their debt and never to lend to an entrepreneur who has failed on his debt. The number of entrepreneurs who are issued credit is determined by the zero profit and the incentive compatibility constraint. We assume that for both the risky and the riskless technology the costs of production are given by:

⁹In the previous section increasing entry to growing markets resulted in an increase in competition which endogenously led to declining prices.

$$C(q) = F + (c + \theta)q + \frac{q^2}{2\phi_t}.$$

F is the fixed cost, c , θ and ϕ are components of the marginal costs. ϕ_t reflects the technological innovations and $\phi_t = \{\phi_0, \dots\}$ is an increasing sequence of technological parameters. As mentioned before, we allow for the possibility that ϕ_t affects the consumers' demand in addition to its effect on costs. We assume that the evolution in ϕ_t is similar to the evolution of G_t in the previous section. We use similar notation as before, and denote the state of the economy by (g_t, ϕ_t) where $g_t = \phi_t/\phi_{t-1}$. The state of the economy evolves as follows: when the growth state is $g_t = g > 1$ the next periods state is $(g, g\phi_t)$ with probability $(1 - \rho)$ and it is (s, ϕ_t) with probability ρ . $s = 1$ and the state (s, ϕ_t) is absorbing. We assume that there exists some upper bound on ϕ such that $\phi_t \leq \bar{\phi}$ where $\bar{\phi} = \phi_0 g^T$ for some finite T . Here the technological innovations decrease the costs of production and increase the optimal plant size.

We assume that the entrepreneurs must borrow the fixed costs F in every period. The lenders and borrowers can ex ante commit to a monitoring technology by which the lender can observe the entrepreneurs profits but at a cost of losing that periods profit. The borrower chooses a quantity $q \in [0, \infty]$ and the level of risk in the marginal costs $\theta \in \{\theta_s, \theta_r\}$. We assume that $\theta_s = 0$, and $\theta_r = \theta_H > 0$ with probability γ and $\theta_r = \theta_L < 0$ with probability $(1 - \gamma)$. We assume that $E\theta_r > 0$.

As before we assume that the lenders do not discount the future within the period that they live and are willing to lend money to entrepreneurs at no cost if they are sure to receive their money back. It is easy to verify that under these conditions

the entrepreneurs optimal quantity choice if he chooses the riskless technology is $q^* = \phi [p - c]$. Let us consider the (interesting) case where the entrepreneurs profit under the optimal production level is strictly negative if $\theta = \theta_H$. The assumption of competitive lending markets guarantees that this will be the case in equilibrium. If the borrower chooses the risky technology and $\theta = \theta_L$ his optimal output is $q^d = \phi [p - c - \theta_L] > q^*$. The associated profits from production are:

$$\pi = \frac{[p - c - \theta]^2 \phi}{2} - F.$$

Because of limited liability, the entrepreneurs payoff is the maximum of π and zero. As before, the entrepreneurs payoff is therefore convex in θ . So by Jensen's inequality, other things equal, the entrepreneur prefers variance in θ . He continues to prefer risk in θ even if the expected value of θ_r is greater than zero, as long as the variance in θ_r is large enough. In fact, in equilibrium the number of firms and price are such that the expected periodic payoff to the entrepreneur is larger under the risky technology than under the riskless technology.

As mentioned, we would like to allow for the possibility that the innovations affect positively the level of demand. Let us therefore assume that the demand is linear in ϕ^α for some $\alpha \geq 0$. If $\alpha > 0$ the demand is increasing in ϕ and if $\alpha = 1$ it is linear. Otherwise we assume a constant elasticity of demand, so that:

$$Q = \phi^\alpha p^{-\eta}, \tag{2.13}$$

where η is the elasticity of demand.

2.4.1 Stationary Markets

For now let us assume that ϕ_t is fixed and look at the market equilibria for different ϕ . The equilibrium conditions for an equilibrium in which the firms always use the safe technology are:

$$\pi(n, \phi) \geq 0 \quad (2.14)$$

$$\frac{\pi(n, \phi)}{(1 - \delta)} \geq (1 - \gamma)\pi^u(n, \phi, \theta_L) + (1 - \gamma)\frac{\delta\pi(n, \phi)}{1 - \delta} \quad (2.15)$$

and a requirement that one of these equations holds as an equality. We can write the incentive compatibility constraint as

$$\frac{1 - \delta(1 - \gamma)}{1 - \delta} \left[\frac{[p - c]^2 \phi}{2} - F \right] \geq (1 - \gamma) \left[\frac{[p - c - \theta_L]^2 \phi}{2} - F \right]$$

\Leftrightarrow

$$\phi \left[\frac{\gamma [p - c]^2}{1 - \delta} + (1 - \gamma) [2[p - c]\theta_L - \theta_L^2] \right] \geq \frac{2\gamma F}{(1 - \delta)} \quad (2.16)$$

It is easy to see that $\pi(n, \phi) = 0$ implies that the left hand side of (2.15) is zero while the right hand side is strictly positive implying that the incentive

compatibility constraint does not hold. We therefore conclude that $\pi^*(n, \phi) > 0$ and that the relevant constraint to look is the incentive compatibility constraint. In equilibrium, therefore, (2.15) holds as an equality. This allows us to solve for $p(\phi)$:¹⁰

$$p(\phi) = c + \frac{-\theta_L(1-\delta)(1-\gamma)}{\gamma} \left[\sqrt{1 + \frac{\gamma}{(1-\delta)(1-\gamma)} + \frac{2\gamma^2 F}{(1-\delta)(1-\gamma)^2 \theta_L^2 \phi}} + 1 \right]$$

From this equation we can see that $p(\phi)$ is decreasing in ϕ . It is also easy to see that $[p - c]^2 \phi$, the firm's profit along the equilibrium path, is increasing in ϕ . In fact as $\phi \rightarrow \infty$, p converges to

$$p \rightarrow \underline{p} \equiv \frac{-\theta_L(1-\gamma)(1-\delta)}{\gamma} \left(1 + \sqrt{\frac{1-\delta(1-\gamma)}{(1-\delta)(1-\gamma)}} \right) + c,$$

so that in the limit the changes in profits are proportional to changes in ϕ . We have therefore shown that the profits are larger in markets with better technology. This corresponds to our earlier result with the model with varying demand: that profits are larger in larger markets. We now look what happens to the number of firms in stationary markets as a function of the market size. Equation (2.13) gives us that:

$$p = \phi^{\frac{\alpha}{\eta}} Q^{-\frac{1}{\eta}} = \phi^{\frac{\alpha}{\eta}} (n [p - c] \phi)^{-\frac{1}{\eta}},$$

or that :

$$n = \frac{\phi^{\alpha-1} p^{-\eta}}{[p - c]}$$

¹⁰Remember that $\theta_L < 0$.

We have seen that p decreases in ϕ . However the effect of ϕ on n is more complicated. If $\alpha \geq 1$ so that the technological innovations have a profound effect on demand, then the number of firms always increases in ϕ . If, however, α is zero, so that the technological innovations affect only the costs, then everything depends on the elasticity of demand η and the magnitude of ϕ . In this case we can see that if η is large then n first increases in ϕ since p decreases in ϕ . However as ϕ keeps increasing p converges to some number and for large ϕ the equilibrium number of firms necessarily decreases in ϕ . For small η , when $\alpha < 1$, the equilibrium number of firms decreases in ϕ right from the start.

2.4.2 Growing Markets

Let us now look at the growing markets. As before we denote by $\psi_{g_{t+1}}(\phi_t)$ the probability of survival to the next period given that the next period's state is $(g_{t+1}, g_{t+1}\phi_{t+1})$. Again we consider two possible exit rules: "Equal probability of exit" and "First in last out." Denote by m_i the measure of firms that have entered before firm i and by k the measure of firms that entered the same time as i . As before, for these exit rules

$$\psi_{g_{t+1}} = \frac{n(g_{t+1}, \phi_{t+1})}{n(g_t, \phi_t)}$$

and

$$\psi_{g_{t+1}}^i = \begin{cases} 0 & \text{if } n(g_{t+1}, \phi_{t+1}) < m_i \\ \frac{n(g_{t+1}, \phi_{t+1}) - m_i}{k} & \text{if } m + k > n(g_{t+1}, \phi_{t+1}) \geq m_i \\ 1 & \text{if } n(g_{t+1}, \phi_{t+1}) \geq m_i + k \end{cases} .$$

respectively. That is, if there are several firms that have entered at the same time and some of these firms have to exit we have assumed “Equal probability of exit” even when the exit rule is “First in last out.”

As before, we can write the incentive compatibility constraint and the zero profit constraint for firm i in state (g, ϕ_t) as:

$$\pi(n, \phi_t) \geq 0$$

and

$$\delta\gamma E^i \psi_{g_{t+1}} V(g_{t+1}, \phi_{t+1}) \geq (1 - \gamma)\pi^d(n, \phi_t, \theta_L) - \pi(n, \phi_t)$$

Further equilibrium requirement is that one of these equations holds as an equality for the entrants in the last cohort. Remember that as before $\pi^* = [p - c]^2 \phi/2 - F$ and $\pi^d = [p - c - \theta_L]^2 \phi/2 - F$. These equations allow us to recover, by backward induction, the prices and the number of firms in all growth states.

By increasing g and δ we can again make the profits $\pi(s, g\phi_t)$ arbitrarily large in relation to $\pi(s, \phi_t)$. Note however that if $\alpha < 1$ we may at the same time reduce the number of firms that can stay in the markets in state $(s, g\phi_t)$. This suggests that for $\alpha \geq 1$ similar results to the previous section can be obtained. For $\alpha < 1$, however, the elasticity of demand as well as the exit rule are crucial in determining the equilibrium number of firms in growing markets and thereby the extent of the shakeout. For instance when $\alpha = 0$, in contrast to the case where $\alpha > 0$, the value of expected future earnings for an incumbent producer do not necessarily increase when we increase g . Comparing stationary markets with different ϕ when $\alpha = 0$ the

profits of the incumbent firms are proportional to ϕ in the limit, but at the same time the number of firms, $n(s, \phi)$, is inversely proportional to ϕ . When $\alpha < 1$ and the exit rule is: “First in last out,” larger g may actually decrease the number of firms in growing markets of any given size.

To conclude, when the industry growth is driven by technological innovations the path of industry evolution is similar to the previous section if the technological innovations simultaneously increase the demand for the product and increase the optimal plant size. By “similar” we mean that even the very extreme results that profits are always zero in growing markets and there is always a shakeout when the growth stops are likely to be obtained in this case. The analysis suggests also that if the technological innovations affect only by reducing costs and increasing the optimal plant size then the industry evolution would be similar to the previous section only when η , the elasticity of demand, is large and ϕ is small.

2.5 Two Types of Entrepreneurs

In this section we study a model where there are two types of entrepreneurs: First there are entrepreneurs who can use the safe technology just as in the previous section. Secondly there are entrepreneurs who despite their effort simply can not. We call these “good” and “bad” entrepreneurs respectively. We assume that the proportion of bad entrepreneurs among the entrepreneur population is μ and that the proportion of good entrepreneurs is $1 - \mu$. The bad entrepreneurs, when using the safe technology, have only an upside risk in their costs in the sense that when they are using the safe technology with probability γ the costs are higher than for the good entrepreneurs with the safe technology and with probability $(1 - \gamma)$

they are the same. The entrepreneurs do not know whether they are good or bad before they experience a bad cost realization when using the safe technology. Given these assumptions, there must now be a positive monetary transaction from the entrepreneur to the creditor in every period whenever the amount of firms is such that bankruptcy is possible, that is, there must be a nonzero interest rate. We must also modify the lender's strategy for the use of the costly monitoring technology. We assume that they use this technology whenever the entrepreneurs payoff is less than the minimum of the scheduled interest payment and the lower support of his profits along the equilibrium path.

In this kind of setting the exit rule is clearly: "First in last out." In contrast to the former model there will now be entry and exit both in growing and stationary markets. In fact, the young and growing markets have a high exit rate in addition to a high entry rate, as many of the producing entrepreneurs are still bad. In stationary markets the bad entrepreneurs are weeded out over time and the pool of entrepreneurs becomes better and better. At this stage, the exit rate as well as the interest rate slowly decline to zero.

In order to calculate the equilibrium, we need the conditional probability that an entrepreneur is bad (good) given his credit history. Clearly in an equilibrium where every entrepreneur attempts to use the safe technology, a failure to pay ones debt is proof of being a bad type. If, on the other hand, a firm has obtained and paid his credit back in τ previous periods, then by Bays rule he is bad with probability:

$$\lambda(\tau) = \frac{\mu(1 - \gamma)^\tau}{1 - \mu + \mu(1 - \gamma)^\tau} < 1.$$

In an equilibrium where all entrepreneurs use the safe technology, or try to use it,

this is also the proportion of bad entrepreneurs in the cohort of entrepreneurs that entered τ periods ago.

Given this, we want to calculate what is the interest rate at which an entrepreneur with such credit history would obtain credit, if he does. Denote by $r(\tau)$ the interest rate than an entrepreneur who has paid his credit τ times would have to pay if he obtains credit. Clearly $r(\tau)$ must satisfy:

$$r(\tau) = \lambda(\tau)\gamma (F + r(\tau) - \pi^d(n, \phi, \bar{c})).$$

So we have

$$r(\tau) = \frac{\lambda\gamma [F - \pi^d(n, \phi, \bar{c})]}{1 - \lambda\gamma} = \frac{\gamma [F - \pi^d(n, \phi, \bar{c})]}{\frac{1-\mu}{\mu(1-\gamma)^\tau} + 1 - \gamma}.$$

Characterizing an equilibrium in this case is more difficult than in the previous case. Here the entrepreneurs incentives are affected besides the growth rate in the markets by also the probability with which he is good, and possibly the probabilities with which the other incumbent entrepreneurs who have entered before him are good. If we are dealing with finitely many incumbents, because the zero profit condition does not hold for an incumbent who has a good credit rating when producing the Cournot quantity, there may be room for him to behave strategically. If we are dealing with a continuum of entrepreneurs, as in the previous section, these strategic issues do not arise. For simplicity let us therefore focus on the framework of the previous section: the case of technological innovations and assume that there is a continuum of entrepreneurs active in markets of any given size.

In stationary markets, $\phi_t = \phi$, in every period, in case an entering entrepreneur is certain to remain in the markets as long as he pays his debt the value of his remaining payoffs when choosing the safe technology are:

$$V_t^S(n_t, \phi) = \sum_{\tau=0}^{\infty} \delta^\tau [(1 - \lambda(0)) + \lambda(0)(1 - \gamma)^\tau] [\pi(n_\tau, \phi) - r(\tau)]$$

Where n_τ is determined in equilibrium and is typically decreasing in stationary markets as bad firms exit over time.¹¹ Assuming that there is no exit except for firms who fail to pay their debt, the number of firms who are active in markets is given by:

$$n_t = \sum_{\tau=0}^t m_\tau [1 - \mu + \mu(1 - \gamma)^{t-\tau}],$$

where m_τ is the number of entrants in period τ . The proportion of good incumbents in period t is then:

$$v_t = \frac{(1 - \mu) \sum_{\tau=0}^t m_\tau}{n_t}$$

Equilibrium, once again is characterized by the zero profit constraint and the incentive compatibility constraint (for entrants and all cohorts of incumbents). For an entrant who entered in period τ the zero profit constraint is:

¹¹Note that given the exit rule and deterministic evolution of different types of entrepreneurs it can be shown by a backward induction argument that only entrepreneurs who have a possibility of remaining in the markets for an infinitely long period of time can ever have an incentive to choose a safe technology and enter the markets.

$$\pi(n_t, \phi) - r(t - \tau) \geq 0,$$

The incentive compatibility constraint for not taking risks is that

$$V_t^S(n_t, \phi, \tau) \geq (1 - \gamma)\pi(n, \phi, \theta_L) + (1 - \gamma)\delta V_{t+1}^S(n_{t+1}, \phi, \tau + 1)$$

\Leftrightarrow

$$(1 - \gamma)\pi^d(n, \phi, \theta_L) - [1 - \gamma\lambda(t - \tau)]\pi(n, \phi) \leq \delta\gamma(1 - \lambda(t - \tau)).$$

$$\sum_{i=t+1}^{\infty} \delta^{\tau-t-1} \left[[(1 - \lambda(t - \tau + 1)) + \lambda(t - \tau + 1)(1 - \gamma)^{\tau-t}] [\pi(n_i, \phi) - r(i - \tau)] \right]$$

We can see that the incentive compatibility constraint and the zero profit constraint are relaxed as we decrease τ . Therefore these constraints are binding to a maximum of one cohort of entrepreneurs. In particular, if they are binding for entrants then they are satisfied for all incumbent cohorts of entrepreneurs. Also, if these conditions are satisfied when the number of incumbents is \bar{n} , they are also satisfied when the number of incumbents is less than \bar{n} .¹²

¹²The above equations do not take into account the second moral hazard problem - namely that the firm wants to pay back its entire loan and does not steal the lenders money by claiming that the cost realization was \bar{c} . This condition is:

$$\delta V^S(n, \phi, \tau) \geq \pi^d(n, \phi, 0) - \pi.$$

We have ignored this constraint above because we want to focus on the other moral hazard problem that we have identified. We therefore have implicitly assumed that the firms experience a very high disutility from stealing.

In principle these two conditions allow us to calculate the equilibrium number of entrants, incumbent firms as well as prices over time in stationary markets given any initial pool of entrepreneurs. First we have to calculate the incentive compatibility constraint for all possible proportions of good firms. Then we use backward induction to figure out what the entry rate is for any particular proportion of bad entrepreneurs in the first state that there is entry. We can then back to the previous period and calculate the entry rate in that period and so on.

We can then proceed to the growing markets. Here the exit rule is “First in last out,” because firms with longer credit history are in a strictly better position to obtain credit in the future. We can write the incentive compatibility constraint now as:

$$\begin{aligned} & \delta\gamma(1 - \lambda(t - \tau)) E^i \left[\psi_{g_{t+1}} V_{t+1}(g_{t+1}, \phi_{t+1}, \tau) \mid v_\tau, m_\tau, n_\tau, v_t, n_t \right] \\ & \geq (1 - \gamma)\pi^d(n, \phi_t, \theta_L) - (1 - \gamma\lambda(t - \tau)) \pi(n, \phi_t). \end{aligned}$$

When the number of firms in any given cohort must be rationed we assume that the exit rule is “Equal probability of exit.” We have now developed algorithms that can be used to determine the entire course of industry evolution in terms of number of firms, entry and exit rates, outputs produced and prices.

We can make a few observations already from the shapes of the incentive compatibility constraints. Other things equal, an uncertainty over the other incumbents abilities increases an entrants incentives to enter and behave prudently because it affects the future exit rate and the number of incumbents with better

credit history in the future periods. Uncertainty over one's own ability, on the other hand, reduces firms willingness to behave prudently. As was discovered in the previous section the technological innovations can result in several types of industry dynamics depending on the magnitude of α , that is, the magnitude by which the innovations affect demand, as well as the elasticity of demand. Major difference to the model of the previous section is that now there is entry and exit in both growing and mature markets. The latest entrants are most likely to exit (Horvath et. al., 1996) and there is most entry and exit in young markets (Klepper and Miller, 1996). As the markets mature and the pool of existing entrepreneurs slowly becomes better, the incentives of a new entrant to behave prudently are reduced and the entry rate to the markets decreases. This observation suggests that the shakeout is now much more gradual than in the single type of entrepreneur model.

2.6 Conclusion

We studied industry evolution when the entrepreneurs must finance their investments by borrowing and there is concern over their incentives to take risks. The demand for industry output was exogenously determined. We assumed that the demand is first small but growing at a high rate. The demand continues to grow until, at some random point in time, the markets mature and growth rate is significantly reduced. The firms have no saving technology and must borrow the funding for their investments in every period. The entrepreneurs incentives to take risks are proportional to their output and the market for lending is competitive. We first analyzed a market with finitely many firms competing in quantities.

Under these conditions the market evolution resembles that documented by

the empirical research: e.g. Gort and Klepper (1982). First the number of firms is small and, due to the lack of competition, the price for industry output is high. As markets grow the number of firms increases, competition increases and price falls towards the marginal costs. When markets mature, there is a shakeout and the number of firms is dramatically reduced. The shakeout occurs because with lower growth the firms incentives to behave prudently are reduced and many of the existing firms are denied credit. We then extended the model to a case where the growth of demand was driven by technological innovations which also decreased the costs of production and made the optimal plant size bigger. Here we allowed for a continuum of firms to enter to the markets and assumed two types of entrepreneurs: "good" and "bad". The good as well as the bad entrepreneurs had a choice between two technologies, safe and risky, but the bad entrepreneurs experienced high cost shocks with a positive probability even when they used the safe technology. For this case we developed algorithms that allow us to track the industry evolution in terms of prices, quantities, entry and exit rates and the number of producers. It was argued that when the technological innovations affect both demand and costs the industry evolution is similar to the basic model, except that now there is entry and exit in both growing and static markets. It was argued that the relative exit rate is highest in the young markets because in these markets the proportion of bad entrepreneurs is the greatest.

Chapter 3

Word-of-Mouth Communication and Community Enforcement

3.1 Introduction

It has long been recognized that community enforcement can make sellers behave cooperatively even when they meet particular buyers only infrequently and have a short term incentive to cheat, e.g., to supply low quality or to shirk in a labor contract. For instance, Klein and Leffler (1981) study the problem of credibly committing to offer high quality in a model where a continuum of buyers are randomly matched with several sellers and each seller has a short term incentive to supply low quality at a lower cost. In their model community enforcement by the buyers, through a coordinated boycott after observing low quality, provides incentives for the seller to produce high quality. The results of Klein and Leffler (1981), along with most of the existing literature on community enforcement, depend upon the assumption that past quality choices of the seller are public information.

When the number of sellers and buyers is large and particular sellers and buyers meet only infrequently or only once, the assumption of public information seems rather demanding. Recently this observation has led to a number of articles looking at community enforcement with less stringent informational assumptions. These papers include Milgrom et. al., (1990), Okuno-Fujiwara and Postlewaite (1995), Kandori (1992) and Ellison (1994). However, partially due to the difficulties of dealing with private information, all of the above mentioned papers have made extreme informational assumptions: either complete anonymity of players together with the assumption that players observe only the actions chosen in their own games, or alternatively, locally complete information, which allows a player to perfectly observe the status of his current opponent, based on the opponents past behavior. See also Greif (1993), Harrington (1995), Greif (1994) and Greif et. al., (1994).

Kandori (1992) and Ellison (1994) study a repeated prisoners' dilemma in a large but finite-population random-matching setting, where players are unable to recognize their opponents. They show that even then there exist sequential equilibria where all players play cooperatively in every period. Cooperation is supported by community enforcement based on contagious strategies: all players who are cheated immediately start cheating their opponents, understanding that the whole society is in a process of switching into non-cooperative actions. In the equilibrium, players behave cooperatively to avoid initiating a general switch to non-cooperative actions. An important factor underlying Kandori's and Ellison's results is that defection is a dominant strategy of the stage game. Whether contagious strategies would work in a repeated random matching game that does not share this property, such as the buyer seller game we are about to study, is still unknown. In addition, at least under their informational assumptions, the cooperative equilibria based on conta-

gious strategies are unstable in the sense that a single “insane” player who does not cooperate can destroy the good equilibrium for all agents (Ellison, 1994).

Okuno-Fujiwara and Postlewaite (1995) and Kandori (1992) consider games with local information processing: 1) Each player carries a label observable to her opponent, 2) when two players are matched they observe each other’s label before choosing their actions, 3) a player and his partner’s actions and labels today determine their labels tomorrow. The information processing is “local” in the sense that the actions chosen by a pair of players are based only on their labels, not on the entire distribution of labels across the population, and the updating of each player’s label depends only on the previous labels and the outcome of the stage game. When the population is large and players are randomly matched, the observability of the current trading partner’s label and the updating of the labels require the existence of some efficient information transmission and processing mechanism. This could be a medieval law judge (Milgrom et. al., 1990) or institutions like credit bureaus which track the transactions of every agent.

In many real life situations, however, social norms and informal information transmission mechanisms can replace formal institutions and still facilitate cooperation. In this paper, we present a model of community enforcement that is based on word-of-mouth communication. The information transmission is highly imperfect in the sense that information about each defection spreads only to part of the player population and defectors can not always be immediately punished. Despite this, since players can be identified, *private* reputations evolve. This allows equilibria where only defectors are punished; making our equilibria more stable with respect to “insane” players, who do not cooperate, than those based on contagious strategies. In fact, when information is privately costly, only some of the sellers can

cooperate in any equilibrium. Nonetheless, word-of-mouth communication is shown to be surprisingly efficient in facilitating cooperation.

We assume there are M sellers and M buyers, where M is large but finite number, and that in each time period $t = 0, 1, 2, \dots$, the sellers and buyers are randomly matched to play a stage game that contains an opportunity for a mutually beneficial trade. The sellers have a short-term incentive to supply low quality and will supply high quality only if the gains from maintaining good reputation outweigh the short-term loss. For simplicity, it is assumed that buyers would not knowingly purchase low quality at any price. We assume that buyers have networks of communication that, roughly speaking, work in the following manner: In each period each buyer observes N trades in addition to his own and N buyers, called spectators, observe his trade and send him signals regarding his current trading partner. We can think of these N spectators as friends of the buyer or just people who happen to pass by. We assume that the identities of the spectators can change from period to period.

Throughout the paper we consider two kinds of strategy profiles, which differ in the informativeness of the signals. The strategy profile with the less informative (actually totally uninformative) signals is equivalent to a model where signalling is not allowed. For these two strategy profiles we provide sufficient conditions on N and the discount factor for a sequential equilibrium where good quality is supplied by all sellers in every period. Assuming the existence of a public randomization device and high enough discount factor, these conditions can be stated as $N \geq N^*$ where N^* is a constant determined by the population size, discount factor and the payoff matrix. As one of our main results, we show that with informative signalling N^* is a diminishing fraction of the population size.

We then study a model where “networking” (i.e., setting up N connections) is costly. In this case, when M is large, we show that there must be a positive probability of sellers producing low quality goods in any equilibrium in order to give buyers an incentive to network. When the costs of networking are below a threshold value, we find a sequential equilibrium in which sellers initially randomize between high and low quality and continue to produce high quality if and only if they produced high quality in the first period. In this equilibrium the probability of buying low quality goods increases in M and the cost of networking. When the cost of networking reaches the threshold value, trade collapses because the probability of low quality goods that would provide agents with sufficient incentive to network is so large that each buyer no longer wishes to experiment with an unknown seller.

In the existing literature on quality provision, it is assumed that agents instantaneously learn about a seller’s defection, see e.g. Klein and Leffler (1981) and Allen (1984). In this paper we show that word-of-mouth communication can spread information rather quickly and make such assumptions reasonable approximations in some settings. When the population is large and information privately costly, our results suggest, however, that both high and low quality would be produced in equilibrium. To further understand the role of institutions in transmitting information, as in Milgrom et. al. (1990), we feel it is useful to understand the workings of informal channels of information transmission. We hope that our formalization of word-of-mouth communication has interest on its own.

The rest of the paper is organized as follows: Section 2 gives the formal description of the model. Section 3 discusses players’ strategies and presents sufficient conditions for sequential equilibria with informative and uninformative signalling, where high quality is produced by every seller in every period. Section 4 shows

that with informative signalling as M goes to infinity, trade can be sustained with buyers networking with a diminishing proportion of other buyers. Section 5 studies costly networking and section 6 concludes the paper.

3.2 The Model

There are two finite sets of players $M_k = \{1, 2, \dots, M\}$, $k = S, B$. Denote by M_S the set of sellers and by M_B the set of buyers. We envisage the sellers as being positioned at fixed locations around a circle, where the locations are numbered clockwise from 1 to M . We refer to seller i as the seller at location i . It is assumed the buyers can identify the sellers by the number of their location, but the sellers can not recognize the identities of the buyers.¹

Let Θ be the set of all permutations of M_B . In each period $t = 0, 1, 2, \dots$, a permutation $\theta_t \in \Theta$ is chosen with uniform probability, independent of previous realizations. Buyer $\theta_t(i) \in M_B$ is placed at location i to play with seller $i \in M_S$ the following simultaneous move “trade” game:

		<i>Buyer</i> $\theta_t(i)$	
		<i>B</i>	<i>NB</i>
<i>Seller i</i>	<i>H</i>	1, 1	0, 0
	<i>L</i>	1 + g , - ℓ	0, 0

where both g and ℓ are taken to be strictly positive numbers. The first (second) number in each entry indicates the seller’s (buyer’s) payoff. Seller’s actions H and

¹The more general assumption that sellers can also recognize the identities of buyers does not change our results. The two strategy profiles that we consider would be equilibria of such a game under conditions slightly different from ours.

L refer to providing “high-quality” and “low-quality” while buyer’s action B refers to “buy” and NB to “not buy”. With g and ℓ strictly positive, L is the dominant strategy for the seller and (L, NB) is the only Nash equilibrium of the trade game. The sellers and buyers have a common discount factor $\delta \in (0, 1)$ and their overall payoffs are the discounted sum of payoffs from the trade games.

In each period $t = 0, 1, 2, \dots$, there is preplay communication among neighboring buyers before the trade games. To be precise, the stage game proceeds as follows:

1. After θ_t is realized, buyer j observes θ_t , recognizes the identity of his opponent, $\theta_t^{-1}(j)$, as well as the identities of his “neighboring” sellers, $\theta_t^{-1}(j) + 1, \theta_t^{-1}(j) + 2, \dots, \theta_t^{-1}(j) + N$, where $N \leq M/2 - 1$.^{2,3,4} Let us denote by $S_j(\theta_t) = \{\theta_t^{-1}(j) + k\}_{1 \leq k \leq N}$ the subset of neighboring sellers, whom buyer j observes at period t , and by $N_j(\theta_t) = \{\theta_t(\theta_t^{-1}(j) + k)\}_{1 \leq k \leq N}$ their period t matches. We call $N_j(\theta_t)$ buyer j ’s neighboring buyers at period t . Also, let us denote by $N_j^s(\theta_t) = \{\theta_t(\theta_t^{-1}(j) - k)\}_{1 \leq k \leq N}$ the subset of buyers, who observe the interaction between j and $\theta_t^{-1}(j)$ at period t . We call $N_j^s(\theta_t)$ the spectators to buyer j ’s game at period t . Notice that the identities of the spectators, neighboring sellers and buyers depend on θ_t .

2. Buyer j sends a payoff-irrelevant signal to each of his neighboring buyers $n \in N_j(\theta_t)$ and receives a message from each spectator in $N_j^s(\theta_t)$. Let us

²The assumption that buyers observe θ_t is made to ease the notation. An alternative assumption that does not change our results would be that buyer j is able to recognize only the identities of his opponent $\theta_t^{-1}(j)$ as well as his neighboring sellers in period t .

We also make the assumption that $N \leq M/2 - 1$. The extension of our analysis to $M/2 \leq N \leq M - 1$ is trivial. The case $N = M - 1$ would then correspond to the game with perfect observability, while the case $N = 0$ to the game where each buyer observes only the outcome of his trade game and the identity of his opponent.

³Sellers are female and buyers are male.

⁴These locations are of Mod M .

introduce the following notation.

- $C = \{\gamma, \beta\}$: the set of possible signals. We can interpret a signal γ or β as meaning respectively “Good” or “Bad”.
 - $m_t^j(\ell) \in \{\gamma, \beta\}$: the signal from buyer j to buyer $\ell \in N_j(\theta_t)$.
 - $m_t^j \in \{\gamma, \beta\}^N$: the N -tuple of the signals from j to each of his neighboring buyers in $N_j(\theta_t)$.
 - $m_t(j) \in \{\gamma, \beta\}^N$: the N -tuple of the signals from j 's spectators, $N_j^s(\theta_t)$, to buyer j .
3. Seller $\theta_t^{-1}(j)$ and buyer j play the 2×2 simultaneous move trade game described above. Denote the outcome (or the realized action profile) of that game by $(a_t^S(\theta_t^{-1}(j)), a_t^B(j))$, where $a_t^S(\theta_t^{-1}(j)) \in A_S = \{H, L\}$, $a_t^B(j) \in A_B = \{B, NB\}$.
4. In addition to his own outcome, buyer j observes the realized action profiles of the period t trade games played by the sellers $i \in S_j(\theta_t)$ and buyers $n \in N_j(\theta_t)$. Denote this observation by $o_t(j) = \left((a_t^S(i), a_t^B(\theta_t(i))) \right)_{i \in S_j(\theta_t) \cup \{\theta_t^{-1}(j)\}} \in (A_S \times A_B)^{N+1}$.

The information that buyer j receives in period t can now be written as $(\theta_t, m_t(j), o_t(j))$. We denote with $H^t(j)$ the set of all possible histories for buyer j up to but not including period t . By convention, let $H^0(j) = \emptyset$. An element $h_t(j) \in H^t(j)$ includes all past realizations of θ_s , all past messages to player j , $m_s(j)$, all past messages from player j , m_s^j , and all past observations of player j , $o_s(j)$, where $0 \leq s < t$. Hence $h_t(j)$ is:

$$h_t(j) = (\theta_\tau, m_\tau(j), m_\tau^j, o_\tau(j))_{\tau=0}^{t-1}.$$

A pure strategy for buyer j is then a sequence $\{\widehat{m}_t^j, \widehat{b}_t^j\}_{t=0}^\infty$, where

$$\begin{aligned}\widehat{m}_t^j &: \Theta \times H^t(j) \rightarrow \{\gamma, \beta\}^N \\ \widehat{b}_t^j &: \Theta \times H^t(j) \times \{\gamma, \beta\}^N \rightarrow \{B, NB\}.\end{aligned}$$

$\widehat{m}_t^j(\theta_t, h^t(j))$ specifies the N -tuple of signals that buyer j with private history $h^t(j)$ sends to his neighboring buyers $n \in N_j(\theta_t)$ in period t . $\widehat{b}_t^j(\theta_t, h^t(j), m_t(j))$ specifies the choice of action for buyer j in the period t trade game against seller $\theta_t^{-1}(j)$, when j has private history $h^t(j)$ and he receives signals $m_t(j)$ in the period t communication stage. Correspondingly, let $\{\widehat{\mu}_t^j, \widehat{\beta}_t^j\}_{t=0}^\infty$ denote a behavioral strategy for buyer j , where

$$\begin{aligned}\widehat{\mu}_t^j &: \Theta \times H^t(j) \rightarrow \Delta\{\gamma, \beta\}^N \\ \widehat{\beta}_t^j &: \Theta \times H^t(j) \times \{\gamma, \beta\}^N \rightarrow \Delta\{B, NB\}.\end{aligned}$$

For seller i we define pure and behavioral strategies as sequences of maps $\{\widehat{s}_t^i\}_{t=0}^\infty$ and $\{\widehat{\sigma}_t^i\}_{t=0}^\infty$, where

$$\begin{aligned}\widehat{s}_t^i &: (\{H, L\} \times \{B, NB\})^t \rightarrow \{H, L\}. \\ \widehat{\sigma}_t^i &: (\{H, L\} \times \{B, NB\})^t \rightarrow \Delta\{H, L\}.\end{aligned}$$

Note that the assumption that a seller does not recognize the identity of a buyer is implicit in this notation.

Because of the private histories that players have, the equilibrium concept that we apply is sequential equilibrium. Sequential equilibrium requires that after any history player's equilibrium strategy maximize his (her) expected payoff, taking as given all other player's strategies and his beliefs about the signals and actions taken by all other players in all previous periods. Furthermore, his beliefs should be "consistent" with the equilibrium strategy profile and private history, in the sense of

Kreps and Wilson (1982).⁵ A trivial sequential equilibrium is one where sellers play L and buyers NB after any history: the repetition of the *only* Nash equilibrium of the trade game. We are interested, however, in sequential equilibria that support the efficient outcome where (H, B) is played by all players in every period.

For the most part we confine our analysis on a particular class of strategy profiles, which we call “unforgiving”. These strategy profiles require sellers to sell high quality in period zero (in section 5 with some probability), and sell high quality thereafter if and only if 1) they have always done so, and 2) buyers have always purchased their goods. Under the unforgiving strategy profile buyers play B , except to punish a seller by playing NB when they are informed of her defection. The strategy profiles are unforgiving in the sense that informed buyers punish a defector whenever they meet her.

There are two reasons for focusing on these strategy profiles. First, they are simple: In fact, because of the private information that players have, it is difficult to imagine other strategies that could support the efficient outcome as a sequential equilibrium in this game. For instance, it is not obvious whether contagious strategy profiles, where a seller’s defection affects how buyers treat other sellers, would be equilibria in this game. Checking the incentives of a buyer to follow such a strategy on off-the-equilibrium paths is very complicated, because his incentives depend on his belief about the previous plays, which in turn depend on his private history.⁶ Strategies with less severe, finite punishments are also difficult to implement because buyers typically do not know the time of the first defection and therefore cannot

⁵In Kreps and Wilson (1982), the definition of sequential equilibrium requires the specification of beliefs system as well as a strategy profile. Because the beliefs system which is consistent with our strategy profiles is simple, we refer only to the strategy profile when describing a sequential equilibrium.

⁶See Kandori (1992) for a discussion on the difficulties of private information.

synchronize the last period of a punishment phase. Unforgiving strategies avoid these problems and the buyers incentives are easily shown to be satisfied. The second reason for focusing on these strategy profiles is that, in the class of non-contagious strategy profiles (i.e., where one seller's action does not affect how the other sellers are treated), these strategy profiles provide the maximum punishment for the seller. This is important because the conditions for the efficient outcome that we derive then characterize the minimum N that is necessary for the efficient outcome in any sequential equilibrium based on non-contagious strategy profiles.

3.3 Exogenous Connections and Trade

In this section we provide sufficient conditions in terms of N for a sequential equilibrium where (H, B) is played by all players in every period. In our model information about sellers' behavior may spread through two possible sources, the effectiveness of which depends on the number of spectators, N . First, by observing the outcomes of $N + 1$ trade games in each period, a buyer receives information about $N + 1$ sellers: he observes their current actions and may infer knowledge of their past defections from the actions of their opponents. Second, the information can be transmitted through direct communication among neighboring buyers. The effectiveness of direct communication depends, however, on the information content of the signals.

We now introduce two unforgiving strategy profiles that differ with respect to the informativeness of the buyer's signals. For obvious reasons we refer to the first as the *Uninformative Strategy Profile* and to the second as the *Informative Strategy Profile*. In all periods $t = 0, 1, 2, \dots$, after θ_t is realized:

The strategy for seller i is same under both strategy profiles and is:

- (I) In the first period play H . After that, if the outcome in seller i 's past games was always (H, B) , play H .
- (II) Play L otherwise.

The *Uninformative Strategy* for buyer j is:

- (III) Signal randomly γ or β with equal probabilities to all $n \in N_j(\theta_t)$, irrespective of the private history $h_t(j)$.
- (IV) If j has ever observed $\theta_t^{-1}(j)$ play L or someone (including himself) play NB against her, play NB regardless of the messages $m_t(j)$.
- (V) Play B otherwise.

The *Informative Strategy* for buyer j is:

- (III)' If j has ever observed $\theta_t^{-1}(n)$, where $n \in N_j(\theta_t)$, play L or someone (including himself) play NB against her, signal β . Signal γ otherwise.
- (IV)' If j previously observed $\theta_t^{-1}(j)$ play L or someone play NB against her, or if he received a message β from any of his current spectators, $h \in N_j^s(\theta_t)$, play NB .
- (V)' Play B otherwise.

Under both strategies, in the beginning of each period t each buyer j categorizes sellers into two different status groups based on his private history $H^t(j)$. If he has observed a seller play L or someone (including himself) play NB against her by period $t - 1$, he gives her the “*Bad*” status β . Otherwise, he gives her the “*Good*”

status γ . During the communication stage in period t (phase 2 of the stage game), buyer j is given the opportunity to signal to his neighboring buyers $n \in N_j(\theta_t)$ the statuses that he has assigned to their opponents $\theta_t^{-1}(n)$, and to revise the status that he assigns to his current opponent by taking into account the signals that he receives from the period t spectators to his game $h \in N_j^s(\theta_t)$.

As can be seen from the conditions (III), (IV) and (V), under the *Uninformative Strategy Profile* buyers merely "babble", disregard their neighbors' signals and base their choices of action against their period t opponents on the statuses that they assigned to them after the period $t - 1$. So the communication stage is totally uninformative. Under the *Informative Strategy Profile*, on the other hand, signalling reveals all the relevant information (about receiver's opponent) of the senders, given the seller's strategy. Under this profile each buyer j sends a signal γ or β to each of his neighboring buyers $n \in N_j(\theta_t)$, depending on the statuses that he assigned to their opponents $\theta_t^{-1}(n)$ after period $t - 1$. He also fully respects the messages that his spectators $h \in N_j^s(\theta_t)$ send to him before the period t trade game, and revises the status that he has assigned to his current opponent $\theta_t^{-1}(j)$ accordingly, basing his period t trade game choice of action on the revised status of $\theta_t^{-1}(j)$. Both strategy profiles are unforgiving since once a buyer assigns a particular seller a status β , he never upgrades her status to γ .

Before proceeding it is convenient to introduce some additional notation. Take any two time periods t' and t , where $t' < t$. Under the *Uninformative Strategy Profile* denote by b the probability that $\theta_t(i)$ was among the $N + 1$ buyers who observed seller i at period t' . That is, let b denote the probability

$$\Pr \left\{ i \in S_{\theta_t(i)}(\theta_{t'}) \cup \{ \theta_{t'}^{-1}(\theta_t(i)) \} \right\}.$$

Correspondingly, under the *Informative Strategy Profile* denote by b the probability

that either $\theta_t(i)$ or some of the N $\theta_t(i)$'s time t spectators, $h \in N_{\theta_t(i)}^s(\theta_t)$, were among the $N + 1$ buyers who observed i at period t' . In this case, b is the probability

$$\Pr \left\{ i \in \bigcup_{j \in \{\theta_t(i)\} \cup N_{\theta_t(i)}^s(\theta_t)} [S_j(\theta_{t'}) \cup \{\theta_{t'}^{-1}(j)\}] \right\}.$$

It is straightforward to check that

$$b = \begin{cases} \frac{N + 1}{M} & \text{with the } \textit{Uninformative Strategy Profile}, \text{ and} \\ 1 - \frac{\binom{M - N - 1}{N + 1}}{\binom{M}{N + 1}} & \text{with the } \textit{Informative Strategy Profile} \end{cases}$$

For both strategies, note that since θ_t is i.i.d., b is time independent and does not depend on t and t' . Note also that for both strategies b increases in the number of spectators, N . In the proofs of Propositions 1 and 2 we need the unconditional probability of $\theta_t(i)$ assigning the seller i a status γ after the period t communication stage, given that seller i has defected in every period $t' \in \{t_D, \dots, t - 1\}$. With our notation, this probability can be written as $(1 - b)^{t - t_D}$.

We are now ready to state our first two propositions. These propositions provide the conditions under which the *Uninformative* and the *Informative Strategy Profiles* are sequential equilibria of the random matching game. Notice that under both strategy profiles (H, B) is played in every period at each location along the equilibrium path.

Proposition 1: Define two constants δ^* and b^* as follows:

$$\delta^* = \left(1 + \frac{\left(\frac{M+g}{M}\right)^2}{4(1+g)g\left(\frac{M-1}{M}\right)} \right)^{-1},$$

$$b^* = \frac{g(1-\delta)}{\delta}.$$

- i) If $\frac{g}{1+g} \leq \delta \leq \delta^*$, the *Uninformative Strategy Profile* is a sequential equilibrium of the above random matching game if $b \geq b^*$.
- ii) If $\delta \geq \max\left[\frac{g}{1+g}, \delta^*\right]$, there exists constants b_L and b_H , where $b^* \leq b_L \leq b_H < 1$, such that the *Uninformative Strategy Profile* is a sequential equilibrium of the above random matching game if either $b^* \leq b \leq b_L$ or $b \geq b_H$. The constants b_L and b_H are given by

$$b_L = \frac{\frac{M+g}{M} - \sqrt{\left(\frac{M+g}{M}\right)^2 - 4(1+g)\frac{g(1-\delta)}{\delta}\left(\frac{M-1}{M}\right)}}{2(1+g)},$$

and

$$b_H = \frac{\frac{M+g}{M} + \sqrt{\left(\frac{M+g}{M}\right)^2 - 4(1+g)\frac{g(1-\delta)}{\delta}\left(\frac{M-1}{M}\right)}}{2(1+g)}.$$

The condition $\delta \geq g/(1+g)$ is necessary because with δ less than this, the efficient outcome could not be sustained by any equilibrium even with $M = 1$.⁷

⁷When $M = 1$, our random matching game is equivalent to a two-player standard repeated game with observable actions. In this case, the *Uninformative Strategy Profile*, which is identical with the *Informative Strategy Profile*, provides the maximum punishment for defection. This profile supports the efficient outcome as a Nash and a subgame perfect equilibrium if and only if $\delta \geq g/(1+g)$.

Buyers' incentives to follow the unforgiving strategy profiles are easily satisfied: A buyer should expect his current opponent with status β to play L , regardless of his beliefs about the outcomes of her previous games, in which case NB is his best choice of action. If, on the other hand, he assigns her a status γ , playing B is optimal both on and off the equilibrium path given the consistent belief that she has never defected and will play H . Also the incentives for signalling are trivially satisfied.

Sellers' incentives to follow the uninformative strategy profile are characterized by two conditions: one preventing her from playing L on the equilibrium path and one that guarantees that sellers who have defected keep defecting irrespective of their private history. To give a seller an incentive to play H along the equilibrium path, the short-term gain from cheating, g , must be outweighed by the long-term loss resulting from the gradual loss of reputation among buyers. Given the buyers' strategies, this occurs if $b(N)$ is sufficiently large that information about a seller's defection spreads quickly enough among the buyers. This results in the condition that $b \geq b^*$. On off-the-equilibrium paths, the strategy profile requires sellers to keep defecting rather than play H in an attempt to slow down the deterioration of her reputation. This off-the-equilibrium path constraint is satisfied when δ is small, $\delta \leq \delta^*$, since the short term gain g from selling low quality will then outweigh the future reward from trying to maintain a good reputation. When $\delta > \delta^*$, the condition is satisfied if either b is very large or b is small enough. If b is very large, $b \geq b_H$, it does not pay to slow down the deterioration of one's reputation, since with several informed buyers already playing NB against the seller, all buyers are soon likely to learn about seller's bad status anyway. On the contrary, if b is small enough, $b \leq b_L$, playing L is better than playing H simply because the informa-

tion about her defections is not spreading very quickly. It can be shown that the off-the-equilibrium path constraint is always satisfied when $b = b^*$, implying that $b_L \geq b^*$.

This strategy profile is not a sequential equilibrium when $b \in (b_L, b_H)$. In proposition 3, however, by using a public randomization device we construct a sequential equilibrium which supports the efficient outcome for any b greater than b^* .

Proof. When (II) holds a seller has incentive to follow (I) if and only if the following inequality holds:

$$\frac{1}{1 - \delta} \geq \frac{1 + g}{1 - (1 - b)\delta}. \quad (3.1)$$

The left-hand side is the payoff from playing H in every period whereas the right-hand side is the payoff from playing L in every period. Since b must be less than one, the inequality can hold only when $\delta \geq g/(1 + g)$. In that case equation (3.1) can be written as:

$$b \geq b^*. \quad (3.2)$$

By the principle of dynamic programming to verify that (II) is optimal, it is enough to check that a one time switch to H is not profitable after any history in which the seller has obtained a bad status, i.e., she has played L or some buyer has played NB against her. We can show that the seller's incentives to follow (II) increase in the number of buyers who are aware of her bad status. Since consistency requires this number to be at least $N + 1$ (in states that (II) is concerned with),

it will be sufficient for us to show that a seller has incentive to follow (II) when exactly $N + 1$ players assign her a bad status.

Define $P_s(K)$ as the probability that $K + s$ buyers know about i 's bad status after period t if i plays H in period t and K players knew about i 's bad status after the period $t - 1$. Let $\alpha = K/M$. It is straightforward to show that:

$$P_0(K) = (1 - \alpha) + \frac{\alpha \binom{K-1}{N}}{\binom{M-1}{N}}$$

and for $\alpha < 1$,

$$P_s(K) = \frac{\alpha \binom{K-1}{N-s} \binom{M-K}{s}}{\binom{M-1}{N}}, \forall 1 \leq s \leq \min[N, M-K].$$

Denote with $u_s(K) = 1 - (K + s)/M$ the associated conditional probability that seller i 's period $t + 1$ match $\theta_{t+1}(i)$ does not know about i 's bad status (after the period $t + 1$ communication stage), given that $K + s$ buyers know about i 's bad status after the period t .

Then, assuming that K buyers are aware of a seller's bad reputation before the current period, a seller has incentive to follow (II) if and only if:

$$\frac{(1+g)(1-\alpha)}{1-(1-b)\delta} \geq (1-\alpha) + \frac{\delta(1+g)}{1-(1-b)\delta} \sum_{s=0}^{\min[N, M-K]} P_s(K) u_s(K). \quad (3.3)$$

The left hand side is the payoff from playing L in each of the remaining periods, whereas the right hand side is the payoff for playing H in one period and then L thereafter. Realizing that s follows a hypergeometric distribution for $s = 1, 2, \dots, N$, it can be shown that

$$\sum_{s=0}^{\min\{N, M-K\}} P_s(K) u_s(K) = (1 - \alpha)^2 + \alpha(1 - \alpha)(1 - b) \left(\frac{M}{M-1} \right).$$

It is now easy to see that equation (3.3) is relaxed as α is increased. The intuition for this result is quite simple: A seller who is matched with a buyer that is not aware of his bad status may by playing H keep his reputation among at most $N + 1$ players and benefit from this reputation later. When α is large many of her current spectators are likely to know about his defection already, which reduces the benefit to playing H . Since $\alpha \geq b$ by consistency, it is sufficient to check that equation (3.3) holds when $\alpha = b$. Setting $\alpha = b$ and rearranging, equation (3.3) can be written:

$$b^2 - \left(\frac{M + g}{(1 + g)M} \right) b + \frac{(1 - \delta)g}{(1 + g)\delta} \frac{M - 1}{M} \geq 0.$$

This quadratic inequality holds if either $\delta \leq \delta^*$ or if $\delta \geq \delta^*$ and either $0 \leq b \leq b_L$ or $b_H \leq b \leq 1$. Combined with equation (3.2) this result implies the equilibrium conditions stated in proposition 1.

(III) is trivially satisfied given (IV) and (V). It is also easy to see that (IV) and (V) are satisfied given (I),(II) and (III). If buyer j observed his current match, $\theta_t^{-1}(j)$, play L in the past or some buyer play NB against her, he should believe she will play L by (II); so playing NB is his best response. If he has never observed $\theta_t^{-1}(j)$ play L , nor some buyer play NB against her, then given (I), (II) and (III), he should believe she will play according to (I) regardless of the messages he has received. This being the case, B is his optimal choice of action. This establishes (IV) and (V) and completes the proof. ■

As one would expect, the equilibrium conditions are very similar for the *Informative Strategy Profile*. In this case, however, it is not possible to state the

off-the-equilibrium path conditions in terms of b as was true for the *Uninformative Strategy Profile*.

Proposition 2: For $\delta \geq g/(1+g)$, the *Informative Strategy Profile* is a sequential equilibrium of the above random matching game if

$$1 - \frac{\binom{M-N-1}{N+1}}{\binom{M}{N+1}} \geq \frac{g(1-\delta)}{\delta} \quad (3.4)$$

and

$$\frac{g}{\delta(1+g)} \geq \left(\frac{1+2g}{1+g}\right) \frac{\binom{M-N-1}{N+1}}{\binom{M}{N+1}} - I_{\{3(N+1) \leq M\}} \frac{\binom{M-2N-2}{N+1}}{\binom{M}{N+1}}, \quad (3.5)$$

where I_A is a function that takes value 1 if A is true and 0 otherwise.

Equation (3.4) concerns on-the-equilibrium path behavior and can be written as $b \geq b^*$. Equation (3.5) is the constraint for off-the-equilibrium path behavior. As before, it is always satisfied when δ is small enough and when δ is large it is satisfied when b is either very large or b is small enough. Although the intuition for both conditions is exactly the same as under the *Uninformative Strategy Profile*, the second condition is not exactly the same in terms of b under the two strategy profiles. This constraint concerns a seller's possible deviation to H when playing on off-the-equilibrium paths, and is different for the two strategy profiles because the dissemination of a seller's bad reputation when playing H is different in terms of b under the two strategy profiles.

Proof. As was shown in the proof of proposition 1, assuming that (II) holds, a seller has incentive to follow (I) if and only if $b \geq b^*$. This is stated in equation (3.4).

Showing that she has incentive to follow (II) when equation (3.5) holds proceeds much as the proof of proposition 1. Denote with α the probability that seller i 's period t opponent $\theta_t(i)$ is aware of i 's bad status after the period t communication stage, if K buyers are aware of sellers i 's bad status after the period $t - 1$. Correspondingly, denote with ξ the probability that $\theta_t(i)$ is aware of i 's bad status after the period t communication stage, if $K + N + 1$ buyers are aware of sellers i 's bad status after period $t - 1$. Then

$$\alpha = 1 - I_{\{(N+1) \leq M-K\}} \frac{\binom{M-K}{N+1}}{\binom{M}{N+1}}$$

and

$$\xi = 1 - I_{\{2(N+1) \leq M-K\}} \frac{\binom{M-K-N-1}{N+1}}{\binom{M}{N+1}}.$$

Let P_s and u_s represent the same probabilities as in the proof of proposition 1.

Seller i has an incentive to follow (II) if and only if:

$$(1+g) \left((1-\alpha) + \frac{\delta((1-\alpha)(1-\xi) - (1-\alpha)^2 + \sum_{s=0}^{\min\{N, M-K\}} P_s u_s)}{1 - (1-b)\delta} \right) \geq$$

$$(1-\alpha) + (1+g) \left(\frac{\delta \sum_{s=0}^{\min\{N, M-K\}} P_s u_s}{1 - (1-b)\delta} \right) \quad (3.6)$$

The left hand side is the payoff from playing L in each of the remaining periods, whereas the right hand side is the payoff from playing H in one period and then L thereafter. Seller i 's action makes a difference only when neither i 's opponent $\theta_t(i)$ nor any of the spectators to i 's game have assigned the bad status to i . This happens with probability $(1 - \alpha)$. In that case, playing L results in a larger payoff by g , but $N + 1$ new buyers learn about i 's bad status, reducing the probability that i receives $(1 + g)$ in the next period from $(1 - \alpha)$ to $(1 - \xi)$. If $\theta_t(i)$ or some of the spectators to i 's game know about i 's bad status, which happens with probability α , i receives nothing in that period and her reputation deteriorates similarly irrespective of the action that he takes.

This inequality can be written as:

$$\frac{g(1 - (1 - b)\delta)}{\delta(1 + g)} \geq \xi - \alpha. \quad (3.7)$$

It is now straightforward to check that equation (3.6) is relaxed as K increases. Since $K \geq N$ by consistency, it is sufficient to check that equation (3.6) holds when $K = N$ or $\alpha = b$. Setting $\alpha = b$ and rearranging gives equation (3.5).

If a buyer j has ever observed a seller $i \in S_j(\theta_t)$ play L or her opponent play NB against her, j is indifferent between signalling γ or β to $\theta_t(i)$, since given (II) he expects i to play L in all his future games, including those with j himself, irrespective of the signal that he sends. So we may assume that he sends a truthful signal β in this case. If j has never observed i 's game or if j has observed i but the outcome in i 's games were always (H, B) he strictly prefers to send the message γ instead of β . Sending the message β would result in $\theta_t(i)$ playing NB against seller i , giving her a bad status and making her play L in the future. This would reduce buyer j 's future payoffs from games where he is matched with seller i . This

establishes condition (III)'. Given (I), (II) and (III)' conditions (IV)' and (V)' are trivial. ■

In order to create sequential equilibria that support trade for all $b \geq b^*$, let us now extend our basic model to include a public randomization device. The idea of using a public randomization device to adjust the severity of punishments is borrowed from Ellison (1994). In particular, we assume that before players choose their actions in period t , they observe a public random variable f_t which is drawn independently from a uniform distribution on $[0, 1]$. Let $f \in [0, 1]$ and consider adjusting the *Uninformative* and *Informative Strategy Profiles* as follows: In period t , sellers play according to the original strategies as long as $f_t \leq f$, but return immediately to the equilibrium path of the original strategies if $f_t > f$; buyers play according to the original strategies, except whenever $f_t > f$, at which point they forget all past actions of sellers and assign each seller status γ . Assuming that such a public randomization device is available, we can state the following proposition.

Proposition 3: For $\delta \geq g/(1 + g)$ there exists a function $f(\delta)$ such that the *adjusted Informative* and *Uninformative Strategy Profiles* with $f = f(\delta)$ are a sequential equilibrium of the random matching game if $b \geq b^*$, where $b^* \equiv g(1 - \delta)/\delta$.

The idea in the proof is that whenever $b \geq b^*$, we can by an appropriate choice of f adjust the severity of punishment for a seller so that she becomes indifferent between playing H (following (I)) and deviating on the equilibrium path. Because at off-the-equilibrium paths a seller has less incentive to protect her reputation than on-the-equilibrium path, as her reputation is deteriorating anyway, this indifference

can be shown to imply that the off-the-equilibrium path condition always holds.

Proof. For any f_t , a seller has incentive to follow (I) if and only if

$$\frac{1}{1-\delta} \geq \frac{1+g}{1-(1-b)f\delta} + \sum_{t=1}^{\infty} f^{t-1}(1-f)\frac{\delta^t}{1-\delta}$$

or

$$\frac{1}{1-\delta f} \geq \frac{1+g}{1-(1-b)f\delta} \quad (3.8)$$

The left-hand side of the first inequality is the seller's payoff from following (I), while the right-hand side is her payoff from deviating and following (II), as long as $f_\tau \leq f$, and following (I) thereafter. For all $\delta \geq g/(1+g)$ and $b \geq b^*$, there exists $f(\delta) \in [0, 1]$ such that equation (3.8) holds as an equality when $f = f(\delta)$. From now on, let us assume that $f = f(\delta)$.

A seller who is playing on the equilibrium path is now indifferent between playing H in every period and deviating. She is also indifferent between playing H in the current period and then deviating and deviating right away. By playing H in the current period she can keep her good reputation among $N + 1$ buyers, until they, in some way, learn about her defections in the future. Now consider a seller who is following (II). If her opponent and all the N spectators to her game happen to assign her a good status, she also can keep her good reputation among $N + 1$ buyers by playing H . This reputation, however, is worth less to her than if she were on the equilibrium path because, with some buyers already assigning her a bad status, these $N + 1$ buyers are more likely to learn about her bad status before playing against her in the future. Since the short term gain from deviating is same in both cases, we conclude that playing L is optimal off the equilibrium path.

More formally, consider the sellers incentives to follow (II) in period t when $f_t \leq f(\delta)$. By the principle of dynamic programming, it is sufficient to show that a single-period deviation to H is unprofitable. Let α , $P_s(K)$, and $u_s(K)$ denote the same probabilities as in the proof of proposition 1. A seller has an incentive to follow (II) if and only if:

$$\begin{aligned} & \frac{(1+g)(1-\alpha)}{1-(1-b)f(\delta)\delta} + \sum_{t=1}^{\infty} f(\delta)^{t-1}(1-f(\delta))\frac{\delta^t}{1-\delta} \geq \\ (1-\alpha) & + \frac{\delta f(\delta)(1+g)\left(\sum_{s=0}^{\min[N,M-K]} P_s u_s\right)}{1-(1-b)f(\delta)\delta} + \sum_{t=1}^{\infty} f(\delta)^{t-1}(1-f(\delta))\frac{\delta^t}{1-\delta} \end{aligned} \quad (3.9)$$

or

$$\frac{(1+g)(1-\alpha)}{1-(1-b)f(\delta)\delta} \geq (1-\alpha) + \frac{\delta f(\delta)(1+g)\left(\sum_{s=0}^{\min[N,M-K]} P_s u_s\right)}{1-(1-b)f(\delta)\delta}. \quad (3.10)$$

We can show that this inequality holds as follows:

$$\begin{aligned} & \frac{(1+g)(1-\alpha)}{1-(1-b)f(\delta)\delta} = \frac{(1-\alpha)}{1-\delta f(\delta)} = \\ & (1-\alpha) + \frac{(1+g)\delta f(\delta)(1-\alpha)}{1-(1-b)f(\delta)\delta} \geq \\ & (1-\alpha) + \frac{(1+g)\delta f(\delta)\left(\sum_{s=0}^{\min[N,M-K]} P_s u_s\right)}{1-(1-b)f(\delta)\delta}. \end{aligned}$$

The first two equalities come from the fact that (3.8) holds as an equality with $f = f(\delta)$. The inequality follows since

$$\sum_{s=0}^{\min[N,M-K]} P_s u_s \leq \sum_{s=0}^{\min[N,M-K]} P_s u_0 = u_0 = 1 - \alpha.$$

When $f_t \leq f(\delta)$ a buyer's problem is similar to that in propositions 1 and 2 so he is better off following the original strategies. On the other hand, given that a past defector plays H after $f_t > f(\delta)$ it is optimal for the buyer to assign her a status γ and treat her like the seller who never defected. ■

3.4 Large Population Results

In this section we study how fast N must grow in relation to M in order to sustain (H, B) as the outcome of the trade game for our strategies. If we denote $N^*(M)$ as the smallest integer N that satisfies the constraint $b \geq g(1 - \delta)/\delta$ (i.e., $b \geq b^*$), then given propositions 1,2 and 3, the question can be reformulated as how fast $N^*(M)$ grows in relation to M .

If seller i has defected in the previous period, the $N + 1$ buyers who observed the defection are the only ones who are informed of the defection before the current period's trade game starts. Then b , the probability that seller i meets a buyer who is informed of her previous defection, is simply $(N + 1)/M$ under the *Uninformative Strategy Profile*. With this strategy profile it is clear that N^* and M must grow at the same rate in the limit.

This is not true, however, with the *Informative Strategy Profile*. In this strategy profile i 's current opponent $\theta_t(i)$ assigns her a bad status if he observed this defection or he received an informative signal based on this defection during the current period's communication stage. With N spectators to $\theta_t(i)$'s game each of whom observes $N + 1$ games, $\theta_t(i)$ obtains information from $N^2 + 2N + 1$ possibly overlapping games and assigns i a bad status if any of these games was played at location i . This suggests that N^* may grow more slowly than M . Below we show

that in order to sustain trade, N^* must grow only at a rate \sqrt{M} . To prove this result formally we need the following lemma.

Lemma 1: Under the *Informative Strategy Profile* $\lim_{M \rightarrow \infty} \frac{N^*(M)}{M} = 0$.

Proof. By the definition of $N^*(M)$ we have the following inequalities

$$\frac{\binom{M - N^*(M) - 1}{N^*(M) + 1}}{\binom{M}{N^*(M) + 1}} \leq (1 - b^*) < \frac{\binom{M - N^*(M)}{N^*(M)}}{\binom{M}{N^*(M)}} \quad (3.11)$$

First note $\limsup_{M \rightarrow \infty} N^*(M)/M < 1/2$. For the subsequences of $N^*(M)/M$ whose limits are $1/2$, the numerator of the right-hand side of the strict inequality in (3.11) approaches 1 and the denominator goes to the infinity, leading to a contradiction since b^* is assumed to be strictly less than one.

Using Stirling's formula

$$\sqrt{2\pi n} n^n e^{-n} \leq n! \leq \sqrt{2\pi n} n^n e^{-n} e^{\frac{1}{12n}}$$

the second inequality in (4.1) implies that

$$(1 - b^*) <$$

$$\frac{(M - N^*)^{2(M - N^*) + 1}}{M^{M + \frac{1}{2}} (M - 2N^*)^{M - 2N^* + \frac{1}{2}}} e^{\frac{1}{6(M - N^*)}} \quad (3.12)$$

Given that $\limsup_{M \rightarrow \infty} N^*/M < 1/2$, it is easy to see that the exponential term that appears at the right-hand side of (3.12) goes to 1 as M approaches infinity.

The nonexponential term then has to be bounded away from zero for large M . In what follows we show this implies $\lim_{M \rightarrow \infty} N^*/M = 0$.

Let $q = N^*/M$. The nonexponential terms in the right hand side of (3.12) can now be written as:

$$\left(\frac{(1-q)^{2-2q}}{(1-2q)^{1-2q}} \right)^M \frac{(1-q)}{(1-2q)^{1/2}}.$$

Since $0 < q < 1/2$ and $\limsup_{M \rightarrow \infty} q < 1/2$ the second term of the above expression is bounded. Furthermore, we can show the first term is strictly decreasing with respect to q for $0 \leq q \leq 1/2$ and for that range of q it is one if and only if $q = 0$. Thus if $\lim_{M \rightarrow \infty} q \neq 0$, the first term is very close to zero for large M , which is a contradiction. Hence it must be that $\lim_{M \rightarrow \infty} N^*/M = 0$. ■

Proposition 4: Under the *Informative Strategy Profile*

$$0 < \lim_{M \rightarrow \infty} \frac{N^*(M)^2}{M} < \infty.$$

Proof. If we rewrite the inequalities (3.13) using Stirling's formula we get

$$\begin{aligned} & \frac{(M - N^* - 1)^{2(M-N^*)}}{M^M (M - 2N^* - 2)^{M-2N^*}} \left(\frac{(M - 2N^* - 2)^3}{M(M - N^* - 1)^2} \right)^{\frac{1}{2}} e^{-\frac{1}{12(M-2N^*-2)} - \frac{1}{12M}} \\ & \leq (1 - b^*) < \\ & \frac{(M - N^*)^{2(M-N^*)}}{M^M (M - 2N^*)^{M-2N^*}} \left(\frac{(M - N^*)^2}{M(M - 2N^*)} \right)^{\frac{1}{2}} e^{\frac{1}{8(M-N^*)}} \end{aligned} \quad (3.13)$$

Since the last two terms of both sides of the inequalities (4.3) approach one as $M \rightarrow \infty$ by lemma 1 and both of the first terms exhibit the same behavior in the limit, we must have

$$\frac{(M - N^*)^{2(M-N^*)}}{M^M(M - 2N^*)^{M-2N^*}} \rightarrow (1 - b^*) \text{ as } M \rightarrow \infty. \quad (3.14)$$

Next we show that $0 < \lim_{M \rightarrow \infty} N^{*2}/M < \infty$ in order for (4.4) to hold.

First of all, note that we can write

$$\begin{aligned} & \frac{(M - N^*)^{2(M-N^*)}}{M^M(M - 2N^*)^{M-2N^*}} = \\ & \left[\left(1 - \left(\frac{N^*}{M - N^*} \right)^2 \right)^{-\left(\frac{M-N^*}{N^*} \right)^2} \right]^{\frac{N^{*2}}{M-N^*}} \left[\left(1 - \frac{2N^*}{M} \right)^{\frac{-M}{2N^*}} \right]^{\frac{-2N^{*2}}{M}} = \\ & (e + \alpha_M)^{\frac{N^{*2}}{M-N^*}} (e + \beta_M)^{\frac{-2N^{*2}}{M}} \end{aligned}$$

where

$$\begin{aligned} \alpha_M &= \left(1 - \left(\frac{N^*}{M - N^*} \right)^2 \right)^{-\left(\frac{M-N^*}{N^*} \right)^2} - e, \\ \beta_M &= \left(1 - \frac{2N^*}{M} \right)^{\frac{-M}{2N^*}} - e. \end{aligned}$$

Since $\lim_{M \rightarrow \infty} N^*/M = 0$, both sequences $\{\alpha_M\}, \{\beta_M\}$ converge to zero from above.

Now define new sequences $\{a_M\}, \{b_M\}$ such that $e^{a_M} = e + \alpha_M$, $e^{b_M} = e + \beta_M$. We can then write

$$\frac{(M - N^*)^{2(M-N^*)}}{M^M(M - 2N^*)^{M-2N^*}} = e^{a_M \frac{N^{*2}}{M-N^*} - b_M \frac{2N^{*2}}{M}}. \quad (3.15)$$

Given (4.4) and (4.5), all that remains to show is that $\{N^{*2}/M\}$ must converge to some positive number in order for $\{a_M N^{*2}/(M - N^*) - b_M 2N^{*2}/M\}$ to converge to $\ln(1 - b^*)$. If $\lim_{M \rightarrow \infty} N^{*2}/M = \infty$, $a_M N^{*2}/(M - N^*) - b_M 2N^{*2}/M$ approaches minus infinity as M goes to infinity. This can be easily shown using the fact that $\{a_M\}$ and $\{b_M\}$ converge to one from above. If $\lim_{M \rightarrow \infty} N^{*2}/M = 0$, we obtain another contradiction since $a_M N^{*2}/(M - N^*) - b_M 2N^{*2}/M \rightarrow 0$ as $M \rightarrow \infty$. Since $\{N^{*2}/M\}$ is bounded, it is straightforward to show $\lim_{M \rightarrow \infty} N^{*2}/M = -\ln(1 - b^*)$. ■

3.5 Endogenous Connections

In this section we extend our model by assuming that networking is costly. Let us say that strategies are non-contagious when only the sellers who have produced low-quality are punished. When either inviting spectators, $N_j^s(\theta_t)$, observing neighboring buyers, $N_j(\theta_t)$, or both are costly to buyer j , the following result holds:

Proposition 5: In any Nash equilibrium of the random matching game with costly networking that uses non-contagious strategies, low quality is produced with positive probability when $M > \delta/[g(1 - \delta)]$.⁸

Proof. The proposition is proved by contradiction. Assume that there is a Nash equilibrium with non-contagious strategies in which every seller produces high quality with probability one in every period. Buyers then do not have any incentive to network and the optimal N_j must be zero for all buyers j . If, however, N_j is zero for all buyers and $M > \delta/[g(1 - \delta)]$ all sellers have incentive to unilaterally deviate and produce low-quality. Contradiction. ■

⁸A similar proposition could be stated allowing for contagious strategies for $M > \bar{M}(\delta, g, \ell)$, where $\bar{M} < \infty$.

There clearly exists the equilibrium where $N_j = 0$ for all buyers and every seller produces low quality. More interestingly, we show that if the costs of networking are small enough, there exist sequential equilibria with strictly positive probability of trade.

Let us concentrate on the case where observing neighboring buyers $n \in N_j(\theta_t)$ is costly to buyer j and where buyers are unable to affect the number of spectators to their game. This assumption corresponds to the idea that buyers network to gather information about their trading environment. An alternative - that leads to similar results - would be that buyers invited other buyers to their games in an attempt to obtain information regarding their current opponents. Clearly the second alternative would make sense only under the *Informative Strategy Profile*. More specifically, let us extend our basic model by assuming that in period zero, before θ_0 is realized, all buyers j can invest in N_j connections that allow them to observe the games in N_j consecutive locations to their own in every future period. To obtain N_j connections j must pay $N_j c$, where $c > 0$, in period zero.

With these assumptions, whenever the costs of networking are less than some threshold value $\bar{c}(M)$, we can find sequential equilibria with slightly modified *Informative and Uninformative Strategy Profiles* such that the sellers initially randomize between high and low quality and produce that level of quality in the future. A positive probability of low quality goods is necessary to provide buyers with an incentive to network. This probability tends to increase with M . When the costs of networking exceed the threshold value $\bar{c}(M)$, trade collapses because the probability of low quality goods that would provide buyers sufficient incentives to network is so high that buyers are unwilling to buy from unknown sellers. For simplicity, let

us confine our analysis to the *Uninformative Strategy Profile*.

Consider the following *modified Uninformative Strategies*:

For the seller i , in all periods $t = 0, 1, 2, \dots$, after θ_t is realized:

- (I) In the first period play H with probability $1 - p$ and L with probability p . After that, if the outcome in all the trade games where seller i played was (H, B) , play H .
- (II) Play L otherwise.

For the buyer j :

- (III) In period 0, before θ_0 is realized, invest in $N^* - 1$ connections with probability r and in N^* connections with probability $1 - r$,

and in all periods $t = 0, 1, 2, \dots$, after θ_t is realized:

- (IV) Signal randomly γ or β with equal probabilities to all $n \in N_j(\theta_t)$, irrespective of the private history $h_t(j)$.
- (V) If j has ever observed $\theta_t^{-1}(j)$ play L or someone (including himself) play NB against her, play NB regardless of the messages $m_t(j)$.
- (VI) Play B otherwise.

Before proceeding, we need some additional notation. If buyer j invested in N_j connections, the probability that he observed $\theta_{t'}^{-1}(j)$ at period t' , where $t' < t$, is $(N_j + 1)/M$. Let us denote this probability by $b^e(N_j)$.⁹ Then the probability that buyer j has never observed $\theta_t^{-1}(j)$ until period t is simply $(1 - b^e(N_j))^t$.

⁹More precisely, $b^e(N_j) = \Pr[\theta_{t'}^{-1}(j) \in S_j(\theta_{t'}) \cup \{\theta_{t'}^{-1}(j)\}]$

Proposition 6: The *modified Uninformative Strategy Profile* is a sequential equilibrium of the above random matching game with

$$p = cM \left(\frac{((1 - \delta(1 - b^e(N^* - 1))) (1 - \delta(1 - b^e(N^*))))}{\ell\delta} \right),$$

where

$$c \leq \bar{c} \equiv \frac{1}{M} \left(\frac{\ell\delta (M(1 - \delta) + \delta)}{(1 - \delta(1 - b^e(N^* - 1))) (1 - \delta(1 - b^e(N^*))) ((\ell + 1)M(1 - \delta) + \delta)} \right).$$

Proof. To have a sequential equilibrium in this extended game for our strategies, the following three equations must hold:

$$\frac{1}{1 - \delta} = \frac{r(1 + g)}{1 - (1 - b^e(N^* - 1))\delta} + \frac{(1 - r)(1 + g)}{1 - (1 - b^e(N^*))\delta} \quad (3.16)$$

$$N^*, N^* - 1 \in \arg \max_{N_j} \frac{1 - p}{1 - \delta} - \frac{p\ell}{1 - \delta(1 - b^e(N_j))} - N_j c, \quad (3.17)$$

$$(1 - p) + \frac{\delta(1 - p)}{M(1 - \delta)} \geq p\ell. \quad (3.18)$$

A seller is willing to randomize in the initial period between providing high quality goods forever and providing low quality goods forever if and only if equation (3.16) holds. The left-hand side of (3.16) is the payoff from providing high quality goods forever and the right-hand side is the expected payoff from providing low quality goods forever. To see this, note that the probability that $\theta_t(i)$ has never observed i is just $(1 - b^e(N_{\theta_t(i)}))^t$. Given this indifference in the initial period, the seller who once provided low quality good can be shown to keep on providing low quality goods; the proof is exactly the same as that of proposition 3. So (I) and (II) are established.

Equation (3.17) requires that buyers are willing to randomize between N^* and $N^* - 1$ connections. We can show that the right hand side of equation (3.17) has a single peak if N_j is treated as a positive real number. Hence if the expected payoff to buyer j is the same with N^* and $N^* - 1$, then N^* and $N^* - 1$ both solve j 's maximization problem. Equation (3.17) is therefore satisfied if

$$\frac{p\ell}{(1 - \delta(1 - b^e(N^*)))} = \frac{p\ell}{(1 - \delta(1 - b^e(N^* - 1)))} + c,$$

or

$$p = cM \left(\frac{((1 - \delta(1 - b^e(N^* - 1))))(1 - \delta(1 - b^e(N^*)))}{\ell\delta} \right) \quad (3.19)$$

Given that other buyer's actions do not depend on the messages, (IV) is obvious. If a buyer has observed his current opponent play L or someone play NB against her, he should play NB against her given (II). And if a buyer has observed his current opponent and the outcomes have always been (H, B) , he should believe she will play H and he should play B . If the buyer has never observed his current opponent, he should believe that she will play H with probability $(1 - p)$. For a buyer to play B against her rather than give her a bad status by playing NB , equation (3.18) has to be satisfied. This equation can be rewritten as

$$p \leq \frac{M(1 - \delta) + \delta}{(\ell + 1)M(1 - \delta) + \delta}. \quad (3.20)$$

Therefore the *modified Uninformative Strategy Profile* with p defined by equation (3.19) is a sequential equilibrium if p satisfies equation (3.20). This happens when $c \leq \bar{c}$. ■

Several interesting results now follow: First, for $c > 0$ equation (3.19) requires that p is strictly positive as was shown in proposition 5. L has to be played with positive probability to provide buyers with the incentive to network. Secondly, this probability is increasing in M . Under the *Uninformative Strategy Profile* approximately proportionally and with *Informative Strategy Profile* (it can be shown) less than proportionally. More striking result is the knife edge property of our equilibria: If even one more buyer invested in one more connection there would be no low quality at all (increasing the utility of all buyers and sellers discontinuously). But networking to reduce production of low quality is a public good and, as usual, everyone wants to free ride in its production. Because of this, the economy is stuck in an inefficient equilibrium.

Another interesting observation is that when equation (3.20) fails, trade collapses even when it might be beneficial for buyers to keep trading. This occurs because a buyer, who considers whether to trade with an unknown seller or to give her a bad status by playing NB , does not take into account the future trading opportunities of other buyers with her. With the *Informative Strategy Profile* there would still be another externality because of the informative signalling to neighbors. When choosing the number of locations to observe the buyers would not take into account the learning by their neighbors, but would only be interested in their own learning.

3.6 Conclusion

In many real life situations particular sellers and buyers trade with each other infrequently, or only once. In such instances, when one or both parties have short-term

incentives to cheat, community enforcement may be needed to facilitate cooperation and trade. This paper studied community enforcement in the absence of institutions to transmit information.

We studied a large population, random matching game between buyers and sellers, where the sellers have a short-term incentive to cheat and supply low quality. We studied informal networks of communication as the mechanism that spreads information about sellers' behavior and facilitates trade. We looked at both informative and uninformative signalling and for the latter we showed that high quality can be sold in a sequential equilibrium with population M where each buyer networks with only $N^*(M)$ players with $\lim_{M \rightarrow \infty} N^*/M = 0$.

We studied the case of costly networking and showed that in this case, when M is large, low-quality goods must be supplied with positive probability in any equilibrium to provide buyers with an incentive to network. When the costs of networking were below a threshold value, we found a sequential equilibrium in which sellers initially randomize between high and low quality with probabilities $(1 - p)$ and p respectively and then continue to produce high quality if and only if they did so in the first period. In this equilibrium p is strictly positive and increasing in both M and the costs of networking.

Appendix A

Appendix For Chapter 1

Proof of Proposition 1: We must show that the portfolio manager is willing to follow this trading rule. To begin with we show that $x_t(R_t)$ is increasing in R_t . Note that equilibrium requires

$$\frac{\pi^e(R_t)}{x(R_t)} = c, \quad (\text{A.1})$$

since investments to the portfolio manager flow in until this equation is satisfied.¹ Substituting for $\pi^e(R_t)$ from equation (1.3) and rearranging we obtain the following identity $x(R_t) = \frac{m}{c} (k\alpha_t - k^2\alpha_t^2)$. Differentiating this identity with respect to R_t gives the result:

$$x'(R_t) = \frac{m(k - 2k^2\alpha_t)[\bar{\alpha} - \underline{\alpha}]}{c} > 0$$

¹Investor $i \in [0, I]$ makes a binary decision whether to delegate his unit wealth to the portfolio manager or not. When making this decision he perceives that the first $x_t(R_t)$ investors, other than himself, are investing through the portfolio manager. In this case his expected profit from investing are $\pi^e/x_t - c$. His expected profits from not investing, on the other hand, are zero. In a rational expectations equilibrium where some but not all investors invest through the portfolio manager these two must be equal.

Given the updating of beliefs that we have assumed, for other values of k and ρ , also other linear trading strategies can be part of an equilibrium in the overall game. Nevertheless, by lemma one and equation (A.1) we note that this trading strategy $y(z_t) = m(1 - k\alpha_t)z_t$ maximizes $x_t(R_t)$ among linear trading strategies.

We must now show that the portfolio manager is willing to follow this trading strategy in all periods t and for all z_t . The proof proceeds by backward induction. Since the portfolio manager's payoff is a constant fraction of the assets under management the portfolio manager is indifferent between different trading rules that result in the same level of reputation. In these cases, we have assumed that he trades according to the linear trading strategy that maximizes the periodic trading profits. This behavior is anticipated by the market maker and investors. Given this, and given that only the sign of the trade matters for the portfolio manager's future reputation, the portfolio manager essentially chooses y_t among three alternatives $y_t \in \{y^*, -y^*, 0\}$, where $y^* = m(1 - k\alpha_t)$. Since there are no commissions after period T , we can assume that the portfolio manager follows the trading rule $y_T = m(1 - k\alpha_T)z_T$ in period T . Since this trading strategy maximizes $\pi^e(R_T)$ among linear trading strategies, it also maximizes $x_T(R_T)$ among those strategies.

We now move to periods $t < T$. We will show that portfolio manager's value function, representing the portfolio manager's remaining payoffs from period t onwards, increases in R_t . Given that the current commission is independent of his current actions, the portfolio manager then chooses $y_t \in \{y^*, -y^*, 0\}$ to maximize his next period's reputation. We first show that trading $y_t = y^*z_t$ is optimal in the event that $z_t \neq 0$. Note that given the updating of beliefs stated in (3.4), R_{t+1} increases when trading on information.² That being the case, it is optimal for the

²In this event the posterior belief $\hat{R}_t = \bar{\alpha}R_t/\alpha_t \equiv \bar{\alpha}R_t/(R_t\bar{\alpha} - (1-R_t)\underline{\alpha}) > R_t$, whereas following

portfolio manager's to trade according to this rule when $z_t \neq 0$ if the value function at period $t + 1$, $V_{t+1}(R_{t+1}, s_{t+1})$, is strictly increasing in R_{t+1} for both $s_{t+1} \in \{g, b\}$. Let $V_t^b(R_t)$ denote the value function of the bad portfolio manager and let $V_t^g(R_t)$ be the same for the good portfolio manager. Since $V_T(R_T) = cx(R_T)$ for both types of portfolio managers, $V_T(R_T)$ is strictly increasing in R_T . For $t < T$ same result follows for both types of manager's, since by backward induction, assuming that the portfolio manager follows his equilibrium strategy, both

$$V_t^b(R_t) = cx(R_t) + E \left[(1 - \varepsilon)V_{t+1}^b(R_{t+1}) + \varepsilon V_{t+1}^g(R_{t+1}) \mid s_t = b \right], \quad (\text{A.2})$$

and

$$V_t^g(R_t) = cx(R_t) + E \left[(1 - \mu)V_{t+1}^g(R_{t+1}) + \mu V_{t+1}^b(R_{t+1}) \mid s_t = g \right],$$

are strictly increasing in R_t . To see this, note that all possible reputations R_{t+1} that may follow at time $t + 1$ (for different values of z_t and a_t) at least weakly increase in R_t (equation 1.4) and given our earlier result $x(R_t)$ is strictly increasing in R_t . Here the expectation is taken with respect to z_t and the public signal a_t .

We now want to show that there exists $\bar{\rho} < 1$ and $\underline{k} > 0$ such that for all k, ρ that satisfy $\rho \geq \bar{\rho}$ and $k \leq \underline{k}$ the portfolio manager does not churn. For this we need to define the probability that $\delta_t \neq 0$ given that $z_t = 0$ for both $s_t \in \{g, b\}$. Let $\alpha_s \in \{\bar{\alpha}, \underline{\alpha}\}$ denote the probability that type s player observes δ_t . In that case, Bayes rule gives

all other trades the inequality goes the other way. The result follows since $R_{t+1} = \hat{R}_t(1 - \varepsilon - \mu) + \varepsilon$.

$$\gamma_s = \text{Prob} [\delta_t \neq 0 \mid z_t = 0, s] = \frac{2k(1 - \alpha_s)}{(1 - 2k\alpha_s)}.$$

Therefore a type s portfolio manager does not want to churn if:

$$\begin{aligned} & \left(1 - \rho + \frac{\rho\gamma_{st}}{2}\right) V_{t+1}^{s_{t+1}} \left(\frac{\bar{\alpha}R_t}{\alpha_t}(1 - \varepsilon - \mu) + \varepsilon\right) + \rho \left(1 - \frac{\gamma_{st}}{2}\right) V_{t+1}^{s_{t+1}}(\varepsilon) < \\ & \rho(1 - \gamma_s)V_{t+1}^{s_{t+1}}(R_t(1 - \varepsilon - \mu) + \varepsilon) + \rho\gamma_s V_{t+1}^{s_{t+1}} \left(\frac{(1 - \bar{\alpha})R_t(1 - \varepsilon - \mu)}{1 - \alpha_t} + \varepsilon\right) \\ & + (1 - \rho) V_{t+1}^{s_{t+1}} \left(\frac{((1 - 2k) + 2k(1 - \bar{\alpha}))R_t}{(1 - 2k) + 2k(1 - \alpha_t)}(1 - \varepsilon - \mu) + \varepsilon\right) \end{aligned}$$

for all t and $R_t > \varepsilon$. To show that this holds when k is small and ρ is large we do the following. We let $m \rightarrow \infty$ while $k \rightarrow 0$ so that km is constant. In that event both π and it's derivative with respect to R_t remain bounded from zero, the latter approaching $[\bar{\alpha} - \underline{\alpha}]km$. Then also V_t remains bounded from zero and infinity and strictly increasing in R_t . Now let also $\rho \rightarrow 1$ and note that as $k \rightarrow 0$, $\gamma \rightarrow 0$. With all these limits taken simultaneously the left hand side approaches $V_{t+1}^{s_{t+1}}(\varepsilon)$ and for some values \underline{k} , and $\bar{\rho}$ the left hand side will be smaller than the value function at any of the events that may occur on the right hand side for all t and $R_t \geq \varepsilon$. The inequality will then hold, as it will hold also for any k and ρ that satisfy $k \leq \underline{k}$, and $\rho \geq \bar{\rho}$. Since V is linear in m , for those values of ρ and k , the inequality will hold for any positive m . ■

Proof of the expression for volume of trade: Let $Y^+ = y^+ + u^+$ and define Y^- accordingly. The following table lists the events where $Y^+ \geq Y^-$, the expected value of Y^+ in that event, and the probability of the event.

Event i	$E[Y^+ i]$	$\Pr[i]$
$y = y^*, -y^* \leq u \leq 0$	y^*	$k\alpha y^*/2m$
$y = y^*, u \geq 0$	$y^* + m/2$	$k\alpha/2$
$y = -y^*, u \geq y^*$	$\frac{m+y^*}{2}$	$k\alpha \frac{m-y^*}{2m}$
$y = 0, u \geq 0$	$m/2$	$(1 - 2k\alpha)/2$

The second table is the breakdown of events where $Y^- \geq Y^+$.

Event j	$E[Y^- j]$	$\Pr[j]$
$y = y^*, u \leq -y^*$	$\frac{m+y^*}{2}$	$k\alpha \frac{m-y^*}{2m}$
$y = -y^*, 0 \leq u \leq y^*$	y^*	$k\alpha y^*/2m$
$y = -y^*, u \leq 0$	$y^* + m/2$	$k\alpha/2$
$y = 0, u \leq 0$	$m/2$	$(1 - 2k\alpha)/2$

The expected trading volume at stage two given Ψ_{t-1} can now be written as:

$$\begin{aligned} \omega_t^o &= \sum_i E[Y^+ | i] \Pr[i] + \sum_j E[Y^- | j] \Pr[j] \\ &= \frac{k\alpha y^{*2}}{m} + k\alpha[y^* + m/2] + \frac{k\alpha}{2m}[1 - y^{*2}] + \frac{m}{2}(1 - 2k\alpha) = \frac{m}{2} + k\alpha \left[y^* + \frac{y^{*2}}{2m} \right] \end{aligned}$$

If we include the volume of trade that takes place in stage five, we obtain the expression for expected volume of trade, ω_t^e , that was given in the text. ■

Proof of proposition 3: It is useful to define the following two events: one where $(P_t - P_{t-1})^2 = 1$ and there is not informed trading, i.e., $y_t = 0$, and the event where $(P_t - P_{t-1})^2 = 1$ and $y_t \neq 0$. The probability of the former is easily obtained as $2k(1 - \alpha_t)\rho$, and the probability of the latter as $2k\alpha_t$. By referring to the tables that were given when we derived the expression on volume, the expected volumes of trade conditional on these two events are easily found to be:

$$E[\omega_t \mid (P_t - P_{t-1})^2 = 1, y_t = 0, \Psi_{t-1}] = m/2$$

and

$$\begin{aligned} & E[\omega_t \mid (P_t - P_{t-1})^2 = 1, y_t \neq 0, \Psi_{t-1}] \\ &= \frac{k\alpha[m + y^* + \frac{y^{*2}}{2m}]}{1 - (1 - 2k\alpha)} + y^* = \frac{1}{2} \left[m + 3y^* + \frac{y^{*2}}{2m} \right]. \end{aligned}$$

We can now write:

$$\begin{aligned} & \text{cov}[(P_t - P_{t-1})^2, \omega_t \mid \Psi_{t-1}] = \\ & E[(P_t - P_{t-1})^2 \omega_t \mid \Psi_{t-1}] - E[(P_t - P_{t-1})^2 \mid \Psi_{t-1}] E[\omega_t \mid \Psi_{t-1}] \\ &= 2k\alpha_t \frac{1}{2} \left[m + 3y^* + \frac{y^{*2}}{2m} \right] + 2k(1 - \alpha_t)\rho \frac{m}{2} \end{aligned}$$

$$\begin{aligned}
& -2k(\rho + (1 - \rho)\alpha_t) \left[\frac{m}{2} + k\alpha_t \left[3y^* + \frac{y^{*2}}{2m} \right] \right] \\
= & k\alpha_t \left[m + [1 - 2k(\rho + (1 - \rho)\alpha_t)] \left[3y^* + \frac{y^{*2}}{2m} \right] \right] \\
& + \rho km(1 - \alpha_t) - km(\rho + (1 - \rho)\alpha_t) \\
= & k\alpha \left[3y^* + \frac{y^{*2}}{2m} \right] [1 - 2k(\rho + (1 - \rho)\alpha_t)] > 0
\end{aligned}$$

Similarly

$$\begin{aligned}
\text{cov} [(P_t - P_{t-1})^2, \omega_t] &= E [(P_t - P_{t-1})^2 \omega_t] - E(P_t - P_{t-1})^2 E\omega_t \\
= & E [E [(P_t - P_{t-1})^2 \omega_t | \Psi_{t-1}]] - E [E [(P_t - P_{t-1})^2 | \Psi_{t-1}]] E [E [\omega_t | \Psi_{t-1}]] \\
= & E \left[2k\alpha_t \frac{1}{2} \left[m + 3y^* + \frac{y^{*2}}{2m} \right] + 2k(1 - \alpha_t)\rho \frac{m}{2} \right] \\
& - E [2k(\rho + (1 - \rho)\alpha_t)] E \left[\frac{m}{2} + k\alpha_t \left[3y^* + \frac{y^{*2}}{2m} \right] \right] \\
= & E \left[k\alpha_t \left[m + [1 - 2k(\rho + (1 - \rho)E\alpha_t)] \left[3y^* + \frac{y^{*2}}{2m} \right] \right] \right] \\
& + \rho km(1 - E\alpha_t) - km(\rho + (1 - \rho)E\alpha_t) \\
= & E \left[k\alpha \left[3y^* + \frac{y^{*2}}{2m} \right] \right] [1 - 2k(\rho + (1 - \rho)E\alpha_t)] > 0
\end{aligned}$$

We now proceed to the proof of the last claim in proposition three. Let us introduce a random variable S_t such that $S_t = 1$ when $s_t = g$ and $S_t = 0$ when $s_t = b$. We can now write the covariance between $(P_t - P_{t-1})^2$ and $(P_{t-1} - P_{t-2})^2$ as:

$$\text{cov} [(P_{t+s} - P_{t+s-1})^2, (P_t - P_{t-1})^2] =$$

$$\begin{aligned}
& \Pr [\widehat{\delta}_{t+s} \neq 0 \text{ and } \widehat{\delta}_t \neq 0] - \Pr [\widehat{\delta}_{t+s} \neq 0] \Pr [\widehat{\delta}_t \neq 0] \\
&= 4k^2 E [(\rho + (1 - \rho) [\underline{\alpha} + S_{t+s}[\bar{\alpha} - \underline{\alpha}]]) \cdot (\rho + (1 - \rho) [\underline{\alpha} + S_t[\bar{\alpha} - \underline{\alpha}]])] \\
&\quad - 4k^2 [(\rho + (1 - \rho) E [\underline{\alpha} + S_{t+s}[\bar{\alpha} - \underline{\alpha}]])]^2.
\end{aligned}$$

Here we have used the fact that $ES_\tau = ES_1 = \varepsilon/(\varepsilon + \mu)$.³ Using the law of iterated expectations we have

$$\begin{aligned}
E [S_{t+1} | S_t] &= S_t(1 - \varepsilon - \mu) + \varepsilon \\
E [S_{t+2} | S_t] &= E [E [S_{t+2} | S_{t+1}] | S_t] = E [(S_{t+1}(1 - \varepsilon - \mu) + \varepsilon) | S_t] \\
&= S_t(1 - \varepsilon - \mu)^2 + \varepsilon(1 - \varepsilon - \mu) + \varepsilon
\end{aligned}$$

and more generally that

$$\begin{aligned}
E [S_{t+s} | S_t] &= S_t(1 - \varepsilon - \mu)^s + \sum_{i=0}^{s-1} \varepsilon(1 - \varepsilon - \mu)^i = \\
&= S_t(1 - \varepsilon - \mu)^s + \frac{\varepsilon - \varepsilon(1 - \varepsilon - \mu)^s}{\varepsilon + \mu}
\end{aligned}$$

Using this result and the fact that $ES_t = ES_t^2$ we have

³This can be proved by induction.

$$\begin{aligned}
& \text{cov} \left[(P_{t+s} - P_{t+s-1})^2, (P_t - P_{t-1})^2 \right] \\
&= 4k^2 E \left[E \left[(\rho + (1 - \rho) [\underline{\alpha} + S_{t+s}[\bar{\alpha} - \underline{\alpha}]]) (\rho + (1 - \rho) [\underline{\alpha} + S_t[\bar{\alpha} - \underline{\alpha}]]) \mid S_t \right] \right. \\
&\quad \left. - 4k^2 [(\rho + (1 - \rho) E [\underline{\alpha} + S_t[\bar{\alpha} - \underline{\alpha}]])]^2 \right] \\
&= 4k^2 E \left(\rho + (1 - \rho) \left[\underline{\alpha} + \left[S_t(1 - \varepsilon - \mu)^s + \frac{\varepsilon - \varepsilon(1 - \varepsilon - \mu)^s}{\varepsilon + \mu} \right] [\bar{\alpha} - \underline{\alpha}] \right] \right) \\
&\quad \cdot (\rho + (1 - \rho) [\underline{\alpha} + S_t[\bar{\alpha} - \underline{\alpha}]]) - 4k^2 \left[\left(\rho + (1 - \rho) \left[\underline{\alpha} + \frac{\varepsilon}{\varepsilon + \mu} [\bar{\alpha} - \underline{\alpha}] \right] \right) \right]^2 \\
&= 4k^2 (1 - \rho)^2 [\bar{\alpha} - \underline{\alpha}]^2 \left[E \left[S_t^2 (1 - \varepsilon - \mu)^s + S_t \frac{\varepsilon - \varepsilon(1 - \varepsilon - \mu)^s}{\varepsilon + \mu} \right] - \left(\frac{\varepsilon}{\varepsilon + \mu} \right)^2 \right] \\
&= \frac{4k^2 \varepsilon \mu (1 - \rho)^2 [\bar{\alpha} - \underline{\alpha}]^2}{(\varepsilon + \mu)^2} (1 - \varepsilon - \mu)^s. \blacksquare
\end{aligned}$$

Proof of Proposition 4: First, note that

$$\begin{aligned}
E_t[\hat{R}_t] &= 2k\alpha_t \frac{\bar{\alpha}R_t}{\alpha_t} + \rho \left[2k(1 - \alpha_t) \left(\frac{(1 - \bar{\alpha})R_t}{(1 - \alpha_t)} \right) + (1 - 2k)R_t \right] + \\
(1 - \rho) &\left[[(1 - 2k) + 2k(1 - \alpha_t)] \left(\frac{[(1 - 2k) + 2k(1 - \bar{\alpha})] R_t}{(1 - 2k) + 2k(1 - \alpha_t)} \right) \right] = \\
&[2k\bar{\alpha} + 2k(1 - \bar{\alpha}) + (1 - 2k)] R_t = R_t,
\end{aligned}$$

and that

$$E_t[R_{t+1}] = E[R_{t+1} | R_t] = E[\widehat{R}_t](1 - \varepsilon - \mu) + \varepsilon = R_t(1 - \varepsilon - \mu) + \varepsilon.$$

Similarly as in the proof of proposition three we can use the law of iterated expectations to derive

$$\begin{aligned} E[R_{t+s} | R_t] &= R_t(1 - \varepsilon - \mu)^s + \sum_{i=0}^{s-1} \varepsilon(1 - \varepsilon - \mu)^i \\ &= R_t(1 - \varepsilon - \mu)^s + \frac{\varepsilon - \varepsilon(1 - \varepsilon - \mu)^s}{\varepsilon + \mu} \end{aligned}$$

Using the law of iterated expectations and Jensen's inequality we have

$$\begin{aligned} \text{cov}[\sigma_{t+s}^2, \sigma_t^2] &= E\left([\sigma_{t+s}^2 - E\sigma_{t+s}^2][\sigma_t^2 - E\sigma_t^2]\right) = \\ &= 2k^2(1 - \rho)^2 E[\alpha_{t+s} - E\alpha_{t+s}][\alpha_t - E\alpha_t] \\ &= 2k^2(1 - \rho)^2 E_{R_t} E[[\alpha_{t+s} - E\alpha_{t+s}][\alpha_t - E\alpha_t] | R_t] \\ &= 2k^2(1 - \rho)^2. \\ E_{R_t} E\left[\left[\alpha_{t+s} - \left(\frac{\varepsilon}{\varepsilon + \mu}[\bar{\alpha} - \underline{\alpha}] + \underline{\alpha}\right)\right]\left[\alpha_t - \left(\frac{\varepsilon}{\varepsilon + \mu}[\bar{\alpha} - \underline{\alpha}] + \underline{\alpha}\right)\right] | R_t\right] \\ &= 2k^2(1 - \rho)^2 E_{R_t} E\left[\left(R_{t+s} - \frac{\varepsilon}{\varepsilon + \mu}\right)[\bar{\alpha} - \underline{\alpha}]\left(R_t - \frac{\varepsilon}{\varepsilon + \mu}\right)[\bar{\alpha} - \underline{\alpha}] | R_t\right] \\ &= 2k^2(1 - \rho)^2[\bar{\alpha} - \underline{\alpha}]^2. \end{aligned}$$

$$E_{R_t} \left[\left(R_t(1 - \varepsilon - \mu)^s + \frac{\varepsilon - \varepsilon(1 - \varepsilon - \mu)^s}{\varepsilon + \mu} - \frac{\varepsilon}{\varepsilon + \mu} \right) \left(R_t - \frac{\varepsilon}{\varepsilon + \mu} \right) \right] >$$

$$2k^2(1 - \rho)^2[\bar{\alpha} - \underline{\alpha}]^2.$$

$$\left[\left(E[R_t](1 - \varepsilon - \mu)^s + \frac{\varepsilon - \varepsilon(1 - \varepsilon - \mu)^s}{\varepsilon + \mu} - \frac{\varepsilon}{\varepsilon + \mu} \right) \left(E[R_t] - \frac{\varepsilon}{\varepsilon + \mu} \right) \right] = 0 \blacksquare$$

Proposition 5 follows directly from the definition of σ_t^2 and the properties of R_t stated in the proof of the proposition four. Proposition 6 follows directly from the above definitions of conditional variance and trading volume and the fact that when $k \leq \frac{8-\sqrt{43}}{3}$ both σ_t^2 and ω_t^c are strictly increasing functions of R_t .⁴

Proof of Proposition 7: Let us assume that the short sales constraint is binding. When it is not, the trading rule that was derived in section three is optimal. We are looking for a semi linear trading strategy:

$$y_t = \begin{cases} 0 & \text{if } z_t \in \{0, -1\} \\ \hat{y} & z_t = 1 \end{cases},$$

where \hat{y} is chosen to maximize the expected trading profits. Assuming that the portfolio manager follows this trading rule, the pricing rule of the market maker is

⁴To prove formally the rather obvious fact that two increasing functions of the same random variable are positively correlated note the following: let $\Psi(x)$ and $\Phi(x)$ be strictly increasing functions. Consider two i.i.d. random variables x and y . Then

$$E[\Psi(x) - \Psi(y)][\Phi(x) - \Phi(y)] > 0$$

\Leftrightarrow

$$E[\Psi(x)\Phi(x)] - E\Psi(x)E\Phi(y) - E\Psi(y)E\Phi(x) + E\Psi(y)E\Phi(y)$$

$$= 2E[\Psi(x)\Phi(x)] - 2E\Psi(x)E\Phi(x) = 2cov[\Psi, \Phi] > 0.$$

$$P_t^o = \bar{F} + \sum_{\tau=0}^{t-1} \hat{\delta}_\tau + \begin{cases} 1 & \text{if } v_t > m \\ 0 & \text{if } \hat{y} - m \leq v_t \leq m \\ \frac{-k\alpha}{1-k\alpha} & \text{if } -m \leq v_t \leq \hat{y} - m \end{cases}$$

and⁵

$$P_t^c = \bar{F} + \sum_{\tau=0}^t \hat{\delta}_\tau.$$

Similar arguments as before show that the expected trading profits for $y \leq \hat{y}$ are:

$$\pi_{f=1}^e = y_t \left[\frac{4 - 3k\alpha}{1 - k\alpha} + \frac{k\alpha\hat{y}}{(1 - k\alpha)2m} \right] - \frac{y_t^2}{2m(1 - k\alpha)}.$$

Given this, we can solve for the optimal trading rule:

$$\hat{y} = \frac{m(4 - 3k\alpha)}{1 - k\alpha}.$$

We can show that $\pi_{f=1}^e$ is strictly increasing in R_t so, as before, equation (1.4) implies that x_t is strictly increasing in R_t in both states of the world: i.e., when there are short sales constraints and when there are not. This together with beliefs similar to (1.3) imply that the portfolio manager wants to trade when he receives private information. Similarly as before, when $\rho \rightarrow 1$ and $k \rightarrow 0$, to avoid a loss of reputation the portfolio manager does not want to churn. The formal proof

⁵Note that while the definition of $\hat{\delta}_t$ is still $\hat{\delta}_t = E[\delta_t | \Psi_t]$, the value of this random variable is now different in the event that there is no public announcement and $y_t = 0$.

of the above arguments proceeds along the lines of the proof in section three and is omitted. It is also possible to show that proposition 2 continues to hold and that all covariances stated in the proposition 3 are strictly positive.

I will now prove proposition seven, taking as given the optimality of the trading rule stated above. Following the same argument, the expected trading volume in the event that $f = 1$ and $z_t = 1$ is

$$E[\omega_t \mid f = 1, z_t = 1, \Psi_{t-1}] = \frac{m}{2} + \frac{\hat{y}}{2} + \frac{\hat{y}^2}{4m}.$$

We have,

$$\begin{aligned} \text{cov}((P_t - P_{t-1}), \omega_t) &= E(P_t - P_{t-1})\omega_t - E(P_t - P_{t-1})E\omega_t = E(P_t - P_{t-1})\omega_t \\ &= E[E[(P_t - P_{t-1})\omega_t \mid R_t, f_t]] \\ &= E\left[k\alpha_t \left[\frac{m}{2} + \frac{\hat{y}}{2} + \frac{\hat{y}^2}{4m}\right] + \frac{m(1 - \alpha_t)k\rho}{2} - \frac{mk\rho}{2} - \frac{mk\alpha(1 - \rho)}{2}\right] \cdot \text{Pr}[f_t = 1] \\ &= E\left[k\alpha_t \left(\frac{\hat{y}}{2} + \frac{\hat{y}^2}{4m}\right)\right] \cdot \text{Pr}[f_t = 1] > 0 \blacksquare \end{aligned}$$

Proof of Proposition 8: As before, the commission $c\alpha(\alpha_t)$ is strictly increasing in α_t . Total commission is therefore increasing in R_t iff α_t increases in R_t . This no longer necessarily follows. In period T , with fixed commission, $q_T = 0$ and $\alpha_T = R_T(\bar{\alpha} - \underline{\alpha}) + \underline{\alpha}$ and trivially increases in R_T . This need not be the case for $T - 1$, however. What is important for our strategies is that V_{t+1} increases in R_{t+1} ,

because then, since $q^* < 1$ and therefore $\alpha_t < \bar{\alpha}$, V_{t+1} increases when trading on information. Since $V_T = cx(\alpha_T)$ is strictly increasing in R_T , and $q_{T-1} < 1$, both types of managers want to trade on information at $T - 1$. And similarly as before, if ρ is large and k is small, when they are not informed, they do not want to trade. Since we have a fixed commission, V_T is independent of the managers type.

Now denote by V_{T-1}^I the value of the bad portfolio managers remaining income stream net of disutility of information acquisition, if he invests in information acquisition in period $T - 1$. Denote by V_{T-1}^N the same if he does not. The value of good portfolio manager's payoff stream is then $V_{T-1}^g = V_{T-1}^I + e$. Clearly $V_{T-1}^b = \max[V_{T-1}^I, V_{T-1}^N]$. We can write these as:

$$\begin{aligned}
V_{T-1}^I(R_{T-1}, b) &= cx(R_{T-1}) + 2kV_T \left(\frac{\bar{\alpha}R_{T-1}(1 - \varepsilon - \mu)}{\alpha_{T-1}} + \varepsilon \right) \bar{\alpha} \\
&+ (1 - \rho)V_T \left(\frac{[(1 - 2k) + 2k(1 - \bar{\alpha})] R_{T-1}}{(1 - 2k) + 2k(1 - \alpha_t)} (1 - \varepsilon - \mu) + \varepsilon \right) [(1 - 2k) + 2k(1 - \bar{\alpha})] \\
&\quad + \rho(1 - 2k)V_T (R_{T-1}(1 - \varepsilon - \mu) + \varepsilon) \\
&\quad + \rho 2k(1 - \bar{\alpha})V_T \left(\frac{(1 - \bar{\alpha})R_{T-1}}{1 - \alpha_t} (1 - \varepsilon - \mu) + \varepsilon \right) - e,
\end{aligned}$$

and

$$\begin{aligned}
V_{T-1}^N(R_{T-1}, b) &= cx(R_{T-1}) + 2kV_T \left(\frac{\bar{\alpha}R_{T-1}(1 - \varepsilon - \mu)}{\alpha_{T-1}} + \varepsilon \right) \underline{\alpha} \\
&+ (1 - \rho)V_T \left(\frac{[(1 - 2k) + 2k(1 - \bar{\alpha})] R_{T-1}}{(1 - 2k) + 2k(1 - \alpha_t)} (1 - \varepsilon - \mu) + \varepsilon \right) [(1 - 2k) + 2k(1 - \underline{\alpha})]
\end{aligned}$$

$$+\rho(1-2k)V_T(R_{T-1}(1-\varepsilon-\mu)+\varepsilon)+\rho 2k(1-\underline{\alpha})V_T\left(\frac{(1-\bar{\alpha})R_{T-1}}{1-\alpha_t}(1-\varepsilon-\mu)+\varepsilon\right)$$

Note that V_T is continuous in R_T . By lemma 1, $V_{T-1}^N \geq V_{T-1}^I$, and $V_{T-1}^N = V_{T-1}^I$ when $q_{T-1}^* > 0$. This equality then determines $q^*(R_{T-1})$ in that region. So when $q^* > 0$,

$$\begin{aligned} & V_T\left(\frac{\bar{\alpha}R_{T-1}(1-\varepsilon-\mu)}{\alpha_{T-1}}+\varepsilon\right)- \\ & (1-\rho)V_T\left(\frac{[(1-2k)+2k(1-\bar{\alpha})]R_{T-1}(1-\varepsilon-\mu)}{(1-2k)+2k(1-\alpha_{T-1})}+\varepsilon\right) \\ & -\rho V_T\left(\frac{(1-\bar{\alpha})R_{T-1}}{1-\alpha_{T-1}}(1-\varepsilon-\mu)+\varepsilon\right)=\frac{c}{(\bar{\alpha}-\underline{\alpha})}. \end{aligned} \quad (\text{A.3})$$

Since V_T is strictly increasing in R_T this equation determines $q^*(R_{T-1})$ uniquely and furthermore, $q^*(R_{T-1})$ is continuous. This follows from the fact that all terms on the left hand side are strictly decreasing in q and continuous in both q and R_{T-1} . Now note that since $\bar{\alpha} > \underline{\alpha}$, this equation can not hold when R_{T-1} is sufficiently close either to one or zero. Therefore $q^*(R_{T-1}) = 0$ when R_{T-1} is close enough to either zero or one. Similar argument to that in the proof of the proposition one shows that the portfolio manager does not want to trade when he receives no information when ρ is large and k is small. ■

Appendix B

Appendix For Chapter 2

We now derive sufficient conditions for the number of firms to be larger in the equilibrium in which the firms are producing with the riskless technology than in the equilibrium in which all firms are using the risky technology. First note that the number of firms must be smaller in the equilibrium with risky technology than in an equilibrium with risky technology where the lenders could somehow persuade the entrepreneurs to pay back their entire profits as interest. We now derive conditions such that even in this case when the number of firms is $n(s, G)$ the expected profits in equilibrium are less than zero. By solving the optimization problem of n entrepreneurs using the risky technology it is easy to show that the expected profits are:

$$\pi^e = \frac{G2a^2(1-\gamma)}{b[n+1-\gamma(n-1)]} - F, \text{ for } n \geq n'$$

$$\pi^e = G \left[(1-\gamma) \frac{(a + \gamma(n-1)\frac{\bar{\epsilon}}{2})^2}{b(n+1)^2} \right] + \gamma \left[\frac{(a + \gamma(n-1)\frac{\bar{\epsilon}}{2})^2}{b(n+1)^2} - \frac{\bar{c}^2}{4b} \right] - F, \text{ for } n < n'$$

where

$$n' = \frac{\frac{2a}{\bar{c}} - (1 + \gamma)}{(1 - \gamma)}.$$

In both cases the expected profits are negative when γ , \bar{c} and F are large enough. The condition on γ , \bar{c} and F for the number of firms to be larger in the equilibrium with safe technology, in stationary markets, is implicitly defined by $\pi^e(n(s, G), \gamma, \bar{c}, F) \leq 0$.

Proof of proposition 1: The incentive compatibility constraint is

$$(1 - \gamma)\pi^d(n, G, 0) - \pi(n, G) \leq \tag{B.1}$$

$$\gamma\delta \left[\rho\psi_s(n, G_t)V^s(G_t) + (1 - \rho)\psi_g(n, G_t)V^g(\min\{G_t g_j, \bar{G}\}) \right],$$

\Leftrightarrow

$$G_t \left[\frac{c^2}{4b} + \frac{(a - c)}{b(n + 1)^2} [(1 - \gamma)(n + 1)c - \gamma(a - c)] \right] \leq \tag{B.2}$$

$$\rho\gamma\delta \psi_s(G_t)V^s(G_t) + \gamma \sum_{i=t+1}^{T-1} [$$

$$((1 - \rho)\delta)^{i-t} \left(\prod_{j=t+1}^i \psi_g(g^{j-t-1}G_t) \right) \left[\pi(g, g^{i-t}G_t) + \rho\delta\psi_s(g^{i-t}G_t)V^s(g^{i-t}G_t) \right] \Big]$$

$$\gamma \left[((1 - \rho)\delta)^{T-t} \left(\prod_{j=t+1}^T \psi_g(g^{j-t-1}G_t) \right) V^s(g^{T-t}G_t) \right]$$

To gain some understanding how this inequality might behave for large g and δ we must write it out more explicitly. We use (2.2) and (2.8) to calculate for $V^s(G_t)$. We obtain:

$$V^s(G_t) = \frac{G_t(a-c)^2}{(1-\delta)b(n+1)^2} - \frac{F}{(1-\delta)} =$$

$$\frac{\frac{2b\gamma Fc^2G^2}{(1-\gamma)} + \frac{c^4G^3(1-\delta)}{4} - 2FG^2bc^2(1-\delta) - \frac{Fbc^2G^2\gamma}{(1-\gamma)} + 2Fbc^2\chi}{2G^2bc^2(1-\delta)^2 + \frac{4Fb^2G\gamma^2}{(1-\gamma)^2} + \frac{bc^2G^2(1-\delta)\gamma}{(1-\gamma)} + 2bc^2\psi}$$

where

$$\chi = \sqrt{(1-\delta)^2G^4 + \frac{G^34b\gamma^2F}{c^2(1-\gamma)^2} + \frac{(1-\delta)^2G^4}{1-\gamma}}$$

and

$$\psi = \sqrt{(1-\delta)^4G^4 + \frac{G^34b\gamma^2F(1-\delta)^2}{c^2(1-\gamma)^2} + \frac{(1-\delta)^3G^4}{1-\gamma}}$$

It is now easy to verify that

$$\lim_{\delta \rightarrow 1} V^s(G_t) = \frac{G_t c^2 (1-\gamma)}{4b\gamma} + \sqrt{\frac{G_t c^2 (1-\gamma)^2 F}{b\gamma^2}}.$$

Let us now look at (B.2) more carefully if ψ_g and ψ_s were both equal to one. Clearly ψ_s is not, but for the moment assume that both of them are equal to one. In that event when $\delta \rightarrow 1$, the right hand side of (B.2) approaches

$$\kappa = \rho \left[\frac{G_t c^2 (1-\gamma)}{4b} + \sqrt{\frac{G_t c^2 (1-\gamma)^2 F}{b}} \right] + \gamma \sum_{i=t+1}^{T-1} ((1-\rho)\delta)^{i-t}.$$

$$\left[\pi(g, g^{i-t}G_t) + \rho \left[\frac{g^{i-t}G_t c^2(1-\gamma)}{4b\gamma} + \sqrt{\frac{g^{i-t}G_t c^2(1-\gamma)^2 F}{b\gamma^2}} \right] \right]$$

$$+ \gamma \left[((1-\rho)\delta)^{T-t} \left[\frac{g^{T-t}G_t c^2(1-\gamma)}{4b\gamma} + \sqrt{\frac{g^{T-t}G_t c^2(1-\gamma)^2 F}{b\gamma^2}} \right] \right]$$

Note that

$$\kappa > \frac{\rho G_t c^2(1-\gamma)}{4b} \left[1 + \sum_{i=t+1}^T ((1-\rho)\delta g)^{i-t} \right] > \frac{\rho G_t c^2(1-\gamma) [1 + (1-\rho)\delta g]}{4b}.$$

On the other hand we have that the left hand side of (B.2) reaches its maximum at

$$n = \frac{2\gamma(a-c)}{(1-\gamma)c} - 1$$

and is therefore bounded by

$$\frac{G_t}{4b} + \frac{G_t(1-\gamma)^2 c^2(a-c)}{4b\gamma}.$$

This implies that if ψ_g and ψ_s were equal to one the incentive compatibility constraint could not be binding for large δ if

$$g > \frac{1}{(1-\rho)(1-\gamma)\rho c^2} + \frac{(1-\gamma)(a-c)}{(1-\rho)\gamma\rho} - \frac{1}{(1-\rho)}.$$

Let \bar{g} be equal to this number. Given our previous results we are then done if we can show that when $g > \bar{g}$ $\psi_s(G_t)$ approaches one as $\delta \rightarrow 1$ and that $\psi_s(G_t) = 1$ for large δ .

Define now $\hat{n}(G_t) = \sqrt{G_t(a-c)^2/Fb-1}$ to be the number of firms such that when there are $\hat{n}(G_t)$ producers in the markets of size $G_t(a-bP)$ competing in quantities and producing with the riskless technology, the profits of all incumbents are equal to zero. Clearly, given our non negative profits constraint the number of firms in state (g, G_t) must be less than $\hat{n}(G_t)$. Similarly the survival probability to state (s, G_t) , $\psi_s(G_t)$ must then be greater than $\underline{\psi} = \frac{n^*(G_t)}{\hat{n}(G_t)}$. (3.7), on the other hand implies that $n_s(G_t) \rightarrow \hat{n}(G_t)$ as $\delta \rightarrow 1$. This implies that $\psi_s(G_t)$ approaches one as $\delta \rightarrow 1$. By backward induction it is then easy to verify that $\psi_g(G_t)$ is equal to one when δ is high enough. ■

Proof of proposition 2: The incentive compatibility constraint is as before:

$$G_t \left[\frac{c^2}{4b} + \frac{(a-c)}{b(n+1)^2} [(1-\gamma)(n+1)c - \gamma(a-c)] \right] \leq \quad (\text{B.3})$$

$$\rho\gamma\delta \psi_s(G_t, m)V^s(G_t) + \gamma \sum_{i=t+1}^{T-1} [((1-\rho)\delta)^{i-t}.$$

$$\left(\prod_{j=t+1}^i \psi_g(g^{j-t-1}G_t, m) \right) \left[\pi(g, g^{i-t}G_t) + \rho\delta\psi_s(g^{i-t}G_t, m)V^s(g^{i-t}G_t) \right]$$

$$+ \gamma \left[((1-\rho)\delta)^{T-t} \left(\prod_{j=t+1}^T \psi_g(g^{j-t-1}G_t, m) \right) V^s(g^{T-t}G_t) \right]$$

Given the exit rule: “First in last out,” the probability of survival decreases in m so that if the incentive compatibility constraint holds for $m = n(\bar{G})$ it will hold for all firms that have entered before the m 'th entrant. For the $n(\bar{G})$ 'th entrant $\psi_s(g^{i-t}G_t, m) = 0$ for all $i < T$. His only hope to remain in the markets is that the growth continues until \bar{G} . So for the $n(\bar{G})$ 'th entrant the incentive compatibility constraint in period $t < T$ is that

$$\left[\frac{c^2}{4b} + \frac{(a-c)}{b(n+1)^2} [(1-\gamma)(n+1)c - \gamma(a-c)] \right] \leq \quad (\text{B.4})$$

$$\frac{\gamma}{G_t} \left[((1-\rho)\delta)^{T-t} \left(\prod_{j=t+1}^T \psi_g(g^{j-t-1}G_t, m) \right) V^s(g^{T-t}G_t) \right]$$

As before the left hand side is bounded by

$$\frac{1}{4b} + \frac{(1-\gamma)^2 c^2 (a-c)}{4b\gamma}.$$

The right hand side, given our earlier result on the limit of $V(\bar{G})$ as $\delta \rightarrow 1$, is, on the other hand, larger than

$$\left[((1-\rho)\delta)^{T-t} \left(\prod_{j=t+1}^T \psi_g(g^{j-t-1}G_t, m) \right) \frac{g^{T-t}c^2(1-\gamma)}{4b} \right],$$

when δ is large. This means that if $\psi_g(g^{j-t-1}G_t, m) = 1$ the incentive compatibility constraint can not be binding for large δ at any t when

$$g > \bar{g} = \left(\frac{\left[1 + \frac{(1-\gamma)^2 c^2 (a-c)}{\gamma} \right]}{c^2 (1-\rho)^T (1-\gamma)} \right).$$

When $g > \bar{g}$ and when δ is sufficiently close to one, up to $n(\bar{G})$ firms are willing to enter the markets and behave prudently, assuming that their profits are greater than zero in growing markets. However, when $\hat{n}(G_t) < n(\bar{G})$ only $\hat{n}(G_t)$ firms can enter because of the zero profit constraint. We can now verify by backward induction that $\psi_g(G_t, m) = 1$ for all G_t .

We still have to show that no additional firms would enter. By backward induction a additional entrant would defect in period T if given credit. Thereby he would not be issued credit in period T . Knowing this he would defect also in period $T - 1$ and $T - 2$ and so on. Therefore no more than $n(\bar{G})$ firms are ever issued credit. ■

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