Political Economy of Institutions and Development: 14.773
Problem Set 1
Due Date: Thursday, February 20th

Question 1

(1) Consider the example of a three-person three-policy society with preferences

$$\begin{array}{ll} 1 & a \succ b \succ c \\ 2 & b \succ c \succ a \\ 3 & c \succ b \succ a \end{array}$$

Voting is dynamic: first, there is a vote between a and b. Then, the winner goes against c, and the winner of this contest is the social choice. Find the subgame perfect Nash equilibrium voting strategy profiles in this two-stage game in "weakly undominated" (recall that each player's strategy has to specify how they will vote in the first round, and how they will vote in the second round as a function of the outcome the first round). [Hint: for "weakly undominated" strategies, first eliminate weakly dominated strategies in the last round, and then eliminate whatever is weakly dominated in the previous round, etc.].

(2) Suppose a generalization whereby there are finite number of policies, $Q = \{q_1, q_2, ..., q_N\}$ and M agents (which you can take to be an odd number for simplicity). Voting takes $N-1$ stages. In the first stage, there is a vote between q_1 and q_2. In the second stage, there is a vote between the winner of the first stage and q_3, until we have a final vote against q_N. The winner of the final vote is the policy choice of the society. Prove that, if preferences of all agents are single peaked (with a unique bliss point for each), then the unique subgame perfect Nash equilibrium in "weakly undominated" implements the bliss point of the median voter.

(3) Why is "weakly undominated" in quotation marks? [Hint: can you construct other equilibria in parts 1 and 2, when you simply focus on weakly dominated strategies (that is, without doing the sequential elimination described in the Hint of part 1)?].

Question 2

Consider a group of countries that differ in their preferences for public goods. The utility function for the representative country i is

$$U_i = y - g_i + \alpha_i H\left(g_i + \beta \sum_{j=1, j\neq i}^{N} g_i\right)$$

where y is income, g_i is national public spending, $H_g(\cdot) > 0$, and $H_{gg}(\cdot) < 0$. The parameter $\alpha_i > 0$ captures how much a country i values public consumption relative to private consumption. Countries are ordered such that $\alpha_1 \leq \alpha_2 \leq ....$ Public spending creates cross-country spillovers captured by $\beta \in (0,1)$. Only members of the union receive the externalities.

1. Characterize the "first best" allocation that maximizes the unweighted sum of utilitiesof all countries, and interpret.

2. Suppose that every member adopts the same policy ("rigid union"). Find the level of spending chosen by a N-sized union. How does it vary with N and β?

3. Now analyze an "initial stage" of union formation in which any country can unilaterally join a single union. [Hint: find the utility of country i in a union of N members $V_i^{in}(\alpha, N)$. An equilibrium union is one in which all members prefer to be members and viceversa for outsiders]. Which countries are going to join? Interpret. Explicitly characterize the equilibrium for $H(g) = \frac{g^{1-\theta}}{1-\theta}$. How does the equilibrium size of the union changes with β?

4. Under which conditions will the members of an existing equilibrium union decide to accept a new candidate country (once and for all)? What are the effects of accepting a new member?

5. Consider now the repeated version of the game. Each country maximizes the discounted sum of stage payoffs. At each point in time members of the union decide which country to accept and the policy to implement. Characterize the Markov Perfect Equilibrium.

6. Consider the one-shot version of the model. Let's now extend the model to consider the case of multiple policies. Imagine F different policies providing different public goods ordered by the intensity of spillovers, $\beta_1 > \beta_2 > ... > \beta_F$. The utility function is now

$$U_i = y - \sum_{k=1}^{F} g_i^k + \alpha_i \sum_{k=1}^{F} H\left[g_i^k + \sum_{j\neq i} \beta_k g_j^k\right]$$

Consider two rules. Rule A: the provision of each public good by the union is voted with majority rule. Rule B: the union can commit to centralize just a subset of policies. In this case, the union votes first on the number of policies to centralize and then a second vote takes place on how much of each public good should be provided. Compare the size of the union under the two rules. Interpret.

Question 3

Consider a society consisting of δ_r rich agents, $\delta_m > \delta_r$ middle-class agents and $\delta_p > \delta_m + \delta_r$ poor agents. All individuals are infinitely lived in discrete time and maximize the net present discounted value of their lifetime income with discount factor $\beta \in (0, 1)$. There are three political states: oligarchy (O), in which the rich agents are in power, limited franchise (L) in which the rich and the middle class vote, and full democracy in which all individuals are enfranchised (D). The society starts with oligarchy. The political game is as follows: in each period the median voter of the prevailing political regime decides a policy $\tau \in T \subset R$, and also what the political regime should be tomorrow (from the set $\{O, L, D\}$). Each agent's income depends directly on the regime (for example, because different economic relationships are possible within different regimes) and on the policy. In particular, let $y_i(S; \tau)$ be the income of individual of class $i \in \{r, m, p\}$ in political state $S \in \{O, L, D\}$ when the policy is τ . Assume that the following are uniquely defined:

$$\begin{aligned} \tau^p &= \arg\max_{\tau \in T}\{y_p(D, \tau)\} \\ \tau^m &= \arg\max_{\tau \in T}\{y_m(L, \tau)\} \\ \tau^r &= \arg\max_{\tau \in T}\{y_r(O, \tau)\} \end{aligned}$$

Individuals do not take any other action than the political actions described above.

1. Define a Markov Perfect Equilibrium (MPE) of this dynamic game.

2. Show that when $y_r\ (O; \tau^r) > max\{y_r\ (D; \tau^p),\ y_r(L; \tau^m)\}$, the unique MPE involves the society remaining in oligarchy forever with policy τ^r. Explain the intuition for this result.

3. Suppose that $y_r\ (O; \tau^r) < y_r\ (L; \tau^m)$ and $y_m\ (D; \tau^p) < y_m\ (L; \tau^m)$. Show that in this case the unique MPE involves the society immediately switching to limited franchise and remaining there forever with policy τ^m. Interpret this result.

4. Suppose that $y_r\ (D; \tau^p) < y_r\ (O; \tau^r) < y_r\ (L; \tau^m)$, that $y_m\ (D; \tau^p) > y_m\ (L; \tau^m)$, and that $y_p\ (D; \tau^p) > max\{\ y_p\ (O; \tau^r)\ , y_p\ (L; \tau^m)\ \}$. Show that there exists β^* such that, when $\beta < \beta^*$, the unique MPE involves the society becoming a limited franchise in the first period, then democracy in the following period, and remaining in democracy

with policy τ^p thereafter. When $\beta > \beta^*$, the unique MPE involves then a society remaining in oligarchy forever with policy τ^r. Explain why limited franchise cannot persist as the equilibrium regime in this case. Why is a higher discount factor making democracy less likely?

Question 4

Consider a society consisting of $|N|$ groups, and denote the set of groups by N. Each group is identified by its political (voting) power $\gamma_i \in \mathbb{R}_+$. Rank the groups, without loss of any generality, in ascending order with respect to γ_i. Throughout suppose that the groups have solved all of their internal collective action problems and act as single agent in the game. The society has a resource normalized to 1, which will be divided between these groups. Let an allocation of resources between the groups be $\{x_i\}_{i \in N}$ such that $x_i \geq 0$ for all i and $\sum_{i=1}^{N} x_i \leq 1$. Each group i has strictly increasing preferences over their share of the resource, x_i. Consider the following game:

- The group with the highest i, group N, makes an allocation offer $\{x_i\}_{i \in N}$.
- All groups vote over this allocation. Each group's vote is equivalent to its power. If the allocation is accepted (i.e., $\sum_{i \in Yes} \gamma_i > \sum_{i \in No} \gamma_i$, where Yes denotes the set of groups that vote yes and No denotes the set of groups that vote no), it is implemented. Otherwise, we move to the next stage, and the next group with the highest power makes an offer.
- The game continues until one of the offers is accepted or the last group makes an offer. If the last offer is also rejected, then all groups receive the allocation $\{x_i^d\}_{i \in N}$ such that $x_i^d \geq 0$ for all i and $\sum_{i=1}^{N} x_i^d \leq 1$. There is no discounting.

1. Explain why this game has a subgame perfect equilibrium. Is it necessarily unique?
2. Next suppose that $x_i^d = 0$ for all i. Also assume that society consists of 5 groups with powers $\gamma_1 = 1$, $\gamma_2 = 2, \gamma_3 = 4, \gamma_4 = 8$ and $\gamma_5 = 9.5$. Characterize the subgame perfect equilibria of this game.
3. Now suppose that groups cannot make offers of the form $\{x_i\}_{i \in N}$. Instead, they can make coalition offers. For example, group 5 can make an offer of $\{1, 2, 5\}$, which would mean that these three groups form a ruling coalition. A ruling coalition divides the resource among its members according to their power. That is, if X is the ruling coalition, each $i \in X$ receives $x_i = \gamma_i / \sum_{i \in X} \gamma_i$. Find the subgame perfect equilibrium of this game with the five groups as specified in part 2.

4. Discuss what would happen in part 4 if once a coalition is formed, there is one more round of voting within the coalition to form yet another ruling subcoalition. For example, if the original coalition was $\{1, 2, 5\}$, group 5 can now offer $\{5\}$ as the ruling coalition, and win against groups 1 and 2 (since it has 5 votes versus their 3 combined). Characterize the equilibrium ruling coalitions in this case.

5. Let us now return to the original game with offers $\{x_i\}_{i \in N}$ but modify the game so that after the least powerful group makes an offer and it is rejected, the cycle of offers starts again. Payoffs are only realized once agreement is reached and at that point the game ends (it continues indefinitely if no offer is accepted). Does there exist a subgame perfect equilibrium. If yes, give an example. If no, explain why not. Is there any relationship between this game and the non-existence of the core in cooperative games of resource division?

6. Now consider an infinite-horizon version of the game discussed in 5 above, where at each stage the current coalition can votes to include further members or exclude existing members. At the end of each date, payoffs are realized as specified in 4 above (i.e., each $i \in X$ receives $x_i = \gamma_i / \sum_{i \in X} \gamma_i$). What types of problems do you foresee if you were to try to characterize the Markov perfect equilibria of this game. [Do not attempt to characterize such equilibria unless you feel particularly inspired. Simply state what the difficulties would be if you tried to characterize them and if you have ideas about how to overcome these difficulties feel free to suggest them.]

7. Discuss whether games of this form may be useful in understanding coalition formation in real-world political environments. For example, consider the following concrete situation. There are three groups; unionized workers, bankers and monopolists, and they are trying to decide over labor market reforms (disliked by unionized workers), financial reforms (disliked by bankers) and product market reforms (disliked by monopolists). Suppose that each reform is liked by groups that do not dislike it and, if two groups agree, they can become the ruling coalition and decide on all the reforms. Could you use a model along the lines of that described above to decide what type of coalition will form and which reforms will pass? How would you enrich or simplify the model here if you wanted to make more progress on this specific question?

Question 5

Suppose the society consists of N individuals ranked $i = 1, 2, ..., N$ and stage payoffs are defined over clubs/states which are of the form $s_k = \{1, 2, ..., k\}$ for some integer k less than N. Suppose that stage payoffs are single peaked with the following structure:

$$s_{k+1} \succ_k s_k \succ_k s_{k-1}.$$

Each individual maximizes the discounted sum of stage payoffs and suppose that the discount factor is close to 1.

All clubs make decisions by absolute majority and can choose to expand or contract. Characterize the set of stable clubs. [Hint: s_2, s_3 and s_4 are stable but s_1 or s_5 is not].