Essays in Applied Game Theory and Political Economy

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Abstract

This dissertation provides a game theoretic study of the incentives and dynamics of government action. In chapter 1, I study how the political incentives for providing legal protections to minority workers is influenced by the mobility of workers across cities. I build a model that shows how cities that provide insufficient protection to minorities can do better at attracting workers and end up being larger overall. To the extent that policy makers care about city size and city growth, my result identifies an incentive that reinforces the inadequate protection of minorities in the workplace.

In chapter 2, I build a model to explain electoral cycling, which is the regularity observed in the data that the longer a party is in power the more likely it loses the next election. I study an infinite horizon model of political competition where parties choose between nominating moderate candidates, who are better at winning elections, and partisan candidates who energize and grow the base which helps to win future elections. Hence, parties face a trade-off between winning current and future elections. In the presence of such a trade-off, parties win elections by sacrificing their future prospects which naturally leads to electoral cycling.

In chapter 3, I explore novel consequences of the learning model of economic voting. Economic voting is the empirical phenomenon that a strong economy makes incumbent re-election more likely while a weak economy makes incumbent defeat more likely. Previous work in the literature explains this phenomenon with voter learning; a strong economy is taken as a positive signal about the incumbent's policy or competence. I introduce the idea that voters who learn should also experiment, that is they should care about how much learning a policy allows when deciding whom to elect. In particular voters will, at times, prefer a policy that creates worse outcomes but that allows for more learning.

Acknowledgements

To my parents.

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Chapter 1

Discrimination and Mobility in a Search Model

1.1 Introduction

Tiebout (1956) developed the idea that individuals can escape undesirable local policies by relocating. This dynamic of "voting with your feet" leads, in principle, to well-managed cities attracting more workers and growing, while poorly managed cities shrink. To the extent that policy makers care about city growth, mobility creates pressure for reform. Even in the absence of reform, poorly-managed cities becomes less relevant as they become smaller and their policies impact fewer people. In democratic societies, mobility is especially important in the protection of minorities who cannot, by themselves, muster the votes for change. Moving may be the only recourse when faced with a discriminating majority.

In this chapter, I analyze the extent to which Tiebout-style competition affects the legal protection of minorities in the labor market. To this end, I consider two cities that differ in the degree to which they enforce the equal protection of minorities in the the workplace. In one city, enforcement is high and minorities are treated equally. In the other city, enforcement is low and firms are tacitly allowed to violate the labor rights of minorities for monetary gain.

My main result is that, for certain parameters, the city with low enforcement will be larger, and, in extreme cases, will contain all of the non-minority workers and half of the minority workers. In other words, I find that mobility has the potential to reward rather than punish inadequate legal protections for minorities. The result requires a tacitly allowed form of discrimination in that firms cannot openly advertise a minority specific wage. This tacit discrimination creates a transfer to non-minority workers that would not occur under more explicit wage discrimination.

Labor law violations are a widespread problem in the United States that disproportionately impacts minorities. In a 2001 study, the U.S. Department of Labor randomly investigated firms within certain industries and found wage violations in as many as 78 percent of garment factories in Los Angeles and 60 percent of nursing homes nationwide (U.S. Department of Labor (2001)). These violations took many forms including under counting hours, failing to pay overtime rates, and paying less than the minimum wage. Moreover, minorities are disproportionately likely to experience these violations. For example, Bernhardt et al. (2013) finds that within low wage occupations, African Americans and immigrants have roughly twice the odds to experience minimum wage violations as white workers[1]. Inadequate enforcement of labor laws is often cited as one of the prime causes of labor law violations[2]. By examining the interaction of worker mobility with legal protections of minorities, this chapter contributes to the understanding of why inadequate protection of minority groups may arise and why it persists.

Consider a mobile workforce living in two cities. Workers are split into two groups, a disadvantaged minority and an advantaged majority. Each worker draws an idiosyncratic city preference that helps ensure no city is ever empty. The distribution from which this preference is drawn treats the cities symmetrically. Each city has its own labor market captured using a search model with frictions, where firms post wages and randomly match with unemployed workers. A free entry condition ensures all firms earn a profit of zero.

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[1]See also Brotherton et al. (2013).

[2]For example see Bobo (2011)

Firms must post the same wage for both types of workers, but can choose whether or not to hire any worker they meet. Upon matching, workers draw a random labor cost specific to that firm, which determines whether they accept the job offer. Without the random labor costs, in equilibrium firms would always offer a wage of zero.

Firms have occasional opportunities to violate the labor rights of the workers they employ. Specifically, firms may fail to give workers their full wage, which will be referred to as wage theft. In both cities wage theft is illegal, but enforcement is imperfect and varies across cities and across types of workers. In what will be called the high enforcement city, all workers are protected equally, and firms are caught often enough to deter them. In the low enforcement city, protection is unequal, and firms are less likely to be caught while underpaying minority workers. This leads to systematic wage theft against the minority, while majority workers remain well protected. In both cities, firms that are caught are forced to pay a penalty, and the match between worker and firm is ended. This enforcement policy will mean that, in the low enforcement city, the minority workers are not only paid less but also have a higher unemployment rate. That minority workers are disadvantaged in both compensation and unemployment is necessary for the results. Capturing both of these dimensions in one mechanism is done for the sake of simplicity.

My main result establishes that, when the average length of employment is sufficiently long, there will exist a low enforcement policy that ensures the low enforcement city is always larger than the high enforcement city. In other words, when workers can vote with their feet, they vote in favor of inadequate protection for the minority. For extreme parameters, the low enforcement city will contain all of the majority workers and half of the minority workers. This case also identifies a somewhat counter intuitive mechanism for segregation. One might think that inadequate protection would drive away minority workers from the low enforcement city. Instead it is the lack of wage theft against minorities that drives away all the majority workers from the high enforcement city. The net result is that a city that

protects its workers equally ends up being segregated, while the city where a minority is inadequately protected attracts a broader cross section of workers.

The mechanism whereby the low enforcement city grows larger works through the equilibrium wage. In the low enforcement city, the possibility of underpaying minority workers grants a monetary gain to the firms. But the free entry condition ensures firms earn zero profits, and hence this monetary gain translates into a higher equilibrium wage. Majority workers, who are safe from wage theft, benefit from this higher wage. However, the minority workers will still have a lower *effective* wage, accounting for the wage theft they experience. In this way, low enforcement acts as a transfer from minority workers to majority workers. This transfer attracts more majority workers to the low enforcement city, while driving away minority workers. The city preference ensures that the number of workers who are attracted or driven away depends continuously on the differential in effective wages across cities. Workers with a strong enough city preference will accept lower compensation to live in their preferred city. Therefore, which city is larger depends on how the benefit to majority workers from low enforcement compares to the cost imposed on minority workers.

The key mechanism is that the wage increases with a differential in the unemployment rate across types. When minority workers have a higher unemployment rate, they make up a larger portion of the unemployment pool, and hence firms are more likely to match with them. And when hiring a minority workers is more likely, firms have a higher expected monetary gain from wage theft, which pushes up the wage in order to satisfy the zero profit condition. A higher wage raises the effective compensation received by minority workers, partially compensating them for the wage theft they experience. Hence, with an unemployment differential, the impacts of wage theft can be more beneficial to majority workers than they are detrimental to minority workers, and the low enforcement city will attract more workers overall.

To further illustrate the mechanism, consider the extreme case where the unemployment pool in the low enforcement city is comprised entirely of minority workers. Firms entering

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the market know they will surely hire a minority worker and will always pay them below the posted rate. In equilibrium, this underpayment becomes priced in and acts only to relabel the meaning of the wages. Therefore, the wage rises until it fully compensates the minority workers for the wage theft they experience. Majority workers, even those not currently unemployed, make their location decisions based on the prevailing wage in each city. Hence, in steady state, majority workers will strictly prefer the low enforcement city, while minority workers will be close to indifferent, so the low enforcement city will attract more workers overall.

One consideration not accounted for in the previous argument is that the differential in unemployment rates is supported by minority workers losing their jobs more often, which makes the low enforcement city less attractive to both firms and d workers. However this effect becomes insignificant when employment length is very long, which is the key parameter in the results. When losing a job is rare, an increase in the job losing rate for minority workers can be small in absolute terms while being large in relative terms. In other words, a minority worker can lose their job at rate that is simultaneously low in absolute terms and many times the rate of the majority workers. And due to discounting, a small job losing rate has little impact on where workers choose to live.

An important ingredient in the above argument is a tacit form of discrimination. Firms are de facto permitted to underpay minority workers but cannot openly advertise a minority specific wage. If firms could offer different wages, the labor market in the low enforcement city would functionally segment into two portions, one for each type of worker. Under such segmentation, wages of the two types of workers would be independent, and the mechanism by which the low enforcement city gains more workers would disappear. Tacit discrimination is needed for majority workers to gain from wage theft against minority workers, which drives my result.

While firms cannot post explicitly different wages, they can decide who to hire. In particular, firms in the low enforcement city could pursue the strategy of only hiring the

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more profitable minority workers. This strategy would create an informal segmentation of the market. But when meeting a majority worker, the firm must either hire the current prospect at the posted wage or continue searching and incur further search costs. As long as the benefit to wage theft is not too large, the firm will prefer to hire the majority worker over searching, and this informal segmentation will not occur.

One limitation of the model is that cities are fixed as either low or high enforcement instead of being allowed to choose their policies. Incorporating an endogenous policy decision would not allow for use steady state analysis. Given any initial state, a city that changes its enforcement policy would not jump instantaneously to a new steady state but instead would converge there over time. To study endogenous policy choice properly, it would be necessary to evaluate the dynamic path out of steady state, which is difficult. My model offers an analysis of the impact of low enforcement, and provides an argument for its persistence regardless of how it arises. And, under the reasonable assumption that cities value long term growth, my analysis identifies an incentive for cities to provide inadequate legal protection for minorities.

This chapter brings together to two distinct literatures: the literature on worker mobility and the literature on the economics of discrimination. Another paper that combines these literatures is Margo (1990), who looks at whether mobility constrained discrimination in the southern United States at the beginning of the twentieth century. However his model of the economy is frictionless and has no unemployment, and so cannot capture the key mechanism of this chapter. Moreover, he finds that mobility did constrain discrimination, coming to the opposite conclusion I do. This chapter also connects to the labor search literature through the modeling techniques employed. I will now discuss the connection of this chapter to the literatures on discrimination, mobility and labor search.

The economics of discrimination literature dates back at least as far as Becker (1971), who argued that discriminating firms will be less profitable and will be bought out in the long run. Goldberg (1982) adds nuance to this observation by pointing out that if firms gain

utility from hiring majority workers rather than dis-utility from hiring minority workers, then it is possible discriminatory firms will not want to sell. They may accept lower profits in order to retain the utility gained from hiring their favorite type of worker.

Black (1995) and Bowlus and Eckstein (2002) embed Becker's argument into a labor search model with firm entry. Black argues that discriminating firms will only be driven out of the market if the pool of potential firms is sufficiently large. In particular, Black assumes that firms differ in their productivity and more productive discriminatory firms remain in business even with entry. However, the productivity cutoff above which firms enter will be lower for non-discriminatory firms. Black use techniques similar to Borjas and Bronars (1989), who study the impact of consumers who dislike buying from self-employed minorities.

Another strain of the economics of discrimination literature starts with Arrow (1973), who showed how discrimination can arise without any taste for discrimination, but instead using incomplete information. In his model, firms infer a worker's private education investment based on whether the worker is a minority. This creates discriminatory equilibria in which all minorities make low investments and are assigned to low paying jobs that have small complementarities with education. In a similar vein, Aigner and Cain (1977) and Lundberg and Startz (1983) show how discrimination will arise if employers are more uncertain about the skill of minority workers than the skill of majority workers.

Coate and Loury (1993) extend Arrow's model in order to consider the impact of affirmative action. They find that, under affirmative action policies, discriminatory equilibria still exist and may even become worse. Rosén (1997) builds a model similar to Arrow's but with labor market search. Rosén (1997) shows that equilibria without discrimination are unstable to small perturbations. Lang et al. (2005) builds a model of discrimination using a labor search market without incomplete information. In their model, when firms choose between hiring two different workers, they break indifference by hiring majority workers over minority workers. Eeckhout (2006) as well as Peski and Szentes (2013) echo Rosén (1997) in

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arguing that equilibria with discrimination tend to be more stable than equilibria without discrimination.

This chapter contributes to the literature on the economics of discrimination by considering the impact of worker mobility. All the papers mentioned above take place in a single labor market and rule out the possibility that minority workers could choose where to live and work.

This chapter also connects to the literature on mobility started by Tiebout (1956). Much of the theoretical literature following Tiebout has focused on the impact of mobility on fiscal policy. Oates (1999) surveys the discussion on whether, given mobility, public good provision should be done at the federal or local level. Ellickson (1971) as well as Epple et al. (1984) and Epple and Romer (1991) have looked at the degree to which mobility constrains redistribution. A contribution this chapter makes to this literature is incorporating a labor search model into a Tiebout style mobility model. However, this chapter does not allow for endogenous policy, which is a feature common to this literature.

There is also a literature that studies how workers locate within cities. Of particular relevance is the spatial mis-match hypothesis, first introduced by Kain (1968) and of which Holzer (1991) provides a good survey. Spatial mis-match studies whether increased suburbanization has worsened unemployment for inner city workers, and in particular for minorities, by moving jobs further away into the suburbs. Gobillon et al. (2007) surveys the theoretical models of the spatial mis-match hypothesis. Unlike in this chapter, none of these models look at differences in the legal protection of minorities across locations. Moreover, typically these models fix wages exogenously, which leaves no scope for the mechanism driving my main result. One exception to this is Brueckner and Zenou (2003) who allow for endogenous wages, but only for the minority group, the majority group wage is still fixed exogenously.

Lastly, this chapter connects to the labor search literature through the modeling techniques employed. My model combines elements of the random search model of Mortensen and Pissarides (1994) as well as models where firms post wages such as Moen (1997). How-

ever, in Moen (1997) workers costlessly and immediately see every distinct posted wage. Each worker decides the wage level at which he wants to apply and engages in costly search among firms at that particular wage. This makes different wages act as effectively different markets. A key departure in my model is that workers instead come across posted wages randomly, similar to Mortensen and Pissarides (1994). Once a worker sees a wage, he decides whether to apply or to keep looking. My modeling of wage theft uses techniques similar to those of Albrecht and Axell (1984) and Burdett et al. (2003). Those papers are interested in workers stealing from firms within the labor market. In my model, however, it is the firms that steals from the workers.

The rest of the chapter is organized as follows. Section 1.2 analyzes a simple example that illustrates how the main mechanism functions. Section 1.3 gives the formal description of the model. Section 1.4 characterizes the optimal behavior of agents and provides a formal definition of steady state equilibrium. Section 1.5 presents the results. Sections 1.6 and 1.7 consider extensions. And section 1.8 concludes.

1.2   A Simple Example

This section analyzes a simple example that illustrates the mechanism underlying the main result of this chapter. There are two cities, labeled 1 and 2, each with a large population of firms and workers. Time is discrete and all agents are risk neutral and discount at rate $\beta \in (0,1)$. Workers live forever and come in two types, an advantaged majority (a) and a disadvantaged minority (d). Both cities are split evenly between the two types of workers. City 1 is the low enforcement city where firms are tacitly allowed to discriminate against d workers, leading to those workers being both paid less and losing their job more often. City 2 is the high enforcement city where a and d workers are treated equally.

Unlike in the full model, workers cannot move. Instead I will consider two new workers, one of each type, who don't yet live in a city and are considering where to locate. These new

workers start their life unemployed after deciding where to live. Each of these new workers draws a idiosyncratic city preference shock η from distribution Γ, which is symmetric around zero and full support. Each η is a payoff that a worker gets for living in city 2. Since η is symmetrically distributed around 0, the baseline expectation is for each city to get one worker. When employment length is sufficiently long, city 1 will have an advantage in attracting workers and will do better than gaining one worker on average. In the full model where workers can move, this will translate into city 1 being larger.

One worker matches with one firm to produce a per period surplus of 1. In city j, workers earn a per period wage of w^j, which will be pinned down by a zero profit condition. In city 1, d workers lose a fixed amount y of their wages due to wage theft by firms. Hence, the *effective* wage of a d worker in city 1 is $w^1 - y$. Employment length is T periods for all workers, except for d types in city 1 who lose their job more often and have employment lasting only $\sqrt{T}$ periods. For simplicity I restrict attention to values of T that are perfect squares. When employment ends, the worker becomes unemployed and the firm dies. Each period, firms enter each city and match randomly with a worker. Enough firms enter to match all the unemployed workers, meaning that unemployment lasts for only one period. Firms pay entry cost $k > 0$, and a free entry condition pins their expected discounted profit to zero. Because matching is random, the probability that an entering firm matches with an i worker equals the proportion of the unemployed pool that is made up of i types.

In city 2, let U^2 denote the lifetime discounted value of an unemployed worker net of the city preference η. The same value in city 1 depends on the worker's type and will be denoted U_a^1 and U_d^1. A new worker of type i moves to city 1 if and only if

$$\eta \le U_i^1 - U^2.$$

Hence the expected number of new workers city 1 will attract is

$$\Gamma\left(U_a^1 - U^2\right) + \Gamma\left(U_d^1 - U^2\right). \tag{1.1}$$

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I now analyze and compute the steady state. Steady state requires the number of workers exiting and entering employment to be equal, which pins down the city 2 unemployment rate to $\frac{1}{T+1}$. Given the different employment lengths, the unemployment rate of d and a type workers will be $\frac{1}{\sqrt{T}+1}$ and $\frac{1}{T+1}$ respectively. Let m_d^1 be the proportion of the unemployment pool made up of d type workers, then m_d^1 is defined by:

$$m_d^1 = \frac{T+1}{T+1+\sqrt{T}+1}.$$

Moreover, m_d^1 equals the probability an entering firm matches with a d type worker. Notice that m_d^1 increases in T and goes to one as T goes to infinity.

Now consider the zero profit condition in city 2, which can be written as

$$\left(1-w^2\right) \frac{\beta\left(1-\beta^T\right)}{1-\beta} = k.$$

The term $1-w^2$ is the firm's per period profit once a match is made. And the term $\frac{\beta\left(1-\beta^T\right)}{1-\beta}$ captures both that employment only lasts T periods and that searching takes one period. In city 1, the zero profit condition can be written as

$$\left(1-m_d^1\right)\left(1-w^1\right) \frac{\beta\left(1-\beta^T\right)}{1-\beta} + m_d^1\left(1-w^1+y\right) \frac{\beta\left(1-\beta^{\sqrt{T}}\right)}{1-\beta} = k. \qquad (1.2)$$

The left-hand side of the above equation has two terms, one for each type of worker. Each of those terms gives the discounted profit the firm earns from employing that type of worker, weighted by the probability of matching with that type. The first term on the left-hand side is the term for the a workers, and it reflects that firms earn a per period profit of $1-w^1$ for T periods from matching an a worker. The second term on the left-hand side is the term for d workers, and it reflects that firms earn a per period profit of $1-w^1+y$ for $\sqrt{T}$ periods from matching a d worker. The a worker term pushes w^1 towards w^2, while the d worker term pushes w^1 towards w^2+y when T is large. To see why, notice that the

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a worker term exactly matches the zero profit condition in city 2, and hence this term by itself would set $w^1 = w^2$. When T is large, the difference between $1 - \beta^T$ and $1 - \beta^{\sqrt{T}}$ is very small, and ignoring this difference the d worker term by itself would set $w^1 = w^2 + y$. The value of w^1 depends on the entire left-hand side of equation 1.2, and hence, for large T the following inequalities hold:

$$w^2 < w^1 < w^2 + y.$$

The above inequality shows us that wage theft acts as a transfer from d workers to a workers. The value of $w^1 - w^2$ is the wage gap for a workers between city 1 and city 2. While $w^1 - y - w^2$ is the gap in the *effective* wage for d workers across cities. Hence, the above inequality implies that a workers earn more in city 1 and are better off, while d workers earn less in city 1 and are worse off.

Whether w^1 is closer to w^2 or $w^2 + y$ depends on m_d^1. As m_d^1 becomes closer to one, the d worker term of equation 1.2 dominates, and that pushes w^1 towards $w^2 + y$. Hence, for larger m_d^1, the wage loss experienced by d workers in city 1 is mitigated, and the wage gain a workers experience increases. In this way, the composition of the unemployment pool plays a crucial role in determining the impacts of wage theft. And since m_d^1 increases in T, for sufficiently large T, a workers gain more than d workers lose, which means city 1 will do better at attracting new workers.

To see this argument more formally, consider the limit as T goes to infinity. The value of m_d^1 goes to one which implies that wages become:

$$\lim_{T\to\infty} w^1 = 1 - \frac{1-\beta}{\beta} k + y, \text{ and } \lim_{T\to\infty} w^2 = 1 - \frac{1-\beta}{\beta} k.$$

In this limit the d workers are earning the same effective wage in both cities, while a workers earn strictly more in city 1. The lifetime value of an unemployed worker in city 2 can be

written as

$$U^2 = \sum_{t=0}^{\infty} \beta^t w^2 - \sum_{t=0}^{\infty} \beta^{t(T+1)} w^2 = w^2 \frac{\beta - \beta^{T+1}}{\left(1 - \beta^{T+1}\right)\left(1 - \beta\right)}.$$

The above expression captures that city 2 workers are employed for T periods at a time earning w^2, interspaced with single periods of unemployment where they earn nothing. And the lifetime values of unemployed d and a workers in city 1 can be written as

$$U_d^1 = \left(w^1 - y\right)\left(\frac{\beta - \beta^{\sqrt{T}+1}}{\left(1 - \beta\right)\left(1 - \beta^{\sqrt{T}+1}\right)}\right),$$

and

$$U_a^1 = w^1 \frac{\beta - \beta^{T+1}}{\left(1 - \beta^{T+1}\right)\left(1 - \beta\right)}.$$

And as T goes to infinity, $U_d^1 - U^2$ goes to zero, and $U_a^1 - U^2$ goes to $\frac{y\beta}{1-\beta}$. And since Γ is symmetric around zero it must be that

$$\Gamma\left(0\right) + \Gamma\left(\frac{y\beta}{1-\beta}\right) > 1.$$

And the left-hand side of this inequality is how many new workers city 1 will attract in the limit as T goes to infinity. Hence, when T is sufficiently large, city 1 will expect more than half of the new workers; that is city 1 will have an edge in attracting workers.

To highlight the importance of the search features of the model, consider what would happen if, in lieu of discrimination by firms, city 1 made lump sum transfers from d workers to a workers. In this case, city 1 gains no advantage in attracting new workers. So now suppose a and d workers earn the same wage and have the same employment length in both cities. However, city 1 imposes a per-period lump sum tax of $t_d > 0$ on d workers and splits the proceeds evenly among all a type workers. So a workers receive a negative lump sum

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tax of $t_a < 0$. Budget balance requires that

$$\frac{1}{2}t_a + \frac{1}{2}t_d = 0.$$

Let U_i^j include the lump sum transfer, while still not including the city preference η. Then it must be that for a worker of type i:

$$U_i^1 + \frac{t_i}{1 - \beta} = U^2.$$

And since the budget balance condition requires $t_d = -t_a$, it must be that

$$\left(U_a^1 - U^2\right) = -\left(U_d^1 - U^2\right).$$

Using the symmetry of Γ around zero and equation 1.1, it turns out that city 1 expects exactly one of the new workers. Under lump sum transfers, the probability a workers move to city 1 goes up by the same amount that the probability that d workers move to city 1 goes down. Hence, on net, lump sum transfers do not help city 1 attract more workers. It is only with the search mechanism included that mobility rewards inadequate protections for minorities. In the full model, this same analysis carries through provided that the distribution of city preferences has an increasing hazard rate and finite expectation. In fact, lump transfers makes city 1 strictly smaller than city 2. This analysis is done in section 1.6 below.

1.3 Model

There are two cities, labeled 1 and 2, each of which contains a large number of workers and firms. Time moves continuously and all agents discount the future at rate $r > 0$. One worker matches with one firm to produce a flow of profit $\pi > 0$. However, workers and firms must search for each other, which involves frictions and takes time. There is a unit mass of

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workers who can choose to reside in either city and live forever. The mass of firms in each city is endogenously determined by a free entry condition that sets profits to zero. I will analyze the steady states of this model.

Workers come in two types, an advantaged majority (a) and a disadvantaged minority (d). The size of these populations are fixed at s_a and s_d respectively, with $s_a + s_d = 1$. Throughout subscripts will refer to a worker's type and superscripts will refer to cities. City 1 will be the low enforcement city where d workers are inadequately protected. City 2 will be the high enforcement city that protects all workers equally at a high level.

Workers can move between cities costlessly at any time. However, when an employed worker moves he give up his current job and becomes unemployed. Workers have a fixed preference regarding where to live. This is modeled by receiving flow payoff $\eta\gamma \in \mathbb{R}$ while living in city 2. The parameter $\gamma > 0$ controls the importance of the city preference. The random variable η is drawn from cdf Γ, which is symmetrically distributed around 0, has full support and is differentiable. Each worker's η is drawn once before the market opens and does not change.

Unemployed workers search costlessly for a job. Vacant firms search for workers at a flow cost of $k > 0$. If there are a measure of υ searching firms and μ searching workers in a given city, then a flow of $m(\upsilon, \mu)$ matches are created. As is standard, assume $m(\cdot, \cdot)$ is continuous, non-negative, increasing in both its arguments and constant returns to scale[3]. Constant returns to scale means the flow rate of matches depends only on $q = \frac{\upsilon}{\mu}$. Firms meets workers at rate

$$\alpha_f(q) = \frac{m(\upsilon, \mu)}{\upsilon} = m\left(1, q^{-1}\right),$$

and workers meet firms at rate

$$\alpha_w(q) = \frac{m(\upsilon, \mu)}{\mu} = m(q, 1).$$

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[3] An example of $m()$ is Cobb-Douglas where the exponents add to one.

Notably, α_w is increasing in q and α_f is decreasing in q. I also impose the standard assumption that

$$\lim_{q \to 0} \alpha_f(q) = \infty \text{ and } \lim_{q \to \infty} \alpha_f(q) = 0.$$

Every worker and firm have the same probability of matching. Hence, when a firm meets a worker, the probability that the worker is of type i is equal to the proportion of unemployed workers who are i types in that city.

A free entry condition determines the number of searching firms in each city by setting expected discounted profit of an entering firm to zero. Upon entering, firms post a wage w which they cannot change. When a match is created the worker observes this wage as well as a random match specific labor cost c distributed according to cdf G. Assume G has support $[0, \infty)$, is strictly concave and is twice continuously differentiable[4]. The firm sees the worker's type and can choose whether to extend a job offer to the worker at the posted wage. Notably, firms can condition their hiring decision, but not the level of the wage, on a worker's type. The worker can also choose whether to accept the offer not. The random labor cost ensures the worker's probability of accepting a job moves continuously with the wage, without which the firms would always have an incentive to slightly lower the wage, leading to an equilibrium wage of zero. Once employment begins, the worker gets flow utility of $w - c$ and the firm gets flow profit of $\pi - w$. Matches break exogenously and randomly with arrival rate $\delta > 0$. When the match ends, workers go back into the unemployment pool of their preferred city and firms die[5].

Firms will have occasional opportunities to engage in wage theft by underpaying the workers they employ. These opportunities arrive at rate $\lambda > 0$ and allow the firm to take a lump sum amount of $x > 0$ from the worker. In both cities, wage theft is illegal, and if caught the firm must return x and pay penalty $P^j > 0$ to the worker. Where the penalty

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[4]Examples of G include the exponential or a normal distribution truncated at 0

[5]Because a searching firm has profit zero, having firms die at the end of the match is equivalent to allowing them to search for a new worker.

P^j is specific to city j. Moreover, the match between the worker and firm is broken by the enforcement agency.

Enforcement, however, is imperfect and varies across cities and across the types of workers. In city 2, both types of workers are protected equally, while in city 1, low enforcement of the equal right of minorities leads to a type workers being more carefully protected than d type workers. Let ρ_i^j denote the probability that wage theft against a type i worker is caught in city j. As a baseline model, I consider a very stark enforcement regime. In particular set

$$\rho_a^1 = \rho_d^2 = \rho_a^2 = 1.$$

Hence, all workers, except d types in city 1, are perfectly protected and therefore firms will not attempt to underpay them. Moreover we'll assume

$$\rho_d^1 < \frac{x\,(r+\delta)}{\pi + (r+\delta)\,(x+P^1)},$$

which ensures that firms in city 1 always take advantage of opportunities to underpay d workers. This stark enforcement regime greatly eases the exposition and is without loss of generality. Section 1.7 allows for more general enforcement regimes and shows that the main results still hold.

Notice that d workers in city 1 are both effectively paid less and lose their job more often. Disadvantage in both these dimension is necessary for the main result. A more expansive model could model these two dimensions separately, but I do it within one mechanism for the sake of simplicity.

Let y_i^j be the expected monetary gain from wage theft against a worker of type i in city j. That is

$$y_i^j = x\left(1 - \rho_i^j\right) - P^j \rho_i^j.$$

One can think of wage theft as reducing the wage by λy_i^j, where λ is the arrival rate of wage theft opportunities. In particular, at an official wage of w, the *effective wage* paid by a firm engaging in wage theft is $w - \lambda y_i^j$. Additionally, wage theft increases the rate at which matches end from δ to $\delta + \lambda \rho_i^j$.

1.4 Steady State Equilibrium

This section analyzes the worker's and firm's behavior in more detail as well as provide a formal definition of steady state equilibrium. Section 1.4.1 specializes the analysis to pure strategy equilibria. Sharper conclusions hold for such equilibria and they play a central role in the proof of the main result. Throughout, superscripts will to refer to cities and subscripts will refer to worker's type. When it is clear a variable is referring to city 2, subscripts will be dropped since city 2 treats both types of workers equally.

Each worker makes two choices: whether to accept a job and which city to live in. These decisions depend on the random labor cost (c) and city preferences (η) respectively, both of which are drawn from a continuous distribution. Hence, indifference occurs with probability zero, and without loss of generality I treat the workers as employing pure strategies that depend on c and η.

Firms choose what wage to post and whether to hire workers that they meet. Since firms learns nothing between posting the wage and deciding whether to hire a worker, I treat them as making both decisions simultaneously. Hence, upon entering the market, each firm chooses a triplet $(w, h_a h_d)$, where w denotes the wage and h_i is the probability they hire a worker of type i. All firms in the same city employ the same mixed strategy over $(w, h_a h_d)$, denoted by F^j. In both cities I can restrict the firm's choice of wage to the interval $[0, \pi + \lambda y_d^1]$, since at any wage above $\pi + \lambda y_d^1$ the firm will always make a negative profit. Let $\mathcal{F}$ be the set of cumulative distribution functions on $[0, \pi + \lambda y_d^1] \times [0, 1] \times [0, 1]$, which implies that $F^j \in \mathcal{F}$.

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The firm also chooses whether to engage in wage theft. Denote by $e_i^j \in [0,1]$ the probability a firm underpays a worker of type i in city j when the opportunity arises. The assumptions on ρ_i^j ensure that

$$e_a^1 = e_a^2 = e_d^2 = 0 \text{ and } e_d^1 = 1.$$

There are also three general equilibrium variables to characterize: the ratio of searching firms to searching workers in city j denoted by q^j, the unemployment rate for a worker of type i in city j denoted by u_i^j, and lastly the proportion of city j's unemployment pool made up of type i workers denoted by m_i^j.

I now examine the worker's problem in more detail. Let U_i^j be the lifetime discounted expected payoff of an unemployed worker of type i in city j. Similarly let $W_i^j(w,c)$ be the discounted lifetime value of an employed worker of a type i worker in city j who is being paid wage w and has realized labor cost c. Both U_i^j and $W_i^j(w,c)$ are net of the city preference η. The value of $W_i^j(w,c)$ is characterized by the following continuous time value function.

$$rW_i^j(w,c) = w - \lambda e_i^j y_i^j - c + \left(\delta + \lambda e_i^j \rho_i^j\right)\left(U_i^j - W_i^j(w,c)\right) \tag{1.3}$$

The worker's payoff flow while employed is $w - \lambda e_i^j y_i^j - c$, which is the effective wage minus the cost of working. The worker becomes unemployed at rate $\delta + \lambda e_i^j \rho_i^j$, either due to an exogenous break or because the firm is caught underpaying. And $U_i^j - W_i^j(w,c)$ captures the utility loss when the worker loses his job.

Now imagine a worker deciding whether to accept a wage offer w, and who draws labor cost c for the job in question. The worker accepts the offer if and only if $W_i^j(w,c) \geq U_i^j$. Hence, the equation 1.3 above generates a cutoff $c_i^j(w)$ such that a worker of type i in city j will accept the offer if and only if c is below $c_i^j(w)$. And $c_i^j(w)$ obeys

$$c_i^j(w) = w - \lambda e_i^j y_i^j - rU_i^j. \tag{1.4}$$

Using $c_i^j(w)$, we characterize U_i^j as follows:

$$rU_i^j = \alpha_w\left(q^j\right) \int h_i G\left(c_i^j(w)\right) \left(E\left[W_i^j(w,c) \,|c \le c_i^j(w)\right] - U_i^j\right) dF^j(w, h_a, h_d). \qquad (1.5)$$

Since searching is free to workers, there is no flow utility term in the above equation. Workers encounters firms at rate $\alpha_w(q^j)$, and that firm will have posted wage w and will use hiring rates h_a, h_d according to the mixed strategy F^j. For a fixed realization of (w, h_a, h_d), the joint probability that the firm hires the worker and the worker accepts is $h_i G\left(c_i^j(w)\right)$. Once hired the term $E\left[W_i^j(w,c)\,|c \le \left(c_i^j(w)\right)\right] - U_i^j$ is the expected value of employment to the worker over being unemployed. The value of employment is conditional on the labor cost being low enough so that the worker will accept the job.

When accounting for the preference shock η, an unemployed worker in city 2 has a lifetime value of $rU^2 + \gamma\eta$. Recall that γ is the parameter that measures the importance of the city preference shock. The preference shock only applies while living in city 2, hence the lifetime utility of an unemployed worker in city 1 is still just rU_i^1. So an unemployed worker will choose to reside in city 1 if and only if η is below η_i where

$$\gamma\eta_i = rU_i^1 - rU^2. \qquad (1.6)$$

And, in steady state, employed workers do not change cities because they lose their job when moving. Since all workers become unemployed eventually, where workers live in steady state is determined by the cutoff rule η_i. The fraction of i workers living in city 1 is $\Gamma(\eta_i)$, and the mass of such workers is $s_i\Gamma(\eta_i)$. Hence the outcome of the competition between cities for workers depends entirely on η_i, which depends entirely on how (U_a^1, U_d^1) compares with U^2.

Now consider the firm's problem. Let V^j be the discounted lifetime value of a searching firm in city j. Let $J_i^j(w)$ be the discounted lifetime value of a city j firm matched with

worker of type i at wage w. $J_i^j(w)$ is characterized by the following continuous time value function:

$$rJ_i^j(w) = \pi - w + \lambda e_i^j y_i^j + \left(\delta + \lambda e_i^j \rho_i^j\right)\left(V^j - J_i^j(w)\right).$$

The firm receives a flow profit of π minus the effective wage $w - \lambda e_i^j y_i^j$. The firm loses its employee with probability $\delta + \lambda e_i^j \rho_i^j$, which causes the firm to experience a loss of $V^j - J_i^j(w)$. Since free entry requires that searching firms make zero profit, set $V^j = 0$ and derive:

$$J_i^j(w) = \frac{\pi - w + \lambda e_i^j y_i^j}{r + \delta + \lambda e_i^j \rho_i^j}.$$

I now analyze the firm's choice of the wage w and when to hire a worker. The firm's optimal decision depends on the composition of the unemployment pool given by m_i^j, and solves the following continuous time bellman equation:

$$rV^j = \max_{w,h_a,h_d}\left\{-k + \sum_{i\in\{a,d\}}\alpha_f(q^j)m_i^j h_i G(c_i^j(w))(J_i^j(w) - V^j)\right\}. \tag{1.7}$$

This equation characterizes the discounted lifetime value of a searching firm, which is the left-hand side of the equation. The searching firm pays flow cost k and meets a worker of type i at rate $m_i^j \alpha_f(q^j)$. For a given (w, h_a, h_d), $h_i G(c_i^j(w))$ is the joint probability that the firm hires the worker and the worker accepts. And the value to the firm of employing a worker over searching is $J_i^j(w) - V^j$. Since $V^j = 0$, and since the firm takes q^j as given, any w, h_a, h_d that solves the above maximization problem also solves:

$$M^j = \max_{w,h_a,h_d}\sum_{i\in\{a,d\}} h_i m_i^j G\left(c_i^j(w)\right) J_i^j(w). \tag{1.8}$$

From this maximization problem one can see that for any fixed wage w, firms set $h_i = 1$ whenever $J_i^j(w) > 0$, and sets $h_i = 0$ when $J_i^j(w) < 0$. When $J_i^j(w) = 0$ the firm is indifferent and can hire at any probability between 0 and 1. The expression maximized in

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the above equation is differentiable except when $c_i^j(w) = 0$, since the cdf G has support $[0, \infty]$ creating a kink at zero. However, the pdf G' jumps up at zero which makes the profit function locally convex. Hence it is impossible for a wage at one of the kinks to be optimal and it follows that, for any fixed choice of h_a, h_d, the firm's choice of wage w must obey the following first order condition.

$$\sum_{i \in \{a,d\}} m_i^j h_i^j \left(G'\left(c_i^j(w)\right) J_i^j(w) - \frac{G\left(c_i^j(w)\right)}{r + \delta + \lambda e_i^j \rho_i^j} \right) = 0 \qquad (1.9)$$

Lastly I verify that the restriction on ρ_d^1 is sufficient to ensure that d types workers in city 1 are always underpaid. A firm paying wage w benefits from underpaying if and only if

$$\frac{\pi - w + \lambda y_i^j}{r + \delta + \lambda \rho_i^j} \geq \frac{\pi - w}{r + \delta}.$$

The left-hand side of the inequality is $J_i^j(w)$ when setting $e_i^j = 1$, while the right-hand side is $J_i^j(w)$ when setting $e_i^j = 0$. Rearranging the inequality and substituting out y_i^j yields

$$\rho_i^j \leq \frac{x(r + \delta)}{\pi - w + (r + \delta)(x + P^j)}.$$

And no worker will ever accept a negative wage so I only consider weakly positive wages. And the right-hand side of the inequality is increasing in w, and at $w = 0$ is equal to the upper bound on ρ_d^1. Hence the assumption on ρ_d^1 ensures that the firms in city 1 set $e_d^1 = 1$.

Now I characterize the three market variables, q^j, m_i^j and u_i^j. I can pin down q^j using the free entry condition by setting V^j to zero in each city and rewriting equation 1.7 as:

$$0 = rV^j = -k + \alpha_f\left(q^j\right) M^j, \qquad (1.10)$$

where M^j is defined in equation 1.8. Since $\alpha_f(\cdot)$ is strictly decreasing, equation 1.10 uniquely pins down q^j. To derive the unemployment rate u_i^j, note that in steady state the number of

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workers leaving employment must equal the number entering which gives that

$$\left(1 - u_i^j\right)\left(\delta + \lambda \rho_i^j e_i^j\right) = u_i^j \alpha_w \left(q^j\right) \int h_i G \left(c_i^j \left(w\right)\right) dF^j \left(w, h_a, h_d\right).$$

The left-hand side of this equation is the rate at which workers of type i lose their job multiplied by the employment rate, while the right-hand side of the equation gives the rate at which workers gain a job multiplied by the unemployment rate. Rearranging the above equation gives that:

$$u_i^j = \frac{\delta + \lambda e_i^j \rho_i^j}{\delta + \lambda e_i^j \rho_i^j + \alpha_w \left(q^j\right) \int h_i G \left(c_i^j \left(w\right)\right) dF^j \left(w, h_a, h_d\right)}. \tag{1.11}$$

To derive the fraction of the unemployment pool made up of d types in city 1, m_d^1, weight the unemployment rates u_i^j by how many workers of each type live in city 1, which is given by $s_i \Gamma \left(\eta_i\right)$.

$$m_d^1 = \frac{s_d \Gamma \left(\eta_d\right) u_d^1}{s_d \Gamma \left(\eta_d\right) u_d^1 + s_a \Gamma \left(\eta_a\right) u_a^1} \tag{1.12}$$

And of course m_a^1 is simply $1 - m_d^1$. The values of m_d^2 and m_a^2 can be similarly derived, but since city 2 treats all workers equally it is unimportant. The city 2 steady state equations can all be written without reference to m_d^2 or m_a^2.

Notice that, given q^j, U_i^j and F^j, one can uniquely derive all the other steady state variables. The values for $c_i^j \left(w\right), \eta_i m_d^1$ are pinned respectively by equations 1.4, 1.6, and 1.12. Hence, I can define a steady state equilibrium in the following way.

Definition 1.1. A steady state equilibrium consists of the variables, $\left(q^1, q^2, U_a^1, U_d^1, U^2\right)$, and a pair of firm's actions $\left(F^1, F^2\right) \in \mathcal{F} \times \mathcal{F}$ such that, having derived $c_i^j \left(w\right), \eta_i, m_d^1$ from the above equations:

1. Every $\left(w, h_a, h_d\right)$ in the support of F^j solves condition 1.8.

2. U_i^j satisfies equation 1.5.

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3. q^j satisfies equation 1.10.

Proposition 1.1. *For all parameters a steady state equilibrium exists.*

Proof. See Appendix. ☐

With the restrictions on ρ_i^j, the equations that define steady state are continuous, and since firms can employ mixed strategies their best response functions are convex valued. Hence, existence can be proved with a standard fixed point argument. In appendix section A.4, I establish existence in the more general case without the assumptions on ρ_i^j. In this more general case, continuity is no longer assured. In particular, the worker's labor cost cutoff $c_i^j(w)$ jumps discontinuously at the wage where the firm is indifferent about engaging in wage theft. I am able to restore continuity by giving firms the ability to cheap talk with the workers. The cheap talk plays a role similar to public randomization. And then I show that an equilibria in which the cheap talk is never used always exists, and so I can remove the cheap talk while maintaining existence.

1.4.1 Pure Strategy Steady States

This section studies steady state equilibria where firms in the same city employ the same pure strategy. In the proof of the main result, we are able to focus on a portion of the parameter space where only these type of steady states exist. So in each city, all firms post the same wage w^j and make the same hiring decisions h_a^j, h_d^j. Therefore, a worker of type i in city j will always use the same labor cost cutoff c_i^j, when choosing whether to accept a job offer. Combining equations 1.3 and 1.5 pins down U_i^j while solving out W_i^j.

$$rU_i^j = \frac{h_i^j \alpha_w(q^j) G(c_i^j)(w^j - \lambda y_i^j e_i^j - E[c|c \leq c_i^j])}{r + \delta + \lambda e_i^j \rho_i^j + \alpha_w(q^j) G(c_i^j)} \tag{1.13}$$

Using the characterization of c_i^j from equation 1.4 U_i^j can be substituted out to get:

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$$w^j - \lambda e_i^j y_i^j = c_i^j + \frac{h_i^j \alpha_w(q^j)}{r + \delta + \lambda \rho_i^j e_i^j} \int_0^{c_i^j} \left(c_i^j - c\right) dG(c). \tag{1.14}$$

The right-hand side of the above equation strictly increases in c_i^j, which means c_i^j can be uniquely determined in terms of $\left(w^j, q^j, h_i^j\right)$. The firm's first order condition then becomes:

$$\sum_{i \in \{a,d\}} \frac{m_i^j h_i^j}{r + \delta + \lambda e_i^j \rho_j^j} \left(G'\left(c_i^j\right)\left(\pi - w^j + \lambda e_i^j y_i^j\right) - G\left(c_i^j\right)\right) = 0. \tag{1.15}$$

The free entry condition can also be rewritten as:

$$\alpha_f\left(q^j\right) \sum_{i \in \{a,d\}} \left(m_i^j h_i^j G\left(c_i^j\right) \frac{\pi - w^j + \lambda e_i^j y_i^j}{r + \delta + \lambda e_i^j \rho_i^j}\right) - k = 0 \tag{1.16}$$

The expression for the unemployment rate becomes simpler as well.

$$u_i^j = \frac{\delta + e_i^j \lambda \rho_i^j}{h_i^j \alpha_w\left(q^j\right) G\left(c_i^j\right) + \delta + e_i^j \lambda \rho_i^j}. \tag{1.17}$$

An important fact that will be used later is that for any city j, fixing m_d^j, h_a^j, h_d^j, there exists a unique solution $\left(q^j, c_a^j, c_d^j, w^j\right)$ to equations 1.14 through 1.16. And given $\left(q^j, c_a^j, c_d^j, w^j\right)$ for both cities, one can uniquely determine the rest of the equilibrium variables including city size. Uniqueness of the solution can then be proven in the following way. First fix any $m_d^j, h_a^j, h_d^j, q^j, w^j$, and notice that there is a unique c_i^j that satisfies equation 1.14, since the right-hand of that equation is strictly increasing in c_i^j. Then fix any m_d^j, h_a^j, h_d^j, q^j, and let c_i^j be defined as a function of w^j, and it turns out that the left-hand side of the firm's first order condition, equation 1.15, is decreasing in w^j even taking into account how c_i^j moves with w^j. This uniquely pins down w^j and c_i^j for any fixed values of m_d^j, h_a^j, h_d^j, q^j. Lastly, fix only m_d^j, h_a^j, h_d^j and make a similar argument on the free entry condition, equation 1.16.

1.5 Results

Now that I have formally defined steady state equilibria, I turn to establishing the main claim of this chapter, that the city that inadequately protects a disadvantaged minority gains an advantage in attracting workers. The following proposition provides a formal statement of this advantage.

Proposition 1.2. *There exists a $\delta^\star > 0$, such that for all $\delta < \delta^\star$ there will be an open set of enforcement regimes (ρ_d^1, P^1) such that in every steady state more workers live in city 1 than in city 2.*

Proof. See Appendix. □

Proposition 1.2 says that, when employment length is sufficiently long, a city can gain an advantage in attracting workers by inadequately protecting minorities. The process of voting with one's feet leads to workers voting in favor of inadequate protection of minorities. Far from protecting disadvantaged minorities, mobility has the perverse effect of promoting discrimination against them. Notably, this result does not involve selecting particular equilibrium since it applies to all steady states at the specified parameters. Also, while not explicitly stated, the open set of (ρ_d^1, P^1) found in the proposition respects the inequality on ρ_d^1 that ensured firms always engage in wage theft against d workers in city 1.

Proposition 1.2 chooses (ρ_d^1, P^1) so that the degree of wage theft is not too severe in city 1. In particular, y_d^1, which measures the firm's net gain from wage theft including penalties paid when caught, cannot be too large. Consider that, when λy_d^1 is larger than π, only wages higher than π gives d workers positive *effective* wages. And at any wage above π, firms earn negative profit from a type workers, and hence never hire them. Therefore, at any wage, either d workers will not accept it or a workers will never be hired. Hence the market will segment, with some firms posting wages appropriate for a types, and some firms posting wages appropriate for d types. And this segmentation prevents the mechanism underlying the main result from working. However, for the correct choice of (ρ_d^1, P^1), the possibility of

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being penalized for wage theft will ensure y_d^1 is small enough to prevent this segmentation from occurring. It should be noted that segmentation can occur when λy_d^1 is less than π, and bounds on y_d^1 sufficient to rule it out are found in the appendix.

To gain an intuition on how city 1 attracts more workers, consider what happens as δ goes to 0. In this limit, the a workers never lose their job and have a negligible unemployment rate. The d workers, on the other hand, still lose their job whenever the firm is caught underpaying. Hence, the unemployment pool in city 1 is made up entirely of d type workers. Entering firms in city 1 only match with d type workers, and they know that any posted wage w will result in an effective wage of $w - \lambda y_d^1$. So one can think of the firms as relabeling the wages and acting as if there is no wage theft. In other words, if $w^\star$ would have been the wage without wage theft, then the firms will post wage $w^\star + \lambda y_d^1$ instead. Hence, d type workers earn the same effective wage in both cities, which induces roughly half of them to move to each city. On the other hand, a type workers in city 1 will earn the full $w^\star + \lambda y_d^1$, inducing a majority of them to live there. Therefore, city 1 will have more workers overall. The one point the above argument glosses over is that enforcement of wage theft causes matches to to end at rate $\delta + \lambda \rho_d^1$ instead of rate δ for d type workers. However, the proposition allows us to choose ρ_d^1, and ρ_d^1 is chosen to be small enough to not interfere with the above analysis.

To complement the intuition given above I now sketch some of the more technical details. The proof proceeds in three main steps. First, focus on pure strategy equilibria where firms hire all willing workers and consider the limit as δ goes to zero. I will show that, in that limit, city 1 is always larger. I show next that, for the correct choice of enforcement regime (ρ_d^1, P^1), as δ goes to zero all equilibria will indeed be in pure strategy equilibria where firms hire all willing workers. I then finish the proof by applying an upper hemicontinuity argument to prove that city 1 must be larger for small, but strictly positive, values of δ.

To study the limit case as δ goes to 0 focus on equations 1.14 through 1.16, those equations apply to pure strategy steady states and are well defined at $\delta = 0$. Recall that these equations uniquely characterize $\left(c_i^j, q^j, w^j\right)$ for any fixed values of h_i^j and m_i^j. Now set $h_i^j = 1$, since

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we are focusing on steady states where firms hire all willing workers. Set $m_d^1 = 1$, since, at the $\delta = 0$ limit, in city 1 a workers never lose their job while d workers still do. And the solution to equations 1.14 through 1.16 does not depend on m_i^2, so it can be set to anything. With $h_i^j = m_d^1 = m_d^2 = 1$ there is a unique solution for $\left(c_i^j, q^j, w^j \right)$, which in turn determines U_i^j and city sizes. It turns out there exists a $\rho^\star > 0$ such that city 1 will have more workers as long as $\rho_d^1 < \rho^\star$. And from here on I will only consider $\rho_d^1 < \rho^\star$.

To ensure that all steady states are pure strategy where firms hire all workers, note that these properties always hold in city 2. In city 2 all workers are equally profitable to firms, which implies all workers must give strictly positive profits so that firms can recover their search costs. Hence firms must hire all willing workers. Similarly, since all workers act identically in city 2, firms will only offer wages that every workers accepts with positive probability, which avoids the kink that occurs when $c_i^j(w) = 0$. Without that kink the firm's profit function is strictly concave and all firms must offer the same wage. And these arguments can be extended to city 1 as long as y_d^1 is not too large and δ is small enough. And for any ρ_d^1, I can choose P^1 to achieve any y_d^1. And since y_d^1 is decreasing in P^1, the fact that we are making y_d^1 small mean the condition that $P^1 \geq 0$ will not be violated.

The last step is to relate the $\delta = 0$ limit to what happens at small positive values of δ. The equations 1.14 through 1.16 used to study the $\delta = 0$ limit are continuous in both the equilibrium variables and the parameter δ. Hence, the set of solutions to that system will change upper hemicontinuously as one moves δ, and city sizes must converge upper hemicontinuously to the $\delta = 0$ limit. And it must follow that city 1 has more workers when δ is sufficiently small, which finishes the proof.

I now examine how much larger city 1 can be compared to city 2. The intensity of the city preference, captured by parameter γ, determines how large a size differential is possible. For example, as γ goes to infinity workers care only about their city preference, and each city must have exactly half of each type of workers. I instead study the case as γ goes to

zero, and see how much larger city 1 can be compared to city 2, for the correct choice of (ρ_d^1, P^1) and δ.

Proposition 1.3. *For any $\varepsilon > 0$, there exists a $\gamma^\star > 0$ such that for all $\gamma < \gamma^\star$ there will exist an open set of the parameters (P^1, ρ_d^1, δ) such that in every steady state $\Gamma(\eta_d) \geq \frac{1}{2} - \varepsilon$ and $\Gamma(\eta_a) \geq 1 - \varepsilon$.*

Proof. See Appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Recall that $\Gamma(\eta_i)$ is the fraction of type i workers that live in city 1. This result says that, in the most extreme case, city 1 will have all of the a workers and half of the d workers. To gain an intuition for this result, recall that I argued above that, in the $\delta = 0$ limit, d type workers earn close to the same wage in both cities while a workers earn strictly more in city 1. And as γ goes to zero, workers care only about their earnings, and so d workers will split themselves evenly between the cities, while all a workers will choose to live in city 1.

Besides demonstrating how much better city 1 can do at attracting workers, proposition 1.3 identifies a somewhat counter-intuitive mechanism for segregation. One might reasonably expect that low enforcement would drive away the d workers, and hence with low γ, city 1 could only attract one type of worker while city 2 could attract both types. Proposition 1.3 turns this on its head and says that only city 1 can attract both types of workers. Instead of inadequate protection of minorities chasing away d workers, it is strong protection of minorities that chases away a workers. The net result is that a city that protects its workers equally ends up being segregated, while the city where a minority is inadequately protected attracts a broader cross section of workers.

One concern raised by proposition 1.2 is it may involve unrealistic unemployment rates. In particular, the unemployment rates may be very low due to the requirement of small δ. Another objection is that, given the intuition of the $\delta = 0$ case where the unemployment pool in city 1 is entirely d workers, the ratio of the unemployment rates may need to be unrealistically skewed. However, the main result is robust enough to address both of these

concerns. In particular one can match any single unemployment rate, or any ratio of unemployment rates in city 1, provided I choose the functional forms and parameters correctly. This is captured by the following result.

Proposition 1.4. *The following two things hold:*

1. *Fix any $\varepsilon > 0$ and $u^\star \in (0,1)$ and fix i,j with associated unemployment rate u_i^j. Then there exists functional forms for $m(\cdot,\cdot), \Gamma, G$ and parameters, such that in every steady state city 1 has more workers than city 2, and $u_i^j \in (u^\star - \varepsilon, u^\star + \varepsilon)$.*

2. *For any $\varepsilon > 0$ and $z > 1$ there exists functional forms for $m(\cdot,\cdot), \Gamma, G$ and parameters, such that in every steady state city 1 has more workers than city 2, and $\frac{u_d^1}{u_a^1} \in (z - \varepsilon, z + \varepsilon).$*

Proof. See Appendix $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

To prove this result I employ the simplest functional forms. In particular set the matching function to be

$$m(\mu, \upsilon) = m\mu,$$

which implies workers always meet firms at the constant rate m. And I set both Γ and G to be uniform distributions. Technically the uniform distribution does not satisfy the requirements that Γ has full support and G has support on $[0, \infty]$. However I can modify the tails of Γ and G to have the correct supports while delivering the same steady states as the uniform distribution. Consider that rU_i^j will never be below zero or above π, which allows us to impose the following bound:

$$|\eta_i| \leq \frac{\pi}{\gamma}.$$

And this bound implies that, while the mass that Γ puts below $\frac{-\pi}{\gamma}$ or above $\frac{\pi}{\gamma}$ matters, the shape of Γ in those regions does not matter. Hence one can modify the uniform distribution

in order to have an infinite tail, without changing any of the features of steady state. And a similar argument can be made for G.

Given the functional forms I am able to solve the model more explicitly and can prove proposition 1.4 using similar methods to the proof of proposition 1.2, except now I allow δ to be arbitrary and vary r instead. As I take r to be large, workers and firms become very impatient and the eventual possibility of losing a job seems increasingly distant. So from the perspective of the agents, a large r and a small δ serve a similar function. However it is δ that matters for the unemployment rate and not r. This allows us to match any unemployment rate will still maintaining city 1 as larger.

1.6   Lump Sum Transfers

In order to highlight the necessity of the search elements of the model to the main result, I consider a more direct form of discrimination. Suppose that, in lieu of allowing discrimination by firms, city 1 directly transfers money from the minority workers to the majority workers via a lump sum tax. I show that, under these transfers, city 1 will always be strictly smaller than city 2, provided that Γ has an increasing hazard rate and a finite expectation. Hence, I get the opposite effect as proposition 1.2: mobility punishes discrimination against a disadvantaged minority. Moreover, the conditions imposed on Γ are satisfied by many of the usual distributions, such as the Normal and Logistic distributions.

Start by setting $\rho_i^j = 1$ for all i, j, which ensures all workers are treated equally in both cities. For simplicity set $\gamma = 1$, but any positive γ can be used while obtaining similar results. Suppose that city 1 imposes a lump sum tax of $t_d > 0$ on d workers and splits the proceeds evenly among the a workers. The a workers receive a negative lump sum tax of $t_a < 0$, determined by the following budget constraint.

$$s_a \Gamma \left(\eta_a \right) t_a + s_d \Gamma \left(\eta_d \right) t_d = 0 \tag{1.18}$$

Since the transfers are lump sum, they only impact the worker's decision of where to live. The wages offered by firms and the speed at which workers find jobs will be the same in both cities for all workers. Hence, in every steady state it must be that

$$rU_a^1 + t_a = rU_d^1 + t_d = rU^2.$$

Recall that U_i^j is the determining factor in how workers make their location decision. And using equation 1.6, one can see that a worker of type i chooses to live in city 1 if and only if $\eta \leq \eta_i$ where

$$\eta_a = -t_a \text{ and } \eta_d = -t_d.$$

And the size of city 1 will be $s_a \Gamma(-t_a) + s_d \Gamma(-t_d)$, which demonstrates that the search elements are now irrelevant to city size. Workers make their location decision based solely on the transfers they receive and their city preference shock.

Recall that Γ has the monotone hazard ratio when $\frac{\Gamma'(\eta)}{1-\Gamma(\eta)}$ is weakly increasing. By symmetry of Γ around zero, that is equivalent to saying that $\frac{\Gamma(\eta)}{\Gamma'(\eta)}$ is weakly increasing. This latter form of the monotone hazard ratio will be used in the proof of the following proposition.

Proposition 1.5. *If Γ has a monotone hazard ratio and a finite expectation, then for every $\tau_d > 0$ city 2 will have more workers than city 1.*

Proof. It will suffice to prove that $t_a = t_d = 0$ is the transfer scheme that makes city 1 as large as possible. Consider the following maximization problem:

$$\max_{(t_a, t_d) \in \mathbb{R}^2} s_a \Gamma(-t_a) + s_d \Gamma(-t_d)$$

such that

$$s_a \Gamma\left(-t_a\right) t_a + s_d \Gamma\left(-t_d\right) t_d = 0.$$

First I will rule out the possibility that this maximization problem tries to push t_i to infinity. Since Γ is full support, the finite expectation assumption requires that $t_i \Gamma'\left(-t_i\right)$ goes to zero as t_i goes to infinity. Using the monotone hazard ratio we can conclude that $t_i \Gamma\left(-t_i\right)$ also goes to zero as t_i goes to infinity. Hence, sending t_d to infinity yields zero revenue and can't be optimal. So I can restrict t_i to a compact set with the property that there is an interior solution. Using the first order conditions of the above maximization problem one can derive that

$$t_a - t_d = \frac{\Gamma\left(-t_a\right)}{\Gamma'\left(-t_a\right)} - \frac{\Gamma\left(-t_d\right)}{\Gamma'\left(-t_d\right)}.$$

Now suppose that the optimal solution involves $t_d > 0$. From the budget constraint it must be that $t_a < 0$, which makes the left-hand side of the above equation strictly negative. However, $-t_d < 0 < -t_a$ implies the right-hand side of the above equation is weakly positive by the monotone hazard property. Hence $t_d > 0$ is not a solution to the maximization problem. And a similar argument rules out $t_d < 0$. And so it must follow that $t_a = t_d = 0$ is the unique transfer scheme that maximizes the size of city 1, which concludes the proof. $\square$

1.7 General Enforcement Regimes

In this section, I relax the restrictions we imposed on the enforcement regimes and show the main result still holds. City 2 still treats all workers equally, and catches firms engaging in wage theft at rate ρ^2, and levies penalty P^2. City 1 catches firms at rates ρ_a^1 and ρ_d^1, depending on whether the firm is underpaying an a worker or a d worker respectively. Firms that are caught in city 1 pay a penalty P^1 that is independent of the worker's type. I show that, as long δ is sufficiently small, there is an enforcement regime $(\rho_a^1, \rho_d^1, P^1)$ that ensures

city 1 is always larger, regardless of the enforcement regime (ρ^2, P^2) used by city 2. The intuition for this result is that allowing wage theft by firms only benefits the city if workers are treated unequally. Hence, city 2 attracts the most workers under perfect enforcement, where ρ^2 equals 1, which is precisely the case considered in proposition 1.2.

Existence of steady state is more difficult to establish with general enforcement regimes. In particular discontinuities will appear in the labor cost cutoff used by the workers. In section 1.7.1, I will show how to restore continuity by granting the firms the ability to cheap talk with the workers, and then showing the cheap talk is actually unnecessary for the steady state. I now state the main result of this section.

Proposition 1.6. *There exists $\delta^\star > 0$, such that for all $\delta < \delta^\star$ there exists an open set of city 1 enforcement regimes $(\rho_a^1, \rho_d^1, P^1)$ such that, for any enforcement regime (ρ^2, P^2) used in city 2, in every steady state city 1 is larger than city 2.*

Proof. See Appendix. □

Proposition 1.7 replicates ther main result using general enforcement regimes. The result says that, with the right enforcement regime and with δ small enough, city 1 is larger than city 2 no matter what enforcement regimes city 2 uses. The key step, a proof of which is found in the appendix, is showing that $\rho^2 = 1$ maximizes U^2. Since city size is directly tied to U_i^j, $\rho^2 = 1$ makes city 2 as large as possible. So simply choose $(\rho_a^1, \rho_d^1, P^1)$ in the same way as in the proof of proposition 1.2, and since city 1 must be larger than city 2 when $\rho^2 = 1$, it must be larger under all other enforcement regimes.

The intuition for why $\rho^2 = 1$ maximizes U^2 is as follows: when all workers are treated equally, wage theft serves only to shorten employment length. Firms will underpay all workers equally, hence they perfectly price this into the original wage. So the only net effect of wage theft is that occasionally the enforcement agency catches the firm and breaks the match.

1.7.1 Existence with general enforcement regimesI

I now establish existence of a steady state for arbitrary enforcement regimes. First notice that whether a firm will want to engage in wage theft will depend on the wage they are paying. There will be a cutoff wage ω_i^j, above which a city j firm employing a type i worker will always engage in wage theft, and below which they will never engage in wage theft. At wage ω_i^j they will be indifferent. The value of ω_i^j is given by

$$\omega_i^j = \pi - \frac{(r + \delta)\, y_i^j}{\rho_i^j}.$$

The value of ω_i^j depends only on exogenous parameters because free entry ensures that the firm's payoff from losing the match is zero. Let $e_i^j(w) \in [0, 1]$ be the probability that a city j firm engages in wage theft against a type i worker each time the opportunity arises. Optimality requires that $e_i^j(w)$ equals 1 for $w > \omega_i^j$, and $e_i^j(w)$ equals 0 for $w < \omega_i^j$. Hence, at the wage of ω_i^j, the labor cost cutoff will experience a discontinuity for worker of type i in city j. To see this recall that the labor cutoff is

$$c_i^j(w) = w - \lambda e_i^j(w)\, y_i^j - U_i^j.$$

No matter what value $e_i^j(\omega_i^j)$ takes, there will be a discontinuity in $c(\cdot)$ at ω_i^j. Hence, one can no longer straightforwardly apply a fixed point theorem to prove existence. This is an example of the type of existence problem that arises in dynamic games with continuous action spaces, as discussed in Harris (1985); Harris et al. (1995). Harris et al. (1995) shows that in multi-stage games public randomization ensures existence. However the model is not a multi-stage game so their results cannot be directly applied. Instead I allow the firms to engage in cheap talk with the workers, which plays a similar role to public randomization and is very much in the spirit of Harris et al. (1995). I will also show that the equilibrium that arises with cheap can be interpreted as equilibrium without cheap talk. Hence cheap

talk only serves as a proof technique and an equilibrium exists in the original model without cheap talk.

To add cheap talk, I expand the model so that when a firm meets a worker, they are able to engage to say how often they will underpay the worker, i.e. how they will set $e_i^j(w)$. At any wage other than ω_i^j, this talk is purely cheap and the worker will ignore it. However, when offering wage ω_i^j, the firm will be indifferent about wage theft, which allows cheap talk to be credible. Hence I can restrict attention to equilibria in which firms commit to e_i^j when posting wage ω_i^j, which will restore continuity. I will show how this works in city 2, and a similar argument for city 1 can be found in the appendix.

In city 2, change the action space of firms so that, instead of a choosing a wage, they choose $z \in [0, \bar{z}^2]$, where $\bar{z}^2 = \pi + \lambda y^2 + 1$. One should interpret $z \in [0, \omega^2)$ as offering wage z, and never engaging in wage theft since $w < \omega^2$. In the range $z \in [\omega^2, \omega^2 + 1]$ the firm offers wage ω^2, and credibly commits to engage in wage theft with probability $z - \omega^2$ each time the opportunity arises. Finally, in the range $z \in (\omega^2 + 1, \bar{z}^2]$ the firm is offering wage $z - 1$ and will always engage in wage theft. I can then define continuous functions $\tilde{w}^2(z)$ and $\tilde{e}^2(z)$, which give the implied wage and wage theft probability as functions of z. Writing out $\tilde{w}^2(z)$ explicitly gives that:

$$
\tilde{w}^2(z) = \begin{cases} z & \text{if } z \in [0, \omega^2] \\ \omega^2 & \text{if } z \in [\omega^2, \omega^2 + 1] \\ z - 1 & \text{if } z \in [\omega^2 + 1, \bar{z}^2]. \end{cases}
$$

And defining $\tilde{e}^2(z)$ explicitly gives:

$$
\tilde{e}^2(z) = \begin{cases} 0 & \text{if } z \in [0, \omega^2] \\ z - \omega^2 & \text{if } z \in [\omega^2, \omega^2 + 1] \\ 1 & \text{if } z \in [\omega^2 + 1, \bar{z}^2]. \end{cases}
$$

The key observation is that $\tilde{e}^2$ and $\tilde{w}^2$ are both continuous in z. The worker's labor cost cutoff can be written as

$$c^2(z) = \tilde{w}^2(z) - \lambda y^2 \tilde{e}^2(z) - rU^2,$$

which is a continuous function of z. In a similar way I can rewrite all of the relevant steady state conditions in terms of $\tilde{w}^2(z)$ and $\tilde{e}^2(z)$ and they will all be continuous. This allows the use of a fixed point theorem to prove that a steady state with cheap talk exists.I

I now show that the cheap talk is not needed. At wage ω^2, firms are only indifferent about engaging in wage theft once the worker has accepted the job. Before the worker accepts, firms will always want to promise not to underpay in order to increase the probability of acceptance. Therefore, optimality will require that any firm that posts wage ω^2 will always use the cheap talk to set $e^2 = 0$. Hence, cheap talk can taken away and it will be simply a feature of equilibrium that firms never engage in wage theft at wage ω^2.

1.8   Conclusion

In this chapter I analyzed the interaction of mobility and discrimination in the labor market. My main result was that inadequate protection of a disadvantaged minority has the potential to give a city an advantage in attracting workers. When policy makers care about long term city growth, this creates an incentive for low enforcement of the equal protection of minorities. Even when policy makers do not respond to these incentives, the growth of a city leads to a larger number of the disadvantaged minority being adversely impacted. The result required a tacit form of discrimination of the minority workers, where the firms could deprive minority workers of their wages, but are not allowed to explicitly advertise a minority specific wage. This chapter contributes to the discussion on why economically exploited minorities arise and why they persist.

Chapter 2

Losing to Win: How Partisan Candidates Help Parties Win Future Elections

2.1 Introduction

In the 1964 U.S. presidential election, Republican nominee Barry Goldwater was widely considered too ideologically extreme to be elected. The New York Times said Goldwater threatened to turn the GOP into a "right-wing splinter group" and had "minimal" chance of winning[1]. As predicted, Goldwater lost in a landslide with less than 40% of the vote, winning only six states. However, conservatives now argue that Goldwater's nomination energized the base and gave the party a momentum that led to future victories (Middendorf (2006) p.xii; Will (1994)). Goldwater himself writes in his autobiography that he never believed he had a chance of winning, yet decided his candidacy could rally the base and give voice to the conservative cause (Goldwater (1988), p. 154).

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[1] "The Goldwater Nomination", *The New York Times*, July 16, 1964, p. 30.

Goldwater's case exemplifies the thesis of this work: partisan candidates are less electorally viable but energize the base, helping to win future elections. This creates a *trade-off between winning today and winning tomorrow*. The goal of this chapter is to formalize this trade-off and explore its consequence. I build an infinite horizon model of two party competition, where in each period both parties nominate a citizen candidate to represent them in an election. In the unique subgame perfect equilibrium moderate candidates always win, and yet partisan candidates are nominated to help the party win future elections.

The model has two key ingredients: that partisan candidates energize the base and that this lasts until the next election. Energizing the base potentially has many dimensions, but for the purposes of this work I specifically mean increasing turnout. Both of these ingredients find strong support in the empirical literature. Partisan candidates increasing turnout among their base finds support in the literature on "abstention due to alienation". The idea that increasing turnout today impacts turnout tomorrow is found in the literature on "habitual voting". Section 2.2 further discusses the evidence underlying these assumptions.

The most striking feature of the equilibrium is that parties cycle in and out of power in a regular and predictable fashion, a pattern which has strong empirical support. This cycling is a natural consequence of the core trade-off of the model. In order to win, a party has to nominate a moderate, but in doing so it neglects its base and sets itself up for a weaker future. Thus the longer a party is in power the weaker and more likely to lose it becomes. This prediction has been strongly empirically verified and given various names: the "cost of ruling" (Paldam (1986); Lewis-Beck and Nadeau (2004)), "the time for a change effect" (Abramowitz (1988, 1996); Abramowitz and Saunders (2008)), the "electoral pendulum" (Norpoth (1996, 2004)), and "realignment theory" (Key (1959); Merrill et al. (2008)).

This chapter provides a new explanation for the observed cycling and provides an equilibrium model that produces this behavior. The most common explanation given is the natural variation in the business cycle. However, the papers cited above include economic controls and still conclude that there is a separate penalty for being in power. Other explanations

given for the cycling are over-reach by the party in power and an innate desire for novelty by voters. In this chapter I give a novel reason: the tension between nominating a viable candidate or one that will energize the base. Moreover, the previous literature has focused on estimating this phenomenon and providing econometric models. Since parties will manipulate and respond to any electoral force of importance, a strategic model is needed, which I provide in this chapter.

The model also predicts cycles in two other areas: ideology of nominees and turnout. In equilibrium, the parties oscillate between partisan and moderate candidates. In particular, a party that just lost power will be more ideologically extreme and gradually move to the center over time. This oscillation will create a companion cycle in the turnout of the ideological base of that party. The longer a party remains in power the less energized its base becomes, and turnout falls. Once the party leaves power it focuses on energizing its base and turnout rises again. These novel and testable predictions highlight potentially interesting empirical relationships produced by the model. As far as I know, little to no empirical work has been done on these predictions, and they present an interesting direction for future work.

As remarked above, the infinite horizon model presented here will have a unique subgame perfect equilibrium. This is important, since it allows for the distinct testable predictions that I emphasized as the key results. Uniqueness holds even as the discount rate goes to one, which is striking considering the model belongs to a class games for which a folk theorem has been proved (Dutta (1995)). This can occur because there is only a single set of payoffs that are both feasible and above the appropriately defined min-max. The intuition behind this and a sketch of the proof will be discussed in more detail in section 2.4.

The rest of the chapter is organized as follows. Section 2.1.1 briefly reviews the formal literature on candidate selection. Section 2.2 motivates and discusses the key mechanism of the model. Section 2.3 formally describes the model. Section 2.4 characterizes the equilibrium and discusses its key features and predictions. Section 2.5 checks robustness by looking at purely office motivated parties with varying levels of patience. Section 2.5 concludes.

2.1.1 Related Literature

The formal literature on candidate selection began with the insights of the median voter theorem, which goes back at least as far as Downs (1957). The median voter theorem delivers two distinct testable implications: moderate candidates perform better in election and both parties should converge to the same position. The empirical evidence strongly supports the first of these claims and strongly disagrees with the second (see Ansolabehere et al. (2001) and Erikson and Wright (2005)). The subsequent literature has worked on modifying the model to explain divergence. To do so it must address the following question: if moderates perform better in elections why are partisans candidates ever nominated?

The most direct way to explain divergence is assuming that parties have an intrinsic preference for partisan candidates, i.e., a policy motive. This tack was first taken by Wittman (1983) and later developed by Calvert (1985) and Roemer (1994). This works if the basic Downsian model is modified to include uncertainty. Another tack, taken by Palfrey (1984), argues that the possibility of entry by a third party cause divergence. Adams and Merrill III (2003) argue that partisan candidates will beat moderates by increasing turnout in the base which compensates for votes lost from the center. However, this explains divergence at the cost of contradicting the evidence that moderate candidates do better in elections. And while the focus on turnout resembles the argument in this chapter, they conclude partisan candidates win in the present while I conclude they help parties win in the future.

The key innovation found in this chapter is the use of a multi period setting to address candidate selection. All of the models discussed above are one period, and in a one period model parties can only care about two things: winning today and policy today. And each of the above models gives one of those two reason as to why partisan candidates are nominated. Because my model is multi-period parties can care about something new: winning and policy in the future. This allows for the central story of this chapter: partisans energize the base which helps parties win in the future. A multi-period model also has the advantage that it can

explain inter-temporal patterns in elections. This allows my model to explore phenomenon such as the electoral cycling highlighted in the introduction.

2.2 Motivating the Model: Evidence and Theory

In this section I discuss and motivate the two key ingredients of the model: that partisan candidates increase the turnout of the base and that this effect lasts until the next election. The first ingredient links to the literature on "abstention due to alienation" and the second to "habitual voting". These are discussed in sections 2.2.1 and 2.2.2 respectively. Crucially these literatures connect in that people who did not vote previously are more likely to have their turnout decision affected by the ideology of the candidate. In other words "habitual voters" are less likely to "abstain due to alienation". This connection leads to the key mechanism of the model: that partisan candidates can influence whether voters acquire the habit of voting. This mechanism is fully discussed in 2.2.3.

2.2.1 Abstention due to Alienation

Zipp (1985) examined how the ideological position of the candidates relative to the voter influences turnout. Zipp scored the ideological position of each individual and each candidate on a range of policy issues. He found that individuals who were far from either candidate voted less, an effect referred to as "abstention due to alienation". Of course the flip side of this effect is that people who are ideologically close to at least one candidate vote more. This captures the idea that a partisan candidate is capable of energizing the base.

More recently Plane and Gershtenson (2004) and Adams et al. (2006) have conducted similar studies and also found a significant "abstention due to alienation" effect. Adams *et al.* assumes the effect works through a threshold, that is an individual votes only if the closest candidate is close enough. Using maximum likelihood they estimated the determinants of this threshold. They find that having voted in the past election significantly reduces how

close the candidate has to be. In other words non-voters are more likely to require a candidate with similar ideology to decide to vote.

2.2.2 Habitual Voting

Plutzer (2002) notes: "[v]irtually all major works on turnout have concluded that voting behavior is, in part, a gradually acquired habit". The observation that underlies this conclusion is that voting is a highly stable behavior. This can be observed from both cross-sectional and panel data. Miller and Shanks (1996) look at cross-sectional cohort data and finds cohort turnout rises for the first two or three election before hitting a long "turnout plateau" and then tailing off at old age. Similarly Plutzer (2002) looks at panel data from U.S. presidential elections and notes that of the 516 respondents who voted in both 1968 and 1972 97% voted again in 1976. More generally numerous studies show previous voting is a robust predicator of current voting in multivariate analysis (e.g. Franklin (2004); Brody (1977)). The interpretation given to these results is that the population can be divided into two groups: voters and non-voters. During young adulthood individual citizens sorts themselves into these groups and, once settled, are high unlikely to shift from one to the other.

For the purposes of this work what matters is that there is a causal link between voting today and voting tomorrow. That such a link exists has been confirmed by both the instrumental variable and experimental approach. Green and Shachar (2000) instrumented for past voting using aggregate variables from the past elections known to influence voting, such as perceived closeness of the election or the ideological gap between candidates. They concluded that past voting causes current voting. Gerber et al. (2003) conducted an experiment where the treatment groups was urged to vote through direct mail or face to face canvassing. The treatment was more likely to vote in the current election and this difference persisted in subsequent elections. In fact those who voted in the treatment group became indistinguishable from those who voted in the control group. These two studies point to something intrinsic to voting that makes future voting more likely.

2.2.3 The Mechanism

The most common explanation given for habitual voting is that voting involves a one time fixed cost. Plutzer (2002) puts it as follows:

> As young citizens confront their first election, all of the costs of voting are magnified: they have never gone through the process of registration, may not know the location of their polling place, and may not have yet developed an understanding of party differences and key issues. Moreover, their peer group consists almost entirely of other nonvoters: their friends cannot assure them that voting has been easy, enjoyable, or satisfying.

Of course there is variation, some people enjoy engaging in the political process, even in their first election. The crucial point is, all else equal, having experience with voting makes it easier, and hence more likely.

This explanation of habitual voting connects in a crucial way with the evidence mentioned above that non-voters have a much higher "abstention due to alienation" threshold. In other words those who still face the fixed cost to voting are more likely to require a candidate with a similar ideology to be convinced to vote. This evidence isn't perfect in that non-voters are defined as someone who didn't vote last period, not someone who has never voted. Nevertheless this highly suggests that a partisan candidate brings first time voters from the political base to the polls, overcoming the initial cost to voting, and pushing them onto the path of habitual voting. More habitual voters in the base leads to higher turnout in the future, enhancing the party's future election prospects. This is the key mechanism by which partisan candidates help parties win in the future.

2.3 The Model

2.3.1 Basics

I start with an overlapping generations model with citizens who live for 2 periods and discount the future at $\beta \in (0, 1)$. Each generation contains a continuum of citizens of measure .5. Each period there is an election in which each citizen has the right, but not the obligation, to vote. Each generation of citizens is split into three equally sized groups based on ideology $\{l, m, r\}$[2].

The l and r groups are called partisan citizens, and form the base of two infinitely lived parties L and R. Type m citizens don't have a party and are referred to as moderate citizens. Each period the parties nominate a citizen candidate from one of the three groups to represent it in the election. For simplicity I suppose the parties will not choose a candidate from the other party's base. Thus parties make a binary choice between a moderate candidate or a partisan candidate from its own base. In order to avoid mixed strategy equilibrium, the party that won last period has to choose its candidate first.

The candidate with a larger measure of votes wins the election. If the party that won last period decides to keep the same candidate, then that candidate is known as an incumbent. In the event of a tie, incumbent candidates win with probability $p \in (.5, 1)$. If there is no incumbent then ties are decided by a 50-50 coin flip.

The parties are infinitely lived with discount rate β and are both policy and office motivated. A party receives payoff $\gamma > 0$ for winning an election and 0 for losing. The policy payoff of a party is 0 if someone from its base wins, $-.25$ if a moderate wins and -1 if someone from the opposite base wins. This is consistent with quadratic loss in a spatial model. To write this out formally, in any period t let w_t, y_t be the party and ideology, respectively,

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[2]The assumption that all three groups are the same size is not essential. It is important that the l and r groups are the same size.

of the winning candidate. Thus $w_t \in \{L, R\}$ and $y_t \in \{l, m, r\}$. Party L maximizes:

$$\sum_{t=0}^{\infty} \beta^t \left(v_L \left(w_t \right) + u_L \left(y_t \right) \right)$$

where $v_L \left(L \right) = \gamma > 0, v_L \left(R \right) = 0$ and $u_L \left(l \right) = 0, u_L \left(m \right) = -.25, u_L \left(r \right) = -1$. R has symmetric preferences.

2.3.2 The Decision to Vote

I follow Riker and Ordeshook (1968) and assume people vote because they gain consumption value from voting. This could take many forms: voters could enjoy expressing their opinion, fulfilling their moral or civic duty (as in Feddersen and Sandroni (2006)) or feel social pressure to vote. The alternative approach would be to assume people vote because they might change the outcome. This view is implausible in a large election and becomes even more implausible if there is a cost to voting. And since the motivation of this model, discussed in section 2.2, involves a fixed cost of voting, the pivotal voter view is an unpalatable choice.

Citizens choose whether to vote and who to vote for after observing which candidates have been nominated. In each period, voting yields utility of $D > 0$ which is net of costs such as time spent driving to the polling place, waiting in line etc. A citizen i, when voting for the first time, face an additional cost of α_i. Candidates energize the ideological group they come from and reduce the first time cost by $\delta > 0$[3].

I restrict α_i to two values $\alpha^L < \alpha^H$. These two types are evenly distributed throughout the three ideological groups and there is strictly positive mass of both. The a^L types find engaging with the political process relatively painless (or even enjoyable) and always vote in both periods of their life. For simplicity set $\alpha^L = 0$. The α^H types need to be energized or else they will "abstain due to alienation" and fail to gain the habit of voting. Specifically, if a candidate from their own ideological group is nominated in their first period of life,

---

[3]Alternatively the citizen utility's from voting could increase by δ when energized.

then they become habitual voters and will always vote. In all other cases they never vote. Formally, this is given by $(1 + \beta)D < \alpha^H < \delta + (1 + \beta)D$.

Conditional on voting, all citizens vote their sincere preference. Every citizen's first choice is a candidate with the same ideology. Partisan citizens (l and r types) prefer a moderate to a candidate with the opposite ideology. And moderate citizens are indifferent between l and r candidates. When faced with two moderate candidates, partisan citizens vote with their own party. Moderates break indifference on ideology using 50-50 randomization. These preferences lead to the following result.

Lemma 2.1. *In every election the following statements hold:*

1. *A moderate candidate will always defeat a partisan candidate.*

2. *Suppose both parties choose moderate candidates. Then if party i chose a partisan candidate last period and party j chose a moderate last period than party i's candidate will win. If both parties chose a partisan candidate last period, or neither did, then the election is a tie.*

Proof. In appendix.□

Lemma 2.1 formalizes the central logic of this work: that partisan candidates aren't as electorally viable as moderate candidates but help parties win in the future. Part 1 says a partisan can never defeat a moderate, which makes specific how they are electorally unviable. Part 2 explains what happens if both parties pick a moderate. In that case, moderate citizens will split their vote evenly and partisan citizen will vote with their own party. Thus the election is won by the party with higher turnout from their base, which in turn is determined by which party energized their base last period. In this way, partisan candidates help parties win future election by drawing in new voters and raising future turnout. In the next section, I study how these forces play out in the strategic interaction of the two parties.

2.4 Equilibrium

In this section I analyze the game described above using the solution concept of subgame perfect equilibrium. The model above will have a unique subgame perfect equilibrium, which allows for sharp predictions. This section is split into three parts. Section 2.4.1 discusses two key assumptions. Section 2.4.2 presents the unique equilibrium in detail and gives a discussion of its important features. Section 2.4.3 provides a sketch of the proof.

2.4.1 Assumptions

Throughout this section the following two assumptions are maintained

$$\textbf{Assumption 1}: \ \gamma < \frac{1+\beta^2}{2\beta(1-\beta)}; \ \ \textbf{Assumption 2}: \beta > \frac{1-p}{p}$$

The first assumption says the parties can't be too office motivated and the second says that they can't be too impatient. If the parties are patient enough ($\beta \to 1$), both conditions are always satisfied. Assumption 1 gives the allowed level of office motivation as a function of the patience of the parties and always holds for β is close enough to 1 or close enough to 0. Moreover any $\gamma \leq 2.4$ satisfies assumption 1 no matter the level of β. Assumption 2 says the parties have to be patient compared to the advantage incumbents get in the case of ties. As $p \to 1$ any level of β works, as $p \to .5$ then β needs to approach 1.

Assumption 1 is not essential; in section 2.5 I show purely office motivated parties still retain the central characteristics of the equilibrium. The main difference with purely office motivated parties is partisan candidates manage to win in equilibrium. Counterintuitively this implies a policy motive makes it less likely for partisan policies to get enacted. On the other hand, assumption 2 is essential. In section 2.5 I show that impatient parties always pick moderate candidates in every period. It is intuitive that short sighted parties won't invest in future elections. And without this forward looking behavior my model largely resembles a standard one shot spatial model and the logic of the median voter theorem applies.

2.4.2 Characterization and Discussion

Here I characterize and discuss the unique subgame perfect equilibrium, with strategies denoted by $\sigma^\star = (\sigma_L^\star, \sigma_R^\star)$. The equilibrium ends up being Markovian, so it can be described using a state that includes the party that won and the positions taken in the previous period. To denote these states I will use the notation (I, x, y) where I is the party that won last period, x is that party's last period position, and y is the losing party's previous position. For example (R, m, l) means the R party won last period with a moderate candidate, and the L party lost with a partisan candidate.

In the equilibrium the parties will use symmetric strategies, so only the states where the L party is the incumbent need to be described. This is done in the following table:

State	L's Position	R's Position	R's Position if L deviates
(L, m, m)	m	r	m
(L, m, r)	m	m	r
(L, l, r)	m	m	m

Note the table doesn't include state (L, l, m) because it is impossible since the moderate candidate picked by party R can't lose to the partisan candidates picked by L.

Proposition 2.1. *Both parties employing strategy $\sigma^\star$ is the unique subgame perfect equilibrium.*

Proof. In appendix.□

By examining the table it can be seen that this equilibrium has the property that after any history the cyclical pattern displayed in figure 1 below will result. Each box in the figure represents a period. The first line in the box is the state, and the second line describes what happens.

The first thing to notice is a moderate candidate wins in every period, and yet partisan candidates are regularly nominated. To see why consider the lower left hand box with state (R, m, m) which means L is out of power and neither party has the advantage of an energized base. R goes first and picks a moderate candidate. L could pick a moderate

Figure 2.1: Cycle in Equilibrium

(R,m,l)	(L,m,m)
L-m Beats R-m	L-m Beats R-r

(R,m,m)	(L,m,r)
R-m Beats L-l	R-m Beats L-m
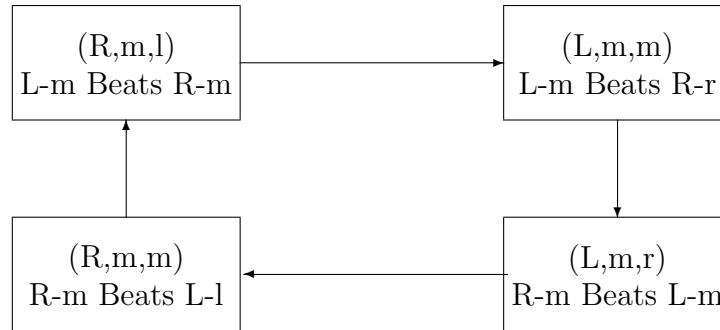
candidate which would result in a tie and give L probability $1 - p < \frac{1}{2}$ of winning. Instead L decides to concede the current election and pick a partisan. This leads to the upper left hand box (state (R, m, l)) where both parties pick moderates, but L wins for certain because they energized their base last period. Hence L trades off a chance of victory (at state (R, m, m)) for certain victory in the next period (at state (R, m, l)). Notice that $1 - p$ represents how much L is giving up by picking a partisan. This makes clear the role of assumption 2, the parties have to be sufficiently patient relative to what they are giving up by conceding the election. In the remaining two boxes, (L, m, m) and (L, m, r), the same story occurs with the role of the two parties switched.

One feature that might seem puzzling is that R chooses a moderate candidate in the upper left box in state (R, m, l). R knows L's base is energized, and can foresee defeat, so why doesn't he choose a partisan candidate to build a stronger future? Well if R chooses a partisan then party L can also choose a partisan and win for sure (recall R has to go first). And since R has a policy motive he prefers to lose to a moderate than lose to a partisan. That R acts in this way is the content of assumption 1. Thus the policy motive of the parties acts to prevent partisan candidates from winning, which is a counterintuitive result. This point is exactly how the equilibrium with purely office motivated differs from the above cycle, and will be discussed in more detail in section 2.5.

The most distinctive result that comes out of the equilibrium is that parties cycle in power in a predictable fashion. This is a result of the fundamental tension between winning today and tomorrow. In equilibrium, only moderates ever win, but they fail to energize the base leaving the winning party with a weakened future. As discussed in the introduction, this result finds strong support in data. This chapter contributes to the literature by providing a novel reason why this cycling occurs as well providing a model with strategic interaction that produces this result.

The equilibrium cycle also creates patterns in the ideology of the candidates nominated and turnout among the parties' bases. The logic of the model requires parties neglect their base while in power. So once out of office they need to reenergize their base before being able to win elections. This implies that newly out of power parties should nominate more partisan candidates and see a surge in turnout among young voter in their base. As far as I know, little empirical work has been done on these question and represents strong testable predictions of the model. Cohen et al. (2008) (pg. 92) provides some suggestive evidence, but this is a direction that warrants further investigation.

A third feature of the equilibrium is that parties only manage to elect moderates despite nominating partisan candidates. The median convergence theorem holds more strongly in policies implemented than in candidates chosen. This can occur because parties nominate partisan candidates to invest in the future, not to win today. There is limited evidence to suggest parties converge in policy outcomes (see Imbeau (2001) for a survey) and quite strong evidence they diverge in candidates nominated (see Ansolabehere et al. (2001) and Erikson and Wright (2005)). But this is another testable prediction that requires further study.

2.4.3   Sketch of Proof

Verifying that $\sigma^\star$ is an equilibrium is simply an application of the one shot deviation principle: see appendix for calculations. The calculations are made manageable by the fact the

equilibrium is Markovian. Additionally the strict inequalities in the assumptions implies the one shot deviation principle holds strictly at every history, which will be used to prove the equilibrium is unique.

The intuition for why the equilibrium is unique is the model is *almost* a zero sum game. To see this note that if a moderate always wins then the only variation in payoffs comes from who get the office motivated payoff, which is zero sum. And as seen in Lemma 2.1 a moderate always defeats a partisan, so a partisan can only win if the other party allows it. And the parties won't allow it as long as they are sufficiently policy motivated, which is guaranteed by assumption 1.

The key step of the proof of uniqueness is showing that $\sigma^\star$ is both strongly efficient and pushes both players to their min-max payoff. Thus the equilibrium is the only arrangement of strategies that is individually rational for both players. Consider any history h that begins a period, so that it is the party in power's turn to move. Let $V_i^\star(h)$ be the continuation payoff party i receives at history h if both parties are employing strategy $\sigma^\star$. The proof establishes the following three statements:

1. At history h, each party can achieve at least $V_i^\star(h)$, no matter the strategy of the other party.

2. $\sigma^\star$ is strongly efficient in the sense that the sum of continuation payoffs at history h can never exceed $V_L^\star(h) + V_R^\star(h)$

3. In any equilibrium the continuation payoff of both parties at history h must be $V_i^\star(h)$.

That statement 3 follows from 1 and 2 is obvious. Statement 2 can be seen by noting that if a partisan candidate wins the parties combined payoff is $\gamma - 1$ and if a moderate candidate wins it is $\gamma - .5$ and these are the only two possibilities. Thus combined payoff is maximized when a moderate always win, which happens in equilibrium $\sigma^\star$ starting from any history that begins a period. To see statement 1 suppose that party L commits to playing strategy $\sigma_L^\star$, then notice that, no matter what R does, $\sigma_L^\star$ ensures an r type will never win.

Thus it is intuitive the worst case for L will have a moderate winning in every period[4]. But when a moderate always wins the game is zero sum, and so the strategy that minimizes L's payoff is the strategy that maximizes R's payoff, which is $\sigma_R^\star$. Thus L's worst case is $V_L^\star(h)$ and hence he can always achieve this payoff by playing $\sigma_L^\star$.

Given statement 3 the rest of the proof proceed as follows. Take any history h that doesn't start a period, i.e. where it is the out of power party's turn to move. Without loss of generality let it be L's turn to move. Once L moves the next history will be one that starts a period. Using statement 3, for any action a, $L's$ continuation payoff must be $V_L^\star((h,a))$. This, plus the fact that the one shot deviation principle holds strictly for $\sigma_L^\star$, means that in any equilibrium L must take the same action as $\sigma_L^\star$ at history h. And this can be used to show the continuation payoffs for both parties at h must be $V_i^\star(h)$. And then this same argument can be applied again on histories that start a period, which completes the proof.

2.5 Office Motivated Parties

In this section I consider purely office motivated parties with varying level of patience. The key feature, that parties concede elections to invest in the future, is preserved with patient office motivated parties but not when they are short sighted. This shows that assumption 2 was essential while assumption 1 was not. The only change to the model is to the parties' payoff. Each party gets a payoff of 1 for winning and 0 for losing and discount the future at β. The needed level of patience is defined as:

$$\beta^\star(p) = \frac{p - \left(p^2 - 4(1-p)\left(p - \frac{1}{2}\right)\right)^{1/2}}{4\left(p - \frac{1}{2}\right)}$$

It can be easily verified that for any $p \in (.5, 1)$ we have that $\beta^\star(p) \in (0, 1)$ and $\beta^\star(p)$ is decreasing in p. When the parties are patient they follow strategy pair $\sigma_{\text{office}}^\star =$

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[4]Showing this step is somewhat involved and requires assumption 1. See Lemma B.1 in the appendix for full details.

$\left(\sigma^{\star}_{L,\text{office}}, \sigma^{\star}_{R,\text{office}}\right)$, which is described in the following matrix. Just as in $\sigma^{\star}$ the equilibrium is symmetric so I only describe the states where L is in power.

State	L's Action	R's Action	R's Action if L deviates
(L, m, m)	m	r	m
(L, m, r)	l	r	m
(L, l, r)	m	m	m

Proposition 2.2. *If $\beta > \beta(p)^{\star}$ then $\sigma^{\star}_{\text{office}}$ is the unique sub game perfect equilibrium. If $\beta < \beta^{\star}(p)$ then in the unique sub game perfect equilibrium, after any history, within two period both parties will nominate moderate candidate forever.*

Proof: In Appendix.□

By examining $\sigma^{\star}_{\text{office}}$ it can be seen that, after any history and within 2 periods, the cycle displayed in Figure 2 below results.

Figure 2.2: Cycle in Equilibrium
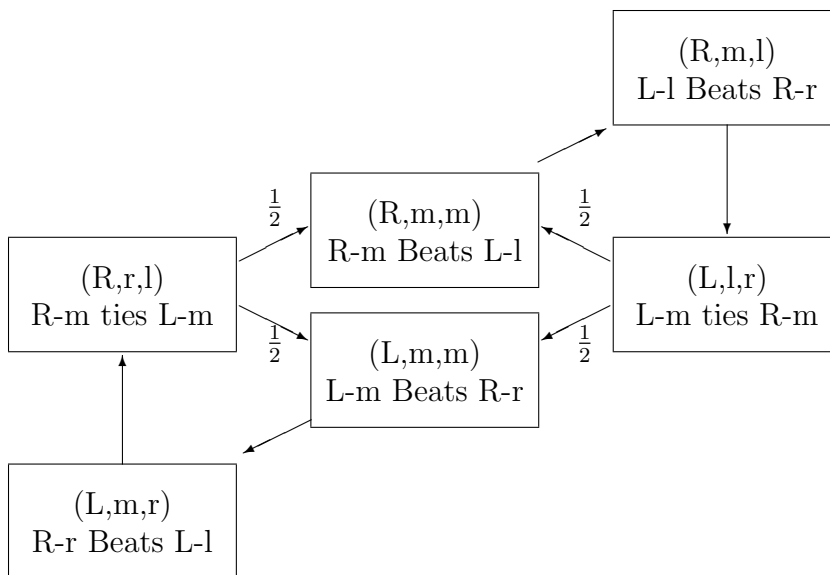
Unlike the previous cycle this one involve ties. Where the ties occur the party in power did not keep their incumbent candidate, so the election is decided by a fair coin flip. The boxes where the ties occur have two arrows coming out to indicate the two possibilities, each

of which occur with probability $\frac{1}{2}$. Conceding the current election to invest in the future still occurs at states (R, m, m) and (L, m, m) (the two center boxes), which is the same states where it happened in the original equilibrium. Thus the central feature of the original equilibrium is maintained with purely office motivated parties.

The main difference from the original equilibrium occurs at states (R, m, l) and (L, m, r), which are the lower left and upper right boxes respectively. Under $\sigma^\star$, in these states both parties move to the middle and the side with the energized base wins with certainty. In $\sigma^\star_{\text{office}}$ the side with the energized base still wins, but both parties choose partisan candidates. The reason for this is the party without the energized base knows they are going to lose no matter what. Their only motive for taking a moderate position is to keep a partisan from the other side out of power. Since there is no policy motive they don't care about that and choose instead to pick a partisan candidate to build a stronger future. This provides the counterintuitive result that a policy motive prevents partisan candidates from winning.

Proposition 2.2 also says that impatient parties converge to the center in every period. Myopic parties do not care about investing in the future and thus have no incentive to pick partisan candidates. If parties ignore the future my model reduces to a Downsian model where a median voter theorem type result holds. This result could be extended to impatient parties with a policy motive.

2.6   Conclusion

In this work I formalized the idea that partisan candidates are less electorally viable, but energize the base which is valuable for future elections. I motivated this mechanism with empirical evidence on habitual voting and abstention due to alienation. The key result of the model is it provides a novel reason for why parties cycle in power in a regular fashion. That the party in power has trouble energizing their base and so their electoral strength erodes over time.

The core innovation of this work is that it builds a strategic model of candidate selection where parties care about future as well as the present. That is parties are patient enough to consider the ramification of their actions on future elections. This work has focused specifically on the mechanism of partisan candidates energizing the base. But this is just one example of a larger principle; exploring other ways forward looking behavior can impact political strategies presents an important direction for future work.

Chapter 3

Economic Voting and Experimentation

3.1 Introduction

Economic events impact elections, a phenomenon referred to as economic voting (Kramer, 1971; Tufte, 1978; Lewis-Beck and Stegmaier, 2000). Specifically, a strong economy makes incumbent re-election more likely, while a weak economy makes it less likely. Previous work such as Rogoff (1990), Persson and Tabellini (1990) and Alesina et al. (1993) explains this behavior using a simple model of voter learning, where a strong economy is taken as a positive signal about the incumbent's policies and/or competence, leading to re-election. In this chapter, I introduce the idea that voters who have the opportunity to learn should care about learning when deciding whom to elect. In particular they may elect a party with a policy that creates worse economic outcomes but that allows for more learning. I refer to this as *experimentation,* and it is the focus of this chapter. I build a stylized model of experimentation in the context of economic voting and explore its consequences.

The papers cited above do not allow for experimentation, but rule it out using a series of assumptions that make their setup analogous to a two-period model with only one election.

The voter can only value learning if there is a future election at which to use the information learned. Hence in these models, the voter does not care about learning at the time of the election and will never experiment.

I introduce experimentation within an infinite horizon model that captures the following story. There are two parties, Left and Right, each of which advocates for a distinct ideological policy. Each period, a median voter elects a party, which then chooses what policy to implement. The parties cannot commit to policies before the election, which limits the voter's control. The voter is indifferent about ideology and cares only about growing the economy. There is an underlying state of the world that determines which ideological policy is actually best for the economy. None of the players can observe the state directly, and it changes over time. All the players attempt to infer the state by observing how the economy performs under different policies. However, economic performance has an i.i.d. noise component that makes this inference difficult. Parties also have the ability to implement a compromise policy that includes some of the ideas of each party.

The process whereby the players learn about the state has two key features: extreme economic outcomes are more informative, and ideological policies allow for more learning than the compromise policy. Spectacular policy failures or successes allow for strong inferences. In contrast, it is unclear what to infer from average economic outcomes. Moreover, the compromise policy blocks inference, since it contains ideas from both parties. As a result, both parties can take credit for good economic outcomes or pass blame for poor ones. In order to generate sharp results and for tractability, I employ assumptions that deliver a very stark learning process. In particular I assume the i.i.d. noise component of the economic outcome is distributed uniformly. This has the implication that learning is all or nothing: either the state is revealed or nothing is learned.

The first insight the model delivers is that a voter who cares only about the economy will, at times, vote for inferior economic policy in order to experiment. For example suppose the voter believes the underlying state likely favors the Left party, but is faced with a Left party

who wants to implement the compromise policy and a Right party who wants to implement its ideological policy. The Right party's policy is inferior for immediate economics outcomes, but it sets up a clean test between the ideologies. Whether it succeeds or fails, the voter will have learned something. Hence a voter who values learning sufficiently will elect the Right party, even though doing so is likely to harm the economy. In other words the voter *experiments* with the Right party. Proposition 3.1 provides a natural definition of the voter's value for learning that fully characterizes whether experimentation occurs in equilibrium. I then show that experimentation will occur if the underlying state is sufficiently persistent but will not occur if the state is sufficiently impersistent.

The second insight the model produces is that when experimentation is possible, economic voting incentives matter greatly even during calm economic times. In the classic formulation economic voting matters more with larger economic fluctuations. The model still captures this channel but also introduces a new one. During times of moderate economic outcomes, the voter has difficulty making inferences about the underlying state. Hence, after a sufficiently long period of calm economic times, the voter becomes highly uncertain about the underlying state. This raises the voter's desire for learning and makes experimenting with the disfavored ideological policy less costly. So it is precisely during calm economic times that the voter's desire for learning is highest. In proposition 3.1, I show that if experimentation occurs at all then it will occur during calm economic times. In this way the incentives underlying economic voting can be important even in the absence of a proximate economic event.

I also explore the electoral outcomes and party behavior that occur in equilibrium. Classic economic voting behavior still takes place in every equilibrium in the model: after any economic event that is large enough to reveal the underlying state, the voter will always elect the party the state favors, and that party will implement their ideological policy. When the state has just been revealed, the favored party and the voter both want the same thing: the favored party's ideological policy.

To further explore electoral outcomes I study equilibria with and without experimentation separately. Proposition 3.3 examines equilibria with experimentation. It says that, in any equilibrium with experimentation, after a sufficiently long period of calm economic times the party in power will switch every period and only ideological policies will be implemented. This continues until the next economic outcome large enough to reveal the state. The implementation of ideological policies is an expression of the voter's desire to learn, which is high during calm economic times because the voter becomes very uncertain about the underlying state. Switching the party in power every period is required to give the parties the proper incentives. Since the party in power knows it will lose the next election unless learning occurs, it will desire to implement ideological policies. Hence, the voter's desire for experimentation causes high political turnover during calm economic times. This reinforces the earlier point that economic voting incentives matter even without a proximate economic event. In equilibria without experimentation the exact opposite outcome occurs: proposition 3.4 says that after a sufficiently long period of calm economic times, the same party will remain in power every period and will always implement the moderate policy.

Recall that experimentation is more likely when the underlying state of the world is persistent. The interpretation of a persistent state is that the policy needs of the economy are stable. Hence, the model has the counter-intuitive implication that, during moderate economic times, stable economic fundamentals lead to higher political turnover and unstable fundamentals lead to lower political turnover.

I solve the model by looking for mixed strategy Markov perfect equilibria that treat the parties symmetrically. The focus on symmetric equilibria is based on the idea that the median voter is not invested in either party. Asymmetric equilibria would allow the voter to coordinate better with one of the parties, and hence prefer that party to be in power. Moreover, the parties in the model are entirely symmetric, so focusing on symmetric equilibria is natural. The Markov assumption, as usual, helps with tractability and to cut through the multiplicity problem prevalent in infinite horizon models. This is especially

important given that the model belongs to a class of games for which Dutta (1995) has proved a folk theorem. Technical considerations aside, the Markov assumption is appropriate in these large election games due to the difficulty in coordinating many voters. Coordinating on complex history-dependent equilibria seems implausible with a large electorate. Of course, the model only has a single median voter, but this is a useful abstraction, and one can think of the voter as representing a large electorate in the background.

The model assumes a high level of voter sophistication, which can be viewed as unrealistic. However, the behavior I examine is interesting independent of the realism of the voter. Exploring optimal voter behavior is important even if just to see how far away it is from actual voter behavior. Additionally this chapter is undertaking the important theoretical task of taking the learning rationale for economic voting seriously and exploring it to its natural end. Moreover, the sophistication required by the model is not altogether implausible. It is true that the calculations required of the voter are fairly complex, but the strategies are quite straightforward. The strategies of the voter depend on something quite simple and plausible: the voter's belief about which party's policy is better for the economy. This can be represented by a single value and is something that does factor into actual voting behavior.

This chapter connects to two distinct literatures. It is most directly linked to the literature cited above relating economic voting to learning (Rogoff, 1990; Persson and Tabellini, 1990; Alesina et al., 1993). However, as previously discussed, these models employ assumptions to ensure the equilibrium is the same as a two period model, with the two periods just continually repeated. They also only allow elections every other period. Hence these models are functionally two period models with only one election. In such a model, the voter learns before the election, but at the time of the election he no longer cares about additional learning since there is no future election. Callander (2011) explores voter learning in a truly dynamic model with a much richer learning environment than employed this chapter, but

he employs myopic players who do not value the future, which rules out any possibility of experimentation.

A second literature studies learning about polices in the absence of parties. Piketty (1995) and Strulovici (2010) look at models where individuals vote directly on policies. Removing the parties removes the agency problem, which is one of the main focuses of this chapter. Also, many of the key results of this chapter are framed in terms of the parties and whom the voter chooses to elect. In both papers mentioned above, agents learn based on their private information and make no inference from aggregate variables. In contrast I focus precisely on learning from aggregate outcomes and how that influences the interaction of the players.

The rest of this chapter is organized as follows. Section 3.2 formally describes the model and discusses how the learning process works. Section 3.3 lays out the value functions of the players and demonstrates a few useful facts about player behavior. Section 3.4 discusses when and why the voter experiments in equilibrium. Section 3.5 studies the electoral outcomes in equilibria both with and without experimentation. Section 3.6 concludes.

3.2   Model

This section presents the formal model. I build a stark model both for tractability and to allow for sharp results. This section starts with a terse, but complete, description of the model and then expands on and discuss the most important element: learning process.

Model Description: There are three infinitely lived players: two parties, L and R, and a voter v. Time is discrete and denoted $t = 1, 2....$ Each period the voter elects a party who then publicly selects a policy $x_t \in \{-1, 0, 1\}$. The parties cannot commit to a policy before the election. The policies -1 and 1 represent extreme or ideological policies associated with one of the parties. Party L prefers policy -1, and party R prefers policy 1. The 0 policy is a compromise, which both parties are lukewarm about. Full preferences will be described below.

After the policy is selected the economic outcome y_t is revealed which represents the growth rate of the economy. The value of y_t follows the process

$$y_t = -(x_t - s_t)^2 + \varepsilon_t.$$

The random variable ε_t is i.i.d. and distributed uniformly on the interval $\left[\frac{-2}{1-r}, \frac{2}{1-r}\right]$, with $r \in (0,1)$. The variable s_t is an underlying state of the world determined at the start of every period. The variable s_t can be either -1 or 1 and follows a Markov switching process where $s_t = s_{t-1}$ with probability γ and $s_t \neq s_{t-1}$ with probability $1 - \gamma$. As a non-triviality condition restrict $\gamma \in \left(\frac{1}{2}, 1\right)$. None of the players observe s_t or ε_t, and all players form beliefs on s_t using the public history of x_t and y_t. Denote by p_t the belief that the state equals 1, and p_t can be defined as:

$$p_t = \Pr\left(s_t = 1 | \{x_j, y_j\}_{j=1}^{t}, s_0\right).$$

There is no asymmetric information so all players always share the belief p_t.

By inspecting the process for y_t it becomes clear that s_t determines which policy is best for the economy. If s_t were known, then setting $x_t = s_t$ would maximize y_t. This captures the idea that each party's ideological policy is right for the economy in some state of the world. In practice the voter is uncertain about s_t, and p_t captures his belief about which policy is more likely to be correct. In other words the L party is *belief-favored* whenever $p_t < \frac{1}{2}$ and the R party is *belief-favored* whenever $p_t > \frac{1}{2}$.

All players discount at $\beta \in (0, 1)$. The voter cares only about the economy and maximizes

$$E\left[\sum_{s=1}^{\infty} \beta^{s-1} y_s\right].$$

The parties are both office and policy motivated while caring nothing about the economy. The L party's policy payoff is $-x_t$, and their office payoff gives them $\lambda > 0$ in each period

they are elected and $-\lambda$ in each period they are not elected. Let $I_t \in \{-1, 1\}$ be an indicator function that takes value 1 if L is elected at t and -1 otherwise. Then the L party maximizes

$$E\left[\sum_{s=1}^{\infty} \beta^{s-1} \left(I_t \lambda - x_t\right)\right].$$

The R party's payoffs are similar: they get policy payoff x_t, and λ for winning and $-\lambda$ for losing. Hence the R party maximizes

$$E\left[\sum_{s=1}^{\infty} \beta^{s-1} \left(-I_t \lambda + x_t\right)\right].$$

Notice that the sum of the two parties' payoffs is 0. However, the game is not zero-sum because of the existence of the voter.

I analyze this model using mixed strategy symmetric Markov perfect equilibrium where the Markov state is the belief p_t. Symmetry imposes that at beliefs p and $1 - p$, L and R act symmetrically and the voter treats them symmetrically. Formally, the strategies of the parties are functions $\sigma_i : [0, 1] \to \Delta\{-1, 0, 1\}$, and the strategy of the voter is a function $\sigma_v : [0, 1] \to \Delta\{L, R\}$. I abuse notation and let $\sigma_i(a; p)$ be the probability that player i takes action a at belief p. The symmetry assumption imposes that

$$
\begin{aligned}
\sigma_L(-1; p) &= \sigma_R(1; 1 - p) \\
\sigma_L(1; p) &= \sigma_R(-1; 1 - p) \\
\sigma_L(0; p) &= \sigma_R(0; 1 - p) \\
\sigma_v(L; p) &= \sigma_v(R; 1 - p).
\end{aligned}
$$

Since the parties can use mixed strategies, standard fixed point arguments ensure equilibrium existence. A formal existence proof is given in the appendix. Throughout, the following

assumptions are maintained:

$$\frac{1}{2-r} < \beta \text{ and } \lambda > \frac{1-\beta+2\beta r}{\beta(1-r)}.$$

The first assumption ensures the players are forward looking enough to care about learning. The second assumption ensures the parties are sufficiently office motivated.

Learning Process: At the start of each period t, all the players have a shared prior about s_t denoted p_t. After observing x_t, y_t, they use this information to update to a new belief denoted by v_t. Note that v_t will not equal p_{t+1}, since the players will update again using their knowledge of the Markov process that s_t follows. The next period belief p_{t+1} is set by

$$p_{t+1} = v_t + (1-\gamma)v_t.$$

The process for updating from p_t to v_t is very stark due to the assumption that ε_t is distributed uniformly. For extreme y_t, the underlying state s_t is completely revealed, and for moderate y_t nothing is learned. Hence v_t can only take one of three values: 0,1 or p_t. To see this, suppose the -1 policy is implemented, then there are two possible distributions for y_t. If $s_t = -1$ then $y_t \sim U\left[\frac{-2}{1-r}, \frac{2}{1-r}\right]$, or if $s_t = 1$ then $y_t \sim U\left[-4+\frac{-2}{1-r}, -4+\frac{2}{1-r}\right]$. Any $y_t < \frac{-2}{1-r}$ is outside the support when $s_t = -1$, and hence for any such y_t it can be inferred that $s_t = 1$. Similarly if $y_t > -4+\frac{2}{1-r}$ then it must be that $s_t = -1$. For intermediate values of y_t nothing can be learned since both of the possible distributions have the same density in this region, and hence $v_t = p_t$. With the compromise policy, y_t has the same distribution regardless of s_t, and hence no matter the outcome nothing can be learned and $v_t = p_t$.

For clarity, I will that say learning occurs if and only if s_t is revealed. Note that learning is distinct from p_t changing. For any ideological policy ($x \in \{-1, 1\}$) learning occurs with probability $1-r$ and no learning with probability r. Conditional on learning, with probability p_t the state is revealed to be 1 which gives $p_{t+1} = \gamma$, and with probability $1 - p_t$ the state is revealed to be -1 which gives $p_{t+1} = 1 - \gamma$. If there is no learning then $p_{t+1} =$

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$\gamma p_t + (1 - \gamma)(1 - p_t)$. If the policy is moderate $(x = 0)$ there is never any learning and $p_{t+1} = \gamma p_t + (1 - \gamma)(1 - p_t)$ with probability 1.

The learning process used here is very simple and yet captures the essence of the story told in the introduction. In particular, it captures the idea that large shocks create learning, and that moderate policies make learning more difficult. Both of these features are fairly robust and would remain in a wide variety of specifications.

I conclude this section with a technical remark on the belief process. As discussed above, the belief moves either by jumping to the extremes, $(\gamma, 1 - \gamma)$ or moving in discrete jumps towards $\frac{1}{2}$. Hence, only a countable number of beliefs are ever possible. This feature simplifies the technical analysis and is the main tractability advantage of the all or nothing learning process. The possible beliefs can be calculated by forward solving the equation $p_{t+1} = \gamma p_t + (1 - \gamma) p_t$ starting from γ and $1 - \gamma$.

3.3   Value Functions and Incentives

In this section I discuss the incentives of the players and highlight some useful facts about their behavior. I do this primarily by examining the value functions of the voter and the parties. Using the Markov assumption, the value functions depend only on the current belief about s_t, denoted by p. Recall that

$$p = \Pr\left(s = 1 \middle| \{x_j, y_j\}_{j=0}^{t}, s_0\right),$$

and I say that L is belief-favored when $p < \frac{1}{2}$ and R is belief-favored when $p > \frac{1}{2}$.

First consider the voter's decision of which party to elect. The Markov assumption ensures that the voter only cares about the implemented policy. In equilibrium the voter forecasts what each party will do, and then elects the party whose policy he prefers. This allows the voter's decision problem to reduced simply comparing different policies. I will abuse notation and let $W(p)$ represent the voter's equilibrium payoff at belief p, and I let

$W(x; p)$ be the equilibrium payoff at belief p if policy x is implemented. I use $\Pr(q|x, p)$ to denote the probability that next period's belief is q given current belief p and policy x. I can characterize $W(x; p)$ as follows:

$$W(x; p) = E[y|x, p] + \beta \sum_q \Pr(q|x, p) W(q).$$

The right-hand side of the above equation has two terms: $E[y|x, p]$ is the expected economic outcome, and $\sum_q \Pr(q|x, p) W(q)$ is the voter's continuation payoff. Notice the policy only impacts the voter's continuation payoff by changing the $\Pr(q|x, p)$ term. In other words, the policy only impacts the voter's future payoff through the learning process. Hence the voter's choice of policy depends on two things: the current economy and learning. The voter's concern about the economy is captured by the following expression:

$$E[y|x, p] = \begin{cases} -4p \text{ if } x = -1 \\ -4(1-p) \text{ if } x = 1 \\ -1 \text{ if } x = 0 \end{cases}$$

If $p > \frac{3}{4}$ or $p < \frac{1}{4}$, then the voter is quite certain of s_t and the belief favored ideological policy is best for the economy today. If $p \in \left(\frac{1}{4}, \frac{3}{4}\right)$ then the voter is uncertain of the state and the compromise policy is best for today's economy[1]. Moreover, the disfavored ideological policy is always the worst policy for today's economy.

Now consider the voter's second concern: the learning consequences of policies captured by $\Pr(q|x, p)$. The process governing $\Pr(q|x, p)$ was laid out in section 3.2. One important point to recall is that the two ideological policies have the same learning consequences. Hence $\Pr(q| - 1, p) = \Pr(q|1, p)$ for all q, p. This implies the voter always prefers the favored

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[1]This analysis clarifies the need for $\gamma > .75$. If $\gamma < .75$ then $p \in \left(\frac{1}{4}, \frac{3}{4}\right)$ will always be true and hence the voter will never be sure enough about the s_t to be willing to gamble on ideological policy. This means they can never value learning and the model becomes quite trivial.

ideological policy over the unfavored one. In other words $W\left(-1;p\right) \geq W\left(1;p\right)$ if and only if $p \leq \frac{1}{2}$.

Whenever learning occurs, s_t is fully revealed, and hence, the voter gets continuation payoff of either $W\left(\gamma\right)$ or $W\left(1-\gamma\right)$. It turns out that the voter's continuation payoff is the same regardless of what was learned, as is captured by the following lemma.

Lemma. *In any symmetric MPE $W\left(\gamma\right) = W\left(1-\gamma\right)$.*

Proof. Follows directly from the symmetry of the model and assumption of a symmetric MPE. $\square$

This result says that the voter has the same continuation value from learning regardless of what s_t is revealed. Intuitively, this represents that the voter is not invested in the ideological debate between the parties. Moreover, this result allows reference to a single unambiguous value of learning for the voter. This will be crucial in the statement and interpretation of my results. Additionally, this simplifies the learning process from the voter's perspective: the compromise policy allows no learning and the ideological policies create learning with probability $1 - r$ and no learning with probability r.

I now turn to the parties' incentives. The only decision the parties make is what policy to implement once elected. So only the party's preferences over policies need to be considered. The value functions of the parties are:

$$V_L\left(x;p\right) = -x + -I_t\lambda + \beta \sum_q \Pr\left(q|x,p\right) V_L\left(q\right) \tag{3.1}$$

$$V_R\left(x;p\right) = x - I_t\lambda + \beta \sum_q \Pr\left(q|x,p\right) V_R\left(q\right) \tag{3.2}$$

For concreteness I will discuss L's value function (equation 3.1) but everything said applies equally to R. The term $-x$ represents L's current policy payoff. The term $I_t\lambda$ captures the office motivated payoff. Recall that I_i takes value 1 if the L party wins and -1 if the R party wins. At the time the party chooses the policy the election has already occurred

and the office payoff is already sunk. However, the parties will account for getting the office payoff in future periods in making their decision. The value $\beta \sum_q \Pr(q|x, p) V_L(q))$ represents L's continuation payoff and, as with the voter, the policy only impacts the continuation payoff through the learning process. The first thing to notice is that the parties will never implement the opposite ideological policy. This follows from the fact that the ideological policies have the same learning consequences. Hence, without loss of generality suppose the parties make a binary choice between the compromise policy and their own ideological policy.

An important implication of the zero sum property is that both parties never want to implement the compromise policy in the same period. Intuitively, blocking learning is the parties' only motivation for the compromise policy, and if blocking learning helps one party it must hurt the other party. Hence both parties can't want to block learning at the same time, which can be seen formally from the following expression:

$$V_L(p, 0) - V_L(p, -1) = -V_R(p, 0) + V_R(p, -1) = V_R(p, 1) - V_R(p, 0) - 2. \qquad (3.3)$$

The first equality is a consequence of the zero sum property. The second equality occurs since policies -1 and 1 have the same learning consequences and differ only in today's policy payoff. Hence $V_R(p\dot{-}1) = V_R(p, 1) - 2$ for all p. Party L desiring to implement policy 0 is the same as the far left of equation 3.3 being positive, which means the far right must be positive. And that can only happen if $V_R(p, 1) - V_R(p, 0) > 0$, which implies party R strictly prefers to implement their ideological policy. And this shows it is impossible for both parties to desire to implement the compromise policy at the same p.

3.4   Experimentation

This section defines what it means for the voter to experiment, and discusses why and when it occurs.

Definition. Experimentation *occurs if the voter elects the belief unfavored party who implements their ideological policy with probability 1. Experimentation occurs in a equilibrium if there exists a belief p where experimentation occurs that is reached with probability 1 as $t \to \infty$.*

To understand this definition recall that the belief unfavored party's ideological policy is the worst possible policy for today's economy (y_t). The economy would have done better with the other party no matter what policy they would have implemented. Hence, the only justification for the voter's choice is a desire to increase learning, i.e. to experiment. An alternative definition would have been to define experimentation as occurring when the voter makes a choice that is worse for today's economy than what the other party would have implemented. However, this alternative definition would require knowing counter-factual information: what the unelected party would have done and whether it would have been better for the economy. On the other hand, the definition given above can be identified from what actually occurred along with beliefs about which party the public trusts more with the economy, a subject of constant polling. Additionally, this definition makes experimentation harder to find, biasing the results against finding experimentation.

I now state my first result about experimentation.

Proposition 3.1. *In any symmetric MPE:*

1. *Experimentation will occur in equilibrium if and only if the voter's valuation of learning $(W(1-\gamma))$ is strictly greater than $\frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$.*

2. *There exists $\gamma_H^\star, \gamma_L^\star \in \left(\frac{1}{2}, 1\right)$ such that for all $\gamma > \gamma_H^\star$ experimentation always occurs and for all $\gamma < \gamma_L^\star$ experimentation never occurs.*

3. *If $W(1-\gamma) > \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$ then experimentation will occur regularly after a sufficiently long period of moderate economic shocks (no learning), and will continue until the next large economic shock. In particular there is a $p^\star < \frac{1}{2}$ such that as long as the belief remains in $(p^\star, 1-p^\star)$ experimentation will occur every other period.*

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Proof. In appendix. □

Experimentation requires that the voter, who cares only about the economy, to vote for the worse economic policy. Proposition 3.1 illuminates why and when this would occur. Part 1 of the proposition says that a voter's willingness to experiment depends on his continuation value after learning occurs, which is captured by $W(1-\gamma)$. With a high enough value of $W(1-\gamma)$, the voter is willing to accept worse economic outcomes today in order to learn more and get better outcomes in the future. The second part of the proposition relates experimentation to the primitives of the model. The value of learning depends on the persistence of the underlying state captured by γ. A high value for γ makes the underlying state very persistent, which makes learning valuable and leads to experimentation. Conversely, a low value for γ prevents experimentation. The interpretation of γ is the stability of the policy needs of the economy. In an economy with rapidly changing policy needs, experimentation serves little purpose. Conversely, if a policy that works today is likely to work far in the future, then learning about the policies is worthwhile.

The third part of proposition 3.1 says that the incentives behind economic voting are important even without a proximate economic event. In the classic formulation, economic voting is a response to immediate economic events and has more salience with larger fluctuations. This part of the proposition broadens the view of when economic voting incentive matter by showing that it is precisely during calm economic times that the voter is most likely to experiment. Moderate economic outcomes block learning which causes the belief p to drift towards $\frac{1}{2}$. After a sufficiently long period of such times the voter becomes very uncertain about the state captured by $p \in (p^\star, 1-p^\star)$, which drives the voter's desire to learn. It also makes experimenting with the disfavored ideological policy less costly since the voter only weakly believes the disfavored policy is wrong for the economy. In this way, economic voting incentives remain salient even during calm economic times. This point will be strengthened in section 3.5 where I show this motive to experiment will also impact election outcomes.

This proposition also yields testable predictions of the model: the party which voters disagree with on the economy will occasionally win with an extreme platform. Moreover, this will occur during calm economic times and to continue regularly until the next large economic shock. This prediction is testable in part because experimentation was defined in an observable way. Part 2 of the proposition can also be tested using the interpretation of γ as the stability of the policy needs of the economy. The prediction would be that experimentation is more common in places or times with more stable policy needs.

The full proof of proposition 3.1 can be found in the appendix, but I provide a brief sketch here. I start with part 1 of the proposition. First, suppose experimentation occurs and we want to show that

$$W\left(1-\gamma\right) > \frac{-1}{1-\beta} + \frac{1}{\beta\left(1-r\right)}.$$

Without loss of generality consider $p < \frac{1}{2}$, which means the L party is belief favored. Experimentation when $p < \frac{1}{2}$ requires the R party to be elected and to implement policy 1. Since the voter prefers -1 to 1 this can only happen if the L offered policy 0. Set p' to be the next period belief when no learning occurs, so that $p' = \left(1-p\right)\left(1-\gamma\right) + p\gamma$. Using the value functions from above, the voter prefers policy 1 over 0 if and only if:

$$0 \le -2\left(1-2p\right) - 1 + \beta\left(1-r\right)\left(W\left(\gamma\right) - W\left(p'\right)\right).$$

This can be rewritten as

$$W\left(p'\right) + \frac{1+2\left(1-2p\right)}{\beta\left(1-r\right)} \le W\left(1-\gamma\right).$$

Since $p < \frac{1}{2}$, it must be that $\frac{1+2(1-2p)}{\beta(1-r)} > \frac{1}{\beta(1-r)}$. So it would suffice to show that $W\left(p'\right) \ge \frac{-1}{1-\beta}$. The formal proof of this point can be found in the appendix but the intuition is as follows: the voter likes learning enough that he prefers even the wrong ideological policy

over the compromise policy. Hence it is intuitive his worst case scenario is always getting the compromise policy and that scenario gives payoff $\frac{-1}{1-\beta}$. So it follows that $W(p') \geq \frac{-1}{1-\beta}$.

For the other direction suppose that

$$W(1-\gamma) > \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)},$$

and we want to show experimentation occurs. For concreteness suppose L is belief advantaged ($p < \frac{1}{2}$). Well, high $W(1-\gamma)$ makes the voter value learning enough that always getting policy -1 is their best case. For p close to $\frac{1}{2}$ the voter is close to indifferent between policies -1 and 1. Recall, at least one of those will always be offered. So the voter can achieve close to their best case by electing whoever offers an ideological policy. To do this, equilibrium incentives will require that the R will be elected for some p close to $\frac{1}{2}$. To see this suppose the L party wins for all p close to $\frac{1}{2}$. Then L has a strong incentive to implement policy 0, which will keep p close to $\frac{1}{2}$ by blocking learning. But L cannot be elected with policy 0, which creates a contradiction. Hence R must win for some p close to $\frac{1}{2}$ and implement policy 1.

Part 2 follows quite naturally from part 1. If γ is very close to 1 then learning lasts for a long time which is valuable to the voter. Also, with high γ the belief advantage party is not worried about learning and is happy to implement their ideological policy which reduces the agency problem. So we can show as $\gamma \to 1$ that

$$W(1-\gamma) \to 0 > \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)},$$

and the result follows from part 1 of the proposition. Note that the inequality $0 > \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$ is guaranteed by the assumptions on β. On the other side, if γ is too low that learning isn't useful to the voter. Recall that for any belief $p \in \left(\frac{1}{4}, \frac{3}{4}\right)$ the compromise policy maximizes $E[y_t]$. And if $\gamma = .75$ the lowest (highest) the belief can ever be is $\frac{1}{4}\left(\frac{3}{4}\right)$. So there must exists an upper-bound for $W(1-\gamma)$ that converges to $\frac{-1}{1-\beta} < \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$ as $\gamma \to .75$.

3.5 Electoral outcomes

In this section I discuss the electoral outcomes that occur in equilibrium. This includes who the voter elects and what policy gets implemented. I start by replicating the basic economic voting result.

Proposition 3.2. *In any symmetric MPE, after any economic shock large enough to cause learning the incumbent party is re-elected if the economic outcome was higher than $E[y_t]$, and the out of power party is elected if the outcome was lower than $E[y_t]$.*

Proof. See Appendix□

Proposition 3.2 replicates the basic economic voting result and with the same reasoning as found in previous works. Large economic shocks allow the voter to make an inference about the underlying state. The voter infers the state favors the incumbent following a positive shock, and infers the state favors the challenger following a negative shock. And when the party is highly confident s_t favors them they have little incentive to block learning which reduces the agency problem between the voter and the favored party. Following learning the voter both strongly believes in the favored party and has a reduced agency problem with them. Both of these forces push the voter to re-elect the favored party leading to the result.

For other electoral outcomes consider equilibria with and without experimentation separately. First I examine equilibria with experimentation.

Proposition 3.3. *In any symmetric MPE with experimentation, there exists a $p^\star < \frac{1}{2}$ such that for all $p \in (p^\star, 1 - p^\star)$ incumbents always lose and the party in power always implements their own ideological policy.*

Proof In appendix.□

To interpret this result recall that the belief drifts towards $\frac{1}{2}$ during moderate economic times and only moves away from $\frac{1}{2}$ with extreme economic outcomes. Hence, proposition 3.3 says that, in any equilibrium with experimentation, after a sufficiently long period of calm

economic times the party in power will switch every period and only ideological policies will be implemented. This continues until the next economic outcome large enough to reveal s_t. Hence, during a calm economy I can expect to see highly ideological policies and high political turnover. This reinforces the earlier point that economic voting incentives matter even without a proximate economic event.

The fact that only ideological policies are implemented is an expression of the voter's high desire to learn during calm economic times. Recall from the previous discussion of proposition 3.1 that p close to $\frac{1}{2}$ makes the voter desire experimentation and learning generally. Moreover, p close to $\frac{1}{2}$ means the voter views the two ideological policies similarly. Hence, the voter will elect whichever party offers an ideological policy with a slight preference for the belief-favored policy. And recall from the discussion in section 3 that at least one party will implement an ideological policy at all times. And so only parties who intend to implement ideological policies will ever be elected.

The switching of who is in power every period is required by equilibrium in order to give parties the proper incentives. If no learning leads to an incumbent being re-elected, then the party in power has a strong incentive to block learning. And when p is close to $\frac{1}{2}$, the parties are naturally wary of learning since they are unsure of s_t. These two incentives will push the party to implement a compromise policy. But the voter would never have elected them in that case. Hence equilibrium requires that the incumbent party always loses. In this way the voter's desire for experimentation causes high political turnover during calm economic times. In the next result I consider equilibria without experimentation.

Proposition 3.4. *In any equilibrium without experimentation there exists a $p^\star < \frac{1}{2}$ such that for all $p \in (p^\star, 1 - p^\star)$ the incumbent always win and implements a moderate policy.*

Proof. In Appendix□.

The intuition behind this result is that when the value of learning is low and p is close to $\frac{1}{2}$, the voter desires to play it safe with a compromise policy. In equilibrium this takes the form of electing the same party for as long as learning doesn't occur. This gives that party who

is being elected a strong incentive to block learning and so they implement the compromise policy. This results directly contrasts to the previous results where calm economic times lead to parties only implementing ideological policies with a high degree of political turnover.

One consequence of the previous two results together is an inverse relationship between the persistence in the economic fundamentals and the stability of political outcomes. Recall that one interpretation of γ is the stability of the policy needs of an economy. And recall from proposition 3.1 that high γ leads to experimentation which leads to high political turnover during calm economic times. Conversely low γ leads to no experimentation and low political turnover during calm economic times. This result is counter-intuitive and is driven by the equilibrium incentives created by the voter's desire to learn.

3.6   Summary

In this chapter I laid out a theory of experimentation in the context of economic voting. The starting point is the idea that economic voting can be explained by voters making inferences from economic outcomes. I extended this idea by allowing voters to care about the learning consequences of policies which was not allowed in previous models. This leads to the idea that a voter who cares only about the economy may vote against their immediate economic interest in order to learn; in the other words the voter may experiment with inferior policies. My model delivers insights about when and why experimentation will occur as well the accompanying electoral patterns. Whether experimentation occur depends on how much the voter values learning given by the expression $W(1 - \gamma)$. This can be linked to the primitives in term of the persistence of the underlying state. With a highly persistence state, learning is naturally valuable and experimentation occurs. Conversely with an impersistent state, learning is not valuable and experimentation won't occur.

I also showed that experimentation is most likely to occur during a calm economy. During a calm economy the voter has a hard time making inferences and becomes uncertain about

the state. This make learning valuable to the voter and makes experimentation with the unfavored policy less costly. Experimentation also occurs during a calm economy and is accompanied by a high degree of political turnover. This is required by equilibrium in order to give the correct incentives to the parties. In equilibria without experimentation calm economic times are associated with low political turnover. Since experimentation occurs when the policy needs of the economy are stable this sets up an inverse relationship between that kind of stability and political stability. In other words experimentation requires stability in the fundamentals and creates instability in political outcomes.

Appendix A

Appendix to Chapter 1

A.1   Proof of proposition 1.2

We start by studying equations 1.14 through 1.16 and setting $\delta = 0, m_d^j = 1$ and $h_i^j = 1$. With m_d^1 and h_i^j pinned down, those equations give a unique solution for the variables $w^1, q^1, c_a^1, c_d^1, w^2, q^2 c^2$, from which all other steady state variables can be derived. We will refer to this solution as the $\delta = 0$ steady state. Now consider what happens to the $\delta = 0$ steady state if we set $\rho_d^1 = 0$. It is clear the equations for city 1 are the same as for city 2 when we replace $w^1 - \lambda y_d^1, q^1, c_d^1$ with w^2, q^2, c^2. Hence it must be that, in the $\delta = 0$ solution when $\rho_d^1 = 0$, we have $w^1 - \lambda y_d^1 = w^2$ as well as $q^1 = q^2$ and $c_d^1 = c^2$. Examining equation 1.13 which defines U_i^j, we can show that $U_d^1 = U^2$ and $U_a^1 > U^2$. To see why $U_a^1 > U^2$, set $c_a^1 = c^2$ and compare the equations. And since the labor cost cutoffs c_i^j are optimally chosen, the a type workers must do better while using cutoff c_a^1 compared to c^2. And hence we get that city 1 has a number of workers equal to:

$$s_d \Gamma \left(\frac{rU_d^1 - rU^2}{\gamma} \right) + s_a \Gamma \left(\frac{rU_a^1 - rU^2}{\gamma} \right) = \frac{s_d}{2} + s_a \Gamma \left(rU_a^1 - U^2 \right) > \frac{1}{2}.$$

So at the $\delta = 0$ steady state, when we set $\rho_d^1 = 0$ city 1 has more workers. And the equations used to define the $\delta = 0$ solution move continuously in ρ_d^1 and give a unique

solution. Hence the $\delta = 0$ solution must also move continuously in ρ_d^1. It follows that we can find a $\rho^\star > 0$ such that for all $\rho_d^1 \in (0, \rho^\star)$, city 1 has more workers at the $\delta = 0$ steady state. Now consider the following lemma.

Lemma A.1. *There exists a $y^\star > 0$ and a $m^\star \in (0,1)$ such that for all $y_d^1 < y^\star$, in every steady state $h_i^j = 1$ for all i, j. If, in addition, $m_d^1 > m^\star$, every steady state is in pure strategies. Moreover $y^\star$ and $m^\star$ are independent of ρ_d^1 and γ.*

The proof of this lemma is given in section A.1.1 below. From now on we fix any y_d^1, ρ_d^1 such that $y_d^1 < \min \left\{ y^\star, \frac{x}{2} \right\}$ and $\rho_d^1 < \min \left\{ \rho^\star, \frac{(r+\delta)(y_d^1)}{\pi - w}, \frac{1}{3} \right\}$. We let P^1 move as necessary to gain our desired y_d^1. And these bounds also ensure for us that $P^1 \geq 0$ and ρ_d^1 is small enough so that firms all underpay d workers in city 1.

From now on we will only vary δ and consider all other parameters fixed. Fix any $\bar{\delta} > 0$ and we will only consider $\delta \in (0, \bar{\delta})$. The rest of the proof will be concerned with proving that, when δ is small enough all steady states must be close enough to the $\delta = 0$ solution so that city 1 has more people, given how we chose (ρ_d^1, P^1).

I first claim that as δ goes to 0, m_d^1 goes to 1 in every steady state. Since we are setting $y_d^1 < y^\star$ we know that $h_i^j = 1$. So in any steady state we can write that:

$$\frac{u_d^1}{u_a^1} = \frac{\delta + \lambda \rho_d^1}{\delta + \lambda \rho_d^1 + \alpha_w(q^1) \int G(c_d^1(w)) dF^1(w)} \left(\frac{\delta + \alpha_w(q^1) \int G(c_a^1(w)) dF^1(w)}{\delta} \right).$$

Define $\bar{q}$ such that $\alpha_f(\bar{q}) \frac{\pi}{r} = k$. Since $J_i(w) < \frac{\pi}{r}$, $\bar{q}$ is an upper bound on q for all values of δ. Define $\bar{U}$ as:

$$r\bar{U} = \frac{\alpha_w(\bar{q}) \pi}{r + \alpha_w(\bar{q})} < \pi.$$

And $\bar{U}$ is an upper bound on U_i^j for all values of δ. We can also find a $L > 0$ such that $\alpha_w(q^1) \int G(c_a^1(w)) dF^1(w) \geq L$, for every $\delta \in (0, \bar{\delta})$, which will show how to derive below. It then follows that:

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$$\frac{u_d^1}{u_a^1} > \frac{\delta + \lambda \rho_d^1}{\delta + \lambda \rho_d^1 + \alpha_w\left(\bar{q}\right)} \left(\frac{\delta + L}{\delta}\right).$$

As δ goes to 0 the right-hand side of the above inequality goes to infinity. From which it follows m_d^1 goes to 1 since:

$$m_d^1 = \frac{s_d \Gamma\left(\eta_d\right) u_d^1}{s_d \Gamma\left(\eta_d\right) u_d^1 + s_a \Gamma\left(\eta_a\right) u_a^1} = \frac{\frac{u_d^1}{u_a^1}}{\frac{u_d^1}{u_a^1} + \frac{s_a \Gamma\left(\eta_a\right)}{s_d \Gamma\left(\eta_d\right)}} > \frac{\frac{u_d^1}{u_a^1}}{\frac{u_d^1}{u_a^1} + \frac{s_a}{s_d \Gamma\left(-\frac{\pi}{\gamma}\right)}}.$$

The inequality follows from the fact that $\Gamma\left(\eta_a\right) \leq 1$ and that $rU_i^j \in [0, \pi]$ which implies $\gamma \eta_i \geq \pi$. And the far right-hand side term converges to 1 as $\frac{u_d^1}{u_a^1}$ converges to infinity, so it must be that m_d^1 converges to 1 as δ goes to zero.

So from now we only consider δ low enough so that $m_d^1 > m^\star$ and hence all steady states have to be pure in strategies with $h_i^j = 1$. Now define $\Phi\left(m_d^1, \delta\right)$ as the unique solution to equations 1.14 through 1.16 at m_d^1, δ while setting $h_i^j = 1$. And we don't worry about m_a^2 since the solution to the equations 1.14 through 1.16 does not depend on it in any way. And since the equations defining Φ are continuous in m_d^1, δ, and Φ is single valued, it must follow that Φ is continuous as well.

Now suppose for contradiction there is no $\delta^\star$ such that, in all steady states, city 1 is bigger. Then we could build a sequence δ_n such that $\delta_n \to 0$ as $n \to \infty$, and at each δ_n there exists steady state with equilibrium values $\left(w_n^1, q_n^1, c_{a,n}^1, c_{d,n}^1, w_n^2, q_n^2, c_n^2, m_{d,n}^1\right)$ where city 2 is weakly larger than city 1. These variables must satisfy equations 1.14 through 1.16 which mean that $\left(w_n^1, q_n^1, c_{a,n}^1, c_{d,n}^1, w_n^2, q_n^2, c_n^2\right) = \Phi\left(\delta_n m_{d,n}^1\right)$.

Hence, we are looking at a sequence $\left\{\left(w_n^1, q_n^1, c_{a,n}^1, c_{d,n}^1, w_n^2, q_n^2, c_n^2, m_{d,n}^1, \delta_n\right)\right\}_{n=1}^{\infty}$ where $\delta_n \to 0$ and $m_d^1 \to 1$. And we can put bounds on all the variables meaning we are in compact space. So that sequence must have a convergent sub-sequence. Without loss of generality let the convergent sub-sequence be the original sequence. And by continuity of Φ, it must follow that $\left(w_n^1, q_n^1, c_{a,n}^1, c_{d,n}^1, w_n^2, q_n^2, c_n^2, m_{d,n}^1, \delta_n\right)$ converges to $\Phi\left(0, 1\right)$. But $\Phi\left(0, 1\right)$

is exactly the $\delta = 0$ steady state solution where we already showed that city 1 was strictly larger. And city size is a continuous function of those variables, hence there must be some point along that sequence where city 1 is larger, which gives us our contradiction. .

Deriving L: We now show to derive the lower bound L we used above. First we use the upper bound $\bar{U}$ to find a lower bound $0 < \underline{q} < q^1$ by setting

$$\underline{J} = \max_w G\left(w - r\bar{U}\right) \frac{\pi - w}{r + \bar{\delta} + \lambda\rho_d^1}.$$

and setting

$$\frac{1}{2}\alpha_f\left(\underline{q}\right)\underline{J} = k$$

This works because in any steady state the firm can take whichever worker type makes up at least half the unemployment pool and plan to only hire those workers. And $\frac{1}{2}\alpha_f\left(\underline{q}\right)\underline{J}$ is the worst case profit from this plan. Similarly, using the fact that $U_i^j \geq 0$, we can derive a $\underline{w} > 0$ such all firms will offer wages above $\underline{w}$. Using that we can derive a lower bound $0 < \underline{U} \leq U_a^1$, where

$$r\underline{U} = \alpha_w\left(\underline{q}\right)G\left(\frac{1}{2}\underline{w}\right)\frac{\underline{w} - E\left[c|c \leq \frac{1}{2}\underline{w}\right]}{r + \bar{\delta}},$$

And in any steady state with equilibrium values of q^1, c_a^1 we must have that:

$$rU_a^1 \leq \frac{\pi}{r}\alpha_w\left(q^1\right)\int G\left(c_a^1\left(w\right)\right)dF^1\left(w\right)$$

And combining this with our the fact that $r\underline{U} \leq rU_a^1$, we get a lower bound $L > 0$ on $\alpha_w\left(q^1\right)\int G\left(c_a^1\left(w\right)\right)dF^1\left(w\right)$ of:

$$L = \frac{r}{\pi}\alpha_w\left(\underline{q}\right)G\left(\frac{1}{2}\underline{w}\right)\frac{\underline{w} - E\left[c|c \leq \frac{1}{2}\underline{w}\right]}{r + \bar{\delta}}$$

So we can find our desired lower bound L, which holds for all $\delta \in \left(0, \bar{\delta}\right)$.

A.1.1 Proof of Lemma A.1

Showing that firms in city 2 all post the same wage and hire all willing workers is immediate and discussed in the main body of the text. We only need to prove the lemma for city 1. Define

$$w^\star (rU) = \arg \max_{w} G (w - rU) (\pi - w)$$

Note that $w^\star (rU)$ does not directly depend on ρ_d^1, δ, P^1 or γ, which becomes crucial later. Also note that the above maximization problem is strictly concave and hence $w^\star$ is pinned down by the following first order condition:

$$G' (w^\star - rU) (\pi - w^\star) - G (w^\star - rU) = 0.$$

By the implicit function theorem we can show that

$$\frac{\partial w^\star}{\partial rU} \in (0, 1).$$

Moreover it is clear that whenever $rU < \pi$ we must have $w^\star (rU) \in (rU, \pi)$. In any equilibrium all wages w offered must obey

$$\min \left\{ w^\star \left(rU_a^1 \right), w^\star \left(rU_d^1 \right) + \lambda y_d^1 \right\} \le w \le \max \left\{ w^\star \left(rU_a^1 \right), w^\star \left(rU_d^1 \right) + \lambda y_d^1 \right\}. \qquad \text{(A.1)}$$

This follows from the firm's first order condition from equation 1.9, since it can be verified that, for any $w < \min \left\{ w^\star \left(rU_a^1 \right), w^\star \left(rU_d^1 \right) + \lambda y_d^1 \right\}$, the first order condition is strictly positive, and at any $w > \max \left\{ w^\star \left(rU_a^1 \right), w^\star \left(rU_d^1 \right) + \lambda y_d^1 \right\}$, the first order condition is strictly negative.

Recall we derived $\bar{U} < \pi$ which is an upper bound on U_i^j for all values of δ, ρ_d^1, y_d^1 and γ. Now define $y^\star$ such that

$$w^\star \left(r\bar{U} \right) + \lambda y^\star = \pi.$$

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Choose any $y_d^1 < y^\star$. And combining this definition with our condition from equation A.1, we get that at any wage w that a firm may post in steady state:

$$w \leq \max \left\{ w^\star \left(rU_a^1 \right), w^\star \left(rU_d^1 \right) + \lambda y_d^1 \right\} < w^\star \left(r\bar{U} \right) + \lambda y_d^1 < \pi$$

And at any wage below π all workers gives the firm positive profits which is sufficient to show that firms hire all willing workers, which is the first thing we wanted to show.

Now we want to find the appropriate $m^\star$ such that for all $m_d^1 > m^\star$ all steady states are in pure strategy. Given that we are fixing $y_d^1 > y^\star$, we know that $h_i^j = 1$ in all steady states. The firm's choice of wage obeys the following maximization problem:

$$\max_w m_d^1 G \left(w - rU_d^1 - \lambda y_d^1 \right) J_d^1 (w) + h \left(w, m_d^1 \right)$$

where

$$h \left(w, m_d^1 \right) = \left(1 - m_d^1 \right) G \left(w - rU_a^1 \right) J_a^1 (w).$$

And for any $\varepsilon > 0$ we can find a m_d^1 close enough to 1 such that $\left| h(w) \right| < \varepsilon$ for all wages $w \in [0, \pi + \lambda y_d^1]$. And the solution to the above maximization problem when $h \left(w, m_d^1 \right) = 0$ is simply $w^\star \left(rU_d^1 \right) + \lambda y_d^1$. By a continuity argument, for any $\varepsilon' > 0$ we can find a m_d^1 close to 1 such that for any wage w posted in steady state must obey:

$$w \in \left(w^\star \left(rU_d^1 \right) + \lambda y_d^1 - \varepsilon', w^\star \left(rU_d^1 \right) + \lambda y_d^1 + \varepsilon' \right).$$

From this we can derive that:

$$c_a^1 (w) = w - rU_a^1 > w^\star \left(rU_d^1 \right) + \lambda y_d^1 - \varepsilon' - rU_a^1.$$

And using our upper bound $\bar{q}$ we get:

$$rU_a^1 < \frac{\alpha_w(\bar{q})}{\alpha_w(\bar{q}) + r} \left(w^\star \left(rU_d^1 \right) + \lambda y_d^1 + \varepsilon' \right).$$

Combining the last two inequalities gives us that:

$$c_a^1(w) > \left(w^\star \left(rU_d^1 \right) + \lambda y_d^1 \right) \left(\frac{r}{(\alpha_w(\bar{q}) + r)} \right) - 2\varepsilon' > w^\star(0) \left(\frac{r}{\alpha_w(\bar{q}) + r} \right) - 2\varepsilon'.$$

Since $w^\star(0) > 0$, we can choose ε' small enough to get that $c_a^1(w) > 0$ is assured. And by a similar proof we can show that $c_d^1(w) > 0$ if ε' is small enough. And that means we can avoid the kinks that occur when $c_i^j(w) = 0$. And away from those kinks the firm's profit function is strictly concave. Hence when m_d^1 is close enough to 1, all firms must post the same wage.

A.2 Proof of Proposition 1.3

The proof of this proposition is a corollary to the proof of proposition 1.2. Nowhere in the proof of lemma A.1 was γ referenced, so we can find the $y^\star$ and $m^\star$ independently of γ. Moreover, the equations that characterize the $\delta = 0$ solution do not rely on γ in any way. Hence we can derive U_i^j in the $\delta = 0$ steady state independently of γ. And the size of city 1 that we derived at the $\delta = 0$ solution was:

$$s_d \Gamma \left(\frac{rU_d^1 - rU^2}{\gamma} \right) + s_a \Gamma \left(\frac{rU_a^1 - rU^2}{\gamma} \right) = \frac{s_d}{2} + s_a \Gamma \left(\frac{rU_a^1 - rU^2}{\gamma} \right)$$

And since $rU_a^1 - rU^2 > 0$, if we choose γ small enough we can get $\Gamma \left(\frac{rU_a^1 - rU^2}{\gamma} \right)$ to be arbitrarily close to 1. And, then we extend this from the $\delta = 0$ steady state to small positive δ using the same upper hemicontinuity argument as in the proof of proposition 1.2.

A.3 Proof of Proposition 1.4

We will show how to match any ratio of unemployment rates. The proof for matching levels of unemployment rate is similar. So we want to match unemployment rate ratio $z = \frac{u_d^1}{u_a^1} > 1$, choosing the functional forms and the parameters. Set the matching function as

$$m\left(\mu, v\right) = m\mu,$$

where μ and v are the measures of searching workers and firms respectively. This functional form implies that $\alpha_w\left(q\right) = m$ for all q. And we set G to be uniform on the interval $[0, N]$ and Γ to be uniform on $[-N, N]$. Where $N > \pi + \lambda y_d^1$. Fix any $\rho_d^1, P^1 > 0$ such that $2y_d^1 \in \left(0, \frac{\pi}{2}\right)$. We set $\gamma = 1$ and we will vary r while choosing k so that $rk = 1$ always holds. The other parameters we take as fixed. Consider the following system of equations.

$$u_d = \frac{\delta + \lambda\rho_d^1}{\delta + \lambda\rho_d^1 + m\frac{\left(\frac{\pi}{2} - (1-m_d)\lambda y_d^1\right)}{N}}$$

$$u_a = \frac{\delta}{\delta + m\frac{\left(\frac{\pi}{2} + m_d y_d^1\right)}{N}}$$

$$m_d = \frac{s_d u_d}{s_a u_a + s_d u_d}$$

This system of equations is simply the definitions of u_i^1, m_d^1 in a pure strategy steady state, putting in our functional forms and setting $w^2 = \frac{\pi}{2}$ and $w^1 = \frac{\pi}{2} + m_d^1 + \lambda y_d^1$. We solve this system for (m_d, u_d, u_a) while considering the other variables fixed. It can be shown that this reduces to a quadratic equation where (m_d, u_d, u_a) has only one solution where m_d is positive. Moreover, as we vary δ from 0 to ∞, the positive solution moves continuously and the ratio $\frac{u_d^1}{u_a^1}$ goes from ∞ to 1. By the intermediate value theorem we can xix δ so that the

solution, which we denote by $(\hat{m}_d, \hat{u}_d, \hat{u}_a)$, has the property $\frac{\hat{u}_d}{\hat{u}_a} = z$. And since $z > 1$ it must follow that $\hat{m}_d > s_d$.

Now I claim there exists an $r^\star$ such that for all $r \geq r^\star$, every steady state is in pure strategies where firms always underpay d workers in city 1 and all firms set $h_i^j = 1$. Recall that our restriction on ρ_d^1 that ensured firms engage in wage theft was

$$\rho_d^1 < \frac{x(r + \delta)}{\pi + (r + \delta)(x + P)}.$$

And as r goes to infinity the right-hand side goes to 1, which makes this always hold. Additionally, note that as r goes to infinity rU_i^j goes to zero for all i, j. And using condition A.1 from above, and the fact that $2y_d^1 \in \left(0, \frac{\pi}{2}\right)$ as well, we can make a similar argument as we did in the proof of lemma A.1 to show that all steady states are pure strategy.

From now we will only consider $r \geq r^\star$. Putting in our functional forms into the firm's maximization problem we get that the firm solves:

$$\max_w \sum m_i \left(\frac{w_i - rU_i - \lambda e_i^j y_i^j}{N}\right) \left(\frac{\pi - w + \lambda e_i^j y_i^j}{r + \delta + \lambda e_i^j \rho_i^j}\right).$$

Taking the first order condition gives us:

$$\sum m_i \frac{1}{N \left(1 + \frac{\delta}{r} + \frac{\lambda e_i^j \rho_i^j}{r}\right)} \left(\pi - 2w + 2\lambda e_i^j y_i^j + rU_i^j\right) = 0$$

As we take $r \to \infty$ we get that $rU_i^j \to 0$ from which it follows that

$$w^1 \to \frac{\pi}{2} + \lambda m_d^1 y_d^1 \text{ and } w^2 \to \frac{\pi}{2}$$

And these converges are exactly what deliver the system of equations we considered earlier. Hence, as r goes to infinity, u_d^1, u_a^1, m_d^1 converge to the $\hat{u}_d, \hat{u}_a, \hat{m}_d$ that we derived above. Hence for r large enough we must close to our desired ratio z. All that remains to be shown is that city 1 is larger than city when r is large. Since Γ is uniform, we can write

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the size of city 1 as:
$$\frac{1}{2} + \frac{s_a \left(rU_a^1 - rU^2 \right)}{2N} + \frac{s_d \left(rU_d^1 - rU^2 \right)}{2N}.$$

Hence city 1 will be larger if and only if

$$s_a r^2 U_a^1 + s_d r^2 U_d^1 - r^2 U^2 > 0,$$

Our definition of U_i^j can be written as:

$$r^2 U_i^j = \frac{mG\left(c_i^j\right)\left(w^j - \lambda y_i^j e_i^j - E\left[c|c \le c_i^j\right]\right)}{1 + \frac{\delta}{r} + \frac{\lambda e_i^j \rho_i^j}{r} + \frac{\alpha_w(q^j)G\left(c_i^j\right)}{r}}.$$

And for any worker of type i in city j, the term c_i^j is chosen to maximize U_i^j. Hence in every steady state we get that

$$r^2 U_i^j \ge \frac{mG\left(c^2\right)\left(w^j - \lambda y_i^j e_i^j - E\left[c|c \le c^2\right]\right)}{1 + \frac{\delta}{r} + \frac{\lambda e_i^j \rho_i^j}{r} + \frac{\alpha_w(q^j)G(c^2)}{r}}.$$

Define $\hat{c}^2 = \frac{\pi}{2}$, which is the limit of c^2 as we take $r \to \infty$. Then we get that limit of $s_a r^2 U_a^1 + s_d r^2 U_d^1 - r^2 U^2$ is bounded below by

$$s_a mG\left(\hat{c}^2\right)\left(w^1 - E\left[c|c \le \hat{c}^2\right]\right) + s_d mG\left(\hat{c}^2\right)\left(w^1 - \lambda y_d^1 - E\left[c|c \le \hat{c}^2\right]\right)$$
$$- mG\left(\hat{c}^2\right)\left(w^2 - E\left[c|c \le \hat{c}^2\right]\right).$$

Using the limit we derived on w^1, w^2 we get that converges to

$$mG\left(\hat{c}^2\right)\left(\lambda \hat{m}_d^1 y_d^1 - s_d \lambda y_d^1\right).$$

And we already showed that $\hat{m}_d^1 > s_d$ so that term is strictly greater than 0. Hence it must follow that for r large enough we have city 1 is larger than city 2. And we are done.

A.4   Proofs from section 1.7

A.4.1   Proof of Existence

We have two enforcement regimes $(\rho_a^1, \rho_d^1, P^1)$ in city 1 and (ρ^2, P^2) in city 2. As usual define $y_i^1 = x(1 - \rho_i^1) - \rho_i^1 P^1$. And similarly $y_i^2 = x(1 - \rho^2) - \rho^2 P^1$. Also define $\omega_i^1 = \pi - \frac{(r+\delta)y_i^1}{\rho_i^1}$ and $\omega^2 = \pi - \frac{(r+\delta)y^2}{\rho^2}$. These are the wage cutoff above which the firm will always engage in wage theft and below which it will never engage in wage theft. For simplicity we will treat the case where $\omega_a^1 \neq \omega_d^2$ in city 1. If $\omega_a^1 = \omega_d^2$ it must be that $\rho_a^1 = \rho_d^2$ and we simply treat city 1 like city 2. Without loss of generality, let $\omega_d^1 < \omega_a^1$ and define $\bar{y} = \max\{y_a^1, y_d^1, y^2\}$. Set $\bar{z}^1 = \pi + 2 + \lambda\bar{y}$ and $\bar{z}^2 = \pi + 1 + \lambda\bar{y}$.

The main technique in this proof is we will reinterpret the firm's action. We already outline how to do this in city 2. For completeness we do the analogous thing for city. In city 1, instead of a wage, the firm will choose $z \in [0, \bar{z}^1]$. We will define $\tilde{w}^1(z), \tilde{e}_a^1(z), \tilde{e}_d^1(z)$ that tells us, for any z, what wage the firm is offering and with what probability he will engage in wage theft. We define $\tilde{w}^1$ as

$$\tilde{w}^1(z) = \begin{cases} z & \text{if } z \in [0, \omega_d^1] \\ \omega_d^1 & \text{if } z \in [\omega_d^1, \omega_d^1 + 1] \\ z - 1 & \text{if } z \in [\omega_d^1 + 1, \omega_a^1 + 1] \\ \omega_a^1 & \text{if } z \in [\omega_a^1 + 1, \omega_a^1 + 2] \\ z - 2 & \text{if } z \in [\omega_a^1 + 2, \bar{z}^1] \end{cases}$$

and define $\tilde{e}_d^1(z)$ as

$$\tilde{e}_d^1(z) = \begin{cases} 0 & \text{if } z \in [0, \omega_d^1] \\ z - \omega_d^1 & \text{if } z \in [\omega_d^1, \omega_d^1 + 1] \\ 1 & \text{if } z \in [\omega_d^1 + 1, \bar{z}^1] \end{cases}$$

and finally

$$
\tilde{e}_a^1 (z) = \begin{cases} 0 & \text{if } z \in [0, \omega_a^1 + 1] \\ z - \omega_a^1 - 1 & \text{if } z \in [\omega_a^1 + 1, \omega_a^1 + 2] \\ 1 & \text{if } z \in [\omega_a^1 + 2, \bar{z}^1] . \end{cases}
$$

And it is clear that $\tilde{w}^1, \tilde{e}_a^1, \tilde{e}_d^2$ are all continuous function. We will also define functions to capture the worker's actions. Set $\bar{U} = \frac{\pi}{r}, \bar{c} = \pi$. Define $\tilde{c}_i^1 : [0, \bar{z}^1] \times [0, \bar{U}] \to [0, \bar{c}]$ as:

$$
\tilde{c}_i^1 (z, U) = -rU + \tilde{w}_i (z) - \tilde{e}_i (z) \lambda y.
$$

Define $\tilde{c}^2$ similarly. Clearly these functions are continuous in z. And these functions represent the cutoff on the labor cost below which the worker will accept an offer of z. Now we can redefine the value functions of the agents. The firm's value from a match at action z becomes:

$$
\tilde{J}_i^j (z) = \frac{\pi - \tilde{w}^j (z) + \tilde{e}_i^j (z) \lambda y_i^j}{r + \delta + \lambda \tilde{e}_i^j (z) \rho_i^j}.
$$

The worker's value from a match given firm action z and labor cost cutoff of c is simply

$$
\tilde{W}_i^j (z, c) = \frac{\tilde{w}^j (z) - c - \tilde{e}_i^j (z) \lambda y_i^j}{r + \delta + \lambda \rho_i^j \tilde{e}_i^j (z)}.
$$

The value of a searching firm becomes:

$$
r\tilde{V}^j = \max_{z, h_a, h_d} \left\{ -k + \sum_{i \in \{a,d\}} \alpha_f(q^j) m_i^j h_i G(\tilde{c}_i^j (z))(\tilde{J}_i^j (z) - \tilde{V}^j) \right\}.
$$

And we redefine the firm's mixed strategy F^j as measures on the borel sets on of $[0, \bar{z}^j] \times [0, 1] \times [0, 1]$ endowed with the weak topology. Then we can give the value of an unemployed worker as:

$$r\tilde{U}_i^j = \alpha_w\left(q^j\right) \int h_i G\left(\tilde{c}_i^j\left(z\right)\right) \left(E\left[\tilde{W}_i^j\left(z,c\right) | c \leq \left(\tilde{c}_i^j\left(w\right)\right)\right] - \tilde{U}_i^j\right) dF^j\left(z, h_a, h_d\right).$$

And all these value functions are now continuous in their arguments. There is a slight difficulty in defining u_i^j which we deal with below. But otherwise our redefinition of the firm's action has made everything continuous, and the existence proof can proceed using standard Kakutani fixed point arguments, which we omit. And the equilibrium we find in this way will be an equilibrium of the original game, for the same reasoning we gave in section 1.7.1 above. In particular firms in city 1 will never find it optimal to choose $z \in (\omega_d^1, \omega_d^1 + 1]$ or $z \in (\omega_a^1 + 1, \omega_a^1 + 2]$. Hence at every wage offered in equilibrium, there will only be one level of wage theft that will be expected to occur. And we can reinterpret the equilibrium with the expanded action space as in an equilibrium of the original model.

We now deal with the difficulty in defining the unemployment rate. The reason for this difficultly is that workers of the same type may now lose their job at a different rates. In fact, since the rate of job loss depend on z, it is possible that workers of the same type lose their job at a infinitely many different rates. So we now have to keep track of the distributions of z in the employed population in order to derive the unemployment rate. Let $\mathcal{B}\left([0, \bar{z}^j]\right)$ be the borel measures on $[0, \bar{z}^j]$. Then in each city we will have a probability measure $L_i^j : \mathcal{B}\left([0, \bar{z}^j]\right) \to [0,1]$ where the value $L_i^j\left(A\right)$ is the percentage of employed workers of type i in j working for a firm who posted any $z \in A$. Using L_i^j, the number of workers of type i in city j who lose their job at any $z \in A$ can be written as:

$$\left(1 - u_i^j\right) \int_A \left(\delta + \lambda\rho_i^1 \tilde{e}_i^1\left(z\right)\right) dL_i^j\left(z\right).$$

And the number of workers of type i in city j gaining a job at any $z \in A$ is

$$u_i^j \alpha_w\left(q^j\right) \int_A h_i G\left(\tilde{c}_i^1\left(z\right)\right) dF^j\left(z, h_a, h\right).$$

And the steady state requirement is that exit and entries equal for all $A \in \mathcal{B}\left([0, \bar{z}^j]\right)$. So we get that for all $A \subset \mathcal{B}\left([0, \bar{z}^j]\right)$

$$u_i^j \alpha_w\left(q^j\right) \int_A h_i G\left(\tilde{c}_i^j(z)\right) dF^j\left(z, h_a, h\right) = \left(1 - u_i^j\right) \int_A \left(\delta + \lambda \rho_i^j \tilde{e}_i^j(z)\right) dL_i^j(A). \qquad (A.2)$$

This defines u_i^j in terms of $L_i^j(A)$, but we would like to derive u_i^j without reference to such an object. Extend L_i^j to have the same domain as F^j and it must be that L_i^j and F_i^j are absolutely continuous with respect to each other. Hence we can apply the Radon-Nikoyn theorem to tell us that:

$$\left(1 - u_i^j\right) L_i^j(A) = u_i^j \alpha_w\left(q^j\right) \int_A \frac{h_i G\left(\tilde{c}_i^j(z)\right)}{\left(\delta + \lambda \rho_i^j \tilde{e}_i^j(z)\right)} dF^j\left(z, h_a, h_d\right).$$

And then we simply set $A = [0, \bar{z}^j]$ and we get that:

$$\left(1 - u_i^j\right) = u_i^j \alpha_w\left(q^j\right) \int_{[0, \bar{z}^j]} \frac{h_i G\left(\tilde{c}_i^j(z)\right)}{\left(\delta + \lambda \rho_i^j \tilde{e}_i^j(z)\right)} dF^j\left(z, h_a, h_d\right),$$

which leads to

$$u_i^j = \frac{1}{\left(1 + \alpha_w\left(q^j\right) \int_{[0, \bar{z}^j]} \frac{h_i G\left(\tilde{c}_i^j(z)\right)}{\left(\delta + \lambda \rho_i^j \tilde{e}_i^j(z)\right)} dF^j\left(z, h_a, h_d\right)\right)}$$

which is our desired definition.

A.4.2 Proof of Proposition 1.6

Given the arguments made in the main body of the text we only need to prove the following lemma.

Lemma. *Fix all the parameters except $\left(\rho^2, P^2\right)$. Then U^2 is maximized whenever $\rho^2 = 1$.*

   Proof: We first define a few useful variables. For any fixed q, let $\tilde{w}(q), \tilde{c}(q)$ be the unique solution to the following system of two equations:

$$\tilde{w} = \tilde{c} + \frac{\alpha_w(q)}{r+\delta} \int_0^{\tilde{c}} (\tilde{c} - c) \, dc \tag{A.3}$$

$$G'(\tilde{c})(\pi - \tilde{w}) - G(\tilde{c}) = 0 \tag{A.4}$$

It can be shown that $\tilde{w}(q)$ is increasing in q, a fact we will use later.

Let $(U^\star, w^\star, q^\star, c^\star)$ be the city 2 steady state variables when $\rho^2 = 1$. It can be shown that $w^\star = \tilde{w}(q^\star)$ and $c^\star = \tilde{c}(q^\star)$. Now fix any ρ^2, P^2 with associated equilibrium variables $U^2, F^2, q^2, e^2(w), c^2(w)$. Recall that $e^2(w) \in [0,1]$ is the probability that a city 2 firms engages in wage theft given the opportunity. Optimality of the firm will ensure there is some cutoff wage ω^2 such that $w > (<) \omega^2 \Rightarrow e^2(w) = 1(0)$.

The rest of the proof works as follows. We suppose for contradiction that $U^2 > U^\star$. We then show that $q < q^\star$, and then we use this inequality show that $w - e^2(w)\lambda y^2 \leq w^\star$ for every w in the support of F^2. Together these inequalities imply that $U^2 < U^\star$, giving us our contradiction and finishing the proof.

Proof that $q^2 < q^\star$

Firm's optimality requires that any wage w posted in city 2 has to solve:

$$\max_w G\left(w - rU^2 - e^2(w)\lambda y^2\right) \frac{\pi - w + e^2(w)\lambda y^2}{r + \delta + \lambda e^2(w)\rho^2} \tag{A.5}$$

And it is clear that the firm's profit is weakly higher if they could commit to never engaging in wage theft by setting $e^2(w) = 0$. Hence we can write that that

$$\max_w G\left(w - rU^2 - e^2(w)\lambda y^2\right) \frac{\pi - w + e^2(w)\lambda y^2}{r + \delta + \lambda e^2(w)\rho^2} \leq \max_w G\left(w - rU^2\right) \frac{\pi - w}{r + \delta}$$

And the right-hand side of that inequality is strictly decreasing in U^2 and using that $U^2 > U^\star$ we get

$$\max_w G\left(w - rU^2 - e^2\left(w\right)\lambda y^2\right)\frac{\pi - w + e^2\left(w\right)\lambda y^2}{r + \delta + \lambda e^2\left(w\right)\rho^2} < \max_w G\left(w - rU^\star\right)\frac{\pi - w}{r + \delta}$$

And the right hand side is the maximization problem the firm's face in the $\rho^2 = 1$ case. Hence, using the free entry condition it follows that $q^2 < q^\star$ as desired.

Proof that $w - e^2\left(w^2\right)\lambda y^2 \leq w^\star$

Note that equation A.5 can have at most one discontinuity at wage ω^2, and on either side of this discontinuity the problem is strictly concave. Hence F^2 can have at most finite wages it mixes over. Let $f^2\left(w\right)$ be the probability of offering any gave wage. Now we can define U^2 using equation equation 1.13 as

$$rU^2 = \frac{\alpha_w\left(q^2\right)\sum f^2\left(w\right)G\left(c^2\left(w\right)\right)E\left[W^2\left(w,c\right)|c \leq c^2\left(w\right)\right]}{r + \sum \alpha_w\left(q\right)f\left(w\right)G\left(c^2\left(w\right)\right)}.$$

Let $\bar{w}$ be the wage in the support of F^2 that gives the highest value for $w - \lambda y^2 e^2\left(w\right)$. Suppose the worker always gets wage offer $\bar{w}$, but still employs cutoff $c^2\left(\bar{w}\right)$. This makes the worker weakly better off, giving us

$$rU^2 \leq \frac{\alpha_w\left(q^2\right)G\left(c^2\left(\bar{w}\right)\right)E\left[W^2\left(\bar{w},c\right)|c \leq c^2\left(\bar{w}\right)\right]}{r + \alpha_w\left(q\right)G\left(c^2\left(\bar{w}\right)\right)}.$$

Which can be further re-arranged into

$$rU^2 \leq \frac{\alpha_w\left(q\right)G\left(c^2\left(\bar{w}\right)\right)\left(w - \lambda y e^2\left(\bar{w}\right) - E\left[c|c \leq c^2\left(\bar{w}\right)\right]\right)}{r + \delta + \lambda e^2\left(\bar{w}\right)\rho^2 + \alpha_w\left(q^2\right)G\left(c^2\left(\bar{w}\right)\right)}$$

And using the definition of $c^2\left(w\right)$ we know that

$$\bar{w} - \lambda y^2 e^2\left(\bar{w}\right) - c^2\left(\bar{w}\right) = rU^2 \leq \frac{\alpha_w\left(q\right)G\left(c^2\left(\bar{w}\right)\right)\left(w - \lambda y e^2\left(\bar{w}\right) - E\left[c|c \leq c^2\left(\bar{w}\right)\right]\right)}{r + \delta + \lambda e^2\left(\bar{w}\right)\rho^2 + \alpha_w\left(q^2\right)G\left(c^2\left(\bar{w}\right)\right)}$$

Which we can rearrange into the following inequality

$$\bar{w} - \lambda y^2 e^2 (\bar{w}) \leq c^2 (\bar{w}) + \frac{\alpha_w (q^2)}{r + \delta} \int_0^{c^2(\bar{w})} \left(c^2 (\bar{w}) - c \right) dG (c)$$

I now claim that the firm's first order condition must always hold in one direction for $\bar{w}$ that is:

$$G' \left(c^2 (\bar{w}) \right) \left(\pi - \bar{w} - \lambda e^2 (\bar{w}) \right) - G \left(c^2 (\bar{w}) \right) \geq 0. \tag{A.6}$$

First notice that $c^2 (\bar{w}) > 0$ because otherwise the wage $\bar{w}$ would give zero profit and hence cannot be optimal for the firm. So the kink that occurs there is avoided. The only other problem that might occur is if $c^2 (w)$ is discontinuous at $\bar{w}$. This can only happen at the wage where the firm is indifferent between underpaying and not underpaying, and it must be that for all $w < \bar{w}$ we have $e^2 (w) = 0$ and for all $w > \bar{w}$ we have $e^2 (w) = 1$. Then optimality for the firm would require that $e^2 (\bar{w}) = 0$, otherwise the firm could offer wage $\bar{w} - \varepsilon$ and have a discontinuous jump in the acceptance rate while giving up almost no profit. Hence we have the firm's profit function is differentiable from the left at $\bar{w}$ and so if inequality A.6 fails to hold the firm could raise their profit by offering a wage slightly below $\bar{w}$.

Now suppose for contradiction that $w^\star < \bar{w} - \lambda y^2 e^2 (\bar{w})$. Well we know that $w^\star = \tilde{w} (q^\star)$ and we can use the above inequality in equation A.6 to show that

$$G' \left(c^2 (\bar{w}) \right) \left(\pi - \tilde{w} (q^\star) \right) - G \left(c^2 (\bar{w}) \right) > 0.$$

And this inequality is decreasing in the c^2 term, so using the equation A.3 it must be that $\tilde{c} (q^\star) > c^2 (\bar{w})$. Then, recalling that $q^2 < q^\star$ we get

$$\bar{w} - \lambda y^2 e^2 (\bar{w})$$

$$\leq c^2 (\bar{w}) + \frac{\alpha_w(q^2)}{r+\delta} \int_0^{c^2(\bar{w})} (c^2 (\bar{w}) - c) \, dG (c) < \tilde{c} (q^\star) + \frac{\alpha_w(q^\star)}{r+\delta} \int_0^{\tilde{c}(q^\star)} (\tilde{c} (q^\star) - c) \, dG (c)$$

$$= \tilde{w} (q^\star) .$$

Which is a contradiction. Hence, we can infer that $w^\star \geq \bar{w} - \lambda y e (\bar{w})$, which is what we wanted to show. $\square$

Appendix B

Appendix to Chapter 2

B.1 Proof of Lemma 2.1

B.1.1 Part 1

WLOG suppose party L chooses a partisan candidate (from l) and party R choose a moderate candidate (from m). We want to show that the m candidates gets a strictly high measure of votes. Well, all of the m and r types citizens will vote for the moderate candidate, all of the l types citizens will vote for the L candidate. So the most votes the partisan candidate can get is $\frac{1}{3}$.

Now to figure out how many votes the moderate candidate gets consider two cases. Let $x \in (0, 1)$ be the fraction of citizens who are α_L type.

Case 1) Last period R choose a moderate. That means both the old and young generation of the m types are energized so they all vote. So R gets measure $\frac{1}{3}$ votes from the m types. Among the r types at least all the citizens with cost α^L will vote, and since $x > 0$ there is a strictly positive measure of them. So R get's strictly more than $\frac{1}{3}$ measure of votes and wins for sure.

Case 2) Last period R choose a partisan. That means the old generation of the r types are energized and young generation of the m types are energized. The α^L types always vote,

hence R gets at least votes of

$$\frac{1}{3}\left(x+\frac{1}{2}\left(1-x\right)\right)+\frac{1}{3}\left(x+\frac{1}{2}\left(1-x\right)\right)=\frac{2}{3}\left(\frac{1}{2}+\frac{1}{2}x\right)$$

And since $x>0$ that value is strictly greater than $\frac{1}{3}$ and hence R wins for sure.

And these two cases exhaust all the possibilities. $\square$

B.1.2 Part 2

If both parties choose a moderate then they split the votes from m evenly. The partisan citizens each vote for their own party, so the outcome is determined by which party has a higher turnout from their base. The young citizens in both bases are not energized, so only the α^L types vote, and each base has the same number of those. So all that matters is the turnout from the old citizen in the base. Of those the α^L always vote, and the α^H types vote iff they were energized last period. If both or neither bases were energized last period then there is a tie. If one base was energized and the other wasn't, the side who energized the base gets a higher turnout and wins. $\square$

B.2 Proof of proposition 2.1

To make this simpler we will introduce some notation. In addition to the states that begin periods such as (L, m, m) we will introduce states in the middle of periods. For example (L, m, m, l) means last period L won last and both parties played m. This period L has already gone and chosen l, and now it is R's turn to play. Also I will let $V_i\left(\sigma_L, \sigma_R | h\right)$ denote the payoff for player i at history h given strategies σ_L, σ_R. As shorthand I will write $V_i^\star\left(h\right)=V_i\left(\sigma_L^\star, \sigma_R^\star | h\right)$, where $\sigma_i^\star$ is the unique equilibrium discussed in the text. And since $\sigma^\star$ is Markovian I can abuse notation and write $V_i^\star\left(s\right)$ for any state s.

B.2.1 Showing $\sigma^\star$ is an equilibrium

This can be verified using the one shot deviation principle, which will hold strictly at every point. I will only consider states where L won last period, all the other states are entirely symmetric.

Calculating Continuation Payoffs

I will do the calculations state by state. I will go through the first two calculations carefully, and then more quickly. Using the cycle depicted in figure 1 we can determine that a m candidate wins in every period. Also L will cycle between winning two periods in a row and losing two periods in a row. So his payoff stream for this state looks like:

$$V_L^\star (L, m, m) = \gamma - .25 - .25\beta - .25\beta^2 + \beta^3(\gamma + -.25) + \beta^4(-.25 + \gamma)...$$
$$= \gamma + \frac{-.25}{1 - \beta} + \gamma\beta^3 \frac{1 + \beta}{1 - \beta^4}$$
$$= \frac{0.25\beta^2 - \gamma + \beta\gamma - \beta^2\gamma + 0.25}{\beta^3 - \beta^2 + \beta - 1}$$

Now consider state (L, l, r). Here both parties will choose an m candidate, and the outcome will be decided by a coin flip sending us to state (L, m, m) or (R, m, m). So we get

$$V_L^\star(L, l, r) = .5(-.25 + \gamma + \beta V_L(L, m, m)) + .5(-.25 + \beta V_L(R, m, m))$$

We need to calculate $V_L^\star(R, m, m)$. Well by symmetry we know $V_R(R, m, m) = V_L(L, m, m)$. And since in the cycle a me a moderate candidate wins in every period we know that:

$$V_R^\star(L, m, m) + V_L(L, m, m). = \frac{-.5 + \gamma}{1 - \beta}$$

So we can calculate that

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$$V_L^\star(L, l, r) = .5(-.25 + \gamma + \beta V_L(L, m, m)) + .5(-.25 + \beta(\frac{-.5 + \gamma}{1 - \beta} - V_L(L, m, m))$$

$$= \frac{1}{4(1 - \beta)}(2\gamma - 1)$$

Now we will go more quickly.

$$V_L^\star(L, m, r) = \frac{-.25}{1 - \beta} + \gamma\beta^2\frac{1 + \beta}{1 - \beta^4} = \frac{0.25\beta^2 - \beta^2\gamma + 0.25}{\beta^3 - \beta^2 + \beta - 1}$$

Now we calculate R's payoff in these states.

$$V_R^\star(L, m, m, m) = \frac{-.5 + \gamma}{1 - \beta} - V_L^\star(L, m, m) = \frac{4\gamma\beta - \beta^2 - 1}{4(1 - \beta)(\beta^2 + 1)}$$

$$V_R^\star(L, m, r, m) = \frac{-.5 + \gamma}{1 - \beta} - V_L^\star(L, m, r) = \frac{4\gamma - \beta^2 - 1}{4(1 - \beta)(\beta^2 + 1)}$$

$$V_R^\star(L, l, r, m) = \frac{-.5 + \gamma}{1 - \beta} - V_L^\star(L, l, r) = \frac{2\gamma - 1}{4(1 - \beta)}$$

$$V_R^\star(L, m, m, l) = -.25 + \gamma + \beta V_R^\star(R, m, l)$$

$$= -.25 + \gamma + \beta V_L^\star(L, m, r) = -.25 + \gamma + \beta V_L^\star(L, m, r) =$$

$$\frac{1}{\beta^3 - \beta^2 + \beta - 1}(0.25\beta^2 - \gamma + \beta\gamma - \beta^2\gamma + 0.25)$$

$$V_R^\star(L, m, r, l) = \gamma + \beta V_R^\star(R, r, l) = \gamma + \beta V_L^\star(L, l, r)$$
$$= \frac{4\gamma - \beta - 2\beta\gamma}{4(1 - \beta)}$$

$$V_R^\star(L, l, r, l) = -.25 + \gamma + \beta V_R^\star(R, m, l) = -.25 + \gamma + \beta V_L^\star(L, m, r)$$
$$= \frac{0.25\beta^2 - \gamma + \beta\gamma - \beta^2\gamma + 0.25}{\beta^3 - \beta^2 + \beta - 1}$$

And that covers all the states. Next we check to see if a deviation in each of these cases is profitable.

Checking (one shot) Deviations for L

Consider the states (L, m, m). If L deviates to l, R will choose m and L gets payoff stream

$$-.25 + \beta V_L^\star(R, m, l) = -.25 + \beta V_R^\star(L, m, r, m)$$
$$= -.25 + \beta V_R^\star(L, m, r, m) = \frac{0.25\beta^2 - \gamma\beta + 0.25}{\beta^3 - \beta^2 + \beta - 1}$$

So we have the equilibrium action is strictly optimal iff

$$\frac{0.25\beta^2 - \gamma + \beta\gamma - \beta^2\gamma + 0.25}{\beta^3 - \beta^2 + \beta - 1} - \frac{0.25\beta^2 - \gamma\beta + 0.25}{\beta^3 - \beta^2 + \beta - 1} > 0 \iff$$
$$\frac{\gamma}{\beta^2 + 1}(1 - \beta) > 0$$

Which always holds given our assumptions on β, γ. Now consider state (L, l, r). If L deviates to pick l he gets payoff

$$-.25 + \beta V_L^\star(R, m, l) = \frac{0.25\beta^2 - \gamma\beta + 0.25}{\beta^3 - \beta^2 + \beta - 1}$$

So the equilibrium action is strictly optimal if and only if

$$\frac{2\gamma - 1}{4(1 - \beta)} - \frac{0.25\beta^2 - \gamma\beta + 0.25}{\beta^3 - \beta^2 + \beta - 1} > 0 \iff 0.5\frac{\gamma}{\beta^2 + 1}(1 - \beta) > 0$$

Which always holds. Now consider state (L, m, r). If L deviates R will pick r and win for sure. By symmetry it must be that $V_L^\star(R, r, l) = V_R^\star(L, l, r) = V_R^\star(L, l, r, m)$ this yields payoff:

$$-1 + \beta V_L^\star(R, r, l) = -1 + \beta V_R^\star(L, l, r, m) = \frac{3\beta + 2\beta\gamma - 4}{4(1 - \beta)}$$

So the equilibrium action is strictly optimal when

$$\frac{0.25\beta^2 - \beta^2\gamma + 0.25}{\beta^3 - \beta^2 + \beta - 1} - \frac{3\beta + 2\beta\gamma - 4}{4(1 - \beta)} > 0 \iff \frac{0.25(3\beta^2 - 2\beta\gamma + 2\beta^2\gamma + 3)}{\beta^2 + 1} > 0 \iff$$

$$\gamma(-2\beta + 2\beta^2) > -3 - 3\beta^2 \iff \gamma < \frac{-3 - 3\beta^2}{-2\beta + 2\beta^2} = \frac{3(1 + \beta^2)}{2\beta(1 - \beta)}$$

And assumption 1 is a sufficient condition to guarantees this holds.

Checking (one shot) Deviations for R

Consider state (L, m, r, m). If R deviates to r he gets payoff

$$-.25 + \beta V_R^\star(L, m, r) = -.25 + \beta V_R^\star(L, m, r, m)$$

And since in equilibrium a moderate candidate always win, and R gets to win at least once, we know that $V_R^\star(L, m, r, m) > \frac{-.25}{1-\beta}$. Thus it must hold that

$$V_R^\star(L, m, r, m) > \beta V_R^\star(L, m, r, m) - .25$$

Which is precisely what we need for this one shot deviation to be strictly unprofitable. Now consider state (L, m, m, m). If R deviates to m he gets payoff

$$(1 - p)(-.25 + \gamma + \beta V_R^\star(R, m, m)) + p(-.25 + \beta V_R^\star(L, m, m))$$

And hence the deviation is strictly unprofitable iff

$$V_R^\star(L, m, m)(1 - p\beta) > -.25 + (1 - p)(\gamma + \beta V_R^\star(R, m, m))$$

And using that by symmetry $V_R^\star(R, m, m) = V_L^\star(L, m, m)$ we get the deviation in unprofitable when:

$$\frac{\gamma}{\beta^2 + 1}(p - 1 + p\beta) > 0 \iff \beta > \frac{1 - p}{p}$$

Which is precisely assumption 2. Now consider state (L, l, r, l). If R deviates to r he gets payoff

$$p(-1 + \beta V_R^\star(L, l, r)) + (1 - p)(\gamma + \beta V_R^\star(R, r, l)) =$$

$$p(-1 + \beta \frac{2\gamma - 1}{4(1 - \beta)}) + (1 - p)(\gamma + \beta \frac{1}{4(1 - \beta)}(2\gamma - 1)) =$$

$$\frac{1}{4(\beta - 1)}(4p + \beta - 4\gamma - 4p\beta + 4p\gamma + 2\beta\gamma - 4p\beta\gamma)$$

So this deviation is strictly unprofitable when;

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$$\frac{0.25\beta^2 - \gamma + \beta\gamma - \beta^2\gamma + 0.25}{\beta^3 - \beta^2 + \beta - 1} - \frac{1}{4(\beta - 1)}(4p + \beta - 4\gamma - 4p\beta + 4p\gamma + 2\beta\gamma - 4p\beta\gamma) > 0$$

$$\iff -\frac{0.25}{\beta^2 + 1}\left(\beta^2 - 4p\gamma - 4p + 2\beta\gamma - 4p\beta^2 + 2\beta^2\gamma - 4p\beta^2\gamma + 1\right) > 0$$

$$\iff \beta^2 - 4p\gamma - 4p + 2\beta\gamma - 4p\beta^2 + 2\beta^2\gamma - 4p\beta^2\gamma + 1 < 0$$

$$\iff \beta^2 + 2\beta\gamma + 2\beta^2\gamma + 1 - 4p\left(\gamma + 1 + \beta^2 + \beta^2\gamma\right) < 0$$

We know $p \in (.5, 1)$ so that $4p > 2$. And clearly $(\gamma + 1 + \beta^2 + \beta^2\gamma) > 0$. Hence a sufficient condition for the above inequality is

$$\beta^2 + 2\beta\gamma + 2\beta^2\gamma + 1 - 2\left(\gamma + 1 + \beta^2 + \beta^2\gamma\right) < 0 \iff -\beta^2 + 2\gamma\beta - 2\gamma - 1 < 0$$

Which holds because $\beta < 1 \Rightarrow 2\gamma\beta - 2\gamma < 0$. Now consider state (L, m, m, l). If R deviates to r he gets payoff

$$.5(-.25 + \gamma + \beta V_R^\star(R, r, l)) + .5(-.25 + \beta V_R^\star(L, l, r))$$

$$= -.25 + .5\gamma + .5\beta(V_R^\star(R, r, l) + V_L^\star(R, r, l))$$

$$= -.25 + .5\gamma + .5\beta\frac{-.5 + \gamma}{1 - \beta} = \frac{2\gamma - 1}{4(1 - \beta)}$$

So the equilibrium action is strictly profitable when

$$\frac{1}{\beta^3 - \beta^2 + \beta - 1}\left(0.25\beta^2 - \gamma + \beta\gamma - \beta^2\gamma + 0.25\right) - \frac{2\gamma - 1}{4(1 - \beta)} > 0 \iff 0.5\frac{\gamma(1 - \beta)}{\beta^2 + 1} > 0$$

Which always holds. Now consider state (L, l, r, m). If R deviates he gets payoff

$$-.25 + \beta V_R^\star(L, m, r) = \frac{0.25\beta^2 - \gamma\beta + 0.25}{\beta^3 - \beta^2 + \beta - 1}$$

So the equilibrium action is strictly optimal when:

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$$\frac{1}{4(1-\beta)}(2\gamma - 1) - \frac{0.25\beta^2 - \gamma\beta + 0.25}{\beta^3 - \beta^2 + \beta - 1} > 0 \iff 0.5\frac{\gamma(1-\beta)}{\beta^2 + 1} > 0$$

Which always holds.

Now the final state (L, m, r, l). If R deviates to m he gets payoff

$$-.25 + \gamma + \beta V_R^\star(R, m, l) = -.25 + \gamma + \beta V_L^\star(L, m, r) = \frac{0.25\beta^2 - \gamma + \beta\gamma - \beta^2\gamma + 0.25}{\beta^3 - \beta^2 + \beta - 1}$$

So the equilibrium action is strictly optimal when

$$\frac{-\beta + 4\gamma - 2\beta\gamma}{4(1-\beta)} - \frac{0.25\beta^2 - \gamma + \beta\gamma - \beta^2\gamma + 0.25}{\beta^3 - \beta^2 + \beta - 1} > 0$$
$$\iff \frac{0.25}{\beta^2 + 1}\left(\beta^2 + 2\beta\gamma + 2\beta^2\gamma + 1\right) > 0$$

Which always holds.

And so we've proved there is no profitable one shot deviation and it follows that $\sigma^\star$ is a subgame perfect equilibrium. Moreover we've shown all one shot deviations are strictly unprofitable, a fact we will use later. $\square$

B.2.2   Proof that $\sigma^\star$ is unique

Two Lemmas

I will start by establishing some lemmas.

Lemma B.1. *Take any history h_0 that starts a period and suppose party L pursues strategy $\sigma_L^\star$ from here on. Then following $\sigma_R^\star$ is the unique strategy of R that minimizes the payoff of L. The same statement holds switching the role of L and R.*

Proof.

Fix any history h_0 that starts a period t. We assume L follows $\sigma^\star$ from here on. Thus R in minimizing L's payoff is trying to solve

$$\min_{\sigma_R} V_L(\sigma_L^\star, \sigma_R | h_0)$$

In this problem we can consider L actions fixed and known, this is identical so solving a standard dynamic programming problem. I claim that to prove Lemma B.1 it will suffice to prove the following statement:

Statement A: Take any history h, when it is R's turn to act, and that follows h_0, and in which L has never deviated from $\sigma^\star$ after h_0, then L is strictly worse off if R plays $\sigma^\star$ as opposed to any one shot deviation from $\sigma^\star$.

That this statement implies Lemma B.1 can be shown through standard dynamic programming arguments. In particular there is a constant discount rate $\beta < 1$, and the per period payoffs are bounded, so the usual theorems apply. The uniqueness of $\sigma_R^\star$ follows from the strictness assumed in the statement.

Proving Statement A

Here we show Statement A holds. Fix the appropriate histories h_0 and h. Let S be the state at history h. We assumed that h was such that it is R's turn to act, and L never deviates, hence it can only be at one of 6 states:

$$S \in \{(R, m, m), (R, m, l), (R, r, l), (L, m, m, m), (L, m, l, m), (L, l, r, m)\}$$

Notice that for every state except (R, m, l), no matter what one shot deviation R pursues a moderate candidate will win every period from here on (this depends on the fact that L is using $\sigma_L^\star$.) And whenever a moderate candidate always win the policy payoff for both parties is always $-.25$ and the only variability is which party gets the γ in each period, which means the parties payoff will sum to $-.5 + \gamma$ in every period. Thus for every state except

(R, m, l) a one shot deviation makes L strictly better off iff it makes R strictly worse off. And we already showed above that every one shot deviation from $\sigma_R^\star$ makes R strictly worse off and hence would make L strictly better off.

So now we have just one state left: (R, m, l). If R doesn't deviate L gets payoff:

$$\frac{4\gamma - \beta^2 - 1}{4(1 - \beta)(\beta^2 + 1)}$$

If R deviates then L gets the $\sigma^\star$ equilibrium payoff from (R, m, l, r) in game G. Which is the same payoff R receives in states (L, m, r, l). Checking from above this is just

$$\frac{4\gamma - \beta - 2\beta\gamma}{4(1 - \beta)}$$

So not deviating makes L strictly worse off when

$$\frac{4\gamma - \beta^2 - 1}{4(1 - \beta)(\beta^2 + 1)} - \frac{4\gamma - \beta - 2\beta\gamma}{4(1 - \beta)} < 0$$
$$\iff -\frac{1}{4(\beta^2 + 1)}\left(\beta^2 - 2\beta\gamma + 2\beta^2\gamma + 1\right) < 0$$
$$\iff \gamma(-2\beta + 2\beta^2) > -1 - \beta^2 \iff \gamma < \frac{1 + \beta^2}{2\beta(1 - \beta)}$$

And this is guaranteed to hold by assumption 1 $\square'$

Lemma B.2. *Fix any equilibrium σ_L, σ_R and any history h that starts a period. Then:* $V_i(\sigma_L, \sigma_R | h) = V_i^\star(h)$

Proof.

Fix an equilibrium (σ_L, σ_R), and a history h that starts a period. Consider party L. Note that $\sigma_L^\star$ is a feasible strategy, which L could pursue. And by Lemma B.1 the lowest payoff

he can get is the case where R pursues $\sigma_R^\star$, which would yield $V_L^\star(h^t)$. Thus by optimality σ_L must do at least as well. We can apply the symmetric argument to R. and we get the following inequalities:

$$V_L(\sigma_L, \sigma_R | h^t) \geq V_L^\star(h^t) \text{ and } V_R(\sigma_L, \sigma_R | h^t) \geq V_R^\star(h^t) \tag{*}$$

Notice if a moderate candidates wins the combined period payoff of the parties must be $\gamma - .5$, if an extreme candidates win it must be $\gamma - 1$. These are the only two possibility so we have the combined period payoff must range between these numbers, which implies that

$$\frac{\gamma - 1}{1 - \beta} \leq V_L(\sigma_L, \sigma_R | h^t) + V_R(\sigma_L, \sigma_R | h^t) \leq \frac{\gamma - .5}{1 - \beta} \tag{**}$$

But notice that in the equilibrium $\sigma^\star$ for any history h that starts a period the moderate candidates will always wins in every period, thus $V_L^\star(h^t) + V_R^\star(h^t) = \frac{\gamma - .5}{1 - \beta}$. So if either inequality in equation * was strict the upper bound on equation ** would be violated. And thus the inequalities in * must hold with equality, which is what we wanted to show.□

Proof of Uniqueness Given Lemmas B.1 and B.2

Take any equilibrium σ_L, σ_R. First take any history h and. WLOG suppose it is R's turn to act. 3 cases.

Case 1: h starts a period.

If $\sigma_R(h)$ recommends a different action than $\sigma_R^\star(h)$ with positive probability then by Lemma B.1 we must have $V_L(\sigma_L^\star, \sigma_R | h) > V_L^\star(h)$. But by optimality of an equilibrium we must have $V_L(\sigma_L, \sigma_R | h) \geq V_L(\sigma_L^\star, \sigma_R | h) > V_L^\star(h)$. And this violates Lemma B.2.

Case 2: h is in the middle of a period. By optimality we must have for any action a

$$g_R(\sigma_R(h)) + V_R(\sigma_L, \sigma_R | (h, \sigma_R(h))) \geq g_R(a) + V_R(\sigma_L, \sigma_R | (h, a))$$

And in particular that must hold for the action $a = \sigma_R^\star (h)$. And for any action c that R takes, the next history will start a period, and so we can apply Lemma A2 to say: $V_i (\sigma_L, \sigma_R | (h, c)) = V_i^\star (h, c)$ for any action c. So we can write that:

$$g_R (\sigma_R (h)) + V_R^\star (h, \sigma_R (h)) \geq g_R (\sigma_R^\star (h)) + V_R^\star (h, \sigma_R^\star (h))$$

But since the one shot deviation principle holds strictly for $\sigma^\star$ this can only happen if $\sigma_R^\star (h) = \sigma_R (h)$ as desired. $\square$

B.3   Proof of Proposition 2.2 (Office Motivated Candidates)

First set

$$\beta^\star (p) = \frac{p - \left(p^2 - 4 (1 - p) \left(p - \frac{1}{2} \right) \right)^{1/2}}{4 \left(p - \frac{1}{2} \right)}$$

And note for any $p \in [.5, 1]$ we have that $\beta^\star (p) \in (0, 1)$ and $\beta \in (0, \beta^\star (p))$ implies $p (1 + \beta - \beta^2) - 1 + \frac{1}{2}\beta^2 < 0$ and $\beta \in (\beta^\star (p), 1)$ implies $p (1 + \beta - \beta^2) - 1 + \frac{1}{2}\beta^2 > 0$.

Define $\sigma_{\text{office}}^\star$ as

State	L's Action	R's Action	R's Action if L deviates
(L, m, m)	m	r	m
(L, m, r)	l	r	m
(L, l, r)	m	m	m

And $\sigma_{\text{short}}^\star$ as:

State	L's Action	R's Action	R's Action if L deviates
(L, m, m)	m	m	m
(L, m, r)	l	r	m
(L, l, r)	m	m	m

An examination of $\sigma^\star_{\text{short}}$ will quickly verify that any state will lead back to (L, m, m) or (R, m, m) within two periods, at which point it will alternate stochastically between those two states forever.

B.3.1 Patient Office Motivated Parties

Here we assume $\beta > \beta^\star(p)$. We want to show $\sigma^\star$ is the unique equilibrium. This is just an application of the one shot deviation principle. I won't go through all the calculations here, it is similar to what was done above. The only difficult calculation is when party R is faced with state (L, m, m, m). In this case it turns out it is profitable to not deviate if and only if $p(1 + \beta - \beta^2) - 1 + \frac{1}{2}\beta^2 > 0$ which is guaranteed by the fact that $\beta > \beta^\star(p)$.

To show $\sigma^\star_{\text{office}}$ is unique as again similar to the proof of proposition 2.1, except now it is even easier. The equivalent of Lemma B.1 is now immediate because the game is zero sum, and the rest of proof proceeds similarly. The proof also relies on the one shot deviation principle holding strictly at every point, which it does.

B.3.2 Impatient Office Motivated Parties

Here we assume $\beta < \beta^\star(p)$ and we want to show that $\sigma^\star_{\text{short}}$ is the unique equilibrium. Again this is very similar to what we just did in the proof of proposition 2.1. At state (L, m, m, m) party finds playing m to be optimal exactly when $\beta < \beta^\star(p)$. Optimality of R playing m at state (L, l, r, l) holds when $\beta < 1/2p$, but it can be easily verified this is guaranteed by the fact that $\beta < \beta^\star(p)$. And uniqueness is proven just as above.

Appendix C

Appendix to Chapter 3

C.1   Notation and Preliminaries

In this section we discuss the notation used in the appendix and show a few useful inequalities that describe the behavior of the players. We use a different notation in the appendix than was employed in the paper. In particular we restrict the beliefs to the countable set of achievable beliefs. Just after learning occurs we know that the belief $p \in \{1 - \gamma, \gamma\}$. And when no learning occurs we the belief evolves as $p' = (1 - p)(1 - \gamma) + p\gamma$. And these are the only two ways the belief can evolve. Hence we can find all possible beliefs by forward solving this equation starting from $1 - \gamma$ and γ. In particular we let $p_1^{-1} = 1 - \gamma$ and $p_1^1 = \gamma$. Then we recursively define $p_{n+1}^i = (1 - \gamma) p_n^i + \gamma p_n^i$. So the set $\{p_n^{-1}\}_{n=1}^{\infty}$ is the set of all reachable beliefs less than $\frac{1}{2}$ and $\{p_n^1\}_{n=\infty}^{\infty}$ is the set of all reachable beliefs greater than $\frac{1}{2}$.

We also show a few inequalities that characterize the players' behavior. The inequalities are derived from the value functions and equalities we discussed in section 3.3. Without loss of generality, we still display these inequalities for beliefs less than $\frac{1}{2}$ but analogous statements would hold for beliefs greater than $\frac{1}{2}$. First at any belief p_n^{-1} the voter prefers

policy -1 over policy 0 if and only if:

$$W\left(p_{n+1}^{-1}\right) + \frac{1 - 2\left(2\gamma - 1\right)^n}{\beta\left(1 - r\right)} \leq W\left(p_1^{-1}\right)$$

.

And the voter prefers policy 1 over policy 0 if and only if:

$$W\left(p_{n+1}^{-1}\right) + \frac{1 + 2\left(2\gamma - 1\right)^n}{\beta\left(1 - r\right)} \leq W\left(p_1^{-1}\right)$$

And the voter always prefers policy the favored ideological policy over the unfavored one so we don't need to rank -1 and 1. Now for the parties. At belief p_n^{-1} party L prefers policy -1 over if and only if

$$\left(2\gamma - 1\right)^n V_L\left(p_1^{-1}\right) - V_L\left(p_{n+1}^{-1}\right) \geq \frac{-1}{\beta\left(1 - r\right)}$$

The L party never implements policy 1 so we don't have state inequalities about 1. Using the zero sum property we can show the R party prefers policy 1 over 0 if and only if

$$\left(2\gamma - 1\right)^n V_L\left(p_1^{-1}\right) - V_L\left(p_{n+1}^{-1}\right) \leq \frac{1}{\beta\left(1 - r\right)}$$

And these four inequalities characterize what policies the players prefer. They will be used extensively in the proofs below.

C.2 Two Useful Lemmas

In this section we prove two very useful lemmas that we will makes use of in the proof of the propositions.

Lemma C.1. *If at state p the voter ever prefers policy 0 over policy -1 then it must be that $W\left(p\right) \leq \frac{-1}{1-\beta}$.*

Proof.

WLOG take $p_n^{-1} < \frac{1}{2}$ and suppose the R party is chosen and implements 0 with positive probability at this belief. We want to show that $W\left(p_n^{-1}\right) \leq \frac{-1}{1-\beta}$. First take the case that for all $m > n$ either $x_m^{-1} = 0$ OR 0 is the voter's most preferred policy. This would give us that: $W\left(p_m^{-1}\right) = -1 + \beta W\left(p_{m+1}^{-1}\right)$ OR $W\left(p_m^{-1}\right) \leq -1 + \beta W\left(p_{m+1}^{-1}\right)$. Either way we can solve forward and show that $W\left(p_n^{-1}\right) \leq \frac{-1}{1-\beta}$.as desired.

Now take the other case where there exists some $m > n$ such that $x_m \neq 0$ and 0 is not the voter's most preferred policy. WLOG let this be the smallest such m. Since 0 is not the most preferred policy, -1 must be. By choice of m as the smallest we can use the same logic as above to derive that: $W\left(p_{n+1}^{-1}\right) \leq -1-\beta+..-\beta^{m-n-2}+\beta^{m-n-1}W\left(p_m^{-1}\right)$. That -1 is optimal for the voter at m requires that $W\left(p_m^{-1}\right) \leq -2+2\left(2\gamma-1\right)^m+\beta\left(1-r\right)W\left(p_1^{-1}\right)+\beta r W\left(p_{m+1}\right)$. Combining this with the above equation we get that:

$$W\left(p_{n+1}^{-1}\right) \leq$$

$$-1-\beta-...-\beta^{m-n-2}+\beta^{m-n-1}\left(-2+2\left(2\gamma-1\right)^m+\beta\left(1-r\right)W\left(p_1^{-1}\right)+\beta r W\left(p_{m+1}\right)\right)$$

Voter's optimality at m also requires that $W\left(p_{m+1}^{-1}\right) + \frac{1-2(2\gamma-1)^m}{\beta(1-r)} \leq W\left(p_1^{-1}\right)$. Plugging in and simplifying yields:

$$W\left(p_{n+1}^{-1}\right) \leq$$

$$-1-\beta-..-\beta^{m-n-2}$$

$$+\beta^{m-n-1}\left(-2+2\left(2\gamma-1\right)^m+\beta W\left(p_1^{-1}\right)+r\left(\frac{-1+2(2\gamma-1)^m}{1-r}\right)\right)$$

By assumption, at n the voter prefers 0 over -1 which has optimality condition of $W\left(p_{n+1}^{-1}\right) + \frac{1-2(2\gamma-1)^n}{\beta(1-r)} \geq W\left(p_1^{-1}\right)$. Plugging in and rearranging gives and using the fact that $m > n$ implies $-2\left(2\gamma-1\right)^n < -2\left(2\gamma-1\right)^m$ gives:

$$W\left(p_{n+1}^{-1}\right)\left(1 - \beta^{m-n}\right) <$$

$$-1 - \beta + .. - \beta^{m-n-2} + \beta^{m-n-1}\left(-1 + 2\left(2\gamma-1\right)^m + \tfrac{1}{1-r}\left(-2\left(2\gamma-1\right)^m\left(1-r\right)\right)\right).$$

Which becomes:

$$W\left(p_{n+1}^{-1}\right) <$$

$$< \frac{-1 - \beta - .. - \beta^{m-n-2} - \beta^{m-n-1}}{1 - \beta^{m-n}}$$

$$= -\frac{1 - \beta^{m-n}}{1-\beta}\left(\frac{1}{1 - \beta^{m-n}}\right) = \frac{-1}{1-\beta}.$$

As desired $\square$

Lemma C.2. *In any symmetric MPE:*

1. *$W\left(p_n^{-1}\right) \geq \frac{-2}{1-\beta}$ for all n*

2. *If $W\left(1-\gamma\right) \geq (>) \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$ then $W\left(p_n^{-1}\right) \geq (>) \frac{-1}{1-\beta}$ for all n,*

3. *If $W\left(1-\gamma\right) \leq \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$ then $W\left(p_n^{-1}\right) \geq \frac{-2}{1-\beta r} + \frac{\beta(1-r)W\left(p_1^{-1}\right)}{1-\beta r}$ for all n.*

Proof of part 1: If the voter only elect the belief-favored party, then the worst case value for $E\left[y_t\right]$ is -2. This is a feasible strategy so it must be that the voter gets payoff greater than $\frac{-2}{1-\beta}$ at all times.

Proof of part 2: Suppose $W\left(1-\gamma\right) \geq \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$. Fix any n. One possible strategy the voter can pursue is always electing the L party at all p_m^{-1} with $m \geq n$ until the next time the state is revealed then revert to the equilibrium strategy. The L party will only

implement -1 or 0 (or some mixture) and I claim the worst possible outcome for the voter in this case can face is $x_m = 0$ for all $m \geq n$. In this case the payoff would be: $W\left(p_m^{-1}\right) = \frac{-1}{1-\beta}$ for all $m \geq n$. The principle of optimality tells us that a sufficient condition for this to be the worst possible path overall is that, for each $m \geq n$, deviating to -1 would improve the voter's payoff. Well we know this holds exactly when: $\frac{-1}{1-\beta} + \frac{1-2(2\gamma-1)^m}{\beta(1-r)} \leq W\left(p_1^{-1}\right)$. And a sufficient condition for that to be true for all $m \geq n$ is that $W\left(p_1^{-1}\right) \geq \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$, which we assumed to be true. So the voter has a strategy available that has a worst case payoff of $\frac{-1}{1-\beta}$, and it follows that $W\left(p_n^{-1}\right) \geq \frac{-1}{1-\beta}$ as desired.

Proof of part 3: Now assume $W\left(1-\gamma\right) \leq \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$. WLOG we consider $p_n^{-1} < \frac{1}{2}$. Again consider the strategy where the voter only elects the belief favored party until the next learning event then returns to his equilibrium strategy. Consider a modified problem where the voter always gets period payoff of -2 for the favored ideological policy until the next learning event. At the next learning event the voter gets continuation payoff $W\left(1-\gamma\right)$. This modified problem is (weakly) worse under every possible path. Hence if we consider the worst possible path in this modified game until the next learning event that path yield payoff weakly below $W\left(p_n^{-1}\right)$. I claim the worst case in this modified problem using the favored party only strategy is when the voter gets the favored ideological policy in every period. In this case the voter's payoff is: $\frac{-2}{1-\beta r} + \frac{\beta(1-r)}{1-\beta r}W\left(1-\gamma\right)$. And to verify this is the worst case we can apply the principle of optimality. All we need to show is that $\frac{-2}{1-\beta r} + \frac{\beta(1-r)}{1-\beta r}W\left(1-\gamma\right) + \frac{1}{\beta(1-r)} \geq W\left(1-\gamma\right)$. Which we can rewrite as: $\frac{-1}{1-\beta} + \frac{1}{\beta(1-r)} \geq W\left(p_1^{-1}\right)$. Which we assumed holds. Hence the worst case payoff in the modified game is $\frac{-2}{1-\beta r} + \frac{\beta(1-r)}{1-\beta r}W\left(p_1^{-1}\right)$, which is a lower bound on the worst case payoff in the original game. $\square$

C.3   Proofs of Propositions

C.3.1   Proof of proposition 3.1

Proof of part 1.

First suppose that $W\left(p_1^{-1}\right) > \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$. I will show that experimentation occurs at some belief $p_n^{-1} < \frac{1}{2}$. Of course by symmetry experimentation occurs at p_n^1. And at least one of these beliefs will occur in any equilibrium with probability 1 as $t \to \infty$.

First I will show that for n large enough, at any belief p_n^{-1} the voter prefers both -1 and 1 to 0. Well $W\left(p_1^{-1}\right) > \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$ immediately ensures ensure the voter's best option is always -1 whenever $p < \frac{1}{2}$. This is just a consequence of lemma C.2 combined with lemma C.1. From that it follows that we can put an upper bound on the voter's payoff for any n: $W\left(p_n^{-1}\right) \le \frac{-2+\beta r(1-r)W\left(p_1^{-1}\right)}{1-\beta r} + \frac{2(2\gamma-1)^n}{1-\beta r(2\gamma-1)}$. Now we want to show that for n large enough the voter strictly prefers 1 over 0 at p_n^{-1}. Suppose for contradiction the voter prefers 0 over 1. Optimality requires that:

$$
\begin{aligned}
W\left(p_1^{-1}\right) & \le W\left(p_{n+1}^{-1}\right) + \frac{1 + 2\left(2\gamma - 1\right)^n}{\beta\left(1 - r\right)} \\
& \le \frac{-2 + \beta r\left(1 - r\right) W\left(p_1^{-1}\right)}{1 - \beta r} + \frac{2\left(2\gamma - 1\right)^{n+1}}{1 - \beta r\left(2\gamma - 1\right)} + \frac{1 + 2\left(2\gamma - 1\right)^n}{\beta\left(1 - r\right)}
\end{aligned}
$$

Rearranging we get:

$$
W\left(p_1^{-1}\right) \le \frac{-1}{1 - \beta} + \frac{1}{\beta\left(1 - r\right)} + \frac{1 - \beta r}{1 - \beta}\left(\frac{2\left(2\gamma - 1\right)^{n+1}}{1 - \beta r\left(2\gamma - 1\right)} + \frac{2\left(2\gamma - 1\right)^n}{\beta\left(1 - r\right)}\right)
$$

And as we take n to ∞ the righthand side converges to $\frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$. And so $W\left(p_1^{-1}\right) > \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$ implies there is an $n^\star$ large enough such that for all $n > n^\star$ the above inequality fails and the voter strictly prefers -1 and 1 to 0 at p_n^{-1}. Notice this implies that at any such belief R can only get elected if it implements policy 1 with probability 1. For if R implemented policy 0 with positive probability that means L would strictly prefers policy -1. And hence the voter would have elected L instead. And by a similar argument L can only get elected if they implement -1 with positive probability.

All that needs to be show now is the the R party wins at p_n^{-1} for some $n > n^\star$. Well suppose not for contradiction. Then the L party has the option of playing $x = 0$ at all such beliefs. This will yield a payoff of $\frac{\lambda}{1-\beta}$. Hence optimality requires that $V_L\left(p_n^{-1}\right) \geq \frac{\lambda}{1-\beta}$ for all $n > n^\star$. Recall that we assumed $\lambda > \frac{1-\beta+2\beta r}{\beta(1-r)} > \frac{1-\beta}{\beta(1-r)}$. Hence it follows that $V_L\left(p_{n+1}^{-1}\right) > \frac{1}{\beta(1-r)}$. But for n sufficiently large that means $V\left(p_{n+1}^{-1}\right) > (2\gamma-1)^n V_L\left(p_1^{-1}\right) + \frac{1}{\beta(1-r)}$. And this is exactly the optimality condition that says party L strictly prefers policy 0 at p_n^{-1}. And we know if party L prefers 0 then party R must strictly prefer 1. But since $n > n^\star$ the voter would elect party R over L in this case. And that gives us our contradiction. Hence R must be elected for some $n > n^\star$. And this finishes the first direction.

Now suppose that experimentation occurs. WLOG there exists an n where R is elected at p_n^{-1} and implements policy 1 with probability 1. This requires that the voter weakly prefers policy 1 to policy 0, otherwise policy 1 is the voter's worst outcome and they would have elected L instead. [1]. Hence the voter's optimality condition requires that $W\left(p_{n+1}^{-1}\right) + \frac{1+2(2\gamma-1)^n}{\beta(1-r)} \leq W\left(p_1^{-1}\right)$. Suppose for contradiction that $W\left(p_1^{-1}\right) \leq \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$. Then by Lemma C.2 we'd have that $W\left(p_{n+1}^{-1}\right) \geq \frac{-2}{1-\beta r} + \frac{\beta(1-r)W\left(p_1^{-1}\right)}{1-\beta r}$. Combining this with the voter's optimality condition yields: $\frac{-2}{1-\beta r} + \frac{\beta(1-r)W\left(p_1^{-1}\right)}{1-\beta r} + \frac{1+2(2\gamma-1)^n}{\beta(1-r)} \leq W\left(p_1^{-1}\right)$. And that can rearranged into:

$$\frac{-1}{1-\beta} + \frac{1}{\beta(1-r)} + \left(\frac{1-\beta r}{1-\beta}\right)\frac{2(2\gamma-1)^n}{\beta(1-r)} \leq W\left(p_1^{-1}\right)$$

Which is the desired contradiction. Hence $\frac{-1}{1-\beta} + \frac{1}{\beta(1-r)} < W\left(p_1^{-1}\right)$ must hold.

Proof of Part 2:

Proof for $\gamma_H^\star$. Here we prove that γ close to 1 implies $W\left(p_1^{-1}\right) > \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$. Fix any equilibrium and let $n_\gamma^\star$ be the lowest n such that L party would not implement policy -1 if elected at p_n^{-1}. If such a period does not exist we set $n_\gamma^\star = \infty$. I claim that $n_\gamma^\star \to 1$ as $\gamma \to 1$. This can be proved using methods similar to proof of proposition 3.2. Well one

---

[1]Recall we know the L party will never give positive probability to policy 1 in equilibrium

partial strategy the voter can pursue is to always elect the ideological dominant party at any p_n^{-1} or p_n^1 where $n < n_\gamma^\star$. This would produce payoff of:

$$W\left(p_1^{-1}\right) = \sum_{k=1}^{n_\gamma^\star} \left(-2 + 2\left(2\gamma - 1\right)^k + \beta\left(1 - r\right) W\left(p_1^{-1}\right)\right) + \left(\beta r\right)^{n_\gamma^\star} W\left(p_{n_\gamma^\star+1}^{-1}\right)$$

Which we can rearrange into:

$$W\left(p_1^{-1}\right)\left(1 - \frac{\beta(1-r)\left(1-(\beta r)^{n_\gamma^\star}\right)}{1-\beta r}\right)$$
$$= \frac{-2\left(1-(\beta r)^{n_\gamma^\star}\right)}{1-\beta r} + \frac{2(2\gamma-1)\left(1-(\beta r(2\gamma-1))^{n_\gamma^\star}\right)}{1-\beta r(2\gamma-1)} + \left(\beta r\right)^{n^\star} W\left(p_n^\star\right)$$

Notice that $\left(\beta r\left(2\gamma - 1\right)\right)^{n_\gamma^\star} \leq 0$ and from lemma C.2 we know that $W\left(p_n^\star\right) \geq \frac{-2}{1-\beta}$. Hence we can write that:

$$W\left(p_1^{-1}\right) \geq$$
$$\left(\left(1 - \frac{\beta(1-r)\left(1-(\beta r)^{n_\gamma^\star}\right)}{1-\beta r}\right)\right)^{-1}\left(\frac{-2\left(1-(\beta r)^{n_\gamma^\star}\right)}{1-\beta r} + \frac{2(2\gamma-1)\left(1-(\beta r)^{n_\gamma^\star}\right)}{1-\beta r(2\gamma-1)} + \left(\beta r\right)^{n^\star}\frac{-2}{1-\beta}\right)$$

And as $\gamma \to 1$ and $n_\gamma^\star \to \infty$ the right hand side of that inequality converges to 0. And recall we assumed $\beta > \frac{2}{2-r}$ which is sufficient to ensure that $\frac{-1}{1-\beta} + \frac{1}{\beta(1-r)} < 0$. And so for γ close enough to 1 it follows that $W\left(p_1^{-1}\right) > \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$ as desired.

Proof for $\gamma_L^\star$: Now we prove that γ close to .75 implies $W\left(p_1^{-1}\right) \leq \frac{-1}{1-\beta} + \frac{1}{\beta(1-r)}$. Well notice that the largest value of $E\left[y\right]$ the voter can ever get is policy -1 at p_1^{-1}, which yields value $-2 + 2\left(2\gamma - 1\right)$, and this represents the largest possible period the voter can ever get. And hence in any equilibrium $W\left(p_1\right) \leq \frac{-2+2(2\gamma-1)}{1-\beta} \to \frac{-1}{1-\beta}$ as $\gamma \to .75$ as desired.

Proof of part 3. In the proof for part 1 we proved there existed a $n^\star$ such that for all $n > n^\star$ at p_n^{-1} the voter prefers policies -1 and 1 to policy 0. Furthermore we showed that R can only win if they implement 1 with probability 1 and L can only win if they implement

-1 with positive probability. Here we will show there exists a $m^\star > n^\star$ such that the same party is not elected with positive probability at p_n^{-1} and p_{n+1}^{-1} for all $n > m^\star$. The other side will be implied by symmetry.

Suppose for contradiction that R is elected with positive probability at beliefs p_n^{-1}, p_{n+1}^{-1} where we can think of n as arbitrarily high. We already showed that L can't be in power at p_n^{-1} for all $n > n^\star$ and a similar proof works for R. Hence WLOG we can suppose R is not in power at p_{n+2}^{-1}. For ease of exposition we will set $(2\gamma - 1)^n = 0$. The exact same proof can be down without making this simplification and just carrying around extra terms.

At p_{n+2}^{-1} we know the L party is elected and must play -1 with positive probability. Which means that either L plays -1 with probability 1 or is indifferent between playing -1 and 0. Either way we can write that $V_L\left(p_{n+2}^{-1}\right) = 1 + \lambda + \beta r V_L\left(p_{n+3}^{-1}\right)$. Combining that with the fact that we know that at p_{n+1} the R party is elected and plays -1 we get:
$$V_L\left(p_{n+1}^{-1}\right) = -\left(1+\lambda\right)\left(1-\beta r\right) + \left(\beta r\right)^2 V_L\left(p_{n+3}^{-1}\right)$$

The optimality condition for L at $n+2$ is $\frac{1}{\beta(1-r)} \geq V_L\left(p_{n+3}^{-1}\right)$. We can use that to get: $V_L\left(p_{n+1}^{-1}\right) \leq -\left(1+\lambda\right)\left(1-\beta r\right) + \frac{(\beta r)^2}{\beta(1-r)}$. And the optimality condition for R at $n+1$ is $\frac{1}{\beta(1-r)} \leq V_L\left(p_{n+1}^{-1}\right)$. This leads to the following inequality:

$$\frac{1}{\beta\left(1-r\right)} \leq V_L\left(p_{n+1}^{-1}\right) \leq -\left(1+\lambda\right)\left(1-\beta r\right) + \frac{\left(\beta r\right)^2}{\beta\left(1-r\right)}$$

We can re-arrange this to get: $\frac{1-(\beta r)^2}{\beta(1-r)} \leq -\left(1+\lambda\right)\left(1-\beta r\right)$. Which gives us our contradiction since the left hand side is negative and the right hand side is positive. And our simplification that $(2\gamma - 1)^n = 0$ is irrelevant because both sides are strictly bounded away from 0 and don't depend on n.

Now suppose that L is in power at p_n^{-1} and p_{n+1}^{-1} with n arbitrarily high. Again we make the simplification that $(2\gamma - 1)^n = 0$. Using similar logic to what we just did we can derive the condition that: $1 + \lambda + \beta r V_L\left(p_{n+2}^{-1}\right) \leq V_L\left(p_{n+1}^{-1}\right) \leq \frac{1}{\beta(1-r)}$. And we know R can never be in power at p_m^{-1} and p_{m+1}^{-1} for $m > n$. And we can use that to calculate a lower bound of $V_L\left(p_{n+2}^{-1}\right) \geq -\frac{1+\lambda}{1+\beta r}$. Using that our inequalities become: $1 + \lambda - \beta r\left(\frac{1+\lambda}{1+\beta r}\right) \leq \frac{1}{\beta(1-r)}$. We

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can rearrange this to get: $\lambda \leq \frac{1+2\beta r-\beta}{\beta(1-r)}$. But we assumed that $\lambda > \frac{1+2\beta r-\beta}{\beta(1-r)}$ giving us our contradiction. And again because our assumption involves a strictly inequality which does not depend on n, our assumption that $(2\gamma - 1)^n = 0$ becomes irrelevant.

And this completely the proof $\square$

C.3.2   Proof of proposition 3.2.

We will show that R cannot win at belief $p_1^{-1} = 1 - \gamma$. The other case is entirely symmetric. First suppose for contradiction that R wins and implements 0 with positive probability. By Lemma C.1 it must follow that $W\left(p_1^{-1}\right) \leq \frac{-1}{1-\beta}$. Additionally we know the L party would have implemented -1. Since -1 is always preferred to 1, electing R can only be optimal for the voter only if the voter prefers 0 to -1. This requires that $W\left(p_2^{-1}\right) + \frac{1-2(2\gamma-1)}{\beta(1-r)} \geq W\left(p_1^{-1}\right)$. Optimality also requires $W\left(p_1^{-1}\right) \geq -2 + 2(2\gamma - 1) + \beta(1-r)W\left(p_1^{-1}\right) + \beta r W\left(p_2^{-1}\right)$ since the voter could have elected L. Combining these together and simplifying we get:

$$W\left(p_1^{-1}\right)(1-\beta) \geq -2 + 2(2\gamma - 1) + r\left(\frac{-1 + 2(2\gamma - 1)}{1-r}\right)$$

Using that $2\gamma - 1 > \frac{1}{2}$ we get that:

$$W\left(p_1^{-1}\right)(1-\beta) > -2 + 1 - r\left(\frac{1-1}{1-r}\right) = -1 \Rightarrow W\left(p_1^{-1}\right) > \frac{-1}{1-\beta}$$

And that is a contradiction, and the R party can't be elected and implement 0 with positive probability.

Now assume for contradiction the R party is elected and implements 1. Well the voter always prefers -1 to 1 so it must be that the L party was offering 0 with positive probability and the voter prefers 1 to 0. Optimality in this case requires that: $W\left(p_2^{-1}\right) + \frac{1+2(2\gamma-1)}{\beta(1-r)} \leq W\left(p_1^{-1}\right)$. And we know that $W\left(p_1^{-1}\right) = -2 - 2(2\gamma - 1) + \beta(1-r)W\left(p_1^{-1}\right) + \beta r W\left(p_2^{-1}\right)$ which implies that $W\left(p_1^{-1}\right) = \frac{-2-2(2\gamma-1)+\beta r W\left(p_2^{-1}\right)}{1-\beta(1-r)}$. Combining that with our optimality

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condition yields:

$$W\left(p_2^{-1}\right) + \frac{1 + 2\left(2\gamma - 1\right)}{\beta\left(1 - r\right)} \leq \frac{-2 - 2\left(2\gamma - 1\right) + \beta r W\left(p_2^{-1}\right)}{1 - \beta\left(1 - r\right)}$$

Rearranging:

$$W\left(p_2^{-1}\right)\left(\frac{1 - \beta}{1 - \beta\left(1 - r\right)}\right) \leq \frac{-1}{1 - \beta\left(1 - r\right)} - \frac{1 + 2\left(2\gamma - 1\right)}{\beta\left(1 - r\right)\left(1 - \beta\left(1 - r\right)\right)}$$

Using again that $(2\gamma - 1) > \frac{1}{2}$

$$W\left(p_2^{-1}\right) \leq \frac{-1}{1 - \beta} - \frac{1 + 2\left(2\gamma - 1\right)}{\beta\left(1 - r\right)\left(1 - \beta\right)} < \frac{-1}{1 - \beta} - \frac{2}{\beta\left(1 - r\right)\left(1 - \beta\right)} < \frac{-3}{1 - \beta}$$

And we have showed in lemma C.2 that $W\left(p_n^{-1}\right) \geq \frac{-2}{1-\beta}$ for all n in any equilibrium so that isn't possible. Hence it follows that the L party must win at p_1^{-1}.

$\square$

C.3.3 Proof of proposition 3.3

The proof of this is implied by the proof of part 3 of proposition 3.1.

C.3.4 Proof of proposition 3.4

It suffices to show that, in any equilibrium without experimentation, for p_n^{-1} close enough to $\frac{1}{2}$ the R party is every elected then they are elected at p_m^{-1} for all $m > n$. If this were true then there are two possible cases, the belief advantage party is always in power for p near $\frac{1}{2}$ or the belief dis-advantaged party is always in power. And in either case it is easy to check the party in power would only play policy 0 since doing so ensures they stay in power forever.

Now we just need to show that statement holds. Well suppose R is in power at p_n^{-1} but L takes power at p_{n+1}^{-1} where we can think of n as being arbitrarily large. Since there is no

experimentation R must implement policy 0 at p_n^{-1} which implies L offered policy -1. A stronger version of lemma C.1 (proof available from the author) can be shown where this implies that for all $m > n$ the voter must prefer 0 over -1 at p_m^{-1}. Hence for L to be elected at p_{n+1}^{-1} it must be that party R weakly prefers policy 1 and L must implement 0 with positive probability. Hence we can write R's optimality conditions at n and $n+1$ as

$(2\gamma - 1)^n V_L \left(p_1^{-1}\right) - V_L \left(p_{n+1}^{-1}\right) \geq \frac{1}{\beta(1-r)}$ and $(2\gamma - 1)^{n+1} V_L \left(p_1^{-1}\right) - V_L \left(p_{n+2}^{-1}\right) \leq \frac{1}{\beta(1-r)}$. We also know that $V_L \left(p_{n+1}^{-1}\right) = \lambda + \beta V_L \left(p_{n+2}^{-1}\right)$ because either L implements policy 0 or they mix which means they are indifferent to doing that and what they actually do. Using that we can rewrite the first inequality as: $(2\gamma - 1)^n V_L \left(p_1^{-1}\right) - \lambda + -\beta V_L \left(p_{n+2}^{-1}\right) \geq \frac{1}{\beta(1-r)}$. Since n is arbitrary large we can again use the approximation $(2\gamma - 1)^n \approx 0$ giving us the two inequalities of: $\frac{-\lambda}{\beta} - \frac{1}{\beta^2(1-r)} \geq +V \left(p_{n+2}^{-1}\right)$ and $V_L \left(p_{n+2}^{-1}\right) \geq \frac{-1}{\beta(1-r)}$. And for that to hold requires that $\frac{-\lambda}{\beta} - \frac{1}{\beta^2(1-r)} \geq \frac{-1}{\beta(1-r)}$ which can be reduced to : $\beta\lambda \leq -\frac{1-\beta}{1-r}$ which is obviously impossible. Hence we have our contradiction and this completes the proof. $\square$

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