Labor Economics, 14.661, Second Part, Problem Set 2

Please answer questions 1, 2, 4 and 6. This problem set is due on or before December 1.

Exercise 1 Consider the following career concerns model. The world lasts two periods. All firms and workers are risk neutral and there is no discounting. Workers are high or low ability, with ability denoted by $\eta \in \{\eta^H, \eta^L\}$, with $\eta^H > \eta^L$. The fraction of high ability workers in the population is $p \in (0, 1)$. Worker ability is observed neither by the worker nor by the firms in the market. Each worker chooses an effort $a \in R_+$ at each date, and with probability $q(\eta, a) \in (0, 1)$, he generates high output $Y^h > 0$ and with the complementary probability, he generates low output Y^l which we normalize to $Y^l = 0$. Assume that $q(\eta, a)$ is continuous, increasing and differentiable in a and increasing in η. The output level of each worker is publicly observed, but his effort level is not observed by potential employers. After the first period output level Y_1 is realized, a large number of firms compete a la Bertrand to hire the workers. Finally, assume that workers have a continuous, differentiable, and convex cost of effort, $c(a)$.

1. Define a Perfect Bayesian Equilibrium for this game.
2. Show that in period 2, a worker will be paid
$$w_2(Y_1) = \pi(Y_1)\, q\left(\eta^H, 0\right) Y^h + (1 - \pi(Y_1))\, q\left(\eta^L, 0\right) Y^h,$$
where $\pi(Y_1)$ is the probability that the market assigns to the worker being high ability after observing his output level $Y_1 \in \{Y^h, Y^l = 0\}$ in the first period.
3. Suppose that all workers choose effort a in the first period and derive $\pi(Y_1)$ from Bayes's rule.
4. Given $w_2(Y_1)$ and $\pi(Y_1)$, derive the best response first period effort a of workers. Show that in equilibrium this effort must satisfy $a = \bar{a}$.
5. Provide conditions such that a symmetric pure strategy equilibrium exists. Can there be multiple equilibria? Provide an economic intuition.
6. Suppose that a unique symmetric pure strategy equilibrium exists. What is the impact of an increase in Y^h on equilibrium effort level? How does this effort depend on the form of the function $q(\eta, a)$? Can you relate this to any real-world labor market facts?
7. Define the "first-best" effort level. Can the equilibrium level of effort be greater than the first-best effort?
8. What is the difference of this model from the Holmstrom's baseline career concerns model? What are the advantages and disadvantages of this model?

Exercise 2 Consider the Shapiro-Stiglitz model where workers and firms are infinitely lived and risk-neutral, both with discount rate r. Effort costs e, and without effort there is no output produced. There are N firms each with production function $AF(L)$ which is increasing and strictly concave where L is the number of workers employed by the firm who exert effort. There is an exogenous separation rate equal to b, and unemployed workers get disutility of leisure (and benefits) equal to z. Unemployed workers are randomly allocated to new job openings (which are due to separations). Firms decide what wage to offer to their workers. Workers who shirk (do not exert effort) are caught with probability q. The difference from the standard model is that q is chosen by the firm. It costs $C(q)$ per worker (thus a total of $C(q)L$) to choose a level of monitoring equal to q.

1. First write the Bellman equations for given q and derive the incentive compatibility condition (or the no shirking constraint).

2. Now find a first-order condition to determine the optimal level of q for a firm (Hint: be careful here; a common mistake is not to distinguish between the "q" of the firm in question, say q_i, and the "q" of all other firms which enters through V^U and which is obviously not controlled by the firm).

3. Show that an increase in A, which reduces unemployment, leaves q unchanged. Explain this result. Is it counter-intuitive?

4. How would you modify the model so that changes in A have an impact on q? Outline, if you can, possible ways of and generating the prediction that $dq/dA > 0$ and that $dq/dA < 0$.

5. Informally discuss whether $C(q)L$ as the cost of monitoring is plausible. In particular, would $C(q)$ be better? What would change in the model if instead of "q" we had workers supervising other workers (Hint: think of wages as costs)?

Exercise 3 The efficiency wage models we analyzed in the lecture were of the moral hazard variety (or effort-elicitation models). Another strand of the efficiency wage literature relies on adverse selection (type-elicitation).

Suppose that there are N workers. ϕN of the workers are low type and have 1 efficiency unit of labor. $(1-\phi)N$ of the workers are high type and they have $\alpha > 1$ efficiency units of labor. The type of the worker is his private information and never observed by any other agent. High type workers have a reservation return u_h and low type workers have a reservation return $u_l < u_h$. There are M firms each with a decreasing returns to scale production function $F(H)$ where H is the efficiency units of labor.

1. Draw the supply and demand curves for labor.

2. Assume that these two curves intersect at $w < u_h$. Show mathematically that it may be profitable for a firm to offer a wage $w = u_h$. Explain the intuition. Characterize diagrammatically the equilibrium in which all firms offer $w = u_h$. Find the unemployment rate of this economy. Is all of this "involuntary"? Why don't the employers cut wages?

3. The implicit assumption that you have used so far is that workers can apply to as many firms as they like. Now assume that each worker can only apply to one firm and choose which firm to apply after seeing the whole distribution of wage offers by firms. Show that starting from the allocation characterized in part (2) where $w = u_h$ for all firms, there is a profitable deviation for a firm.

4. Can you guess the form of the equilibrium in this case where each worker can only apply to one firm?

Exercise 4 Consider the following economy. At $t = 0$, the firm decides how much to invest in its employee's general skills. The cost of an investment τ is $c(\tau)$, which is incurred by the firm. A worker with general skills τ produces $1+\tau$ output in period $t = 1$. At this point, he can also move to a different firm where his wage will be $1 + \tau - \theta$ where θ is the cost of moving to a different firm. θ is a random variable, drawn from a uniform distribution $[0, 1]$, and is the private information of the worker (i.e., the firm does not observe it). The exact sequence of events is as follows: at $t = 0$, the firm chooses τ and makes a wage offer (w) to the worker; next, the worker, knowing her own θ, decides whether to quit or to stay.

1. Characterize the firm's wage offer as a function of τ. In particular, is $w'(\tau)$ positive, negative, zero, or ambiguously determined? Why?

2. Solve for the firm's level of training and wage offer that maximize expected profit. Explain why the firm is not investing in τ?

3. Suppose now that the worker can finance his own training investment. Solve for the worker's choice of training and the firm's wage offer.

4. Suppose again that the worker cannot finance her training, but that her wage, if she quits the firm, is given by $1+\tau(1-\theta)$. Explain why the mobility cost might take this form. Solve for τ and w. Why is the firm investing in training in this case? Contrast these results with those obtained in part 2.

5. Contrast these results with the Becker view of training (in particular, contrast how the costs of training are shared between firms and workers in the two different views).

Exercise 5 This question asks you to think about a three-period training model. Consider the following timeline:

- In period 1, firm-specific investments in human capital are made by the worker.
- In period 2, investment in general human capital is made by the firm.
- In period 3, the firm makes a wage offer.

Assume that the production function has the following form:

$$f(\tau, s) = (1+\tau)(1+s)$$

in which τ is general human capital and s is specific human capital. The production function outside the firm is

$$g(\tau, s) = 1+\tau$$

Finally, the cost of general human capital, incurred by the firm in period 2 is τ^2, and the cost of specific human capital is s^2 and is incurred by the worker in period 2.

1. What is the wage offer the firm will make to the worker in period 3? Explain.

2. Assume that the firm cannot invest in any general human capital in period 2 (or ever). Solve for s and w. Interpret.

3. Assume instead that the worker cannot invest in specific human capital. Solve for τ and w. Interpret.

4. Now, assume that both parties can the make investments as described above. What incentive does the firm have to invest in general human capital? What incentive does the worker have to invest in specific human capital? (Hint: use backward induction.)

5. Explain how and why your answer would change if $f(\tau, s) = 1+\tau+s$. Why is this the case?

Exercise 6 Consider a firm with two ex ante identical employees, $i = 1, 2$. At time $t = 0$ each employee decides whether to invest in his firm-specific skills at private cost c. If worker i makes this investment, we denote it by $s_i = 1$ and otherwise by $s_i = 0$. At the beginning of time $t = 1$, the firm decides the allocation of the two workers to tasks. There are two tasks, production and management. If both workers are assigned to the production task, then total output of the firm is

$$y^P(s_1) + y^P(s_2),$$

where $y^P(1) > y^P(0)$. If worker 1 is assigned to management and worker 2 to production, then the total output of the firm is

$$y^M(s_1) + y^P(s_2),$$

where

$$y^M(1) \geq y^P(1)\, y^P(0) \geq y^M(0).$$

Both workers cannot be assigned to management. Suppose that firm-specific skills and investments are observable (by the firm) but not contractible (i.e., neither task assignments nor wages can be conditioned on firm-specific skills), but the firm can commit to different wages for different tasks (w^M for workers employed in management and w^P for workers employed in production; thus can commit to a "wage structure" (w^P, w^M)). A worker can quit at any point and receive an outside option normalized to 0.

1. Define a subgame perfect equilibrium. [Hint: this should include an assignment function g for the firm that determines as a function of (s_1, s_2) which worker, if any, will be assigned to the management task].

2. Determine the equilibrium assignment of the firm as a function of (s_1, s_2) and the wage structure (w^P, w^M).

3. Show that if $y^M(1) = y^P(1)$, then there exists no wage structure (w^P, w^M) that will induce either employee to undertake investments in firm specific skills in any subgame perfect equilibrium. Provide an intuition for this result. [Hint: distinguish it from the "holdup problem" discussed in the lecture].

4. Now suppose that
$$y^P(1) + 2c > y^M(1) > y^P(1) + c.$$
Show that there exists a wage structure (w^P, w^M) such that given this wage structure, one of the workers invests in firm-specific skills and the other one does not. At $t = 1$, the firm promotes the worker who has invested in firm-specific skills to the managerial position. [Hint: show that both workers do not want to invest in skills]. Provide an intuition for why this wage structure is providing incentives for firm-specific skills investment.

5. Now suppose that
$$y^M(1) > y^P(1) + 2c.$$
Show that the firm can choose a wage structure (w^P, w^M) that encourages both workers to invest in firm-specific skills (and then promote one of two workers who have invested in firm-specific skills and management if both of them have done so). Determine the wage structure to achieves this.2

6. Do you find the possibility that the firm can manipulate the organizational structure to encourage firm-specific investments plausible? How else could the firm encourage firm-specific investments in this model?