

14.461: Technological Change, Lectures 7 and 8 Innovation, Reallocation and Growth

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Motivation (I)

- Recent economic recession has reopened the debate on industrial policy.
- In October 2008, the US government bailed out GM and Chrysler. (Estimated cost, \$82 Billion)
- Similar bailouts in Europe: Estimated cost €1.18 trillion in 2010, 9.6% of EU GDP.
- Many think that this was a success from a short-term perspective, because these interventions
 - protected employment, and
 - encouraged incumbents to undertake greater investments.

Motivation (II)

- More generally, what are the implications of “industrial policy” for R&D, reallocation, productivity growth, and welfare?
- Bailouts or support for incumbents could increase growth if there is insufficient entry or if they support incumbent R&D.
 - In fact, this is recently been articulated as an argument for industrial policy.
- They may reduce growth by
 - preventing the entry of more efficient firms and
 - slowing down the reallocation process.
- Reallocation potentially important, estimated sometimes to be responsible for up to 70-80% of US productivity growth.

Motivation (III)

- What's the right framework?
 - 1 endogenous technology and R&D choices,
 - 2 rich from dynamics to allow for realistic reallocation and matched the data (and for selection effects),
 - 3 different types of policies (subsidies to operation vs R&D),
 - 4 general equilibrium structure (for the reallocation aspect),
 - 5 exit for less productive firms/products (so that the role of subsidies that directly or indirectly prevent exit can be studied).
- Starting point: Klette and Kortum's (2004) model of micro innovation building up to macro structure.

Motivating Facts

- R&D intensity is independent of firm size.
- The size distribution of firms is highly skewed.
- Smaller firms have a lower probability of survival, but those that survive tend to grow faster than larger firms. Among larger firms, growth rates are unrelated to past growth or to firm size.
- Younger firms have a higher probability of exiting, but those that survive tend to grow faster than older firms.
- Gibrat's law holds approximately (but not exactly): firm growth rate roughly independent of size, though notable deviations from this at the top and the bottom.

Model I

- Representative household maximizes

$$U = \max \int_0^{\infty} e^{-\rho t} \log C_t dt$$

- All expenses are in terms of labor. Hence $C_t = Y_t$.
- The household owns all the firms including potential entrants. Therefore the total income is

$$Y_t = w_t L + r_t \mathcal{A}_t$$

where \mathcal{A} is the total asset holdings and r_t is the rate of return on these assets.

Model II

- Final good production

$$\ln Y_t = \int_0^1 \ln y_{jt} dj$$

- y_j : quantity of intermediate j
- A fixed mass L of labor

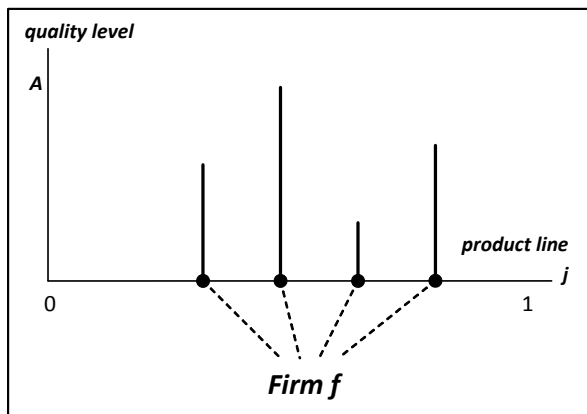
$$L_P + S_E + S_I = L$$

- L_P : production
- S_E : scientists working for outsiders
- S_I : scientists working for incumbent firms.
- All workers receive w_t
- Normalize the price of the final good to 1.

Profits I

- A firm is defined as a **collection of product lines**.

FIGURE 3: EXAMPLE OF A FIRM



Profits II

- n will denote the number of product lines that the firm operates.
- Each intermediate is produced with a linear technology

$$y_{jt} = A_{jt} l_{jt}$$

- This implies that the marginal cost is

$$w_t / A_{jt}$$

where w_t is the wage rate in the economy at time t .

- Innovations in each product line improves the productivity by $\lambda > 0$ such that

$$A_{jt+\Delta t} = \begin{cases} (1 + \lambda) A_{jt} & \text{if successful innovation} \\ A_{jt} & \text{otherwise} \end{cases}$$

Profits III

- Bertrand competition \implies previous innovator will charge at least her marginal cost: $\frac{(1+\lambda)w_t}{A_{jt}}$.
- Hence the latest innovator will charge the marginal cost of the previous innovator

$$p_{jt} = \frac{(1 + \lambda) w_t}{A_{jt}}.$$

- Recall that the expenditure on each variety is Y_t (since $P_t = 1$).
- Then the profit is

$$\begin{aligned} \pi_j &= y_j (p_j - MC_j) \\ &= \frac{A_{jt} Y_t}{(1 + \lambda) w_t} \left(\frac{(1 + \lambda) w_t}{A_{jt}} - \frac{w_t}{A_{jt}} \right) \\ &= \pi Y_t \end{aligned}$$

where $\pi \equiv \frac{\lambda}{1+\lambda}$.

Innovation Technology I

- Innovations are undirected across product lines.
- Innovation technology

$$X_i = \left(\frac{S_i}{\zeta} \right)^{1-\gamma} n^\gamma$$

where $\gamma < 1$, X_i is the innovation flow rate, S_i is the amount of R&D workers, n is the number of product lines to proxy for the firm specific (non-transferable, non-tradable) knowledge stock.

Innovation Technology II

- Alternatively, the cost of innovation:

$$\begin{aligned}
 C(X, n) &= wS_i \\
 &= \zeta wn \left[\frac{X_i}{n} \right]^{\frac{1}{1-\gamma}} \\
 &= \zeta wn x_i^{\frac{1}{1-\gamma}}
 \end{aligned}$$

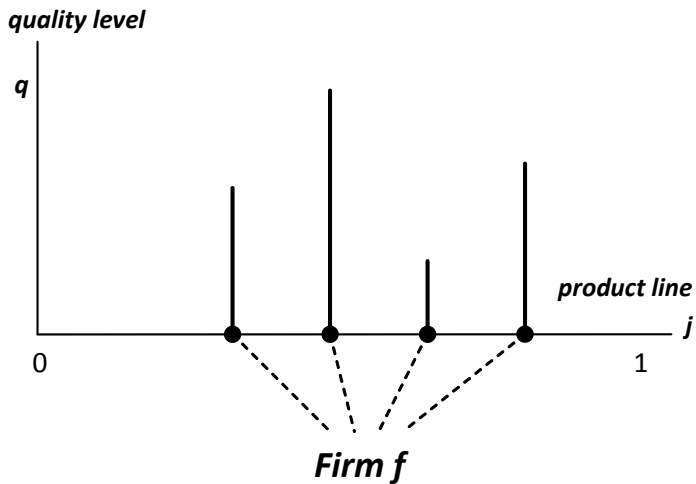
where $x_i \equiv X_i/n$ is the innovation intensity (per product line).

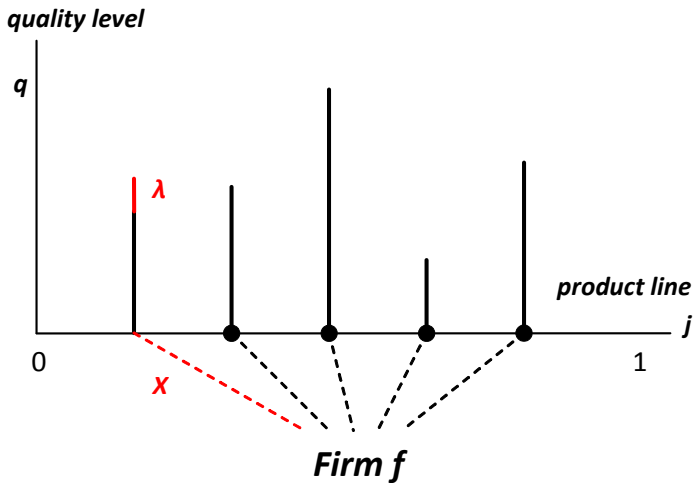
- Let x denote the aggregate innovation rate in the economy.
- Innovation rate by entrants is x_e .
- Aggregate innovation rate is

$$\tau = x_i + x_e.$$

Innovation Technology III

- When a firm is successful in its current R&D investment, it innovates over a random product line $j' \in [0, 1]$.
 - ① Then, the productivity in line j' increases from $A_{j'}$ to $(1 + \lambda)A_{j'}$.
 - ② The firm becomes the new monopoly producer in line j' and thereby increases the number of its production lines to $n + 1$.
- At the same time, each of its n current production lines is subject to the *creative destruction* τ by new entrants and other incumbents.
- Therefore during a small time interval dt ,
 - ① the number of production units of a firm increases to $n + 1$ with probability $X_i dt$, and
 - ② decreases to $n - 1$ with probability $n\tau dt$.
- A firm that loses all of its product lines exits the economy.





Value Function I

- Relevant firm-level state variable: number of products in which the firm has the leading-edge technology, n .
- Then the value function of a firm as a function of n is

$$rV_t(n) - \dot{V}_t(n) = \max_{x_i \geq 0} \left\{ \begin{array}{l} n\pi_t - w_t \zeta n^{\frac{1}{1-\gamma}} \\ + nx_i [V_t(n+1) - V_t(n)] \\ + n\tau [V_t(n-1) - V_t(n)] \end{array} \right\}$$

- This can be rewritten as

$$\rho v = \pi - \tau v + \max_{x_i \geq 0} \left\{ x_i v - \omega \zeta x_i^{\frac{1}{1-\gamma}} \right\}$$

where $v \equiv V_t(n)/nY_t$ is normalized per product value and $\omega \equiv w_t/Y_t$ is the labor share and constant in steady state.

Value Function II

- First-order condition of R&D choice gives:

$$x_i = \left(\frac{v}{\eta \zeta \omega} \right)^{\frac{1-\gamma}{\gamma}}. \quad (1)$$

- Or substituting it back:

$$v = \frac{\pi - \zeta \omega x_i^{\frac{1}{1-\gamma}}}{\rho + \tau - x_i}. \quad (2)$$

Value Function III

Proposition Per-product line value of a firm v can be expressed as a sum of production value v_P and R&D “innovation option” value v_R :

$$v = v_P + v_R$$

where

$$v_P = \frac{\pi}{\rho + \tau}$$

$$v_R = \frac{1}{(\rho + \tau)} \max_{x_i \geq 0} \left\{ x_i (v_R + v_P) - \omega \zeta x_i^{\frac{1}{1-\gamma}} \right\}.$$

Entry I

- A mass of potential entrants.
- In order to generate 1 unit of arrival, entrants must hire a team of ψ researchers, i.e., production function for entrant R&D is

$$x_e = \frac{S_E}{\psi}.$$

- The free-entry condition equates the value of a new entry $V_t(1)$ to the cost of innovation ψw_t such that

$$v = \omega \psi.$$

- Thus, together with (1) and (2) :

$$x_e = \frac{\pi}{\omega \psi} - (1 - \gamma) \left(\frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \rho \quad \text{and} \quad x_i = \left(\frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}}.$$

Labor Market Clearing I

- Production workers

$$L_P = \frac{Y_t}{A_j p_j} = \frac{1}{(1 + \lambda) \omega}$$

- Incumbent R&D workers

$$S_I = \zeta \left(\frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1 - \gamma}{\gamma}}$$

- Entrant R&D workers

$$S_E = \frac{\pi}{\omega} - \zeta \left(\frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1 - \gamma}{\gamma}} - \psi \rho$$

Labor Market Clearing II

- Therefore labor market clearing determines the normalized wage rate

$$\begin{aligned}
 L &= \frac{1}{(1+\lambda)\omega} + \zeta \left(\frac{(1-\gamma)\psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} \\
 &\quad + \frac{\pi}{\omega} - \zeta \left(\frac{(1-\gamma)\psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \psi\rho \\
 &\implies \\
 \omega &= \frac{1}{L + \rho\psi}
 \end{aligned}$$

Equilibrium Growth I

- Recall the final good production function

$$\begin{aligned}
 \ln Y_t &= \int_0^1 \ln y_{jt} dj \\
 &= \int_0^1 \ln A_{jt} l_{jt} dj \\
 &= \ln \frac{Y_t}{(1 + \lambda) w_t} + \int_0^1 \ln A_{jt} dj \\
 &= \ln \frac{L + \rho \psi}{1 + \lambda} + \int_0^1 \ln A_{jt} dj
 \end{aligned}$$

Equilibrium Growth II

- Define

$$\begin{aligned} Q_t &\equiv \exp\left(\int_0^1 \ln A_{jt} dj\right) \\ &\implies \\ \ln Q_t &\equiv \int_0^1 \ln A_{jt} dj \end{aligned}$$

- Thus

$$g = \frac{\dot{C}_t}{C_t} = \frac{\dot{Q}_t}{Q_t}$$

Equilibrium Growth III

- Moreover

$$\begin{aligned}
 \ln Q_{t+\Delta t} &= \int_0^1 [\tau \Delta t \ln(1 + \lambda) A_{jt} + (1 - \tau \Delta t) \ln A_{jt}] dj + o(\Delta t) \\
 &= \tau \Delta t \ln(1 + \lambda) + \ln Q_t + o(\Delta t) \\
 &\iff \\
 g &= \tau \ln(1 + \lambda)
 \end{aligned}$$

- Hence

$$g = \left[\left(\frac{\lambda}{1 + \lambda} \right) \frac{L}{\psi} + \frac{1 - \gamma}{\gamma} \left(\frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1 - \gamma}{\gamma}} - \frac{\rho}{1 + \lambda} \right] \ln(1 + \lambda)$$

Moments

- Consider a firm with n product lines. The “approximate” growth rate is

$$\begin{aligned}
 n_{t+\Delta t} &= n_t + nx_i\Delta t - n\tau\Delta t \\
 &\implies \\
 \frac{\dot{n}_t}{n_t} &= x_i - \tau
 \end{aligned}$$

- R&D spending/intensity

$$\frac{R\&D}{Sales} = \frac{\zeta wn x_i^{\frac{1}{1-\gamma}}}{n} = \zeta w x_i^{\frac{1}{1-\gamma}}$$

- Both of these are independent of firm size (consistent with “Gibrat’s law”).

Firm Size Distribution

- Firm size distribution: fraction of firms with n leading-edge products, μ_n , given by:

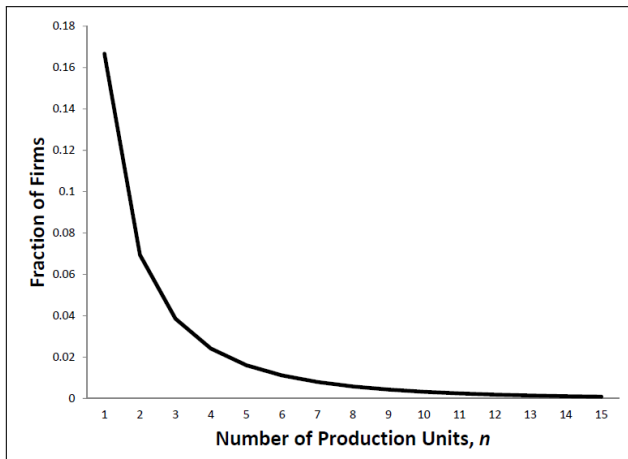
	<i>Outflow</i>		<i>Inflow</i>
entry&exit:	$\mu_1 \tau$	=	x_e
1-product:	$(x_i + \tau) \mu_1$	=	$\mu_2 2\tau + x_e$
n-product:	$(x_i + \tau) n \mu_n$	=	$\mu_{n+1} (n+1) \tau + \mu_{n-1} (n-1) x_i$

- This implies the following simple firm size distribution:

$$\begin{aligned} \mu_1 &= x_e / \tau \\ \mu_2 &= \frac{x_e}{2\tau^2} x_i \\ \mu_3 &= \frac{x_e x_i}{3\tau^3} \\ \dots &= \dots \\ \mu_n &= \frac{x_e x_i}{n\tau^n} \end{aligned}$$

Firm Size Distribution

FIGURE 4: FIRM SIZE DISTRIBUTION

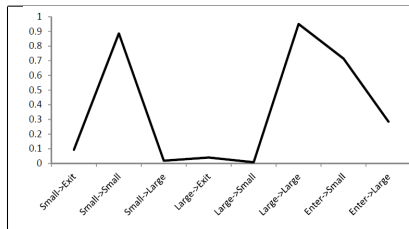


What's Missing?

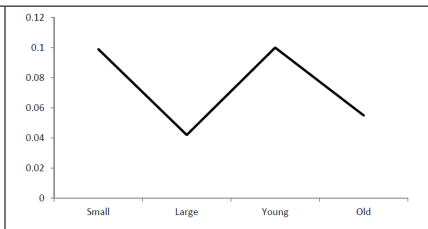
- A nice and tractable model, but:
 - *no reallocation* (all firms that previous in equilibrium are equally good at using all factors of production);
 - *no endogenous exit* of less productive firms;
 - limited heterogeneity (see next slide).
- All of these together imply very little room for endogenous selection which could be impacted by policy.
- We now consider a model that extended this framework to introduce these features.

Why Heterogeneity Matters

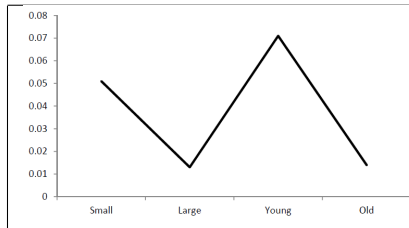
1A: TRANSITION RATES



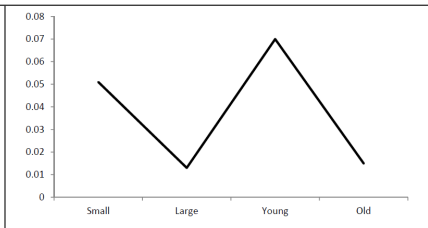
1B: R&D INTENSITY



1C: SALES GROWTH



1D: EMPLOYMENT GROWTH



Baseline Model: Preferences

- Simplified model (abstracting from heterogeneity and non-R&D growth).
- Infinite-horizon economy in continuous time.
- Representative household:

$$U = \int_0^{\infty} \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt.$$

- Inelastic labor supply, no occupational choice:
 - Unskilled for production: measure 1, earns w^u
 - Skilled for R&D: measure L , earns w^s .
- Hence the budget constraint is

$$C(t) + \dot{A}(t) \leq w^u(t) + w^s(t) \cdot L + r(t) \cdot A(t)$$

- Closed economy and no investment, resource constraint:

$$Y(t) = C(t).$$

Final Good Technology

- Unique final good Y :

$$Y = \left(\int_{\mathcal{N}} y_j^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

- $\mathcal{N} \subset [0, 1]$ is the set of *active* product lines.
- The measure of \mathcal{N} is less than 1 due to
 - 1 exogenous destructive shock
 - 2 obsolescence

Intermediate Good Technology

- As usual, each intermediate good is produced by a **monopolist**:

$$y_{j,f} = q_{j,f} l_{j,f},$$

$q_{j,f}$: worker productivity, $l_{j,f}$: number of workers.

- Marginal cost :

$$MC_{j,f} = \frac{w^u}{q_{j,f}}.$$

- Fixed cost of production, ϕ in terms of skilled labor.
- Total cost

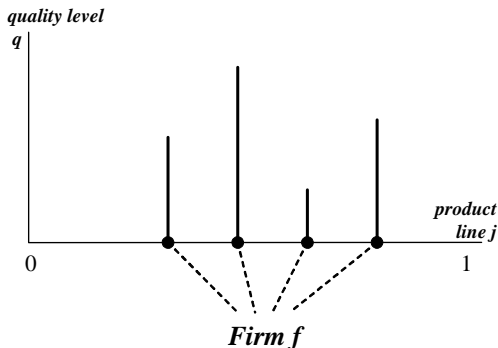
$$TC_{j,f}(y_{j,f}) = w^s \phi + w^u \frac{y_{j,f}}{q_{j,f}}.$$

Definition of a Firm

- A firm is defined as a collection of product qualities as in Klette-Kortum

$$\text{Firm } f = Q_f \equiv \{q_f^1, q_f^2, \dots, q_f^{n_f}\}.$$

$n_f \equiv |Q_f|$: is the number of product lines of firm f .



Relative Quality

- Define *aggregate quality* as

$$Q \equiv \left(\int_{\mathcal{N}} q_j^{\varepsilon-1} dj \right)^{\frac{1}{\varepsilon-1}}.$$

- In equilibrium,

$$Y = C = Q,$$

- Define *relative quality*:

$$\hat{q}_j \equiv \frac{q_j}{w^u}.$$

R&D and Innovation

- Innovations follow a “controlled” Poisson Process

$$X_f = n_f^\gamma h_f^{1-\gamma}.$$

X_f : flow rate of innovation

n_f : number of product lines.

h_f : number of researchers (here taken to be regular workers allocated to research).

- This can be rewritten as *per product* innovation at the rate

$$x_f \equiv \frac{X_f}{n_f} = \left(\frac{h_f}{n_f} \right)^{1-\gamma}.$$

- Cost of R&D as a function of per product innovation rate x_f :

$$w^s G(x_f) \equiv w^s n_f x_f^{\frac{1}{1-\gamma}}.$$

Innovation by Existing Firms

- Innovations are again *undirected* across product lines.
- Upon an innovation:
 - 1 firm f acquires another product line j
 - 2 if technology in j is active:

$$q(j, t + \Delta t) = (1 + \lambda) q(j, t).$$

- 3 if technology in j is not active, i.e., $j \notin \mathcal{N}$, a new technology is drawn from the steady-state distribution of relative quality, $F(\hat{q})$.

Entry and Exit

- A set of potential entrants invest in R&D.
- Exit happens in three ways:
 - 1 **Creative destruction.** Firm f will lose each of its products at the rate $\tau > 0$ which will be determined endogenously in the economy.
 - 2 **Obsolescence.** Relative quality decreases due to the increase in the wage rate, at some point leading to exit.
 - 3 **Exogenous destructive shock** at the rate φ .

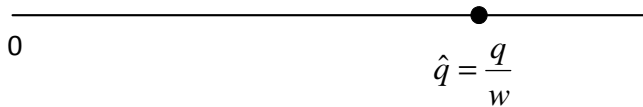
Static Equilibrium

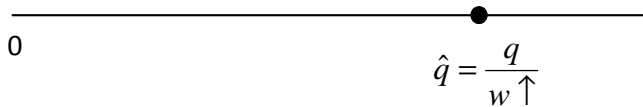
- Drop the time subscripts.
- Isoelastic demands imply the following monopoly price and quantity

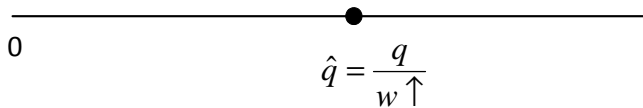
$$p_{j,f}^* = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{1}{\hat{q}_j} \text{ and } c_j^* = \left(\frac{\varepsilon - 1}{\varepsilon} \hat{q}_j \right)^\varepsilon Y$$

- Gross equilibrium (before fixed costs) profits from a product with relative quality \hat{q}_j are:

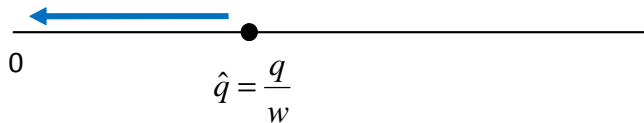
$$\pi(\hat{q}_{j,f}) = \hat{q}_j^{\varepsilon-1} \left(\frac{(\varepsilon - 1)^{\varepsilon-1}}{\varepsilon^\varepsilon} \right) Y.$$

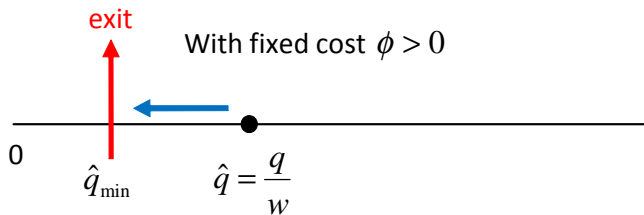


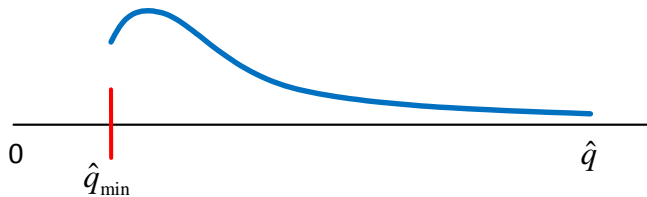




Without a fixed cost







Dynamic Equilibrium

- In equilibrium,

$$Y = C = Q$$

and

$$w^u = \frac{\varepsilon - 1}{\varepsilon} Q.$$

- Let us also define *normalized values* as

$$\tilde{V} \equiv \frac{V}{Y}, \quad \tilde{\pi}(\hat{q}_{j,f}) = \frac{\pi(\hat{q}_{j,f})}{Y}, \quad \tilde{w}^u \equiv \frac{w^u}{Y} \quad \text{and} \quad \tilde{w}^s \equiv \frac{w^s}{Y}.$$

Dynamic Equilibrium (continued)

$$r^* \tilde{V}(\hat{Q}_f) = \left[\sum_{\hat{q}_{j,f} \in \hat{Q}_f} \left\{ \begin{array}{c} \tilde{\pi}(\hat{q}_{j,f}) - \tilde{w}^s \phi_j \\ + \dot{\tilde{V}} \\ + \tau [\tilde{V}(\hat{Q}_f \setminus \{\hat{q}_{j,f}\}) - \tilde{V}(\hat{Q}_f)] \end{array} \right\} + \right. \\ \left. |\hat{Q}_f| \max_{x_f} \left\{ \begin{array}{c} -\tilde{w}G(x_f) \\ + x_f [\mathbb{E}_{\hat{q}} \tilde{V}(\hat{Q}_f \cup (1+\lambda)\hat{q}_{j',f}) - \tilde{V}(\hat{Q}_f)] \\ + \varphi [0 - \tilde{V}(\hat{Q}_f)] \end{array} \right\} \right]$$

τ : creative destruction rate in the economy.

Dynamic Equilibrium (continued)

$$r^* \tilde{V}(\hat{Q}_f) = \left[\sum_{\hat{q}_{j,f} \in \hat{Q}_f} \left\{ \begin{array}{l} \tilde{\pi}(\hat{q}_{j,f}) - \tilde{w}^s \phi_j \\ + \frac{\partial \tilde{V}}{\partial \hat{q}_{j,f}} \frac{\partial \hat{q}_{j,f}}{\partial w^u(t)} \frac{\partial w^u(t)}{\partial t} \\ + \tau [\tilde{V}(\hat{Q}_f \setminus \{\hat{q}_{j,f}\}) - \tilde{V}(\hat{Q}_f)] \end{array} \right\} + \right. \\ \left. |\hat{Q}_f| \max_{x_f} \left\{ \begin{array}{l} -\tilde{w} G(x_f) \\ + x_f [\mathbb{E}_{\hat{q}} \tilde{V}(\hat{Q}_f \cup (1+\lambda)\hat{q}_{j',f}) - \tilde{V}(\hat{Q}_f)] \\ + \varphi [0 - \tilde{V}(\hat{Q}_f)] \end{array} \right\} \right]$$

τ : creative destruction rate in the economy.

Franchise and R&D Option Values

Lemma *The normalized value can be written as the sum of franchise values:*

$$\tilde{V}(\hat{Q}_f) = \sum_{\hat{q} \in \hat{Q}_f} Y(\hat{q}),$$

where the franchise value of a product of relative quality \hat{q} is the solution to the differential equation (iff $\hat{q} \geq \hat{q}_{\min}$):

$$rY(\hat{q}) - \frac{\partial Y(\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u(t)} \frac{\partial w^u(t)}{\partial t} = \tilde{\pi}(\hat{q}) - \tilde{w}^u \phi + \Omega - (\tau + \varphi) Y(\hat{q}),$$

where Ω is the R&D option value of holding a product line,

$$\Omega \equiv \max_{x_f \geq 0} \left\{ -\tilde{w}^s G(x_f) + x_f \left(\mathbb{E}_{\hat{q}} \tilde{V}(\hat{Q}_f \cup (1 + \lambda) \hat{q}_{j'f}) - \tilde{V}(\hat{Q}_f) \right) \right\},$$

Moreover, exit follows a cut-off rule: $\hat{q}_{\min} \equiv \pi^{-1}(\tilde{w}^s \phi - \Omega)$.

Equilibrium Value Functions and R&D

Proposition

Equilibrium normalized value functions are:

$$Y(\hat{q}) = \frac{\tilde{\pi}(\hat{q})}{r + \tau + \varphi + g(\varepsilon - 1)} \left[1 - \left(\frac{\hat{q}_{\min}}{\hat{q}} \right)^{\frac{r + \tau + \varphi + g(\varepsilon - 1)}{g}} \right] + \frac{\Omega - \tilde{w}^s \phi}{r + \tau + \varphi} \left[1 - \left(\frac{\hat{q}_{\min}}{\hat{q}} \right)^{\frac{r + \tau + \varphi}{g}} \right],$$

and equilibrium R&D is

$$x^*(\hat{q}) = x^* = \left[\frac{(1 - \gamma) \mathbb{E}_{\hat{q}} Y(\hat{q})}{\tilde{w}^s} \right]^{\frac{1 - \gamma}{\gamma}}.$$

Entry

- Entry by outsiders can now be determined by the free entry condition:

$$\max_{x^{entry} \geq 0} \left\{ -w^s \phi + x^{entry} \mathbb{E} V^{entry}(\hat{q}, \theta) - w^s G(x^{entry}, \theta^E) \right\} = 0$$

where $G(x^{entry}, \theta^E)$, as specified above, gives a number of skilled workers necessary for a firm to achieve an innovation rate of x^{entry} (with productivity parameter θ^E).

- $X^{entry} \equiv m x^{entry}$ is the total entry rate where
 - m is the equilibrium measure of entrants, and
 - x^{entry} innovation rate per entrant.

Labor Market Clearing

- Unskilled labor market clearing:

$$1 = \int_{\mathcal{N}(t)} l_j(w^u) dj.$$

- Skilled labor market clearing

$$L^s = \int_{\mathcal{N}(t)} [\phi + h(w^s)] dj + m \left[\phi + G(x^{\text{entry}}, \theta^E) \right].$$

Transition Equations

- Finally, we need to keep track of the distribution of relative quality \rightarrow stationary equilibrium distribution of relative quality F .
- This can be done by writing transition equations describing the density of relative quality.
- These are more complicated than in Klette-Kortum because there is no strict Gibraltar's law anymore.

Preferences and Technology in the General Model

- Same preferences.
- Introduce managerial quality affecting the rate of innovation of each firm.
- Some firms start as more innovative than others, over time some of them lose their innovativeness.
 - Young firms are potentially more innovative but also have a higher rate of failure.
- Introduce non-R&D growth (so as not to potentially exaggerate the role of R&D and capture potential advantages of incumbents).

Definition of a Firm

- A firm is again defined as a pair of technology set and “management quality” θ :

$$\text{Firm } f \equiv (Q_f, \theta_f),$$

where

$$Q_f \equiv \{q_f^1, q_f^2, \dots, q_f^n\}.$$

- $n_f \equiv |Q_f|$: is the number of product lines owned by firm f .

R&D and Innovation

- Innovations follow a controlled Poisson Process.
- Flow rate of innovation for leader and follower given by

$$\lambda_f = (n_f \theta_f)^\gamma h_f^{1-\gamma}.$$

n_f : number of product lines.

θ_f : firm type (management quality).

h_f : number of researchers.

Innovation Realizations

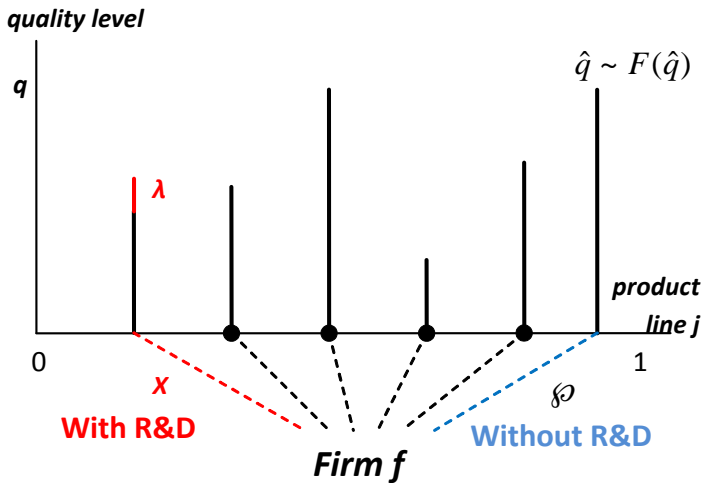
With R&D

- Innovations are *undirected* within the industry.
- After a successful innovation, innovation is realized in a random product line j . Then:
 - 1 firm f acquires product line j
 - 2 technology in line j improves

$$q(j, t + \Delta t) = (1 + \lambda) q(j, t).$$

Without R&D

- Firms receive a product line for free at the rate ϱ .



Entry and Exit

- There is a measure of potential entrants.
- Successful innovators enter the market.
- At the time of initial entry, each firm draws a management quality θ :

$$\begin{aligned}\Pr(\theta = \theta^H) &= \alpha \\ \Pr(\theta = \theta^L) &= 1 - \alpha,\end{aligned}$$

where $\alpha \in (0, 1)$ and $\theta^H > \theta^L > 0$.

- Exit happens in three ways as in the baseline model.

Maturity Shock

- Over time, high-type firms become low-type at the rate $\nu > 0$:

$$\theta^H \rightarrow \theta^L.$$

- Convenient to capture the possibility of once-innovative firms now being inefficient (and the use of skilled labor).

Equilibrium

- Equilibrium definition and characterization similar to before (with more involved value functions and stationary transition equations).

Data: LBD, Census of Manufacturing and NSF R&D Data

- Sample from combined databases from 1987 to 1997.
- Longitudinal Business Database (LBD)
 - Annual business registry of the US from 1976 onwards.
 - Universe of establishments, so entry/exit can be modeled.
- Census of Manufacturers (CM)
 - Detailed data on inputs and outputs every five years.
- NSF R&D Survey.
 - Firm-level survey of R&D expenditure, scientists, etc.
 - Surveys with certainty firms conducting \$1m or more of R&D.
- USPTO patent data matched to CM.
- Focus on “continuously innovative firms”:
 - I.e., either R&D expenditures or patenting in the five-year window surrounding observation conditional on existence.

Data Features and Estimation

- 17,055 observations from 9835 firms.
- Accounts for 98% of industrial R&D.
- Relative to the universal CM, our sample contains over 40% of employment and 65% of sales.
- “Important” small firms also included:
 - of the new entrants or very small firms that later grew to have more than 10,000 employees or more than \$1 billion of sales in 1997, we capture, respectively, 94% at 80%.
- We use Simulated Method of Moments on this dataset to estimate the parameters of the model.

Creating Moments from the Data

- We target 21 moments to estimate 12 parameters.
- Some of the moments are:
 - Firm entry/exit into/from the economy by age and size.
 - Firm size distribution.
 - Firm growth by age and size.
 - R&D intensity (R&D/Sales) by age and size.
 - Share of entrant firms.

RESULTS

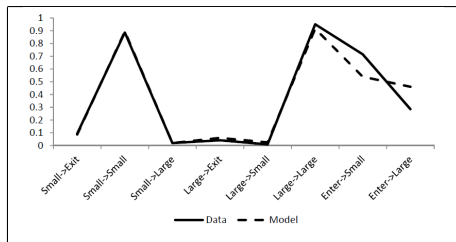
TABLE 1. PARAMETER ESTIMATES

#	Parameter	Description	Value
1.	ε	CES	1.701
2.	ϕ	Fixed cost of operation	0.032
3.	L^S	Measure of high-skilled workers	0.078
4.	θ^H	Innovative capacity of high-type firms	0.216
5.	θ^L	Innovative capacity of low-type firms	0.070
6.	θ^E	Innovative capacity of entrants	0.202
7.	α	Probability of being high-type entrant	0.428
8.	ν	Transition rate from high-type to low-type	0.095
9.	λ	Innovation step size	0.148
10.	γ	Innovation elasticity wrt knowledge stock	0.637
11.	φ	Exogenous destruction rate	0.016
12.	ϱ	Non-R&D innovation arrival rate	0.012

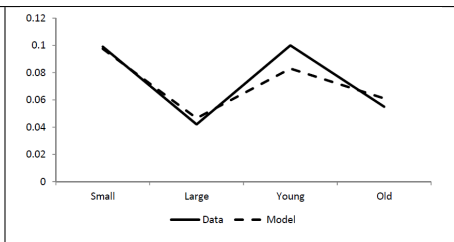
TABLE 2. MOMENT MATCHING

#	Moments	model	data		#	Moments	model	data
1.	Firm Exit (small)	0.086	0.093		12.	Sales Gr. (small)	0.115	0.051
2.	Firm Exit (large)	0.060	0.041		13.	Sales Gr. (large)	-0.004	0.013
3.	Firm Exit (young)	0.078	0.102		14.	Sales Gr. (young)	0.070	0.071
4.	Firm Exit (old)	0.068	0.050		15.	Sales Gr. (old)	0.030	0.014
5.	Trans. large-small	0.024	0.008		16.	R&D/Sales (small)	0.097	0.099
6.	Trans. small-large	0.019	0.019		17.	R&D/Sales (large)	0.047	0.042
7.	Prob. small	0.539	0.715		18.	R&D/Sales (young)	0.083	0.100
8.	Emp. Gr. (small)	0.063	0.051		19.	R&D/Sales (old)	0.061	0.055
9.	Emp. Gr. (large)	-0.007	0.013		20.	5-year Ent. Share	0.363	0.393
10.	Emp. Gr. (young)	0.040	0.070		21.	Aggregate growth	0.022	0.022
11.	Emp. Gr. (old)	0.010	0.015					

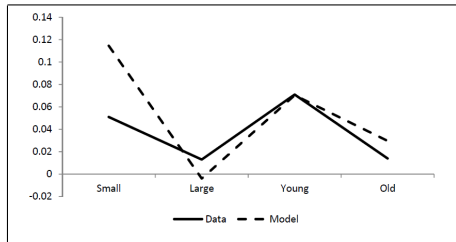
2A: TRANSITION RATES



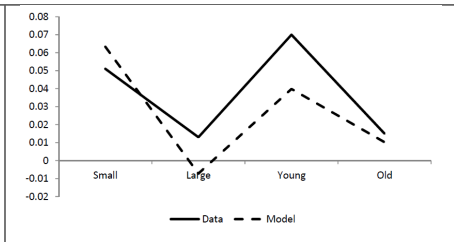
2B: R&D INTENSITY



2C: SALES GROWTH



2D: EMPLOYMENT GROWTH



Non-Targeted Moments

TABLE 3: NON-TARGETED MOMENTS

Moments	Model	Data
Corr(exit prob, R&D intensity)	0.04	0.05
Exit prob of low-R&D-intensive firms	0.36	0.32
Exit prob of high-R&D-intensive firms	0.37	0.34
Corr(R&D growth, emp growth)	0.48	0.19
Share firm growth due to R&D	0.77	0.73
Ratio of top 7.2% to bottom 92.8% income	13.4	9.3

Comparison to Micro Estimates

- Estimates of the elasticity of patents (innovation) to R&D expenditures (e.g., Griliches, 1990):
 - [0.3, 0.6]
 - This corresponds to $1 - \gamma$, so a range of [0.4, 0.7] for γ .
 - Our estimate is in the middle of this range.
- Use IV estimates from R&D tax credits.
 - US spending about \$2 billion with large cross-state over-time variation.
 - Literature estimates:

$$\log(R\&D_{i,t}) = \alpha_i + \beta_t + \gamma \log(R\&D_Cost_of_Capital_{i,t})$$

- Bloom, Griffith and Van Reenen (2002) find -1.088 (0.024) on a cross-country panel. Similar estimates from Hall (1993), Baily and Lawrence (1995) and Mumuneas and Nadiri (1996).
- In the model, $\ln R\&D = \frac{\gamma-1}{\gamma} \ln(c_{R\&D}) + \text{constant}$.
- So approximately $\gamma \approx 0.5$, close to our estimate of $\gamma = 0.637$.

Baseline Results

TABLE 4. BASELINE MODEL

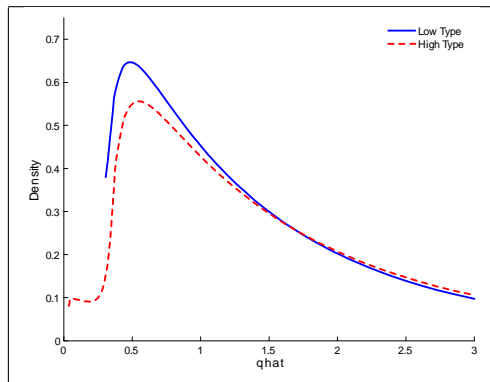
x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

Note: All numbers except wage ratio and welfare are in percentage terms.

g :	growth rate	Φ^{high} :	fraction of high p. lines
x^{out} :	entry rate	$\hat{q}_{l,min}$:	low-type cutoff quality
x^{low} :	low-type innv rate	$\hat{q}_{h,min}$:	high-type cutoff quality
x^{high} :	high-type innv rate	wel :	welfare in cons equiv.
Φ^{low} :	fraction of low p. lines		

Relative Quality Distribution

FIGURE 3



- Explains why very little obsolescence of high-type products.

Policy Analysis: Subsidy to Incumbent R&D

TABLE 4. BASELINE MODEL

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- Use 1% and 5% of GDP, resp., to subsidize incumbents R&D:

TABLE 5A. INCUMBENT R&D SUBSIDY ($s_i = 15\%$)

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	3.05	10.56	68.1	70.74	24.96	13.40	0.00	2.23	99.86

TABLE 5B. INCUMBENT R&D SUBSIDY ($s_i = 39\%$)

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	3.61	13.04	49.8	69.58	25.97	13.15	0.00	2.16	98.48

Policy Analysis: Subsidy to the Operation of Incumbents

TABLE 4. BASELINE MODEL

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- Use 1% of GDP to subsidize operation costs of incumbents:

TABLE 6. OPERATION SUBSIDY ($s_o = 6\%$)

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	2.80	9.59	73.7	71.30	24.52	11.74	0.00	2.22	99.82

- Now an important negative selection effect.

Policy Analysis: Entry Subsidy and Selection

TABLE 4. BASELINE MODEL

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- Use 1% of GDP to subsidize entry:

TABLE 7. ENTRY SUBSIDY ($s_e = 5\%$)

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	2.73	9.30	75.3	71.16	24.41	15.91	0.00	2.26	100.15

Social Planner's Allocation

TABLE 4. BASELINE MODEL

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- What would the social planner do (taking equilibrium markups as given)?

TABLE 8. SOCIAL PLANNER

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	2.55	10.47	80.9	54.06	27.76	118.6	1.02	3.80	106.5

Optimal Policy (I)

TABLE 4. BASELINE MODEL

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- Optimal mix of incumbent R&D subsidy, operation subsidy and entry subsidy:

TABLE 9. OPTIMAL POLICY ANALYSIS AND WELFARE

INCUMBENT & ENTRY POLICIES ($s_i = 17\%$, $s_o = -246\%$, $s_e = 6\%$)

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	3.04	10.21	75.5	62.19	25.53	96.28	55.88	3.12	104.6

Optimal Policy (II)

TABLE 4. BASELINE MODEL

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	2.80	9.58	73.6	71.16	24.53	13.90	0.00	2.24	100

- Optimal mix of incumbent R&D subsidy and operation subsidy:

TABLE 9. OPTIMAL POLICY ANALYSIS AND WELFARE

INCUMBENT POLICIES ($s_i = 12\%$, $s_o = -264\%$)

x^{entry}	x^l	x^h	m	Φ^l	Φ^h	$\hat{q}_{l,min}$	$\hat{q}_{h,min}$	g	Wel
8.46	3.04	10.21	75.3	62.31	25.53	91.38	54.85	3.11	104.6

Summing up

- Industrial policy directed at incumbents has negative effects on innovation and productivity growth—though small.
- Subsidy to entrants has small positive effects.
- But not because R&D incentives are right in the laissez-faire equilibrium.
- The social planner can greatly improve over the equilibrium.
- Similar gains can also be achieved by using taxes on the continued operation of incumbents (plus small R&D subsidies).
 - This is useful for encouraging the exit of inefficient incumbents who are trapping skilled labor that can be more productively used by entrants and high-type incumbents.

Robustness

- These results are qualitatively and in fact quantitatively quite robust.
- The remain largely unchanged if:
 - $\gamma = 0.5$.
 - $\varrho = 0$.
 - entry margin much less elastic.