STATISTICAL DISCRIMINATION IN THE LABOR MARKET: PUBLIC POLICY AND EMPIRICAL EVIDENCE

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A i miei genitori Agnese e Cordiano, ed a Cristina

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ABSTRACT

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Statistical discrimination in the labor market occurs when employers have only incomplete information about workers' productivity, and use aggregate statistics to make their decisions. This dissertation expands our understanding of how statistical discrimination is generated, explains how public policy can affect it, and explores to what extent race differentials in the U.S. have been generated by this source. After the introductory chapter, the second chapter, Affirmative Action in a Competitive Economy (joint with Peter Norman), analyzes how incentives for human capital acquisition are affected by affirmative action in a model of statistical discrimination with endogenous human capital. It is shown that affirmative action can fail, in the sense that there may still be discrimination in equilibrium. However, the incentives for agents in the disadvantaged group to invest are better in any equilibrium under affirmative action than in the most discriminatory equilibrium without the policy. The welfare effects are ambiguous: it is demonstrated that the policy may hurt the intended beneficiaries even when the initial equilibrium is the worst equilibrium for the targeted group. The third chapter, The Effects of Statistical Discrimination on Black-White Wage Differentials: Estimating a Model with Multiple Equilibria, structurally estimates an extension of the model presented in the second chapter. Although the model is capable of displaying multiple equilibria, an estimation strategy that identifies both the fundamental parameters and the equilibrium selected by the agents is developed. The model is estimated using U.S. black and white male wage data from 1963 to 1992. In addition to recovering the selected equilibrium, all the equilibria that the model could have displayed are computed. A comparison between the equilibria that were selected over time and the other potential equilibria reveals that the outcome with essentially the lowest wage inequality have always been selected. This implies that self-fulfilling expectations did not exacerbate wage differentials in the U.S. and that the decline in wage inequality experienced in the U.S. in the last decades cannot be attributed to changes in the equilibrium selection.

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1. Introduction

Statistical discrimination in the labor market occurs when employers have only incomplete information about workers' productivity, and use aggregate by-group statistics to make their decisions. This dissertation expands our understanding of how statistical discrimination is generated, explains how public policy can affect it, and explores to what extent race differentials in the U.S. have been generated by this source.

The dictionary definition of discrimination is "to show prejudice", or, "to make a difference in treatment on a basis other than individual merit". Therefore, one might question the rightfulness of the use of this term in the context of statistical discrimination. Nevertheless, it will be used throughout the dissertation, because the terminology has been commonly adopted in economic research in the context of group inequality generated by the use of statistical averages.

Theoretical research on discrimination perhaps began with the essay by Becker (1957), who showed how wage differentials can be generated by employers who care about the race of their employees. In an important article, Arrow (1973) raised some doubts on Becker's framework suggesting that in a competitive environment prejudiced employers should be driven out of the market by (economically more efficient) non-discriminating employers. Arrow proposed a model that generates differentials without resorting to preferences, or group disparity. As in the models that will be described in this dissertation, wage inequality can be an outcome when employers have incomplete information about workers' productivity. If employers expect members of one group to be less productive, then this information will be used in the formation of expectations for a worker's productivity, and workers of different groups will be judged differently productive. Phelps (1972) showed how discrimination can be generated in a different framework. In his model, individuals are differently productive, but groups have the same distribution over productivity. Employers have incomplete information about a workers' productivity, and observe productivity with noise. If the signal is more informative for members of one group (that is, if the noise has smaller variance), then wage distributions will be different. The unreliability of the signal is not sufficient for generating different average wages, but several extensions of the model have been proposed. For example, including employers' risk aversion (Aigner and Cain (1977)), workers' output dependent on job assignment (Rothschild and Stiglitz (1982)), pre-labor market investment by workers (Lundberg and Startz (1983)) one can generate outcome with groups receiving different average wages. While Phelps' argument differs substantially from Arrows', both models have received the "statistical discrimination" label. In fact, they share the basic feature of being incomplete information models where statistical averages are used in order to form expectations.

While the theoretical literature has explained different factors responsible for generating discrimination, quite surprisingly very little has been done to analyze the effect of public policy on discrimination. In the first essay of this dissertation, titled Affirmative Action in a Competitive Economy (joint with Peter Norman), we develop a model where incentive effects and distributional consequences of affirmative action can be studied. The model builds on Arrow (1973) and Coate and Loury (1983) but incorporates some unique features, mostly because in this model it is in some sense harder to discriminate against the smaller group. Introducing an affirmative action policy consisting of an employment quota we find that we in general cannot rule out the possibility of discriminatory equilibria. The most striking result is perhaps that there is a possibility that the discriminated group is made worse off in terms of welfare by the policy, while the other group may gain. One of the main difficulties in evaluating the effectiveness of public policy is that the model is in general capable of displaying multiple equilibria, therefore only set-based comparisons can be performed.

From an empirical perspective, the theoretical literature raises several interesting questions. Multiplicity of equilibria, and the qualitative dependence of the results on the parameters of the model make theory incapable of delivering sharp predictions. Empirical literature has tended to lag behind theory: quantitative research on discrimination has mainly been conducted by estimating race and sex wage differentials after controlling for proxies for productivity (see for example the surveys in Smith and Welch (1989) and Donohue and Heckman (1991)). Figure 1.1 reports one measure of wage differentials: the Black to White median wage ratio from 1965 to 1993 (3-year moving average). The figure is consistent with results obtained from the literature: blacks earn less than whites, and this remains true after controlling for observable characteristics. However, over the last 30 years the racial wage differential has narrowed considerably, especially between 1965 and 1980. Among the reason cited for the reduction in the differential are (1) a convergence in years of schooling; (2) a convergence in the quality of schooling, (3) the selective decline in

Figure 1.1: Black-White median wage ratio (3-year moving average)

the labor force participation of low-skilled blacks, (4) migration of black workers out of southern regions, (5) affirmative action and other anti-discrimination legislation. Much of the debate has centered on the relative contributions of each of these explanations.

It is interesting to note that in their survey, Donohue and Heckman (1991) argue that the first four explanations can account for at most 65% of the reduction in earnings inequality. Although better measurement of these factors could enhance their combined explanatory power, an additional explanation is that the unexplained portion of the decline reflects movements between different economy-wide equilibria that might result from the same fundamentals. From the theoretical literature, we know that different levels of inequality might be possible even under the same set of fundamentals. Therefore, one could conjecture that the economy experienced a movement from an equilibrium with high differentials to one with low differentials that implied a reduction of inequality greater than the reduction of the measured difference in fundamentals.

Chapter 3 attempts to fill the gap between theoretical and empirical literature by developing a methodology capable of verifying this conjecture. This method consists in structurally estimating an extension of the statistical discrimination model presented in chapter 2. The technique allows identification not only of the parameters of the model, but also of the equilibrium selected by the economic agents. This allows the investigator to compare the equilibria that have been chosen over time with the other equilibria that could have been selected. Results show that the economy selected always the outcome with essentially the lowest wage inequality. This implies that statistical discrimination generated by self-fulfilling expectations did not exacerbate wage differentials in the U.S. labor market, ant that the decline in wage inequality experienced in the last thirty years cannot be attributed to changes in the equilibrium selection.

2. Affirmative Action in a Competitive Economy

(joint with Peter Norman)

2.1. Introduction

The purpose of this chapter is to develop a model where incentive effects and distributional consequences of affirmative action can be studied. While there are other important aspects, we believe that these are key issues for a proper evaluation of these programs, a view that seems to be shared by many participants in the public debate[1]

Many opponents of affirmative action argue that affirmative action makes it easier for unskilled female and minority workers to obtain good jobs. This, it is claimed, reduces the incentives to acquire skills and may therefore be counterproductive since skill differences, which are viewed as the real problem, may increase as a result. Proponents, on the other hand, argue that minorities and women are excluded from certain attractive parts of the labor market, so they do not have the same incentives to invest in human capital as white male workers. Affirmative action, it is argued, improves incentives to invest in skills by forcing firms to hire workers from disadvantaged groups. Distributional effects are also heavily debated: opponents often claim that affirmative action only helps already well-situated members of the target groups, while proponents argue that this is not the case. However, it is more or less taken for granted that the policy redistributes towards the groups the quotas are intended to help, a view that we challenge here.

Our model is a straightforward extension of models studied in the literature on statistical discrimination, pioneered by Arrow(1973) and Phelps(1972), but the most closely related paper is Coate and Loury (1993) [2]. Their motivation is similar to ours, but since their model has

[1] See for example Bergman (1996), Bolick (1996) and Sowell (1990).

[2] See also Foster and Vohra (1992) who independently proposed a similar model, Acemoglu

exogenously fixed wages they cannot address effects on the income distribution.

Coate and Loury (1993) assume that output can be produced using two different technologies and workers face a costly human capital investment, which, if undertaken, makes a worker productive in the advanced technology. The sole decision made by firms' is how to allocate randomly drawn workers between the technologies based on a noisy signal of productivity. Given that the model has multiple equilibria, there are equilibria with discrimination when payoff irrelevant group characteristics are introduced into the model. There are circumstances under which affirmative action removes all discriminatory equilibria. However, under equally plausible circumstances there are still equilibria where groups behave differently and employers rationally perceive members of one of the groups to be less capable. In fact, one of the main points with the paper is to show that group disparity in investment behavior may increase as a result of affirmative action.

Wages as well as the workforce for any particular firm are fixed exogenously in Coate and Loury (1993). In a world where firms compete to attract workers these assumptions do not make much sense[3]. Indeed, one may even think that the possibility of equilibria with identical workers treated differently disappears if firms are engaged in wage competition for workers. After all, a standard neoclassical textbook argument holds that if identical workers have different wages, then no firms would demand any of the higher paid workers, so wages must adjust.

We make two important changes relative Coate and Loury (1993). Most important is that we assume that firms are engaged in wage competition for workers, which implies that all workers are paid according to their expected marginal productivity conditional on observable characteristics. The second change, which is important for the interpretation of discrimination, but not for the policy analysis, is that there are complementarities in the production technology. Production requires input of labor in two tasks, a complex task and a simple task, that are complementary in the standard neoclassical sense. Only workers who have undertaken the investment are productive in the complex task, whereas any worker can perform the simple task. This means that there is a potential mismatch between workers and jobs.

First of all we verify that discrimination between ex ante identical groups is sustainable in equilibrium, so wage competition does not guarantee a "color-blind" equilibrium. At first this may

(1995) who studies labor market quotas in a search framework and Lundberg (1991) and Lundberg and Startz (1983) who study efficiency properties of equal oppurtunities policies.

[3] In fact, it is hard to reconcile the model in Coate and Loury with rational behavior by the firms. Firms move after the workers, so if the firms don't compete it is hard to see why firms would not offer the reservation wage to all workers. See Friedman (1992) for an elaboration on this point.

seem surprising and one may wonder where the textbook argument for equal treatment fails. The reason is that the investment decision is unobservable. Instead, firms must use whatever indicators of productivity that are available and if these are noisy, a rational firm will use group averages when interpreting them. For workers from groups were few agents invest in their human capital this means that a high signal is more likely to be the result of a lucky draw rather than actually indicating that the worker invested. Firms takes this into account when wage offers are made, so groups with few investors face flatter wage schemes. It should be observed that agents with the same expected marginal productivities are paid equal wages in the model. Still, workers from discriminated groups have lesser incentives to invest due to the negative externality created by the informational problem.

There are other models, some with a competitive labor market, that can explain discrimination of identical groups. However, the driving force in our model is distinct from the earlier literature. The standard approach is to view discrimination as a coordination failure, as in for example Spence (1974), Akerlof (1976) and Coate and Loury (1993). While differing on details, these papers have in common that they start from a "base model" with multiple equilibria and generate discrimination by assuming that different groups coordinate on different equilibria.

In our model discrimination is driven by specialization of the groups. In fact, discrimination is possible even if the corresponding "single group model" has a unique equilibrium. The reason is that an increase in the fraction of investors in one of the groups makes qualified workers a less scarce resource, which makes the wage scheme flatter for the other group[4]. Thus, benefits of investment decreases in the fraction of investors in the other group, which opens up the possibility of equilibria where groups specialize as high and low quality workers respectively. This hurts the group that specializes as low quality workers and creates inefficiencies in investment behavior, but reduces the informational problem for the firms[5].

In contrast to models where discrimination is a coordination failure there are conflicts of interests between groups: if discrimination occurs because a group is coordinating on a bad equilibrium, other groups need not be affected if the coordination failure is solved. In our model, removing

[4] These effects on marginal productivities would of course not occur unless the tasks are complementary.

[5] It is easy to visualize a version of our model where agents choose different types of human capital investment that enhances the productivity in different types of jobs. In such a model, discrimination may be efficiency enhancing. However, contrary to our framework, discrimination would be voluntary in the sense that it would be incentive compatible to truthfully announce group identity if this were unobservable.

discrimination will in general affect the dominant group adversely.

Introducing an affirmative action policy consisting of an employment quota in the model we find that we in general cannot rule out the possibility of discriminatory equilibria. This exactly parallels the findings in Coate and Loury (1993). Still, the human capital investment only matters for productivity in the complex task, so the expected marginal productivity vary across agents in the complex task only. This suggests that if more agents from the discriminated group would be in the complex task, incentives would improve. Thus, it seems that wage competition would rule out the perverse effects on investment behavior considered in Coate and Loury (1993). Our analysis gives some support for this intuition. Comparing the most discriminatory equilibrium under each regime we find that the incentives to invest and the fraction of investors in the discriminated group are higher with affirmative action[6].

Turning to the welfare effects of affirmative action we get the most striking result. It is possible that the discriminated group is made worse off, while the other group may gain. These perverse distributional effects may seem counterintuitive, but follow in a straightforward way from the characterization of competitive wages with and without the policy. The partial equilibrium effect of the quota is to push down the wage in the bad job for the discriminated group and push it up for the other group. This indeed changes incentives in the desired direction, but the immediate effect on the income distribution is to increase group inequality. The full equilibrium effect depends on how workers respond to this change in incentives and if the response is too small, the policy increases the difference in earnings between groups.

The rest of the chapter is structured as follows. Section 2.2 describes the basic model and section 2.3 characterizes the equilibria. In section 2.4 we extend the model by introducing two identical groups of workers and in section 2.5 we analyze the consequences of affirmative action. The discussion in section 2.6 presents some concluding remarks. All proofs are in section 2.7.

2.2. The Model

To capture the idea that human capital is more important in qualified jobs we assume that firms need workers performing two separate tasks, the complex task and the simple task. Workers on the labor market are of two different types: some workers, called qualified, are able to perform the complex task and others are not. The effective input of labor in the complex task, denoted

[6] Thus, to get perverse incentive effects at least two equilibria where a particular group is discriminated are needed.

C; is taken to be the quantity of qualified workers employed in the complex task[7]. We assume that the human capital investment does not matter for the productivity in the simple task, so the input of labor, S; is equal to the quantity of workers employed to perform the simple task. The output of the firm is then given by $y(C,S)$ where $y: \mathbb{R}^2_+ \to \mathbb{R}_+$ satisfies the standard neoclassical assumptions, i.e. it is a twice continuously differentiable function, strictly concave in both arguments, homogenous of degree one, satisfies the Inada conditions[8] and both factors are essential[9].

2.2.1. The Game

The timing is as follows: In Stage 1 individual workers decide whether to invest or not in their human capital. After the investment decisions (Stage 2) each worker is assigned a signal θ by nature. In Stage 3 firms simultaneously announce wages (as functions of the signal) and in Stage 4 workers choose which firm to work for. Finally, in Stage 5 firms decide how to allocate the workers between the two tasks. The model will now be described in detail.

Stage 1. For tractability we do not want actions of any individual worker to have effects on aggregate variables, so we assume that there is a continuum of agents with heterogeneous costs of investment. Each agent has to choose an action $e \in \{e_q, e_u\}$, where $e = e_q$ means that the agent invests in his human capital (and becomes a qualified worker) and $e = e_u$ that he does not. If agent c undertakes the investment he incurs a cost of c while no cost is incurred if the investment is not undertaken. The agents are distributed on $[\underline{c}, \overline{c}] \subseteq \mathbb{R}$ according to a continuous and strictly increasing distribution function G. We assume that $\underline{c} \leq 0$ and $\overline{c} > 0$:

Stage 2. Each worker is assigned a signal $\theta \in [0,1]$, distributed according to density f_q if the worker invested in Stage 1 and f_u otherwise. It is assumed that f_q and f_u are continuously differentiable, bounded away from zero and satisfy the strict monotone likelihood ratio property $f_q(\theta)/f_u(\theta) < f_q(\theta')/f_u(\theta')$ for all θ, θ' such that $\theta < \theta'$. This implies that qualified workers

[7] We are abstracting from heterogeneity in intrinsic abilities. This is purely for expositional convenience (see Chapter 3), as is the assumption of only two tasks in production.

[8] We assume $\lim_{C \to 0} y_1(C,S) = \infty$ for any $S > 0$ and $\lim_{S \to 0} y_2(C,S) = \infty$ for any $C > 0$. We will consistently use subscribts to denote partial derivatives

[9] The limiting case with y being linear corresponds to the technology considered in Coate and Loury (1993).

are more likely to get higher values of θ than unqualified workers. We let F_q and F_u denote the associated cumulative distributions.

Stage 3. There are two firms, $i = 1,2$. The firms simultaneously announce wages, which are allowed to depend on the signal, so a (pure) action of firm i is a measurable function $w_i : [0,1] \to \mathbb{R}_+$. We assume that the firms cannot observe the distribution of signals at this stage.[10].

Stage 4. Workers observe the wage schedules w_1 and w_2 and decide which firm to work for.

Stage 5. Firms allocate the available workers by using a task assignment rule which is a measurable function $t_i : [0,1] \to \{0,1\}$. The interpretation is that $t_i(\theta) = 1$ (0) means that firm i assigns all workers with signal θ to the complex (simple) task.

We assume that the risk neutral workers' payoffs are additively separable in money income and the cost of investment and that workers do not care directly about which task they are employed in. Thus, once the investment cost is sunk, the worker will rationally choose the firm that offers the higher wage for his particular realization of θ. To save on notation we immediately impose optimal behavior by workers in Stage 4 and write payoffs as

$$E_\theta[\max\{w_1(\theta), w_2(\theta)\} \mid e] - c(e), \tag{2.1}$$

where $c(e_q) = c$ and $c(e_u) = 0$.

Next we want to express the firms' profits as a function of the actions and to do this we need frequency distributions over realized values of the signals. We take these distributions to be given by F_q and F_u, so a strong law of large numbers holds in the model[11].

[10] This is not crucial, but simplifies the description of the strategy sets.

[11] See Judd (1985) and Feldman and Gilles (1985) for discussions on the technical issues corresponding to law of large numbers for continuum random variables. In our model, the simplest way to bypass these technical issues is to use "aggregate shocks" rather than assume that signals are i.i.d. draws from F_q and F_u: the investment decisions by the agents induce distributions of qualified workers and unqualified workers on $[\underline{c}, \overline{c}]$. Call these distributions H_q and H_u. Now let the random variable x be uniformly distributed on $[0,1]$ and let $\theta_c(x)$ denote the test-score for a qualified agent c; where $\theta_c(x) = F_q^{-1}(H_q(c) + x)$ if $H_q(c) + x \leq 1$ and $\theta_c(x) = F_q^{-1}(H_q(c) + x - 1)$ if $H_q(c) + x > 1$. Straightforward, but tedious calculations verify that $\Pr[\theta_c(x) \leq \theta \mid e_q] = F_q(\theta)$ for all $c \in [\underline{c}, \overline{c}]$ and all $\theta \in [0,1]$ and that $\int_{c \in A(x,\theta)} dH_q(c) = F_q(\theta)$ for all $x \in [0,1]$ and all

A single firm does not care directly about the realized frequency distributions for the whole population. Only the distributions of available workers are relevant and these depend on actions by individual workers in Stage 4. Thus, to evaluate the profits of a single firm we need to aggregate the behavior of the workers. We define

$$I^i_{\langle w_i, w_j\rangle}(\theta) = \begin{cases} 1 & \text{if } w_i(\theta) > w_j(\theta) \\ \frac{1}{2} & \text{if } w_i(\theta) = w_j(\theta) \\ 0 & \text{if } w_i(\theta) < w_j(\theta) \end{cases}. \tag{2.2}$$

The interpretation is that $I^i_{\langle w_i, w_j\rangle}(\theta) = 1$ means that a worker with signal θ chooses to work for firm i. Besides the arbitrary tie-breaking rule, each worker goes to the firm that offers the higher wage for her particular signal, so (2.2) simply aggregates workers' rational responses to a pair of wage schemes. No additional equilibrium outcomes can be supported by other tie-breaking rules, so the restriction in (2.2) is innocuous.

Given that a fraction π of the workers invests and wage schedules $\langle w_1, w_2\rangle$ the quantity of qualified workers available for firm i with a signal $\theta \leq \bar{\theta}$ is $\int_0^{\bar{\theta}} I^i_{\langle w_1, w_2\rangle}(\theta)\pi f_q(\theta)d\theta$ and the quantity of unqualified workers is computed symmetrically. The effective input of labor in the two tasks given task assignment rule t_i are then given by

$$\begin{aligned} C_i(w_1, w_2, t_i) &= \int I^i_{\langle w_1, w_2\rangle}(\theta) t_i(\theta)\pi f_q(\theta)d\theta \\ S_i(w_1, w_2, t_i) &= \int I^i_{\langle w_1, w_2\rangle}(\theta)(1 - t_i(\theta))(\pi f_q(\theta) + (1-\pi) f_u(\theta))d\theta \end{aligned} \tag{2.3}$$

respectively. The profits of firm i can then be expressed as

$$\Pi^i(\sigma) = y(C_i(w_1, w_2, t_i), S_i(w_1, w_2, t_i)) - \int I^i_{\langle w_1, w_2\rangle}(\theta) w_i(\theta)(\pi f_q(\theta) + (1-\pi) f_u(\theta))d\theta. \tag{2.4}$$

After the workers' decisions in Stage 4 have been replaced by the sequentially rational allocation rule (2.2) a pure strategy for a worker is simply to decide to invest or not. We will summarize the behavior of all workers as a map $i: [\underline{c}, \bar{c}] \to \{e_q, e_u\}$. A pure strategy for a firm is a pair $\langle w_i, \tau_i\rangle$ where w_i is a measurable function from $[0,1]$ into R_+, $\tau_i: M \times M \to T$, M denotes the set of measurable functions from $[0,1]$ into R_+ and T denotes the set of measurable functions from $[0,1]$ into $\{0,1\}$. The interpretation is that if $\tau_i(w_1, w_2)(\theta) = 1$ then firm i assigns workers with signal

$\theta' \in [0,1]$; where $A(x;\theta) = \{c \in [\underline{c}, \bar{c}] \mid \theta_c(x) < \theta\}$. Unqualified agents can be treated identically.

θ to the complex task given that the pair of wage schedules offered in Stage 3 is $(w_1; w_2)$:

2.3. Characterization of Equilibria

Although the model is dynamic, refinements such as perfect Bayesian equilibrium will not give sharper predictions than Nash Equilibrium in terms of equilibrium outcomes (see Section 2.3.1). Therefore we take Nash equilibrium as our solution concept. First we characterize the firms' "equilibrium responses", which determine a unique wage schedule such that both firms are playing best responses given any fixed investment behavior by the workers. In equilibrium, investment decisions by workers must also be best responses to the wages and this generates a fixed point equation in a single variable that fully characterizes the equilibria of the model.

Consider the decision problem for the firm in the final stage after a history when a fraction $\pi > 0$ of the workers invested. The monotone likelihood ratio implies that any optimal task assignment rule for firm i must agree with some cutoff-rule of the form $t_i(\theta) = 0$ for $\theta < \hat{\theta}_i$ and $t_i(\theta) = 1$ for $\theta \geq \hat{\theta}_i$, except possibly for a set of signals with measure zero. Since firms have already committed to wages, the cut-off must maximize output. On the Equilibrium path both firms must offer the same wage for almost all θ, so the quantity of qualified (unqualified) workers available for firm i with realized signal less than θ is $F_q(\theta)/2$ $(F_u(\theta)/2)$. The output-maximizing cutoff solves

$$\max_{\theta_i, C_i, S_i} y(C_i; S_i) \tag{2.5}$$
$$\text{subj. to } 2C_i \leq \pi(1 - F_q(\theta_i))$$
$$2S_i \leq \pi F_q(\theta_i) + (1 - \pi) F_u(\theta_i):$$

The constraint set is convex and there is a unique, interior solution (see the proof of Proposition 1). Since both firms face a identical problems with a unique solution we drop the indices from now on. Eliminating C and S from the problem we can write the necessary and sufficient condition for θ to solve (2.5) as

$$p(\theta; \pi) y_1(\pi(1 - F_q(\theta)); F_\pi(\theta)) = y_2(\pi(1 - F_q(\theta)); F_\pi(\theta)); \tag{2.6}$$

where $F_\pi(\theta)$ is shorthand notation for $\pi F_q(\theta) + (1-\pi)F_u(\theta)$ and

$$p(\theta;\pi) \equiv \frac{\pi f_q(\theta)}{\pi f_q(\theta) + (1-\pi) f_u(\theta)} \tag{2.7}$$

is the posterior probability that a randomly drawn agent with test score θ is qualified given prior π. The interpretation of (2.6) is that the cutoff for the "critical worker" is set so that the expected marginal productivity is equalized across tasks. Agents with lower (higher) signals are more productive in the simple (complex) task and are assigned accordingly.

For each $\pi > 0$ the solution to (2.6) is denoted by $\hat\theta(\pi)$. The implicit function theorem applies, so $\hat\theta$ is a smooth function of π. For notational convenience we define $r(\pi)$ as the ratio of effective units of complex labor over units of simple labor implied by $\hat\theta(\pi)$;

$$r(\pi) = \frac{\pi\left(1 - F_q(\hat\theta(\pi))\right)}{\pi F_q(\hat\theta(\pi)) + (1-\pi) F_u(\hat\theta(\pi))}. \tag{2.8}$$

Since firms are involved in Bertrand competition for workers we conjecture that workers are paid according to their expected marginal productivities: given a fraction π of investors we take the candidate "labor market equilibrium" wage schedule to be given by

$$w(\theta) = \begin{cases} y_2(r(\pi);1) & \text{for } \theta < \hat\theta(\pi) \\ p(\theta;\pi)\, y_1(r(\pi);1) & \text{for } \theta \geq \hat\theta(\pi) \end{cases}. \tag{2.9}$$

The main content of our first result is that our conjecture about equilibrium wages is correct. Denote by t the cutoff rule with critical point $\hat\theta(\pi)$ and let t_i be the task assignment rule on the outcome path for firm $i = 1,2$ for an arbitrary strategy profile.

Proposition 1. *A necessary and sufficient condition for both firms to play best responses when a fraction π of the workers invest is that $w_i(\theta) = w(\theta)$ and $t_i(\theta) = t(\theta)$ for $i = 1,2$ and for all almost all $\theta \in [0,1]$.*

The intuition for the sufficiency part is that a deviating firm has to pay at least the expected marginal productivity (given task assignments according to candidate equilibrium) for all workers and strictly more if it wants to attract any additional workers. This could only be profitable if the deviating firm could allocate the workers more efficiently between tasks, which is impossible since the original task assignment rule maximizes output.

So far we have kept investment behavior fixed. In a full equilibrium the additional condition that each worker maximizes (2.1) given the wage schedules must hold as well. We denote by $B(\pi)$ the gross benefits of investment, the difference in expected earnings for an agent who invests and an agent who does not. By substitution from (2.9) we find that

$$B(\pi) = y_2(r(\pi),1)\left(F_q(\hat{\theta}(\pi)) - F_u(\hat{\theta}(\pi))\right) + y_1(r(\pi),1)\int_{\hat{\theta}(\pi)} p(\theta;\pi)\left(f_q(\theta) - f_u(\theta)\right)d\theta. \quad (2.10)$$

Given a wage schedule w a worker with investment cost c is better off by investing if and only if the gross benefits of investments are higher than the costs, so $G(B(\pi))$ is the fraction of workers who invest as best responses to the labor market equilibrium wages when a fraction π invests. The equilibria of the model are thus fully characterized by the solutions to

$$\pi = G(B(\pi)). \quad (2.11)$$

Summing up these observations we have:

Proposition 2. The fraction of investors in any equilibrium solves (2.11). Moreover, any solution to (2.11) corresponds with an equilibrium of the model[12].

Proposition 2 implies that the question of existence of equilibria reduces to the question of existence of a fixed point of $G \circ B$. This gives us a relatively easy proof of existence of equilibria:

Proposition 3. If $G(0) > 0$[13], then there exists a non-trivial equilibrium of the model.

From Figure 2 we see that all there is to show is that $G \circ B$ is continuous, but for future reference we also note that $B(0) = B(1) = 0$. The intuition for this is simple: since we assume that both factors are essential, output must be zero if nobody invests and it follows that $w(\theta) = 0$ for all θ. Hence $B(0) = 0$. If on the other hand everybody invests we have that the signal is uninformative, so the wage must be a constant function of the signal, implying $B(1) = 0$.

[12] The second part means that if π solves (2.11) we can construct an equilibrium $\left\{ i; \langle w_i, \xi_i \rangle_{i=1,2} \right\}$ where $i(c) = e_q$ for all $c < G^{-1}(\pi)$ and $i(c) = e_u$ for all $c > G^{-1}(\pi)$; $w_1(\theta) = w_2(\theta) = w(\theta)$ given by (2.9) and $\xi_1(w_1, w_2)(\theta) = \xi_2(w_1, w_2)(\theta)$ for almost all $\theta \in [0,1]$.

[13] In order for a non-trivial equilibrium to exist when $G(0) = 0$ the slope of $G \circ H$ evaluated at 0 must be larger than unity. Given that the density associated with G is bounded away from zero, a sufficient condition for this is that $\lim_{C \to 0} y_{11}(C,S)y(C,S)/(y_1(C,S))^2 > -1$, in which case the slope of $G \circ H$ is unbounded at zero. While not easy to interpret, the condition holds for several common parametric production functions (for example Cobb-Douglas).

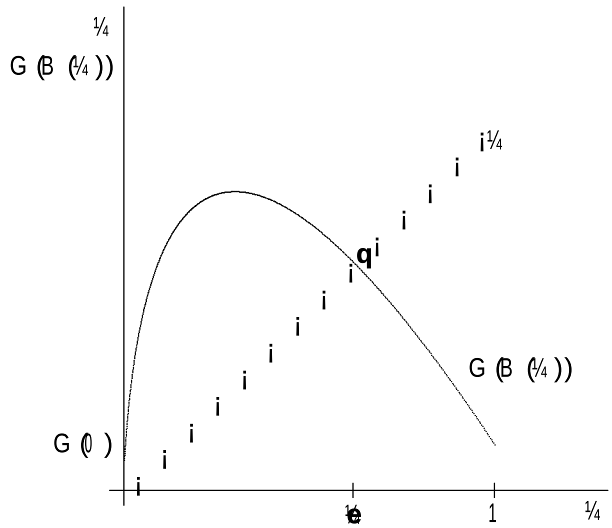

Figure 2.1: An example with a unique interior equilibrium

2.3.1. Some Modeling Issues

The equilibrium conditions have a flavor of competitive equilibrium, so one may conjecture that the model could be formalized as a model with price taking agents. As far as we understand this is indeed possible, but one runs into some technical and conceptual difficulties[14]. Modeling competition as a Bertrand game rather than assuming that firms are price-takers help overcome these difficulties and makes the policy analysis in later sections easier to handle.

The "reduced game" obtained by assuming that each firm's distribution of available workers is determined by (2.2) is still a dynamic game and we have nevertheless ignored unreached information sets: Propositions 1 and 2 do not even specify what firms are supposed to do when choosing task assignment rules at information sets where the wage schedules differ on a set of points with positive measure. The reader may therefore worry that we consider equilibria supported by non-credible threats off the equilibrium path, but this is not the case. The intuitive reason is that the last stage of the game is non-strategic in the sense that the task assignments by the other firm have no

[14] Maximizing over "quantities" the firm has to choose a distribution on the support of the noisy signal. To write down market clearing conditions it turns out that a strong law of large numbers is needed. Unlike our model, there is no simple "trick" that can be used, so one has to rely on somewhat arbitrary probability measures (see Judd (1985) and Feldman and Gillles (1985)). The conceptual problem is that even with a strong law of large numbers it is not clear how the firms should evaluate profits out of equilibrium.

impact on the best responses in Stage 3. Hence it is impossible to enlarge the set of equilibrium outcomes compared to the set of perfect Bayesian equilibrium outcomes by committing to task assignment rules that are suboptimal off the equilibrium path.

2.4. The Model with Two Identifiable Groups of Workers

We now assume that each worker belong to one of two identifiable groups. The main points of the section is that unobservable human capital investment together with complementarities in production creates negative externalities between groups (Proposition 4) and that equilibria with discrimination exist under the not implausible assumption that not too may agents have negative costs of investment (Proposition 5).

2.4.1. The Extended Model

We index the groups by $j = a, b$; where a fraction λ^a of the workers belongs to group a and a fraction $\lambda^b = 1 - \lambda^a$ to group b. The distribution of investment costs is given by G in each group and the probability density over signals is given by f_q for a worker (from any group) who invests and f_u otherwise. The groups are thus ex ante identical in terms of investment costs and signals are unbiased[15].

The game is as in Section 2.2.1, except that wages may depend on group identity. Hence, a strategy for firm i is a quadruple $< w_i^a, w_i^b, \tau_i^a, \tau_i^b >$ where for $j = a, b$ the function $w_i^j : [0, 1] \to R_+$ denotes a generic wage schedule and τ_i^j maps wage schedules to task assignment rules for group j.

In equilibrium, both firms must post the same wage for almost all signals, so the distribution of available workers for each firm is a scaling of the population distribution. As before, task assignments must be done according to cut-off rules and since wage costs are sunk the cutoffs must be chosen so as to maximize output (given investment behavior), that is solve

$$\max_{(\theta^a,\theta^b)\in[0,1]^2} y\left(\sum_{j=a,b} \lambda^j \pi^j \left[1 - F_q\left(\theta^j\right)\right], \sum_{j=a,b} \lambda^j F_{\pi^j}\left(\theta^j\right)\right). \tag{2.12}$$

Qualitatively, the only difference between this problem and the task assignment problem in Section 2.3 is that corner solutions now are possible (and interesting). One shows that there is a unique solution to (2.12) for any $\pi \neq (0, 0)$; which is characterized by the Kuhn-Tucker conditions. We

[15] As in the basic model we will use f_u and f_q as realized frequency distributions as well. This can be justified by a direct generalization of the exact stochastic model described in footnote 2.2.1.

let γ_j be the multiplier associated with the constraint $\theta^j \geq 0$ and η_j be associated with $1 - \theta^j \geq 0$. The optimality conditions are, after some rearranging, given by

$$-p\left(\theta^j; \pi^j\right) y_1(\cdot) + y_2(\cdot) + \frac{\gamma_j - \eta_j}{f_{\pi^j}\left(\theta^j\right)} = 0; \text{ for } j = a, b \tag{2.13}$$

together with complementary slackness conditions. We let $\tilde{\theta}(\pi) = (\tilde{\theta}^a(\pi), \tilde{\theta}^b(\pi))$ denote the unique solution and, as in the basic model, the implicit function theorem implies that $\tilde{\theta}$ is continuous. To economize on notation we let $\tilde{k}(\pi)$ denote the factor ratio implied by $\tilde{\theta}(\pi)$; that is

$$\tilde{k}(\pi) = \frac{\sum_{j=a,b} \lambda^j \pi^j (1 - F_q(\tilde{\theta}^j(\pi)))}{\sum_{j=a,b} \lambda^j (\pi^j F_q(\tilde{\theta}^j(\pi)) + (1 - \pi^j) F_u(\tilde{\theta}^j(\pi)))} \tag{2.14}$$

In equilibrium, each worker is paid according to his expected marginal productivity, i.e.

$$w^j(\theta) = \begin{cases} y_2(\tilde{k}(\pi), 1) & \text{for } \theta < \tilde{\theta}^j(\pi) \\ p\left(\theta; \pi^j\right) y_1(\tilde{k}(\pi), 1) & \text{for } \theta \geq \tilde{\theta}^j(\pi) \end{cases} \tag{2.15}$$

The gross benefits of investment given any investment behavior π is

$$B^j(\pi) = y_2(\tilde{k}(\pi), 1)(F_q(\tilde{\theta}^j(\pi)) - F_u(\tilde{\theta}^j(\pi))) + y_1(\tilde{k}(\pi), 1) \int_{\tilde{\theta}^j(\pi)} p\left(\theta; \pi^j\right)(f_q(\theta) - f_u(\theta)) d\theta \tag{2.16}$$

and the fraction of agents in group j who invests is still given by the fraction with investment cost lower than the benefits, so the relevant system of fixed-point equations that fully characterize the equilibrium set is

$$\pi^j = G(B^j(\pi)) \text{ for } j = a, b. \tag{2.17}$$

The most natural way to define discrimination is in this framework is to call an equilibrium discriminatory if, say, the average wage is lower for one group than the other. However, from (2.15) we see that if $\pi^j > \pi^k$; then the wage scheme for group j is uniformly above the wage scheme for group k; while the wage schemes are identical if the fractions of investors are the same. Therefore, we call an equilibrium discriminatory if $\pi^a \neq \pi^b$ and refer to the group with the lower fraction of investors as the discriminated group.

If $\pi^a = \pi^b$ the conditions for optimal task assignments (2.13) simplify to the optimality condition for the single group model (2.6). Thus, the cutoffs, the implied factor ratio and the gross benefits of investment will be as in the single group model, so if $(\pi;\pi)$ is an equilibrium, then π is an equilibrium in the single group model. The converse is also true, so the set of non-discriminatory equilibria corresponds directly with the set of equilibria in the single group model, which guarantees existence of equilibria under the same conditions as before.

We are mainly interested in discriminatory equilibria and will demonstrate below that discriminatory equilibria exist under the rather plausible assumption that not too many agents have negative costs of investment. However, to highlight that the driving force behind discrimination is distinct from the earlier literature, we will first discuss a comparative statics result that says that increased investments in one group affect incentives to invest negatively for the other group.

Proposition 4. Fix $\pi^j > 0$: Then $B^j(\pi^j;\pi^k)$ is decreasing in π^k over the whole unit interval and strictly decreasing for all π^k such that $0 << \hat{r}(\pi) << 1$:

The intuitive explanation is as follows: if π^b increases the firms will respond by replacing some a-workers by b-workers in the complex task and increase the factor ratio (2.14). Since labor inputs in the two jobs are complementary, the marginal productivity in the complex task decreases while it increases in the simple task. Hence, from the a-workers point, the cutoff signal is increased, the wage in the simple task increases and the wage in the complex task decreases for all realizations of θ. Taken together this unambiguously reduces incentives to invest.

To construct an example where, say, group a is discriminated in equilibrium the first step is to show that there exists π with $\pi^a < \pi^b$ such that $B^a(\pi) < B^b(\pi)$. Once existence of such π is established we can pick any distribution function G that satisfies $\pi^a = G(B^a(\pi))$ and $\pi^b = G(B^b(\pi))$ and the example is complete. Inspecting (2.13) we see that if $\pi^b > 0$ is held fixed and π^a is small enough, then all workers from group a will be assigned to the simple task[16], while some workers from the other group must be assigned to the complex task. It follows that for any fixed $\pi^b > 0$ there is always some $\varepsilon \in (0;\pi^b)$ such that $0 = B^a(\pi^a;\pi^b) < B^b(\pi^a;\pi^b)$ for all $\pi^a \leq \varepsilon$: This means that irrespective of the other fundamentals of the model it is always possible to find a distribution of costs such that there is at least one discriminatory equilibrium of the model.

[16] To see this, note that $\lim_{\pi^a \to 0} r(\pi^a;\pi^b) > 0$; for any $\pi^b > 0$ since otherwise $\lim_{\pi^a \to 0} \hat{r}^b(\pi^a;\pi^b) = 0$ and $\lim_{\pi^a \to 0} p(\hat{r}^b(\pi^a;\pi^b);\pi^b)y_1(\cdot) = 1$; which condradicts (2.13) for π^a sufficiently small. Next, $\lim_{\pi^a \to 0} r(\pi^a;\pi^b) > 0$ implies that $y_1(\cdot)$ stays bounded, so $\lim_{\pi^a \to 0} p(\hat{r}^a(\pi^a;\pi^b);\pi^b)y_1(\cdot) = 0$: From (2.13) we then conclude that η_a must be strictly positive for π^a small enough, which means that $\hat{r}^a(\pi^a;\pi^b) = 1$:

While the argument above shows that it is possible to find distributions of costs such that the model allows discriminatory equilibria[17] it does not say anything about what conditions on the primitives are needed. The next proposition is intended to give some intuition: if the fraction of agents who derive positive utility from the investment is small enough there will be an equilibrium where all agents from one of the groups are assigned to the simple task at a constant wage. To make this precise we now let G be some strictly increasing distribution function over $[0, \bar{c}]$ satisfying $G(0) = 0$; where we assume that the single-group model has a non-trivial equilibrium. To get a parametric sequence of distribution functions where the fraction of agents with negative investment cost approaches zero we let G_α be defined as $G_\alpha(c) = \alpha + (1 - \alpha)G(c)$ for each $c \in [0, \bar{c}]$:

Proposition 5. Fix $y; f_q; f_u; \lambda^a$ and λ^b and assume that $\lambda^a; \lambda^b > 0$. Then there exists $\bar{\alpha} > 0$ such that there exists an equilibrium where no workers from one of the groups, j; are assigned to the simple task and $\pi^j = \alpha = G_\alpha(0)$ for any $\alpha \leq \bar{\alpha}$: Moreover, in this equilibrium the wage schedule for group j is uniformly below the wage scheme for the other group.

The intuition is straightforward: if all agents in, say, group a, are assigned to the simple task the equilibrium conditions for the other group are qualitatively as in the single group model. Applying the same steps as in the proof of Proposition 3 we establish that a fraction $\pi^b > \alpha$ must invest in the other group and that some of these workers must be used in the complex task. Let θ^b be the implied cutoff for group b in the "restricted model" where it has been assumed that all agents from group a are used in the simple task. To verify that $(\alpha; \pi^b)$ is an equilibrium of the full model we only need to check that $(1; \theta^b)$ satisfies the conditions in (2.13) given fractions $(\alpha; \pi^b)$; which is the case for α small enough.

It is not very surprising that (2.16), the benefits of investment for group j; converges pointwise to (2.10) as the relative group size λ^j goes towards unity. Although this result is simple to understand it has some rather interesting consequences. Most notable perhaps is that it means that if the single-group model has a unique equilibrium, π^*; and group j is large enough, then the fraction of investors in group j must be "near" π^* in any equilibrium of the model with multiple

[17] Formal genericity analysis is out of the scope for this paper, but it is easy to see that a "large" set of $\pi \in [0,1]^2$ with $\pi^a \neq \pi^b$ can be supported as equilibria by some distribution G: For each of these π there is obviously a multitude of distributions that work since only the value at two points matters. More interesting is maybe that for any π such that the group with the lower fraction of investors has less incentives to invest we may pick G such that any G^0 that satisfies $|G^0(c) - G(c)| < \varepsilon$ for all c; also has an equilibrium with discrimination if $\varepsilon > 0$ is sufficiently small and that this is true for an arbitrary choice of G that supports a discriminatory equilibrium such that $B^j(\pi)$ is decreasing in π^j for the group with the higher fraction of investors.

groups. While this does not rule out equilibria where the fraction of investors in the small group is strictly larger than π^* we note that only the small group can be in a situation where no workers from the group are assigned to the complex task.

2.5. Affirmative Action

In this section we will use our framework to analyze the effects of affirmative action. We characterize the equilibrium wages when the policy is introduced (Proposition 6) and use this characterization to show that discrimination may persist (Proposition 7), but that affirmative action raises the lower bound on the fraction of investors in the discriminated group (Proposition 8). Finally, we consider a pair of examples. In the first example we demonstrate that it is possible that affirmative action has perverse distributional consequences and in the second we show that it is also a possibility that affirmative action guarantees that the economy moves to a color blind equilibrium.

An alternative intervention would be an equal opportunities law, requiring firms to offer wages that do not depend on group identity. In our simple framework this would mean that the firms would be constrained to offer identical wage schedules to both groups. Since the incentives to invest would be the same for both groups this would eliminate discrimination. The problem with this policy is that the regulator must observe all information the employer has available in order to implement it. Since hiring decisions are often based on intangible variables such as the outcome of an interview, verbal recommendations etc. we think that this is unrealistic[18]. Quotas on the other hand don't require detailed information about the workers and can be implemented as long as the policymaker can observe the number of workers from each group in each job category.

2.5.1. The Model with Affirmative Action

We model affirmative action as a requirement for each firm to hire workers for each task in accordance with the population fractions. While the constraint is straightforward to write down given arbitrary wages it is more transparent for the case where $w_1^j = w_2^j$ for $j = a; b;$ which will be the case in equilibrium. The affirmative action policy requires that

$$\frac{\int t_i^a(\theta)\lambda^a f_{\pi^a}(\theta)d\theta}{\int t_i^b(\theta)\lambda^b f_{\pi^b}(\theta)d\theta} = \frac{\lambda^a}{\lambda^b} \text{ and } \frac{\int \left(1 - t_i^a(\theta)\right)\lambda^a f_{\pi^a}(\theta)d\theta}{\int \left(1 - t_i^b(\theta)\right)\lambda^b f_{\pi^b}(\theta)d\theta} = \frac{\lambda^a}{\lambda^b}: \tag{2.18}$$

[18] For example, it seems that a policy requiring equal pay for equal qualifications in the academic market would be more or less impossible to implement.

Other numerical goals can be handled as long as full employment is feasible, but it is important that there is a quota in each task (see Section 2.5.3).

The payoffs as functions of actions and the timing of the actions are as before and the only difference compared to the model in Section 2.4 is that the task assignment rule chosen in the final stage of the game must satisfy the affirmative action constraint[19].

As in earlier sections the first step in the equilibrium characterization is to note that firms must offer wage schedules that are identical almost everywhere and that task assignments must be done according to cut off rules. We can characterize the optimal task assignment rule after any history where both firms have offered (essentially) the same wage schedules by solving

$$\begin{aligned} \max_{\theta^a,\theta^b} \; & y\left(\sum_{j=a,b} \lambda^j \pi^j \left(1 - F_q\left[\theta^j\right]\right); \sum_{j=a,b} \lambda^j \left(\pi^j F_q\left[\theta^j\right] + \left[1 - \pi^j\right] F_u\left[\theta^j\right]\right)\right) \\ \text{s.t} \quad & \pi^a F_q(\theta^a) + (1 - \pi^a) F_u(\theta^a) = \pi^b F_q(\theta^b) + (1 - \pi^b) F_u(\theta^b) \end{aligned} : \qquad (2.19)$$

This problem is just adding a constraint, which is the affirmative action requirement (2.18) when the task-assignment rules are cutoff rules, to the problem (2.12) in Section 2.4. The necessary and sufficient conditions for optimality are

$$\begin{aligned} -y_1(\cdot) p(\theta^a; \pi^a) + y_2(\cdot) - \frac{\mu}{\lambda^a} &= 0 \\ -y_1(\cdot) p(\theta^b; \pi^b) + y_2(\cdot) + \frac{\mu}{\lambda^b} &= 0 \end{aligned} ; \qquad (2.20)$$

where $\mu > 0$ if $\pi^a < \pi^b$. Using the constraint, the multiplier and one of the decision variables can be eliminated and the remaining equation has all the qualitative properties of (2.6). By arguments more or less identical to the ones used in the single group model one can show that for each $\pi = (\pi^a; \pi^b)$ such that either π^a or π^b is strictly positive there is a unique $\hat{\theta}(\pi) = (\hat{\theta}^a(\pi^a; \pi^b); \hat{\theta}^b(\pi^a; \pi^b)) >> 0$ that solves (2.19) and that the implicit function theorem applies. The solution will consequently be a smooth function of π: For notational convenience we let $\hat{\ }(\pi)$ denote the implied factor ratio, which is defined as $\tilde{\ }(\pi)$ in Section 2.4 (equation (2.14)), but with cutoffs given by $\hat{\theta}(\pi)$ instead of $\tilde{\theta}(\pi)$:

While the characterization of equilibrium task-assignment rules is not significantly harder than earlier, the determination of wages is somewhat counterintuitive. It is tempting to guess that

[19] Alternatively, the strategy sets can be kept as before and affirmative action can be imposed by charging penalties on firms that violates the numerical goals on employment stipulated by the policymaker. If the penalties are sufficiently high the two approaches are equivalent.

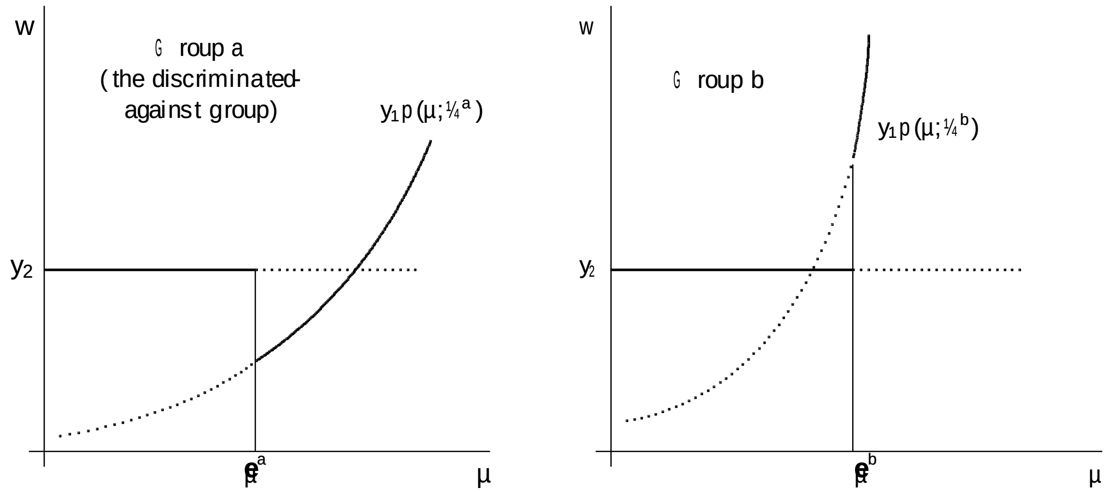

Figure 2.2: Not an equilibrium

wages still are given by expected marginal productivities, that is to take (2.15) as the candidate equilibrium wage function, using the unique cut-off points determined above. This is however not consistent with equilibrium since (assuming $\pi^a \neq \pi^b$) some agents of the discriminated group employed in the complex tasks are paid less than agents from the same group who are in the simple task (see Figure 2). Hence a firm could steal some of these cheaper workers from the other firm to replace some of the workers previously in the simple task. As long as the total quantity of workers in the simple task is held constant, the affirmative action constraint is not affected. Output is unchanged and total wage payment has decreased, so this is a profitable deviation.

The unique wage schedules consistent with equilibrium are

$$w^j(\theta) = \begin{cases} y_1(b(\pi);1)\,p(\hat{\theta}^j(\pi);\pi^j) & \text{for } \theta < \hat{\theta}^j(\pi) \\ y_1(b(\pi);1)\,p\left(\theta;\pi^j\right) & \text{for } \theta \geq \hat{\theta}^j(\pi) \end{cases}; \tag{2.21}$$

so the wage in the simple task is now determined by the marginal agent's productivity in the complex task rather than the productivity in the simple task. These wage schemes together with the task assignment rules derived above characterize the firms' equilibrium responses. In analogue with the single group model we let t^j denote the task-assignment rule with cutoff $\hat{\theta}^j(\pi^a;\pi^b)$ for $j = a;b$ and let t_i^j be the task assignment rule on the outcome path for an arbitrary strategy

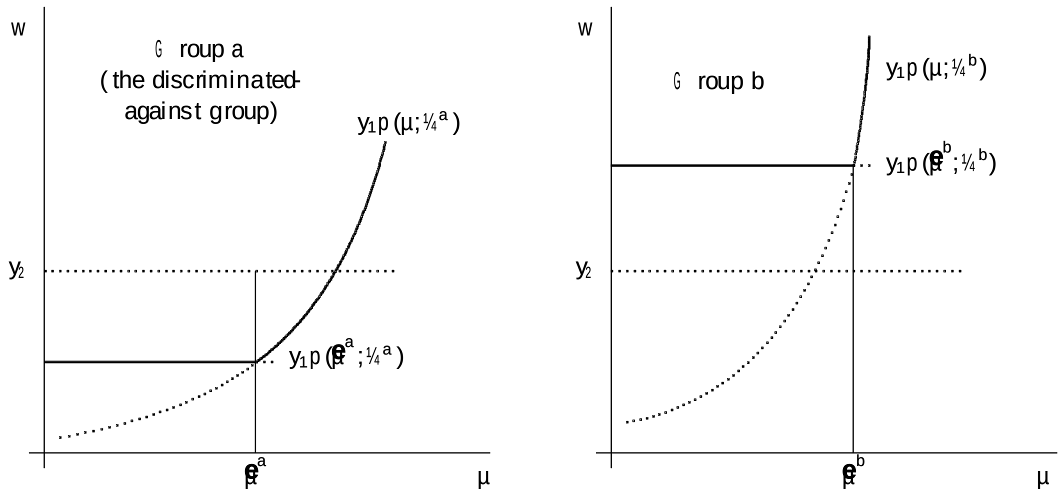

Figure 2.3: Equilibrium wage schedules under affirmative action

profile.

Proposition 6 A necessary and sufficient condition for both firms to play best responses when fractions $\pi = (\pi^a; \pi^b)$ of the workers invest is that $w_i^j(\theta) = w^j(\theta)$ and $t_i^j(\theta) = t^j(\theta)$ for $i = 1;2$ and $j = a;b$ and for almost all $\theta \in [0;1]$.

We now give a brief intuition: workers employed in the complex task are paid their expected marginal productivities, so there is no incentive to deviate to change the allocation of workers in the complex task. Consulting Figure 3, which depicts the equilibrium wages in the case with $\pi^a < \pi^b$ we see that workers from the group with the lower fraction of investors are paid less than their marginal contribution to output in the simple task, while workers from the other group are paid more[20]. However, the affirmative action policy means that a firm can not change the ratio of $a-$workers to $b-$workers in the simple task and eliminating the multiplier from (2.20) we get

$$y_1(\mathbf{b}(\pi);1)\sum_{j=a;b}\lambda^j p(\hat{\theta}^j(\pi);\pi^j) = y_2(\mathbf{b}(\pi);1); \qquad (2.22)$$

which means that the weighted average of the expected marginal productivities in the complex task for the critical agents equals the marginal productivity in the simple task. At this point we

[20] If $\pi^a < \pi^b$ then $\hat{\theta}^a(\pi) < \hat{\theta}^b(\pi)$; since otherwise the affirmative action constraint can not be satisfied. Hence $p(\hat{\theta}^a(\pi);\pi^a) < p(\hat{\theta}^b(\pi);\pi^b)$ and by inspection of (2.20) we see that $w^a(\theta) < y_2(\mathbf{b}(\pi);1)$ for $\theta < \hat{\theta}^a(\pi)$ and $w^b(\theta) > y_2(\mathbf{b}(\pi);1)$ for $\theta < \hat{\theta}^b(\pi)$:

note that $F_{\pi^a}(\hat\theta^a(\pi)) = F_{\pi^b}(\hat\theta^b(\pi))$ by the affirmative action constraint, so the left hand side of (2.22) is also the average wage in the simple task. Hence the average wage in the simple task is given by the marginal productivity, so there is no incentive to change the factor ratio. Finally, since the average wage in both tasks is the expected marginal productivity, firms make zero profits and have no incentives to change the scale of operation.

Using the wage schedules (2.21) we can express the expected gross benefits from undertaking the investment for an agent in group j when $\pi = (\pi^a;\pi^b)$ as

$$\begin{aligned} H^j(\pi) &= y_1(\hat\theta(\pi);1)\,p(\hat\theta^j(\pi);\pi^j)(F_q(\hat\theta^j(\pi)) - F_u(\hat\theta^j(\pi))) \\ &\quad + y_1(\hat\theta(\pi);1)\int_{\hat\theta^j(\pi)}^{1} p\left[\theta;\pi^j\right](f_q(\theta) - f_u(\theta))d\theta. \end{aligned} \tag{2.23}$$

Arguing as in the single group model we see that $\pi = (\pi^a;\pi^b)$ is an equilibrium if and only if

$$\pi^j = G(H^j(\pi)) \text{ for } j = 1;2 \tag{2.24}$$

From these expressions it is easily seen that any non-discriminatory equilibrium in the extended model is an equilibrium under affirmative action. This should be fairly obvious since if the groups behave the same way then the employers voluntarily treat the groups identically. To see it formally we observe that the multiplier in the conditions (2.20) must be zero when $\pi^a = \pi^b$; so both equations in (2.24) reduces to the fixed point equation for the single group model.

If there are asymmetric equilibria under affirmative action, inspection of (2.21) reveals that the wage schedule for the group with the lower fraction of investors will be uniformly below the wage schedule for the other group. Hence, wage discrimination persists in our model unless the policy forces the economy to an equilibrium where the fractions of agents who invest are the same in both groups. For this reason we will say that an equilibrium is discriminatory unless $\pi^a = \pi^b$.

Intuitively, affirmative action makes it harder to sustain discrimination since it pushes up wages in the simple task for workers from the group with the higher fraction of investors and pushes down wages in the simple task for the discriminated group. However, as we show next, equilibria with discrimination may exist even under affirmative action.

Proposition 7. *Fix $y; f_q\ f_u$ and $(\lambda^a;\lambda^b) \in (\mathrm{int}\,\Delta^2)$: Then there exists some strictly increasing distribution function G with $G(0) > 0$ such that the model with affirmative action has an equilibrium $(\hat\pi^a;\hat\pi^b)$ with $\hat\pi^a < \hat\pi^b$:*

The result can be strengthened in several directions. First, it should be clear that we get multiplicity for a large set of distribution functions. To see this one notes that $\mu^a < \mu^b$ only if $H^a(\mu) < H^b(\mu)$; which since H^a and H^b are continuous means that there is an open set U containing $(\mu^a; \mu^b)$ such that the expected benefits of investment for members in group b exceeds the benefits for members in group a for all $\pi \in U$. Also, the argument only relies on existence of a function G that takes on particular values at a few points, which means that assumptions about its curvature will not be enough to get any sufficient conditions for ruling out discriminatory equilibria.

This result has the important implication that we have to compare equilibria setwise. A natural way to make such a setwise comparison is to compare the "worst case scenarios", that is, compare the most discriminatory equilibrium under each regime with each other.

If we take "most discriminatory equilibrium" to mean the equilibrium where the distance between π^a and π^b is the largest, it follows from Proposition 4 that this is the equilibrium with the lowest fraction of investors in the discriminated group and the highest fraction in the other group. Now, if the model allows a "corner equilibrium", it follows trivially that the fraction of investors in the discriminated group must increase relative to this equilibrium if affirmative action is introduced: if $G(0) > 0$ there is a positive fraction of agents from both groups who invest in any equilibrium. Hence the wage schedule for both groups will be strictly increasing under affirmative action and there will consequently be some (possibly very small) monetary incentives to invest for agents in both groups in any equilibrium under affirmative action.

One may conjecture that this holds true in general. After all, comparing the wage schemes under the different regimes it looks as if affirmative action increases the incentives to invest for the discriminated group and decreases the incentives for the other group. The complementarities between groups should then guarantee that this holds true even if one takes the full equilibrium effects into account. The second part of this argument turns out to be right: given that the direct effect of affirmative action on incentives (holding the fraction of investors fixed in each group) increases (decreases) the benefits of investment in the discriminated (other) group, then the fraction of investors in the discriminated group must be higher under affirmative action than in the "worst" equilibrium without the policy. What fails in general is the first part: due to changes in the factor ratio the direct effect may go the wrong way for one of the groups (but not for both).

However, if the discriminated group is small enough, the effect on the factor ratio is negligible and in this case we can show that the direct effect of affirmative action is what we intuitively

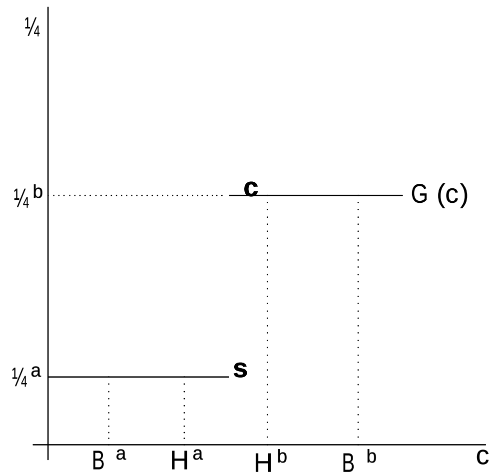

Figure 2.4: Cost distribution with no effect on behavior

expect and the logic above goes through.

Proposition 8. Let $(\pi^a;\pi^b)$ be the most discriminatory equilibrium in the model of Section 2.4 and let $(\hat{\pi}^a;\hat{\pi}^b)$ be the most discriminatory equilibrium in the model with affirmative action, where group a is the discriminated group in both cases. Then $\pi^a < \hat{\pi}^a$ if λ^a is sufficiently small.

2.5.2. Examples: Welfare Effects of Affirmative Action

We now consider simple distribution functions of the form:

$$G(c) = \begin{cases} \pi^a & \text{if } c < \underline{c} \\ \pi^b & \text{if } c \in [\underline{c}; \bar{c}] \\ 1 & \text{if } c > \bar{c} \end{cases} \tag{2.25}$$

where $\bar{c}$ will be larger than any possible benefits of investment.

Example 1: The point of this example is that the disadvantaged group may be made worse off by affirmative action even if the starting point is the most severe form of discrimination possible. The construction is illustrated in Figure 4.

Using the notation from Section 2.4 we let $B^j(\pi)$ denote the expected gross benefits of investment in the model without affirmative action. Now, if we fix $\pi^b > 0$ and π^a is sufficiently small the solution to the task assignment problem for the firm is to assign all workers from group a to the simple task, while some workers from group b will be assigned to the complex task. Hence

$\mathcal{B}^a(\pi^a,\pi^b) = 0 < \mathcal{B}^b(\pi^a,\pi^b)$ for π^a sufficiently small. Thus, if $\bar{c}$ in (2.25) is in between 0 and $\mathcal{B}^b(\pi^a,\pi^b)$ we have that (π^a,π^b) is an equilibrium in the model without affirmative action. Next we proceed in the spirit of the argument of the proof of Proposition 6 and argue that for π^a small enough and for the right choice of $\bar{c}$ this will also be an equilibrium with affirmative action. To see this we note that $H^a(\pi^a,\pi^b) \to 0$ as $\pi^a \to 0$ while $H^b(\pi^a,\pi^b) \to H^b(0,\pi^b) > 0$ as $\pi^a \to 0$. Hence there exists some $\pi^a > 0$ such that

$$0 = \mathcal{B}^a(\pi^a,\pi^b) < H^a(\pi^a,\pi^b) < \min(\mathcal{B}^b(\pi^a,\pi^b), H^b(\pi^a,\pi^b)). \tag{2.26}$$

Choosing $\bar{c}$ in (2.25) between $H^a(\pi^a,\pi^b)$ and the minimum of $\mathcal{B}^b(\pi^a,\pi^b)$ and $H^b(\pi^a,\pi^b)$ we have that (π^a,π^b) is an equilibrium both with and without the policy. The change in expected utility for an agent of group j who invests is

$$\begin{aligned}\Delta W^j_{INV} = {} & \left[p(\hat{\theta}^j(\pi),\pi^j)\, y_1(\hat{\theta}(\pi),1) F_q(\hat{\theta}^j(\pi)) + \int_{\hat{\theta}^j(\pi)}^1 y_1(\hat{\theta}(\pi),1)\, p(\theta,\pi^j) f_q(\theta)\, d\theta \right] - \\ & \left[y_2(\tilde{\theta}(\pi)) F_q(\tilde{\theta}^j(\pi)) + \int_{\tilde{\theta}^j(\pi)}^1 y_1(\tilde{\theta}(\pi),1)\, p(\theta,\pi^j) f_q(\theta)\, d\theta \right];\end{aligned} \tag{2.27}$$

and the change in expected utility for agents who do not invest is derived symmetrically[21]. Note that the factor ratio in general changes when the policy is introduced. This effect may go either way depending on the production function and the distribution functions for the signal. However, for this example this creates no problems: no matter what happens to the factor ratio we can always rely on the fact that under affirmative action $w^a(\theta) \leq y_1(\hat{\theta}(\pi),1)p(1,\pi^a)$ for all θ; so for π^a small enough the policy must decrease the expected utility for all agents in group a. The welfare effect for the other group is ambiguous, but since the set of feasible production plans with the policy is a strict subset of the feasible plans without the policy (remember that investment behavior is unchanged) output unambiguously decreases.

Example 2: In this example we show that it is indeed possible that affirmative action removes equilibria with discrimination and that the target group may be made better off.

Consider the benefits of investment if a fraction π^b would invest in both groups, $\mathcal{B}^j(\pi^a,\pi^b)$ in the model without the quota and $H^j(\pi^a,\pi^b)$ in the model with; both which equals the benefits of investment in the single group model if a fraction π^b invests, $\mathcal{B}(\pi^b)$. It is straightforward to

[21] Since we constructed the equilibria so that the fraction of investors remains the same we do not need to worry about agents who change their behaviour when the policy is introduced.

show that the benefits of investment for agents in one of the groups is monotonically decreasing in the fraction of investors in the other group, so $B^a(\pi^a;\pi^b) < B^a(\pi^b;\pi^b) = B(\pi^b) < B^b(\pi^a;\pi^b)$: As we argued above $B^a(\pi^a;\pi^b) = 0$ for some small enough $\pi^a > 0$; so for $\bar{c}$ chosen in between 0 and $B^b(\pi^a;\pi^b)$ and π^a small enough $(\pi^a;\pi^b)$ is an equilibrium. But if $\bar{c}$ is chosen in the interval $[0;H^a(\pi^a;\pi^b))$ then $(\pi^a;\pi^b)$ is not an equilibrium when affirmative action is introduced. Thus (assuming a is the smaller group we cannot switch to discrimination of the other group) the only remaining equilibrium candidate is one where a fraction of π^b invests in both groups. Since $\bar{c}$ can be chosen so that it is smaller than $B(\pi)$ there is a range of parameter values so that this is indeed an equilibrium. One can show that the wage for agents employed in the simple task will be higher in the symmetric equilibrium and this means that all agents in group a benefits from the policy. Since production increases the other group may or may not be made worse off.

By relatively standard continuity arguments both arguments extend to strictly increasing distributions as well.

2.5.3. Why a Quota in the Simple Task?

Most people probably think of real world quotas as targeted towards increasing the representation of minorities and women in good jobs and not as increasing white male employment in bad jobs. In our model however, the constraints in (2.18) impose restrictions both ways, so it may be argued that our analysis does not correspond to real world quotas.

Unfortunately, it is not possible to have a quota in one task only in our model. Labor market equilibrium wages simply fail to exist with such a quota. To understand this, note that whatever necessary conditions we have for an equilibrium with quotas in both tasks remains necessary for an equilibrium if one quota is removed: removing the quota in the simple task just increases the possibilities for deviations, but an equilibrium, if there is one, must still be immune against all deviations that are feasible with quotas in both tasks. But, consulting Figure 3 we see that if there is no quota in the simple task, then a firm would want to deviate by getting rid of the higher paid workers from group b; so there is no labor market equilibrium if $\pi^a \neq \pi^b$: Since only non-discriminatory (Nash) equilibria remains in the full game one could in principle interpret this as saying that affirmative action guarantees a color blind equilibrium. We are however uncomfortable with this interpretation. The logic relies on non-existence of equilibria in all continuation games where groups differ in their investment. We think that this is "abusing" the notion of Nash equilibrium and note the similarities to the use of "integer games" in the Nash implementation literature.

While we recognize that it would be desirable to have a model that can handle quotas in good jobs only we note that differences between the racial composition of the workforce and the racial composition of the community where the firm is located as well as differences in the racial composition in the workforce across job categories within the firm have been taken as evidence of discrimination in lawsuits[22]. Taken together this effectively implies a quota in each task.

2.6 Concluding Remarks

We have developed a framework where the effects of affirmative action policies on wages can be studied. There are at least two reasons why a model with endogenously determined wages is desirable when studying the impact of labor market quotas. First of all, if we are interested in distributional consequences of the policy, we need an idea about how wages respond. Second, it is difficult to reconcile exogenously fixed wages with rational behavior by the firms.

With competitively determined wages affirmative action seems more promising than in Coate and Loury (1993) in terms of improving incentives to invest in human capital for the discriminated group. Although a color blind equilibrium is not guaranteed, the partial equilibrium effect is typically to increase the incentives for the disadvantaged group[23]. General equilibrium effects are somewhat blurred by the fact that we compare sets of equilibria, but if the initial equilibrium without the policy is the most discriminatory, then the fraction of investors in the discriminated group must increase and group disparity will typically decrease[24].

Since wages are fixed exogenously, affirmative action always has the desired distributional effect to reduce group inequality in Coate and Loury (1993). The major lesson of this chapter is that this is not necessarily true with competitively determined wages, so inequality in earnings across groups may increase as a consequence of quotas in the labor market.

Our model has the novel feature that discrimination may occur in equilibrium even if there is a unique equilibrium in the corresponding model without observable group characteristics. We

[22] See Bolick (1996).

[23] It is a logical possibility that this conclusion is reversed if the input of labor in the complex task relative to the simple increases when the quotas are introduced. However, if the discriminated group is small, changes in factor proportions will be small, so in this case the incentives are unambiguously improved for the disadvantaged group.

[24] Again, changes in factor proportions may produce counterintuitive results. Thus, it is logically possible that the incentives to invest for the dominant group also increases.

interpret this as saying that discrimination is driven by specialization rather than being a coordination failure. One may object that economics of specialization as we know it is in the interest of all participants in the market, while agents in the discriminated group are worse off in our model. Still, if we think of the respective groups as "countries" selling labor on the world market we see that the forces at play are similar to forces that drive specialization in trade models with increasing returns: the existence of other groups with a high fraction of investors is what drives down the returns to investment relative a situation where the groups were separated.

Since discrimination has this specialization interpretation and since specialization is usually efficiency enhancing we may ask whether discrimination is efficient or not. This question is studied in Norman (1997), where it is shown that while all market equilibria in the model are constrained sub-optimal, discrimination as a phenomenon need not be. In particular, a utilitarian planner may want to discriminate between identical groups. Hence, given that transfers between groups are feasible and that the regulator cares about equality in income rather than equality in behavior, the regulator may actually want to discriminate.

2.7. Proofs

Proof of Proposition 1: (sufficiency) Suppose a firm deviates and plays strategy $\langle w_i', \xi_i' \rangle$ with actions on the implied outcome path $\langle w_i', t_i' \rangle$ different from $\langle w, t \rangle$ given by (3.12) and the cutoff rule with critical point solving (3.12) on a set of positive measure. Define $\Theta^h = \{\theta : w'(\theta) > w(\theta)\}$; $\Theta^l = \{\theta : w'(\theta) < w(\theta)\}$; $\Theta^e = \{\theta : w'(\theta) = w(\theta)\}$. Letting C' and S' denote the implied factor inputs for the deviator given that the other firm plays according to the equilibrium strategies we can express profits for the deviator as (using equations (2.3) and (2.4))

$$\Pi_d^i = y(C', S') - \int_{\theta \in \Theta^h} w'(\theta) f_\pi(\theta) d\theta - \frac{1}{2} \int_{\theta \in \Theta^e} w(\theta) f_\pi(\theta) d\theta; \text{ where} \quad (2.28)$$

$$C' = \int_{\theta \in \Theta^h} t'(\theta) \pi f_q(\theta) d\theta + \frac{1}{2} \int_{\theta \in \Theta^e} t'(\theta) \pi f_q(\theta) d\theta$$

$$S' = \int_{\theta \in \Theta^h} (1 - t'(\theta)) f_\pi(\theta) d\theta + \frac{1}{2} \int_{\theta \in \Theta^e} (1 - t'(\theta)) f_\pi(\theta) d\theta$$

We now multiply both sides in the second equation of (2.28) by $y_1(C,S)$ and substitute the identity $\pi f_q(\theta) = p(\theta;\pi) f_\pi(\theta)$. Similarly we multiply the third equation by $y_2(C,S)$ and get

$$\begin{aligned} y_1(C,S)C' &= \int_{\theta\in\Theta^h} t'(\theta)p(\theta;\pi)y_1(C,S)f_\pi(\theta)d\theta + \frac{1}{2}\int_{\theta\in\Theta^e} t'(\theta)p(\theta;\pi)y_1(C,S)f_\pi(\theta)d\theta \qquad (2.29)\\ y_2(C,S)S' &= \int_{\theta\in\Theta^h} (1-t'(\theta))y_2(C,S)f_\pi(\theta)d\theta + \frac{1}{2}\int_{\theta\in\Theta^e} (1-t'(\theta))y_2(C,S)f_\pi(\theta)d\theta. \end{aligned}$$

But $w(\theta) = \max\{y_2(C,S), p(\theta;\pi)y_1(C,S)\}$, where C,S are the implied factor inputs in the proposed equilibrium. Furthermore, $y(C',S') \le y_1(C,S)C' + y_2(C,S)S'$ by concavity and constant returns. Summing the equalities in (2.29) and using these two facts we find that $y(C',S') \le \int_{\theta\in\Theta^h} w(\theta)f_\pi(\theta)d\theta + \frac{1}{2}\int_{\theta\in\Theta^e} w(\theta)f_\pi(\theta)d\theta$ and substituting into (2.28) we get

$$\Pi^i_d \le \int_{\theta\in\Theta^h} (w(\theta) - w'(\theta))f_\pi(\theta)d\theta \le 0.$$

■

The necessity part will be proved from a sequence of intermediate results:

Lemma 1. Suppose $\langle w_i, \xi_i\rangle_{i=1,2}$ is a pair of best responses. Then $w_1(\theta) = w_2(\theta)$ for almost all $\theta \in [0,1]$

Proof. Suppose $\langle w_i, \xi_i\rangle_{i=1,2}$ are best responses and there is a set $\Theta \subseteq [0,1]$ with positive measure such that $w_i(\theta) - w_j(\theta) > 0$ for all $\theta \in \Theta$. Consider a deviation $\langle w_i', \xi_i'\rangle$ such that $\xi_i'(w_i', w_j)(\theta) = \xi_i(w_i, w_j)(\theta)$ for all θ; $w_i'(\theta) = w_i(\theta)$ for $\theta \in [0,1]\setminus\Theta$ and $w_i'(\theta) = (w_i(\theta) + w_j(\theta))/2$ for $\theta \in \Theta$. The distribution of available workers and task assignments on the outcome path are unchanged, so output is unchanged. The difference in expected profits is thus the difference in the total wage costs, $1/2\int_{\theta\in\Theta}(w_i(\theta) - w_j(\theta))f_\pi(\theta)d\theta > 0$; contradicting the hypothesis that $\langle w_i, \xi_i\rangle_{i=1,2}$ is a pair of best responses. ■

Lemma 2. Let t_i denote the implied task assignment rule on the equilibrium path for firms $i = 1,2$. Then there exists some $\hat{\beta}^i \in (0,1)$ such that $t_i(\theta) = 1$ for almost all $\theta > \hat{\beta}^i$ and $t_i(\theta) = 0$ for almost all $\theta < \hat{\beta}^i$ and for $i = 1,2$:

Proof. If the claim is false there are sets $\Theta^h, \Theta^l \subseteq [0,1]$ with positive measure such that $\theta^h > \theta^l$ for all $\theta^h, \theta^l \in \Theta^h \times \Theta^l$, $t_i(\theta^h) = 0$ for all $\theta^h \in \Theta^h$ and $t_i(\theta^l) = 1$ for all $\theta^l \in \Theta^l$. f_q and f_u are continuous, so f_π is continuous and we may w.l.o.g. assume $\int_{\theta\in\Theta^h} f_\pi(\theta)d\theta = \int_{\theta\in\Theta^l} f_\pi(\theta)d\theta > 0$. Consider the alternative task assignment rule,

$$t_i^a(\theta) = \begin{cases} 1 & \text{if } \theta \in \Theta^h \\ 0 & \text{if } \theta \in \Theta^l \\ t_i(\theta) & \text{otherwise} \end{cases}$$

Let $S_i; C_i$ and S_i^a, C_i^a be the factor inputs implied by t_i, t_i^a respectively. Since $\int_{\theta\in\Theta^h} f_\pi(\theta)d\theta = \int_{\theta\in\Theta^l} f_\pi(\theta)d\theta$ it follows that $S_i = S_i^a$. Since the deviation assigns to the complex task workers who are more likely to be productive it is intuitively rather clear that $C_i^a > C_i$. To see this we let $l(\theta) = f_q(\theta)/f_u(\theta)$ denote the likelihood ratio and note that

$$2(C_i^a - C_i)/\pi = \int_{\theta\in\Theta^h} f_q(\theta)d\theta - \int_{\theta\in\Theta^l} f_q(\theta)d\theta = \int_{\theta\in\Theta^h} l(\theta)f_u(\theta)d\theta - \int_{\theta\in\Theta^l} l(\theta)f_u(\theta)d\theta; \quad (2.30)$$

By the monotone likelihood ratio there exists θ^* such that $l(\theta) \geq l(\theta^*)$ for all $\theta \in \Theta^h$ and $l(\theta) \leq l(\theta^*)$ for all $\theta \in \Theta^l$, with at least one inequality strict. Hence $C_i^a - C_i > \pi l(\theta^*)(S_i^a - S_i)/2 = 0$, so output, and therefore also profits, are higher under t_i^a. ■

Lemma 3. *Let t_i denote the implied task assignment rule on the equilibrium path for firms $i = 1;2$ and let t be cutoff rule with critical point $\hat{\theta}(\pi)$. Then $t_i(\theta) = t(\theta)$ almost everywhere.*

Proof. By Lemmas 1 and 2 the problem of finding an optimal task assignment rule reduces to finding a solution to problem (2.5) in the main text. Since firms are facing identical problems we drop indices. Changing variables by defining $C = \pi(1 - F_q(\theta))$ and $S = \pi F_q(\theta) + (1-\pi)F_u(\theta)$ the problem can be restated as

$$\max_{c;s} y(C;S)$$
$$\text{subj. to } g(C;S) \equiv \pi - C - S + (1-\pi)F_u\left(F_q^{-1}\left(\frac{\pi - C}{\pi}\right)\right) \geq 0 \quad (2.31)$$

Using the inverse function theorem and differentiating we find that $\partial g/\partial C = -1 - (1-\pi)/\pi \cdot l(F_q^{-1}(\frac{\pi - C}{\pi}))$ and taking second derivatives we find that $\partial^2 g/\partial C^2 < 0$ while all other elements of the Hessian is zero by the linearity in S. Hence g is concave. Since y is concave the Kuhn-Tucker conditions are sufficient for a solution to (2.31) and necessity follows since concavity of g is sufficient for constraint qualification. Invoking the boundary conditions we see that any solution must be interior. Since the programs (2.5) and (2.31) are equivalent this completes the proof. ■

Lemma 4. Suppose $\langle w_1, w_2 \rangle$ is a pair of equilibrium wage schedules and let $\hat{\theta}(\pi)$ be the solution to (2.6). Then there is a pair (k_s, k_c) such that $w_i(\theta) = k_s$ for $i = 1, 2$ and for almost all $\theta < \hat{\theta}(\pi)$ and $w_i(\theta) = p(\theta; \pi) k_c$ for $i = 1, 2$ and for almost all $\theta > \hat{\theta}(\pi)$.

Proof. First we show that $w_i(\theta) = k_s$ for almost all $\theta < \hat{\theta}$. For contradiction assume that there are sets $\Theta^a, \Theta^b \subset [0, \hat{\theta}(\pi)]$ with strictly positive measure such that $w_i(\theta) < k$ for all $\theta \in \Theta^a$ and $w_i(\theta) \geq k$ for all $\theta \in \Theta^b$. Consider a unilateral deviation $\langle w_i^0, \xi_i^0 \rangle$ by firm i where

$$w_i^0(\theta) = \begin{cases} w_i(\theta) + \varepsilon & \text{for } \theta \in \Theta^a \\ 0 & \text{for } \theta \in \Theta^b \\ w_i(\theta) & \text{otherwise} \end{cases}. \tag{2.32}$$

By continuity of f_π we may w.l.o.g. assume $\int_{\theta \in \Theta^a} f_\pi(\theta) d\theta = \int_{\theta \in \Theta^b} f_\pi(\theta) d\theta > 0$, which implies that the input of both factors remains constant if ξ_i^0 is chosen so that task assignments on the outcome path are unchanged, which we assume. The difference in payoffs for the deviating firm is then just the difference in wage payments, i.e.

$$\Delta(\varepsilon) = \frac{1}{2} \left[\int_{\theta \in \Theta^b} w_i(\theta) f_\pi(\theta) d\theta - \int_{\theta \in \Theta^a} (w_i(\theta) + \varepsilon) f_\pi(\theta) d\theta \right]$$

Since $\lim_{\varepsilon \to 0} \Delta(\varepsilon) > 0$ there exists $\varepsilon > 0$ such that $\Delta(\varepsilon) > 0$, so for ε small enough the deviation is profitable. Symmetrically, suppose there are sets $\Theta^a, \Theta^b \subset [\hat{\theta}(\pi), 1]$ with strictly positive measure (where we again w.l.o.g. may assume $\int_{\theta \in \Theta^a} f_q(\theta) d\theta = \int_{\theta \in \Theta^b} f_q(\theta) d\theta$) such that $(w_i(\theta)/p(\theta; \pi)) < k$ for all $\theta \in \Theta^a$ and $(w_i(\theta)/p(\theta; \pi)) \geq k$ for all $\theta \in \Theta^b$. Again we consider a deviation according to (2.32). Since $p(\theta; \pi) = (\pi f_q(\theta)/f_\pi(\theta))$ we find that output is unchanged by a symmetric argument the deviation profitable for ε small enough. ■

Proof of Proposition 1: (necessity) Only the characterization of wages remains. From Lemma 4 the firms must offer wages that are identical almost everywhere and satisfy $w(\theta) = k_s$ for $\theta < \hat{\theta}(\pi)$ and $w(\theta) = p(\theta; \pi) k_c$ for $\theta > \hat{\theta}(\pi)$, for some real numbers k_s, k_c. It remains to be shown that $k_s = y_2(C, S)$ and $k_c = y_1(C, S)$. It is easy to show that if $k_s < y_2(C, S)$ and $k_c < y_1(C, S)$, then firms are making positive profits and a deviation where firm i offers $w_i^0(\theta) = w_i(\theta) + \varepsilon$ for all θ would be profitable for ε small enough. Also, if both inequalities would go the other way firms would make negative profits and a deviation to $w_i(\theta) = 0$ for all θ would be profitable. The two

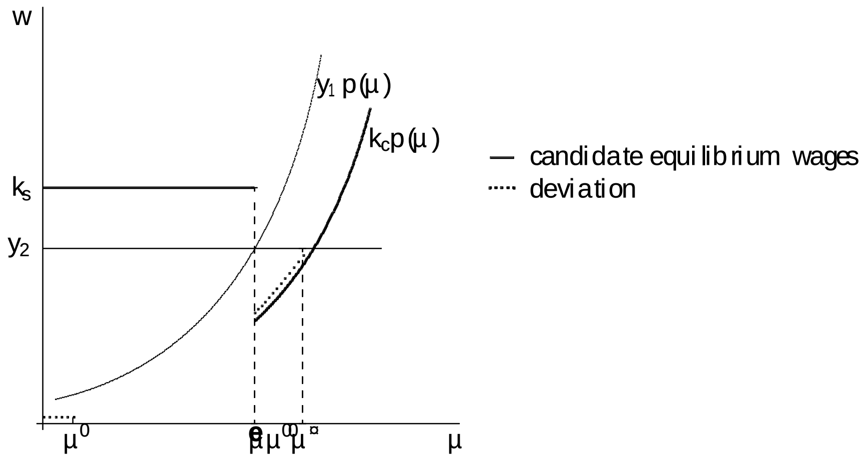

Figure 2.5: A profitable deviation

cases that require some work are when the inequalities work in opposite directions. The arguments are symmetric and we will only consider the case with $k_s > y_2(C;S)$ and $k_c < y_1(C;S)$. The idea is illustrated in Figure 6.

Recall that $y_1(C;S)p(\hat\theta(\pi);\pi) = y_2(C;S)$ by (2.6). Hence if $k_s > y_2(C;S)$ and $k_c < y_1(C;S)$ then $k_c p(\hat\theta(\pi);\pi) < k_s$ and there is an interval $(\hat\theta(\pi);\theta^*)$ such that $w_i(\theta) = p(\theta;\pi)k_c < k_s$ for all θ in this interval. We will show that it is better to dispose of some of the workers being paid k_s and attract cheaper workers with $\theta \in (\hat\theta(\pi);\theta^*)$ from the other firm. Let θ' solve $F_\pi(\theta') = F_\pi(\theta^*) - F_\pi(\hat\theta(\pi))$ and θ'' as solve $F_\pi(\theta'') - F_\pi(\hat\theta(\pi)) = (F_\pi(\theta^*) - F_\pi(\hat\theta(\pi)))/2$ and consider the deviation:

$$w_i'(\theta) = \begin{cases} 0 & \text{for } \theta \in [0;\theta'] \\ w_i(\theta) + \varepsilon & \text{for } \theta \in [\hat\theta(\pi);\theta^*) \\ w_i(\theta) & \text{for } \theta \in [\theta^*;\infty) \cup [\theta';\hat\theta) \end{cases} \quad \text{and } t_i'(\theta) = \begin{cases} 0 & \text{for } \theta \in [0;\theta'') \\ 1 & \text{for } \theta \in [\theta'';\infty) \end{cases}$$

By construction, the input of simple labor is unchanged[25]. The change in effective units of complex labor is given by $C' - C = \frac{\pi}{2}(F_q(\theta^*) - 2F_q(\theta'') + F_q(\hat\theta))$ and using $F_\pi(\theta^*) - 2F_\pi(\theta'') + F_\pi(\hat\theta) = 0$ it is easy to show that $C' - C > 0$. Thus, output increases and the difference in profits must be

[25] $F_\pi(\theta'') - F_\pi(\hat\theta(\pi)) + 1/2(F_\pi(\hat\theta(\pi))) - F_\pi(\theta')$ is the input of labor in the simple task after the deviation.

larger than the difference in the wage costs, so

$$\Delta(\varepsilon) > \frac{1}{2}\int_0 w_i(\theta) f_\pi(\theta)d\theta - \frac{1}{2}\int_{\theta^0} w_i(\theta) f_\pi(\theta)d\theta - \int_{\hat\theta}^{\theta^*} (w_i(\theta) - \varepsilon) f_\pi(\theta)d\theta - \frac{1}{2}\int_{\theta^*} w_i(\theta) f_\pi(\theta)d\theta =$$

$$= \frac{1}{2} k_s F_\pi\left(\theta^0\right) - \frac{1}{2}\int_{\hat\theta}^{\theta^*} p(\theta;\pi) k_c f_\pi(\theta)d\theta - \int_{\hat\theta}^{\theta^*} \varepsilon f_\pi(\theta)d\theta$$

Recall that $p(\theta;\pi)k_c < k_s$ for $\theta \in (\hat\theta;\theta^*)$ and $F_\pi\left(\theta^0\right) = F_\pi(\theta^*) - F_\pi(\hat\theta)$. Thus:

$$\frac{1}{2} k_s F_\pi\left(\theta^0\right) = \frac{1}{2}\int_{\hat\theta}^{\theta^*} k_s f_\pi(\theta)d\theta > \frac{1}{2}\int_{\hat\theta}^{\theta^*} p(\theta;\pi) k_c f_\pi(\theta)d\theta.$$

Hence, $\lim_{\varepsilon\to 0}\Delta(\varepsilon) > 0$ and there exists $\varepsilon > 0$ such that the deviation is profitable. The case with $k_s < y_2(C;S)$ and $k_c > y_1(C;S)$ can be treated symmetrically and Proposition 1 follows. ■

Proposition 3: We will prove this result from a pair of intermediate results:

Lemma 5. Suppose that $y : \mathbb{R}^2_+ \to \mathbb{R}$ is strictly concave in both arguments and homogenous of degree 1. Then for each $\pi \in (0;1]$ there exists a unique $\hat\theta(\pi) \in (0;1)$ such that (2.6) is satisfied.

Proof. Let $D : (0;1) \times (0;1] \to \mathbb{R}$ be defined as

$$D(\theta;\pi) = p(\theta;\pi) - \frac{y_2(\rho(\theta;\pi);1)}{y_1(\rho(\theta;\pi);1)}, \tag{2.33}$$

where $\rho(\theta;\pi) = \pi(1 - F_q(\theta))/F_\pi(\theta)$. Since y is homogenous of degree one $\hat\theta(\pi)$ solves the first order condition for the task assignment problem, equation (2.6) in Section 2.3, if and only if $D(\hat\theta(\pi);\pi) = 0$. It follows from the (strict) monotone likelihood ratio property that $p(\theta;\pi)$ is strictly increasing in θ for any $\pi > 0$. Fixing S, $y_1(C;S)$ is strictly decreasing and $y_2(C;S)$ is strictly increasing in C. Since F_q and F_u are strictly increasing $\rho(\theta;\pi)$ is strictly decreasing in θ. Consequently, $y_1(\rho(\theta;\pi);1)$ is strictly increasing and $y_2(\rho(\theta;\pi);1)$ is strictly decreasing in θ. Hence, the ratio $y_2(\rho(\theta;\pi);1)/y_1(\rho(\theta;\pi);1)$ is strictly decreasing, implying that $D(\theta;\pi)$ is strictly increasing in θ. Thus, there can be at most one solution $D(\hat\theta(\pi);\pi) = 0$. To show that a solution exists for any $\pi > 0$ we note that $0 < p(0;\pi) < p(1;\pi) < 1$ for any $\pi > 0$. Since F_q and F_u are cdfs it is easy to check that $\lim_{\theta\to 0}\rho(\theta;\pi) = \infty$ and $\lim_{\theta\to 1}\rho(\theta;\pi) = 0$. Using the Inada conditions, constant returns to scale and standard limit laws

$$\lim_{\theta\to 0}\frac{y_2(\rho(\theta;\pi);1)}{y_1(\rho(\theta;\pi);1)} = \lim_{\theta\to 0}\frac{y_2\left(1;\frac{1}{\rho(\theta;\sigma)}\right)}{y_1(\rho(\theta;\pi);1)} = \frac{\lim_{x\to 0} y_2(1;x)}{\lim_{z\to\infty} y_1(z;1)} = \infty :$$

Symmetrically, $\lim_{\theta\to\infty} y_2(\rho(\theta;\pi);1)/y_1(\rho(\theta;\pi);1) = 0$. Thus, $\lim_{\theta\to 0} D(\theta;\pi) = -\infty$ and $\lim_{\theta\to\infty} D(\theta;\pi) = p(\infty;\pi) > 0$: The result follows. ■

Lemma 6. $\hat{\theta}$ and r (defined in equation (2.8)) satisfy the following properties:

1. $\hat{\theta}$ is continuously differentiable on $(0;\infty)$:
2. $\lim_{\pi\to 0+} r(\pi) = 0$
3. r is monotonically increasing in π

Proof. 1) Since $D_1(\theta;\pi) > 0$ for each $\pi > 0$ this follows from the implicit function theorem. 2) $p(\hat{\theta}(\pi);\pi) - \frac{y_2(r(\pi);1)}{y_1(r(\pi);1)} = 0$ must hold for each $\pi > 0$. But $\lim_{\pi\to 0} p(\hat{\theta}(\pi);\pi) \leq \lim_{\pi\to 0} p(\infty;\pi) = 0$ so $\lim_{\pi\to 0}\frac{y_2(r(\pi);1)}{y_1(r(\pi);1)} = 0$; which implies that $\lim_{\pi\to 0} r(\pi) = 0$: 3) For contradiction suppose $r(\pi) < r(\pi')$ for $\pi > \pi'$: Since (2.6) must hold for both π and π' and y is strictly concave in both arguments it follows that $p(\hat{\theta}(\pi);\pi) < p(\hat{\theta}(\pi');\pi')$: But $p(\theta;\pi)$ is increasing π, so $\hat{\theta}(\pi) < \hat{\theta}(\pi')$: Plugging this into equation (2.8), it follows that $r(\pi) > r(\pi')$; a contradiction. ■

Proof of Proposition 3: The equilibria are characterized as fixed points of $G \circ \mathcal{B} : [0;1] \to [0;1]$; where $\mathcal{B}$ is defined by (2.10) in Section 2.3. By Lemma 6, $\hat{\theta}$ (and therefore also r) are continuously differentiable on $(0;\infty)$. Since $\mathcal{B}$ is a composition of continuously differentiable functions, $\mathcal{B}$ is a continuous in π on $(0;\infty)$: Rewriting (2.6) in terms of $\hat{\theta}(\pi)$ and $r(\pi)$ we see that

$$p(\hat{\theta}(\pi);\pi)y_1(r(\pi);1) = y_2(r(\pi);1) \tag{2.34}$$

must hold for each $\pi \in (0;\infty)$: For $\pi = 1$ $p(\theta;1) = 1$ for all θ; so task assignments are indeterminate. However, $r(1)$ must nevertheless satisfy $y_1(r(1);1) = y_2(r(\pi);1)$ (one way of achieving this is by a cutoff rule). Since all workers are equally productive in both tasks, $w(\theta) = y_1(r(\pi);1) = y_2(r(\pi);1)$ for all θ and the benefits of investment is $\mathcal{B}(1) = 0$: It is easy to verify that $\lim_{\pi\to 1} r(\pi) = r(1)$ by use of (2.34) and using (2.10) it follows that $\lim_{\pi\to 1}\mathcal{B}(\pi) = \mathcal{B}(1)$: The case with $\pi = 0$ is taken care of in the same way. No matter how workers are allocated, output is zero, implying $w(\theta) = 0$ for all θ. Hence $B(0) = 0$ and from (2.34) we have that

$$0 = \lim_{\pi \to 0} p(\hat{\theta}(\pi);\pi) = \lim_{\pi \to 0} \frac{y_2(r(\pi);1)}{y_1(r(\pi);1)}. \tag{2.35}$$

Using the boundary conditions on y we see that the only possibility for this to be satisfied is if $\lim_{\pi \to 0} r(\pi) = 0$; $\lim_{\pi \to 0} y_2(r(\pi);1) = 0$ and $\lim_{\pi \to 0} y_1(r(\pi);1)p(\hat{\theta}(\pi);\pi) = 0$. $F_q(\theta) - F_u(\theta)$ and $\int_{\hat{\theta}(\pi)}^{1} p(\theta;\pi)(f_q(\theta) - f_u(\theta))d\theta$ are bounded below and above, so it follows from (2.10) that $\lim_{\pi \to 0} B(\pi) = 0 = B(0)$ establishing continuity of $G \circ B$ on $[0,1]$. For $\pi \in (0,1)$, $0 < \hat{\theta}(\pi) < 1$ and $p(\theta;\pi)$ is strictly increasing in θ, so

$$\int_{\hat{\theta}(\pi)} p(\theta;\pi)(f_q(\theta) - f_u(\theta))d\theta > p(\hat{\theta}(\pi);\pi)\left[F_u(\hat{\theta}(\pi)) - F_q(\hat{\theta}(\pi))\right].$$

Hence $B(\pi) > 0$ for all $\pi \in (0,1)$. Since $G(0) > 0$ the intermediate value theorem guarantees that there is at least one fixed point of $G \circ B$ and, directly from the assumption that $G(0) > 0$ this must be a non trivial equilibrium. ■

Proposition 4: Again we will proceed by proving a sequence of intermediate result first:

Lemma 7. Suppose $\pi^a > \pi^{a'}$. Then $\hat{\theta}^b(\pi^a,\pi^b) \geq \hat{\theta}^b(\pi^{a'},\pi^b)$.

Proof. Let $r, r', \hat{\theta}^j$ and $\hat{\theta}^{j'}$ denote $r(\pi^a,\pi^b)$, $r(\pi^{a'},\pi^b)$, $\hat{\theta}^j(\pi^a,\pi^b)$ and $\hat{\theta}^a(\pi^{a'},\pi^b)$ respectively. Suppose for a contradiction that $\pi^a > \pi^{a'}$, but $\hat{\theta}^b < \hat{\theta}^{b'}$. This implies that $\hat{\theta}^b > 0$ and $\hat{\theta}^{b'} < 1$ and from the optimality condition (2.13) it follows that $p(\hat{\theta}^b;\pi^b)y_1(r;1) \geq y_2(r;1)$ and $p(\hat{\theta}^{b'};\pi^b)y_1(r';1) \leq y_2(r';1)$. Combining these conditions we get

$$\frac{y_2(r;1)}{y_1(r;1)} \leq p(\hat{\theta}^b;\pi^b) < p(\hat{\theta}^{b'};\pi^b) \leq \frac{y_2(r';1)}{y_1(r';1)},$$

which implies that $r < r'$. Using (2.14) we see that if $\hat{\theta}^b < \hat{\theta}^{b'}$, then $\hat{\theta}^a > \hat{\theta}^{a'}$ in order for $r < r'$. But then $p(\hat{\theta}^a;\pi^a) > p(\hat{\theta}^{a'};\pi^{a'})$, $y_1(r;1) > y_1(r';1)$ and $y_2(r;1) < y_2(r';1)$. Since $\hat{\theta}^a > \hat{\theta}^{a'}$ implies that $\hat{\theta}^a > 0$ and $\hat{\theta}^{a'} < 1$ we also have that $p(\hat{\theta}^a;\pi^b)y_1(r;1) \geq y_2(r;1)$ and $p(\hat{\theta}^{a'};\pi^{a'})y_1(r;1) \leq y_2(r;1)$

from (2.13) and combining we get

$$0 \ge p(\bar{\theta}^a;\pi^a) - \frac{y_2(r;\infty)}{y_1(r;\infty)} > p(\bar{\theta}^{a\prime};\pi^a) - \frac{y_2(r';\infty)}{y_1(r';\infty)} \ge 0;$$

which is a contradiction. ■

Lemma 8. $r(\pi^a;\pi^b)$ is increasing in both arguments.

Proof. Suppose $\pi^a > \pi^{a\prime}$; but $r < r'$. Then $y_1(r;\infty) > y_1(r';\infty)$ and $y_2(r;\infty) < y_2(r';\infty)$ and, by Lemma 7, $\pi^a > \pi^{a\prime} \Rightarrow \bar{\theta}^b \ge \bar{\theta}^{b\prime}$; so

$$p(\bar{\theta}^b;\pi^b)y_1(r;\infty) - y_2(r;\infty) > p(\bar{\theta}^{b\prime};\pi^b)y_1(r';\infty) - y_2(r';\infty).$$

Suppose $1 \ge \bar{\theta}^b > \bar{\theta}^{b\prime} \ge 0$. Then, the left hand side must (applying (2.13)) be less than or equal to zero, while the right hand side must be as least zero, which is a contradiction. Next, suppose that $1 > \bar{\theta}^b = \bar{\theta}^{b\prime} > 0$. Now both the left and right hand side must be equal to zero, which again is a contradiction. We conclude that the only possibilities left are if $\bar{\theta}^b = \bar{\theta}^{b\prime} = 1$ or $\bar{\theta}^b = \bar{\theta}^{b\prime} = 0$. Hence, if $r < r'$ and $\pi^a > \pi^{a\prime}$; this can only happen if $\bar{\theta}^a > \bar{\theta}^{a\prime}$, so

$$p(\bar{\theta}^a;\pi^a)y_1(r;\infty) - y_2(r;\infty) > p(\bar{\theta}^{a\prime};\pi^a)y_1(r';\infty) - y_2(r';\infty);$$

which is a contradiction since the left hand side must be less than or equal to zero and the right hand side must be at least zero in order for (2.13) to hold. ■

Lemma 9. $r(\pi^j;\pi^k)$ is strictly increasing in π^j in the range where $\bar{\theta}^j(\pi^j;\pi^k) < 1$ for $j,k = a,b$.

Proof. Arguing as in Lemma 8 we find that if $\pi^a > \pi^{a\prime}$ and $r = r'$; then $\bar{\theta}^b = \bar{\theta}^{b\prime}$. We note from (2.14) that if $r = r'$, $\bar{\theta}^b = \bar{\theta}^{b\prime}$ and $\pi^a > \pi^{a\prime}$; then it must be that either $\bar{\theta}^a = \bar{\theta}^{a\prime} = 1$ or $\bar{\theta}^a < \bar{\theta}^{a\prime} \le 1$; but the second case is a contradiction for the same reasons as in the proof of Lemma 8. ■

Proof of Proposition 4: Suppose $\pi^a > \pi^{a\prime}$ and consider the difference in gross benefits of investment given by (2.16), $\Delta B^b = B^b(\pi^a;\pi^b) - B^b(\pi^{a\prime};\pi^b)$. By Lemma 8, $r \ge r'$, so $y_1(r;\infty) \le y_1(r';\infty)$ and $y_2(r;\infty) \ge y_2(r';\infty)$. Hence

$$B^b(\pi^a;\pi^b) = y_2(r;\infty)(F_q(\bar{\theta}^b) - F_u(\bar{\theta}^b)) + y_1(r;\infty)\int_{\bar{\theta}^b} p(\theta;\pi^b)(f_q(\theta) - f_u(\theta))d\theta \le$$

$$\leq y_2(r';1)(F_q(\hat{\theta}^b) - F_u(\hat{\theta}^b)) + y_1(r';1)\int_{\hat{\theta}^b} p\left(\theta;\pi^b\right)(f_q(\theta) - f_u(\theta))d\theta;$$

which implies that

$$\Delta B^b \leq y_2(r';1)[F_q(\hat{\theta}^b) - F_u(\hat{\theta}^b) - F_q(\hat{\theta}^{b'}) + F_u(\hat{\theta}^{b'})] - y_1(r';1)\int_{\hat{\theta}^{b'}}^{\hat{\theta}^b} p\left(\theta;\pi^b\right)(f_q(\theta) - f_u(\theta))d\theta;$$

where we have used that $\hat{\theta}^b \geq \hat{\theta}^{b'}$. Note that $p\left(\theta;\pi^b\right) \geq p(\hat{\theta}^{b'};\pi^b)$ and if $\hat{\theta}^{b'}$ is interior we have that the first order condition $p(\hat{\theta}^{b'};\pi^b)y_1 = y_2$ must hold[26]. Thus

$$\Delta B^b \leq y_2(r';1)\left(F_q(\hat{\theta}^b) - F_u(\hat{\theta}^b) - F_q(\hat{\theta}^{b'}) + F_u(\hat{\theta}^{b'}) - \int_{\hat{\theta}^{b'}}^{\hat{\theta}^b}(f_q(\theta) - f_u(\theta))d\theta\right) = 0;$$

which proves that B^j is weakly decreasing in π^k. To show that B^j is strictly decreasing in the range where solutions are interior one notes that $r > r'$ in this case. ■

Proof of Proposition 5: Assume that there is an equilibrium with $\pi^a = \alpha$. Such an equilibrium exists if and only if there is some π^b such that $(\alpha,\pi^b) = (G_\alpha(B^a(\alpha,\pi^b)), G_\alpha(B^b((\alpha,\pi^b))))$. Fixing π^b we note that $\alpha = G_\alpha(B^a(\alpha,\pi^b))$ is satisfied if and only if $\hat{\theta}^a(\alpha,\pi^b) = 1$. Now fix $\pi^a = \alpha$ and note that $G_\alpha(B^b(\alpha,0)) = G_\alpha(B^b(\alpha,1)) = \alpha$ (the arguments are exactly as in the proof of Proposition 3) and that any $\pi^b \in (\alpha,1)$ will imply strictly positive benefits of investment so that $G_\alpha(B^b(\alpha,\pi^b)) > \alpha$. Let $\pi^b(\alpha) > \alpha$ be the largest fraction in group b such that $G_\alpha(B^b(\alpha,\pi^b(\alpha))) = \pi^b(\alpha)$. It only remains to show that for α sufficiently small, $\hat{\theta}^a(\alpha,\pi^b(\alpha)) = 1$ is indeed optimal for the firms. To see this note that (2.13) for group a then reduces to

$$p(1;\alpha) \leq \frac{y_2\left(e\left(\alpha,\pi^b(\alpha)\right);1\right)}{y_1\left(e(\alpha,\pi^b(\alpha));1\right)} = p(\hat{\theta}^b(\alpha,\pi^b(\alpha));\pi^b(\alpha)))$$

Since $p(\alpha;1) \to 0$ as $\alpha \to 0$ the only way the result could fail would be if $e\left(\alpha,\pi^b(\alpha)\right) \to 0$. But then $\pi^b(\alpha) \to 0$ as $\alpha \to 0$. However, $\pi^b(0) > 0$ by the assumption that the single group model admits

[26] If $\hat{\theta}^{b'}$ is not interior it follows directly from the inequality abve that the claim is true.

a larger equilibrium and one verifies that $G_{\alpha}\left(\hat{B}^b\left(\alpha;\pi^b(0)\right)\right) > \pi^b(0)$: Hence $\pi^b(\alpha) > \pi^b(0)$; for each $\alpha > 0$; which is a contradiction against hypothesis that $\pi^b(\alpha) \to 0$: ■

Proof of Proposition 6 (sufficiency) Consider a deviation $< w^a_d; w^b_d; \xi^a_d; \xi^b_d >$ and let the implied task assignment rules on the outcome path be t^a_d and t^b_d. Define $\Theta^h_j = \{\theta : \hat{w}^a(\theta) > w^a(\theta)\}$, $\Theta^l_j = \{\theta : \hat{w}^a(\theta) < w^a(\theta)\}$; $\Theta^e_j = \{\theta : \hat{w}^a(\theta) = w^a(\theta)\}$ for $j = a;b$: Let C and S the factor inputs in the candidate equilibrium and C_d, S_d be the factor inputs for the deviator (computed in analogy to (2.28)). The profits for the deviator, Π_d; are,

$$\Pi_d = y(C_d;S_d) - \sum_{j=a;b} \lambda^j \left[\int_{\theta\in\Theta^h_j} w^j_d(\theta) f_{\pi}(\theta)d\theta + \frac{1}{2}\int_{\theta\in\Theta^e_j} w^j(\theta) f_{\pi}(\theta)d\theta \right]$$

$$C_d = \sum_{j=a;b} \lambda^j \left[\int_{\theta\in\Theta^h_j} t^j(\theta)\pi^j f_q(\theta) + \frac{1}{2}\int_{\theta\in\Theta^e_j} t^j(\theta)\pi^j f_q(\theta) \right]$$

$$S_d = \sum_{j=a;b} \lambda^j \left[\int_{\theta\in\Theta^h_j} \left(1 - t^j(\theta)\right) f_{\eta_j}(\theta) + \frac{1}{2}\int_{\theta\in\Theta^e_j} \left(1 - t^j(\theta)\right) f_{\eta_j}(\theta) \right]$$

$w^j(\theta) = y_1(C;S)p\left(\theta;\pi^j\right)$ for $\theta \geq \hat{\theta}^j$ and $w^j(\theta) = y_1(C;S)p(\hat{\theta}^j;\pi^j) > y_1(C;S)p\left(\theta;\pi^j\right)$ for $\theta < \hat{\theta}^j$; so we have that $y_1(C;S)\pi^j f_q(\theta) = y_1(C;S)p\left(\theta;\pi^j\right) f_{\pi_j}(\theta) \leq w^j(\theta) f_{\pi_j}(\theta)$: Hence

$$y_1(C;S)C_d \leq \sum_{j=a;b} \lambda^j \left[\int_{\theta\in\Theta^h_j} t^j_d(\theta)w^j(\theta) f_{\pi_j}(\theta)d\theta + \frac{1}{2}\int_{\theta\in\Theta^e_j} t^j_d(\theta)w^j(\theta) f_{\pi_j}(\theta)d\theta \right] \qquad (2.36)$$

Let $S^j_d = \lambda^j \int_{\theta\in\Theta^h_j} \left(1 - t^j(\theta)\right) f_{\eta_j}(\theta) + \frac{1}{2}\int_{\theta\in\Theta^e_j} \left(1 - t^j(\theta)\right) f_{\eta_j}(\theta)$; so that $S^a_d + S^b_d$: From the affirmative action constraint it follows that any feasible deviation must have $S^a_d/S^b_d = \lambda^a/\lambda^b$: Note that $y_1(C;S)p(\hat{\theta}^j;\pi^j) \leq w^j(\theta)$; so

$$y_1(C;S)p\left(\hat{\theta}^j;\pi^j\right) S^j_d \leq \int_{\theta\in\Theta^h_j} w^j(\theta)\left(1 - t^j_d(\theta)\right) f_{\pi_j}(\theta)d\theta + \frac{1}{2}\int_{\theta\in\Theta^e_j} w^j(\theta)\left(1 - t^j_d(\theta)\right) f_{\pi_j}(\theta)d\theta$$

$$\geq \qquad (2.37)$$

>From the affirmative action constraint it follows that $y_2(C;S)S_d = y_2(C;S)\left(\lambda^a + \lambda^b\right) S^b_d/\lambda^b$ and combining with (2.37) and (2.22) we get:

$$y_2(C;S)S_d \le \sum_{j=a;b} \lambda^j \left[\int_{\theta \in \Theta_j^h} w^j(\theta)\left(1 - t_d^j(\theta)\right) f_{\pi^j}(\theta)d\theta + \frac{1}{2}\int_{\theta \in \Theta_j^e} w^j(\theta)\left(1 - t_d^j(\theta)\right) f_{\pi^j}(\theta)d\theta \right] \tag{2.38}$$

Summing over (2.36) and (2.38) we get

$$y_1(C;S)C_d + y_2(C;S)S_d \le \sum_{j=a;b} \lambda^j \left[\int_{\theta \in \Theta_j^h} w^j(\theta) f_{\pi^j}(\theta)d\theta + \frac{1}{2}\int_{\theta \in \Theta_j^e} w^j(\theta) f_{\pi^j}(\theta)d\theta \right] \tag{2.39}$$

At this point it is just to observe that concavity and constant returns imply that $y(C_d;S_d)$ is less than or equal to the right hand side of (2.39) to finish the proof. ■

The necessity part is proved using the following steps.

Lemma 10. Suppose $\left\langle w_i^a; w_i^b; \tau_i^a; \tau_i^b \right\rangle_{i=1,2}$ is a pair of best responses. Then;

1. $w_1^j(\theta) = w_2^j(\theta)$ for almost all $\theta \in [0;1]$; $j = a;b$.
2. Firms earn zero profits.
3. $\tau_1^j(w_1^j; w_1^j) = \tau_2^j(w_2^j; w_2^j) = t^j(\theta)$; $j = a;b$; for almost all $\theta \in [0;1]$; where $t^j(\cdot)$ is the cutoff task assignment rule with critical value $\hat{\theta}^j$:

Proof. (1) Same argument as Lemma 1. (2) Follows directly from constant returns. (3) Arguments are simple extensions of proofs of Lemmas 2 and 3 and omitted. ■

Lemma 11. Suppose $\left\langle w_1^j; w_2^j \right\rangle_{j=a;b}$ is a pair of equilibrium wage schedules and $\hat{\theta} = (\hat{\theta}^a; \hat{\theta}^b)$ is the solution to (2.20). Then there is a pair $\left\{ k_c^j; k_s^j \right\}$ for each group $j = a;b$ such that $w_i^j(\theta) = k_s^j$ for almost all $\theta \le \hat{\theta}^j$ and $w_i^j(\theta) = k_c^j p(\theta;\pi^j)$ for almost all $\theta > \hat{\theta}^j$; $j = a;b$:

Proof. To prove $w_i^j(\theta) = k_s^j$ for almost all $\theta \le \hat{\theta}^j$, $j = a;b$ we can use exactly the same argument as in Lemma 4 (using "within group deviations"), but since only deviations that satisfy the affirmative action constraint are feasible it turns out that it is more cumbersome to prove that $w_i^j(\theta) = k_c^j p(\theta;\pi^j)$ for almost all $\theta > \hat{\theta}^j$:

Suppose for contradiction that $w_i^j(\theta)/p(\theta;\pi^j)$ is not constant in θ for one group, which we w.l.o.g. take to be group a. Then we can find a positive measure set $\Theta^a \subset [\hat{\theta}^a;1]$ such that

$w_i^a(\theta)/p(\theta;\pi^a) > w_i^a(\theta^0)/p(\theta^0;\pi^a)$ for all $\theta \in \Theta^a;\ \theta^0 \in [\hat{\theta}^a, 1]\setminus\Theta^a$. Now let $\Theta^b \subset [\hat{\theta}^b, 1]$ be a measurable set such that $w_i^b(\theta)/p(\theta;\pi^b) \geq w_i^b(\theta^0)/p(\theta^0;\pi^b)$ for all $\theta \in \Theta^b;\ \theta^0 \in [\hat{\theta}^b, 1]\setminus\Theta^b$ and $\lambda^a \int_{\theta\in\Theta^a} f_{\pi^a}(\theta)d\theta = \lambda^b \int_{\theta\in\Theta^b} f_{\pi^b}(\theta)d\theta$. Given that the affirmative action constraint was initially satisfied such set always exists.

The proposed deviation consists in "firing" workers belonging to sets Θ^a and Θ^b (offering them wages lower than other firm) while reducing proportionally workers in the simple task to keep the factor ratio at the same level as in the candidate equilibrium. By construction, affirmative action constraint remains satisfied and qualified workers have been reduced by $R_C = \sum_{i=a;b} \lambda^i \int_{\theta\in\Theta^i} \pi^i f_q(\theta)d\theta$. To keep the factor ratio constant, the deviation must reduce workers in the simple task by $R_S = SR_C/C$. Let the initial output be denoted by $y = y(C;S)$; where $C;S$ are the factor inputs in the supposed equilibrium. By use of constant returns, the decrease in output by this downscaling is $yR_C/C = yR_S/S$.

Because of the affirmative action constraint the decrease in input of workers in the simple task must be $\lambda^j R_S$ in groups $j = a;b$. There is an infinity of ways of achieving this: one is to let θ_d^a, θ_d^b solve $\int_{[0,\theta_d^j]} f_{\pi^j}(\theta)d\theta = \lambda^j R_S$; $j = a;b$ and consider the deviation,

$$w_d^j(\theta) = \begin{cases} 0 & \text{if } \theta \in \Theta^j \cup [0;\theta_d^j]; \\ w^j(\theta) & \text{otherwise} \end{cases} \qquad j = a;b$$

Since the factor ratio is constant and we got rid of workers that were "expensive per productive unit" it is now intuitive, but cumbersome, to show that average wage per unit of production decrease, so that the deviation is profitable. In the simple task average wage per worker is constant by the first part of this lemma. Define the average wage per qualified worker in the candidate equilibrium as

$$\bar{k}_c^j = \int_{\theta\in[\hat{\theta}^j,1]} w(\theta) f_{\pi^j}(\theta)d\theta / [\pi^j(1 - F_q(\hat{\theta}^j))]$$

and let

$$\bar{k}_d^j = \int_{\theta\in[\hat{\theta}^j,1]\setminus\Theta^j} w_d^j(\theta) f_{\pi^j}(\theta)d\theta \Big/ \int_{\theta\in[\hat{\theta}^j,1]\setminus\Theta^j} \pi^j f_q(\theta)d\theta$$

be the average wage under the deviation. Using $w(\theta) f_{\pi^i}(\theta) = \pi^i f_q(\theta) \cdot w(\theta)/p(\theta;\pi^j)$ and $w_d^j(\theta) = w_i^j(\theta)$ for $\theta \in [\hat{\theta}^j, 1]\setminus\Theta^j$; $j = a;b$ we can rewrite average wages and derive the following inequality from the fact that the ratio $w(\cdot)/p(\cdot,\pi^j)$ is higher for $\theta \in \Sigma^j$, $j = a,b$

$$\frac{\int_{\theta\in[\hat\theta^j,1]\setminus\Sigma^j}\pi^j f_q(\theta)\frac{w(\theta)}{p(\theta)}d\theta+\int_{\theta\in\Sigma^j}\pi^j f_q(\theta)\frac{w(\theta)}{p(\theta)}d\theta}{\int_{\theta\in[\hat\theta^j,1]\setminus\Sigma^j}\pi^j f_q(\theta)d\theta+\int_{\theta\in\Sigma^j}\pi^j f_q(\theta)d\theta} > \frac{\int_{\theta\in[\hat\theta^j,1]\setminus\Sigma^j}\pi^j f_q(\theta)\frac{w(\theta)}{p(\theta)}d\theta}{\int_{\theta\in[\hat\theta^j,1]\setminus\Sigma^j}\pi^j f_q(\theta)d\theta}$$

i.e. $\bar{k}^j > \bar{k}^j_d$. But then the total wage costs for qualified workers is

$$W_d=\sum_i \bar{k}_d^i\int_{\theta\in[\hat\theta^i,1]\setminus\Sigma^i}\pi f_q(\theta)d\theta<(1-R_c)\sum_i \bar{k}^i\int_{\theta\in[\hat\theta^i,1]}\pi f_q(\theta)d\theta.$$

The deviation is profitable because wages decrease proportionally more than production. ■

Lemma 12. $k_s^j = k_c^j p(\hat\theta^j,\pi^j)$

Proof. If not, then $k_s^j > k_c^j p(\hat\theta^j,\pi^j)$ or $k_s^j < k_c^j p(\hat\theta^j,\pi^j)$ for some group j. In the first case it is simply to proceed as in the proof of Proposition 1: steal workers currently in the complex task from the other firm to replace some workers currently paid $w^j(\theta) = k_s^j$. The affirmative action constraint is unaffected, output constant and wage costs decrease, so the deviation is profitable.

Suppose instead $k_s^j < k_c^j p(\hat\theta^j,\pi^j)$ for, say, $j = a$. We then construct a deviation involving both groups that keeps output constant and respects the affirmative action constraint: let $\theta^{a\prime} < \hat\theta^a$ be such that $k_s^a/p\left(\theta^{a\prime},\pi^a\right) < k_c^a p(\hat\theta^a,\pi^a)$ and define $\theta^{a\prime\prime} \in (\theta^{a\prime},\hat\theta^a)$ as the value that divides the mass of workers with $\theta \in [\theta^{a\prime},\hat\theta^a]$ in equal parts ($\theta^{a\prime\prime}$ solves $[F_{\pi^a}(\hat\theta^a) - F_{\pi^a}(\theta^{a\prime})]/2 = F_{\pi^a}(\theta^{a\prime\prime}) - F_{\pi^a}(\theta^{a\prime})$). The deviation assigns workers on $[\theta^{a\prime},\theta^{a\prime\prime}]$ to the simple task, workers on $[\theta^{a\prime\prime},\hat\theta^a]$ to the complex task and, to keep mass of productive workers in the complex task constant, workers with $\theta \in [\hat\theta^a,\theta^{a*}]$, (where $\int_{\theta^{a\prime\prime}}^{\hat\theta^a} f_q(\theta)d\theta = \left(\int_{\hat\theta^a}^{\theta^{a*}} f_q(\theta)d\theta\right)/2$) are no longer hired. More exactly, we consider the deviation:

$$w_d^a(\theta)=\begin{cases}k_s^a+\varepsilon & \text{for } \theta\in[\theta^{a\prime},\hat\theta^a]\\ 0 & \text{for } \theta\in[\hat\theta^a,\theta^{a*}]\\ w_i^a & \text{otherwise}\end{cases}\quad\text{and}\quad t_d^a(\theta)=\begin{cases}0 & \text{for } \theta\in[0,\theta^{a\prime\prime}]\\ 1 & \text{for } \theta\in(\theta^{a\prime\prime},1]\end{cases}$$

Labor input in each task is unchanged and since some expensive workers have been replaced by workers that are "cheaper per productive unit" profits increase. However, this deviation alone would violate the affirmative action constraint: brute force computations show that the increase in the quantity of worker from group a hired in the complex task is

$$\lambda^{a\prime} - \lambda^{a} = \lambda^{a}\left(\int_{\theta^{a\prime\prime}}^{\bar\theta^{a}} f_{\pi^a}(\theta)d\theta - \frac{\int_{\bar\theta^{a}}^{\theta^{a*}} f_{\pi^a}(\theta)d\theta}{2}\right)$$

To satisfy the affirmative action constraint the deviation must also involve wages for group b. There are two cases to consider: 1) $k_s^b < k_c^b p(\bar\theta^b;\pi^b)$ 2) $k_s^b = k_c^b p(\bar\theta^b;\pi^b)$. In the first case it is simply to consider an identical deviation for group b and chose $\theta^{a\prime};\theta^{b\prime}$ in such a way that the affirmative action constraint is satisfied to get the result.

The second case is harder. Here the idea is that the quantity of workers employed can be increased without changing the effective input by attracting workers with relatively low signals. Formally, compute $\theta^{b\prime}$ and θ^{b*} in order to satisfy the following set of equations:

$$\int_{\bar\theta^{b}}^{\theta^{b\prime}} f_q(\theta)d\theta = \int_{\theta^{b*}}^{1} f_q(\theta)d\theta \qquad (2.40)$$

$$\int_{\bar\theta^{b}}^{\theta^{b\prime}} f_{\pi^b}(\theta)d\theta - \int_{\theta^{b*}}^{1} f_{\pi^b}(\theta)d\theta = \frac{\lambda^{a\prime}-\lambda^{a}}{\lambda^{a}} \qquad (2.41)$$

($\theta^{a\prime}$ sufficiently close enough to $\bar\theta^a$ guarantees existence of $\theta^{b\prime}$ and θ^{b*} solving the system). Let

$$w_d^b(\theta) = \begin{cases} k_s^b + \varepsilon & \text{for } \theta \in [\bar\theta^b;\theta^{b\prime}] \\ 0 & \text{for } \theta \in [\theta^{b*};1] \\ w_i^b & \text{otherwise} \end{cases}$$

Since output is constant, change in profits will depend only on change in wages. Letting ε terms go to zero, we have:

$$\lim_{\varepsilon\to 0}\Delta w_d = \frac{1}{2}\int_{\theta^{a\prime}}^{\bar\theta^{a}} w_i^a(\theta)f_{\pi^a}(\theta)d\theta - \frac{1}{2}\int_{\bar\theta^{a}}^{\theta^{a*}} w_i^a(\theta)f_{\pi^a}(\theta)d\theta + \frac{1}{2}\int_{\bar\theta^{b}}^{\theta^{b\prime}} w_i^b(\theta)f_{\pi^b}(\theta)d\theta - \frac{1}{2}\int_{\theta^{b*}} w_i^b(\theta)f_{\pi^b}(\theta)d\theta$$

Observe now that if $\theta^{a\prime}$ is close enough to $\bar\theta^a$ there is $h < k_c^a$ satisfying $k_s^a + \varepsilon < hp(\theta;\pi^a)$ for every $\theta \in [\theta^{a\prime};\bar\theta^a]$. We can then conclude using the usual relation $f_{\pi^j}(\theta) = \pi^j f_q(\theta)$:

$$\lim_{\varepsilon\to 0}\Delta w_d(\varepsilon) < \frac{1}{2}\int_{\theta^{a\prime}}^{\bar\theta^{a}} h\pi^a f_q(\theta)d\theta - \frac{1}{2}\int_{\bar\theta^{a}}^{\theta^{a*}} k_c^a\pi^a f_q(\theta)d\theta + \frac{1}{2}k_c^b\pi^b\left(\int_{\bar\theta^{b}}^{\theta^{b\prime}} f_q(\theta)d\theta - \int_{\theta^{b*}} f_q(\theta)d\theta\right) \qquad (2.42)$$

The last term on the right hand side is equal to zero by construction, so (2.42) reduces to $2\lim_{\epsilon \to 0} \Delta w_d(\epsilon) < \pi^a (h - k_c^a) \int_{\theta^{a\prime}}^{\hat{\theta}^a} f_q(\theta) d\theta < 0$. Hence the deviation is profitable for ϵ small enough. ∎

Proof of Proposition 6 (Necessity) Using lemmas 11 and 12, the total wage costs for workers from group j is $k_c^j = [p(\hat{\theta}^j; \pi^j) F_{\pi^j}(\hat{\theta}^j) + 1 - F_q(\hat{\theta}^j)]$, which is strictly increasing in k_c^j. Also, profits must be zero in any equilibrium. To show that a wage schedule with $k_c^j \neq y_1(C;S)$ cannot be an equilibrium, we suppose that $k_c^a > y_1(C;S)$, which by zero profits implies $k_c^b < y_1(C;S)$. We will construct a deviation that deals only with workers assigned to the complex task, building on the observation that the cost of labor per productive worker is higher in group a than in group b: substitute high test workers in group a with low test workers (as in Lemma 12). Keeping the number of workers employed in the skilled task constant this reduces the input of qualified labor from group a. Doing exactly the opposite in the other group b restores the original quantity of qualified workers. Furthermore, the total wage costs are lower than in the candidate equilibrium while output is kept constant and the affirmative action constraint is still satisfied. Formally, define $\theta^{a\prime}$ and θ^{a*} with $\hat{\theta}^a < \theta^{a\prime} < \theta^{a*} < 1$ as the solution to $F_{\pi^a}(\theta^{a\prime}) - F_{\pi^a}(\hat{\theta}^a) = 1 - F_{\pi^a}(\theta^{a*})$ and consider a deviation where group a workers are paid according to

$$w_d^a(\theta) = \begin{cases} w_i^a(\theta) + \epsilon & \text{for } \theta \in [\hat{\theta}^a; \theta^{a\prime}] \\ 0 & \text{for } \theta \in [\theta^{a*}; 1] \\ w_i^a(\theta) & \text{otherwise} \end{cases}$$

The effective input of qualified workers from group a decrease since we substitute workers with high test result with an equal mass of workers with low test result. We can quantify the loss in productive workers employed in the complex task as $\Delta C^a = \frac{a}{2}\left(\int_{\theta^{a*}}^{1} \pi^a f_q(\theta) - \int_{\hat{\theta}^a}^{\theta^{a\prime}} \pi^a f_q(\theta)\right)$. Next, let wages in group b be given by

$$w_d^b(\theta) = \begin{cases} 0 & \text{for } \theta \in [\hat{\theta}^b; \theta^{b\prime}] \\ w_i^b(\theta) + \epsilon & \text{for } \theta \in [\theta^{b*}; 1] \\ w_i^b(\theta) & \text{otherwise} \end{cases};$$

where $\theta^{b\prime}$ and θ^{b*} satisfying the following equations:

$$F_{\pi^b}(\theta^{b\prime}) - F_{\pi^b}(\hat{\theta}^b) = 1 - F_{\pi^b}(\theta^{b*}) \quad (2.43)$$

$$\int_{\theta^{b*}} \pi^b f_q(\theta) - \int_{\hat\theta^b}^{\theta^{b0}} \pi^b f_q(\theta) = \frac{\lambda^a}{\lambda^b}\left(\int_{\theta^{a*}}^{1} \pi^a f_q(\theta) - \int_{\hat\theta^a}^{\theta^{a0}} \pi^a f_q(\theta)\right)$$

If θ^{a0} is close enough to $\hat\theta^a$ a solution to (2.43) exists. The first equation guarantees that the number of employed workers remains constant, and the second that the effective input of qualified workers and therefore also output is constant. Thus, the change in profits depends only on the change in wage costs,

$$\begin{aligned} W(\varepsilon) - W &= \frac{\lambda^a}{2}\int_{\hat\theta^a}^{\theta^{a0}} (w_i^a(\theta) + 2\varepsilon) f_{\pi^a}(\theta) d\theta - \frac{\lambda^a}{2}\int_{\theta^{a*}}^{1} w_i^a(\theta) f_{\pi^a}(\theta) d\theta \\ &\quad - \frac{\lambda^b}{2}\int_{\hat\theta^b}^{\theta^{b0}} w_i^b(\theta) f_{\pi^b}(\theta) d\theta + \frac{\lambda^b}{2}\int_{\theta^{b*}}^{1} \left(w_i^b(\theta) + 2\varepsilon\right) f_{\pi^b}(\theta) d\theta \\ &= \left(k_c^b - k_c^a\right)\Delta C^a + \varepsilon\left[\lambda^a \int_{\hat\theta^a}^{\theta^{a0}} \pi^a f_q(\theta) d\theta + \lambda^b \int_{\theta^{b*}}^{1} \pi^b f_q(\theta) d\theta\right] \end{aligned}$$

Since $k_c^b < y_1(C;S) < k_c^a$ and $\Delta C^a < 0$ there is an ε small enough such that the deviation is profitable. ■

Proof of Proposition 7: Suppose $\pi^a = 0$ and $0 < \pi^b < 1$. The optimality conditions for (2.19) are then

$$\begin{aligned} y_2(\cdot) - \frac{1}{\lambda^a} &= 0 \\ -y_1(\cdot)p(\theta^b;\pi^b) + y_2(\cdot) + \frac{1}{\lambda^b} &= 0 \end{aligned} \qquad (2.44)$$

we observe that the unique solution $(\hat\theta^a;\hat\theta^b)$ must still be interior. Using (2.23) we also see that $H^a(0;\pi^b) = 0 < H^b(0;\pi^b)$. It is straightforward to verify that H^j is continuous at $(0;\pi^b)$ for $j = a;b$ and it follows that there must exist some $(\hat\pi^a;\hat\pi^b)$ where $0 < \hat\pi^a < \hat\pi^b$ and, since H^a is initially increasing, $0 < H^a(\hat\pi^a;\hat\pi^b) < H^b(\hat\pi^a;\hat\pi^b)$. There must therefore exist some strictly increasing function G such that $G(0) > 0$, $G(H^a(\hat\pi^a;\hat\pi^b)) = \hat\pi^a$ and $G(H^b(\hat\pi^a;\hat\pi^b)) = \hat\pi^b$; i.e. $(\hat\pi^a;\hat\pi^b)$ is an equilibrium in the economy with fundamentals $\{y; f_q; f_u; (\lambda^a;\lambda^b); G\}$. ■

Proposition 8: We first prove the following lemma.

Lemma 13. Let $\pi = (\pi^a;\pi^b)$ be such that $\pi^a < \pi^b$. Then there exists $\lambda^a > 0$ such that $B^a(\pi) < H^a(\pi)$.

Proof. Using (2.13) and (2.22) we see that $\widetilde{r}(\pi^a,\pi^b) \to r(\pi^b) = r^*$ and $\widehat{r}(\pi^a,\pi^b) \to r(\pi^b) = r^*$ as $\lambda^a \to 0$; the factor ratio under both regimes goes towards the factor ratio that would occur if $\pi = \pi^b$ in the single group model. This implies (from the definitions of $\widetilde{r}(\pi^a,\pi^b)$ and $\widehat{r}(\pi^a,\pi^b)$) that $\tilde{\theta}^b(\pi^a,\pi^b) \to \theta(\pi^b)$ and $\hat{\theta}^b(\pi^a,\pi^b) \to \theta(\pi^b)$; which means that the effect on the large group is negligible in the limit. The effect on the small group is however non-negligible. To see this note that $\tilde{\theta}^a(\pi^a,\pi^b) < \tilde{\theta}^b(\pi^a,\pi^b)$ and $F_{\pi^a}(\hat{\theta}^a(\pi^a,\pi^b)) = F_{\pi^b}(\hat{\theta}^b(\pi^a,\pi^b))$ for any π where $\pi^a < \pi^b$ (holds irrespective of λ^a). Thus,

$$\hat{\theta}^a(\pi^a,\pi^b) \to F^{-1}_{\pi^a}(F_{\pi^b}(\hat{\theta}(\pi^b))) > \hat{\theta}^b(\pi^b);$$

which means that $\tilde{\theta}^{a*} = \lim_{\lambda^a \to 0} \tilde{\theta}^a(\pi^a,\pi^b) < \lim_{\lambda^a \to 0} \hat{\theta}^a(\pi^a,\pi^b) = \hat{\theta}^{a*}$. Hence

$$\begin{aligned}
\lim_{\lambda^a \to 0} B^a(\pi^a,\pi^b) &= y_2(r^*,1)(F_q(\tilde{\theta}^{a*}) - F_u(\tilde{\theta}^{a*})) + \int_{\tilde{\theta}^{a*}}^1 p(\theta;\pi^a)(f_q(\theta) - f_u(\theta))d\theta < \\
&< y_2(r^*,1)(F_q(\hat{\theta}^{a*}) - F_u(\hat{\theta}^{a*})) + \int_{\hat{\theta}^{a*}}^1 p(\theta;\pi^a)(f_q(\theta) - f_u(\theta))d\theta = \\
&= \lim_{\lambda^a \to 0} H^a(\pi^a,\pi^b)
\end{aligned}$$

Proof of Proposition 8: Let λ^a be small enough so that $B^a(\hat{\pi}^a,\hat{\pi}^a) < H^a(\hat{\pi}^a,\hat{\pi}^b)$. Now for $j = 1,2$ let $\pi_0^j = \hat{\pi}^j$ and construct the sequences $\{\pi_t^j\}_{t=0}^{\infty}$ by letting

$$\begin{aligned}
\pi_t^a \text{ be the smallest solution to } \pi_t^a &= G\left(B^a\left(\pi_t^a,\pi_{t-1}^b\right)\right) \\
\pi_t^b \text{ be the largest solution to } \pi_t^b &= G\left(B^a\left(\pi_{t-1}^a,\pi_t^b\right)\right)
\end{aligned}$$

for $t = 1,2,....$ For $t = 1$ one verifies that $\pi_1^a < \pi_0^a$ by using the intermediate value theorem ($G(B^a(\hat{\pi}^a,\hat{\pi}^a))$ is below the diagonal and $G(0)$ is above). π_1^b may on the other hand be larger or smaller than π_0^b. Hence for the next step we could potentially get a problem (we want to construct monotone sequences). However, if we instead let $\pi_0^b = \hat{\pi}^b - \varepsilon$ it follows by continuity that $\pi_1^a < \pi_0^a$ for ε sufficiently small and for λ^a small enough we get $\pi_1^b > \pi_0^b$.

For $t > 1$ we see that if $\pi_t^b > \pi_{t-1}^b$ then it follows from Proposition 4 that

$$G\left(B^a\left(\pi_t^a,\pi_t^b\right)\right) < G\left(B^a\left(\pi_t^a,\pi_{t-1}^b\right)\right) = \pi_t^a$$

and since $G(B^a(0,\pi_t^b)) = 0$ we can again apply the intermediate value theorem to conclude that

$\pi^a_{t+1} < \pi^a_t$[27]. Similarly, we have that if $\pi^a_t < \pi^a_{t-1}$ then $\pi^b_{t+1} > \pi^b_t$. Hence, $\{\pi^a_t\}$ is a monotonically decreasing sequence and $\{\pi^b_t\}$ is a monotonically increasing sequence. Since $\pi^j_t \in [0,1]$ for each t it follows that both sequences are converging to some limits (π^{a*}, π^{b*}) in $[0,1]$. Clearly, the (π^{a*}, π^{b*}) is an equilibrium of the model without policy and since $\pi^{a*} < \hat{\pi}^a$, where $\hat{\pi}^a$ is the fraction of investors from a in the most discriminatory equilibrium with affirmative action. ■

[27] In the arguments we are presuming that $\pi^a_t > G(0)$. If for some t $\pi^a_t = G(0)$, then the process stops and we have reached a corner equilibrium, a case where we know the result holds anyway.

3. The effect of Statistical Discrimination on Black-White Wage Differentials: Estimating a Model with Multiple Equilibria

3.1. Introduction

The empirical literature on discrimination has consistently documented that blacks earn less than whites, even after controlling for observable characteristics. However, over the last 30 years the racial wage differential has narrowed considerably. Among the reason cited for the reduction in the differential are (1) a convergence in years of schooling; (2) a convergence in the quality of schooling, (3) the selective decline in the labor force participation of low-skilled blacks, (4) migration of black workers out of southern regions[1], (5) affirmative action and other anti-discrimination legislation. Much of the debate has centered on the relative contributions of each of these explanations[2].

In their survey of the empirical literature, Donohue and Heckman (1991) argue that the first four explanations can account for at most 65% of the reduction in earnings inequality. Although better measurement of these factors could enhance their combined explanatory power, an additional explanation is that the unexplained portion of the decline reflects movements between different economy-wide equilibria that might result from the same fundamentals. In an environment where multiple equilibria co-exist, a reduction in inequality can be experienced if the economy moves from an equilibrium with high differentials to one with low differentials. Donohue and Heckman conjecture that such a phenomenon might have occurred in the past 30 years, perhaps as a result of anti-discrimination policy. But, they did not provide a structure within which an analysis of this conjecture could be conducted.

[1] Migration contributed significantly to the reduction in inequality earlier this century, but does not seem to have played an importan role in the last decades.

[2] There is a vast literature on this subject. See, for example Card and Krueger (1991), Heckman (1989), Leonard (1984), Smith and Welch (1984, 1989).

Statistical theories of discrimination, pioneered by Arrow (1973) and Phelps (1972), provide a setting within which different outcomes are possible under the same set of primitives. These models do not provide an equilibrium selection mechanism. Nonetheless, recent studies have focused on the efficacy of anti-discrimination policies in reducing group disparities. They show that even if the effect of such policies is in general ambiguous, there are cases where such policies can eliminate the equilibria with discrimination and lead the economy towards group equality[3].

The starting point of this essay is the observation that while the empirical literature cannot account for all of the reduction in wage inequality based on changes in the fundamentals, the theoretical literature has demonstrated that under some circumstances wage inequality can be reduced even without such changes. This chapter proposes and implements an estimation strategy capable of identifying both the fundamentals and the equilibrium chosen by the economic agents in a statistical discrimination model in which there may be multiple outcomes. By estimating the model in different time periods, it is possible to compare the pattern of the equilibria selected in the economy with the other equilibria that the model could have generated under the same fundamentals. This exercise can provide an answer about whether the reduction in wage inequality can be explained, at least in part, by the way equilibria were selected over time[4].

The model presented here builds on Arrow (1973), and is an extension of the model presented in Chapter 2. Production takes place using two different job-tasks of different complexity. Workers face a costly human capital investment decision, which, if undertaken, makes a worker productive in the complex task. Workers are heterogeneous in the cost of investment. Therefore, the fraction of workers who invest depends on how big the benefits from investing are. Firms decide how much to pay workers and how to allocate them between the technologies on the basis of a noisy signal of productivity. Equilibria with wage inequality can exist because if employers hold asymmetric expectations about aggregate human capital investment of members of different groups, workers belonging to the group with less productive people will be expected to be less productive than workers of the other group carrying the same signal. In this situation, higher signals of productivity will be differently rewarded for members of the two groups. This generates different incentives to invest. Therefore, one group will have less workers willing to undertake the costly human capital investment, which fulfills the asymmetric expectations from the employers. In this context it is

[3] See Coate and Loury (1993), Lundberg (1991), and Moro and Norman (1997)

[4] Foster and Rosenzweig (1996), and Oettinger (1996) propose different methods to estimate the presence of statistical discrimination, but none of them possesses the structure to answer the question posed in this paper.

possible that multiple equilibria coexist with different levels of investment disparity, which implies different wage inequality and incentives to invest.

In general, the problem of estimating models with multiple equilibria stems from the fact that the equilibrium selection mechanism is unspecified. Although this is true here as well, the structure of the model is such that the selected equilibrium is nonetheless estimable. The idea behind the estimation strategy is the following. For a given set of fundamental parameters, the model yields different equilibrium wage distributions. However, given workers' human capital investment decision, and firms optimal responses in terms of wages and job allocation, the wage distribution is unique. The distinctive feature that allows the identification of the equilibrium is that there exists a representation of the model that uniquely relates the equilibrium wage distribution of one group of workers to the set of equilibrium choices made by the agents and a subset of the fundamental parameters. In the first stage of the estimation, this subset of the fundamentals and the sufficient statistics for workers and firms' equilibrium choices are considered as estimable parameters of the wage distribution. The other fundamental parameters, which refer essentially to the distribution of workers over investment costs, are recovered in a second stage. Identification requires merging the first stage results obtained for black and white male workers, and adopting functional form assumptions that replicate the equilibrium estimated in the first stage. Among these assumption, it is required that the two groups of workers possess the same distribution over costs of investment. After completing the estimation, it is possible to compute whether there are other equilibria that the model can generate under the same set of fundamentals and to evaluate whether there is a pattern in the way equilibria were selected over time.

The estimation was performed using wage data for ten three-year periods from 1963-65 to 1990-92. Results from the first estimation stage show that the estimated wage distributions essentially match the basic facts, such as the increased within-group wage inequality that occurred after the eighties. It is found that blacks invest in human capital on average 12.3% less than whites, although the difference declines over time. Consequently, relatively more blacks are employed in the simple task (92.9% vs. 79.1%). These differences cause a 23.6% average difference in mean wage between the two groups. As in the data, the predicted average wage difference between groups declined considerably until 1980, and stabilized afterwards.

For every year in which the estimation is performed, the model generates several equilibria. The equilibria that were selected in the economy over time (which from now on will be referred to as the "selected" equilibria) were those in which the mean wage differential was essentially the smallest. However, at the estimated parameters equilibria with higher wage inequality always existed. In some of them, blacks invest less than in the estimated equilibrium and receive a lower average wage, but are better off in terms of welfare than in the selected equilibrium because the reduced cost of investment compensates for the smaller wage. In another set of equilibria, blacks are better off than whites, because they invest more. In almost all of the equilibria of this type, whites are at a "corner solution", where none of them invest in human capital, and therefore are all placed in the simple task. Even though whites in these equilibria save the investment cost, their welfare is on average lower than in the selected equilibrium. This is a consequence of the fact that the larger group is not investing, which strongly affects the aggregate output of the economy.

These findings imply that statistical discrimination and self-fulfilling expectations about groups average productivities did not exacerbate wage differences in the U.S., and that the decline in wage differentials cannot be explained by a change in the way equilibria were selected over time.

The remainder of the chapter is organized as follows. After describing the model in Section 3.2, we characterize the set of equilibria in section 3.3. In section 3.4 we describe the estimation strategy and the datasets used in the estimation. Section 3.5 presents the results, and section 3.6 summarizes the implications of the results and outlines extensions for future research. Proofs and other tables are in sections 3.7 – 3.10.

3.2. The model

This model is an extension of the statistical discrimination model presented in Chapter 2. There is a competitive labor market where firms produce from a technology which consists of two types of job-tasks. One, which will be referred to as the simple task, can be performed by any worker, whereas the other one, the complex task, requires qualified workers[5].

Firms cannot observe with certainty whether or not a worker is qualified. However, workers can be distinguished by an observable signal (a test result), as well as by their race, which is indicated with b and w: The size of each group is λ^b and λ^w; respectively. The informational asymmetry between tasks (i.e. the fact that productivity of a worker is known with certainty only in the simple task) is certainly not an innocuous assumption, but it seems quite reasonable on empirical grounds.

[5] Ideally, one would allow for more tasks and more groups; with appropriate assumptions the model can be easily extended to incorporate more tasks and groups. The simplification adopted in this paper is needed because of computational problems and data limitations.

On the labor market there is a continuum of workers with heterogeneous skill endowment[6]. A worker with skill a is said to have "a efficiency-units". Each worker belongs to one of two identifiable groups, indexed by b and w, of size λ^b and λ^w, respectively. Skill endowment is distributed in each group according to the same distribution $h(a)$[7]. The efficiency units of workers will contribute to the total size of the factor inputs. The key assumption of the model is that only workers who invested in human capital contribute to the input in the complex task, whereas all workers are productive in the simple task. Investment in human capital does not affect workers' skill a, but only makes the worker's efficiency units productive in the complex task. Cost of investment in human capital c depends deterministically on skill according to the cost function $k(a)$, with $k'(a) < 0$ (i.e. more able workers have less cost of investment). The inverse of $k(\cdot)$, which relates efficiency to costs, is denoted with $e(\cdot)$. Distribution $h(\cdot)$ induces a distribution of costs $g(c) = h(e(c))de(c)/dc$. Denote the cumulative distribution of costs with $G(c)$

The output of the firm is given by

$$y(C, S) = C^{\alpha} S^{1-\alpha} \tag{3.1}$$

where C and S denote the input of efficiency units of labor in the complex and simple task, respectively. Assume $C = C^b + C^w$ and $S = S^b + S^w$, where S^b, S^w represent the mass of efficiency units of all workers employed in the simple task, belonging respectively to group b and w, and C^b, C^w represent the effective input of qualified workers employed in the complex task belonging to group b and w. We can express these quantities as follows: let $h_f(a)$ be the distribution over efficiency units of workers hired by firm f[8], and denote with K^j and H^j the set of workers of group j employed respectively in the complex and simple task, and with Q^j the set of qualified workers from group j. Then we compute the inputs for this firm as follows:

$$C^j = \rho^j \int_{a \in Q^j \cap K^j} a h_f(a) da \tag{3.2}$$

[6] This is the main difference between this model and the one presented in Moro and Norman (1997).

[7] This assumption will be relaxed in the estimation. The description of the model and the characterization of equilibria can be easily extended to the case where groups have different distributions of skill endowment. In fact, the parametrization of the wage distribution does not change at all.

[8] In equilibrium this will be the same for all firms, and equal to $h(a)$

$$S^j = \int_{a \in H^j} a h_f(a) da \qquad (3.3)$$

Parameters ρ^j represent commonly known exogenous differences between groups in productivity in the complex task relative to productivity in the simple task[9]. We can without loss of generality re-scale the model so that $\rho^w = 1$. Its role is to allow for a comparative advantage in employing blacks in one of the tasks.

It has already been mentioned that a worker's qualification is only imperfectly observed by employers, which implies that there is a potential mismatch: some workers without human capital investment will in general end up employed in the complex task, and some qualified workers will be employed in the simple task.

The timing of events is as follows: in Stage 1 workers choose whether to invest or not in human capital to become qualified for the skilled job; after the investment decision, each worker performs a test (Stage 2) that results in a signal θ. This signal is a summary of all information the employers have available when they hire one worker. In stage 3 firms simultaneously announce wage schedules (i.e. wages as a function of the signal and group identity) and in Stage 4 workers choose what firm to work for. Finally, in Stage 5, firms allocate workers between the two tasks.

This is a formal description of the game:

Stage 1. Each agent chooses an action $e \in \{e_q, e_u\}$, where $e = e_q$ means that the agent undertakes an investment in his human capital (and becomes a qualified worker) and $e = e_u$ that he does not. If agent a undertakes the investment he incurs a cost of $c = k(a)$ while no cost is incurred if the investment is not undertaken.

Stage 2. Each worker performs a test resulting in an observable signal $\theta \in [0,1]$. The signal θ is distributed according to density f_q^j for group j workers who invested in Stage 1 and f_u^j for group j workers who did not invest, where $j = b, w$. It is assumed that f_q^j and f_u^j are continuously differentiable, bounded away from zero and satisfy:

$\frac{f_q^j(\theta)}{f_u^j(\theta)} > \frac{f_q^j(\theta')}{f_u^j(\theta')}$ if $\theta > \theta'$; $j = b, w$. [strictly monotone likelihood ratio property]

This assumption implies that qualified workers are more likely to get higher values of θ than unqualified workers. Let F_q and F_u denote the associated cumulative distributions. Since the "testing technology" is allowed to be different between groups, the same signal can be differently

[9] They can also represent a "taste" parameter representing biased preferences of the employers. Under this interpretation employers care about both production and race of their employees, and function (3.1) is a reduced form representation of employers' utility function.

informative for workers of different groups, even when groups are equal in all other aspects.

Stage 3. The firms simultaneously announce wage schedules. Wages are allowed to be dependent on the signal and group identity so that a (pure) action of firm $i = 1 ... n$ in stage 3 is a set of measurable functions $w_i^j : [0,1] \to R_+ ; j = b, w$.

Stage 4. The workers observe w_i^j; $i = 1, ..., n$; $j = b, w$; and decide which firm to work for.

Stage 5. In this final stage of the game the firms allocate the available workers by using a task assignment rule which is a measurable function $t_i^j : [0,1] \to \{0,1\}$. The interpretation is that $t_i^j(\mu) = 1$ (0) means that firm i assigns all workers of group j with signal μ to the complex (simple) task.

It is assumed that workers do not care directly what task they are employed in; thus, once the investment cost is sunk, the worker will rationally choose the firm that offers the highest wage for his particular realization of μ and the group he belongs to. Therefore, imposing optimal behavior in stage 4, the payoff of a worker belonging to group j can be written as

$$E_\mu[\max_{i=1...n} w_i^j(\mu)] - c(e)$$

where $c(e_q) = k(a)$; and $c(e_u) = 0$.

Firms' objective is to maximize output minus the wage bill.

3.3. Characterization of Equilibria

The basic structure of the model is similar to the model presented in Chapter 2, therefore the equilibria will be characterized in a similar fashion. One important difference introduced here is worker heterogeneity, which allows for a wage function that is strictly increasing in the signal. Without heterogeneity, all workers in the simple task would receive the same wage. In addition, the model presented in this chapter allows the parameters of the "testing technology" to be different.

The Nash equilibria of the game are computed by characterizing first the firms' best responses (in terms of wage schedules and task assignment rules) to workers' investment choices. These responses will determine a unique set of wage schedules given investment behavior by the workers; imposing optimal worker behavior in the first stage we get a set of fixed point equations that characterize the set of equilibrium outcomes.

First, notice that the wage schedule offered by each firm must be identical almost everywhere in any Nash equilibrium of the game. If one firm were to offer a higher wage to a positive mass of workers, then it could slightly lower wages for those workers. If the wage cut is small enough, those workers will not choose other firms, but profits will increase. Because workers do not care directly what task they are assigned to, they are indifferent among which firm to work for; it is assumed that firms equally share workers whenever they post the same wage for workers of a particular group and test result θ.

Next, consider the problem faced by the firm in the last stage, when fractions π^b and π^w of b and w workers invested in Stage 1 and firms offered wage schedules $w_i^j(\theta) = w(\theta)$; $j = b, w$ for all firms $i = 1..n$ and signals $\theta \in [0, 1]$. In this case, group j workers' distribution over θ is $f_\pi^j(\theta) = \pi^j f_q^j(\theta) + (1 - \pi^j) f_u^j(\theta)$.

Once workers are hired, the wage cost is sunk. Therefore firms will choose in stage 5 the task assignment rule that maximizes output. Notice also that this stage is non-strategic in the sense that choices here do not affect the other firms' profits, so there is no reason to choose any rule that is not output maximizing. It is possible to show (see Section 3.7) that this rule must have the "cutoff" property, i.e. that each firm will choose for each group j of workers a cutoff signal $\hat{\theta}^j$ such that

$$t^j(\theta) = \begin{cases} 0 & \text{if } \theta \leq \hat{\theta}^j \\ 1 & \text{if } \theta > \hat{\theta}^j \end{cases} \qquad (3.4)$$

Given this rule it is possible to express firms' inputs for a given choice of $\hat{\theta}^b$ and $\hat{\theta}^w$ as follows.

Because f_q^j and f_u^j are independent of the cost of investment, the joint distribution of group j workers over costs and signals is

$$g^j(c; \theta) = \begin{cases} g(c) f_q^j(\theta) & \text{if the worker invested,} \\ g(c) f_u^j(\theta) & \text{if the worker did not invest.} \end{cases} \qquad (3.5)$$

From (3.5) the conditional distribution of costs can be written as

$$g^j(c|\theta) = \begin{cases} \frac{g(c) f_q^j(\theta)}{f_\pi^j(\theta)} & \text{if the worker invested,} \\ \frac{g(c) f_u^j(\theta)}{f_\pi^j(\theta)} & \text{if the worker did not invest.} \end{cases}$$

Suppose that firm use the "cutoff rule" (3.4) to assign workers between tasks. Using the fact that in equilibrium all workers with cost of production below a certain level B^j will invest in human capital, and all workers with cost above B^j will not (B^j satisfies $\pi^j = \int_{c<B^j} g(c)dc$); it is possible to "count" the expected number of efficiency units assigned to each task. Rewriting (3.2)

and (3.3),

$$C^j = \frac{\lambda^j}{n}\rho^j \int\int_{\theta \geq \bar{\theta}^j,\ c < B^j} a(c) g^j(c;\theta)\,dc\,d\theta; \tag{3.6}$$

$$S^j = \frac{\lambda^j}{n}\int_0^{\bar{\theta}^j}\int_c a(c) g^j(c;\theta)\,dc\,d\theta: \tag{3.7}$$

Given that the expected number of efficiency units of one qualified and one unqualified worker of group j

$$E^j(a|q) = \frac{\int_{c<B^j} a(c)g(c)dc}{\pi^j};$$
$$E^j(a|u) = \frac{\int_{c\geq B^j} a(c)g(c)dc}{1-\pi^j};$$

it is possible to use these expression along with (3.5) to rewrite (3.6) and (3.7) as

$$C^j = \frac{\lambda^j}{n}\rho^j\pi^j\left(1 - F_q^j(\bar{\theta}^j)\right)E^j(a|q); \tag{3.8}$$
$$S^j = \frac{\lambda^j}{n}\left(\pi^j F_q^j(\bar{\theta}^j)E^j(a|q) + (1-\pi^j)F_u^j(\bar{\theta}^j)E^j(a|u)\right): \tag{3.9}$$

An interpretation of these equations is that nC^j/λ^j is the proportion of qualified workers employed in the complex task (belonging to group j) $\pi(1 - F_q^j(\bar{\theta}^j))$ multiplied by the expected number of efficiency units of one qualified j worker $E^j(a|q)$, and nS^j/λ^j is the sum of the proportions of qualified and unqualified workers employed in the simple task, respectively multiplied by the expected number of efficiency units of one qualified and one unqualified j worker.

It is now possible to write the problem of generic firm i at stage 5:

$$\max_{\theta_i^b,\theta_i^w \in [0,1]^2} \quad y\left(C_i^b + C_i^w; S_i^b + S_i^w\right) \tag{3.10}$$
$$\text{subj. to} \quad \frac{n}{\lambda^j}C^j \leq \rho^j\pi^j\left(1 - F_q^j(\theta^j)\right)E^j(a|q); \quad j = b,w$$
$$\frac{n}{\lambda^j}S^j \leq \pi^j F_q^j(\theta^j)E^j(a|q) + (1-\pi^j)F_u^j(\theta^j)E^j(a|u); \quad j = b,w$$

It is possible to show that there exist a unique solution to (3.10) for any $\pi \neq (0;0)$ and that the solution satisfies the Kuhn-Tucker conditions. Let γ^j and η^j be the multipliers associated with the boundaries for θ_i^j. After some manipulations, these conditions can be expressed as follows (dropping the subscript for firm i):

$$-y_1(\cdot)\pi^j f_q^j(\theta^j)E^j(a|q) + y_2(\cdot)\left[\pi^j f_q^j(\theta^j)E^j(a|q) + (1-\pi^j)f_u^j(\theta^j)E^j(a|u)\right] + (\gamma^j - \eta^j) = 0 \quad (3.11)$$

for $j = b,w$; together with complementary slackness conditions. Denote the solution with $\hat{\theta} = (\hat{\theta}^b, \hat{\theta}^w)$.

The next task is to compute equilibrium wage schedules. It is natural to assume that factors of production are paid their expected marginal productivity. Then, we can denote the "labor market equilibrium" wage function, given $\pi = (\pi^a, \pi^b)$ as follows

$$w^j(\theta) = \begin{cases} y_1(\cdot)p(\theta;\pi^j)E^j(a|q)\rho^j & \theta \geq \hat{\theta}^j; \\ y_2(\cdot)E^j(a|\theta) & \text{otherwise.} \end{cases} \quad (3.12)$$

where $p(\theta;\pi^j) = \pi^j f_q^j(\theta)/f_{\pi^j}(\theta)$ is the probability that a worker with signal θ invested in human capital given that the proportion of investors in his group is π; and $E^j(a|\theta) = \int a(c)g^j(c|\theta)$ is the expected number of efficiency units carried by a worker with a given θ.

Proposition 9. *Let the fraction of agents who invest in each group be given by $\pi = \{\pi^b, \pi^w\}$, and assume that only workers with cost less than B^j invested, where B^j satisfies $\pi^j = \int_{c<B^j} g(c)dc$. Let $t^j : [0,1] \to \{0,1\}$; $j = b,w$ be a pair of cutoff rules with cutoff point $\hat{\theta}^j(\pi)$ determined by (3.10) and let $w^j : [0,1] \to \mathbb{R}$ be given by (3.12). Then, firms are playing best responses if and only if they choose wages and task assignment rule in accordance to $w^j(\theta)$ and $t^j(\theta)$; $j = b,w$.*

The proof is in the section. Given the task assignments, firms have to pay at least marginal productivities, and strictly more if they want to attract more workers. But then the only way to make a profitable deviation by attracting additional workers would be to reallocate them more efficiently between tasks, which is impossible given that, as observed before, the task assignment rule is chosen in a non strategic stage of the game to maximize output.

The interpretation of the components of the wage in the complex task is the following: $\rho_c^j y_1(\cdot)$ is the marginal productivity of one efficiency unit of labor, $E^j(a|q)$ is the average number of efficiency units carried by a productive worker; and $p(\theta;\pi)$ is the probability of a worker being qualified for the complex task. A similar interpretation can be performed for the simple task wage, which can

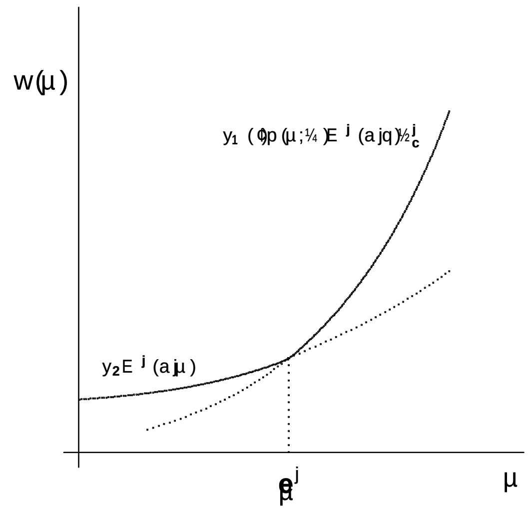

Figure 3.1: The wage function

be written:

$$y_2(\cdot)\left(E^j(a|q)\frac{\pi^j f_q(\theta)}{f^j_\pi(\theta)} + E^j(a|u)\frac{(1-\pi^j)f^j_u(\theta)}{f^j_\pi(\theta)}\right): \tag{3.13}$$

In this case the marginal product is multiplied by the weighted average of the expected number of efficiency units of a qualified and unqualified worker, weighted by the probability of being respectively qualified and unqualified. Using the first order condition for the task allocation problem, we can see that the wage function is continuous at $\hat{\theta}^j$, that is the cutoff signal is chosen so that the worker who carries it has the same expected marginal productivity in both tasks. Notice also that the monotone likelihood ratio property implies that $w^j(\theta)$ is strictly increasing.

Next, we have to consider optimal worker behavior in the first stage. Denote $B^j(\pi)$ to be the gross benefits of investing for a worker belonging to group j when wages are computed in the labor-market equilibrium assuming $(\pi^b, \pi^w) = \pi$: Then, we have

$$B^j(\pi) = \int_\theta w^j(\theta)\left[f^j_q(\theta) - f^j_u(\theta)\right]d\theta; \tag{3.14}$$

where the dependence on π is implicit in the wage function. The equilibria of the model are characterized by the solutions to the set of fixed point equations

$$\begin{aligned} \pi^b &= G(B^b(\pi^b, \pi^w)); \\ \pi^w &= G(B^w(\pi^b, \pi^w)): \end{aligned} \tag{3.15}$$

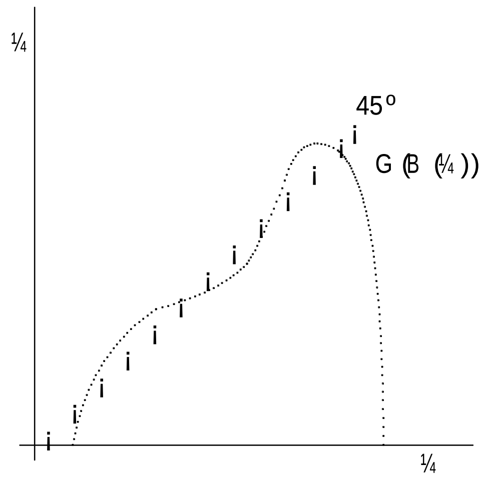

Figure 3.2: Symmetric equilibria with ex-ante identical groups

We can sum up the previous discussion and state the following proposition:

Proposition 10. A pair of fraction of investors $\langle \pi^b; \pi^w \rangle$ in equilibrium solves (3.15). Moreover, any solution to (3.15) corresponds to an equilibrium of the model.

The necessity part of the previous proposition requires showing that in equilibrium only workers with cost less than a certain level, invest in human capital, and no worker with cost above the same level invests (a maintained assumption in the characterization of the "labor-market equilibrium". This must be true since the ex-ante benefits of investing are the same for each worker of group j. Existence of equilibria therefore reduces to the existence of a fixed point of $G \circ B$.

If groups do not possess exogenous differences (i.e. if they shared the same testing technology, and $\rho^b = \rho^w$) then existence of a symmetric equilibrium can be easily established since the problem becomes one-dimensional. It requires existence of a fixed point of the equation $\pi = G(B(\pi))$; where $B(\cdot)$ are the incentives to invest derived as in (3.14). When $\pi = 0$ then production and wages will be zero, so $B(0) = 0$. When $\pi = 1$; then wages will be constant in θ since there is no value in hiring a worker with higher signal when everybody is known to have invested. Therefore, $B(1) = 0$. If $G(0) > 0$ then a fixed point exists from simple application of the intermediate value theorem. Complementarity of the factor inputs guarantees that function $B(\pi)$ is "steep" near $\pi = 0$; therefore if $G(0) = 0$; a non-trivial fixed point exists provided that the minimum cost of investment is not too high (see figure 3.2).

When groups are not identical, then equilibria cannot in general be symmetric: when $\pi^b = \pi^w$;

marginal product will be different because of the exogenous differences (for example, a different testing technology), therefore incentives will be different as well.

3.4. Estimation strategy

One way of estimating the fundamental parameters of a model that generates multiple equilibria is to compute the set of all equilibria that can be generated given a set of fundamentals, compute the likelihood under each of these equilibria, choose the equilibrium that maximizes this likelihood and then maximize over the set of fundamental parameters. This exercise is not only computationally expensive, but it is also prone to failure given that numerical procedures might sometimes miss one or more of the equilibria of the model.

The two-stage estimation procedure pursued here does not require the computation of all the feasible equilibria. The interesting feature of the model that is exploited in the estimation procedure is that given workers' human capital investment decisions and firms' optimal responses in terms of wages and job allocation, the wage distribution is unique. What is needed to represent the wage distribution for a group of workers j is the fraction of investors in human capital π^j, the task assignment rule θ^j, the testing technology γ^j, and the marginal products of that group's qualified and unqualified workers in the complex and simple task $(y_1 \rho^j E^j(a|q); y_2 E^j(a|u); y_2 E^j(a|q))$. These six variables uniquely determine the wage distribution, and can therefore be considered as estimable parameters. The first stage of the estimation consists in finding the set of these parameters that best matches the empirical wage distribution. Notice that no additional information about the distribution over cost $g(\cdot)$ and the cost-skill endowment relation $e(\cdot)$ is needed to compute the likelihood of the distribution, and in general, different parametrization for these functions can be consistent with the same wage distribution.

In the second stage, the parameters obtained separately for the two groups in the first stage of the estimation are merged, and some restrictions of the model are used in order to recover moments of the functions $g(\cdot)$ and $e(\cdot)$. Then, it is possible to adopt functional form assumption that replicate the equilibrium estimated in the first stage, and use the computed moments of $g(\cdot)$ and $e(\cdot)$ to uniquely determine their parameters. At this stage, it is necessary to drop the assumption that the relation between cost and ability $e(\cdot)$ is unique for the two groups.

3.4.1. First stage: the wage distribution

Given a set of wages representative of the group population, it is possible to formulate the likelihood function for a vector of observations about wages of workers of group $j = b, w$ collected in a specific time period. To do that requires the specification of an expression for the wage density. To begin, assume that the testing technology distributions belong to the one-parameter family:

$$\begin{aligned} f_q^j(\theta) &= \gamma^j \theta^{(\gamma^j - 1)}; \\ f_u^j(\theta) &= \gamma^j (1-\theta)^{\gamma^j - 1} \end{aligned}$$

for $j = b, w$. This functional form imposes the monotone likelihood ratio property for any value of the parameter $\gamma^j > 1$[10]. Now, rewrite the wage function (3.12), using also (3.13) and continuity of the wage function at $\hat{\theta}^j$ as

$$w^j(\theta) = \begin{cases} \left(y_q^j + y_u^j \left(\frac{1-\hat{\theta}^j}{\hat{\theta}^j} \right)^{\gamma^j - 1} \right) \frac{\pi^j \theta^{(\gamma^j-1)}}{\pi^j \theta^{(\gamma^j-1)} + (1-\pi^j)(1-\theta)^{\gamma^j-1}} & \theta \geq \hat{\theta}^j; \\ y_q^j \frac{\pi^j \theta^{(\gamma^j-1)}}{\pi^j \theta^{(\gamma^j-1)} + (1-\pi^j)(1-\theta)^{\gamma^j-1}} + y_u^j \frac{(1-\pi^j)(1-\theta)^{\gamma^j-1}}{\pi^j \theta^{(\gamma^j-1)} + (1-\pi^j)(1-\theta)^{\gamma^j-1}} & \text{otherwise,} \end{cases}$$

where

$$\begin{aligned} y_q^j &= y_2 E^j(a|q) \\ y_u^j &= y_2 E^j(a|u) \end{aligned} \tag{3.16}$$

Since wages are strictly increasing in θ, the wage function is invertible. Denote with $t^j(\cdot): R_+ \to [0,1]$ this inverse. Given that θ is distributed in the population according to $f_{\pi^j}^j = \pi^j \theta^{\gamma^j - 1} + (1-\pi^j)(1-\theta)^{\gamma^j-1}$ the wage distribution x^j will satisfy:

$$x^j(\omega) = f_{\pi^j}(t^j(\omega)) \frac{dt^j(\omega)}{d\omega}$$

where $dt(\omega)/d\omega$ can be computed analytically using $\left[dw^j(t^j(\omega))/dt^j(\omega) \right]^{-1}$. It is therefore possible to derive the likelihood function given a vector of wages for group j taken from the population survey $\omega \equiv [\omega_i]_{i=1}^N$ and population weights $k \equiv [k_i]_{i=1}^N$.

To simplify the problem it is possible to find data analogs to 3.16, so that the wage distribution for group j can be expressed as a function of π^j, γ^j, and $\hat{\theta}^j$ only. First, notice that from the model

[10] This assumption is without loss of generality since when $\gamma < 1$ the roles of f_q and f_u are reversed: the lower the signal, the lower the probability that the worker who carries it is productive in the complex task.

the minimum wage $w^j(0)$ is equal to y_u^j. Assuming that ω_1 is the minimum reported wage[11]:

$$y_u^j = w^j(0) = \omega_1 \qquad (3.17)$$

Secondly, using the fact that data on wages used in the analysis are topcoded, y_q can be expressed as a function of the other parameters. Letting ω_N be the maximum reported wage (i.e. the topcode level), y_q can be computed by solving the following equation[12]:

$$\omega_N = \left(y_q^j + y_u^j \left(\frac{1-\beta^j}{\beta^j} \right)^{(\gamma^j - 1)} \right) \frac{\pi^j \theta_N^{(\gamma^j-1)}}{\pi^j \theta_N^{(\gamma^j-1)} + (1-\pi^j)(1-\theta_N)^{(\gamma^j-1)}}; \qquad (3.18)$$

where θ_N is computed by matching the right tail of the theoretical wage distribution with the fractoin of observations at the topcode level. Formally, denote this fraction by K; then, θ_N is computed as the unique root of the equation $F_{\pi^j}(\theta_N) = K/\sum_{i=1}^N k_i$; i.e.:

$$\pi^j (\theta_N)^{\gamma^j} + (1-\pi^j)\left(1-(1-\theta_N)^{\gamma^j-1}\right) = 1 - \frac{K}{\sum_{i=1}^N k_i} \qquad (3.19)$$

The left-hand side of (3.19) is strictly increasing in θ_N when $\gamma^j > 1$; and the right-hand side of (3.18) is strictly increasing in y_q^j; hence a unique solution is guaranteed. Having used the first and the last observations to get y_u and y_q; the likelihood function can be computed using observations from 2 to $N-1$. Imposing (3.17) and (3.19) the wage function depends only on: β^j; π^j; and γ^j. Therefore the likelihood of dataset $(\omega; k)$ is conditional only on β^j; π^j; and γ^j. Specifically,

$$l(\omega; k|\beta^j; \pi^j; \gamma^j) = \sum_{i=2}^{N-1} \left[\log\left(\pi^j \gamma^j (\theta_i)^{\gamma^j-1} + (1-\pi^j)\gamma^j (1-\theta_i)^{\gamma^j-1} \right) + \log(j_i) \right] \cdot k_i \qquad (3.20)$$

where j_i is the derivative of the inverse of the wage function at ω_i. Note that θ_i and j_i depend on the parameters $\beta^j; \pi^j; \gamma^j$.

For any value of the parameters $\beta^j; \pi^j; \gamma^j$; the value of the likelihood for dataset $(\omega; k)$ can be

[11] Only wages at or above the mimimum wage set by Federal legislation are used in the estimation. This implies that $y_u^b = y_u^w$; provided there is at least one observation in each group at the Federal minimum wage.

[12] This expression is valid only if ω_N has been earned by at least an individual employed in the complex task. One could always verify this by checking that the maximum likelihood obtained with a model where everybody is employed in the simple task is less than the likelihood obtained with the more general model. If everybody is employed in the simple task, than the wage equation implies $y_q = \omega_N$

computed as follows:

1. Obtain y_u^j and y_q^j from (3.17) and (3.19)

2. Compute signals θ_i by solving numerically the inverse wage function $\theta_i = t^j(\omega_i)\ \forall i = 2, \ldots, N-1$.

3. Compute the derivatives of the inverse wage function j_i using $j_i = 1/(dw^j(\theta_i)/d\theta_i)$; $\forall i = 2, \ldots, N-1$ (the actual formula is omitted).

4. Compute the log-likelihood using (3.20)

The first stage is completed by maximizing the likelihood using a numerical method.

3.4.2. Second stage: cost and skill endowment

The estimated parameters from the first step include sufficient statistics for the equilibrium choices made by the agents which characterize the equilibrium selected in the economy $(\pi^j; \hat{\beta}^j; y_q^j; y_u^j)$. However, the only fundamental parameter which is estimated is γ^j. To characterize the other equilibria that the model could potentially have generated, the other fundamental parameters have to be derived: α; ρ^b; and the parameters of $g(\cdot)$ and $e(\cdot)$. This section describes a procedure that not only recovers the remaining fundamental parameters, but also imposes that the equilibrium estimated in the first stage remains an equilibrium of the model under the estimated set of fundamentals. This enables comparisons to be made between the equilibrium that the estimation characterizes in the first stage (which can be though of as the equilibrium that agents selected among the available equilibria), and the other equilibria that the model can generate.

The procedure is performed according to the following steps:

1. Parameters α, ρ^b; are recovered by merging the results obtained from the first stage of the estimation from the two racial groups. In addition, the moments of the skill endowment $E(a|q)$; $E(a|u)$; $E(a|q)$; $E(a|u)$ are obtained. These moments will be used in step 3 to obtain the parameters of the cost-skill endowment function. This step is performed by using restrictions implied by the model, and does not require imposing any additional assumptions.

2. Assuming a uniform cost distribution $g(\cdot)$; its parameters are computed using restrictions between the wage distribution and the estimated proportion of investors π^j implied by the model.

3. In the final step the parameters of the cost-skill endowment function are estimated. The assumption that groups have identical cost-skill relations $e(\cdot)$ must be dropped in order that the equilibrium estimated in the first step is an equilibrium under the estimated set of fundamentals. As mentioned in footnote 7, this can be done without any change in the parametrization of the wage distribution. Then, assuming linearity, all the parameters of the two functions can be computed using the information computed in (1) and (2).

Denote all the parameters computed in the first stage of the estimation with carets. Step 1 is performed by solving a system of eight independent equations in eight unknowns: $E(a|q)$; $E(a|u)$; $E(a|q)$; $E(a|u)$; ρ^b; α; y_1 and y_2; which has a unique solution[13]. The first two equations, (3.21) and (3.22), are obtained using the restriction imposed by the linearity of the wage function at $\hat\theta^w$ (see equation (3.12), and recall that the model is normalized with $\rho^w = 1$)); equations (3.23)-(3.26) are equivalent to (3.16), and the last two equations, (3.27) and (3.28), are restrictions implied by the Cobb-Douglas production function.

$$\hat y_q^w + \hat y_u^w \left(\frac{(1-\hat\theta^w)}{\hat\theta^w}\right)^{\hat\gamma^w - 1} = y_1 E^w(a|q); \tag{3.21}$$

$$y_q^b + y_u^b \left(\frac{(1-\hat\theta^b)}{\hat\theta^b}\right)^{\hat\gamma^b - 1} = y_1 \rho^b E^b(a|q); \tag{3.22}$$

$$\hat y_q^b = y_2 E^b(a|q); \tag{3.23}$$

$$\hat y_q^w = y_2 E^w(a|q); \tag{3.24}$$

$$\hat y_u^b = y_2 E^b(a|u); \tag{3.25}$$

$$\hat y_u^w = y_2 E^w(a|u) \tag{3.26}$$

$$y_1 = \alpha (C^b + C^w)^{\alpha - 1}(S^b + S^w)^{1-\alpha} \tag{3.27}$$

$$y_2 = (1-\alpha)(C^b + C^w)^{\alpha}(S^b + S^w)^{-\alpha} \tag{3.28}$$

where: $C^j = \lambda^j \hat\pi^j \rho^j (1 - F_q^j(\hat\theta^j)) E^j(a|q)$; $S^j = \lambda^j(\hat\pi^j F_q^j(\hat\theta^j) E^j(a|q) + (1-\hat\pi^j) F_u^j(\hat\theta^j) E^j(a|u))$[14]; and λ^j is the number of employed male workers of group j.

[13] It can be written as a system of linear equations.

[14] $F_q^j(\hat\theta^j) = \hat\theta^{\hat\gamma^b}$; and $F_u^j(\hat\theta^j) = 1 - (1 - \hat\theta)^{\hat\gamma^b}$

In step 2, assuming a uniform distribution for $g(\cdot)$ implies that the cumulative can be parametrized as a linear function $G(c) = p_1 + p_2 \cdot c;\ c \in [-p_1/p_2; (1 - p_1)/p_2];\ p_2 > 0.$ The first stage of the estimation provides enough information about the distribution over costs $g(\cdot)$ to recover p_1 and p_2. To see this, using (3.14) compute the maximum estimated costs $\hat{B}^b; \hat{B}^w$ paid by workers of group b and w, that is $\hat{B}^j = \int_\theta \hat{w}^j(\theta)(\hat{f}_q^j(\theta) - \hat{f}_u^j(\theta))d\theta$. Then, one can write the following identities

$$\hat{\pi}^b = p_1 + p_2 \hat{B}^b \tag{3.29}$$

$$\hat{\pi}^w = p_1 + p_2 \hat{B}^w \tag{3.30}$$

from which p_1 and p_2 can be computed. The computed parameters have an economic meaning only if $\hat{p}_2 > 0$. Notice that assuming that $g(\cdot)$ is the same for the two groups is crucial in this procedure (if distributions were different than this information would not be enough to compute all of their parameters). Furthermore, even with a unique distribution this procedure is not guaranteed to work because the solution to the system (3.29)-(3.30) might yield an inadmissible value of p_2.[15]

Finally, step 3 is performed by assuming different skill-cost functions for the two groups, parametrized as follows: $e^b(c) = p_3^b + p_4^b \cdot c$; and $e^w(c) = p_3^w + p_4^w \cdot c$. The parameters can be computed by solving the following systems of equations:

$$\begin{cases} \hat{E}^b(a|u) = \int_{c > \hat{B}^b} e^b(c)\hat{g}(c)dc \\ \hat{E}^b(a|q) = \int_{c \leq \hat{B}^b} e^b(c)\hat{g}(c)dc \end{cases} \tag{3.31}$$

and

$$\begin{cases} \hat{E}^w(a|u) = \int_{c > \hat{B}^w} e^w(c)\hat{g}(c)dc \\ \hat{E}^w(a|q) = \int_{c \leq \hat{B}^w} e^w(c)\hat{g}(c)dc \end{cases} \tag{3.32}$$

where $\hat{E}^j(a|q); \hat{E}^j(a|u)$ $j = b; w$ are computed in step 1, $\hat{B}^j$ is the estimated gross returns to investment computed as indicated in step 2 and $\hat{g}(c) = \hat{p}_2$ from step 2. Systems of equations (3.31) and (3.32) are linear, and have two unknowns each. Their solution will be economically meaningful if $\hat{p}_4^w < 0$ and $\hat{p}_4^b < 0$; as assumed in the description of the model.

This concludes the estimation of the model. If admissible parameter values can be obtained for the cost distribution and the cost-skill function, then the equilibrium estimated in the first stage

[15] For example, if the group that invests less has higher incentives $\hat{B}$; then the solution will include $p_2 < 0$; which implies that it is not possible to maintain that groups have the same distribution over costs.

will be also an equilibrium under the estimated set of fundamentals.

3.4.3. Data

The model is estimated using weekly wage data from 1963 to 1992 from the Current Population Surveys. Data were merged over ten three year intervals using weekly wage data of black and white males. Each sample is referred to with the last year of the three-year interval. Weekly data are used because information on hours worked was not collected before 1976, which makes it impossible to compute the hourly wage. Therefore, only workers working full time (full or part year) are considered. Observations are merged over three year periods in order to perform the estimation with larger samples. The number of observations ranges from 3995 (1965) to 6284 (1992) for blacks, and from 49087 (1965) to 76319 (1980) for whites. To perform consistent intertemporal comparisons, wages are standardized to the level 1982-84=100 using to the Consumer Price Index.

Even though CPS wages are topcoded, for many years there are no observations of black workers at the topcode level. Ignoring this fact would create an artificial asymmetry between the estimation of the parameters relative to black and white workers. Therefore, observations are topcoded at the same level of $1000 for the two groups, which guarantees that at least one percent of observations is always on the topcoded tail of the wage distribution.

3.5. Results

The model is estimated over three year intervals from 1963 to 1992. Table 3.1 shows the values of the parameters estimated in the first stage[16] (standard errors are in parenthesis). Other parameters computed in the first stage from the estimated parameters are reported in section 3.8, table 3.6. The proportion of blacks investing in human capital has always been lower than the proportion of whites, on average by 12.3%, but the difference has been declining over time. It is also interesting that the testing technology is estimated to be similar for blacks and whites, even though a formal test rejects the hypothesis of equality in each year.

In order to provide evidence about how well these estimates match the empirical distribution, the theoretical and estimated distributions of wages below the topcoded level is plotted in figure 3.3. In Section 3.9, table 3.9 reports summary statistics of these distributions from 1965 to 1992.

[16] The parameters are obtained by minimizing the log likelihood function using a version of the downhill simplex method.

All statistics are conditional on the wage being lower than the topcode level. This evidence seem to confirm that the model performs fairly well in representing the wage distribution

Table 3.2 presents information computed from the estimated parameters. The second column reports the difference between black and white's aggregate proportion of investors. The third and fourth columns present the implied incentives to invest in human capital computed using equation (3.14); predicted average wages computed integrating the wage function are in columns 5 and 6. The last two columns report the proportions of black and white workers employed in the simple task. All monetary variables are in dollars per week normalized at the 1982-84=100 level.

	Prop. of investors		Threshold signal		Testing technology	
Year	π^b	π^w	$\hat{\theta}^b$	$\hat{\theta}^w$	σ^b	σ^w
1965	.179 (.34E-02)	.333 (.10E-03)	.868 (.17E-01)	.822 (.29E-03)	1.68 (.14E-02)	1.67 (.20E-03)
1968	.222 (.12E-02)	.360 (.88E-04)	.885 (.87E-03)	.763 (.91E-04)	1.71 (.21E-02)	1.67 (.17E-03)
1971	.257 (.23E-02)	.387 (.96E-04)	.857 (.80E-03)	.687 (.96E-04)	1.70 (.26E-02)	1.69 (.21E-03)
1974	.274 (.20E-02)	.404 (.94E-04)	.900 (.28E-02)	.648 (.93E-04)	1.67 (.30E-02)	1.70 (.22E-03)
1977	.265 (.26E-02)	.383 (.92E-04)	.865 (.25E-02)	.757 (.10E-03)	1.77 (.27E-02)	1.77 (.20E-03)
1980	.266 (.24E-02)	.377 (.81E-04)	.906 (.24E-02)	.777 (.68E-04)	1.77 (.25E-02)	1.77 (.17E-03)
1983	.257 (.24E-02)	.368 (.75E-04)	.872 (.18E-01)	.764 (.24E-08)	1.79 (.25E-02)	1.80 (.19E-03)
1986	.262 (.20E-02)	.384 (.82E-04)	.866 (.19E-01)	.750 (.13E-02)	1.82 (.23E-02)	1.84 (.18E-03)
1989	.270 (.10E-02)	.381 (.76E-04)	.895 (.40E-02)	.748 (.13E-02)	1.77 (.21E-02)	1.81 (.18E-03)
1992	.264 (.15E-02)	.373 (.73E-04)	.853 (.15E-01)	.704 (.11E-02)	1.80 (.20E-02)	1.84 (.16E-03)

Table 3.1: Estimation results

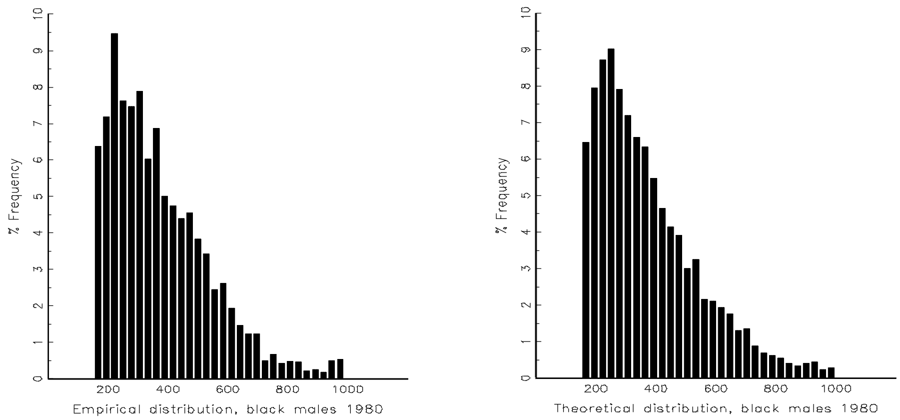

Figure 3.3: Empirical and Estimated distributions, black males 1980

Observe that the estimated differences in the proportion of investors declines between 1965 and 1980 from 15.3% to 11.1%, and remains approximately stable afterwards. The estimated rise in incentives to invest over time, and the decline in estimated average wages in each group after the seventies is consistent with the increase in within-group wage inequality occurred after 1970, and documented in Juhn et al. (1993) and Levy and Murnane (1992). The last two columns show that on average, 92.9% of blacks and 79.1% of whites were employed in the simple task.

As described in section 3.4, results in table 3.2 are obtained without making specific parametric assumptions about the distribution over costs and abilities. When the results from black and whites are merged in each year, and the restrictions discussed in section 3.4.2 are imposed, the other parameters of the model can be obtained (see tables 3.7 and 3.8) and welfare analyses can be performed.

Table 3.3 presents the computed average cost of investment borne by workers. Average welfare is simply the average wage minus the average cost. The last two columns report the black to white average welfare and wage ratios. Notice that the pattern of reduction in black/white differentials is not substantially different when looking at welfare instead of wages, and confirms that discrimination, measured in terms of either welfare or wage inequality, decreased sharply before 1980, but remained approximately stable afterwards.

Table 3.4 reports the proportion of investors, average wage, and average welfare in the equilibria that were not selected of the first and the last year the estimation has been performed. In

		Gross returns to Invest		Predicted average wage		% workers in simple task	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Year	$\pi^w - \pi^b$	Black	White	Black	White	Black	White
1965	.154	153.1	162.0	322.4	430.3	.935	.869
1968	.138	153.6	170.5	338.3	457.7	.939	.811
1971	.130	149.6	184.4	357.3	482.4	.914	.732
1974	.130	155.9	195.9	369.0	488.1	.940	.688
1977	.118	181.2	206.3	371.6	474.2	.919	.801
1980	.111	181.5	207.7	375.2	471.1	.946	.821
1983	.111	186.1	218.6	348.5	450.2	.925	.812
1986	.122	202.0	239.4	347.1	460.2	.920	.794
1989	.111	195.2	239.5	351.9	459.8	.938	.793
1992	.109	202.6	243.4	349.1	447.0	.911	.756

Table 3.2: Computed results

	Average cost of investment		Average welfare		Black/White ratio	
Year	Black	White	Black	White	Welfare	Wage
1965	26.5	50.8	295.9	379.5	.779	.744
1968	31.1	53.5	307.2	404.2	.760	.740
1971	29.7	51.5	327.6	430.8	.761	.741
1974	31.1	54.0	337.8	434.1	.778	.756
1977	40.5	63.4	331.1	410.8	.805	.785
1980	40.0	61.6	335.2	409.5	.817	.796
1983	41.3	60.3	307.1	389.9	.787	.776
1986	42.4	69.4	304.7	390.9	.787	.754
1989	38.2	62.3	313.7	397.5	.779	.763
1992	40.6	64.9	308.5	382.1	.806	.781

Table 3.3: Aggregate welfare, actual equilibrium

addition, the computation is reported for 1968 because it has unique features that will be described below; computations for the other years are presented in section 3.10. Equilibria are sorted by the proportion of black investors; the equilibrium which has been selected in the economy is in italics.

Notice that the economy has always been capable of generating either four or five equilibria (including the selected equilibrium). These equilibria can be classified into three categories: (1) corner equilibria, where all blacks or all whites are employed in the simple task, (2) interior equilibria where blacks receive a smaller average wage than whites (3) interior equilibria where whites receive a smaller average wage than blacks

The first observation is that the economy always selected equilibria of the second type, and that

Investors		Average wage		Average welfare	
π^b	π^w	Black	White	Black	White
1965*					
.000	.410	323.5	433.8	323.5	370.3
.103	.155	305.4	405.9	290.4	383.0
.179	.333	322.4	430.3	295.9	379.5
.191	.401	324.6	433.4	296.3	371.5
.800	.000	951.3	347.3	818.7	347.3
1968*					
.000	.366	336.3	458.2	336.3	403.7
.222	.360	338.3	457.7	307.2	404.2
.532	.245	475.6	443.1	391.0	408.5
.590	.174	556.1	424.5	460.2	400.8
.823	.000	1183.9	295.4	1038.4	295.4
1992*					
.000	.376	346.2	447.3	346.2	381.7
.264	.373	349.1	447.0	308.5	382.1
.715	.017	1152.0	238.8	982.4	237.0
.737	.000	1248.3	220.2	1070.2	220.2

*Selected equilibria in italics

Table 3.4: Equilibria, 1965, 1968, 1992

no equilibrium with substantially smaller differences in terms of average wage inequality existed in the economy[17]. For example, in 1965, three interior equilibria exist, but their black to white average wage ratios differ at most by 0.8 percentage points.

The implication of this result is that statistical discrimination generated by self-fulfilling expectations about workers' productivity did not exacerbate wage inequality in the U.S. Therefore, the observed reduction in wage inequality must be an effect of changes in the fundamental parameters of the model.

In the corner equilibria the group which is not investing is obviously worse-off in terms of wages than in the selected equilibrium. It is interesting to notice the effect of group size on the performance of the economy in this type of equilibria. In the corner equilibria where all blacks are employed in the simple task, they perform better in terms of welfare with respect to the selected equilibrium despite receiving a smaller average wage. This is because they save the investment cost, while there are not big effects on wages because only less than 10% of them are employed in

[17] There is an exception in 1977, when an equilibrium existed where blacks invest more than whites. In this equilibrium, blacks' average wage is about 8% better than in the actual equilibrium (but still 15.3% less than whites' average wage). Blacks are worse in terms of welfare than in the selected equlibrium since they have to pay a higher cost of investment.

the complex task in the selected equilibrium (and they account only for about 10% of the labor force). On the contrary, there are big effects in the overall performance of the economy if all whites have to be employed in the simple task. Employers must in this type of equilibria hire almost all blacks in the complex task, and this generates high incentives for them to invest. Since the large group is not investing, there is a very high marginal productivity in the complex task, and low marginal productivity in the simple task. Therefore, while blacks are better off than in the selected equilibrium whites are penalized in terms of both wages and welfare.

Finally, observe that equilibria of type (3) are infrequent: only in 1968 (see table 3.4) there exists an interior equilibrium where whites invest less than blacks and receive a smaller average wage. Still, whites' welfare is larger because blacks are paying a substantial cost of investment in order to sustain high levels of investment.

In table 3.5 a measure of discrimination is computed by calculating what equilibrium the economy would display in a color-blind society, where employers cannot distinguish the race of their employees. While this does not generate a society without differentials (since some parameters are different for the two groups), differences in terms of wages and welfare are substantially reduced. The direct effect of this experiment is that employers cannot compute by-group statistics to assess individual productivity, therefore the average expected productivity of a worker will be a weighted average of a black and a white's worker productivity. This induces an indirect effect of creating similar incentives to invest in human capital. Since the testing technology is different, incentives are different as well, and for some years they are such that black investment is higher than whites investment in equilibrium. Despite the burden of a higher investment cost, on average blacks gain 24.5% in terms of welfare with respect to the selected equilibrium. Whites instead lose only 2.2%, since their wage loss is partially compensated by the smaller cost of investment paid.

It is important to stress that this experiment has been performed under the estimated set of parameters, which are conditional on the existing policy adopted in the U.S. at the time data were collected. After 1965, the U.S. labor market experienced several waves of enforcement of anti-discrimination policies. While these policies are not explicitly modelled here, they probably affect the estimation of the fundamentals. Since a color blind society is normally intended as a society without preferential treatment, it would be more appropriate to discount the effect of preferential policies on welfare before performing the experiment. To the extent that preferential policies improve blacks' welfare, the assessment of the gains from a color blind society performed here overstates the gains for blacks.

	Investors		Average	Welfare		Gains*	
Year	Black	White	wage	Black	White	Black	White
1965	.274	.268	415.8	374.6	375.4	126.6	98.9
1968	.371	.359	446.9	391.5	393.7	127.5	97.4
1971	.381	.378	470.2	420.0	420.4	128.2	97.6
1974	.389	.395	476.9	425.9	424.6	126.1	97.8
1977	.375	.375	464.4	402.5	402.5	121.6	98.0
1980	.369	.370	461.1	401.2	401.0	119.7	97.9
1983	.358	.359	439.3	381.0	380.6	124.1	97.6
1986	.371	.375	448.1	382.0	380.9	125.4	97.4
1989	.364	.373	448.6	390.4	388.3	124.4	97.7
1992	.357	.365	436.7	375.7	373.8	121.8	97.8

* Welfare percentage gains with respect to the selected equilibrium

Table 3.5: A color-blind society

3.6 Conclusions

In this chapter, a model of statistical discrimination with multiple equilibria is presented, and a procedure capable of identifying both the parameters of the model and the equilibrium selected in the economy is developed and implemented. The results obtained by estimating the model over time show that the U.S. economy has always been close to the equilibrium with the smallest racial wage differential. The other equilibria that the model is capable of generating display larger differences in terms of wage and job assignment differentials. This implies that self-fulfilling expectations generated by statistical discrimination did not exacerbate wage differentials in the U.S. Moreover, there is no evidence that equilibrium selection played a role in reducing wage discrimination in the last thirty years. This seems particularly interesting because one of the effects that the theoretical literature attributes to anti-discrimination policies such as affirmative action is to change the set of equilibria of the economy[18]. In principle, policy can be capable of eliminating equilibria with inequality, thus leading the economy towards group equality[19]. For example if at the introduction of affirmative action policies in 1965 the economy had been at the "corner" equilibrium with blacks employed only in the less skilled task (see table 3.4) then theory predicts that a quota would have been enough to move the economy towards an equilibrium with

[18] This effect is also discussed in the public debate, see Bergman (1996) and Sowell (1990).

[19] One of the important points of the literature is actually that the policy may not be able to eliminate such equilibria. See Coate and Loury (1993), and Moro and Norman (1997). Moreover, it is problematic to assess the effects of policies when starting from an interior equilirium because of the difficulties in comparing sets of equilibria.

lower wage differentials[20].

Can we draw more precise policy implications from these results? To answer this question it is important to note that the estimation has been performed conditionally on the anti-discrimination policies enforced at the time the data were collected. If those policies changed the wage distribution, their effect should be incorporated in the estimated parameters. This makes it is impossible to distinguish to what extent changes in the fundamentals are caused by policy and to what extent they are caused by other changes in, for example, the quality of education, years of schooling, migration, etc... For this reason, it is problematic to interpret the results of counterfactual experiments such as introducing a color-blind society (see the exercise performed at the end of section 3.5)

Nonetheless, the methodology proposed in this chapter can in principle be generalized. The idea is to propose a "parametrization" for specific anti-discrimination policies, and to estimate the more general model in order to provide a precise assessment of the effect of such policies. Then, counterfactual experiments associated with varying policies could be consistently estimated.

3.7. Proof of proposition 9

The proof assumes that only two firms exist, and it is easily generalizable to the case of N firms

(sufficiency) Suppose a firm deviates and plays strategies with actions on the outcome path $\left\langle w^{j\prime}, t^{j\prime}\right\rangle_{j=b,w}$ different from $\langle w, t\rangle$ given by (3.12) and the cutoff rule with critical point solving (3.10). Define sets

$$\Theta^{jh} = \left\{\theta : w^{j\prime}(\theta) > w^j(\theta)\right\};\ \Theta^{jl} = \left\{\theta : w^{j\prime}(\theta) < w^j(\theta)\right\};\ \Theta^{je} = \left\{\theta : w^{j\prime}(\theta) = w^j(\theta)\right\}:$$

Letting $C^{j\prime}$ and $S^{j\prime}$ denote the implied factor inputs for the deviator given that the other firms play according to the equilibrium strategies we can express profits for the deviator as (using equations (3.6) and (3.7))

$$\begin{aligned}\Pi_d^i = {} & y\left(C^{b\prime}+C^{w\prime}, S^{b\prime}+S^{w\prime}\right) - \sum_{j=b,w}\int_{\theta\in\Theta^{jh}} w^{j\prime}(\theta) f_{\pi_j}(\theta)\,d\theta \\ & - \frac{1}{2}\int_{\theta\in\Theta^{je}} w^j(\theta) f_{\pi_j}(\theta)\,d\theta;\ \text{where}\end{aligned} \tag{3.33}$$

[20] This result can be derived as in the similar model by Moro and Norman (1997)

$$C^{j'} = \rho^j E^j(a|q)\left(\int_{\theta\in\Theta^h} t'(\theta)\pi f_q(\theta)\,d\theta + \frac{1}{2}\int_{\theta\in\Theta^e} t'(\theta)\pi f_q(\theta)\,d\theta\right)$$

$$S^{j'} = \int_{\theta\in\Theta^{jh}} \left(1 - t^{j'}(\theta)\right) E^j(a|\theta)\,d\theta + \frac{1}{2}\int_{\theta\in\Theta^{je}} \left(1 - t^{j'}(\theta)\right) E^j(a|\theta)\,d\theta$$

We now multiply both sides in the second equation of (3.33) by $y_1(C,S)$, where C,S are the implied factor inputs in the proposed equilibrium, and substitute the identity $\pi^j f_q^j(\theta) = p^j(\theta,\pi^j) f_{\pi^j}(\theta)$. Similarly we multiply the third equation by $y_2(C,S)$ and get

$$\begin{aligned} y_1(C,S)C^{j'} &= \rho^j E^j(a|q)\left(\int_{\theta\in\Theta^h} t'(\theta)p^j(\theta,\pi^j)\, y_1(\cdot) f_\pi(\theta)\,d\theta \right. \\ &\quad \left. + \frac{1}{2}\int_{\theta\in\Theta^e} t'(\theta)p^j(\theta,\pi^j)\, y_1(\cdot) f_\pi(\theta)\,d\theta\right) \qquad (3.34) \\ y_2(C,S)S^{j'} &= \int_{\theta\in\Theta^h} (1 - t'(\theta))y_2(\cdot)E^j(a|\theta)\,d\theta + \frac{1}{2}\int_{\theta\in\Theta^e} (1 - t'(\theta))y_2(\cdot)E^j(a|\theta)\,d\theta. \end{aligned}$$

But $w^j(\theta) = \max\left\{y_2(C,S), p^j(\theta,\pi^j)\, y_1(C,S)\right\}$. Furthermore, $y\left(C^{j'},S^{j'}\right) \le y_1(C,S)C^{j'} + y_2(C,S)S^{j'}$ by concavity and constant returns. Summing the equalities in (3.34) and using these two facts we find that

$$y\left(C^{j'},S^{j'}\right) \le \sum_{j=b,w}\left(\int_{\theta\in\Theta^{jh}} w^j(\theta) f_{\pi^j}(\theta)\,d\theta + \frac{1}{2}\int_{\theta\in\Theta^{je}} w^j(\theta) f_{\pi^j}(\theta)\,d\theta\right)$$

and substituting into (3.33) we get $\Delta_d \le \sum_{j=b,w}\left(\int_{\theta\in\Theta^{jh}}\left(w^j(\theta) - w^{j'}(\theta)\right) f_{\pi^j}(\theta)\,d\theta\right) \le 0$. ■

The necessity part will be proved from a sequence of intermediate results:

Lemma 14. On the equilibrium path, wage schedule posted by firms are the same almost everywhere.

Proof. Suppose there is a positive measure set of signals where firms post different wages. Then, this implies that there is a set of signals of positive measure where one firms post wages higher than the wages posted by the other firms. Than, this firm can reduce wages on this set, while keeping them above wages posted by the other firms. Output for this firm will not change, but the deviation is profitable since the total wage bill paid is smaller. ■

Lemma 15. Let $\left\langle t_i^j \right\rangle_{j=b,w}$ denote the implied task assignment rules on the equilibrium path for firm $i = 1.N$. Then, this rules have the "cutoff property", that is there exists some $\hat{\theta}_i^j \in (0,1)$ such that $t_i^j(\theta) = 1$ for almost all $\theta > \hat{\theta}_i^j$ and $t_i^j(\theta) = 0$ for almost all $\theta < \hat{\theta}_i^j$ and for $i = 1.N$.

Proof. If the claim is false there are sets $\Theta^h, \Theta^l \subseteq [0,1]$ with positive measure such that $\theta^h > \theta^l$ for all $\theta^h, \theta^l \in \Theta^h \times \Theta^l$, $t_i^j(\theta^h) = 0$ for all $\theta^h \in \Theta^h$ and $t_i^j(\theta^l) = 1$ for all $\theta^l \in \Theta^l$, and at least one group $j = b,w$. f_q^j and f_u^j are continuous, so $f_{\pi j}^j$ is continuous. Assume w.l.o.g. $\int_{\theta\in\Theta^h}\int_c a(c)g^j(c;\theta)dcd\theta = \int_{\theta\in\Theta^l}\int_c a(c)g^j(c;\theta)d\theta > 0$. Consider the alternative task assignment rule for group j,

$$t_i^0(\theta) = \begin{cases} 1 & \text{if } \theta \in \Theta^h \\ 0 & \text{if } \theta \in \Theta^l \\ t_i^j(\theta) & \text{otherwise} \end{cases} \tag{3.35}$$

Let S_i^j, C_i^j and S_i^0, C_i^0 be the factor inputs implied by t_i^j, t_i^a respectively. Since $\int_{\theta\in\Theta^h}\int_c a(c)g^j(c;\theta)dcd\theta = \int_{\theta\in\Theta^l}\int_c a(c)g^j(c;\theta)d\theta$ it follows that $S_i^0 = S_i^a$. Since the deviation assigns to the complex task workers who are more likely to be productive we have that $C_i^0 > C_i$. To see this we let $l(\theta) = f_q^j(\theta)/f_u^j(\theta)$ denote the likelihood ratio and note that

$$2(C_i^0 - C_i^j)/\rho^j \int_{\theta\in\Theta^h}\int_{c<H^j} a(c)g(c)f_q(\theta)dcd\theta - \int_{\theta\in\Theta^l}\int_{c<H^j} a(c)g(c)f_q(\theta)dcd\theta$$

By the monotone likelihood ratio there exists θ^* such that $l(\theta) \geq l(\theta^*)$ for all $\theta \in \Theta^h$ and $l(\theta) \leq l(\theta^*)$ for all $\theta \in \Theta^l$, with at least one inequality holding strictly. Hence $C_i^0 - C_i^j > 0$, so output, and therefore also profits, are higher under t_i^a. ■

Lemma 16. Let $\left\langle t_i^j \right\rangle_{j=b,w}$ denote the implied task assignment rule on the equilibrium path for firms $i = 1.N$ and let t^j be cutoff rule with critical point $\hat{\theta}^j\left(\pi_b,\pi_w\right)$. Then $t_i^j(\theta) = t^j(\theta)$; $j = b,w$ almost everywhere.

Proof. By Lemmas 14 and 15 the problem of finding an optimal task assignment rule reduces to finding a solution to problem (3.10) in the main text. It is straightforward but tedious to show that this is a concave problem, and that the Kuhn-Tucker conditions are necessary and sufficient. The solution can possibly be a corner solution ■

Lemma 17. Suppose $\{w^j\}_{j=b,w}$ is a pair of equilibrium wage schedules and let $\bar{\theta}^j(\pi)$ be the solution to (3.10). Then there are pairs $\{k_s^j;k_c^j\}_{j=b,w}$ such that $w^j(\theta)=k_s^j E^j(a|\theta)$ for $j=b,w$ and for almost all $\theta<\bar{\theta}^j(\pi)$ and $w^j(\theta)=k_c^j\rho^j p^j(\theta;\pi^j)E^j(a|q)$ for $j=b,w$ and for almost all $\theta>\bar{\theta}^j(\pi^j)$.

Proof. First it is shown that $w^j(\theta)=k_s^j E^j(a|\theta)$ for almost all $\theta<\bar{\theta}^j$. For contradiction assume that for group j there are sets $\Theta^a;\Theta^b\subseteq[0;\bar{\theta}^j(\pi)]$ with strictly positive measure such that $w^j(\theta)/E^j(a|\theta)<k$ for all $\theta\in\Theta^a$ and $w_i(\theta)\geq k$ for all $\theta\in\Theta^b$: Consider a unilateral deviation $\{w_i^0;\xi_i^0\}$ by firm i where

$$\frac{w_i^0(\theta)}{E^j(a|\theta)}=\begin{cases}k+\epsilon & \text{for } \theta\in\Theta^a\\ 0 & \text{for } \theta\in\Theta^b\\ k & \text{otherwise}\end{cases}\quad: \tag{3.36}$$

Assume w.l.o.g. $\int_{\theta\in\Theta^b}\int_c a(c)g^j(c;\theta)dcd\theta=\int_{\theta\in\Theta^a}\int_c a(c)g^j(c;\theta)d\theta>0$; which implies that the input of both factors remains constant if the task assignment is unchanged, which one can assume. The difference in payoffs for the deviating firm is then just the difference in wage payments, i.e.

$$\begin{aligned}\Delta(\epsilon)&=\left[\int_{\theta\in\Theta^b}w^j(\theta)f^j_{\pi^j}(\theta)d\theta-\int_{\theta\in\Theta^a}w_i^0(\theta)f^j_{\pi^j}(\theta)d\theta\right]=\\&=\int_{\theta\in\Theta^b}kE(a|\theta)f^j_{\pi^j}(\theta)d\theta-\int_{\theta\in\Theta^a}(k+\epsilon)E(a|\theta)f^j_{\pi^j}(\theta)d\theta=\\&=\int_{\theta\in\Theta^b}k\int_c a(c)g^j(c|\theta)f^j_{\pi^j}(\theta)dcd\theta-\int_{\theta\in\Theta^a}(k+\epsilon)\int_c a(c)g^j(c|\theta)f^j_{\pi^j}(\theta)dcd\theta\\&=k\int_{\theta\in\Theta^b}\int_c a(c)g^j(c;\theta)dc-(k+\epsilon)\int_{\theta\in\Theta^a}\int_c a(c)g^j(c;\theta)dcd\theta\end{aligned}$$

Since $\lim_{\epsilon\to 0}\Delta(\epsilon)>0$ there exists $\epsilon>0$ such that $\Delta(\epsilon)>0$, so for ϵ small enough the deviation is profitable. Symmetrically, suppose there are sets $\Theta^a;\Theta^b\subseteq[\bar{\theta}^j(\pi);1]$ with strictly positive measure (where we again w.l.o.g. may assume $\int_{\theta\in\Theta^a}\int_{c\in H^j}a(c)g^j(c;\theta)dcd\theta=\int_{\theta\in\Theta^b}\int_{c\in H^j}a(c)g^j(c;\theta)d\theta)$ such that $(w_i(\theta)/(\rho^j p^j(\theta;\pi^j)E(a|\theta))<k$ for all $\theta\in\Theta^a$ and $(w_i(\theta)/(\rho^j p^j(\theta;\pi^j)E(a|\theta))\geq k$ for all $\theta\in\Theta^b$: Again consider a deviation according to (2.32). Since $p^j(\theta;\pi^j)=(\pi^j f_q^j(\theta)/f^j_{\pi^j}(\theta))$ output is unchanged and by a symmetric argument the deviation profitable for ϵ small enough. ■

(necessity) From Lemma 17 the firms must offer wages that are identical almost everywhere and satisfy $w(\theta)/E^j(a|\theta)=k_s^j$ for $\theta<\bar{\theta}(\pi)$ and $w(\theta)=p^j(\theta;\pi^j)\rho^j E^j(a|q)k_c^j$ for $\theta>\bar{\theta}(\pi)$, for some real numbers k_s^j, k_c^j; $j = b, w$. It remains to be shown that $k_s^j = y_2(C,S)$ and $k_c^j = y_1(C,S)$. It is easy to show that if for some group $k_s^j < y_2(C,S)$ and $k_c^j < y_1(C,S)$; then firms are making positive profits and a deviation where firm i offers $w_i'(\theta) = w_i(\theta) + \varepsilon$ for all θ would be profitable for ε small enough. Also, if both inequalities would go the other way firms would make negative profits and a deviation to $w_i(\theta) = 0$ for all θ would be profitable. The two cases that require some work are when the inequalities work in opposite directions. The arguments are symmetric and we will only consider the case with $k_s^j > y_2(C,S)$ and $k_c^j < y_1(C,S)$

Recall that $y_1(C,S)p^j(\hat{\theta}^j(\pi),\pi)E^j(a|q)\rho^j = y_2(C,S)E^j(a|\hat{\theta}^j(\pi))$ by (3.11). Hence if $k_s^j > y_2(C,S)$ and $k_c < y_1(C,S)$ then $k_c^j p^j(\hat{\theta}^j, \pi^j)E^j(a|q)\rho^j < k_s^j E^j(a|\hat{\theta}^j(\pi))$ and there is an interval $(\hat{\theta}^j(\pi), \theta^*)$ such that $w^j(\theta) = k_c^j p^j(\theta, \pi^j)E^j(a|q)\rho^j < k_s^j E^j(a|\theta)$ for all θ in this interval. We will show that it is better to dispose of some of the workers being paid $k_s^j E^j(a|\hat{\theta}^j(\pi))$ and attract cheaper workers with $\theta \in (\hat{\theta}^j(\pi), \theta^*)$ from the other firm. To do so, choose $\theta', \theta'', \theta^*$ such the following equalities are satisfied: (a) $\int_0^{\theta'}\int_c a(c)g(c;\theta)dcd\theta = \int_{\hat{\theta}^j}^{\theta^*}\int_c a(c)g(c;\theta)dcd\theta$; and (b) $\int_{\hat{\theta}^j}^{\theta''}\int_c a(c)g(c;\theta)dcd\theta = \int_{\theta''}^{\theta^*}\int_c a(c)g(c;\theta)dcd\theta$ and consider firm i's deviation $w_i'(\theta)$ from wages given to group j such that:

$$\frac{w_i'(\theta)}{p^j(\theta,\pi^j)\rho^j E^j(a|q)} = \begin{cases} 0 & \text{for } \theta \in [0, \theta') \\ k_c^j + \varepsilon & \text{for } \theta \in [\hat{\theta}^j, \theta^*) \\ k_c^j & \text{for } \theta \in [\theta^*, \infty) \cup [\theta', \hat{\theta}^j) \end{cases}$$

$$t_i'(\theta) = \begin{cases} 0 & \text{for } \theta \in [0, \theta'') \\ 1 & \text{for } \theta \in [\theta'', \infty) \end{cases}$$

By construction, the input of simple labor is unchanged. The change in effective units of complex labor from group j is given by $C' - C^j = \frac{\pi^j}{2}E^j(a|q)\rho^j\left(F_q^j(\theta^*) - 2F_q^j(\theta'') + F_q^j(\hat{\theta})\right)$. Using inequality (b) above it is easy to show that $C' - C > 0$. Thus, output increases and the difference in profits must be larger than the difference in the wage costs, so

$$\begin{aligned} \Delta(\varepsilon) > {} & \frac{1}{2}\int_0 w^j(\theta)f^j_{\pi^j}(\theta)d\theta - \frac{1}{2}\int_{\theta'} w^j(\theta)f^j_{\pi^j}(\theta)d\theta \\ & - \int_{\hat{\theta}^j}^{\theta^*}(w'(\theta))f^j_{\pi^j}(\theta)d\theta - \frac{1}{2}\int_{\theta^*} w^j(\theta)f^j_{\pi^j}(\theta)d\theta \\ = {} & \frac{1}{2}k_s^j\int_0^{\theta'}E(a|\theta)f^j_{\pi^j}(\theta')d\theta - \frac{1}{2}\rho^j E(a|q)\int_{\hat{\theta}^j}^{\theta^*}p^j(\theta,\pi^j)\left(k_c^j + 2\varepsilon\right)f^j_{\pi^j}(\theta)d\theta \end{aligned}$$

Recall that $k_c^j p^j\left(\theta;\pi^j\right) E^j(a|q)\rho^j < k_s^j E^j(a|\theta)$ for $\theta \in (\underline{\theta}^j;\theta^*)$; and $\int_0^{\theta^o}\int_c a(c)g(c;\theta)dcd\theta = \int_{\underline{\theta}^j}^{\theta^*}\int_c a(c)g(c;\theta)dcd\theta$. Thus:

$$\frac{1}{2}\int_{\underline{\theta}}^{\theta^*} k_s^j E^j(a|\theta) f_{\pi^j}^j(\theta)\,d\theta > \frac{1}{2}E^j(a|q)\rho^j \int_{\underline{\theta}}^{\theta^*} p^j\left(\theta;\pi^j\right) k_c^j f_{\pi^j}^j(\theta)\,d\theta.$$

Hence, $\lim_{\epsilon\to 0}\Delta(\epsilon) > 0$ and there exists $\epsilon > 0$ such that the deviation is profitable. The case with $k_s^j < y_2(C;S)$ and $k_c^j > y_1(C;S)$ can be treated symmetrically and Proposition 9 follows. ■

3.8. Other parameters

The following tables report the parameters obtained in the estimation not reported in the main text

Year	y_u^b	y_u^w	y_q^b	y_u^w
1965	150.5	150.6	1093.9	976.4
1968	150.0	150.0	988.4	983.9
1971	158.0	158.0	920.7	961.6
1974	130.2	130.3	995.8	983.4
1977	151.8	151.8	971.9	974.5
1980	150.5	150.5	990.9	984.0
1983	134.5	134.5	956.0	975.7
1986	122.3	122.3	971.2	984.7
1989	108.1	108.1	1005.9	1015.4
1992	116.3	116.3	987.9	977.7

Table 3.6 Other parameters obtained in the first step

Year	$E^b(ajq)$	$E^w(ajq)$	$E^b(aju)$	$E^w(aju)$	$½^b$	®
1965	342.9	570.3	216.5	176.0	1.057	.228
1968	419.7	678.3	223.2	183.6	1.001	.303
1971	489.7	772.6	244.1	200.8	.991	.400
1974	573.7	834.9	198.8	163.3	.957	.452
1977	499.0	723.8	216.7	181.9	.999	.325
1980	495.9	698.9	208.1	176.5	.980	.294
1983	473.0	691.0	192.6	163.8	.997	.320
1986	500.0	742.8	177.0	147.7	.998	.350
1989	532.3	758.2	154.5	131.0	.986	.351
1992	532.4	745.0	174.6	148.7	.985	.407

Table 3.7: Other parameters

Year	p_1	p_2	p_3^b	p_3^w	p_4^b	p_4^w
1965	-2.480	.0174	10420.0	9377.42	-57.47	-50.30
1968	-1.030	.0082	5571.3	5763.54	-26.28	-26.14
1971	-.306	.0038	3284.8	3662.11	-11.90	-12.55
1974	-.231	.0032	3433.2	3620.55	-11.79	-11.62
1977	-.587	.0047	4178.1	4379.30	-14.97	-15.02
1980	-.507	.0043	3891.6	4037.16	-13.48	-13.47
1983	-.377	.0034	3443.0	3698.75	-10.79	-11.15
1986	-.396	.0033	3665.2	3926.36	-10.87	-11.05
1989	-.218	.0025	3215.0	3444.19	-8.81	-8.91
1992	-.279	.0027	3479.3	3633.47	-9.54	-9.43

Table 3.8: Parameters of the cost and skill distributions

3.9. Summary statistics for the theoretical and empirical distributions

Blacks		Mean	Std. Dev.	Skewness	Kurtosis
1965	Empirical	314.7	127.5	146.1	202.4
	Estimated	322.3	151.5	193.0	261.4
1968	Empirical	333.2	142.8	163.7	227.1
	Estimated	338.3	153.0	178.6	240.4
1971	Empirical	357.3	152.6	170.8	230.6
	Estimated	355.1	145.9	162.3	225.9
1974	Empirical	370.1	162.7	167.3	235.5
	Estimated	368.9	166.1	173.5	239.1
1977	Empirical	369.9	163.6	168.3	234.7
	Estimated	371.6	174.1	189.0	254.0
1980	Empirical	375.7	171.8	179.9	247.4
	Estimated	375.2	174.7	185.4	250.5
1983	Empirical	350.4	167.4	176.7	243.2
	Estimated	347.8	174.1	189.8	253.9
1986	Empirical	351.8	184.5	196.7	266.5
	Estimated	347.1	185.3	199.1	265.9
1989	Empirical	354.9	185.1	191.8	262.2
	Estimated	351.9	187.7	196.1	265.4
1992	Empirical	357.2	189.7	201.9	272.6
	Estimated	349.0	188.8	202.6	271.1

Whites		Mean	Std. Dev.	Skewness	Kurtosis
1965	Empirical	432.6	180.8	186.4	258.6
	Estimated	430.3	177.8	172.9	244.9
1968	Empirical	458.1	189.5	186.5	263.0
	Estimated	457.7	188.8	177.2	254.8
1971	Empirical	479.9	196.5	184.5	264.0
	Estimated	482.3	199.6	181.6	263.1
1974	Empirical	485.5	207.2	182.1	270.9
	Estimated	488.1	212.1	181.7	273.1
1977	Empirical	465.2	200.8	181.5	265.5
	Estimated	474.2	207.8	183.5	269.0
1980	Empirical	459.8	202.0	184.6	267.6
	Estimated	471.1	208.1	184.3	269.8
1983	Empirical	441.0	213.3	191.9	276.7
	Estimated	450.3	207.5	193.9	276.6
1986	Empirical	449.5	219.3	196.3	285.4
	Estimated	460.2	227.2	194.4	287.5
1989	Empirical	445.8	222.9	198.9	289.2
	Estimated	459.8	231.9	198.3	294.4
1992	Empirical	434.2	222.3	207.3	292.4
	Estimated	447.0	229.9	204.3	293.8

Table 3.9: Summary statistics for the empirical and estimated distributions

3.10. Computed equilibria

The equilibrium that has been selected in the economy is reported in italics

	Investors		Average wage		Average welfare	
	¼^{b}	¼^{w}	Black	White	Black	White
1965	.000	.410	323.5	433.8	323.5	370.3
	.103	.155	305.4	405.9	290.4	383.0
	.179	.333	322.4	430.3	295.9	379.5
	.191	.401	324.6	433.4	296.3	371.5
	.800	.000	951.3	347.3	818.7	347.3
1968	.000	.366	336.3	458.2	336.3	403.7
	.222	.360	338.3	457.7	307.2	404.2
	.532	.245	475.6	443.1	391.0	408.5
	.590	.174	556.1	424.5	460.2	400.8
	.823	.000	1183.9	295.4	1038.4	295.4
1971	.000	.390	353.7	482.7	353.7	430.7
	.257	.387	357.3	482.4	327.6	430.8
	.335	.383	364.0	481.7	321.8	431.0
	.766	.023	1226.7	253.6	1086.4	251.7
	.795	.000	1385.4	225.7	1236.5	225.7
1974	.000	.406	367.3	488.4	367.3	434.1
	.160	.405	367.5	488.3	352.1	434.1
	.274	.404	369.0	488.1	337.8	434.1
	.744	.017	1308.7	211.8	1170.3	210.5
	.766	.000	1430.2	190.1	1285.0	190.1
1977	.000	.386	368.7	474.5	368.7	410.3
	.264	.383	371.6	474.2	331.1	410.8
	.436	.360	400.2	472.6	325.4	413.9
	.752	.038	1037.7	331.1	883.6	326.2
	.808	.000	1282.6	287.5	1112.3	287.5

	Investors		Average wage		Average welfare	
	¼^b	¼^w	Black	White	Black	White
1980	.000	.380	373.7	471.4	373.7	409.2
	.266	.377	375.2	471.1	335.2	409.5
	.354	.370	379.1	470.7	322.2	410.5
	.696	.048	893.9	364.9	754.1	358.9
	.761	.000	1116.1	324.0	957.5	324.0
1983	.000	.371	345.4	450.5	345.4	389.3
	.274	.367	348.5	450.2	307.1	389.9
	.701	.031	968.8	317.0	819.0	313.4
	.744	.000	1123.1	288.5	959.5	288.5
1986	.000	.387	344.8	460.4	344.8	390.4
	.262	.384	347.1	460.2	304.7	390.9
	.723	.024	1057.8	294.2	889.8	291.1
	.757	.000	1193.2	268.1	1013.2	268.1
1989	.000	.383	350.5	459.9	350.5	397.3
	.121	.383	350.6	459.9	337.1	397.3
	.270	.381	351.9	459.8	313.7	397.5
	.695	.017	1145.0	280.8	987.8	279.3
	.715	.000	1227.7	266.3	1063.2	266.3
1992	.000	.376	346.2	447.3	346.2	381.7
	.264	.373	349.1	447.0	308.5	382.1
	.715	.017	1152.0	238.8	982.4	237.0
	.737	.000	1248.3	220.2	1070.2	220.2

Table 3.10: Equilibria, 1965-92

Bibliography

[1] Acemoglu, Daron, "Public Policy in a Model of Long-term Unemployment", Economica, 62 (1995), 161-78.

[2] Aigner, Dennis J. and Glen G. Cain "Statistical Theories of Discrimination in the Labor Market", Industrial and Labor Relations Review, 30 (1977), 175-187

[3] Akerlof, George, "The Economics of Caste and the Rat Race and Other Woeful Tales." Quarterly Journal of Economics, November 1976, 90, 599-617.

[4] Arrow, Kenneth J., "The Theory of Discrimination," in O. Ashenfelter and A. Rees, ed, Discrimination in Labor Markets, Princeton, NJ: Princeton University Press, 1973, 3-33.

[5] Becker, Gary S., The Economics of Discrimination, Chicago, The University of Chicago press, 1957.

[6] Bergman, Barbara, In Defense of Affirmative Action, Basic Books, 1996.

[7] Bolick, Clint, The Affirmative Action Fraud: Can We Restore the American Civil Rights Vision?, Washington, D.C, Cato Institute and National Book Netwotk, 1996.

[8] Coate, Stephen and Glenn C. Loury, "Will Affirmative Action Policies Eliminate Negative Stereotypes?," American Economic Review 83, December 1993, 1220-40.

[9] Card, David and Alan B. Krueger, "School Quality and Black-White Relative Earning: a Direct Assessment", Quarterly Journal of Economics, February 1992, 151-200

[10] Donohue, John J. III and James Heckman, "Continuous vs Episodic change: the Impact of Civil Rights Policy on the Economic Status of Blacks", Journal of Economic Literature XXIX, December 1991, 1603-43.

[11] Feldman, M. and C. Gilles, "An Expository Note on Individual Risk without Aggregate Uncertainty", Journal of Economic Theory, 35, 1985, 26-32.

[12] Foster, D. and R. Vohra, "An Economic Argument for Affirmative Action", Rationality and Society, 4, 1992, 176-188.

[13] Foster, Andrew D. and Mark R. Rosenzweig, "Comparative Advantage, Information and the Allocation of Workers to Tasks: Evidence from an Agricoltural Labour Market", Review of Economic Studies, 63, 1996, 347-374

[14] Friedman, David D., "Comment on "An Economic Argument for Affirmative Action"", Rationality and Society, 4, 1992, 356-360.

[15] Judd, Kenneth L., "The Law of Large Numbers with a Continuum of I.I.D. Random Variables", Journal of Economic Theory 35, 1985, 19-25.

[16] Heckman, James, "The impact of government on the Economic Status of Black Americans" in The question of discrimination. Middletown, CT: Welseyan U. Press, 1989, pp 50-80.

[17] Juhn, Chinhui; Murphy, Kevin M; Pierce, Brooks "Wage Inequality and the Rise in Returns to Skill", Journal of Political Economy, 101, June 1993, 410-42.

[18] Leonard, Jonathan S. "Wage Disparities and Affirmative Action in the 80's", AEA Papers and Proceedings, 86, May 1996, 285-289.

[19] Levy, Frank and Richard J. Murnane, "U.S. Earnings Levels and Earnings Inequality: A Review of Recent Trends and Proposed Explanations", Journal of Economic Literature, 30, September 1992, 1333-81.

[20] Lundberg, Shelly J. "The Enforcement of Equal Opportunity Laws Under Imperfect Information: Affirmative Action and Alternatives", Quarterly Journal of Economics, CVI, February 1991, 309-26. Lundberg, Shelly J. and Richard Startz, "Private Discrimination and Social Intervention in Competitive Labor Markets", American Economic Review 73 (1983), 340-47.

[21] Norman, Peter, "Statistical Discrimination and Efficiency", mimeo, University of Wisconsin, 1997.

[22] Phelps, Edmund S., "The Statistical theory of Racism and Sexism," American Economic Review 62 (September 1972), 659-61.

[23] Rotschild, Michael and Joseph E. Stiglitz, "A Model of Employment Outcomes Illustrating the Effect of the Structure of Information on the Level and the Distribution of Income", Economics Letters, 10, 231-236

[24] Smith, James P. and Finis Welch, "Affirmative Action and Labor Markets", Journal of Labor Economics, 2, April 1984, 269-301.

[25] Smith, James P. and Finis Welch, "Black Economic Progress After Myrdal", Journal of Economic Literature, XXVII, June 1989, 519-564.

[26] Spence, Michael A., Market Signaling: Information Transfer in Hiring and Related Screening Processes, Cambridge, MA, Harvard University Press, 1974.

[27] Sowell, Thomas, Preferential Policies: An International Perspective. New York, William Morrow & Co., 1990.