14.461: Problem Set 2

Due: October 3, 2014.

1. Consider a variant of the Ricardian model of technology and skills presented in Lecture 4. Suppose that there are only two types of workers, L and H, and a continuum of tasks $i \in [0,1]$. Suppose that each task is produced with the production function:

$$y(i) = A_L \alpha_L(i)[l(i)^{\beta(i)} o_L(i)^{1-\beta(i)}] + A_H \alpha_H(i)[h(i)^{\beta(i)} o_H(i)^{1-\beta(i)}] :$$

   where o denotes "oil" used as input and is supplied elastically at an exogenous price $p_0 > 0$. High-skill workers have comparative advantage in tasks with higher i. Assume also that the share of oil $(1-\beta(i))$ is higher for tasks with lower i. Derive the effect of increase in oil prices on the skill premium and interpret this result.

2. Consider the Ricardian model of technology and skills from Lecture 4, with three types of skills, L, M and H. Construct a parametric example (in terms of the $\alpha_L(i)$, $\alpha_M(i)$ and $\alpha_H(i)$ schedules) where a factor-augmenting increase in the productivity of H workers, i.e., an increase in A_H, reduces the wages of M workers.

3. Consider a model of direct technological change with two sectors, and the final good produced from these two sectors as follows:

$$Y(t) = \left[\gamma_L Y_L(t)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma_H Y_H(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

   Sector $j = L, H$ has the production function

$$Y_j(t) = \frac{1}{1-\beta}\left(\int_0^{N_j(t)} x_j(\nu,t)^{1-\beta} d\nu\right) Q_j^{\beta},$$

   with $Q_L = L$ and $Q_H = H$. The innovation possibilities frontier of the economy takes the form

$$\dot{N}_L(t) = \eta_L Z_L(t) \text{ and } \dot{N}_H(t) = \eta_H Z_H(t),$$

   and the resource constraint is $C(t) + X(t) + Z_L(t) + Z_H(t) \leq Y(t)$. The household side is represented by a representative household with CRRA preferences. Throughout, the total supplies of the two factors, L and H, are taken as constants.

(a) Suppose that final good producers face a tax rate of τ when purchasing Y_H (i.e., instead of p_H, they pay $(1+\tau)\,p_H$), the proceeds of which are rebated lump-sum to the representative household.Solve the maximization problem of sectors L and H and show:

$$Y_j\left(t\right) = \frac{1}{1-\beta} p_j(t)^{\frac{1-\beta}{\beta}} N_j(t) Q_j$$

(b) Taking N_L and N_H as given (and constant), find the relation between τ and p_H/p_L, the relative price of sector H to sector L . Does a greater τ increase or decrease p_H/p_L? Provide the intuition for this result.

(c) Now consider the benchmark $\tau = 0$ and endogenize N_L and N_H from the R&D decisions.

(d) Starting from a balanced growth path with $\tau = 0$, now consider a permanent increase to $\tau > 0$. Show that this will at first change p_H/p_L but then in finite time p_H/p_L will return to its initial level (the balance growth path benchmark with $\tau = 0$).

(e) Is taxing sector H causing H-biased technological change? Explain and provide the intuition.

4. Consider the model with directed technological change, again with two sectors and CES technology for the final good. But suppose now that the two factors of production are capital and labor, or in other words $j = L, K$, with each sector having the production function

$$Y_j\left(t\right) = \frac{1}{1-\beta}\left(\int_0^{N_j(t)} x_j\left(\nu, t\right)^{1-\beta} d\nu\right) Q_j\left(t\right)^{\beta},$$

with $Q_L\left(t\right) = L$ and $Q_K\left(t\right) = K\left(t\right)$. The innovation possibilities frontier of the economy takes the form

$$\dot{N}_L\left(t\right) = \eta_L N_L\left(t\right) S_L\left(t\right) \text{ and } \dot{N}_K\left(t\right) = \eta_K N_K\left(t\right) S_K\left(t\right).$$

Suppose also that capital accumulates following an exogenous savings rate, i.e., $\dot{K}\left(t\right) = sY\left(t\right) > 0$, while labor is supplied inelastically.

(a) Show that there exists a unique balanced growth path (BGP) in which the two sectors grow at the same rate (and thus their prices are constant) and in this BGP, all technological change is purely labor augmenting. Show also that in this BGP, factor shares are independent of initial conditions in terms of technology and capital and involve a constant share of national income going to capital. Provide an intuition for these results.

(b) Sketch an argument for this BGP being locally saddle path stable when $\varepsilon < 1$.

(c) What happens when $\varepsilon > 1$? Provide an intuition for this result.

(d) Suppose that $\varepsilon < 1$. Augment the model with a reduced form labor supply which determines $L(t)$ as a (increasing) function of total labor income divided by total capital income in the economy (why would labor supply depend on this?). Characterize the BGP in this case. Then suppose that this labor supply relationship shifts up (meaning that less labor is supplied at every level of labor income divided by capital income). Following this shock, trace out the adjustment to a new BGP in terms of employment, factor shares in national income and technology. Interpret these dynamics and if possible, provide parallels to any real-world dynamics you think these might be relevant for.

5. *Porter hypothesis* claims that tighter environmental regulations will spur faster innovation and increase productivity. This question investigates the Porter hypothesis in a framework of directed technological change along the lines of the model of the effects of labor scarcity on technological progress.

Consider economy M presented in the lecture. Suppose that θ corresponds to "green technology" and let us replace labor L with pollution p, with the main difference that pollution is not input but a byproduct of production. In particular:

$$Y^i = \alpha^{-\alpha}(1-\alpha)^{-1}G(Z^i,\theta)^{\alpha}q^i(\theta)^{1-\alpha}$$

and this also generates pollution:

$$p = \alpha^{-\alpha}(1-\alpha)^{-1}P(Z^i,\theta)^{\alpha}q^i(\theta)^{1-\alpha}.$$

The rest of the framework is the same as in the lecture.

(a) Assume $C(\theta)$ the cost of producing green technology is increasing in θ and final good producers pay a tax equal to τ units of final good on each unit of pollution they generate. Derive the demand for machines from the final good producers as a function of τ.

(b) Define the equilibrium and derive the condition for equilibrium choice of green technology.

(c) Provide conditions such that starting from $\tau = 0$, an increase in the pollution tax rate increases productivity along the lines of the Porter hypothesis.

(d) Show that this result becomes harder to obtain when the initial pollution tax is positive, i.e., $\tau > 0$. Discuss the intuition for this result.