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ORGANIZATIONAL FORMS AND INCENTIVES

Ayşe Mumcu


A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania in Partial Fulfillment
of the Requirements for the Degree of Doctor of Philosophy

1997


Supervisor of Dissertation


Graduate Group Chairperson

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AYŞE MUMCU

University of Pennsylvania

August 1997

ABSTRACT

ORGANIZATIONAL FORMS AND INCENTIVES

AYŞE MUMCU

George J. Mailath

In the first chapter, we extend the results of the Coase theorem to the relationships where, due to contractual incompleteness, agents are unable to bargain over all aspects of the transaction. We show that the initial allocation of ownership rights is irrelevant if a sufficiently large surplus is created by cooperation. Our result contrasts with Grossman and Hart (1986), who, using a similar model, obtain that the ownership rights should be allocated to minimize ex-ante inefficiencies in production. The critical element behind these two different results is that while Grossman and Hart model uses the Nash bargaining solution treating status quo payoffs as disagreement points, here they are treated as outside options.

In the second chapter, we study a firm's choice between employing a worker and using an independent contractor to carry out a task. If the firm hires a worker, all residual rights reside with the firm. In contrast, when the firm deals with an independent contractor, it cannot interfere with the way the task is undertaken. The firm's future actions may impose non-pecuniary costs to the worker, and as a result the worker requires an ex-ante compensation. The firm can economize on the up-front cost by hiring an independent contractor. Independent contracting

is a commitment device which ensures that the principal will not intervene in the future. However, when the firm has superior private information that is relevant to the execution of the task, the firm faces a trade-off between paying lower costs by hiring an independent contractor and keeping the option of value-enhancing intervention in employment relationship.

In the third chapter, we study the bargaining relationship between a firm and its incumbent worker who possesses firm-specific human capital. We show that, in the contract renewal stage, the worker's ability to strategically disclose his skills increases his bargaining power vis-a-vis the firm. The firm can threaten to fire the worker and hire a new inexperienced worker, but this threat is not always credible. Even though the bargaining takes place in an environment with perfect information, the game has inefficient equilibria where delays occur in real time.

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Chapter 1

The Allocation of Ownership Rights in the Presence of Non-Cooperative Bargaining

1.1 Introduction

The Coase theorem asserts that the initial distribution of ownership rights is irrelevant when transaction costs are zero, as rational agents internalize all externalities and reach Pareto efficiency through costless bargaining. Efficiency is achieved because agents can bargain over all aspects of the transaction before implementation. However, many relationships involve elements of contractual incompleteness which prevent the agents from bargaining over all aspects of transaction. For example, actions related to the use of human capital can not be contracted because of enforcement problems.¹ In general, the degree of contractual incompleteness is

¹The principal-agent model is based on this premise.

time-dependent. For example, before a relevant state of the world is realized, two parties involved in a relationship may not be able to sign a state-contingent price contract, while once the state is realized, a price can be contracted.² Since the initial contract is incomplete, the ownership rights can be used as an alternative method of governance, by giving the owner the right to make decisions regarding the use of the asset in uncontracted states of nature.³ Once the contractual incompleteness is resolved, the agents can renegotiate and implement a new contract. However, ex-post bargaining creates a strategic role to the ownership, in addition to the functional one, by allowing the agents to strategically use the assets they own to enhance their ex-post bargaining power.

The purpose of this paper is to extend the result of the Coase theorem to the relationships in which agents are unable to bargain over all aspects of the transaction, due to contractual incompleteness. Similar relationships have been considered in transaction costs models which explain the existence and the boundaries of a firm.⁴ The model in this paper draws on Grossman and Hart [15] where a theory

²Consider the relationship between an electricity-generating plant that is located next to a coal mine in order to use the mine's coal to make electricity. Ex-ante, the quality of coal delivered may not be contractible if there are many potential impurities. Ex-post, however, it may be clear what the relevant impurity is, such as high ash content. Although both firm cannot write state contingent contracts on the quality level ex ante, once the relevant impurity is known, the price can be contracted on the particular quality. (This example is taken from Grossman and Hart [15], page 699.)

³Other such methods include third party arbitration, contract law as interpreted by courts, reputation in long-term relationships, and social norms.

⁴Coase [10], Klein, Crawford and Alchian [21], and Williamson [40] argue that institutions, such as firms, facilitate exchange more efficiently when parties make relationship-specific investment that are not ex-ante contractible. Since comprehensive contracts cannot be written, the division of the surplus between the separately owned buyer and seller is determined via ex-post bargaining. As ex-ante investments are more valuable within the relationship than outside, parties are locked in each other and the non-competitive bargaining leads to opportunistic behavior. Although these theories explain the benefits of integration in terms of avoiding ex-post bargaining, they fail to capture the costs of integration. In these models, it does not matter who owns the assets, as long as the exchange takes place in an integrated firm. As a result, these models do not make any predictions about the distribution of ownership rights.

of ownership rights is developed and is applied to a firm's decision to integrate vertically or horizontally. They consider the relationship between two firms whose productive activities are dependent on each other. Ex-ante both firms make a relationship-specific investment and, ex-post they make a decision regarding the production process. Due to high transaction costs, ex-ante contracts contingent on the choice variables cannot be written. However, once the ex-ante investments have been made, the ex-post production decision becomes contractible. Thus the agents can bargain over the division of surplus before the production decisions are made. In the model, they define a firm as a set of property rights over the physical assets that it owns. Ownership confers *residual control rights* over the assets in the sense that, the owner of the asset has the right to use it in whichever way he desires unless specific rights are contracted away. Since none of the variables are ex-ante contractible, the initial contract only specifies the allocation of the residual control rights. Through its effect on the use of the asset in uncontracted states, ownership rights influence agent's bargaining power and the division of ex-post surplus, which in turn affects the parties' incentives to invest in that relationship. If there is a reciprocal dependency between the production of both firms, integration improves the incentives of the new owner while it weakens the incentives of the acquired firm's ex-owner. This trade-off between the costs and benefits of ownership determines the optimal allocation of control rights, hence ownership. The Coase theorem fails to apply in this model, presumably because of the existence of transaction costs. These transaction costs are created by the agents' opportunistic behavior during bargaining.

The main conclusion of Grossman and Hart [15] is that the ownership rights should be allocated to minimize the ex-ante inefficiencies in production. This result

is driven by the way they incorporate the status quo payoffs into the solution of Nash bargaining. The status quo payoff, which is the payoff received by an agent prior to bargaining, is treated as the disagreement point to the Nash solution. In the model, the agents do not receive an income flow in the course of the bargaining or there is no exogenous risk of breakdown in the bargaining game. Therefore, the disagreement payoff, which is the payoff from a perpetual negotiation without an agreement, should be zero instead of being the status quo payoffs. The parties can obtain their status quo payoff if they quit the bargaining game unilaterally. Therefore, in this setup it is more natural to treat the status quo payoffs as outside options rather than disagreement points. In this paper, we replace the Nash bargaining with an explicit alternating offers bargaining game where status quo payoffs are treated as outside options.

As it has been previously argued in Binmore, Shaked, and Sutton [6], Sutton [36], Shaked and Sutton [34], a dynamic bargaining game differentiates between a disagreement point and an outside option. When the status quo payoffs are taken as the disagreement point of the Nash solution, the agent's equilibrium payoff, which we call as the "*split-the-difference*" payoff, is the sum of his status quo payoff and half of the difference between the total surplus and both agents' status quo payoffs. When, however, the status quo payoffs are taken as outside options, they determine the range of validity for the Nash solution. When neither agents' outside option is binding, both receive half of the total surplus, which we call "*split-the-surplus*" payoff. When only one agent's outside option is binding, he receives his outside option and the opponent claims the residual.

In Grossman and Hart [15] model, the status quo payoffs are treated as agents' disagreement points. Thus, they directly influence the division of ex-post

surplus. The extent to which ownership affects the value of one's status quo payoff, it influences the incentives to invest in the relationship. The optimal allocation of ownership minimizes these ex-ante distortions. We consider the status quo payoffs as outside options. In this case, the agent receives his status quo payoff only if his outside option is binding. If the cooperation generates a large surplus, the status quo payoffs are not binding. Thus, they do not affect the division of surplus. Therefore ownership has no effect on the parties' incentives to invest in the relationship.

It is important to separate the sources of inefficiency in the model. Regardless of the allocation of ownership rights, when the size of the surplus is endogenous, the bargaining results in an inefficient equilibrium because of the free rider problem. As the parties do not receive the full benefit of their actions, their incentives to invest are distorted. In this paper, we are not interested in inefficiencies of this kind but in those that are solely driven by the allocation of ownership rights. Our model predicts that, while the ex-ante investments are inefficient, the initial allocation of ownership rights is irrelevant. As long as the parties can costlessly bargain ex-post over the division of surplus and the surplus generated through cooperation is sufficiently large, the status quo payoffs do not create a hold up problem as they do not constitute a credible threat. To the extent that the status quo payoffs are determined by the initial distribution of ownership rights, the ownership is irrelevant.

In this model, we adopt the definition of ownership, which is the power to exercise control, used in Grossman and Hart [15]. Ownership, however, can also be identified with the rights to the residual income stream. As argued by Holmstrom and Tirole [19], the definition of ownership can be a critical element in analyzing the efficiency properties of the initial allocation of ownership rights. Several papers,

such as Holmstrom and Tirole [19], Bolton and Whinston [7], find that the initial allocation of ownership rights over physical assets have efficiency implications. In these papers, however, ownership is defined as the rights to the both residual control and return stream. It would be interesting to examine the extent to which the irrelevance result depends on the definition of ownership.

The paper is organized as follows. In Section 1.2 the formal model is introduced. In section 1.3 the equilibrium to the induced bargaining subgame is derived in the case of non-integration. Section 1.4 contains the equilibrium of the investment-choice game in the case integration. Section 1.5 considers integration, in particular we analyze the case in which firm 1 owns firm 2. In Section 1.6 we look at the comparative statics as we change the level of complementarity between the two firm, in order to characterize when the equilibrium exist. Section 1.7 contains concluding remarks.

1.2 The Model

We consider two firms, 1 and 2, that are engaged in a relationship which lasts 2 periods. Each firm is managed by an agent who receives the full return of the firm where he is employed. At the beginning of date 1, the two agents sign a contract that specifies the distribution of ownership rights over each firm's assets. After the contract is signed, the two agents make a relationship-specific investment which is denoted by a_i for $i = 1, 2$. We assume that the relationship-specific investments require special skills so that the investment a_i in firm i can only be made by agent i . At date 2, the investments become observable to both agents and some further decisions regarding the production process are made, which are denoted by q_i . Al-

though a_i is chosen by agent i , the ex-post decision, q_i , is made by the agent who owns firm i . If the firms are separately owned, that is if agent i owns firm i , each agent is an owner-manager who has residual control rights over its firm's physical assets, so agent i chooses a_i and q_i of firm i . If the firms are integrated under i 's ownership then agent i owns both firms 1 and 2 then agent j becomes his employee. For example, under 1's ownership, agent 1 chooses a_1 , q_1 and q_2 and agent 2 chooses a_2 . The private benefit to agent i is written as $B_i[a_i, \phi_i(q_1, q_2)]$. The function ϕ_i can be thought of as a monetary payoff from second stage production net of costs. There is a disutility associated with ex-ante investment, which is given by $v_i(a_i)$.⁵ All costs and benefits are measured in date 1 dollars. The benefits and costs are the same under any ownership structure. Moreover, ownership does not provide any additional benefit.

None of the variables a_i , q_i and B_i is contractible ex-ante. We assume that the non-contractibility of the variables arises either as a result of high transaction costs associated with writing comprehensive contracts, or because of enforcement problems.⁶ We regard a_i as the non-verifiable managerial effort which is non-contractible because of the enforcement problem. The variable q_i is ex-ante non-contractible because it stands for complex production decision and it is difficult to describe ex-ante.

Since the decision variables are ex-ante non-contractible, the date 0 contract can only allocate ownership rights between the two agent. Ownership of an asset grants the beholder the right to use it in any way he desires unless these rights are contracted away. In Grossman and Hart's [15] terminology, the owner of the

⁵In Grossman and Hart [15] model, $B_i[a_i, \phi_i(q_1, q_2)]$ denotes the benefits net of all costs including the disutility of *ex ante* investment. In our model we preferred to separate the disutility from the benefit function to simplify the analysis. This structural change in the payoff function would not change the Grossman and Hart [15] result.

⁶This is a standard assumption in theory of incomplete contracts. Another explanation is that is that the agents are boundedly rational so they cannot foresee the future.

asset has the *residual control rights* over that asset.⁷ Although q_i is ex-ante non-contractible, once the state of the world is observed, q_i becomes contractible and the owner of firm i may give up his residual control rights in exchange of a side-payment.

A summary of the sequence of events is as follows. At date 0 a contract is signed. After that, a_1 and a_2 are chosen simultaneously and independently. At date 1, each agent learns the amount invested by his opponent. Before the actual choices of q_i are made they become contractible. If there is no further negotiation, the agent who owns firm i chooses q_i independently. The second stage decision, q , however, become contractible at date 1. Thus, a new contract may be negotiated that implements different choices of q_1 and q_2 , and specifies how the surplus is divided. Then B_1 and B_2 are realized and the necessary transfers are made between the two agents according to the new contract.

The following technical assumptions guarantee that the optimization problems have unique solutions and they can be derived using the first order approach. We assume that $B_i [a_i, \phi_i(q_1, q_2)]$ and $v_i(a_i)$ are twice continuously differentiable and satisfy the following conditions for all $a_i \in A_i$ and $q_i \in Q_i$.

Assumption 1 $B_i(\cdot)$ is increasing in ϕ_i and a_i . $B_1[\cdot] + B_2[\cdot]$ is strictly concave in its four arguments, (a_1, a_2, q_1, q_2) .

Assumption 2 The cost function $v_i(a_i)$ is increasing and convex in a_i .

Assumption 1 simply states that agent i 's private payoff is increasing in ex-ante investment and second stage payoff and the total surplus is increasing at a decreasing rate in ex-ante investments and ex-post actions. Assumption 2 states that the disutility from ex-ante investments are increasing at an increasing rate.

⁷Note that in this model, financial returns are not transferable with ownership. For an example of this, see Holmstrom and Tirole [19].

Assuming that monetary transfers between agents are available, the optimal contract maximizes the total ex-ante net benefits of the two agents,

$$W = B_1 [a_1, \phi_1 (q_1, q_2)] + B_2 [a_2, \phi_2 (q_1, q_2)] - v_1 (a_1) - v_2 (a_2) \quad (1.1)$$

If we assume that a_1 and a_2 are verifiable, and q_1 and q_2 are ex-ante contractible, the first best solution which is obtained by maximizing (1.1) with respect to a_i and q_i for $i = 1, 2$, can be implemented. We denote a_1^F , a_2^F , q_1^F , and q_2^F as the unique maximizers of W subject to $a_i \in A_i$, and $q_i \in Q_i$ for $i = 1, 2$.

Since we have assumed that all date 1 variables are non-contractible as of date 0, the first-best cannot be implemented. The initial contract only allocates ownership rights. There are three cases to consider. In the first case which we call non-integration, the firms are separately owned. In the second and third cases the firms are integrated under the ownership of a single agent, 1 and 2 respectively. We perceive ownership as a discrete variable which takes the value either 0 or 1 for each agent. Either agent 1 or agent 2 owns the firm. Two agent cannot own the same firm at the same time. Therefore we do not consider any type of joint ownership structure.

1.3 Non-integration

We solve for the subgame perfect Nash equilibrium of the full game which is characterized by a vector of $(a, q) \in A \times Q$ and transfer payments. Each vector $a = (a_1, a_2)$ induces a proper subgame where agent 1 and 2 bargains over the division of total surplus. We call these subgames as the induced bargaining subgames. In the next section, we characterize the equilibrium payoffs in these bargaining subgames.

1.3.1 The induced bargaining subgames

In the case where the firms are separately owned, agent i has the right to choose q_i . At date 1, the two agents choose q_1 and q_2 to maximize $B_1 [a_1, \phi_1 (q_1, q_2)]$ and $B_2 [a_2, \phi_2 (q_1, q_2)]$, respectively. We assume that there exists a unique Nash equilibrium to the simultaneous q -choice subgame which is,

$$\begin{aligned}\hat{q}_1 &= \arg \max_{q_1 \in Q_1} \phi_1 (q_1, \hat{q}_2) \\ \hat{q}_2 &= \arg \max_{q_2 \in Q_2} \phi_2 (\hat{q}_1, q_2).\end{aligned}\tag{1.2}$$

In general, the non cooperative solution (\hat{q}_1, \hat{q}_2) is ex-post inefficient.⁸ Therefore the two parties can gain from negotiating a new contract that specifies $(q_1(a), q_2(a))$ as the actions to be taken, where

$$(q_1(a), q_2(a)) = \arg \max_{(q_1, q_2) \in Q_1 \times Q_2} \{B_1 [a_1, \phi_1 (q_1, q_2)] + B_2 [a_2, \phi_2 (q_1, q_2)]\}\tag{1.3}$$

is the equilibrium of the cooperative q -choice subgame. The vector of equilibrium actions $(q_1(a), q_2(a))$ is unique given assumption 1. The new contract is feasible, since q_1 and q_2 are ex-post contractible. Let $B[a, q(a)]$ denote the value function of this problem. The division of $B[a, q(a)]$ among the two agents is determined by an alternating offers bargaining game. In the next section we explain the details of the game.

Bargaining Game

In an alternating offers bargaining game, a disagreement point and an outside option is treated differently. A player obtains his disagreement payoff when an agreement has not been reached. He receives his outside option when he quits the bargaining

⁸The noncooperative choices are efficient when ϕ_i is a function of only q_i or when $\phi_i = \phi_j$, that is the both agents have the same payoff function.

game unilaterally. While a disagreement point directly influence the division of the surplus in the equilibrium, an outside option influences the division of the surplus only when it is a credible threat. In other words, if a player obtains a higher payoff from exercising his outside option than the equilibrium payoff he receives when he continues to bargain, his outside option constitutes a credible threat. Otherwise, quitting is not a credible threat. In the former case, he should at least receive the value of his outside option in any subgame-perfect equilibrium of the bargaining game. In the latter case, his outside option does not influence the equilibrium of the game.⁹ Our model differs from Grossman-Hart [15] model in the way the status quo payoffs are incorporated into the Nash solution. The Grossman-Hart [15] model assumes a Nash bargaining solution as the equilibrium to the negotiation game where the status quo payoffs are treated as disagreement points. We consider them as outside options.

The sequence of moves in the alternating offers bargaining game that takes place among the two firms is as follows. The game begins with agent 1 making an offer. Let B_i^j denote the payment offered by agent j to agent i , where $i, j = 1, 2$. In the first period agent 1 offers B_2^1 to agent 2, keeping $B_1^1 (= B[a, q(a)] - B_2^1)$ for himself. Agent 2 can accept or reject the offer or quit the bargaining game. If she¹⁰ accepts the offer, then the game ends with payoffs B_2^1 and $B[a, q(a)] - B_2^1$ received by agent 2 and 1 respectively. If she rejects the offer, the game continues in the second period where agent 2 can make a counter-offer to agent 1. If she chooses to quit the bargaining game she receives the pre-negotiation payoff, $B_2(a_2, \hat{\phi}_2) = B_2[a_2, \phi_2(\hat{q}_1, \hat{q}_2)]$, by implementing the status quo action under the conditions of the

⁹This issue has been discussed by Binmore, Shaked and Sutton [6]. See also Sutton [36], and Shaked and Sutton [34].

¹⁰We refer to agent 1 as “he” and agent 2 as “she” in the paper.

initial contract. Note that in this game the disagreement, which is characterized by one party rejecting the offer of the other, results in the continuation of the bargaining game. Since there are no income flows accruing to the parties and there is no exogenous risk of breakdown of the bargaining game, the disagreement payoff is zero for both parties. If the game continues in the second period, agent 2 offers B_1^2 to agent 1, keeping $B_2^2 = B[a, q(a)] - B_1^2$ for herself. If agent 1 accepts the offer then the game ends with payoffs B_1^2 and B_2^2 . If he rejects, then the game moves to the third period. If he chooses to quit both receive the status quo level of benefit, $B_1(a_1, \hat{\phi}_1)$ and $B_2(a_2, \hat{\phi}_2)$ respectively. The game continues until either they reach an agreement or one of the agents decides to quit.

Each agent discounts the payoffs received in later periods by a common discount factor δ per period, where $0 < \delta < 1$. In the limiting case when the time interval between successive offers approaches zero, there exists a unique equilibrium to the bargaining game which is characterized below.

Lemma 1 *Given the initial ownership structure, and the vector $a = (a_1, a_2)$ of ex-ante investment levels, the induced bargaining subgame has a unique equilibrium in which the agreement is reached at $\tau = 0$, and firm 1 receives B_1^1 , given by*

$$B_1^1 = \begin{cases} \frac{B[a, q(a)]}{2} & \text{if } B_1(a_1, \hat{\phi}_1), B_2(a_2, \hat{\phi}_2) \leq \frac{B[a, q(a)]}{2} \\ B_1(a_1, \hat{\phi}_1) & \text{if } B_1(a_1, \hat{\phi}_1) > \frac{B[a, q(a)]}{2} \\ B[a, q(a)] - B_2(a_2, \hat{\phi}_2) & \text{otherwise} \end{cases}$$

Proof. See Appendix A ■

When both outside options are small relative to the “*split-the-surplus*” solution, as in the first case, both agents prefer to continue bargaining than quitting. This would generally be the case when the surplus created by cooperation is large.

In the second case agent 1 quits because he receives greater payoff in the status quo than if they split the surplus. In other words, his outside option imposes a credible threat, so that he receives his outside option in a perfect equilibrium. In the third case, agent 2 prefers quitting. She receives a share equal to her outside option, while agent 1 claims the residual. In general, when $\delta \in (0, 1)$, regardless of the preferences of agent 1, agent 2 has the first mover advantage in using her outside option as a credible threat. When δ approaches 0 this advantage disappears. When agent 1's outside option is binding, agent 2's outside option cannot be binding. This contradicts with the assumption that cooperation generates greater surplus.

As opposed to Grossman and Hart's [15] "*split-the-difference*" solution, we find that the outside option has no effect on the bargaining outcome if it does not constitute a credible threat. In other words, if the benefit the parties can obtain from negotiation is greater than their outside option, then quitting is an empty threat and will not affect the division of surplus. In this formalized non-cooperative bargaining model, the outside option acts as a constraint on the valid range of the Nash bargaining solution. Since the optimal allocation of ownership rights heavily depends on the outcome of the negotiation, the way outside option is incorporated into the model is critical.

Given the initial ownership structure and the ex-ante choice of (a_1, a_2) , we let $\Pi_i(a_1, a_2)$ denote the overall payoff to agent i obtained from the induced bargaining subgame. In the rest of the paper, we analyze the game from agent i 's perspective, where j denotes the opponent. Hence all definitions apply for $i, j = 1, 2$ and $i \neq j$. We define

$$H_i(a_1, a_2) = B[a, q(a)] - B_j(a_j, \hat{\phi}_j)$$

as the agent i 's residual payoff after paying the agent j the value of her outside

option, and

$$C_i(a_1, a_2) = \frac{B[a, q(a)]}{2}$$

as the agent i 's share in the “*split-the-surplus*” solution. Then, using Lemma 1.1 we obtain

$$\Pi_i(a_1, a_2) = \begin{cases} H_i(a_1, a_2) - v_i(a_i) & \text{if } j\text{'s outside option is binding} \\ C_i(a_1, a_2) - v_i(a_i) & \text{if neither outside options are binding} \\ B_i(a_i, \hat{\phi}_i) - v_i(a_i) & \text{if } i\text{'s outside option is binding.} \end{cases}$$

There is a qualitative difference in the way the ex-ante investments affect the payoffs of the parties this model compared to the Grossman-Hart [15] model. In the Grossman-Hart [15] model, the two agent receives the “*split-the-difference*” payoff in the equilibrium. As the opponent's action changes agent i responds by maximizing the “*split-the-difference*” payoff. In our model, the opponent's level of investment first determines the payoff function that agent i is facing. Then it influences the value of this function. The opponent's investment does not affect the value of agent i 's outside option because the second period payoff $\hat{\phi}_i$ is independent of *ex-ante* investment choices of both agents. We fix an a_j , such that agent i 's outside option gives him the highest payoff. As we increase the opponent's investment, the agent i 's response remains constant until the “*split-the-surplus*” payoff exceeds the value of his outside option. At this point, agent i is indifferent between maximizing the value of his outside option and the “*split-the-surplus*” payoff. As the opponent's investment continues to increase it is more profitable for agent i to maximize the “*split-the-surplus*” payoff until the region where the opponent's outside option is binding is reached. From this point on, the agent responds by maximizing the residual payoff. It is worth to note that the status quo payoffs do not affect the agents' payoffs when neither of the firm's outside option is binding because both

receive the “*split-the-surplus*” payoff. On the other hand, in a region where the opponent’s outside option is binding, the status quo payoff both influences the level of payoff the agent i receives and also constrains on the validity of the payoff function.

In finding the agents’ response functions, we first need to characterize the three regions of interest in $\Pi_i(a_1, a_2)$ as a function of a .

Lemma 2 *If $C_i(0, 0) > B_i(0, \hat{\phi}_i)$ and*

$$\max_{\phi_i} \frac{\partial B_i(a_i, \phi_i)}{\partial a_i} < 2 \min_{\phi_i} \frac{\partial B_i(a_i, \phi_i)}{\partial a_i} \quad (1.4)$$

for every $a_i \in A_i$, then there exist a monotonically increasing function $\alpha_i : A_j \rightarrow A_i$ such that

- i. j ’s outside option is binding if $a_i \leq \alpha_j^{-1}(a_j)$,*
- ii. neither outside option is binding if $\alpha_j^{-1}(a_j) \leq a_i \leq \alpha_i(a_j)$,*
- iii. i ’s outside option is binding if $\alpha_i(a_j) < a_i$.*

The first assumption is automatically satisfied when the firms are symmetric. Otherwise, cooperation generates a smaller total surplus than non-cooperation. The second assumption requires that the marginal benefit from a_i does not change much with ϕ_i . In other words, the marginal private benefit of *ex-ante* investment must not be very sensitive to the second period payoff.

Proof. See Appendix B. ■

The α function divides the (a_1, a_2) plane into three regions. In the northwest corner, agent 1’s outside option is binding, in the southeast corner, agent 2’s outside option is binding and in the between region, neither agent’s outside option is binding (see figure 1). Consider the case when the firms are symmetric. On the 45° line

both agents invest the same amount, $a_1 = a_2$. Given that they are symmetric, the non-cooperative choices of q 's will be the same, so will be the value of the status quo payoffs. If agent 1's outside option is binding then agent 2's outside option has to be binding because of symmetry. Both outside options, however, cannot be binding at the same time. Therefore, on the 45° line neither outside options are binding. We now consider keeping a_2 at the same level as before but increasing a_1 . Since $B_1(\cdot)$ is increasing in a_1 , if we increase a_1 enough we will reach the region where agent 1's outside option is binding. That's why the region where agent 1's outside option is binding should be on the northwest corner. The similar argument applies for agent 2; the region where her outside option is binding should be on the southeast corner.

1.4 Equilibria to the investment-choice game

Given the solution to the induced bargaining subgame, we have defined $\Pi_i(a_1, a_2)$ as the reduced form payoff to the bargaining subgame. The ex-ante investments a_1 and a_2 are chosen simultaneously and independently at date 0 taking into account the outcome of the negotiation between agents 1 and 2. Given the reduced form payoffs obtained from bargaining subgame, the Nash equilibrium to the investment choice game is a perfect subgame Nash equilibrium of the full game. We will concentrate on the investment-choice game and characterize its equilibrium. A Nash equilibrium in date 0 investments is a pair $(a_1^N, a_2^N) \in A_1 \times A_2$ such that,

$$\begin{aligned} \Pi_1(a_1^N, a_2^N) &\geq \Pi_1(a_1, a_2^N) \text{ for all } a_1 \in A_1 \\ \Pi_1(a_1^N, a_2^N) &\geq \Pi_1(a_1^N, a_2) \text{ for all } a_2 \in A_2 \end{aligned}$$

We introduce some further assumptions into the model before we proceed.

Assumption 3 *Firms are symmetric.*

Assumption 4 *q_1 and q_2 are complementary activities. ϕ_i is increasing in q_j .*

Assumption 5 *The marginal benefit of a_i is increasing in second period payoff, ϕ_i . In other words, a_i and ϕ_i are complementary.*

We first derive the agents' reaction functions. We define $\rho_i : A_j \rightarrow A_i$ to be the agent i 's reaction function, where

$$\rho_i(a_j) = \operatorname{argmax}_{a_i \in A_i} \Pi_i(a_1, a_2) .$$

Since $\Pi_i(a_1, a_2)$ depends on the region of choice space considered, it is convenient to separately analyze these regions, find the optimal action in each, and then determine the optimal action which maximizes the overall payoff.

1.4.1 Agent i 's outside option is binding

In the region where agent i 's outside option is binding his best response is defined as

$$\beta_i(a_j) = \max \{ \hat{a}_i, \alpha_i(a_j) \}$$

where

$$\hat{a}_i = \operatorname{argmax}_{a_i \in A_i} \{ B_i(a_i, \hat{\phi}_i) - v_i(a_i) \} .$$

\hat{a}_i is the agent i 's optimal investment choice when the initial contract is not renegotiated. Given the non-cooperative choices of (\hat{q}_1, \hat{q}_2) , agent i chooses a_i to maximize his net benefit.

Definition 1 *There exists $\bar{a}_j \in A_j$ such that, $\alpha_i(\bar{a}_j) = \hat{a}_i$.*

\bar{a}_j is the level of ex-ante investment made by agent j so that, agent i 's outside option is just binding at its optimum. We assume that $\hat{a}_i > \alpha_i(0)$ so that there exists an $\bar{a}_j > 0$. Now we can rewrite $\beta_i(a_j)$ as

$$\beta_i(a_j) = \begin{cases} \hat{a}_i & \text{if } a_j \leq \bar{a}_j \\ \alpha_i(a_j) & \text{if } a_j > \bar{a}_j \end{cases} .$$

For $a_j \leq \bar{a}_j$, agent i 's outside option is binding hence he chooses \hat{a}_i . For $a_j > \bar{a}_j$, where his outside option is not binding at its optimum, agent i chooses $\alpha_i(a_j)$ so that his outside option just binds.

Claim 1 $\alpha_i(a_j) > a_i$. *The area in which agent i 's outside option is binding always lies above 45° line.*

Proof. We prove the claim for $a_i = \bar{a}_i$. The same argument, however, can easily be extended to all $a_i \in A_i$. Assume that $\bar{a}_i \geq \alpha_i(\bar{a}_j)$. Then $B_i(\bar{a}_i, \hat{\phi}_i) \geq C_i(\bar{a}_i, \bar{a}_j)$ by definition of $\alpha_i(a_j)$ and $B_j(\bar{a}_j, \hat{\phi}_j) \geq C_j(\bar{a}_i, \bar{a}_j)$ by symmetry. By adding the two inequality we obtain $B_i(\bar{a}_i, \hat{\phi}_i) + B_j(\bar{a}_j, \hat{\phi}_j) \geq B[a, q(a)]$ which contradicts the assumption that cooperation generates greater total surplus than non-cooperation. Therefore $\bar{a}_i < \hat{a}_i$. ■

1.4.2 Neither outside option is binding

In the region where neither agent's outside option is binding, agent i 's best response is defined as

$$\eta_i(a_j) = \operatorname{argmax}_{\alpha_j^{-1}(a_j) < a_i \leq \alpha_i(a_j)} \{C_i(a_1, a_2) - v_i(a_i)\} .$$

In this case, the agents share the total surplus, thus agent i maximizes half of the surplus net of cost of ex-ante investment. We define

$$\delta_i(a_j) = \operatorname{argmax}_{a_i \in A_i} \{C_i(a_1, a_2) - v_i(a_i)\}$$

as the i 's best response to the unconstrained maximization problem.

Claim 2 $\delta_i(a_j)$ is increasing in a_j .

Proof. Applying the implicit function theorem, we obtain

$$\frac{\partial \delta_i(a_j)}{\partial a_j} = - \frac{\frac{1}{2} \frac{\partial^2 B_i(a_i, \phi_i^c)}{\partial a_i \partial \phi_i} \frac{\partial \phi_i^c}{\partial a_j}}{\frac{1}{2} \frac{\partial^2 B_i(a_i, \phi_i^c)}{\partial a_i^2} - \frac{\partial^2 v_i(a_i)}{\partial a_i^2}}.$$

The denominator is negative because of the second order conditions on the B and v functions (assumptions 1 and 2). In the numerator, the first term is positive. We assume that $\frac{\partial q_i(a_i)}{\partial a_j}$ is positive for $i, j = 1, 2$, which implies that $\frac{\partial \phi_i^c}{\partial a_j}$ is positive. This assumption implies that the cooperative choice of q is increasing in both the agent's and the opponent's ex-ante investment. ■

Claim 3 $\delta_i(a_j) < \hat{a}_i$ for all a_j . The best response to the “split-the-surplus” payoff is always smaller than the best response to the status quo payoff.

Proof. $\delta_i(a_j)$ is defined by the following first order condition,

$$\frac{1}{2} \frac{\partial B_i(\delta_i(a_j), \phi_i^c)}{\partial a_i} = \frac{\partial v_i(\delta_i(a_j))}{\partial a_i}.$$

By 1.9

$$\frac{1}{2} \frac{\partial B_i(\delta_i(a_j), \phi_i^c)}{\partial a_i} < \frac{\partial B_i(\delta_i(a_j), \hat{\phi}_i)}{\partial a_i}$$

for any $a_i \in A_i$. Therefore,

$$\frac{\partial v_i(\delta_i(a_j))}{\partial a_i} < \frac{\partial B_i(\delta_i(a_j), \hat{\phi}_i)}{\partial a_i}.$$

The right hand side is decreasing and the left hand side is increasing in a_i . To reach to an equilibrium a_i has to increase, hence $\delta_i(a_j) < \hat{a}_i$ for all a_j . ■

Essentially, a increases the value of the second period payoff, ϕ . When the agent receives the status quo payoff he obtains the full benefit of his actions so he has greater incentive to invest. When, however, he receives half of the total surplus, he receives only half of the benefit so his incentive to invest is distorted downwards.

We have defined $\eta_i(a_j)$ as the best response to $C_i(a_1, a_2) - v_i(a_i)$ when $\alpha_j^{-1}(a_j) \leq a_i \leq \alpha_i(a_j)$. Next we will define the critical values of a_j within which $\delta_i(a_j)$ is relevant.

Definition 2 *There exists $a'_j \in A_j$ such that $\delta_i(a'_j) = \alpha_i(a'_j)$, and $a''_j \in A_j$ such that $\delta_i(a''_j) = \alpha_j^{-1}(a''_j)$.*

a'_j is the level of ex-ante investment of agent j where agent i 's outside option is just binding when he maximizes the “*split-the-surplus*” payoff. We assume that $\delta_i(0) > \alpha_i(0)$ so that there exists a'_j . a''_j is the level of ex-ante investment of agent j at which his outside option is just binding when agent i maximizes the “*split-the-surplus*” payoff. In other words, a'_j and a''_j limit the range where $\delta_i(a_j)$ is valid. Since $\delta_i(a_j)$ is increasing in a_j , then $a'_j < a''_j$.

Now we can rewrite agent i 's best response when neither outside options are binding as,

$$\eta_i(a_j) = \begin{cases} \alpha_i(a_j) & \text{if } a_j > a'_j \\ \delta_i(a_j) & \text{if } a'_j \leq a_j \leq a''_j \\ \alpha_j^{-1}(a_j) & \text{if } a''_j \leq a_j \end{cases} .$$

When agent j invests at small levels, $a_j < a'_j$, agent i chooses along $\alpha_i(a_j)$ so that his outside option just binds. For $a'_j < a_j < a''_j$, he chooses $\delta_i(a_j)$, where neither of the firm's outside option is binding. When agent j invests at large levels, $a_j > a''_j$, agent i chooses along $\alpha_i^{-1}(a_j)$ so that the opponent's outside option just binds.

1.4.3 Agent j 's outside option is binding

In the region where agent j 's outside option binds, i 's best response is defined as

$$\xi_i(a_j) = \operatorname{argmax}_{\alpha_j^{-1}(a_j) \geq a_i} \{H_i(a_1, a_2) - v_i(a_i)\} .$$

Agent i 's outside option does not bind whenever agent j 's outside option binds.

Agent i claims the residual and chooses a_i to maximize the total surplus net of the cost of ex-ante investment and the payment to the agent j . We define

$$\epsilon_i(a_j) = \operatorname{argmax}_{a_i \in A_i} H_i(a_1, a_2) - v_i(a_i)$$

to be the maximizer of the unconstrained problem.

Claim 4 $\epsilon_i(a_j)$ is increasing in a_j .

$$\frac{\partial \epsilon_i(a_j)}{\partial a_j} = - \frac{\frac{\partial^2 B_i(a_i, \phi_i^c)}{\partial a_i \partial \phi_i} \frac{\partial \phi_i^c}{\partial a_j}}{\frac{\partial^2 B_i(a_i, \phi_i^c)}{\partial a_i^2} - \frac{\partial^2 v_i(a_i)}{\partial a_i^2}} .$$

The same reasoning as in the proof of claim 2 applies. ■

Definition 3 There exists $\tilde{a}_j \in A_j$ such that $\alpha_j^{-1}(\tilde{a}_j) = \epsilon_i(\tilde{a}_j)$.

\tilde{a}_j is the level of ex-ante investment of agent j at which his outside option just binds when agent i maximizes the “split-the-surplus” payoff. Hence, we can rewrite agent i 's response function when agent j 's outside option is binding as

$$\xi_i(a_j) = \begin{cases} \alpha_j^{-1}(a_j) & \text{if } a_j > \tilde{a}_j \\ \epsilon_i(a_j) & \text{if } a_j < \tilde{a}_j \end{cases} .$$

For small a_j 's agent i chooses along $\alpha_j^{-1}(a_j)$ to make j 's outside option just binding.

For large a_j 's, he chooses $\epsilon_i(a_j)$.

1.4.4 The best-response function

We evaluate the payoff function $\Pi_i(a_1, a_2)$ at the optimum of each region and compare them to find the best response function of agent i . We have shown that $\hat{a}_i > \delta_i(a_j)$ for all a_j . The function $C_i(a_i, a_j) - v_i(a_i)$ reaches its maximum at $\delta_i(a_j)$. Then the function $C_i(a_i, a_j) - v_i(a_i)$ must be decreasing for all $a_i > \delta_i(a_j)$. Since $\hat{a}_i > \delta_i(a_j)$, $\frac{\partial C_i(\hat{a}_i, a_j)}{\partial a_i} - \frac{\partial v_i(\hat{a}_i)}{\partial a_i} < 0$. This implies that the value of agent i 's “*split-the-surplus*” payoff at its maximum, $C_i(\delta_i(\bar{a}_j), \bar{a}_j) - v_i(\delta_i(\bar{a}_j))$, is higher than the value of his outside option at its maximum, $B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$, when agent j invests at \bar{a}_j . Since $C_i(\cdot) - v_i(\cdot)$ is increasing in a_j , agent i is indifferent between choosing \hat{a}_i or $\delta_i(a_j)$ at levels of ex-ante investment which are lower than \bar{a}_j . In other words, at some level of ex-ante investment chosen by agent j , say \check{a}_j , agent i 's outside option at its maximum just equals his “*split-the-surplus*” payoff evaluated at its maximum. Thus we have

Definition 4 *There exists $\check{a}_j \in A_j$, such that,*

$$C_i(\delta_i(\check{a}_j), \check{a}_j) - v_i(\delta_i(\check{a}_j)) = B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i).$$

We next need to locate \check{a}_j . Below we show that $\check{a}_j > a'_j$.

Claim 5 $\check{a}_j > a'_j$. *The jump in agent i 's response function occurs at the region where his outside option is not binding.*

Proof. At a'_j , $\delta_i(a'_j) = \alpha_i(a'_j)$. Therefore, the following condition

$$C_i(\delta_i(a'_j), a'_j) - v_i(\delta_i(a'_j)) = B_i(\delta_i(a'_j), \hat{\phi}_i) - v_i(\delta_i(a'_j))$$

is satisfied. Then,

$$C_i(\delta_i(a'_j), a'_j) - v_i(\delta_i(a'_j)) < B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$$

since \hat{a}_i is the unique maximum. By using the definition of \check{a}_j , we replace the right hand side of the inequality by $C_i(\delta_i(\check{a}_j), \check{a}_j) - v_i(\delta_i(\check{a}_j))$ and obtain

$$C_i(\delta_i(a'_j), a'_j) - v_i(\delta_i(a'_j)) < C_i(\delta_i(\check{a}_j), \check{a}_j) - v_i(\delta_i(\check{a}_j)).$$

Since $\frac{\partial C_i}{\partial a_j} > 0$, then it must be true that $a'_j < \check{a}_j$. ■

Since \hat{a}_i is greater than $\delta_i(a_j)$ for all a_j , $\delta_i(a_j)$ can only be equal to $\alpha_i(a_j)$ when $B_i(a_i, \hat{\phi}_i) - v_i(a_i)$ is increasing. This means that $C_i(\delta_i(a'_j), a'_j) - v_i(\delta_i(a'_j))$ is less than $B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$. In order to increase the value of $C_i(\cdot) - v_i(\cdot)$ to be equal to $B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$, a_j has to increase. Thus, $a'_j < \check{a}_j$. In other words, when agent j 's investment is small, agent i can obtain a higher payoff in status quo than the “*split-the-surplus*” payoff by investing at high levels. However, as agent j 's investment increases, “*split-the-surplus*” payoff increases because of the complementarity assumption and generates higher payoffs than the status quo payoff. Therefore, for small levels of a_j , agent i continues to choose \hat{a}_i even though his outside option is not binding.

Whether \check{a}_j is greater or smaller than a''_j depends on the gains from cooperation. a''_j is the point where agent j 's outside option is just binding when agent i chooses $\delta_i(a_j)$. In other words, a''_j is the agent j 's investment level beyond which agent i chooses to maximize the residual. If \check{a}_j is smaller than a''_j , then agent i responds by choosing along $\delta_i(a_j)$ for $a_j \in [\check{a}_j, a''_j]$. If \check{a}_j is greater than a''_j and agent i responds with $\delta_i(a_j)$, agent j 's outside option becomes binding that implies that agent i does not receive the “*split-the-surplus*” payoff but claims the residual. In fact, he maximizes his payoff if he continues to choose \hat{a}_i for values of $a_j < a^*_j$. At a^*_j , the maximum value of agent i 's status quo payoff, $B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$, just equals the maximum value of his payoff when he receives the residual. For any value $a_j \geq a^*_j$, agent i responds by maximizing the residual, $H_i(\cdot) - v_i(\cdot)$.

Definition 5 *There exists $a_j^* \in A_j$ such that,*

$$C_i \left(\alpha_j^{-1} (a_j^*), a_j^* \right) - v_i \left(\alpha_j^{-1} (a_j^*) \right) = B_i \left(\hat{a}_i, \hat{\phi}_i \right) - v_i \left(\hat{a}_i \right).$$

Before we present the agent i 's response function it is important to note that both \tilde{a}_j and a_j^* are smaller than \bar{a}_j .

Claim 6 $\tilde{a}_j < \bar{a}_j$.

Proof. Using the definitions of \tilde{a}_j and \bar{a}_j ,

$$B_i \left(\hat{a}_i, \hat{\phi}_i \right) - v_i \left(\hat{a}_i \right) = C_i \left(\hat{a}_i, \bar{a}_j \right) - v_i \left(\hat{a}_i \right) = C_i \left(\delta_i \left(\bar{a}_j \right), \bar{a}_j \right) - v_i \left(\delta_i \left(\bar{a}_j \right) \right)$$

which is less than $C_i \left(\delta_i \left(\bar{a}_j \right), \bar{a}_j \right) - v_i \left(\delta_i \left(\bar{a}_j \right) \right)$. This implies that $\tilde{a}_j < \bar{a}_j$. ■

Claim 7 $a_j^* < \bar{a}_j$.

Proof. At \bar{a}_j , $C_i \left(\hat{a}_i, \bar{a}_j \right) = B_i \left(\hat{a}_i, \hat{\phi}_i \right)$ by definition. a_j^* cannot be equal to \bar{a}_j because $H_i \left(a_i, a_j \right) > B_i \left(a_i, \hat{\phi}_i \right)$ for all (a_i, a_j) . Since $H_i \left(a_i, \bar{a}_j \right)$ cannot intersect $C_i \left(a_i, \bar{a}_j \right)$ at \hat{a}_i , it must intersect it at some a_i which is less than \hat{a}_i . Hence $\alpha_j^{-1} \left(\bar{a}_j \right) < \hat{a}_i$. Then at $\alpha_j^{-1} \left(\bar{a}_j \right)$ the following must hold

$$C_i \left(\alpha_j^{-1} \left(\bar{a}_j \right), \bar{a}_j \right) - v_i \left(\alpha_j^{-1} \left(\bar{a}_j \right) \right) > B_i \left(\hat{a}_i, \hat{\phi}_i \right) - v_i \left(\hat{a}_i \right).$$

Since $\alpha_j^{-1} \left(a_j \right)$ is increasing in a_j , $a_j^* < \bar{a}_j$. ■

The above analysis can be summarized in the following lemma that describes the agent i 's response function.

Lemma 3 *If $\tilde{a}_j < a_j''$, then agent i 's reaction function is*

$$\rho_i \left(a_j \right) = \begin{cases} \hat{a}_i & \text{if } a_j \leq \tilde{a}_j \\ \delta_i \left(a_j \right) & \text{if } \tilde{a}_j \leq a_j \leq a_j'' \\ \alpha_j^{-1} \left(a_j \right) & \text{if } a_j'' \leq a_j \leq \bar{a}_j \\ \epsilon_i \left(a_j \right) & \text{if } \bar{a}_j \leq a_j \end{cases}.$$

If $\tilde{a}_j > a_j''$, then agent i 's response function is

$$\rho_i(a_j) = \begin{cases} \hat{a}_i & \text{if } a_j \leq a_j^* \\ \alpha_j^{-1}(a_j) & \text{if } a_j^* \leq a_j \leq \tilde{a}_j \\ \epsilon_i(a_j) & \text{if } \tilde{a}_j \leq a_j \end{cases}$$

Agent i can have two types of response function depending on whether or not he switches from maximizing the status quo payoff to maximizing the “*split-the-surplus*” payoff in the region where the opponent’s outside option is binding. If $\tilde{a}_j < a_j''$, then the jump in the response function occurs in the region where agent j 's outside option is binding. For small a_j , agent i chooses \hat{a}_i . At \tilde{a}_j there is a downward jump in the response function. From this point on, agent i chooses along $\delta_i(a_j)$ until a_j'' is reached. At a_j'' , agent j 's outside option becomes binding. Agent i responds by choosing along $\alpha_j^{-1}(a_j)$ so that agent j 's outside option just binds. After \tilde{a}_j is reached, agent i responds by choosing $\epsilon_i(a_j)$ (see figure 2 and 3 for the graph of agent i 's response function.).

If $\tilde{a}_j > a_j''$, that is, when the jump occurs in the region where agent j 's outside option is binding, agent i chooses \hat{a}_i for small a_j . For $a_j \in [a_j^*, \tilde{a}_j]$ he responds along $\alpha_j^{-1}(a_j)$ so that agent j 's outside option is just binding. For $a_j > \tilde{a}_j$, he responds along $\epsilon_i(a_j)$ (see figure 4.). Whether \tilde{a}_j is smaller or greater than a_j'' depends on the B and v functions.

In general, the game will have either a unique pure strategy Nash equilibrium in which both agents maximize the “*split-the-surplus*” payoff (this occurs if $\tilde{a}_j < \delta_i(\tilde{a}_j)$) or no pure strategy equilibrium (when $\tilde{a}_j > \delta_i(\tilde{a}_j)$). The following proposition describes the equilibrium of the investment-choice game.

Proposition 4 *If there exists a Nash equilibrium to the investment-choice game in which neither agent's outside option is binding, then it is the unique equilibrium (in*

pure strategies).

Proof. Let (a_1^N, a_2^N) be the Nash equilibrium in which neither agent's outside option is binding. First we show that regardless of the existence of (a_1^N, a_2^N) , $(\delta_i(\hat{a}_j), \hat{a}_j)$ and $(\alpha_j^{-1}(\hat{a}_j), \hat{a}_j)$ cannot be equilibria.

Since $\bar{a}_j > \tilde{a}_j$, it is also true by symmetry that $\bar{a}_i > \tilde{a}_i$. This implies that agent j switches to $\delta_j(a_i)$ at some investment level, a_i , which is below \bar{a}_i . Therefore, $\alpha_j^{-1}(a_j)$ never intersects the response function at \hat{a}_j . Moreover, $\delta_j(a_i)$ is part of the response function when it is above $\alpha_j^{-1}(a_j)$. Since $\hat{a}_j < \alpha_j^{-1}(\hat{a}_j)$, then $\hat{a}_j < \delta_i(\hat{a}_j)$. Thus, $(\delta_i(\hat{a}_j), \hat{a}_j)$ can never be an equilibrium, too. Given that (a_1^N, a_2^N) is the Nash equilibrium of the game, it must be true that $\tilde{a}_i < \delta_i(\tilde{a}_j) < \delta_i(\hat{a}_j)$ since $\delta_i(a_j)$ is monotonically increasing in a_j . It is also true that $\delta_i(\hat{a}_j) < \epsilon_i(\hat{a}_j)$ which in turn implies that $\hat{a}_j < \epsilon_i(\hat{a}_j)$. Thus, $(\epsilon_i(\hat{a}_j), \hat{a}_j)$ cannot be an equilibrium. ■

The uniqueness of the Nash equilibrium depends on the positive slope of the $\delta_i(\cdot)$ function which arises from the complementarity assumption that we made. As a_j increases, there is a direct effect on C_i , but also an indirect effect since the second period payoff to both firms, ϕ_i , increases in response to the increase in a_j . The increase in ϕ_i , in return, causes a_i to increase.

Proposition 4 refers to the uniqueness, but not the existence of the pure strategy Nash equilibrium. In order to determine if the equilibrium exists we perform a comparative statics, the result of which we present its results in Section 1.6. It is apparent that the divergence between the cooperation and non-cooperation is completely driven by an interdependency in the second period production. In other words, depending on the degree of the complementarity the total surplus may rise through negotiation. In the case when there is no complementarity, the cooperative and non-cooperative solutions are identical. Thus there is no need for negotiation.

1.5 Integration

We consider two types of ownership structures under integration. Under agent 1's ownership, agent 1 owns both firms and agent 2 becomes his employee. Thus, agent 1 has the residual control rights over both firms' assets. In our model, this amounts to agent 1 choosing both q_1 and q_2 at date 1 under the provisions of the initial contract. It is, however, still necessary that both agents make the relationship-specific investment at date 0. Under agent 2's ownership, firm 2 owns both firms and agent 1 becomes her employee. Regardless of the ownership structure, each agent receives the full private benefit of the firm where they are employed. Being an owner does not change the structure of the payoff function or change the ex-ante distribution of surplus. Ownership only entitles the beholder the right to control assets in unspecified contingencies. We will only examine the equilibrium under agent 1's ownership. The case for agent 2's ownership is symmetric.

1.5.1 1's Ownership

In this case agent 1 owns both firm 1 and 2. Besides having the right to choose both q_1 and q_2 at date 1, agent 1 is also the only agent in the bargaining game who can credibly use his outside option. The residual control rights give him the right to both choose and implement q_1 and q_2 . Agent 2 can bribe agent 1 to choose her favorite q but she cannot quit the bargaining game and implement the status quo choices of q_1 and q_2 . Essentially her outside option is not a credible threat in the bargaining game. Now the variable a_{ki} denotes the ex-ante investment of agent i under k 's ownership.

At date 1, agent 1 chooses q_1 and q_2 to maximize $B_1 [a_1, \phi_1 (q_1, q_2)]$. We

assume that there exists a unique equilibrium to the q -choice subgame under 1's ownership. Let

$$(\hat{q}_{11}, \hat{q}_{12}) = \arg \max_{q_1 \in Q_1, q_2 \in Q_2} \phi_1(q_1, q_2) \quad (1.5)$$

be the unique Nash equilibrium to this game. In general, the non cooperative solution $(\hat{q}_{11}, \hat{q}_{12})$ is ex-post inefficient. Therefore, the two parties can gain from negotiating a new contract. The rest of the analysis is similar to the case of non-integration. The payoff function for agent 1 is given by

$$\Pi_1(a_1, a_2) = \begin{cases} C_1(a_1, a_2) - v_1(a_1) & \text{if neither outside options is binding} \\ B_1(a_1, \hat{\phi}_{11}) - v_1(a_1) & \text{if 1's outside option is binding.} \end{cases}$$

and for agent 2 it is

$$\Pi_2(a_1, a_2) = \begin{cases} C_2(a_1, a_2) - v_2(a_2) & \text{if neither outside options is binding} \\ H_2(a_1, a_2) - v_2(a_2) & \text{if 1's outside option is binding.} \end{cases}$$

The assumptions of Lemma 2 are sufficient to prove the existence of $\alpha_{11}(a_2)$ which divides the space of (a_1, a_2) into two regions such that, for $a_1 > \alpha_{11}(a_2)$ agent 1's outside option is binding and $a_1 < \alpha_{11}(a_2)$ it is not binding. The following lemma describes the agents' response functions under agent 1's ownership.

Lemma 5 *Agent 1's reaction function is the following,*

$$\rho_{11}(a_2) = \begin{cases} \hat{a}_{11} & \text{if } a_2 \leq \tilde{a}_{12} \\ \delta_{11}(a_2) & \text{if } \tilde{a}_{12} \leq a_2 \end{cases}$$

and agent 2's response function is,

$$\rho_{12}(a_1) = \begin{cases} \delta_{12}(a_1) & \text{if } a_1 \leq a''_{11} \\ \alpha_{11}^{-1}(a_1) & \text{if } a''_{11} < a_1 \leq \tilde{a}_{11} \\ \epsilon_{12}(a_1) & \text{if } \tilde{a}_{11} \leq a_1. \end{cases}$$

Proof. See the proof of Lemma 3. ■

Under 1's ownership, agent 1 has a unique response function for any parameter values. This is because the agent 2's outside option is never binding. The jump in the response function always occurs at \tilde{a}_{12} . For small ex-ante investment levels, agent 1 responds by choosing \hat{a}_{11} . At \tilde{a}_{12} , there is a downward jump in the response function. For values greater than \tilde{a}_{12} , agent 1 chooses $\delta_{11}(a_2)$. Agent 2's response function is $\delta_{12}(a_j)$ for small values of a_1 . At a''_{11} there is a jump in her response function. For values greater than a''_{11} , agent 2 chooses $\alpha_{11}^{-1}(a_1)$, so that agent 1's outside option is just binding. For $a_1 > \tilde{a}_{11}$, she chooses $\epsilon_{12}(a_1)$.

As in the case of non-integration the game has either a unique pure strategy Nash equilibrium in which both agents maximize the “*split-the-surplus*” payoff or no equilibrium in pure strategies. The unique Nash equilibrium exists if $\tilde{a}_{12} < \delta_{11}(\tilde{a}_{12})$, that is, if the jump in agent 1's response function occurs to the left of 45° line (see figure 5). An argument similar to that used in the proof of proposition 4 shows that if there exists a Nash equilibrium to the investment-choice game in which neither agent's outside option is binding, then it is a unique equilibrium in pure strategies.

1.6 On the Existence of the Equilibrium

We introduce a parameter into the second period payoff function ϕ to capture the level of complementarity among the two firms. Let γ be an index of complementarity where $\gamma \in [0, 1]$. When $\gamma = 0$ there is no production complementarity. In that case, the second period payoff function ϕ_i depends solely on q_i . As γ increases the degree of complementarity in the production of the two firms increases. As a benchmark we first examine the equilibrium when there is no complementarity, i.e. $\gamma = 0$. We

then analyze how the equilibrium evolves as complementarity is introduced. We make following assumptions:

Assumption 6 $\frac{\partial q_i(a)}{\partial \gamma} > 0$ for $i = 1, 2$. *The cooperative choice of ex-post production increases as complementarity increases.*

Assumption 7 $\frac{\partial B_i(a_i, \hat{\phi}_i)}{\partial \gamma} < \frac{\partial C_i(a_1, a_2)}{\partial \gamma}$, i.e., *as the complementarity between the two firm's production increases the "split-the-surplus" payoff increases by more than the non-cooperative payoff.*

1.6.1 No Complementarity (The case of Non-integration)

Assume that $\gamma = 0$, that is, the second period payoff is independent of the opponent's production decision. Then the optimal cooperative and non-cooperative choices of q_i are the same and in both cases the value of the second period payoff, $\hat{\phi}_i$ and ϕ_i^c are identical. As a result, regardless of whether or not he cooperates, the payoff to agent i when he claims the residual is the same as the status quo payoff. Thus, $H_i(\cdot) = B_i(a_i, \hat{\phi}_i)$, and they are maximized at the same level of ex-ante investment, $\hat{a}_i = \epsilon_i > \delta_i$. Even though δ_i is independent of the opponent's ex-ante investment level, it is still lower than \hat{a}_i since in the "split-the-surplus" solution the agent does not receive the full benefit of his actions. In this non-complementarity case, $\alpha_i(a_j) = \alpha_j^{-1}(a_j) = a_j$, which implies that the agent i 's outside option is binding in the area above 45° line while agent j 's is binding below. In other words there is no region in which neither of the agents' outside option is binding. Below we characterize the equilibrium to this game.

Lemma 6 *If there is no complementarity between the two firms' production then (\hat{a}_1, \hat{a}_2) is the unique equilibrium of the above game.*

Proof. Note that $\bar{a}_i = \hat{a}_i$ by the fact that $\alpha_i(a_j)$ is the 45° line and by the definition of \bar{a}_i . In claim 6 we have shown that $\check{a}_j < \bar{a}_j$. Thus it follows that, $\check{a}_j < \hat{a}_j$.

We next show that $\check{a}_i > \delta_i$. We consider the opposite, that is $\check{a}_i \leq \delta_i$. Then since $\alpha_i(a_i) = a_i$ in the case of no complementarity, $B_i(\check{a}_i, \hat{\phi}_i) - v_i(\check{a}_i) = C_i(\check{a}_i, \check{a}_j) - v_1(\check{a}_i)$. In other words, C_i intersects B_i at \check{a}_i . By the single crossing property in lemma 2, $C_i(a_i, \check{a}_j) - v_1(a_i) \leq B_i(a_i, \hat{\phi}_i) - v_1(a_i)$ for all $a_i \geq \check{a}_i$. Hence $C_i(\delta_i, \check{a}_j) - v_1(\delta_i) \leq B_i(\delta_i, \hat{\phi}_i) - v_1(\delta_i) < B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$, which contradicts with the definition of \check{a}_i . Therefore it must be true that $\check{a}_i > \delta_i$.

Given that \check{a}_j is in the region where the opponent's outside option is binding, the jump must occur at a_j^* . a_j^* is equal to \hat{a}_j because of the fact that $H_i(\cdot) = B_i(a_i, \hat{\phi}_i)$. The best response of agent i is to always play \hat{a}_i . In fact the jump in the response function is fictitious. Because of symmetry \hat{a}_i intersects 45° line at \hat{a}_j , so we have an equilibrium (see Figure 6). ■

As we introduce complementarity, all the relevant functions and critical points in the response function change.

Lemma 7 *As γ increases, \hat{a}_i , $\delta_i(a_j)$, $\alpha_i(a_j)$, and $\epsilon_i(a_j)$ increase.*

Proof. See Appendix C. ■

As we introduce complementarity into the model we obtain an area in which neither agents' outside option is binding.

We argue that for low levels of complementarity, there is no equilibrium in pure strategies. When γ is small, the response function is

$$\rho_i(a_j) = \begin{cases} \hat{a}_i & \text{if } a_j \leq a_j^* \\ \alpha_j^{-1}(a_j) & \text{if } a_j^* \leq a_j \leq \check{a}_j \\ \epsilon_i(a_j) & \text{if } \check{a}_j \leq a_j \end{cases} .$$

When γ is zero a_i'' is smaller than \tilde{a}_i . Thus, for a small degree of complementarity a_i'' is still smaller than \tilde{a}_i by continuity. For that reason, the best response function will be the one above where the agent switches at a_i^* rather than \tilde{a}_i . The only possible equilibrium is the one in which neither parties' outside option is binding. Because of the reasons discussed in the proof of proposition 4, none of them chooses $\alpha_j^{-1}(a_j)$ in the equilibrium. For small γ , \tilde{a}_i is greater than a_i'' which means that $\epsilon_i(a_j)$ is a part of the response function when it is greater than $\alpha_j^{-1}(a_j)$. Since $\alpha_j^{-1}(a_j)$ cannot be an equilibrium then $\epsilon_i(a_j)$ cannot be an equilibrium either. With small complementarity, however, \tilde{a}_i will be greater than $\delta_i(\tilde{a}_j)$. That implies that the jump in the response function occurs to the right of 45° line so there does not exist an equilibrium where neither firm's outside option is binding.

Proposition 8 *For low levels of complementarity, there is no equilibrium to the investment-choice game in pure strategies. If the complementarity between the two firm is sufficiently large, then there is a unique Nash equilibrium (see proposition 1).*

Proof. In the case where there is no complementarity, $\gamma = 0$, the equilibrium to this game is (\hat{a}_1, \hat{a}_2) . Now suppose that we force the agents to receive the “split-the-surplus” payoff. The unique equilibrium of this forced game is (δ_1, δ_2) . Let \mathcal{A}_i be the payoff to agent i in this forced equilibrium and \mathcal{B}_i be the payoff to agent i from deviating to a point which enforces outside option. \mathcal{B}_i is greater than \mathcal{A}_i since $B_i(\cdot)$ is in a_i and $\hat{a}_i > \delta_i$. When the complementarity is small, an interior equilibrium, if it exists, has to be close to the equilibrium of the forced division game when there is no complementarity.¹¹ Let \mathcal{C}_i denote the payoff to agent i in an

¹¹It follows from that the response function has a closed graph.

equilibrium where both agents receive the “*split-the-surplus*” payoff when $\gamma > 0$. Finally let \mathcal{D}_i denote the payoff to agent i from deviating to a point which enforces outside option. We know that \mathcal{B}_i is greater than \mathcal{A}_i . \mathcal{A}_i is close to \mathcal{C}_i and \mathcal{B}_i is close to \mathcal{D}_i which implies that \mathcal{D}_i is greater than \mathcal{C}_i . This implies that agent i has an incentive to deviate from the (δ_1, δ_2) equilibrium when there is small a complementarity. Therefore (δ_1, δ_2) cannot be an equilibrium. There also cannot be an equilibrium where both agents’ outside options are binding. Therefore there is no equilibrium when the firms’ production exhibits small complementarity. The second part of the lemma is proved in proposition 1. ■

The intuition behind proposition 8 is the following. When the agent receives the “*split-the-surplus*” payoff, his incentives are distorted downwards. If we keep the opponent’s action fixed, it is profitable for the agent to deviate and choose \hat{a}_i to maximize the status quo payoff. This is true for both agents because of symmetry. We cannot, however, have an equilibrium where both agents’ outside options are binding. If we do, this would imply that cooperation generates a smaller surplus than non-cooperation. In fact, the only case when we obtain an equilibrium where both outside options are binding is when there is no complementarity between the two firms. In this case, the surplus under cooperation and non-cooperation is identical. As complementarity increases, the agents’ outside options become non-binding, so the deviations described above do not occur. Then the game has the unique equilibrium where neither of the agents’ outside options are binding.

1.7 Concluding Remarks

In this paper we analyze the role of the initial allocation of ownership rights in transactions where parties make relationship-specific investments and the contracts are incomplete. We compare two ownership structures. First, we consider the case where the firms are separately owned by agent 1 and 2 respectively. Then we analyze the case where agent 1 owns both firms and agent 2 is employed in firm 2. In both cases, if the degree of complementarity between the two firms' production is high, cooperation generates large surplus. In this case, the investment-choice game has a unique Nash equilibrium where neither agents' outside option is binding. When the agents' outside options are not binding, agent 1 and 2 split the total surplus in the equilibrium of the bargaining game. Since we obtain the same equilibrium regardless of the ownership structure, the distortions in the ex-ante investments are independent of the initial allocation of ownership rights. Thus, we conclude that the initial allocation of ownership rights does not lead to ex-ante inefficiencies in the production. If, however, the degree of complementarity between the two firms' production is low, then the equilibrium in pure strategies does not exist.

Our conclusion that the allocation of initial ownership rights is irrelevant extends the result of the Coase theorem to the relationships in which agents are unable to bargain ex-ante over all aspects of the transaction, due to contractual incompleteness. This irrelevance result also contrasts the results of Grossman and Hart [15] who obtain that the initial allocation ownership rights have efficiency implications. They argue that even though ex-post bargaining is costless the impossibility of ex-ante bargaining leads to the inefficiencies by distorting parties' incentives to invest in the relationship. The ownership rights should be allocated

to minimize these distortions. Thus, the Coase theorem fails to apply if there is contractual incompleteness. The critical element behind these two different results is that while Grossman and Hart [15] model uses the Nash bargaining solution treating status quo payoffs as disagreement points, in our model they are treated as outside options. It is worthwhile to note that, there is an ex-ante inefficiency in our model, too. This inefficiency, however, does not arise from the initial allocation of ownership rights but as a result of free-rider problem. In particular, the bargaining over an endogenous surplus results in an inefficient equilibrium. As the parties do not receive the full benefit of their actions, their incentives to invest are distorted. In this paper, we show that the ex-ante inefficiencies are not driven by the initial allocation of ownership rights.

An implicit assumption in our model is regarding the definition of ownership. In this paper we define ownership as the power to exercise control. It would be interesting to examine the extent to which the irrelevance result depends on the definition of ownership. In other words, if we broaden this definition to include the rights to the residual income stream, does the irrelevance result continue to hold? Another assumption in the model is that the relationship lasts only two periods. If, however, the relationship lasts longer and the bargaining takes place concurrently with the production, our results may differ. If the bargaining game takes place concurrently with production, the status quo payoffs become the income flow accruing to the agents in the course of the bargaining. In this case, status quo payoffs can be interpreted as the disagreement points. This bargaining game, however, may have many equilibria, some of which are inefficient (see Mumcu [28]). In any case, when the status quo payoffs are treated as the disagreement points Grossman and Hart result [15] can be reestablished.

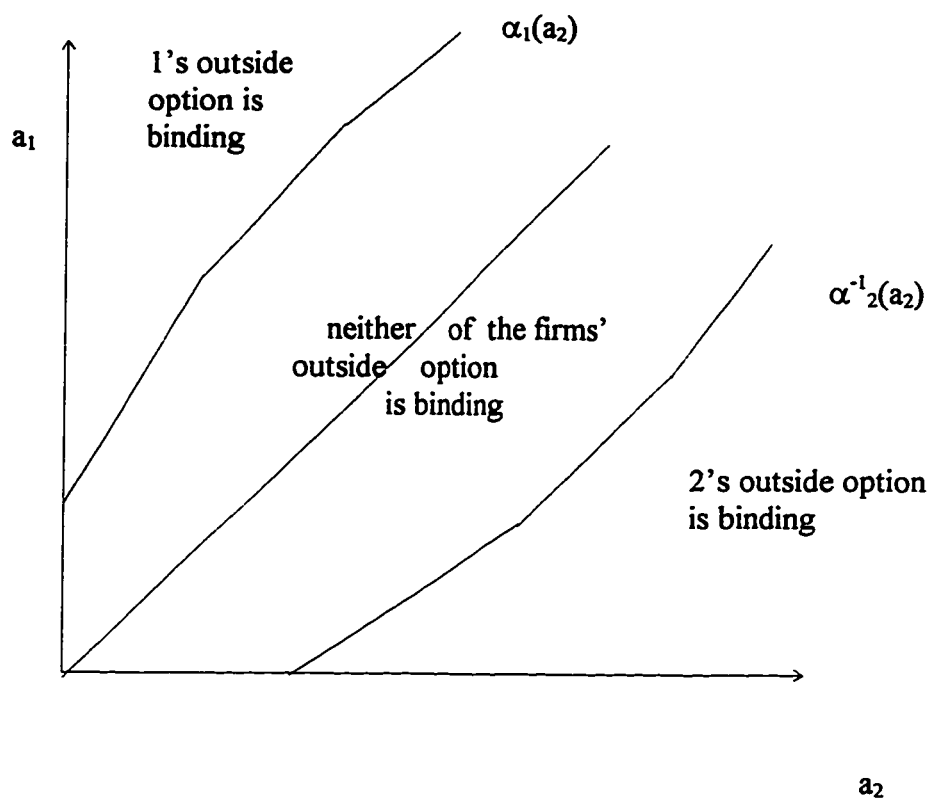


Figure 1. The Functions $\alpha_1(a_2)$ and $\alpha_2^{-1}(a_2)$

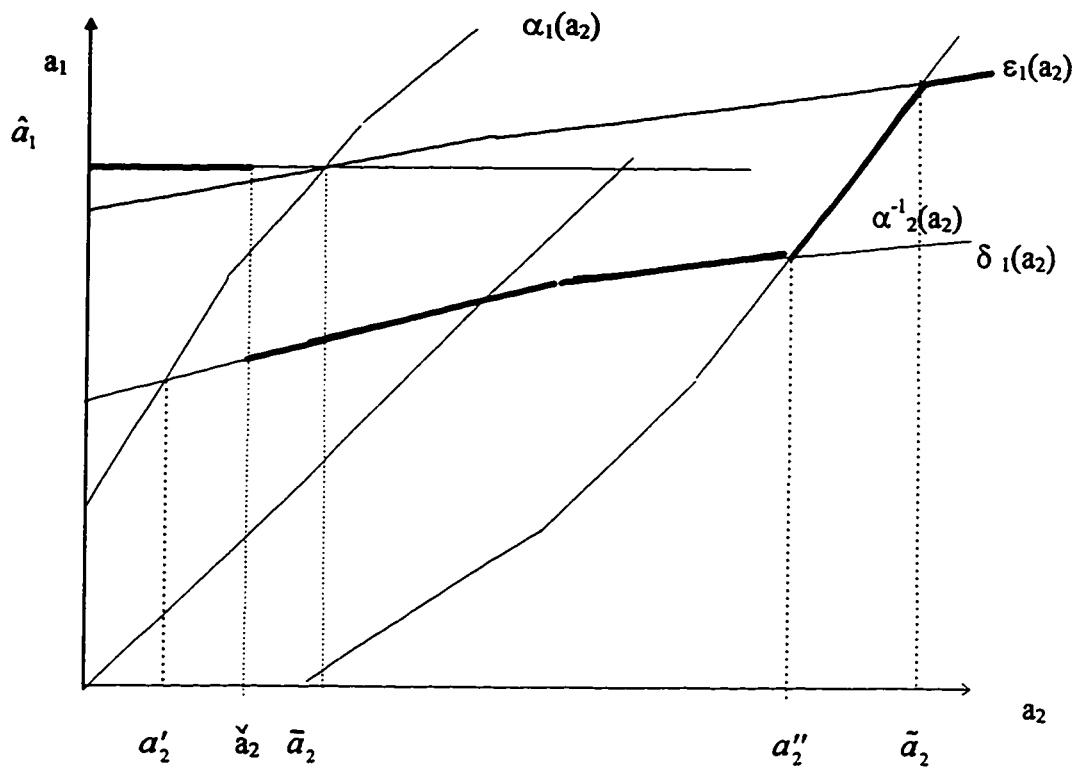


Figure 2. Player 1's response function.
 γ is large and $\check{a} < a''_2$

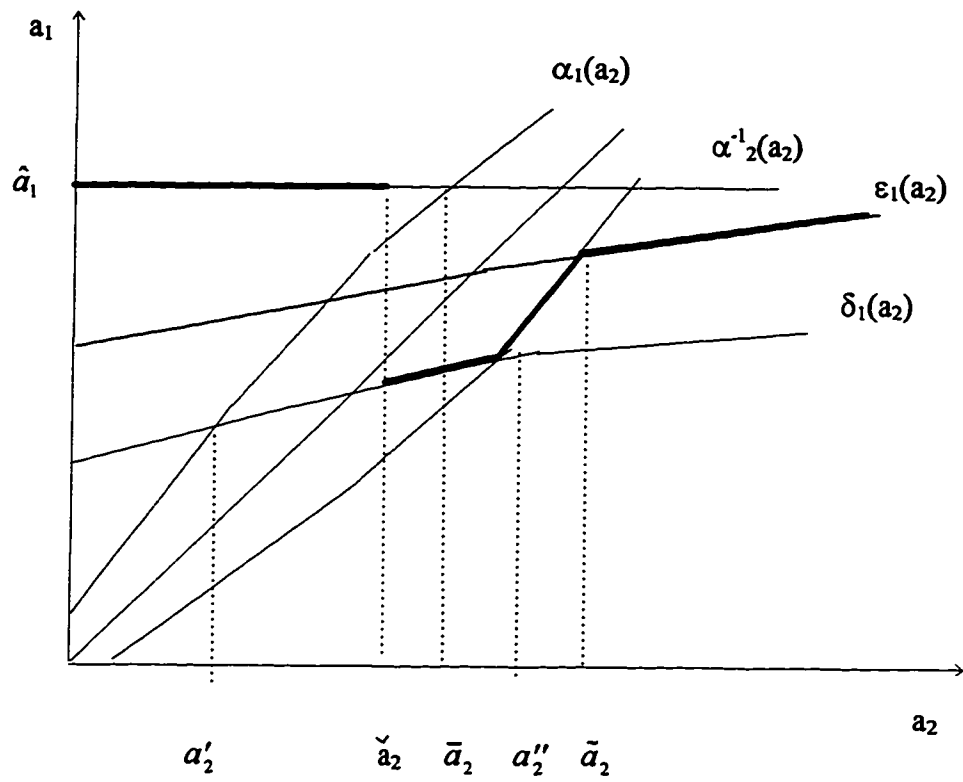


Figure 3. Player 1's response function when γ is small

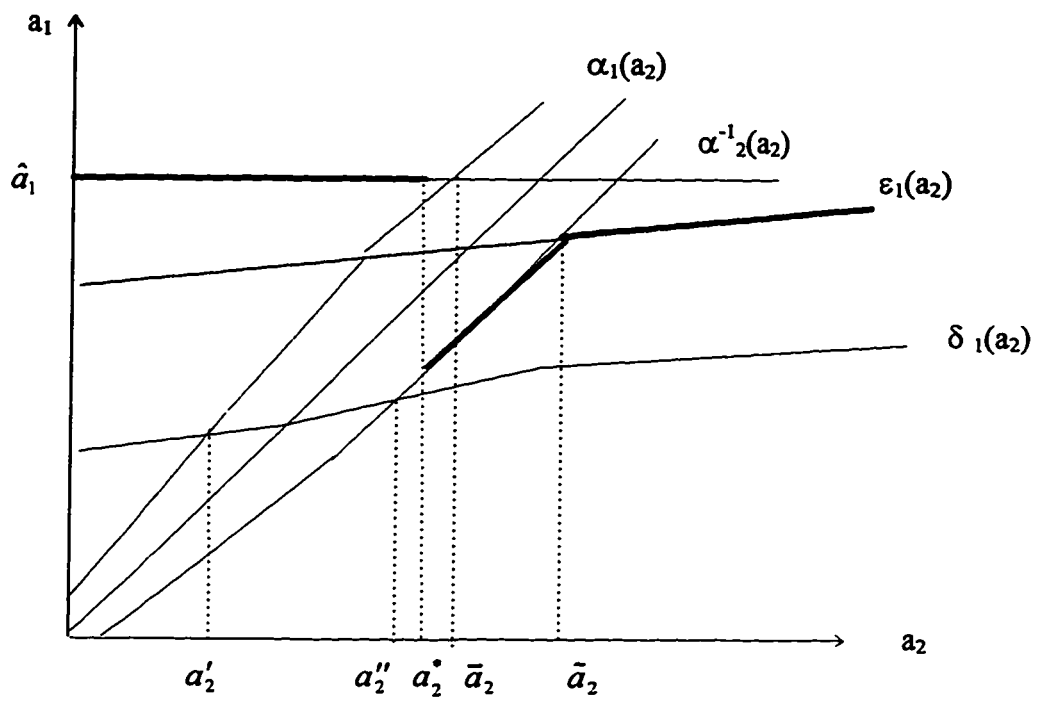


Figure 4. Player 1's response function when γ is small and $\check{a}_2 > a_2''$

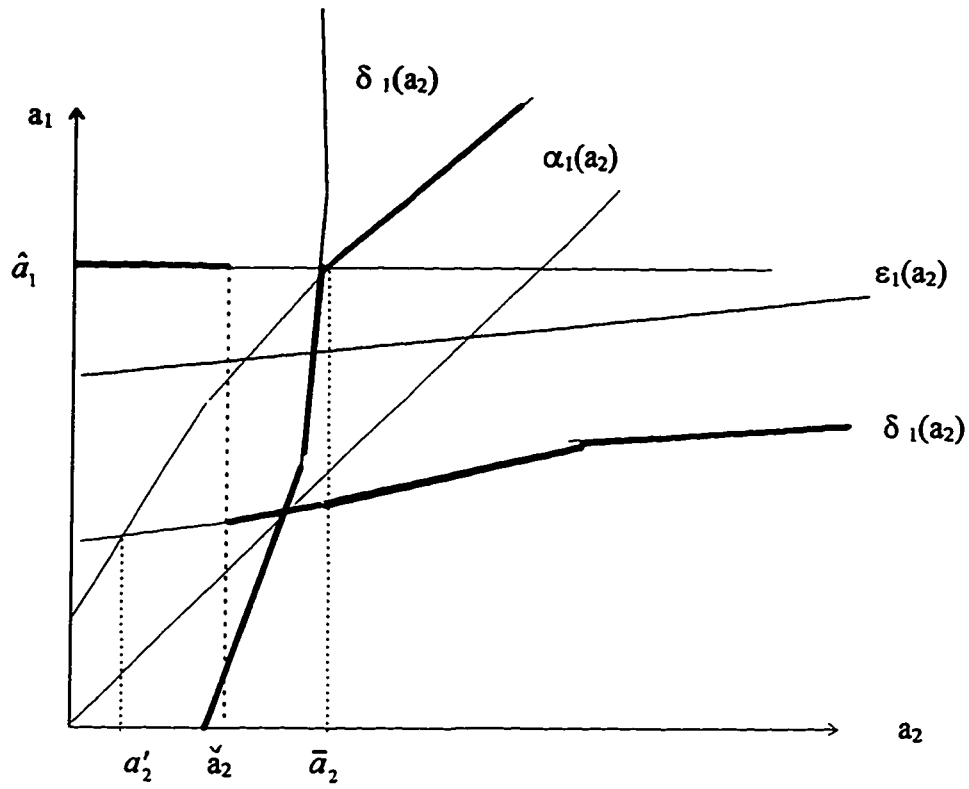


Figure 5. Players' response functions under player 1's ownership

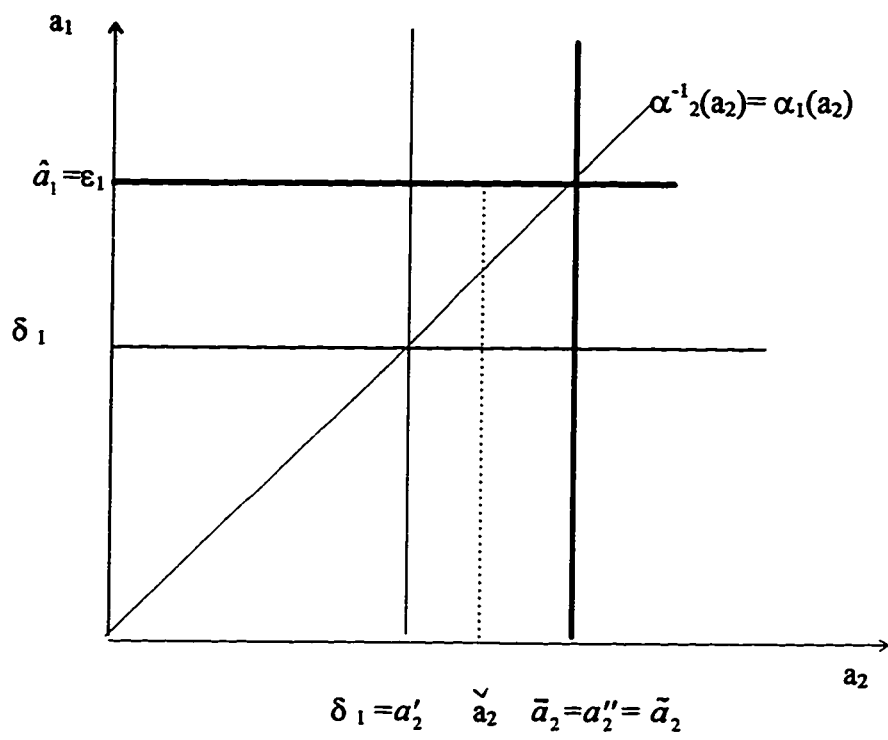


Figure 6. Players' response functions when $\gamma = 0$

1.8 Appendices

1.8.1 Appendix A

Lemma 9 *Given the initial ownership structure and the vector $a = (a_1, a_2)$ of ex-ante investment levels, the induced q -choice/bargaining subgame has a unique equilibrium in which the agreement is reached at $\tau = 0$ and firm 1 receives B_1^1 , given by*

$$B_1^1 = \begin{cases} \frac{B[a, q(a)]}{1+\delta} & \text{if } B_1(a_1, \hat{\phi}_1), B_2(a_2, \hat{\phi}_2) \leq \frac{B[a, q(a)]}{1+\delta} \\ (1-\delta) B[a, q(a)] + \delta B_1(a_1, \hat{\phi}_1) & \text{if } B_1(a_1, \hat{\phi}_1) > \frac{\delta B[a, q(a)]}{1+\delta} \text{ and} \\ & B_2(a_2, \hat{\phi}_2) < \delta B[a, q(a)] - \delta B_1(a_1, \hat{\phi}_1) \\ B[a, q(a)] - B_2(a_2, \hat{\phi}_2) & \text{otherwise} \end{cases}$$

Proof. We solve for the Perfect Equilibrium of the game using the method introduced by Shaked and Sutton [34].

We consider a subgame that starts at period τ with agent 1 making an offer to agent 2. Let M denote the supremum of the payoffs which agent 1 can obtain in any perfect equilibrium of this game. At the preceding period, $\tau - 1$, it is agent 2's turn to make an offer to agent 1. For any offer B_1^2 , agent 1 can accept the offer, or reject it and wait for one period and receive M , which has a present value of δM , or quit and receive the status quo benefit, $B_1(a_1, \hat{\phi}_1)$. Agent 1 will accept any offer that gives him more than $\max\{\delta M, B_1(a_1, \hat{\phi}_1)\}$. Thus, there is no perfect equilibrium in which agent 1 receives more than $\max\{\delta M, B_1(a_1, \hat{\phi}_1)\}$ and agent 2 receives at least $B[a, q(a)] - \max\{\delta M, B_1(a_1, \hat{\phi}_1)\}$. In fact, $B[a, q(a)] - \max\{\delta M, B_1(a_1, \hat{\phi}_1)\}$ is the infimum of the payoffs received by agent 2 in the subgame beginning from that point.

We now consider the offer made by agent 1 at $\tau - 2$. For any offer B_2^1 , agent

2 can accept the offer, or reject it and wait for a period and receive $B[a, q(a)] - \max \{ \delta M, B_1(a_1, \hat{\phi}_1) \}$, which has a present value of

$$\delta \left(B[a, q(a)] - \max \{ \delta M, B_1(a_1, \hat{\phi}_1) \} \right),$$

or quit and receive the status quo benefit, $B_2(a_2, \hat{\phi}_2)$. Agent 2 will not accept any offer that gives her less than

$$\max \left\{ \delta \left(B[a, q(a)] - \max \{ \delta M, B_1(a_1, \hat{\phi}_1) \} \right), B_2(a_2, \hat{\phi}_2) \right\}.$$

Hence, agent 1 receives at most

$$B[a, q(a)] - \max \delta \left\{ \left(B[a, q(a)] - \max \{ \delta M, B_1(a_1, \hat{\phi}_1) \} \right), B_2(a_2, \hat{\phi}_2) \right\}$$

which is, in fact, the supremum of the payoffs he receives in the subgame beginning from that point. However, the game at τ is identical to the game at $\tau - 2$ apart from the discounting of all payoffs by the factor δ^2 . Hence

$$M = B[a, q(a)] - \max \left\{ \delta \left(B[a, q(a)] - \max \{ \delta M, B_1(a_1, \hat{\phi}_1) \} \right), B_2(a_2, \hat{\phi}_2) \right\}. \quad (1.6)$$

If

$$\max \{ \delta M, B_1(a_1, \hat{\phi}_1) \} = \delta M$$

and

$$\max \{ \delta (B[a, q(a)] - \delta M), B_2(a_2, \hat{\phi}_2) \} = \delta (B[a, q(a)] - \delta M),$$

then nobody's outside option is binding and agent 1 receives $B_1^1 = \frac{B[a, q(a)]}{1+\delta}$ in the equilibrium. If

$$\max \{ \delta M, B_1(a_1, \hat{\phi}_1) \} = B_1(a_1, \hat{\phi}_1)$$

and

$$\max \left\{ \delta \left(B[a, q(a)] - B_1(a_1, \hat{\phi}_1) \right), B_2(a_2, \hat{\phi}_2) \right\} = \delta \left(B[a, q(a)] - B_1(a_1, \hat{\phi}_1) \right),$$

then only agent 1's outside option is binding and he receives $B_1^1 = (1 - \delta) B[a, q(a)] + \delta B_1(a_1, \hat{\phi}_1)$. If

$$\max \left\{ \delta \left(B[a, q(a)] - \max \left\{ \delta M, B_1(a_1, \hat{\phi}_1) \right\} \right), B_2(a_2, \hat{\phi}_2) \right\} = B_2(a_2, \hat{\phi}_2)$$

then agent 2's outside option is binding and agent 1 receives $B_1^1 = B[a, q(a)] - B_2(a_2, \hat{\phi}_2)$. ■

The solution takes a simple form in the limit. If we change the time interval between successive offers from 1 to Δ and replace the discount factor δ by δ^Δ and take the limit as Δ goes to zero we obtain

$$B_1^1 = \begin{cases} \frac{B[a, q(a)]}{2} & \text{if } B_1(a_1, \hat{\phi}_1), B_2(a_2, \hat{\phi}_2) \leq \frac{B[a, q(a)]}{2} \\ B_1(a_1, \hat{\phi}_1) & \text{if } B_1(a_1, \hat{\phi}_1) > \frac{B[a, q(a)]}{2} \\ B[a, q(a)] - B_2(a_2, \hat{\phi}_2) & \text{otherwise} \end{cases} \quad (1.7)$$

1.8.2 Appendix B

Lemma 10 *If $C_i(0, 0) > B_i(0, \hat{\phi}_i)$ and*

$$\max_{\phi_i} \frac{\partial B_i(a_i, \phi_i)}{\partial a_i} < 2 \min_{\phi_i} \frac{\partial B_i(a_i, \phi_i)}{\partial a_i} \quad (1.8)$$

for every $a_i \in A_i$, then there exist a monotonically increasing function $\alpha_i : A_j \rightarrow A_i$ such that

- i- j 's outside option is binding if $a_i \leq \alpha_j^{-1}(a_j)$,*
- ii- neither outside option is binding if $\alpha_j^{-1}(a_j) \leq a_i \leq \alpha_i(a_j)$,*
- iii- i 's outside option is binding if $\alpha_i(a_j) < a_i$.*

Proof. The lemma is proven for the case of $i = 1$, and it is symmetric for the case $j = 1$. It is first shown that condition 1.8 implies that for every a_2 , there exist a $\delta_1(a_2)$ and a unique a_1 , such that,

$$\frac{1}{2} \frac{\partial B_1(a_1, \phi_1^c)}{\partial a_1} + \delta_1(a_2) \leq \frac{\partial B_1(a_1, \hat{\phi}_1)}{\partial a_1} \quad (1.9)$$

where ϕ_1^c is the value of function ϕ evaluated at the cooperative choices. Note that ϕ_1^c is a function of q_1 and q_2 .

We define $M^* = \max_{\phi_1} \frac{\partial B_1(a_1, \phi_1)}{\partial a_1}$ and $m^* = \min_{\phi_1} \frac{\partial B_1(a_1, \phi_1)}{\partial a_1}$. Then by definition $\frac{\partial B_1(a_1, \phi_1^c)}{\partial a_1} \leq M^*$ and $\frac{\partial B_1(a_1, \hat{\phi}_1)}{\partial a_1} \geq m^*$. We have assumed that $\frac{1}{2}M^* < m^*$. One can find a sufficiently small $\delta_1(a_2)$ for each $\phi_1^c(a_1, a_2)$ such that $\frac{1}{2}M^* + \delta_1(a_2) < m^*$. Then by substitution we obtain 1.9.

Next we show that condition 1.9 implies that for every a_2 there exist a unique a_1 such that $B_1(a_1, \hat{\phi}_1) = C_1(a_1, a_2)$. We define $D(a_1, a_2) = C_1(a_1, a_2) - B_1(a_1, \hat{\phi}_1)$. By 1.9, $D(a_1, a_2)$ is a monotonically decreasing function of a_1 and a_2 and $D(0, a_2) > 0$. We show that there exists a sufficiently large a_1 for which $D(a_1, a_2) < 0$. We rewrite $B_1(a_1, \hat{\phi}_1)$ as

$$B_1(a_1, \hat{\phi}_1) = B_1(0, \hat{\phi}_1) + \int_0^{a_1} \frac{\partial B_1(a_1, \hat{\phi}_1)}{\partial a_1} da_1.$$

Substituting 1.9, we obtain

$$B_1(a_1, \hat{\phi}_1) \geq B_1(0, \hat{\phi}_1) + \int_0^{a_1} \left[\frac{\partial C_1(a_1, a_2)}{\partial a_1} + \delta_i(a_j) \right] da_1.$$

which can be rewritten as

$$B_1(a_1, \hat{\phi}_1) \geq B_1(0, \hat{\phi}_1) - C_1(0, a_2) + C_1(a_1, a_2) + \delta_i(a_j) a_1.$$

If

$$B_1(0, \hat{\phi}_1) - C_1(0, a_2) + \delta_i(a_j) a_1 > 0$$

then

$$B_1(a_1, \hat{\phi}_1) > C_1(a_1, a_2).$$

Thus we find an $a_1^H > \frac{B_1(0, \hat{\phi}_1) - C_1(0, a_2)}{\delta}$, such that, $D(a_1^H, a_2) < 0$. Using the intermediate value theorem, there exist a point $a_i^* \in [0, a_1^H]$ such that $D(a_i^*, a_2) = 0$. It is unique since $D(a_1, a_2)$ is monotonically decreasing for all $a_i \in A_i$.

Having shown the existence of a unique a_i^* for all a_2 , we define a function $\alpha_1 : A_2 \rightarrow A_1$ such that

$$B_1(\alpha_1(a_2), \hat{\phi}_1) = C_1(\alpha_1(a_2), a_2) \quad (1.10)$$

By 1.9, $\frac{\partial C_1(a_1, a_2)}{\partial a_1} - \frac{\partial B_1(a_1, \hat{\phi}_1)}{\partial a_1} \neq 0$, hence we can apply the implicit function theorem.

Differentiating both sides of 1.10 with respect to a_2 we obtain

$$\frac{\partial \alpha_1(a_2)}{\partial a_2} = - \frac{\frac{1}{2} \frac{\partial B_2(a_2, \phi_2(q_1(a), q_2(a)))}{\partial a_2}}{\frac{1}{2} \frac{\partial B_1(a_1, \phi_1(q_1(a), q_2(a)))}{\partial a_1} - \frac{\partial B_1(a_1, \hat{\phi}_1)}{\partial a_1}}$$

which is strictly greater than zero, since the numerator is positive and the denominator is negative.

The existence of $\alpha_2(a_1)$ can be shown in a similar manner. Since it is a monotonic function, its inverse, $\alpha_2^{-1}(a_2)$, is a well defined function. By definition, $D(\alpha_1(a_2), a_2) > 0$ if $a_1 > \alpha_1(a_2)$, hence agent 1 maximizes $B_1(a_1, \hat{\phi}_1) - v_1(a_1)$. For $a_1 < \alpha_1(a_2)$, agent 1's outside option is not binding and for $a_1 > \alpha_2^{-1}(a_2)$, agent 2's outside option is also not binding. Thus, agent 1 receives $C_1(a_1, a_2) - v_1(a_1)$. For $a_1 \leq \alpha_2^{-1}(a_2)$, agent 2's outside option binds, therefore agent 1 claims the residual and receives $H_1(a_1, a_2) - v_1(a_1)$. ■

1.8.3 Appendix C

Lemma 6.2. *As γ increases, \hat{a}_i , $\delta_i(a_j)$, $\alpha_i(a_j)$ and $\epsilon_i(a_j)$ increase.*

Proof. 1- \widehat{a}_i increases as γ increases.

$$\frac{\partial \widehat{a}_i(\gamma)}{\partial \gamma} = - \frac{\frac{\partial^2 B_i(a_i, \widehat{\phi}_i)}{\partial a_i \partial \phi_i} \frac{\partial \widehat{\phi}_i}{\partial \gamma}}{\frac{\partial^2 B_i(a_i, \widehat{\phi}_i)}{\partial a_i^2} - \frac{\partial^2 v_i(a_i)}{\partial a_i^2}} > 0$$

The numerator is positive if $\frac{\partial \widehat{\phi}_i}{\partial \gamma}$ is positive. $\frac{\partial \widehat{\phi}_i}{\partial \gamma}$ is positive if the firms' activities are complementary.

2- $\delta_i(a_j)$ increases as γ increases.

$$\frac{\partial \delta_i(a_j, \gamma)}{\partial \gamma} = - \frac{\frac{1}{2} \frac{\partial^2 B_i(a_i, \phi_i^c)}{\partial a_i \partial \phi_i} \frac{\partial \phi_i^c}{\partial \gamma}}{\frac{1}{2} \frac{\partial^2 B_i(a_i, \phi_i^c)}{\partial a_i^2} - \frac{\partial^2 v_i(a_i)}{\partial a_i^2}} > 0$$

The denominator is negative and $\frac{\partial \phi_i^c}{\partial \gamma}$ is positive if $\frac{\partial q_i}{\partial \gamma}$ is positive.

3- $\alpha_i(a_j)$ increases as γ increases.

As complementarity increases, the outside option becomes less binding.

$$\frac{\partial \alpha_i(a_j)}{\partial \gamma} = - \frac{\frac{\partial B_i(a_i, \widehat{\phi}_i)}{\partial \phi_i} \frac{\partial \widehat{\phi}_i}{\partial \gamma} - \frac{1}{2} \frac{\partial B_i(a_i, \phi_i^c)}{\partial \phi_i} \frac{\partial \phi_i^c}{\partial \gamma} - \frac{1}{2} \frac{\partial B_j(a_j, \phi_j^c)}{\partial \phi_j} \frac{\partial \phi_j^c}{\partial \gamma}}{\frac{\partial B_i(a_i, \widehat{\phi}_i)}{\partial a_i} - \frac{1}{2} \frac{\partial B_i(a_i, \phi_i^c)}{\partial a_i}} > 0$$

The denominator is positive by the assumption in Lemma 2 and the numerator is positive if $\frac{\partial B_i(a_i, \widehat{\phi}_i)}{\partial \gamma} < \frac{\partial C_i(a_1, a_2)}{\partial \gamma}$. This condition implies that as the complementarity between the two firm's production increases, the "split-the-surplus" payoff increases by more than the non-cooperative payoff.

4- $\epsilon_i(a_j)$ increases as γ increases.

$$\frac{\partial \epsilon_i(a_j, \gamma)}{\partial \gamma} = - \frac{\frac{\partial^2 B_i(a_i, \phi_i^c)}{\partial a_i \partial \phi_i} \frac{\partial \phi_i^c}{\partial \gamma}}{\frac{\partial^2 B_i(a_i, \phi_i^c)}{\partial a_i^2} - \frac{\partial^2 v_i(a_i)}{\partial a_i^2}} > 0$$

The numerator is positive if $\frac{\partial q_i}{\partial \gamma}$ is positive and the denominator is negative. ■

Chapter 2

The Employment Relationship versus Independent Contracting: On the Organizational Choice and Incentives

2.1 Introduction

In the provision of intermediate inputs or services, a firm can either hire labor to produce within the firm, or subcontract with another firm to deliver the finished product. In many contexts, the choice is determined by the relative cost of these two types of transaction. These costs include not only the cost of hiring, firing, and training, but also the transaction costs associated with bargaining, contracting, and monitoring performance. The transaction costs are zero if the agents are fully informed and the contracts are complete and enforceable. In this case, the organi-

zation of production is irrelevant since efficiency can be achieved in both cases. If, however, the transaction takes place in an imperfect environment, the transaction costs will differ depending on the organizational structure. As it has been argued by Coase [10] and Williamson [40], different organizational forms emerge in the market economy in order to minimize these costs. Similarly, when a firm decides whether to organize production as an employment relationship or as independent contracting, it considers the transaction costs associated with each of them.

An organizational structure is a set of rules that govern a relationship. Each organization adopts different rules which, in turn, influence in a different manner incentives of the agents. In this paper, we study two ways of organizing production: in-house production, which we refer to as the employment relationship, and independent contracting. These two organizational structures differ in terms of the allocation of ownership rights over physical assets (Grossman and Hart [15], Klein, Crawford and Alchian [21]), the monitoring instruments used in the relationship (Khalil and Lawarrée [22]), and the compensation (Alchian and Demsetz [2], and Holmstrom [17]).¹ However, the fundamental distinction between the employment relationship and independent contracting is the allocation of residual control rights over production. As noted by Coase [10], and Simon [36], in the employment relationship, the employer has the authority to direct the activities of the employee. This observation has often been criticized on the grounds that the sources of this authority remain unexplained.² However, Masten [26] argues that there is a clear

¹Holmstrom and Milgrom [18] provide an analysis of how these choices are intertwined in the firm's decision.

²For example, Alchian and Demsetz [2], and Jensen and Meckling [23] disagree with the view that the firm has superiority in terms of authority. The former argues that transactions are organized within a firm as a result of technological inseparabilities which require team production. The latter views the firm as the nexus of contractual relationship, rejecting any advantages or limitations that arise from internal organization.

difference in the legal treatment of the employment relationship and the independent contracting because of the allocation of the authority among the parties. On legal grounds, these two types of transaction are perceived as being different in terms of obligations, sanctions, and procedures governing the exchange.³ In this paper, we analyze how differences in the allocation of authority influence the firm's choice between the employment relationship and independent contracting.

The use of the authority in the employment relationship is redundant in an environment where complete contracts can be written. Since the initial contract specifies the obligations of each party in every conceivable state of the nature, there is no need for ex-post interventions in the relationship. We assume, however, that writing comprehensive contracts is not feasible due to high transaction costs (see Coase [10], Klein, Alchian and Crawford [21], and Williamson [40]). Thus, when an unexpected contingency arises the initial contract must specify what is to be done. One way to accomplish this is by assigning residual control rights to the parties (as in Grossman and Hart [15]) in the initial contract. Alternatively, the firm⁴ can choose either the employment relationship, so that she retains the residual control rights, or independent contracting, so that she forgoes these rights which are given to the contractor.⁵

³The following quote from Masten ([26], p.158) supports this view:

Upon entering an employment relationship, for example, every employee accepts an implied duty to "yield obedience to all reasonable rules, orders, and instructions of the employer"...

... whereas, "an 'independent' contractor is generally defined as one who in rendering services exercises an independent employment or occupation and represents his employer only as a results of his work and not as the means whereby it is to be done"...

⁴Throughout the paper we will refer to the firm/principal as "she" and the employer/contractor/agent as "he".

⁵In this paper we are not, modelling the ex-post bargaining problem. We assume that the firm

The model is an extension of the classical principal-agent model. The firm (principal) contracts with a risk-neutral agent to carry out a project. The principal cannot observe the agent's action, which can be thought of, as his exerted effort level. Therefore, she has to offer him an incentive contract to induce him to choose the efficient level of effort. After the agent chooses an effort level, both the principal and the agent receive private signals of the project's future profitability. It is under the discretion of the party with the control rights to take an action conditional upon the signal he/she observes. These actions not only affect the distribution of profits but also impose a non-pecuniary cost to the agent. We assume that neither the signal nor the action is contractible. Therefore, the initial contract only specifies how the production is organized. In other words, if the employment relationship is chosen, the principal has the right to decide the second stage action based on the private signal she receives. On the other hand, if independent contracting is chosen, the agent makes this decision based on his information. The principal's problem in an employment relationship is to design a contract that will induce the agent to exert the optimal effort level in the first stage. In independent contracting, she also wants to align the agent's incentives regarding the second stage action with her incentives.

In addition to the moral hazard problem, in the employment relationship, there also exist a commitment problem. Since the principal cannot commit ex-ante to a second stage decision, she may have to compensate the agent for the unexpected intervention. The firm can economize on the up-front cost by hiring an independent contractor. If there is no informational asymmetry over the signals received, then the principal prefers independent contracting over the employment

has all the bargaining power, hence in the equilibrium of the bargaining game, the firm pays the worker his reservation utility and receives the residual.

relationship. Independent contracting can be viewed as a commitment device that ensures that in the future the principal will not intervene in the production. As it has been discussed earlier by Williamson [40], the inability of the parties to intervene selectively may be the cause of organizing production in the market rather than internally. When we introduce an informational asymmetry, and in particular, as the agent's information is inferior to the principal's information, the benefits from having residual control rights outweighs the cost of compensating the agent. Thus, the employment relationship is the preferred organizational form.

Our model improves the employment relationship model developed by Simon [36], by adding the moral hazard problem. Simon [35] compares the employment relationship, where the employer has the flexibility to postpone decisions regarding production until after the uncertainty is resolved, to contingent contracting. His model, however, ignores the moral hazard problem that may exist in the employment relationship. Even though the principal has given the authority to direct the agent's actions in the employment relationship, some dimensions of his actions, such as the effort he exerts, cannot be monitored and therefore cannot be contracted upon. We use a principal-agent model to describe the employment relationship in order to emphasize the impact of the contractual incompleteness that exists in relationships involving human capital. Our model is also related to Grossman and Hart [15]. Their paper examines the relationship between the two firms and the allocation of residual control rights over physical assets when there is contractual incompleteness. Our paper can be viewed as an application of the incomplete contracts framework to an employment relationship. While they study the role of *ownership* over physical assets, we study the role of *authority* over human assets. We define authority as the residual control rights over the production process and analyze its implications

in a principal-agent setting. Their model focuses on the hold-up problem, and consequently on the distortions that arise in relationship-specific investments, in an environment where contracts are incomplete in every respect. In this model we focus on the contracting problems when there is partial incompleteness.

The remainder of the paper is organized as follows: section 2.2 presents the basic model. The pareto efficient contract is analyzed in section 2.2.1, the employment relationship is presented in section 2.2.2 and the independent contracting is presented in section 2.2.3. Section 2.3 analyzes the model where normality of all random variables is assumed. The pareto optimal contract is analyzed in section 2.3.1. In section 2.3.2 the employment relationship is discussed for two cases; when the principal can commit to an intervention rule and when she cannot. Section 2.3.3 presents the optimal contract in independent contracting. The organizational forms are compared in section 2.4. Conclusions are presented in section 2.5.

2.2 The Basic Model

We consider a principal-agent relationship in which the principal contracts with the agent to carry out a one-time project. The project generates profit π which is partly determined by the agent's actions. After they sign a contract the agent chooses his effort level which is assumed to take two values $e \in \{e_L, e_H\}$ with $0 < e_L < e_H$. The choice of e determines the distribution of π which is given by the (twice differentiable) cumulative distribution function $F(\pi | e)$. Before profits realized, both parties privately observe a noisy signal s_i of profit where i stands for the principal, P , or the agent, A . The distribution of s_i is also determined by e and given by the function $G(s_i | e)$.

After the private signals are received, the party with control rights chooses an action A which affects the distribution of future profits. In the simplest form we can assume that there are two possible actions available to the party with residual control rights; “intervene” and “not intervene”, i.e.: $A = \{I, NI\}$. Intervention, which can be in the form of partial liquidation of firm’s assets, reorganization of production or redirection of the project, reduces the project’s risk. Let $f_A(\pi | s_i, e)$ denote the conditional probability density function of profits when action A is chosen. We make the following assumption:

Assumption 8 ⁶ For each $s \in \mathfrak{R}$, there exists $\hat{\pi}(s)$ such that

$$F_I(\pi | s, e) < F_{NI}(\pi | s, e) \quad \text{for} \quad \pi(s) < \hat{\pi}(s)$$

and

$$F_I(\pi | s, e) > F_{NI}(\pi | s, e) \quad \text{for} \quad \pi(s) > \hat{\pi}(s)$$

The assumption implies that for each signal s , intervening is safer than not intervening, which has fatter lower and upper tails.

We assume that none of the variables, e , s or A are contractible. The non-contractibility of e requires the principal to offer an incentive contract to the agent in order to induce him to exert high levels of effort. This contract can only be written contingent on the verifiable realized profits. Moreover, the non-contractibility of s and A necessitates the allocation of residual control rights in the initial contract, which in turn, determines the organizational form chosen.⁷ In the employment rela-

⁶ See Dewatripont and Tirole [12].

⁷ Notice that if either s or A were is contractible then we can either write the initial contract contingent upon the signal observed or the action taken. In other words, if s is contractible, it is feasible for the principal to offer a contract in the following form. The agent is paid a linear compensation and the cost of intervention δ , if s is less than a particular cutoff point. Otherwise he will only receive the linear compensation. On the other hand if s is not contractible but A is, then we can made the additional payment of δ contingent upon the action I being chosen.

2.2.1 Pareto Optimal Contract

Before examining the optimal incentive scheme under different organizational forms, we first examine the Pareto optimal contract under full information and perfect commitment. In this case, the principal observes the action the agent is taking and ex-ante commits to an intervention rule. Thus, the principal maximizes her net expected profits subject to the agent's participation constraint.

$$\begin{aligned} \max_{I, w(\pi)} \quad & \int_I \int_{\pi} [\pi - w(\pi)] f_I(\pi | s, e_H) g(s | e_H) d\pi ds \\ & + \int_I \int_{\pi} [\pi - w(\pi)] f_{NI}(\pi | s, e_H) g(s | e_H) d\pi ds \end{aligned}$$

subject to

$$\begin{aligned} & \int_I \int_{\pi} w(\pi) f_I(\pi | s, e_H) g(s | e_H) d\pi ds \\ & + \int_I \int_{\pi} w(\pi) f_{NI}(\pi | s, e_H) g(s | e_H) d\pi ds \\ & - v(e_H) - \delta \int_I g(s | e_H) ds \geq U_0. \end{aligned}$$

The optimal contract is a fixed wage contract since the action is observable and verifiable. From the individual rationality constraint, $w = U_0 + v(e_H) + \delta \int_I g(s | e_H) ds$ and the program can be written as

$$\max_I \int \{ [F_{NI}(\pi | s, e_H) - F_I(\pi | s, e_H)] d\pi - \delta \} g(s | e_H) ds - U_0 - v(e_H).$$

The first term in brackets is the monetary gain from intervention and $\delta \int_I g(s | e_H) ds$ is the cost of intervention. The principal chooses an intervention rule that maximizes the net gain from intervention.

Lemma 11 : *If*

1. $\frac{\partial}{\partial s} [\int [F_{NI}(\pi | s, e_H) - F_I(\pi | s, e_H)] d\pi] \leq 0$ for all s and π and
2. $\exists \hat{s}$ such that $E_I[\pi | \hat{s}, e_H] = E_{NI}[\pi | \hat{s}, e_H]$

then the optimal intervention rule is a cut-off rule.⁸

Proof. From condition (2) we have

$$\int [F_{NI}(\pi | \hat{s}, e_H) - F_I(\pi | \hat{s}, e_H)] d\pi = 0.$$

Together with the condition (1) it follows that

$$\begin{aligned} \int [F_{NI}(\pi | s, e_H) - F_I(\pi | s, e_H)] d\pi &\geq 0 \quad \text{for } s < \hat{s} \\ &< 0 \quad \text{for } s > \hat{s}. \end{aligned}$$

Because of the monotonicity assumption, there exists s^* that is smaller than \hat{s} and satisfies

$$F_{NI}(\pi | s^*, e_H) - F_I(\pi | s^*, e_H) - \delta = 0$$

Thus, the set signals in which an intervention occurs is $I = \{s | s < s^*\}$ and the optimal intervention rule is a cut-off rule. We assume that, given the optimal intervention rule s^* , it is socially efficient to implement high effort levels for the agent.

Note that the optimal contract can also be written contingent on the intervention. In other words it is feasible to write a contract that promises to pay the agent different amounts depending on whether or not the intervention takes place. Given this contract, the optimal intervention rule is the same as before. The agent is paid $U_0 + v(e_H)$ if there is no intervention and $U_0 + v(e_H) + \delta$ if there is an intervention. .

2.2.2 Employment Relationship

In the employment relationship, the party in control is the principal. She wants to maximize her expected profits subject to the agent's individual rationality and the

⁸As a convention, we use indefinite integral when integral is taken over the entire domain of a variable.

incentive compatibility constraint. This case is more complicated than the simple principal-agent problem. Each contract that the principal proposes to the agent induces a subgame in which the agent chooses an action and the principal decides whether to intervene or not. The game is one of imperfect information. At the time the principal decides whether to intervene or not, she does not know which action the agent has taken. The principal solves the following program.

$$\begin{aligned}
\max_{w(\pi)} E\Pi &= \int_{-\infty}^{s^{ER}} \int_{\pi} [\pi - w(\pi)] f_I(\pi | s, e_H) g(s | e_H) d\pi ds \\
&\quad + \int_{s^{ER}}^{\infty} \int_{\pi} [\pi - w(\pi)] f_{NI}(\pi | s, e_H) g(s | e_H) d\pi ds \\
s.t \quad EW(e_H, s^{ER}) - v(e_H) &\geq U_0 \quad IR \quad (2.1) \\
EW(e_H, s^{ER}) - v(e_H) &\geq EW(e_L, s^{ER}) - v(e_L) \quad IC
\end{aligned}$$

where

$$\begin{aligned}
EW(e_i, s) &= \int_{-\infty}^{s^{ER}} \int_{\pi} w(\pi) f_I(\pi | s, e_i) g(s | e_i) d\pi ds + \\
&\quad \int_{s^{ER}}^{\infty} \int_{\pi} w(\pi) f_{NI}(\pi | s, e_i) g(s | e_i) d\pi ds
\end{aligned}$$

is the expected compensation to the agent when he exerts effort j where $j = H, L$ and

$$s^{ER} = \inf_s \left\{ s \mid \int \pi f_I(\pi | s, e) d\pi \geq \int \pi f_{NI}(\pi | s, e) d\pi \right\}.$$

According to Lemma 11, s^{ER} exists.

The principal maximizes the expected net profits subject to agent's individual rationality constraint, the incentive compatibility constraint, and the principal's ex

post intervention rule. The principal's ex-post intervention rule states that it is optimal to intervene when the principal receives a signal which is lower than s^{ER} . For the signals that are greater than s^{ER} , the principal does not intervene since the expected profits are lower when he intervenes than when he does not.

2.2.3 Independent Contracting

In this case the control rights are given to the agent. The agent's decision rule constitutes an additional constraint to the classical principal-agent problem. The program is

$$\begin{aligned}
\max_{w(\pi)} E\Pi &= \int_{-\infty}^{s^{ER}} \int_{\pi} [\pi - w(\pi)] f_I(\pi | s, e_H) g(s | e_H) d\pi ds \\
&\quad + \int_{s^{ER}}^{\infty} \int_{\pi} [\pi - w(\pi)] f_{NI}(\pi | s, e_H) g(s | e_H) d\pi ds \\
s.t \quad &EW(e_H, s^{ER}) - v(e_H) \geq U_0 \quad IR \\
&EW(e_H, s^{ER}) - v(e_H) \geq EW(e_L, s^{ER}) - v(e_L) \quad IC \\
s^{IC} &= \arg \max \int w(\pi) [f_{DI}(\pi | s, e_H) - f_I(\pi | s, e_H)] d\pi - v(e) - \\
&\quad \delta \int_{-\infty}^{s^{IC}} g(s | e) ds
\end{aligned} \tag{2.2}$$

In this program, in addition to the incentive compatibility and the individual rationality constraints, the principal's problem is also constrained by the agent's ex-post intervention rule (the third constraint). When deciding whether or not to intervene, the agent compares his net earnings in each event. The cost of interven-

tion, that is borne by the agent, is thus internalized in independent contracting.

In the next section we present an example where the contracts are linear. We examine how the solution to the simple principal-agent model changes with the introduction of a non-contractible action, and how the allocation of control rights influences the optimal contracts.

2.3 Example

Let $v(e_L) = 0$ and $v(e_H) = K > 0$. Consider a linear compensation scheme for the agent in the form of $w(\pi) = \alpha\pi + \beta$ where $\alpha \in [0, 1]$ and $\beta \in \mathfrak{R}$. The distribution of π , conditional on e is assumed to be a normal with mean “ e ” and variance “ e^2 ”. Therefore “low effort” generates low, but safer profits, while “high effort” generates high, but riskier profits. The principal observes the signal $s_P = \pi + \varepsilon$ and the agent observes $s_A = \pi + \varepsilon + \eta$, where the noise term ε is assumed to be normally distributed with mean zero and variance σ_ε^2 , and η is assumed to be normally distributed with mean zero and variance σ_η^2 . We set up the model in such a way that the agent’s information is a garbling of the principal’s information. After the signal is observed, the party in control decides whether or not to intervene. For simplicity, intervention is assumed to scale down profits by a factor λ , where $\lambda \in [0, 1]$. Hence, both the expected profitability, and the riskiness of the project are reduced after intervention.

Given the marginal distributions of π and s conditional on e , the distribution of π conditional on s and e is derived using Bayes’ rule, and it is

$$(\pi \mid s_i, e) \sim N \left(\frac{e\sigma_i^2 + e^2s_i}{e^2 + \sigma_i^2}, e^2\sigma_i^2 \right)$$

where $i = P, A$ and $\sigma_P^2 = \sigma_\varepsilon^2$ and $\sigma_A^2 = \sigma_\varepsilon^2 + \sigma_\eta^2$. Note that the conditional mean of profits, given the signal, is a convex combination of e and s . As the noise term

increases, the signal becomes uninformative about future profits, and the conditional mean of profits approaches the unconditional mean, e . As the noise becomes smaller, the signal becomes informative, and the conditional mean approaches the realized profits. The following lemma states that the values of s , for which the intervention takes place, is strictly lower-tailed.

Lemma 12 *The optimal intervention rule is a cut-off rule.*

Proof. It is sufficient to examine whether the conditions of Lemma 11 are satisfied. The derivative of $\int [F_{NI}(\pi | s_i, e_H) - F_I(\pi | s_i, e_H)] d\pi$ with respect to s is $(\lambda - 1) \left(\frac{e\sigma_i^2 + e^2 s_i}{e^2 + \sigma_i^2} \right)$ which is negative since $\lambda \in [0, 1]$. There exists \hat{s}_i which is equal to $-\frac{\sigma_i^2}{e}$. Therefore the intervention rule is a cut-off rule. ■

2.3.1 Pareto Optimal Contract

The first-best is achieved if the principal has perfect information and there is no a commitment problem. In this environment, the pareto optimal contract is obtained by maximizing the principal's expected net profits subject to the agent's participation constraint. Since the principal's signal is a sufficient statistic for the agent's signal, the beliefs about the distribution of profits is updated by the principal's signal, s_P . Let

$$E(\pi | s_P, e_H) = e_H - (1 - \lambda) \int_{-\infty}^{s_P} \left(\frac{e\sigma_P^2 + e^2 s_P}{e^2 + \sigma_P^2} \right) g(s_P | e_H) d\pi ds$$

denote the expected profits when the effort level e_H is chosen by the agent and the cut-off rule for intervention is s_P . The first term is the unconditional expected profit and the second term, which in the future we will denote as $l(s_{PH}, e_H)$, is the difference in the expected profits due to the intervention. The value of $l(s_{PH}, e_H)$

is negative for $s_P < -\frac{\sigma_e^2}{e}$, therefore, it can be interpreted as the expected losses recovered by intervention. The Pareto optimal contract is generated by the program:

$$\begin{aligned} & \max_{\alpha, \beta, s_P} (1 - \alpha) E(\pi | s_P, e_H) - \beta \\ \text{s.to } & \alpha E(\pi | s_P, e_H) + \beta - K - \delta G(s_P | e_H) \geq w_0 \end{aligned}$$

where w_0 is the agent's expected outside wage. The following lemma provides a condition under which there exists an optimal contract that implements e_H and the first-best intervention rule.

Lemma 13 *The Pareto optimal intervention rule is*

$$D(s_P^*, e) = \begin{cases} \text{intervene if} & s_P \leq s_P^* \\ \text{do not intervene if} & s_P > s_P^* \end{cases}.$$

$$\text{where } s_P^* = -\frac{\delta(e_H^2 + \sigma_e^2)}{(1-\lambda)e_H^2} - \frac{\sigma_e^2}{e_H}$$

The agent's individual rationality constraint is binding, which gives us the total compensation the agent is paid, $w(\pi)$. Substituting $w(\pi)$ into the maximand and solving for s_P , yields s_P^* , the cut-off point for the optimal intervention rule. The following proposition describes the optimal contracts.

Proposition 14 *Suppose that*

$$e_H - (1-\lambda)l(s_H^*, e_H) - K - \delta G(s_{PH}^*, e_H) \geq e_L - (1-\lambda)l(s_{PL}^*, e_L) - \delta G(s_{PL}^*, e_L) \quad (2.3)$$

holds. There exists a continuum of first-best incentive schemes (α, β) that implement e_H , such that, the agent is paid $w^(\pi) \equiv w_0 + K + \delta \int_{-\infty}^{s_{PH}^*} g(s | a_H) ds = \alpha E(\pi | s_{PH}^*, e_H) + \beta$.*

Condition (2.3) implies that it is socially desirable to choose “high effort”.⁹ The optimal contract pays the agent $w^*(\pi)$ which is the sum of his reservation wage, the disutility from exerting high levels of effort, and the expected cost of intervention. The fixed payment contract where

$$\alpha = 0 \text{ and } \beta = w_0 + K + \delta \int_{-\infty}^{s_{PH}^*} g(s | a_H) ds$$

is one of the solutions to the problem. With full information there is no moral hazard problem. A contract that pays $w^*(\pi)$ to the agent, if he exerts high effort, can be implemented. Since there is also no commitment problem, the allocation of residual control rights is irrelevant. The optimal intervention rule that the intervention will take place when a signal $s_{PH} \leq s_{PH}^*$ is observed, can be specified in the initial contract.

2.3.2 Employment Relationship

In an employment relationship, the principal has the residual control rights over production and decides whether or not to intervene depending on the signal, s_P , she receives. There are two problems that cause the employment relationship model to deviate from the pareto optimal case. First, there is a moral hazard problem, due to the unobservability of the agent’s actions by the principal. Therefore, the principal has to offer an incentive payment scheme to the agent. Second, there is a commitment problem, due to non-contractibility of intervention. In order to correctly identify the sources of deviations from the first-best solution correctly we solve the problem in two stages, adding one friction at a time.

⁹In a simple principal-agent model, condition (2.3) corresponds to the condition that $e_H - e_L \geq K$.

Principal can commit to an intervention rule

We first assume that the principal can commit to an intervention rule. In other words, we assume that s is ex-ante contractible. Then the optimal contract solves the following program:

$$\begin{aligned}
 & \max_{\alpha, \beta, s} \quad (1 - \alpha) E(\pi \mid s_P, e_H) - \beta \\
 & \text{subject to} \\
 & \quad \alpha E(\pi \mid s_P, e_H) + \beta - K - \delta G(s \mid a_H) \geq w_0 \\
 & \text{and} \\
 & \quad \alpha E(\pi \mid s_P, e_H) + \beta - K - \delta G(s \mid a_H) \geq \\
 & \quad \alpha E(\pi \mid s_P, e_H) + \beta - \delta G(s \mid a_L)
 \end{aligned}$$

Proposition 15 *If it is socially optimal to implement e_H (i.e. condition (2.3) holds), then there exists a continuum of first best incentive schemes (α, β) that implement (e_H, s_{PH}^*) , such that, the agent is paid a total compensation of $w^*(\pi)$ and $\alpha \in \left[\frac{K + \delta [G(s_{PH}^*, e_H) - G(s_{PH}^*, e_L)]}{e_H - e_L - (1 - \lambda) [l(s_{PH}^*, e_H) - l(s_{PH}^*, e_L)]}, 1 \right]$.*

Since the principal cannot observe the effort level of the agent, she has to offer him an incentive contract. It is a well known result in principal-agent theory, that when the agent is risk neutral, making the agent residual claimant is an optimal solution. Since s is assumed to be contractible, there is no commitment problem. Giving the agent residual claimancy with a fee of $F = E\Pi(s_{PH}^*, e_H) - w^*(\pi)$ (i.e.: $\alpha = 1$ and $\beta = w^*(\pi) - E\Pi(s_{PH}^*, e_H) < 0$ which is in fact the payment to the principal) is one of the optimal solutions to the program. Again the individual rationality constraint is binding. The incentive compatibility constraint is not binding since the principal can adjust β accordingly as long as α is greater than the lower bound. There is a constraint on the values that α is allowed to take in order for $\alpha \in (0, 1)$ to

exist. This constraint which is derived from the incentive compatibility constraint is

$$e_H - (1 - \lambda)l(s_{PH}^*, e_H) - K - \delta G(s_{PH}^*, e_H) \geq e_L - (1 - \lambda)l(s_{PH}^*, e_L) - \delta G(s_{PH}^*, e_L). \quad (2.4)$$

Condition (2.3) is sufficient for condition (2.4) to hold (see Appendix). Therefore, as long as e_H is efficient level of effort, there exists a linear contract that would induce the agent to choose e_H .

Principal cannot commit to an intervention rule

Now we consider the case where the principal cannot commit ex-ante to an intervention rule contingent on s_P . After the principal observes the signal s_P , she updates her belief about the distribution of profits π , and decides whether or not to intervene in the project. The expected value of profits, given the signal is, $E[\pi | s_P, e] = \frac{e\sigma_e^2 + e^2 s_P}{e^2 + \sigma_e^2}$ which is a convex combination of e and s_P . Since the contract has already been signed, $w(\pi)$, the compensation to the agent, is a sunk cost from principal's point of view. Therefore, when she decides whether or not to intervene she is concerned only about the overall expected profits. Solving $E[\pi | s_P, e] = 0$, yields $s_P^{ER} = -\frac{\sigma_e^2}{e}$ as the cut-off point for intervention. Then, the principal's decision rule is

$$D(s_P^{ER}, e) = \begin{cases} \text{intervene if} & s_P \leq s_P^{ER} \\ \text{do not intervene if} & s_P > s_P^{ER} \end{cases}.$$

It is worthwhile to note that $s_P^{ER} > s_P^*$. Thus, if the principal cannot commit ex ante to an intervention rule, she intervenes more often than the socially optimal rule.

At the beginning of the game, when the principal offers a contract to the agent

both parties will take the principal's ex-post intervention rule into consideration. As we discussed earlier, every contract induces a subgame between the principal and the agent, in which the agent chooses an action, and the principal decides whether or not to intervene. The equilibria of these subgames are reflected in the principal's problem which is as follows:

$$\begin{aligned}
& \max_{\alpha, \beta} \quad (1 - \alpha) E \left(\pi \mid s_{PH}^{ER}, e_H \right) - \beta \\
& \text{subject to} \\
& \quad \alpha E \left(\pi \mid s_{PH}^{ER}, e_H \right) + \beta - K - \delta G \left(s_{PH}^{ER} \mid e_H \right) \geq w_0 \\
& \text{and} \\
& \quad \alpha E \left(\pi \mid s_{PH}^{ER}, e_H \right) + \beta - K - \delta G \left(s_{PH}^{ER} \mid e_H \right) \geq \\
& \quad \alpha E \left(\pi \mid s_{PH}^{ER}, e_H \right) + \beta - \delta G \left(s_{PH}^{ER} \mid e_L \right)
\end{aligned}$$

The solution to this problem is presented in the following proposition.

Proposition 16 *If*

$$e_H - (1 - \lambda) l \left(s_{PH}^{ER}, e_H \right) - K - \delta G \left(s_{PH}^{ER}, e_H \right) \geq e_L - (1 - \lambda) l \left(s_{PL}^{ER}, e_{PL} \right) - G \left(s_{PL}^{ER}, e_{PL} \right) \quad (2.5)$$

holds, then there exist a continuum of linear contracts (α, β) that implement (s_{PH}^{ER}, e_H) , such that the agent is paid the total compensation of $w^{ER}(\pi) = w_0 + K + \delta G \left(s_{PH}^{ER} \mid e_H \right)$ and $\alpha \in \left[\frac{K + \delta [G(s_{PH}^{ER}, e_H) - G(s_{PH}^{ER}, e_L)]}{e_H - e_L - (1 - \lambda) [l(s_{PH}^{ER}, e_H) - l(s_{PH}^{ER}, e_L)]}, 1 \right)$.

Condition (2.5) states that e_H is the principal's preferred action under her optimal intervention rule. Condition (2.5) is a sufficient but not a necessary condition for condition (2.3). In other words, e_H may not be an optimal action for the principal, even if it is socially optimal. The lower bound for α which is derived from

the incentive compatibility constraint, requires that

$$e_H - (1 - \lambda)l(s_{PH}^{ER}, e_H) - K - \delta G(s_{PH}^{ER}, e_H) \geq e_L - (1 - \lambda)l(s_{PH}^{ER}, e_L) - \delta G(s_{PH}^{ER}, e_L) \quad (2.6)$$

holds. The condition (2.5) is sufficient for the condition (2.6). In other words, there exists a linear contract that would implements e_H , if it is the principal's preferred effort level.

Note that we deliberately excluded the case of $\alpha = 1$ from the solution set. If the agent becomes residual claimant, the principal's incentive to interfere in the project is distorted, since she gets a fixed rent from the agent in every state. Then the problem is reduced to a simple principal-agent problem without intervention. In this case, the expected value of the principal's payoff is $e_H - w_0 - K$ as opposed to $e_H - (1 - \lambda)l(s_{PH}^{ER}, e_H) - K - w_0 - \delta G(s_{PH}^{ER}, e_H)$ which she would have received if $\alpha < 1$. We will assume that $-(1 - \lambda)l(s_{PH}^{ER}, e_H) \geq \delta G(s_{PH}^{ER}, e_H)$, that is intervention provides positive gains. Therefore, the principal prefers to set $\alpha < 1$ and intervene in the project.

Corollary 17 *If a linear contract exist that implements (s_{PH}^{ER}, e_H) , then the expected payment to the agent is higher, and the principal's net surplus is lower, in the non-commitment case than in the commitment case.*

In both cases the agent's compensation is the sum of his outside wage, w_0 , the disutility from exerting high levels of effort, K , and the expected cost of intervention, $\delta G(s_{PH}, e_H)$. Since the cut-off point for intervention is greater in the case of non-commitment the expected costs are higher and the worker is paid a higher compensation. The principal pays a premium to the agent in the non-commitment case because she intervenes more often.

The principal's net surplus, $e_H - (1 - \lambda)l(s_{PH}, e_H) - K - \delta G(s_{PH}, e_H) - w_0$, is increasing in the values of the signal s that are less than s_{PH}^* . Since $s_{PH}^* < s_{PH}^{ER}$, her net surplus is greater under s_{PH}^* than s_{PH}^{ER} .

2.3.3 Independent Contracting

In the case of independent contracting, the principal has no further role after the contract is signed. We first characterize the optimal intervention rule after the agent observes his signal. The agent observes a signal s_A , which is noisier than the principal's signal. After observing his signal, the agent decides whether or not to intervene taking into account his expected compensation rather than the project's expected profits. Having the residual control rights, the agent trades off the cost of intervention with its benefit. The agent will intervene in the project if the expected compensation after intervention is greater than the one without intervention,

$$\alpha \lambda E(\pi | s_A, e) + \beta - K - \delta \geq \alpha E(\pi | s_A, e) + \beta - K.$$

Solving the above for s_A yields $s_A^{IC} = -\frac{\sigma_\varepsilon^2 + \sigma_\eta^2}{e} - \frac{\delta(\sigma_\varepsilon^2 + \sigma_\eta^2 + e^2)}{e^2 \alpha (1 - \lambda)}$, as the cut-off for intervention. The agent's decision rule is

$$D(s_A^{IC}; e) = \begin{cases} \text{intervene if} & s_A \leq s_A^{IC} \\ \text{do not intervene if} & s_A > s_A^{IC} \end{cases}. \quad (2.7)$$

When designing an incentive scheme, the principal takes into account the agent's optimal decision rule. The optimal contract not only induces the agent to choose e_H but also aligns his incentives to intervene with hers. The optimal contract is

generated by the following program:

$$\begin{aligned}
& \max_{\alpha, \beta} \quad (1 - \alpha) E(\pi | s_{AH}^{IC}, e_H) - \beta \\
& \text{subject to} \\
& \quad \alpha E(\pi | s_{AH}^{IC}, e_H) + \beta - K - \delta G(s_{AH}^{IC} | e_H) \geq w_0 \\
& \text{and} \\
& \quad \alpha E(\pi | s_{AH}^{IC}, e_H) + \beta - K - \delta G(s_{AH}^{IC} | e_H) \geq \\
& \quad \alpha E(\pi | s_{AH}^{IC}, e_H) + \beta - \delta G(s_{AH}^{IC} | e_L)
\end{aligned}$$

The following lemma provides the solution to this program.

Lemma 18 *If it is socially optimal to implement e_H (i.e.: condition (2.3) holds), then there exists an optimal incentive scheme that makes the agent the residual claimant and implements (e_H, s_{AH}^{IC}) . This contract is given by $\alpha = 1$ and $\beta = w^*(\pi) - E(\pi | s_{AH}^{IC}, e_H)$.*

Proof. See Appendix. ■

Note that s_H^{IC} depends on α . The choice of α will not only induce the agent to exert high levels of effort but it will also influence the agent's decision to intervene. If only linear contracts are available, then there does not exist a contract that perfectly aligns the agent's incentives with the principal's. In other words, there does not exist an $\alpha \in [0, 1]$ that equalizes s_{AH}^{IC} with s_{AH}^{ER} .

2.4 Comparison of the Two Organizational Form

In any organizational form, the critical value of s , which determines the intervention rule, is decreasing in σ^2 , the variance of the noise term in the signal. As the signal becomes noisier, the probability of intervention goes down. This reduces the

expected cost of intervention and also increases the losses recovered by intervention. Thus, the net benefit from intervention goes up. In the model, we assume that the principal and the agent observe different signals. In particular, the agent's signal is a garbling of the principal's signal. As a benchmark, we now consider the case in which both the principal and the agent receive the same signal before the actual profits are realized. This is a special case of the model where σ_η^2 , the additional noise term in the agent's signal equals zero. Then, the optimal contract in independent contracting is Pareto efficient, since the cut-off point for the agent's optimal intervention rule becomes s_{PH}^* which is the first best intervention rule and the optimal contract implements (e_H, s_{PH}^*) . Given Lemma (17), the principal prefers independent contracting over the employment relationship as the organizational form, since in the former her net surplus is greater. In independent contracting the principal saves on the up-front payment to the agent which she would have to pay in the employment relationship.

As the agent's signal becomes noisier, intervention in independent contracting is inefficient. The agent intervenes less than the optimal level which results with "underinvestment". Even though he is the residual claimant under the optimal contract, his incentives are distorted because his information is noisier than the principal's information. As the variance of the signal increases, the losses from underinvestment outweighs the gains from the compensation paid to the worker, and the principal finds the employment relationship more desirable. The following proposition summarizes this result.

Proposition 19 *When $\sigma_\eta^2 = 0$, the principal prefers independent contracting over the employment relationship. In this case, independent contracting is also Pareto efficient. For small values of σ_η^2 , independent contracting continues to dominate the*

employment relationship. As the agent's signal becomes noisier, the employment relationship is the preferred organizational form.

Proof. Let $B(s_P^{ER}, e_H)$ be equal to

$$-(1 - \lambda)l(s_P^{ER}, e_H) - \delta G(s_P^{ER}, e_H),$$

the net benefit from intervention under the employment relationship and let $A(s_A^{IC}, e_H)$ be the corresponding function under independent contracting. When $\sigma_\eta^2 = 0$, both the principal and the agent observe the same signal. From the result of the lemma 18 the independent contracting implements the first best. From the result of the lemma 17, the net surplus of the principal is higher under independent contracting than under the employment relationship, thus $A(s_A^{IC}, e_H) > B(s_P^{ER}, e_H)$.

We show in the appendix D that as σ_η^2 increases $A(s_A^{IC}, e_H)$ decreases while $B(s_P^{ER}, e_H)$ stays constant. As σ_η^2 approaches ∞ , $A(s_A^{IC}, e_H)$ approaches to 0 from right. Thus, the principal's expected profits under independent contracting, $e_H - K + A(s_A^{IC}, e_H)$ is bounded away from $e_H - K$. As σ_ϵ^2 approaches ∞ , $B(s_P^{ER}, e_H)$ also approaches 0. We can find σ_ϵ^2 which is sufficiently small so that $B(s_P^{ER}, e_H)$ is greater than 0. Therefore for each values of σ_ϵ^2 there exists $\bar{\sigma}_\eta^2$ such that $A(s_A^{IC}, e_H) = B(s_P^{ER}, e_H)$ and for $\sigma_\eta^2 > \bar{\sigma}_\eta^2$ $A(s_A^{IC}, e_H) < B(s_P^{ER}, e_H)$. In other words, if the agent's signal is very noisy, then the principal's profits under employment relationship is greater than her profits under independent contracting.

■

If the principal's signal is perfectly informative, i.e.: $\sigma_\epsilon^2 = 0$, then for small σ_η^2 , the independent contracting continues to be the principal's preferred organizational form. When $\sigma_\epsilon^2 = 0$, the principal intervenes whenever the signal received is less than 0. However the optimal intervention rule trades off the benefit from intervention

with its cost in the margin. The efficient intervention rule proposes that intervention takes place when $s \leq -\frac{\delta}{1-\lambda}$. When the agent's information is not very noisy, the intervention rule under independent contracting is closer to the efficient intervention rule than the intervention rule under the employment relationship. As σ_η^2 becomes larger, however, the agent intervenes very infrequently so that the principal prefers the employment relationship.

2.5 Concluding Remarks

We study a firm's decision to choose between employing a worker and using an independent contractor to carry out a task. We analyze this problem using a two-stage principal-agent model. We derive conditions under which an optimal contract, which implements high effort level, e_H , of the worker, exists. When we restrict the set of feasible contracts to those that are linear in profits, e_H can be implemented under independent contracting, as long as it is socially efficient. In the employment relationship, however, the linear contracts that implements e_H exist only for certain parameter values of the model. The intervention decision remains inefficient under both organizational structures. The inefficiency in the employment relationship arises because the principal cannot commit *ex ante* to an intervention rule. When she makes the second stage decision, she does not take into account the costs incurred by the agent as a result of her intervention. Thus, in the employment relationship, there is "too much" intervention. Even though the commitment problem is avoided in independent contracting by delegating the intervention decision to the agent, there is "too little" intervention. The distortion in the agent's intervention decision is created by the agents's inferior information.

When both parties receive the same signal, thus, there is no informational asymmetry, independent contracting is Pareto efficient organizational form. The optimal contract implements both the first-best effort level and the intervention rule. The principal prefers independent contracting because she receives higher net profits. As the signal of the agent becomes noisier, the agent's intervention rule becomes more distorted and the cost-saving advantages of the independent contracting dissipate. Even if the principal's signal is perfectly informative about the profitability of the project, for small noise in the agent's signal, the principal finds independent contracting more desirable than the employment relationship. In the model, we assume that the agent's information is worse than the principal's information. If the agent possesses better information, then independent contracting always Pareto dominates the employment relationship. These results support the empirical evidence presented by Masten [27] who examines the firm's integration decision with the upstream firm in the aerospace industry. He finds that the specificity of the component is a detrimental factor in this decision. As the component becomes more specific, the firm prefers in-house production to independent contracting. The specificity of the component can be interpreted in our model as the principal having superior information about the project.

In this model we assume that the players observe their signal privately and there is no communication between them. In the employment relationship the principal makes the second stage decision. Since the principal's signal is a sufficient statistic for the agent's signal, communication does not improve efficiency. In independent contracting, however, the agent makes the second stage decision based on his signal which is noisier than the principal's signal. In fact the main source of inefficiency in independent contracting is that the agent has inferior information.

A possible extension to the model above would be to allow communication in independent contracting. With communication, the principal announces the signal she observes and based on that announcement the agent decides whether or not to intervene. If the principal's announcement is verifiable, then the outcome of the employment relationship can be replicated in independent contracting. The fact that the principal can write a contract that is contingent on her announcement avoids the non-contractibility problem. This, in turn, eliminates the need for the allocation of residual control rights. If the principal's announcement, however, is not contractible, a contract that is contingent on the announcement cannot be implemented. When the principal announces the signal she observes, she also takes into account how the agent's incentives are affected by this announcement. In particular, the cost of compensating the agent, when information is revealed, may exceed the benefits. Then, the principal may find it more desirable not to announce her information. The complications that may arise in the model is similar to the problem of incentive contracting with informed principal which was originally introduced by Myerson [30].

2.6 Appendices

2.6.1 Appendix A

Claim: C1 implies C2.

Proof. Rewriting C1 gives

$$e_H - e_L \geq K + (1 - \lambda)l(s_H^*, e_H) + \delta G(s_H^*, e_H) - (1 - \lambda)l(s_L^*, e_L) - \delta G(s_L^*, e_L)$$

and rewriting C2 gives

$$e_H - e_L \geq K + (1 - \lambda)l(s_H^*, e_H) + \delta G(s_H^*, e_H) - (1 - \lambda)l(s_H^*, e_L) - \delta G(s_H^*, e_L)$$

If $(1 - \lambda)l(s_H^*, e_L) + \delta G(s_H^*, e_L) > (1 - \lambda)l(s_L^*, e_L) + \delta G(s_L^*, e_L)$, then C1 implies C2. Since $s_H^* > s_L^*$, it is sufficient to prove that $(1 - \lambda)l(s, e) + \delta G(s, e)$ is increasing in s since $s_H^* > s_L^*$. Taking derivatives with respect to s yields $(1 - \lambda) \left(\frac{e\sigma^2 + e^2s}{e^2 + \sigma^2} \right)$ which is positive for $s > s^*$. ■

2.6.2 Appendix B

Given $\pi | e \sim N(e, e^2)$, $\varepsilon | e \sim N(0, \sigma_\varepsilon^2)$ and $s_P = \pi + \varepsilon$, first we will derive the pdf of $\pi | s_P, ae$.

Using Bayes rule $h(\pi | s, e) = \frac{f(\pi, s|e)}{g(s|e)}$ and assuming that $\text{cov}(\pi, \varepsilon | e) = 0$,

$$f(\pi, s | e) = \frac{1}{\sqrt{2\pi e}} \exp \left\{ -\frac{1}{2} \left(\frac{\pi - e}{e} \right)^2 \right\} \cdot \frac{1}{\sqrt{2\pi\sigma_\varepsilon}} \exp \left\{ -\frac{1}{2} \frac{(s - \pi)^2}{\sigma_\varepsilon^2} \right\} \quad (2.8)$$

and

$$\begin{aligned} g(s | e) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi e}} \exp \left\{ -\frac{1}{2} \left(\frac{\pi - e}{e} \right)^2 \right\} \cdot \frac{1}{\sqrt{2\pi\sigma_\varepsilon}} \exp \left\{ -\frac{1}{2} \left(\frac{s - \pi}{\sigma_\varepsilon} \right)^2 \right\} d\pi \\ &= \frac{1}{\sqrt{2\pi} (\sigma_\varepsilon^2 + e^2)^{\frac{1}{2}}} \exp \left\{ -\frac{1}{2} \frac{(s - e)^2}{\sigma_\varepsilon^2 + e^2} \right\} \end{aligned} \quad (2.9)$$

then

$$\begin{aligned} h(\pi | s, e) &= \frac{\frac{1}{\sqrt{2\pi e}} \exp \left\{ -\frac{1}{2} \left(\frac{\pi - e}{e} \right)^2 \right\} \cdot \frac{1}{\sqrt{2\pi\sigma_\varepsilon}} \exp \left\{ -\frac{1}{2} \left(\frac{s - \pi}{\sigma_\varepsilon} \right)^2 \right\}}{\frac{1}{\sqrt{2\pi} (\sigma_\varepsilon^2 + e^2)^{\frac{1}{2}}} \exp \left\{ -\frac{1}{2} \frac{(s - e)^2}{\sigma_\varepsilon^2 + e^2} \right\}} \\ &= \frac{(\sigma_\varepsilon^2 + e^2)^{\frac{1}{2}}}{\sqrt{2\pi e\sigma_\varepsilon}} \exp \left\{ -\frac{1}{2} (\pi - e)^2 \frac{\sigma_\varepsilon^2 + e^2}{(e\sigma_\varepsilon)^2} \right\} \end{aligned} \quad (2.10)$$

where $b = \frac{e\sigma_\pi^2 + se^2}{\sigma_\pi^2 + e^2}$ is the conditional mean of profits given the signal. Rewriting $b = \left(\frac{\sigma_\pi^2}{\sigma_\pi^2 + e^2}\right) e + \left(\frac{e^2}{\sigma_\pi^2 + e^2}\right) s$, we see that it is a convex combination of $e = E[\pi]$ and s . As $\sigma_\pi^2 \rightarrow \infty$, the signal becomes uninformative and $b \rightarrow e$ while as $\sigma_\pi^2 \rightarrow 0$, the signal becomes informative and $b \rightarrow \pi$.

2.6.3 Appendix C

Let μ and γ be the multipliers of the individual rationality and incentive compatibility conditions respectively. The first order conditions are

$$\begin{aligned} \frac{\partial L}{\partial \alpha} &= E\left(\pi \mid s_{AH}^{IC}, e_H\right) \frac{\partial E(\pi \mid s_{AH}^{IC}, e_H)}{\partial \alpha} (\mu + \gamma - 1) - \\ &\quad \left(\delta \frac{\partial G(s_{AH}^{IC}, e_H)}{\partial \alpha}\right) (\mu + \gamma) + \\ \gamma \left[E\left(\pi \mid s_{AL}^{IC}, e_L\right) \frac{\partial E(\pi \mid s_{AL}^{IC}, e_L)}{\partial \alpha} - \delta \frac{\partial G(s_{AL}^{IC}, e_L)}{\partial \alpha} \right] &\geq 0 \quad \text{if } \alpha = 0 \\ &= 0 \quad \text{if } \alpha \in (0, 1) \\ &\leq 0 \quad \text{if } \alpha = 1 \end{aligned}$$

and

$$\frac{\partial L}{\partial \beta} = -1 - \mu = 0$$

Substituting $\mu = 1$ into $\frac{\partial L}{\partial \alpha}$ and setting $\eta = 0$ (assuming that incentive compatibility constraint is not binding) yields

$$\frac{\partial L}{\partial \alpha} = -\delta \left(\frac{\delta (e_H^2 + \sigma_\eta^2 + \sigma_\epsilon^2)}{(1 - \lambda) e_H^2} \right) g(s_H^{IC}, e_H)$$

which is negative, hence $\alpha = 1$. When $\alpha = 1$, $s_{PH}^{IC} = -\frac{\sigma_\epsilon^2 + \sigma_\eta^2}{e} - \frac{\delta(\sigma_\epsilon^2 + \sigma_\eta^2 + e^2)}{e^2(1-\lambda)}$ which is less than s_{PH}^* . Therefore the optimal contract in independent contracting is not Pareto efficient.

2.6.4 Appendix D

Let

$$B(s_P^{ER}, e_H) = -(1 - \lambda)l(s_P^{ER}, e_H) - \delta G(s_P^{ER}, e_H),$$

the net benefit from intervention under the employment relationship and

$$A(s_A^{IC}, e_H) = -(1 - \lambda)l(s_A^{IC}, e_H) - \delta G(s_A^{IC}, e_H)$$

be the net benefit from intervention under independent contracting. We first substitute the values of s_P^{ER} and s_A^{IC} , and then rewrite the integrals by replacing s with

$$z = \frac{s - e}{e^2 + \sigma^2}.$$

Then we obtain

$$B(e_H, \sigma_\varepsilon^2) = - \int^{z^{ER}} \left((1 - \lambda) \left(e + \frac{e^2 z}{(e^2 + \sigma_\varepsilon^2)^{\frac{1}{2}}} \right) + \delta \right) f(z) dz$$

and

$$A(e_H, \sigma_\varepsilon^2, \sigma_\eta^2) = - \int^{z^{IC}} \left((1 - \lambda) \left(e + \frac{e^2 z}{(e^2 + \sigma_\varepsilon^2 + \sigma_\eta^2)^{\frac{1}{2}}} \right) + \delta \right) f(z) dz$$

Taking the derivative of $A(e_H, \sigma_\varepsilon^2, \sigma_\eta^2)$ with respect to σ_η^2 we obtain

$$\frac{\partial A(e_H, \sigma_\varepsilon^2, \sigma_\eta^2)}{\partial \sigma_\eta^2} = - \int^{z^{IC}} \left(\frac{e^2 z}{2(e^2 + \sigma_\varepsilon^2 + \sigma_\eta^2)^{\frac{3}{2}}} \right) f(z) dz$$

which is negative. Therefore as σ_η^2 increases the net benefit from intervention in independent contracting decreases.

Chapter 3

Firm-Specific Skills, Wage Bargaining, and Efficiency

3.1 Introduction

Employment contracts are inherently incomplete. In a typical employment contract the worker agrees to carry out the instructions of the employer, within broad limits, in return of a prespecified wage. In the absence of comprehensive contracts, productive efficiency requires that successive adaptations to the changing job and market conditions take place. In the implementation of these adaptations, parties may find it profitable to bargain ex-post over the terms of the contract within the contract period as well as in the contract renewal stage. If the labor market is competitive, the ex-post bargaining between the firm and its employees results in an efficient allocation, as the firm can replace its employees costlessly without any disruption in production. However, most jobs involve non-trivial firm-specific skills and information which develop during the course of the worker's employment. Employees

such as, high level managers, sales representatives, key product engineers, and blue-collar workers in production teams possess firm-specific human capital, and the firm cannot replace them with new inexperienced workers at the spot labor market. Although the firm's initial hiring decision takes place in a competitive labor market, once the worker's skills are developed as a result of experience, the employment relationship resembles a bilateral monopoly. Therefore, in the ex-post bargaining game the hold-up problem may arise.¹ This, in turn, may create inefficiencies both ex-ante and ex-post.

In this paper, we study the bargaining relationship between a firm and its incumbent worker who possesses firm-specific human capital. We show that, in the contract renewal stage, the worker's ability to strategically disclose his skills increases his bargaining power vis-à-vis the firm. The firm can threaten to fire the worker and hire a new inexperienced worker, but this threat is not always credible. Even though the bargaining game takes place in an environment with perfect information, there exist inefficient equilibria in which delays occur in real time. The wage bargaining between the firm and its skilled workers results in ex-post inefficiency in the production. This supports the arguments in Williamson et. al. [41] that sequential spot contracting in the labor market is not efficient when firm-specific human capital is important.²

The specialized skills and information which we call firm-specific human

¹There is an extensive literature on the hold-up problem in bilateral relationships and its remedies. Among these, Grossman and Hart [15] studied the incentives to invest in relationship-specific investment when there is contractual incompleteness. Rogerson [33], Chung [?], MacLeod and Malcomson [24] are among those who studied the contractual solutions to ex-ante inefficiency that is created by ex-post hold-up problem.

²Instead, hierarchical organization of labor such as internal labor markets promotes efficiency by avoiding individual bargaining. The internal labor markets paradigm which is pioneered by Doeringer and Piore [13], argues that there are hierarchical career structure within a firm. The wages are attached to jobs rather than workers which eliminates the inefficient bargaining between the firm and its incumbent workers.

capital, develop either as a part of on-the-job training or accrue naturally during the course of employment. In most jobs, especially those involving “idiosyncratic tasks”³ the firm-specific human capital is an important input for the firm. Familiarity with the physical environment (Doeringer and Piore [13]), customer relationships (Anderson and Schmittlein [3]), the ability to communicate and work effectively with the members of a team (Mailath and Postlewaite [25], and Klein [20]) are examples of firm-specific human capital. When the firm-specific skills develop as a result of on-the-job training that is, an investment in human capital, the possibility of ex-post bargaining creates both ex-ante and ex-post inefficiencies. The ex-ante inefficiency arises because the parties’ incentives to invest in specific human capital are distorted.⁴ The ex-post inefficiency arises because the worker may strategically disclose his specialized skills during the ex-post bargaining. In this model we focus on the firm-specific human capital that accrues naturally to the worker during the course of his employment without a significant cost to either him or the firm. In this way, we isolate the effects of the ex-ante investment decisions and study only the ex-post inefficiencies that may arise in the relationship.

It is a well known result in the bargaining literature that if there is an informational asymmetry between the negotiating parties, then in equilibrium delays occur in real time (for example, see Admati and Perry [1]). In these models, the delay serves as a signalling device. Recently, the works of Fernandez and Glazer [14], Haller and Holden [16], and Busch and Wen [8] show that delays can also be observed in equilibrium in bargaining games with perfect information. The alternating offers

³Williamson et. al. [41] discusses the underlying factors that give rise to job idiosyncracies and the efficiency implications of alternative organizational frameworks in which idiosyncratic exchange can be accomodated.

⁴Becker [5] is among the first who considered incentives to invest in specific human capital. For a theoretical model of how ex-post bargaining creates distortions in parties’ incentives to invest, see Grossman and Hart [15] and Mumcu [28]

bargaining game with constant disagreement payoffs has unique equilibrium. If the disagreement payoffs are endogenous, that is, if the value of disagreement payoffs depend on the actions taken by players in each period, then the players' offers in the bargaining game depend not only on the past rejected offers, but also on the actions taken in the disagreement stage. As in repeated games, multiple equilibria exist because the history-dependent strategies can be used to punish the players for deviations from the proposed equilibrium actions, thus deterring deviations. If the game has multiple equilibria, inefficient equilibrium where delays occur can easily be constructed.

In this model, the firm and the incumbent worker bargain over the total surplus. The bargaining game takes place concurrently with the production. If the firm and the worker do not reach an agreement, production takes place and the worker is paid according to the initial contract. The worker can either exert high effort levels and produce the maximum feasible output or shirk, thus, produce less than the maximum. Since the worker does not bear any disutility if he shirks, he receives a higher payoff if he shirks during every period when an agreement is not reached. In fact, by committing to shirk in every period, the worker guarantees himself the highest equilibrium payoff. If, however, the total output is sufficiently large, the firm can support the action "not shirk" in the disagreement game by compensating the worker's loss from choosing this action by offering him a higher wage in the next period. If the worker deviates from his prescribed equilibrium strategy, he is punished in the next period as the firm proposes a smaller wage. By using history-dependent strategies, we show that the game has many equilibria some of which are inefficient. In inefficient equilibria the agreement is delayed and the worker shirks in every period up to agreement.

This paper contributes to the literature on non-Walrasian wage bargaining where the existence of wage differentials in labor market is attributed to the higher bargaining power of insiders compared to outsiders. In these models, however, the way in which the worker's firm-specific skills increases his bargaining power is not explicitly modelled. In Shaked and Sutton [34] this bargaining power is characterized by the firm's inability to replace its current workforce on the spot. The firm's current workforce enjoys a bargaining advantage because it is time-consuming for the firm to replace them. During the wage bargaining, the firm bargains with the incumbent worker for a number of periods before it makes an offer to an outsider. The game has a unique equilibrium. If the time period during which the firm is forced to bargain with the insider decreases, the equilibrium approaches a Walrasian solution. If the time period increases, then the outsider does not represent a threat to the insider and the equilibrium is similar to the one in bilateral monopoly. Recently, Stole and Zwiebel [37], [38] developed a model of intra-firm bargaining between the firm and its skilled employees in order to explain the firm's input and organizational design decisions. In their model of intra-firm bargaining, the firm has many employees but it bargains with each one individually. The worker's bargaining power stems from his threat to quit. This threat is credible insofar as it deprives the firm from the worker's contribution, thus, weakens its position against the remaining workers. The bargaining game has a unique subgame-perfect equilibrium. An extension of this model is studied in Wollinsky [42] where the firm has the opportunity to replace the existing workers.

All of these models of bargaining between the firm and the incumbent workers assume that the production takes place after a new agreement is reached. In contrast, in our model the intra-firm bargaining game takes place concurrently with

production. The worker's decision in the disagreement stage involves how much effort to exert. If the worker chooses to strike in the disagreement stage, hence, produce nothing, both players' disagreement payoffs are zero. In this sense the intra-firm bargaining models described in these papers can be seen as a special case of ours.

The paper is organized as follows. Section 3.2 presents the model. Section 3.2.1 solves for the equilibrium of the bargaining game when the firm does not have an outside option. Section 3.2.2 presents the bargaining game when the firm can exercise its outside option after rejecting an offer and section 3.3 concludes.

3.2 The Model

In this model, a firm is randomly matched with a worker from a competitive market of identical workers. They sign a contract that specifies the wage that the agent will be paid for a day's work. This wage is equal to the competitive wage which is denoted w_0 . This relationship produces an amount of output that is normalized to 1 in each production period. We assume that initially the worker is unskilled and he does not need to exert effort to perform the job. However, as the worker continues to work in the same firm, his productivity increases as he acquires firm-specific human capital. He develops these skills without exerting any effort. We assume that, after some time, the worker is able to produce $A > 1$ units of output when he combines his firm-specific skills with high levels of effort. High effort levels imposes disutility c to the worker. Although the incumbent worker was drawn from a pool of identical workers before the initial contract was signed, he gradually becomes more productive than the outsiders. Hence the employment relationship resembles a

bilateral monopoly. Once the initial contract expires, the worker can negotiate with the firm to raise his wage above the competitive wage.⁵ The incumbent worker's ability to increase his salary depends on his ability to strategically disclose his firm-specific skills during the contract negotiation.

We characterize the wage negotiation as an alternating offers bargaining game between the firm and the incumbent worker which takes place concurrently with the production. The structure of the game is as follows. In each odd-numbered period the worker proposes a new wage contract x_t . The firm then responds by either accepting or rejecting the offer. If the firm accepts the offer, the negotiation game ends. In the new wage contract the worker receives the average payoff x_t , and the firm receives the average payoff $A - x_t$, thereafter. If the firm rejects the offer, then the players receive their disagreement payoffs which depend on the actions taken by each player. The firm faces a choice between hiring a new unskilled worker from the competitive market, or continuing to bargain with the incumbent worker. In the former case, the firm obtains the average payoff $1 - w_0$. We assume that if the incumbent worker is fired, he earns the competitive wage, w_0 , elsewhere. If the firm chooses not to fire the incumbent worker following a rejection, the worker is paid w_0 during that period. The worker can either work hard and produce the output A , or shirk and produce $1 - \varepsilon$. If the worker works hard he incurs disutility c in monetary terms. Therefore, his utility when he works is $w_0 - c$. If he shirks he does not expend any effort, hence his utility is w_0 .⁶ The worker's decision is observed by the firm and time advances one period.

⁵If there is no breach penalties, the worker could also ask for the raise before the initial contract has expired.

⁶Since the unskilled worker does not choose his effort level, the initial contract does not specify payments contingent on effort. Once the worker becomes skilled, he can choose whether or not to work hard. Regardless of his decision, however he is paid w_0 .

In every even-numbered period, the firm offers a wage contract, y_t , to the worker. The worker then responds by either accepting or rejecting the offer. The acceptance of the offer implements a binding contract between the firm and the agent that holds forever. If the worker rejects the offer, then the worker chooses between shirking and not shirking.⁷ The same rules as described above govern the consequences of these decisions. We assume that

$$A - c > 1 - \varepsilon \tag{3.1}$$

This implies that agreement is strictly preferred to disagreement. We also assume that

$$A > w_0 + c. \tag{3.2}$$

This condition implies that the total output is sufficiently large so that the firm can afford to pay the worker his disutility of work above the competitive wage. If this condition does not hold, then the production is not efficient and an agreement between the incumbent worker and the firm is never reached. In the unique equilibrium of the game, the firm quits the bargaining game and hires a new worker. Both the firm and the worker have the same discount factor $0 < \delta < 1$. The worker's objective is to maximize the discounted sum of net earnings,

$$\sum_{t=0}^{\infty} \delta^t w_t$$

and the firm's objective is to maximize the discounted sum of profits,

$$\sum_{t=0}^{\infty} \delta^t (z_t - w_t)$$

⁷Note that we only allow the firm to opt out after responding an offer to simplify the analysis. If the firm can opt out in every period then the game has multiple equilibria also when the firm's outside option is binding.

where $z_t = A$ if the firm continues to bargain or reaches an agreement with the incumbent worker, and $z_t = 1$ if the firm hires a new worker.

We study the subgame-perfect equilibria of the game described above. Subgame-perfect equilibrium strategies induce a Nash equilibrium in every proper subgame. This game has four typical subgames: at the beginning of each period t when a player makes an offer, right after a proposal is made, right after a rejection and right after the firm decides whether to stay with the incumbent worker. We assume that each player observes every past action. A strategy for player i , where i stands for either the worker, w , or the firm, f , is a function σ_i , which assigns an appropriate action to every possible history.

The model has the characteristics of a repeated game in an alternating offers bargaining game. In a simple bargaining model (Rubinstein [32]), the equilibrium strategies are a function of only past proposals and rejections. In this model, the players have a richer set of actions. The strategies are also a function of whether or not the worker has shirked in the past. Therefore, the firm can use strategies, such as punishing the worker if he shirked in any of the previous periods or compensating him in the next period if he has not shirked. When such reward and punishment mechanisms are available to the players, the game, in general, has multiple equilibria and also inefficient equilibria.

3.2.1 Firm has no outside option

We call G_0 , the game where the firm has no outside option. In the G_0 game, the disagreement payoffs are determined solely by the actions of the worker. We consider a subgame following a rejection. In this subgame, shirking yields to the worker a higher payoff in that period. If the worker commits himself to shirking in

every period following a rejection, then the game resembles a Rubinstein game with disagreement payoffs $(w_0, 1 - \varepsilon - w_0)$. There exists a unique equilibrium where the worker receives $\bar{w} = w_0 + \frac{A-1+\varepsilon}{1+\delta} + \frac{\delta c}{1+\delta}$, the firm receives $A - \bar{w}$, and the agreement is reached in the first period. The firm immediately accepts the wage proposal \bar{w} . If she does not accept, she receives $1 - \varepsilon - w_0$ this period and $A - \hat{w}$ from the next period onward which is equal to the average payoff $A - \bar{w}$. If an offer is rejected, the worker does not deviate from shirking, since “not shirking” yields a higher payoff in that period, and the continuation strategies are not affected by a deviation.

The wage contract \bar{w} is not the only equilibrium of the game. Given the strategy profile above, the firm obtains $A - w_0$ if the worker does not shirk in the disagreement stage. This is not attainable as “not shirk” is a suboptimal action for the worker. If, however, the strategy profile is changed in such a way so that the continuation payoff to the worker depends on the action he chooses in the disagreement stage, the firm can increase her payoff in the disagreement stage. By obtaining a higher payoff in the disagreement stage, the firm can also obtain a higher equilibrium payoff from the game by inducing the worker to choose not to shirk. If in the next period the firm compensates the worker for the loss generated by not shirking by proposing a higher share so that the worker’s discounted payoff remains the same, he is indifferent between shirking and not shirking. In other words, if the firm offers to the worker y_{t+1} if he shirked in period t , and $y_{t+1} + \frac{(1-\delta)c}{\delta}$ if he did not shirk, the worker is indifferent between shirking and not shirking in period t following rejection. Since he is indifferent, he chooses a suboptimal action, “not shirk”, in the disagreement stage. The firm is willing to offer this additional payment because her net average payoff if the worker does not shirk, $(A - w_0 - c)(1 - \delta) + \delta y_{t+1}$, is greater than her average payoff if the worker shirks, $(1 - \varepsilon - w_0)(1 - \delta) + \delta$

y_{t+1} . This is true since $A - c > 1 - \varepsilon$ by assumption in 3.1. If we rewrite assumption 3.1 as

$$A - 1 + \varepsilon > c$$

the gains from “not shirking” exceeds the costs. Therefore, it is feasible for the firm to support the action “not shirk” in the disagreement stage. Given the player’s highest and lowest disagreement payoffs, we describe the equilibrium of the game in the following proposition.

Proposition 20 *Any wage contract w such that,*

$$w_0 + c \leq w \leq w_0 + \frac{A - 1 + \varepsilon}{1 + \delta} + \frac{\delta c}{1 + \delta}$$

can be generated as an equilibrium wage contract with agreement reached in the first period.

Proof. The formal definition of the equilibrium strategies are presented in the Appendix. We also prove that these strategies are subgame-perfect and generate $\bar{w} = w_0 + \frac{A-1+\varepsilon}{1+\delta} + \frac{\delta c}{1+\delta}$ as the maximum wage and $\underline{w} = w_0 + c$ as the minimum wage the worker can obtain in equilibrium. ■

Note that \bar{w} and \underline{w} are the lowest and highest wages the worker can obtain. His lowest utility is w_0 , which is equal to his reservation utility. His highest utility is $w_0 - \frac{c}{1+\delta} + \frac{A-1+\varepsilon}{1+\delta}$, which is greater than w_0 under assumption 3.1.

The subgame-perfect equilibrium strategies that generate \bar{w} as an equilibrium are the following. The worker’s strategy is to shirk after every rejection, offer \bar{w} in every odd-numbered period, and in every even-numbered period accept an offer y_t such that, $y_t \geq \hat{w}$, where $\hat{w} = w_0 + \frac{c}{1+\delta} + \frac{\delta(A-1+\varepsilon)}{1+\delta}w$, and reject otherwise. The firm’s

corresponding subgame-perfect equilibrium strategy is to propose \hat{w} in every even-numbered period and accept any offer that pays $x_t \leq \bar{w}$ in every odd-numbered period.

The minimum wage equilibrium, \underline{w} , is generated by the following pair of strategies. The firm proposes $\underline{w} + \frac{(1-\delta)c}{\delta}$ in every even-numbered period if the worker did not shirk in the previous period and proposes \underline{w} otherwise. She accepts any offer that pays $x_t \leq \underline{w}$ in every odd-numbered period. If the firm deviates from her strategy, she is punished by the maximum wage equilibrium, \bar{w} . The worker's strategy is to offer at least \underline{w} in every odd-numbered periods and accept any offer $y_t \geq \underline{w}$ in every even-numbered period. In every even-numbered period he shirks if he is not offered at least \underline{w} or he does not accept an acceptable offer and in every odd-numbered period he shirks if he asks more than \underline{w} or his proposal of \underline{w} is rejected.

The equilibrium strategy that we propose calls for the worker to shirk in every even-numbered period and not to shirk in the odd-numbered periods. A strategy that calls for the worker not to shirk in every period generates the same result. We consider an alternating offers bargaining game with constant disagreement payoffs in every period. A player's equilibrium payoff is increasing in his disagreement payoff only during periods that he responds to an offer. If his disagreement payoff is high, his acceptable offer will be high, hence he can obtain a higher share from the bargaining game. When the player makes a proposal and his offer is accepted, he collects the residual. In this case the size of his payoff depends negatively on the opponent's disagreement payoff. As long as agreement is preferred to disagreement, the residual he obtains exceeds his disagreement payoff. Thus, his disagreement payoff is irrelevant. In the game we analyze, the worker receives the

same disagreement payoff regardless of whether or not he shirks. He receives w_0 in even-numbered periods because he shirks. In odd-numbered periods he does not shirk and receives $w_0 - c$, but he is compensated in the next period, so that on the average he obtains w_0 in odd-numbered periods as well. From the firm's point of view, the actions taken by the worker during periods when the firm makes an offer do not affect the equilibrium payoff that he receives. However, during periods when the worker responds to an offer, the firm guarantees herself the highest sustainable disagreement payoff and thus receives the highest equilibrium payoff of the game if she supports the "not shirk" action of the worker. (For a more detailed discussion on this see Busch and Wen [8].) Any wage contract w such that $\underline{w} < w_0 < \bar{w}$ can be supported by subgame-perfect equilibrium strategies by punishing the worker with the minimum wage equilibrium if he deviates and the firm with the maximum wage equilibrium if she deviates.

The wage increase the worker can capture from the bargaining game ranges between c and $\frac{\delta c}{1+\delta} + \frac{A-1+\varepsilon}{1+\delta}$. The minimum wage contract equalizes the worker's utility to his reservation utility, w_0 . Thus, the worker is indifferent between working in this firm or elsewhere. The maximum wage contract is increasing in the parameter ε . ε can be interpreted as a measure of the worker's ability to hold-up the firm in wage negotiation and ε takes values between $[0, 1)$. We deliberately exclude the case in which $\varepsilon = 1$. If $\varepsilon = 1$ and the worker chooses to shirk, he produces nothing which is in fact striking. If the worker strikes, the principal observes that nothing is produced and thus, refuses to pay w_0 . The worker receives $w_0 + \frac{\delta c}{1+\delta}$ in the equilibrium which is smaller than $w_0 + \frac{\delta c}{1+\delta} + \frac{A-1+\varepsilon}{1+\delta}$ for any $0 < \varepsilon < 1$. Thus, shirking ($0 < \varepsilon < 1$) is a better strategy for the worker than striking.

Inefficient Equilibria

Besides the efficient subgame-perfect equilibria in which the agreement is reached at the first period, the bargaining game also has inefficient equilibria in which delays occur in real time before an agreement is reached. The following proposition characterizes these inefficient equilibria.

Proposition 21 *If \tilde{w} is such that*

$$w_0 + c \leq \tilde{w} \leq \frac{\delta^T - 1}{\delta^T} (A - 1 + \varepsilon + w_0) + \frac{1}{\delta^T} \bar{w}$$

then there is a subgame-perfect equilibrium in which the worker shirks for T periods followed by an agreement of \tilde{w} .

Proof. We provide conditions that are sufficient for deviations not to occur. Along the equilibrium path, the player's strategies are as follows. In every odd-numbered period up to period $T + 1$, the worker makes a non-serious offer to the firm and the firm rejects his offer. In every period up to the period $T + 1$, the worker shirks. In period $T + 1$, the worker offers \tilde{w} if it is an odd-numbered period, and accepts any offer that pays him at least \tilde{w} , if it is an even-numbered period. The firm makes a non-serious offer to the worker in every even-numbered period up to $T + 1$. In $T + 1$, the firm offers \tilde{w} , if it is even numbered period, and accepts any offer that pays at least \tilde{w} , if it is an odd-numbered period.

The worker can always obtain \underline{w} in the first period. In order for the worker to be willing to shirk for T periods and receive \tilde{w} in period $T + 1$, he should prefer to receive w_0 for T periods and \tilde{w} thereafter. Hence,

$$\frac{w_0}{1 - \delta} \leq \frac{w_0 - \delta^T w_0}{1 - \delta} + \frac{\delta^T (\tilde{w} - c)}{1 - \delta}$$

or

$$w_0 + c \leq \tilde{w}$$

In the same manner, the firm can obtain her lowest equilibrium payoff, $A - \bar{w}$ in the first period. In order for the firm not to deviate from the equilibrium strategy, she should prefer to receive $(1 - \varepsilon - w_0)$ for T periods and \tilde{w} thereafter. Hence,

$$1 - w_0 \leq (1 - \varepsilon - w_0) + \dots + \delta^{T-1} (1 - \varepsilon - w_0) + \delta^T \tilde{w}$$

which is equivalent to

$$\tilde{w} \leq \frac{\delta^T - 1}{\delta^T} (A - 1 + \varepsilon + w_0) + \frac{1}{\delta^T} \bar{w}$$

In order for \tilde{w} to exist, $w_0 + c$ has to be smaller than $\frac{\delta^T - 1}{\delta^T} (A - 1 + \varepsilon + w_0) + \frac{1}{\delta^T} \bar{w}$ which holds always given assumption 3.1. The player's deviations from the strategies described above is eliminated by "equilibrium switching". If the worker deviates, he is punished with the minimum wage contract. If the firm deviates, then she is punished with the maximum wage contract.

3.2.2 Bargaining with outside option

In every odd-numbered period, the rejection of the worker's proposal leaves the firm with the choice of firing the incumbent and hiring a new worker, or continuing to bargain with the incumbent worker. If the new worker is hired, he is paid the competitive wage, w_0 , and produces 1 unit of output. We assume that, once the new unskilled worker is hired, the firm and the worker sign an infinitely-lived contract and the worker remains permanently unskilled. We call this the G_1 game. In an alternating offers bargaining game, the outside option changes the equilibrium of

the game if and only if the firm obtains a higher payoff by exercising this option than by continuing to bargain with the incumbent worker.⁸ If the firm rejects the worker's offer and hires a new worker, her discounted total payoff is $\frac{1-w_0}{1-\delta}$. If she stays with the incumbent worker, she obtains at least $\frac{A-\bar{w}}{1-\delta}$. If the lowest equilibrium payoff the firm obtains in G_0 is greater than her outside option, then the firm never exercises this option. Since it does not constitute a credible threat, the value of the outside option does not affect the distribution of the surplus in the G_1 game. Thus, the set of equilibria of the G_1 game is the same as the set of equilibria of the G_0 game. If, however, the firm's outside option is binding, then the G_1 game has a unique subgame-perfect equilibrium in which the firm receives her outside option. The following proposition describes the set of equilibria of the bargaining game when the firm can quit the bargaining game only after rejecting an offer.

Proposition 22 *If $A - c - \frac{\epsilon}{\delta} > 1$, then the firm's outside option is never binding and the set of equilibria of the G_1 game is the same as the set of equilibria of the G_0 game. If $1 \leq A - c < 1 + \frac{\epsilon}{\delta}$, then the firm's outside option is binding for some equilibria of the G_0 game. Hence, the G_1 game has multiple equilibria in which the firm receives at least her outside option, $1 - w_0$. If $A - c \leq 1$, then the firm's outside option is always binding and the G_1 game has a unique equilibrium in which the firm fires the incumbent worker and hires a new worker and receives $1 - w_0$.*

Proof. A formal proof of this proposition can be found in Osborne and Rubinstein [31]. ■

We now discuss the derivation of the conditions in proposition 22. In any subgame-perfect equilibrium of this game the firm receives, at least, $A - \bar{w}$, and at

⁸See Shaked and Sutton [34] more on this.

most, $A - c - w_0$. The firm's outside option is never binding if the average payoff she receives from the outside option is less than the lowest average equilibrium payoff she obtains from the G_0 game. In other words, if

$$1 - w_0 < A - \bar{w}$$

or, equivalently,

$$A - c - \frac{\varepsilon}{\delta} > 1$$

then the firm never exercises her outside option in any subgame-perfect equilibrium of the game. Thus, the set of equilibria of the game G_1 coincides with the set of equilibria of the game G_0 .

If the average payoff that the firm receives from the outside option is greater than her lowest average equilibrium payoff, but smaller than her highest average equilibrium payoff, then the firm's outside option is binding for some of the equilibria of the G_0 game. This happens if

$$1 < A - c < 1 + \frac{\varepsilon}{\delta}.$$

Then the lowest equilibrium payoff the firm obtains in the G_1 game is $1 - w_0$ and the highest payoff is, as before, $A - c - w_0$.

The firm's outside option is always binding if the average payoff from the outside option is greater than her highest average equilibrium payoff. In this case the firm asks at least $1 - w_0$. If the worker accepts this offer, he receives $A - 1 + w_0$. However, the wage that the worker receives must satisfy his individual rationality constraint, i.e. $A - 1 + w_0 > w_0 + c$. Otherwise, the worker does not accept the contract. This condition implies that $A - c > 1$, which contradicts the assumption that the outside option is binding. Therefore, if the outside option is binding, the

unique equilibrium of the game is the one where the firm quits the bargaining game in the first period and hires an unskilled worker from the competitive labor market. This occurs if c is too high. Even though the firm can compensate the worker for exerting high effort, i.e. $A > c + w_0$, it is not profitable for her to do so. As her net surplus from hiring an unskilled worker, $1 - w_0$, is higher than her net surplus from hiring the skilled worker, $A - c - w_0$. The equilibrium of the game is inefficient because 1 unit of output is produced, instead of $A > 1$.

It can be easily shown that inefficient equilibria always exist in the second case of the lemma 22 where the firm's outside option is binding only for some equilibria of the G_0 game. Both players can obtain their lowest payoff in a perfect equilibrium where the agreement is reached in the first period. The worker can always obtain \underline{w} in the first period. In order for the worker to be willing to shirk for T periods and receive w^* in period $T + 1$, he should prefer to receive w_0 for T periods and w^* thereafter. Hence,

$$\frac{w_0}{1 - \delta} \leq \frac{w_0 - \delta^T w_0}{1 - \delta} + \frac{\delta^T (w^* - c)}{1 - \delta}.$$

or

$$w_0 + c \leq w^*$$

In the same manner, the firm can obtain her lowest equilibrium payoff, $1 - w_0$, in the first period. In order for the firm not to deviate from the equilibrium strategy, she should prefer to receive $(1 - \varepsilon - w_0)$ for T periods and w^* thereafter. Hence, we have

$$1 - w_0 \leq (1 - \varepsilon - w_0) + \dots + \delta^{T-1} (1 - \varepsilon - w_0) + \delta^T w^*$$

which is equivalent to

$$w^* \leq A - \varepsilon \left(\frac{1 - \delta^T}{\delta^T} \right) - (1 - w_0).$$

In order for w^* to exist, $w_0 + c$ has to be smaller than $A - \varepsilon \left(\frac{1 - \delta^T}{\delta^T} \right) - (1 - w_0)$ which holds always given assumption 3.1.

3.3 Concluding Remarks

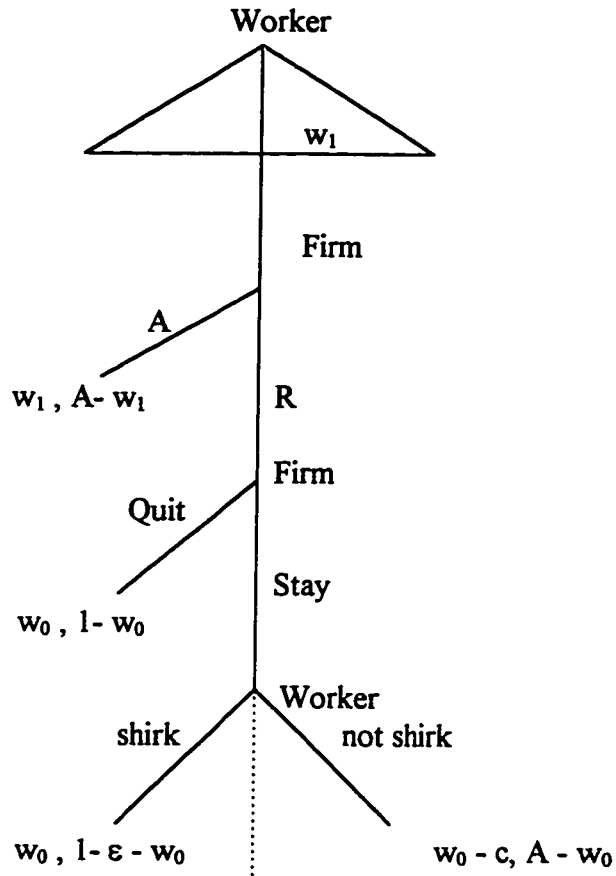
We study the bargaining relationship between a firm and its incumbent worker who possesses firm-specific human capital. The incumbent worker is more productive than the outsiders because of the special skills and information he acquires during his employment. During the contract renewal stage, the worker can strategically disclose these skills in order to increase his bargaining power. The firm can threaten to fire the worker and hire an unskilled worker in the competitive market, but this threat is not always credible. When the firm's outside option is not binding, the bargaining game has multiple equilibria, some of which are inefficient. In the minimum wage equilibrium, the worker is paid a wage so that he is indifferent between working in this firm or elsewhere at the competitive wage. In the maximum wage equilibrium, the worker is able to capture part of the surplus created by his increased productivity. The worker's rent is increasing in his ability to strategically disclose his skills. In the inefficient equilibria, the agreement is reached in period $T > 1$ and the worker shirks in every period prior to agreement.

If the firm's outside option is binding for some equilibria of the game, then the lowest equilibrium payoff the firm receives is bounded by the value of her outside option. If the firm's outside option is always binding, then in the unique equilibrium of the game the firm fires the incumbent worker and hires an unskilled worker. In

this case, even though the production is efficient, it is not profitable for the firm to compensate the worker because his disutility of exerting effort is very high. Thus, in the equilibrium the amount of output produced is less than the efficient amount.

The existing literature on the wage bargaining between the firm and its skilled workers emphasizes that firm-specific human capital is the source of the worker's increased bargaining power. However, these models fail to capture the ex-post inefficiency that may arise as a result of the bargaining. In our model, the ex-post inefficiency arises because of the workers' opportunistic behavior during the bargaining. By shirking during the periods in which the agreement has not been reached the worker is able to capture a rent by obtaining a higher wage. Our results support the conclusions of Williamson et al [41] who argue that when jobs involve specialized skills and information that can be learned by on-the-job training, the market fails to efficiently carry out the exchange between the firm and its employee.

t=1



t=2

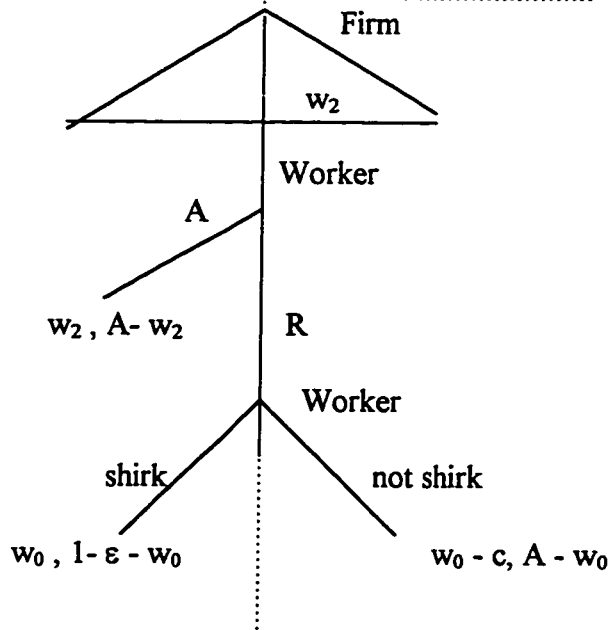


Figure 7. Bargaining Game Tree

3.4 Appendix

We define $H_1(t)$ be the history at the beginning of period t , which consists of all the rejected offers and the disagreement outcomes up to date, $H_2(t)$ is the history after an offer has been made in period t , and $H_3(t)$ is the history after a rejection in period t . We denote σ_i to be the player i 's strategy which assigns an appropriate action to every possible history.

For every period t , we define D_t to be the function of all actions taken in that period excluding the worker's decision whether to shirk, such that,

$$D_t = \begin{cases} fd & \text{if } t \text{ is even and } y_t < \sigma_f(h_1(t)) \\ & \text{if } t \text{ is odd, } x_t \leq \sigma_w(h_1(t)) \text{ but the firm rejected.} \\ wd & \text{if } t \text{ is odd and } x_t > \sigma_w(h_1(t)) \\ & \text{if } t \text{ is even, } y_t > \sigma_f(h_1(t)) \text{ but the worker rejected.} \\ nod & \text{otherwise.} \end{cases}$$

D_t indicates whether or not the firm or the worker has deviated in period t prior to the worker's decision to shirk. If $D_t = fd$, the firm has deviated because she either made an incorrect offer if t is an even-numbered period, or she did not accept an acceptable offer when t is an odd-numbered period. If $D_t = wd$, the worker has deviated because he either made an incorrect offer when t is an odd-numbered period, or he did not accept an acceptable offer when t is an even-numbered period. If neither the firm nor the worker has deviated, then $D_t = nod$.

For every period t , let FD_t be a function of actions taken in period t such

that

$$FD_t = \begin{cases} d & \text{if } D_t = fd \\ nd & \text{otherwise} \end{cases}$$

FD_t indicates whether or not the firm has deviated in period t . Similarly, let WD_t be a function of all actions taken in period t such that

$$WD_t = \begin{cases} d & \text{if } D_t = wd; \text{ or} \\ & \text{if } \tau \text{ is odd, } D_t = nod \text{ for any } \tau \leq t \text{ but} \\ & \text{the worker shirked in that period} \\ nd & \text{otherwise} \end{cases}$$

WD_t indicates whether or not the worker has deviated in period t . The proposed equilibrium strategy in the odd-numbered periods prescribes that the worker shirks only if either the firm or the worker deviated from their equilibrium strategy profiles in the previous stage of the game. Therefore, the worker has deviated in period t if he shirks even though neither he nor the firm has deviated in previous stages of the game. We also define I_t for every period t and $\tau, \tau' < t$ as

$$I_t = \begin{cases} f & \text{if } \sup \{ \tau \mid FD_\tau = d \} > \sup \{ \tau' \mid WD_{\tau'} = d \} \\ w & \text{if } \sup \{ \tau \mid FD_\tau = d \} \leq \sup \{ \tau' \mid WD_{\tau'} = d \} \\ nod & \text{if } \sup \{ \tau \mid FD_\tau = d \} = \sup \{ \tau' \mid WD_{\tau'} = d \} = 0 \end{cases}$$

I_t is a function that indicates the identity of the player who last deviated from his/her strategy profile in history up to period t . τ is the last period prior to period t in which the firm deviated and τ' is the last period prior to period t in which

the worker deviated. If $\tau > \tau'$ then the firm is the last deviant. If $\tau < \tau'$ then the worker is the last deviant. If $\tau = \tau'$ then the worker is again the last deviant since he makes the last move within a period, by choosing between shirking and not shirking.⁹ Then the following strategies generate w as an equilibrium wage, for any $w \in [\underline{w}, \bar{w}]$. For period 1

$$\begin{aligned} \sigma_w(h_1(1)) &= w \\ \sigma_f(h_2(1)) &= \begin{cases} N & \text{if } \sigma_w(h_2(1)) > \bar{w} \\ Y & \text{otherwise} \end{cases} \\ \sigma_w(h_3(1)) &= \begin{cases} ns & \text{if } D_1 = nod \\ s & \text{otherwise} \end{cases} \end{aligned}$$

For t odd and greater than 1,

⁹If $\tau = \tau'$ and τ is even, the firm deviated by making an incorrect offer. If the worker deviates by accepting the firm's incorrect offer the game is over. Therefore, the only possible deviation for the worker is to make an incorrect shirking decision and he becomes the last deviant. If $\tau = \tau'$ and τ is odd then the only way the firm has deviated is by rejecting a correct offer. The only way the worker can deviate is by making an incorrect shirking decision. Since the worker's shirking decision takes place after the firm rejects the offer, the worker is the last deviant.

$$\begin{aligned} \sigma_w(h_1(t)) &= \begin{cases} \bar{w} & \text{if } I_t = f; \text{ or} \\ \underline{w} & \text{if } I_t = w \\ w & \text{otherwise} \end{cases} \\ \sigma_f(h_2(t)) &= \begin{cases} Y & \text{if } \sigma_w(h_2(t)) < \underline{w}; \text{ or} \\ & \text{if } \sigma_w(h_2(t)) \leq \bar{w} \text{ but } I_t = f \\ N & \text{otherwise} \end{cases} \\ \sigma_w(h_3(t)) &= \begin{cases} ns & \text{if } D_t = nod \text{ and } I_t = nod \\ s & \text{otherwise} \end{cases} \end{aligned}$$

For t even,

$$\begin{aligned} \sigma_f(h_1(t)) &= \begin{cases} \hat{w} & \text{if } I_t = f; \text{ or} \\ \underline{w} & \text{if } I_t = w \\ w + \frac{c(1-\delta)}{\delta} & \text{otherwise} \end{cases} \\ \sigma_w(h_2(t)) &= \begin{cases} Y & \text{if } \sigma_f(h_1(t)) > \hat{w}; \text{ or} \\ & \text{if } y_t \geq \underline{w} \text{ and } I_t = w \\ N & \text{otherwise} \end{cases} \\ \sigma_w(h_3(t)) &= s \end{aligned}$$

where $\hat{w} = \frac{\delta(A-1+\epsilon)}{1+\delta} + \frac{c}{1+\delta} + w_0$.

In order to show that the proposed equilibrium strategies profile generates a subgame-perfect equilibrium, we prove that there is no one-shot profitable deviation

from this strategies profile in any proper subgame of the game. First, we show that this is true for the first period. The bargaining game starts with the worker making an offer. We claim that any wage contract w can be generated as a subgame-perfect equilibrium wage of the game, for any $w \in [\underline{w}, \bar{w}]$. If the worker follows the proposed strategy and asks for w and the firm accepts it, the worker receives the discounted total payoff $\frac{w-c}{1-\delta}$ and the firm receives $\frac{A-w}{1-\delta}$. If the worker deviates and asks for x_1 higher than w , the firm rejects the offer. Then the worker shirks and receives w_0 this period. Next period, the firm offers w to the worker. If the worker accepts this offer he obtains $\frac{w_0}{1-\delta}$, which is less than or equal to $\frac{w-c}{1-\delta}$. Hence, the worker does not gain from asking a higher wage. If the worker asks for w and the firm rejects, the worker shirks next period and the firm obtains $1 - \varepsilon - w_0$ this period. Next period the firm proposes \hat{w} to the worker and the worker accepts. The firm's total discounted payoff from this deviation is $1 - \varepsilon - w_0 + \frac{\delta}{1-\delta} (A - \hat{w})$ which is less than or equal to $\frac{A-w}{1-\delta}$. Hence, the firm does not gain from rejecting the worker's proposal. If the worker asks for x_1 , which is higher than w , and the firm deviates from the proposed strategy by accepting the offer, then the firm receives $\frac{A-x_1}{1-\delta}$ which is less than $\frac{A-w}{1-\delta}$. Thus, the firm will not accept such an offer. If the worker ask for x_1 which is higher than w and the firm rejects his offer, the worker obtains $\frac{w_0}{1-\delta}$ if he shirks following the firm's refusal. If he does not shirk, however, he obtains $w_0 - c$ this period, he is offered \underline{w} next period and his total discounted payoff is $\frac{w_0}{1-\delta} - c$. Thus, not shirking is not a profitable deviation.

Next we show that there is no profitable one-shot deviation from the proposed equilibrium strategies profile for either of the players in any proper subgame. There are two possible histories for each subgame. The first type is one where the firm is the last deviant and the second type is one where the worker is the last deviant. We

consider a subgame in the beginning of period t . Regardless of the path that led the game to this point, the proposed strategies are prescribed contingent on the identity of the player who last deviated in that history. We now consider two histories, $h_1(t)$ and $h'_1(t)$ that lead to the subgame in period t . As long as the identity of the last deviant is the same in both histories, the strategies, as a function of these histories, will be the same. Thus, it is sufficient to check the subgame perfection for these two types of histories. We show that for any wage w , such that, $\underline{w} \leq w \leq \bar{w}$, there is no profitable one-shot deviation from the proposed strategies profile for either player in any subgame following the two possible histories.

We consider a subgame that starts in an odd-numbered period with a history where the worker deviated last. The proposed equilibrium strategy prescribes that the worker asks for \underline{w} and the firm accepts it. The worker obtains $\frac{w_0}{1-\delta}$ and the firm receives $\frac{A-c-w_0}{1-\delta}$ by playing this strategy. If the worker asks for x_t higher than \underline{w} , the firm rejects the offer. Then the worker shirks and receives w_0 this period. Next period, the firm offers \underline{w} to the worker. The worker's total discounted payoff from deviating is $\frac{w_0}{1-\delta}$. Thus, the worker does not gain by deviating from this strategy. If the worker asks \underline{w} and the firm rejects the proposed wage, then the worker shirks. The firm receives $1 - \varepsilon - w_0$ this period and next period she offers \hat{w} since she deviated by not accepting the offer. The firm's total discounted payoff is $1 - \varepsilon - w_0 + \frac{\delta}{1-\delta} (A - \hat{w})$ which is less than $\frac{A-c-w_0}{1-\delta}$. Thus, rejecting the worker's offer is not a profitable deviation for the firm. If the worker asks for x_t that is larger than \underline{w} and the firm deviates from the proposed equilibrium strategy by accepting the offer, the firm receives $\frac{A-x_t}{1-\delta}$, which is less than $\frac{A-c-w_0}{1-\delta}$. Thus, the firm will not accept such an offer. If the worker asks for $x_t > \underline{w}$ and the firm rejects, then the worker obtains $\frac{w_0}{1-\delta}$ if he shirks following the firm's refusal. The worker receives w_0

this period and is offered \underline{w} thereafter since he deviated by asking a higher wage. If the worker does not shirk, however, he obtains $w_0 - c$ this period and \underline{w} from the next period onward. Thus, his total discounted payoff from not shirking is $\frac{w_0}{1-\delta} - c$. Therefore, the worker does not deviate.

We now consider a subgame beginning in an odd-numbered period where the firm was the last deviant. The proposed equilibrium strategy calls for the worker to propose \bar{w} and the firm to accept the offer. Following this strategy, the worker receives $\frac{\bar{w}-c}{1-\delta}$ and the firm receives $\frac{A-\bar{w}}{1-\delta}$. If the worker asks for x_t that is larger than \bar{w} , the firm rejects the offer. Then the worker shirks and receives w_0 this period. Next period, the firm offers the worker \underline{w} because the worker has deviated by asking a higher wage. The worker's total discounted payoff is $\frac{w_0}{1-\delta} - c$, which is smaller than $\frac{\bar{w}-c}{1-\delta}$. Thus the worker will not deviate. If the worker offers \bar{w} and the firm rejects, then the worker shirks. The firm receives $1 - \varepsilon - w_0$ this period and next period the firm offers \hat{w} since she deviated by not accepting the offer. The firm's total discounted payoff is $1 - \varepsilon - w_0 + \frac{\delta}{1-\delta} (A - \hat{w})$, which is less than $\frac{A-\bar{w}}{1-\delta}$. Thus, rejecting the worker's offer is not a profitable deviation for the firm. If the worker asks for x_t that is larger than \bar{w} and the firm deviates from the proposed strategy by accepting the offer, the firm receives $\frac{A-x_t}{1-\delta}$, which is less than $\frac{A-\bar{w}}{1-\delta}$. Thus, the firm will not accept such an offer. If the worker ask for $x_t > \bar{w}$ and the firm rejects, then the worker obtains $\frac{w_0}{1-\delta}$ if he shirks following the firm's refusal. If he does not shirk, he obtains $w_0 - c$ this period and is offered \underline{w} next period, thus he obtains $\frac{w_0}{1-\delta} - c$, which is less than $\frac{w_0}{1-\delta}$. Hence, he does not deviate from his proposed equilibrium strategy (shirk) if his offer is not accepted.

Next we consider a subgame beginning in an even-numbered period where the worker is the last deviant. The proposed equilibrium strategy prescribes that the

firm offers \underline{w} and the worker accepts it. The worker obtains $\frac{w_0}{1-\delta}$ and the firm receives $\frac{A-c-w_0}{1-\delta}$ by playing this equilibrium strategy. If the firm offers y_t that is smaller than \underline{w} , the worker rejects the offer and shirks. The firm receives $1 - \varepsilon - w_0$ this period. Next period, the worker asks for \bar{w} because the firm deviated by making an incorrect offer. The firm's total discounted payoff from deviating is $1 - \varepsilon - w_0 + \frac{\delta}{1-\delta} (A - \bar{w})$, which is smaller than $\frac{A-c-w_0}{1-\delta}$. Thus the firm does not deviate from his proposed strategy. If the firm offers \underline{w} and the worker rejects it, then the worker shirks. The worker receives w_0 this period and next period the firm offers \underline{w} . The worker's total discounted payoff is $\frac{w_0}{1-\delta}$, which is the same as what he would have received if he accepted the initial offer in the first place. Thus the worker does not deviate. If the firm offers y_t that is smaller than \underline{w} and the worker deviates from the proposed strategy by accepting the offer, the worker receives $\frac{y_t-c}{1-\delta}$, which is less than $\frac{w_0}{1-\delta}$, the payoff he obtains by rejecting the offer. Thus, the worker will not accept such an offer. If the firm offers $y_t > \underline{w}$ and the worker rejects, then the worker receives $w_0 - c + \frac{\delta}{1-\delta} (\bar{w} - c)$ if he does not shirk following the firm's refusal. Note that the firm is the last deviant by making an incorrect proposal. Since the worker receives $w_0 + \frac{\delta}{1-\delta} (\bar{w} - c)$ if he shirks, he does not deviate from his proposed strategy in this subgame.

We now consider a subgame beginning in an even-numbered period where the firm is the last deviant. The proposed equilibrium strategy prescribes that the firm offers \hat{w} and the worker accepts it. The worker obtains $\frac{\hat{w}-c}{1-\delta}$ and the firm receives $\frac{A-\hat{w}}{1-\delta}$ by playing this proposed strategy. If the firm offers wage y_t that is smaller than \underline{w} , the worker rejects the offer and shirks. The firm receives $1 - \varepsilon - w_0$ this period. Next period, the worker asks for \bar{w} because the firm deviated by making an incorrect offer. The firm's total discounted payoff from deviating is $1 - \varepsilon - w_0 + \frac{\delta}{1-\delta} (A - \bar{w})$, which

is smaller than $\frac{A-\hat{w}}{1-\delta}$. Thus, the firm does not deviate from his proposed strategy. If the firm offers \hat{w} and the worker rejects it, then the worker shirks. The worker receives w_0 this period and next period the worker asks for \underline{w} because the worker has deviated by not accepting the correct offer. The worker's total discounted payoff is $\frac{w_0}{1-\delta}$ which is less than $\frac{\hat{w}-c}{1-\delta}$. Thus rejecting the offer is not a profitable deviation. If the firm offers a wage y_t which is smaller than \hat{w} and the worker deviates from the proposed strategy by accepting the offer, the worker receives $\frac{y_t-c}{1-\delta}$. If the worker rejects the offer he obtains $w_0 + \frac{\delta}{1-\delta}(\bar{w} - c)$ which is equal to $\frac{\hat{w}-c}{1-\delta}$, hence, greater than $\frac{y_t-c}{1-\delta}$. Thus, the worker will not accept such an offer. If the firm offers $y_t < \hat{w}$ and the worker rejects it, then the worker obtains $w_0 - c + \frac{\delta}{1-\delta}(\bar{w} - c)$ if he does not shirk following the firm's refusal. Since he receives $w_0 + \frac{\delta}{1-\delta}(\bar{w} - c)$ if he shirks, the worker will not deviate from the proposed strategy in this subgame and shirk following the firm's refusal.

We have shown that the strategy profile described above generates a subgame-perfect equilibrium wage w , for any $w \in [\underline{w}, \bar{w}]$. Next we show that \underline{w} and \bar{w} are indeed the lowest and highest wages the worker can obtain in the equilibrium generated by these strategies. We prove this by contradiction. Suppose that \underline{w} is not the lowest equilibrium wage and there exist $w' < \underline{w}$ that can be generated as an equilibrium wage by the same strategies. If the agreement is reached in the first period, the worker receives the total discounted payoff, $\frac{w'-c}{1-\delta}$ and the firm receives $\frac{A-w'}{1-\delta}$. The following deviation is profitable for the worker. If the worker deviates by asking a wage $w > w'$, then the firm rejects and the worker shirks. The worker receives w_0 this period and next period the firm proposes \underline{w} to the worker. If the worker accepts his total discounted utility is $w_0 + \frac{\delta}{1-\delta}(w - c)$, which is greater than $\frac{w'-c}{1-\delta}$. Since the worker gains from deviating, w' can not be an equilibrium wage.

This is true for any $w' < \underline{w}$.

Now we consider a wage contract w'' that pays more than \bar{w} . The following is a profitable deviation for the firm in the first period. The firm deviates by rejecting the worker's offer and the worker shirks. The firm receives $1 - w_0 - \epsilon$ this period and offers \hat{w} next period. The worker accepts this offer and the firm receives a total discounted payoff $1 - w_0 - \epsilon + \frac{\delta(A - \hat{w})}{1 - \delta}$, which is greater than $\frac{A - w''}{1 - \delta}$ for any w'' that is greater than \bar{w} . Therefore any wage contract w'' that pays more than \bar{w} cannot be generated by the particular strategy profile we presented.

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