

Labor Economics, 14.661, Second Part, Problem Set 3

This problem set is due on or before the recitation on Friday, December 9.

Please answer the following questions:

Exercise 1 Consider the following economy. There are two firms and one worker. They are all risk neutral and do not discount between the periods.

- Each firm offers a wage W to hire the worker (competing a la Bertrand).
- The firm that hires the worker decides whether to provide general training to the worker, denoted by $\tau \in \{0, 1\}$. The cost of training is $c(\tau)$, with $c(0) = 0$, $c(1) = c > 0$. The entire cost is borne by the firm. The training of the worker is not observed by the other firm.
- The incumbent firm (initial employer) makes a wage offer $w(\tau)$ to the worker after training. Outside firm makes a wage offer w^o to the worker at this point (it does not observe the wage offer of the initial employer, $w(\tau)$, either).
- The worker decides whether to stay with his current employer at the wage $w(\tau)$ or take the outside wage offer.
- The worker produces

$$y + \alpha\tau,$$

for some $\alpha > c$. This production level is regardless of the identity of the employer (since training is general).

1. Explain why outside firms' wage offers cannot be conditioned upon the training level.
2. Define a symmetric pure strategy Perfect Bayesian Equilibrium of this game (Hint: symmetric here means that both firms use the same strategy; be careful and specific in defining strategies and beliefs).
3. Focus on a symmetric equilibrium, where all firms use the same training strategy. Suppose that a symmetric pure strategy equilibrium exists. Show that if this involves training level τ^e , then the sequentially rational strategy profile must necessarily involve

$$w^o = y + \alpha\tau^e \text{ for } \tau^e \in \{0, 1\}.$$

4. Using the result in 3, show that there does not exist a symmetric pure strategy Perfect Bayesian Equilibrium (Hint: show that $\tau^e = 1$ cannot be part of the equilibrium; then show that if $\tau^e = 0$, then the deviation to a positive level of training is profitable for initial employers).
5. Argue why there will exist a mixed strategy equilibrium and why this will involve some amount of training.
6. Show also that there cannot exist a mixed strategy equilibrium in which firms mix over finite number of wage levels. Characterize a mixed strategy Perfect Bayesian Equilibrium. (Hint: suppose that the incumbent firm mixes between training and no training with probabilities q and $1 - q$, and offers a wage of $w(0) = y$, and if $\tau = 1$, it mixes with probability distribution $H(w)$ with support over $[y, \bar{w}]$, and the outside firm mixes over the same support with probability distribution $F(w)$. Then show that by definition of a mixed strategy equilibrium, we must have $F(w)(y + \alpha - w) = c$ and $qH(w)(y + \alpha - w) + (1 - q)(y - w) = K$ for all $w \in [y, \bar{w}]$ for some constant K . Then argue that H cannot have not atom, and find q and K from the appropriate boundary conditions. Finally, determined the initial wage W).

7. Relate your findings in parts 5 and 6 to the result that firm-sponsored general training investments are possible if and only if there are labor market imperfections and wage compression. Explain this result and show whether your finding here fits with this result (if not, explain why not; if yes, show in what sense there is wage compression here).
8. Does this model provide useful insights about training in practice? How would you modify it to make it more attractive theoretically or more realistic empirically? (Hint: be very brief).

Exercise 2 Consider the following economy. At $t = 0$, the firm decides how much to invest in its employee's general skills. The cost of an investment τ is $c(\tau)$, which is incurred by the firm. A worker with general skills τ produces $1 + \tau$ output in period $t = 1$. At this point, he can also move to a different firm where his wage will be $1 + \tau - \theta$ where θ is the cost of moving to a different firm. θ is a random variable, drawn from a uniform distribution $[0, 1]$, and is the private information of the worker (i.e., the firm does not observe it). The exact sequence of events is as follows: at $t = 0$, the firm chooses τ and makes a wage offer (w) to the worker; next, the worker, knowing her own θ , decides whether to quit or to stay.

1. Characterize the firm's wage offer as a function of τ . In particular, is $w'(\tau)$ positive, negative, zero, or ambiguously determined? Why?
2. Solve for the firm's level of training and wage offer that maximize expected profit. Explain why the firm is not investing in τ ?
3. Suppose now that the worker can finance his own training investment. Solve for the worker's choice of training and the firm's wage offer.
4. Suppose again that the worker cannot finance her training, but that her wage, if she quits the firm, is given by $1 + \tau(1 - \theta)$. Explain why the mobility cost might take this form. Solve for τ and w . Why is the firm investing in training in this case? Contrast these results with those obtained in part 2.
5. Contrast these results with the Becker view of training (in particular, contrast how the costs of training are shared between firms and workers in the two different views).

Exercise 3 This question asks you to think about a three-period training model. Consider the following timeline:

- In period 1, firm-specific investments in human capital are made by the worker.
- In period 2, investment in general human capital is made by the firm.
- In period 3, the firm makes a wage offer.

Assume that the production function has the following form:

$$f(\tau, s) = (1 + \tau)(1 + s)$$

in which τ is general human capital and s is specific human capital. The production function outside the firm is

$$g(\tau, s) = 1 + \tau$$

Finally, the cost of general human capital, incurred by the firm in period 2 is τ^2 , and the cost of specific human capital is s^2 and is incurred by the worker in period 2.

1. What is the wage offer the firm will make to the worker in period 3? Explain.
2. Assume that the firm cannot invest in any general human capital in period 2 (or ever). Solve for s and w . Interpret.

3. Assume instead that the worker cannot invest in specific human capital. Solve for τ and w . Interpret.
4. Now, assume that both parties can the make investments as described above. What incentive does the firm have to invest in general human capital? What incentive does the worker have to invest in specific human capital? (Hint: use backward induction.)
5. Explain how and why your answer would change if $f(\tau, s) = 1 + \tau + s$. Why is this the case?

Exercise 4 Consider a firm with two ex ante identical employees, $i = 1, 2$. At time $t = 0$ each employee decides whether to invest in his firm-specific skills at private cost c . If worker i makes this investment, we denote it by $s_i = 1$ and otherwise by $s_i = 0$. At the beginning of time $t = 1$, the firm decides the allocation of the two workers to tasks. There are two tasks, production and management. If both workers are assigned to the production task, then total output of the firm is

$$y^P(s_1) + y^P(s_2),$$

where $y^P(1) > y^P(0)$. If worker 1 is assigned to management and worker 2 to production, then the total output of the firm is

$$y^M(s_1) + y^P(s_2),$$

where

$$y^M(1) \geq y^P(1) y^P(0) \geq y^M(0).$$

Both workers cannot be assigned to management. Suppose that firm-specific skills and investments are observable (by the firm) but not contractible (i.e., neither task assignments nor wages can be conditioned on firm-specific skills), but the firm can commit to different wages for different tasks (w^M for workers employed in management and w^P for workers employed in production; thus can commit to a “wage structure” (w^P, w^M)). A worker can quit at any point and receive an outside option normalized to 0.

1. Define a subgame perfect equilibrium. [Hint: this should include an assignment function g for the firm that determines as a function of (s_1, s_2) which worker, if any, will be assigned to the management task].
2. Determine the equilibrium assignment of the firm as a function of (s_1, s_2) and the wage structure (w^P, w^M) .
3. Show that if $y^M(1) = y^P(1)$, then there exists no wage structure (w^P, w^M) that will induce either employee to undertake investments in firm specific skills in any subgame perfect equilibrium. Provide an intuition for this result. [Hint: distinguish it from the “holdup problem” discussed in the lecture].
4. Now suppose that

$$y^P(1) + 2c > y^M(1) > y^P(1) + c.$$

Show that there exists a wage structure (w^P, w^M) such that given this wage structure, one of the workers invests in firm-specific skills and the other one does not. At $t = 1$, the firm promotes the worker who has invested in firm-specific skills to the managerial position. [Hint: show that both workers do not want to invest in skills]. Provide an intuition for why this wage structure is providing incentives for firm-specific skills investment.

5. Now suppose that

$$y^M(1) > y^P(1) + 2c.$$

Show that the firm can choose a wage structure (w^P, w^M) that encourages both workers to invest in firm-specific skills (and then promote one of two workers who have invested in firm-specific skills and management if both of them have done so). Determine the wage structure to achieves this.²

6. Do you find the possibility that the firm can manipulate the organizational structure to encourage firm-specific investments plausible? How else could the firm encourage firm-specific investments in this model?

Exercise 5 Consider an economy consisting of a large number of workers and firms. Each worker is infinitely lived in discrete time and maximizes the expected discounted value of income, with a discount factor $\beta < 1$. There is no ex ante heterogeneity among the workers, but the quality of the match between a worker and its employer is random, and is not directly observed by either. Suppose that the worker is a good match to its employer with probability $\mu_0 \in (0, 1)$. A worker who is a good match to its employer produces output y_h with probability p and output $y_l < y_h$ with probability $1 - p$. A worker who is not a good match to its employer always produces y_l . Let μ be the posterior probability that the worker is a good match and suppose that the wage of the worker as a function of this posterior is given by

$$w(\mu) = \phi [\mu y_h + (1 - \mu) y_l],$$

where $\phi \in (0, 1]$. At any point in time, the worker can decide to quit. If he does so, he becomes unemployed. Unemployed workers receive an income of $b < y_l$ and find a new employer with probability q .

1. Determine how beliefs about worker-firm match quality evolve over time (as long as the workers employed with a given firm).
2. Write down a Bellman equation describing the value of a worker with belief μ . Show that there exists a belief $\mu^* > 0$ such that whenever $\mu < \mu^*$, the worker will quit her job.
3. Show that a worker who is a good match may initially experience wage decline, but will on average experience wage growth. Show that there exists $T < \infty$ such that a worker who is a good match and remains with its employer for more than T periods will have a constant wage.
4. Show that workers that quit will be unemployed for a while but when they become employed again their income will be greater than when they quit the job.
5. List two “stylized facts” that the model does a good job of matching and two stylized facts that contradict this version of the model. How would you modify the model so that it does a better job in these dimensions.