14.461: Technological Change, Lectures 5-7
Directed Technological Change

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Introduction

- Thus far have focused on a single type of technological change (e.g., Hicks-neutral).
- But, technological change is often not neutral:
  1. Benefits some factors of production and some agents more than others. Distributional effects imply some groups will embrace new technologies and others oppose them.
  2. Limiting to only one type of technological change obscures the competing effects that determine the nature of technological change.
- *Directed technological change*: endogenize the direction and bias of new technologies that are developed and adopted.

Skill-biased technological change

- Over the past 60 years, the U.S. relative supply of skills has increased, but:
  1. there has also been an increase in the college premium, and
  2. this increase accelerated in the late 1960s, and the skill premium increased very rapidly beginning in the late 1970s.
- Standard explanation: skill bias technical change, and an acceleration that coincided with the changes in the relative supply of skills.
- Important question: skill bias is endogenous, so, why has technological change become more skill biased in recent decades?

Skill-biased technological change

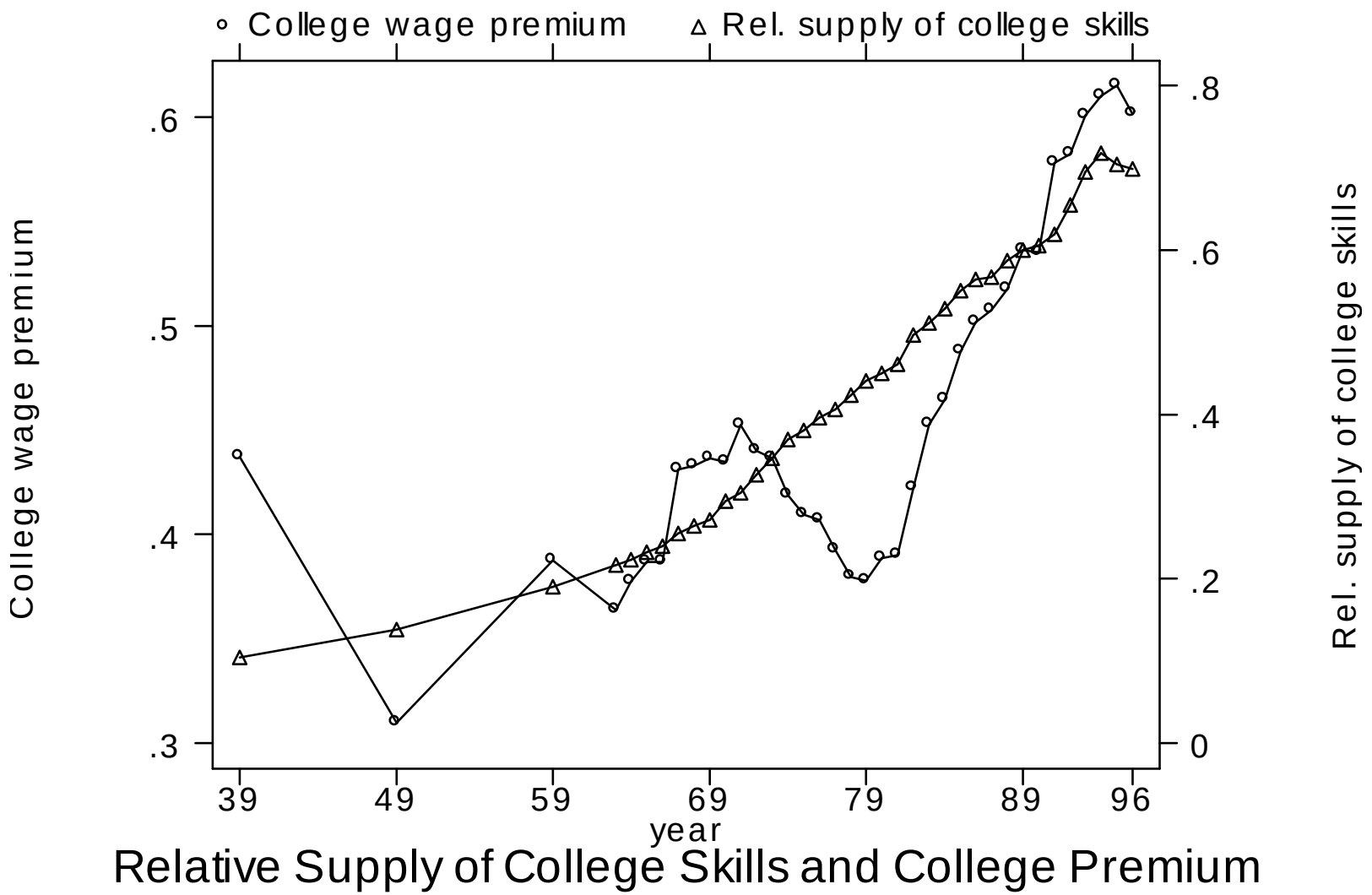


Figure:

Unskill-biased technological change

- Late 18th and early 19th *unskill-bias:*
  "First in firearms, then in clocks, pumps, locks, mechanical reapers, typewriters, sewing machines, and eventually in engines and bicycles, interchangeable parts technology proved superior and replaced the skilled artisans working with chisel and file." (Mokyr 1990, p. 137)
- Why was technological change unskilled-biased then and skilled-biased now?

Wage push and capital-biased technological change

- First phase. Late 1960s and early 1970s: unemployment and share of labor in national income increased rapidly continental European countries.
- Second phase. 1980s: unemployment continued to increase, but the labor share declined, even below its initial level.
- Blanchard (1997):
  - Phase 1: wage-push by workers
  - Phase 2: *capital-biased* technological changes.

  Is there a connection between capital-biased technological changes in European economies and the wage push preceding it?

Importance of Biased Technological Change: more examples

- Balanced economic growth:
  - Only possible when technological change is asymptotically Harrod-neutral, i.e., purely labor augmenting.
  - Is there any reason to expect technological change to be endogenously labor augmenting?
- Globalization:
  - Does it affect the types of technologies that are being developed and used?

Directed Technological Change: Basic Arguments I

- Two factors of production, say L and H (unskilled and skilled workers).
- Two types of technologies that can complement either one or the other factor.
- Whenever the profitability of H-augmenting technologies is greater than the L-augmenting technologies, more of the former type will be developed by profit-maximizing (research) firms.
- What determines the relative profitability of developing different technologies? It is more profitable to develop technologies...
  1. when the goods produced by these technologies command higher prices (*price effect*);
  2. that have a larger market (*market size effect*).

Equilibrium Relative Bias

- Potentially counteracting effects, but the market size effect will be more powerful often.
- Under fairly general conditions:
  - *Weak Equilibrium (Relative) Bias*: an increase in the relative supply of a factor always induces technological change that is biased in favor of this factor.
  - *Strong Equilibrium (Relative) Bias*: if the elasticity of substitution between factors is sufficiently large, an increase in the relative supply of a factor induces sufficiently strong technological change biased towards itself that the endogenous-technology relative demand curve of the economy becomes *upward-sloping.*

Equilibrium Relative Bias in More Detail I

- Suppose the (inverse) relative demand curve:

$$w_H / w_L = D\left(H/L, A\right)$$

  where w_H / w_L is the relative price of the factors and A is a technology term.
- A is H-biased if D is increasing in A, so that a higher A increases the relative demand for the H factor.
- D is *always* decreasing in H/L.
- Equilibrium bias: behavior of A as H/L changes,

$$A\left(H/L\right)$$

Equilibrium Relative Bias in More Detail II

- Weak equilibrium bias:
  - $A(H/L)$ is increasing (nondecreasing) in H/L.
- Strong equilibrium bias:
  - $A(H/L)$ is sufficiently responsive to an increase in H/L that the total effect of the change in relative supply H/L is to increase w_H/w_L.
  - i.e., let the endogenous-technology relative demand curve be

$$w_H/w_L = D(H/L, A(H/L)) \equiv \tilde{D}(H/L)$$

$\rightarrow$*Strong equilibrium bias:* $\tilde{D}$ **increasing in** H/L.

Factor-augmenting technological change

- Production side of the economy:

$$Y(t) = F(L(t), H(t), A(t)),$$

where $\partial F / \partial A > 0$.

- Technological change is *L-augmenting* if

$$\frac{\partial F(L, H, A)}{\partial A} \equiv \frac{L}{A} \frac{\partial F(L, H, A)}{\partial L}.$$

- Equivalent to:
  - the production function taking the special form, $F(AL, H)$.
  - Harrod-neutral technological change when L corresponds to labor and H to capital.
- H-augmenting defined similarly, and corresponds to $F(L, AH)$.

Factor-biased technological change

- Technological change change is *L-biased*, if:

$$\frac{\partial \frac{\partial F(L,H,A)/\partial L}{\partial F(L,H,A)/\partial H}}{\partial A} \geq 0.$$

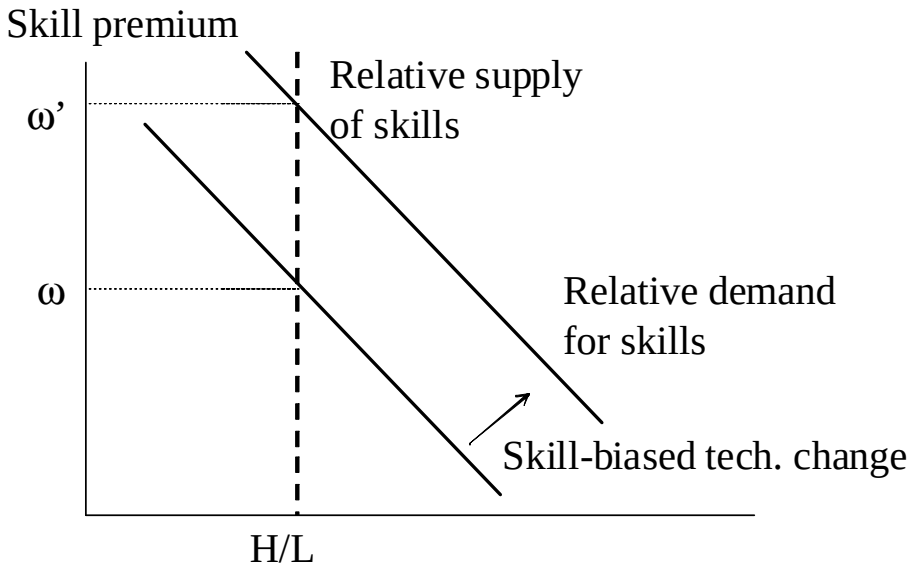


Figure: The effect of *H*-biased technological change on relative demand and relative factor prices.

Constant Elasticity of Substitution Production Function I

- CES production function case:

$$Y(t) = \left[\gamma_L \left(A_L(t) L(t)\right)^{\frac{\sigma-1}{\sigma}} + \gamma_H \left(A_H(t) H(t)\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

  where

  - $A_L(t)$ and $A_H(t)$ are two separate technology terms.
  - γ_is determine the importance of the two factors, $\gamma_L + \gamma_H = 1$.
  - $\sigma \in (0, \infty)$=elasticity of substitution between the two factors.
    - $\sigma = \infty$, perfect substitutes, linear production function is linear.
    - $\sigma = 1$, Cobb-Douglas,
    - $\sigma = 0$, no substitution, Leontieff.
    - $\sigma > 1$, "gross substitutes,"
    - $\sigma < 1$, "gross complements".

- Clearly, $A_L(t)$ is L-augmenting, while $A_H(t)$ is H-augmenting.
- Whether technological change that is L-augmenting (or H-augmenting) is L-biased or H-biased depends on σ.

Constant Elasticity of Substitution Production Function II

- Relative marginal product of the two factors:

$$\frac{MP_H}{MP_L} = \gamma \left(\frac{A_H(t)}{A_L(t)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H(t)}{L(t)} \right)^{-\frac{1}{\sigma}}, \tag{1}$$

  where $\gamma \equiv \gamma_H / \gamma_L$.
- *substitution effect:* the relative marginal product of H is decreasing in its relative abundance, $H(t)/L(t)$.
- The effect of $A_H(t)$ on the relative marginal product:
  - If $\sigma > 1$, an increase in $A_H(t)$ (relative to $A_L(t)$) increases the relative marginal product of H.
  - If $\sigma < 1$, an increase in $A_H(t)$ reduces the relative marginal product of H.
  - If $\sigma = 1$, Cobb-Douglas case, and neither a change in $A_H(t)$ nor in $A_L(t)$ is biased towards any of the factors.
- Note also that σ is the elasticity of substitution between the two factors.

Constant Elasticity of Substitution Production Function III

- Intuition for why, when $\sigma < 1$, H-augmenting technical change is L-biased:
  - with gross complementarity ($\sigma < 1$), an increase in the productivity of H increases the demand for labor, L, by more than the demand for H, creating "excess demand" for labor.
  - the marginal product of labor increases by more than the marginal product of H.
  - Take case where $\sigma \longrightarrow 0$ (Leontieff): starting from a situation in which $\gamma_L A_L(t) L(t) = \gamma_H A_H(t) H(t)$, a small increase in $A_H(t)$ will create an excess of the services of the H factor, and its price will fall to 0.

Equilibrium Bias

- **Weak equilibrium bias** of technology: an increase in H/L, induces technological change biased towards H. i.e., given (1):

$$\frac{d\left(A_H(t)/A_L(t)\right)^{\frac{\sigma-1}{\sigma}}}{dH/L} \geq 0,$$

  so $A_H(t)/A_L(t)$ is biased towards the factor that has become more abundant.
- **Strong equilibrium bias:** an increase in H/L induces a sufficiently large change in the bias so that the relative marginal product of H relative to that of L increases following the change in factor supplies:

$$\frac{dMP_H/MP_L}{dH/L} > 0,$$

- The major difference is whether the relative marginal product of the two factors are evaluated at the initial relative supplies (weak bias) or at the new relative supplies (strong bias).

Evidence

- Various different pieces of evidence suggest that technology is "directed" to words activities with greater profitability.
- In the environmental context:
  - Evidence that technological change and technology adoption respond to profit incentives
  - Newell, Jaffe and Stavins (1999): energy prices on direction of technological change in air conditioning
  - Popp (2002): relates energy prices and energy saving innovation
- In the health-care sector:
  - Finkelstein (2004): government demand for vaccines leads to more clinical trials.
  - Acemoglu and Linn (2004): demographic changes increasing the demand for specific types of drugs increase FDA approvals and new molecular entities directed at these categories.

Market Size and Innovation: Market Size

- Market size for different drug categories driven by demographic changes:

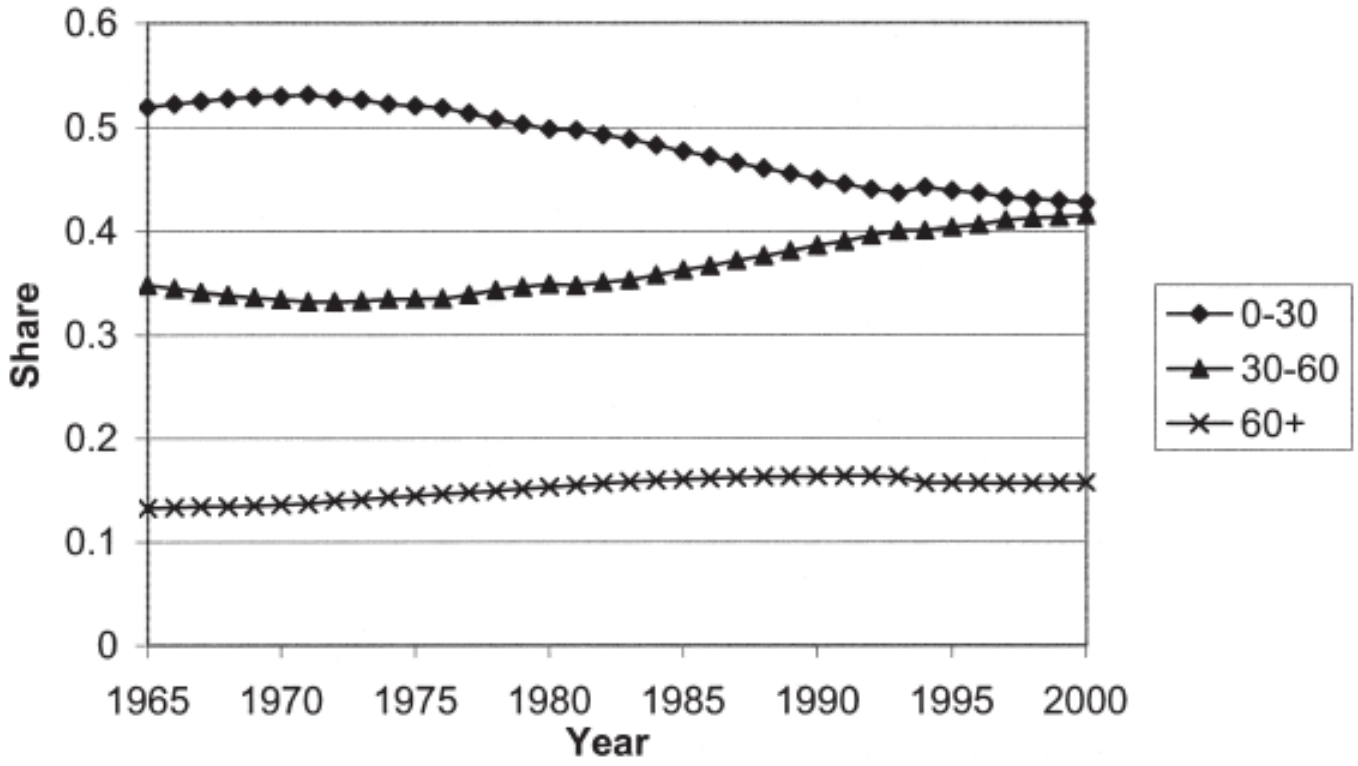


FIGURE I
Share of Population by Age Group from CPS, 1965–2000

Market Size and Innovation: Market Size with Income

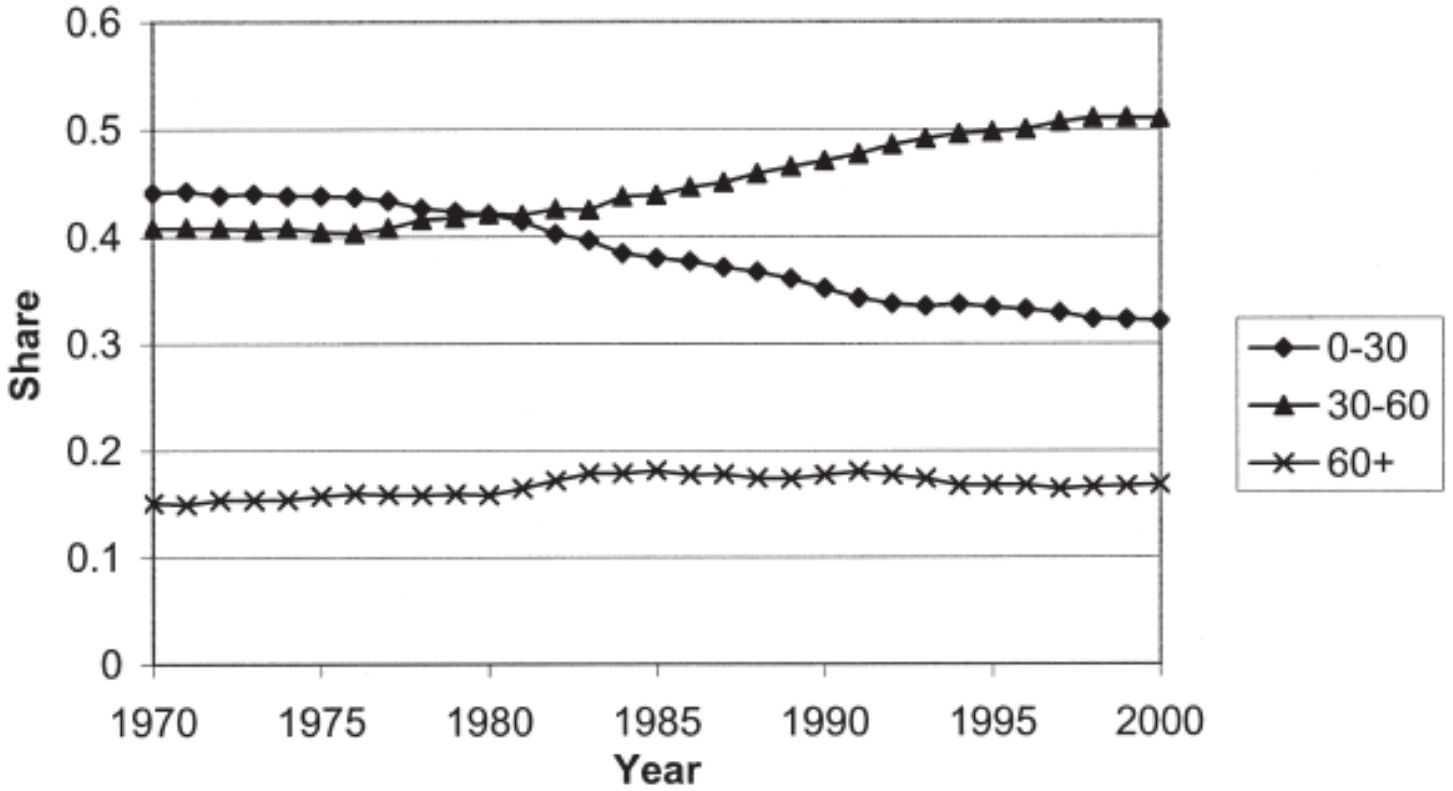


FIGURE II
Share of Income by Age Group from CPS, 1970–2000

Market Size and Innovation: Innovation Response

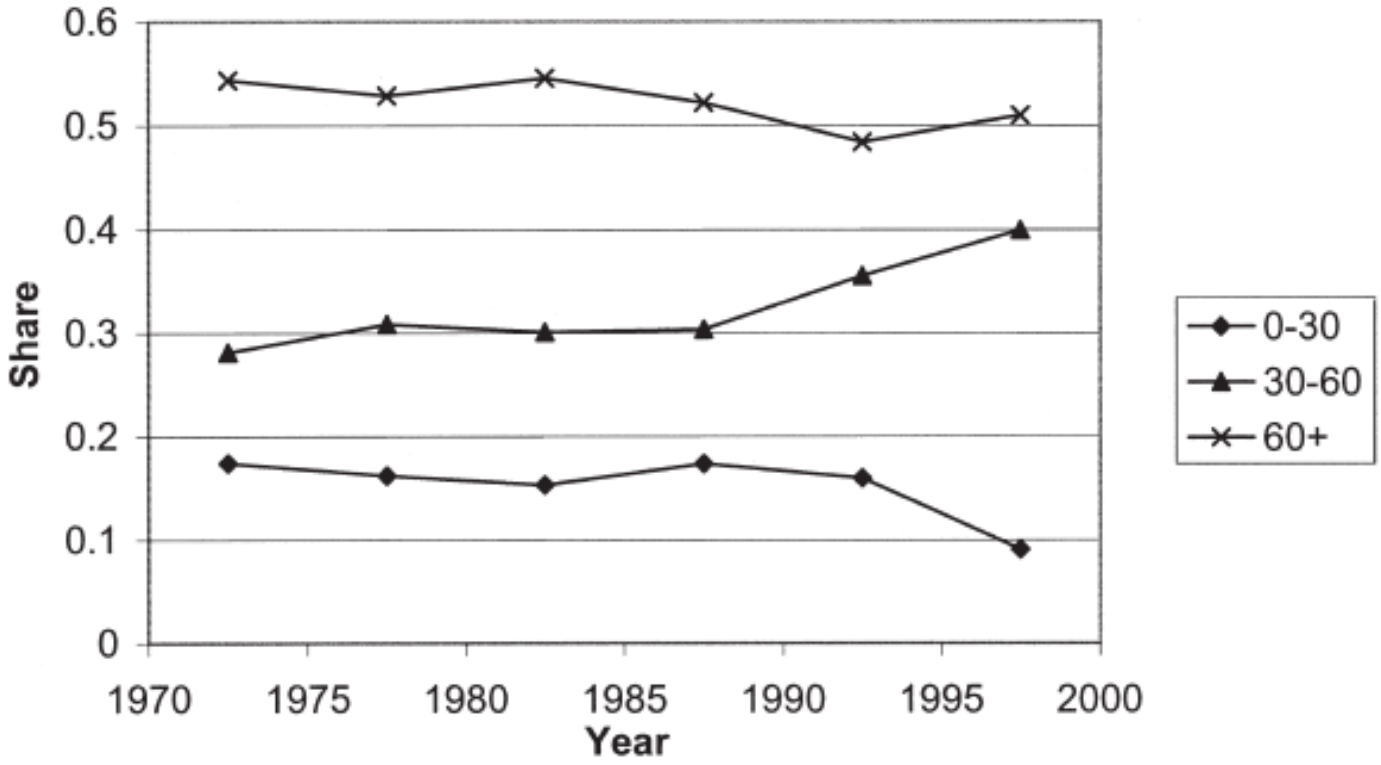


FIGURE III
Share of FDA Approvals by Age Group, 1970–2000

Market Size and Innovation: More Detailed Evidence

TABLE II
EFFECT OF CHANGES IN MARKET SIZE ON NEW DRUG APPROVALS

	(1)	(2)	(3)	(4)
Panel A: QML for Poisson model, dep var is count of drug approvals				
Market size	6.15	6.84	−2.22	
	(1.23)	(4.87)	(4.12)	
Lag market size		−0.61		
		(3.85)		
Lead market size			10.16	7.57
			(4.28)	(1.99)
Panel B: QML for Poisson model, dep var is count of nongeneric drug approvals				
Market size	3.82	6.72	2.91	
	(1.15)	(7.63)	(5.31)	
Lag market size		−2.49		
		(5.97)		
Lead market size			−1.77	1.73
			(6.94)	(2.02)
Panel C: QML for Poisson model, dep var is count of new molecular entities				
Market size	3.54	5.79	−1.38	
	(1.19)	(6.66)	(5.16)	
Lag market size		−1.99		
		(5.28)		
Lead market size			7.35	5.75
			(5.11)	(2.37)

Baseline Model of Directed Technical Change I

- Framework: expanding varieties model with lab equipment specification of the innovation possibilities frontier (so none of the results here depend on technological externalities).
- Constant supply of L and H.

- Representative household with the standard CRRA preferences:

$$\int_0^\infty \exp\left(-\rho t\right) \frac{C\left(t\right)^{1-\theta}-1}{1-\theta} dt, \tag{2}$$

- Aggregate production function:

$$Y\left(t\right) = \left[\gamma_L Y_L\left(t\right)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma_H Y_H\left(t\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where intermediate good $Y_L\left(t\right)$ is L-intensive, $Y_H\left(t\right)$ is H-intensive.

Baseline Model of Directed Technical Change II

- Resource constraint (define $Z(t) = Z_L(t) + Z_H(t)$):

$$C(t) + X(t) + Z(t) \leq Y(t), \tag{4}$$

- Intermediate goods produced competitively with:

$$Y_L(t) = \frac{1}{1-\beta}\left(\int_0^{N_L(t)} x_L(\nu, t)^{1-\beta} d\nu\right) L^{\beta} \tag{5}$$

and

$$Y_H(t) = \frac{1}{1-\beta}\left(\int_0^{N_H(t)} x_H(\nu, t)^{1-\beta} d\nu\right) H^{\beta}, \tag{6}$$

where machines $x_L(\nu, t)$ and $x_H(\nu, t)$ are assumed to depreciate after use.

Baseline Model of Directed Technical Change III

- Differences with baseline expanding product varieties model:
  1. These are production functions for intermediate goods rather than the final good.
  2. (5) and (6) use different types of machines–different ranges $[0, N_L(t)]$ and $[0, N_H(t)]$.
- All machines are supplied by monopolists that have a fully-enforced perpetual patent, at prices $p_L^x(\nu, t)$ for $\nu \in [0, N_L(t)]$ and $p_H^x(\nu, t)$ for $\nu \in [0, N_H(t)]$.
- Once invented, each machine can be produced at the fixed marginal cost ψ in terms of the final good.
- Normalize to $\psi \equiv 1 - \beta$.

Baseline Model of Directed Technical Change IV

- Total resources devoted to machine production at time t are

$$X(t) = (1-\beta)\left(\int_0^{N_L(t)} x_L(\nu,t)\,d\nu + \int_0^{N_H(t)} x_H(\nu,t)\,d\nu\right).$$

- Innovation possibilities frontier:

$$\dot{N}_L(t) = \eta_L Z_L(t) \text{ and } \dot{N}_H(t) = \eta_H Z_H(t), \tag{7}$$

- Value of a monopolist that discovers one of these machines is:

$$V_f(\nu,t) = \int_t^{\infty} \exp\left[-\int_t^s r(s')\,ds'\right] \pi_f(\nu,s)ds, \tag{8}$$

where $\pi_f(\nu,t) \equiv p_f^x(\nu,t)x_f(\nu,t) - \psi x_f(\nu,t)$ for $f = L$ or H.

- Hamilton-Jacobi-Bellman version:

$$r(t)\,V_f(\nu,t) - \dot{V}_f(\nu,t) = \pi_f(\nu,t). \tag{9}$$

Baseline Model of Directed Technical Change V

- Normalize the price of the final good at every instant to 1, which is equivalent to setting the ideal price index of the two intermediates equal to one, i.e.,

$$\left[\gamma_L^{\varepsilon}\left(p_L\left(t\right)\right)^{1-\varepsilon}+\gamma_H^{\varepsilon}\left(p_H\left(t\right)\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}=1 \text{ for all } t, \qquad (10)$$

  where $p_L\left(t\right)$ is the price index of Y_L at time t and $p_H\left(t\right)$ is the price of Y_H.
- Denote factor prices by $w_L\left(t\right)$ and $w_H\left(t\right)$.

Equilibrium I

- Allocation. Time paths of
  - $[C(t), X(t), Z(t)]_{t=0}^{\infty}$,
  - $[N_L(t), N_H(t)]_{t=0}^{\infty}$,
  - $\left[p_L^x(\nu,t), x_L(\nu,t), V_L(\nu,t)\right]_{t=0,\ \nu\in[0,N_L(t)]}^{\infty}$ and $\left[\chi_H(\nu,t), x_H(\nu,t), V_H(\nu,t)\right]_{t=0,\ \nu\in[0,N_H(t)]}^{\infty}$, and
  - $[r(t), w_L(t), w_H(t)]_{t=0}^{\infty}$.
- Equilibrium. An allocation in which
  - All existing research firms choose $\left[p_f^x(\nu,t), x_f(\nu,t)\right]_{t=0,\ \nu\in[0,N_f(t)]}^{\infty}$ for $f = L, H$ to maximize profits,
  - $[N_L(t), N_H(t)]_{t=0}^{\infty}$ is determined by free entry
  - $[r(t), w_L(t), w_H(t)]_{t=0}^{\infty}$, are consistent with market clearing, and
  - $[C(t), X(t), Z(t)]_{t=0}^{\infty}$ are consistent with consumer optimization.

Equilibrium II

- Maximization problem of producers in the two sectors:

$$\max_{L,[x_L(\nu,t)]_{\nu\in[0,N_L(t)]}} p_L(t) Y_L(t) - w_L(t) L - \int_0^{N_L(t)} p_L^x(\nu,t) x_L(\nu,t) d\nu, \quad (11)$$

and

$$\max_{H,[x_H(\nu,t)]_{\nu\in[0,N_H(t)]}} p_H(t) Y_H(t) - w_H(t) H - \int_0^{N_H(t)} p_H^x(\nu,t) x_H(\nu,t) d\nu. \quad (12)$$

- Note the presence of $p_L(t)$ and $p_H(t)$, since these sectors produce intermediate goods.

Equilibrium III

- Thus, demand for machines in the two sectors:

$$x_L(\nu,t)=\left[\frac{p_L(t)}{p_L^x(\nu,t)}\right]^{1/\beta}L \quad \text{for all } \nu\in[0,N_L(t)] \text{ and all } t, \quad (13)$$

and

$$x_H(\nu,t)=\left[\frac{p_H(t)}{p_H^x(\nu,t)}\right]^{1/\beta}H \quad \text{for all } \nu\in[0,N_H(t)] \text{ and all } t. \quad (14)$$

- Maximization of the net present discounted value of profits implies a constant markup:

$$p_L^x(\nu,t)=p_H^x(\nu,t)=1 \text{ for all } \nu \text{ and } t.$$

Equilibrium IV

- Substituting into (13) and (14):

$$x_L(\nu, t) = p_L(t)^{1/\beta} L \quad \text{for all } \nu \text{ and all } t,$$

and

$$x_H(\nu, t) = p_H(t)^{1/\beta} H \quad \text{for all } \nu \text{ and all } t.$$

- Since these quantities do not depend on the identity of the machine profits are also independent of the machine type:

$$\pi_L(t) = \beta p_L(t)^{1/\beta} L \text{ and } \pi_H(t) = \beta p_H(t)^{1/\beta} H. \qquad (15)$$

- Thus the values of monopolists only depend on which sector they are, $V_L(t)$ and $V_H(t)$.

Equilibrium V

- Combining these with (5) and (6), *derived* production functions for the two intermediate goods:

$$Y_L(t) = \frac{1}{1-\beta} p_L(t)^{\frac{1-\beta}{\beta}} N_L(t) L \tag{16}$$

and

$$Y_H(t) = \frac{1}{1-\beta} p_H(t)^{\frac{1-\beta}{\beta}} N_H(t) H. \tag{17}$$

Equilibrium VI

- For the prices of the two intermediate goods, (3) imply

$$\begin{aligned} p(t) &\equiv \frac{p_H(t)}{p_L(t)} = \gamma \left(\frac{Y_H(t)}{Y_L(t)} \right)^{-\frac{1}{\varepsilon}} \\ &= \gamma \left(p(t)^{\frac{1-\beta}{\beta}} \frac{N_H(t) H}{N_L(t) L} \right)^{-\frac{1}{\varepsilon}} \\ &= \gamma^{\frac{\varepsilon\beta}{\sigma}} \left(\frac{N_H(t) H}{N_L(t) L} \right)^{-\frac{\beta}{\sigma}}, \end{aligned} \tag{18}$$

where $\gamma \equiv \gamma_H / \gamma_L$ and

$$\begin{aligned} \sigma &\equiv \varepsilon - (\varepsilon - 1)(1 - \beta) \\ &= 1 + (\varepsilon - 1)\beta. \end{aligned}$$

Equilibrium VII

- We can also calculate the relative factor prices:

$$\begin{aligned}\omega(t) &\equiv \frac{w_H(t)}{w_L(t)} \\ &= p(t)^{1/\beta} \frac{N_H(t)}{N_L(t)} \\ &= \gamma^{\frac{\varepsilon}{\sigma}} \left(\frac{N_H(t)}{N_L(t)}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}. \end{aligned} \tag{19}$$

- σ is the (derived) elasticity of substitution between the two factors, since it is exactly equal to

$$\sigma = -\left(\frac{d\log\omega(t)}{d\log(H/L)}\right)^{-1}.$$

Equilibrium VIII

- Free entry conditions:

$$\eta_L V_L(t) \leq 1 \text{ and } \eta_L V_L(t) = 1 \text{ if } Z_L(t) > 0. \tag{20}$$

and

$$\eta_H V_H(t) \leq 1 \text{ and } \eta_H V_H(t) = 1 \text{ if } Z_H(t) > 0. \tag{21}$$

- Consumer side:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho), \tag{22}$$

and

$$\lim_{t\to\infty}\left[\exp\left(-\int_0^t r(s)\,ds\right)\left(N_L(t)V_L(t) + N_H(t)V_H(t)\right)\right] = 0, \tag{23}$$

where $N_L(t)V_L(t) + N_H(t)V_H(t)$ is the total value of corporate assets in this economy.

Balanced Growth Path I

- Consumption grows at the constant rate, g^*, and the relative price $p(t)$ is constant. From (10) this implies that $p_L(t)$ and $p_H(t)$ are also constant.
- Let V_L and V_H be the BGP net present discounted values of new innovations in the two sectors. Then (9) implies that

$$V_L = \frac{\beta p_L^{1/\beta} L}{r^*} \text{ and } V_H = \frac{\beta p_H^{1/\beta} H}{r^*}, \tag{24}$$

- Taking the ratio of these two expressions, we obtain

$$\frac{V_H}{V_L} = \left(\frac{p_H}{p_L}\right)^{\frac{1}{\beta}} \frac{H}{L}.$$

Balanced Growth Path II

- Note the two effects on the direction of technological change:
  1. The price effect: V_H/V_L is increasing in p_H/p_L. Tends to favor technologies complementing scarce factors.
  2. The market size effect: V_H/V_L is increasing in H/L. It encourages innovation for the more abundant factor.
- The above discussion is incomplete since prices are endogenous. Combining (24) together with (18):

$$\frac{V_H}{V_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{\sigma}} \left(\frac{N_H}{N_L}\right)^{-\frac{1}{\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{\sigma}}. \tag{25}$$

- Note that an increase in H/L will increase V_H/V_L as long as $\sigma > 1$ and it will reduce it if $\sigma < 1$. Moreover,

$$\sigma \gtreqless 1 \iff \varepsilon \gtreqless 1.$$

- The two factors will be gross substitutes when the two intermediate goods are gross substitutes in the production of the final good.

Balanced Growth Path III

- Next, using the two free entry conditions (20) and (21) as equalities, we obtain the following BGP "technology market clearing" condition:

$$\eta_L V_L = \eta_H V_H. \tag{26}$$

- Combining this with (25), BGP ratio of relative technologies is

$$\left(\frac{N_H}{N_L}\right)^* = \eta^\sigma \gamma^\varepsilon \left(\frac{H}{L}\right)^{\sigma-1}, \tag{27}$$

where $\eta \equiv \eta_H/\eta_L$.

- Note that relative productivities are determined by the innovation possibilities frontier and the relative supply of the two factors. In this sense, this model totally endogenizes technology.

Summary of Balanced Growth Path

Proposition Consider the directed technological change model described above. Suppose

$$\beta \left[\gamma_H^{\varepsilon} \left(\eta_H H \right)^{\sigma-1} + \gamma_L^{\varepsilon} \left(\eta_L L \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} > \rho \quad (28)$$

$$\text{and } (1-\theta) \beta \left[\gamma_H^{\varepsilon} \left(\eta_H H \right)^{\sigma-1} + \gamma_L^{\varepsilon} \left(\eta_L L \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} < \rho.$$

Then there exists a unique BGP equilibrium in which the relative technologies are given by (27), and consumption and output grow at the rate

$$g^* = \frac{1}{\theta} \left(\beta \left[\gamma_H^{\varepsilon} \left(\eta_H H \right)^{\sigma-1} + \gamma_L^{\varepsilon} \left(\eta_L L \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} - \rho \right). \quad (29)$$

Transitional Dynamics

- Differently from the baseline endogenous technological change models, there are now transitional dynamics (because there are two state variables).
- Nevertheless, transitional dynamics simple and intuitive:

Proposition Consider the directed technological change model described above. Starting with any $N_H(0) > 0$ and $N_L(0) > 0$, there exists a unique equilibrium path. If $N_H(0)/N_L(0) < (N_H/N_L)^*$ as given by (27), then we have $Z_H(t) > 0$ and $Z_L(t) = 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$. If $N_H(0)/N_L(0) < (N_H/N_L)^*$, then $Z_H(t) = 0$ and $Z_L(t) > 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$.

- Summary: the dynamic equilibrium path always tends to the BGP and during transitional dynamics, there is only one type of innovation.

Directed Technological Change and Factor Prices

- In BGP, there is a positive relationship between H/L and N_H^*/N_L^* only when $\sigma > 1$.
- But this does not mean that depending on σ (or ε), changes in factor supplies may induce technological changes that are biased in favor or against the factor that is becoming more abundant.
- Why?
  - N_H^*/N_L^* refers to the ratio of factor-augmenting technologies, or to the ratio of *physical* productivities.
  - What matters for the bias of technology is *the value of marginal product* of factors, affected by relative prices.
  - The relationship between factor-augmenting and factor-biased technologies is reversed when σ is less than 1.
  - When $\sigma > 1$, an increase in N_H^*/N_L^* is relatively biased towards H, while when $\sigma < 1$, a *decrease* in N_H^*/N_L^* is relatively biased towards H.

Weak Equilibrium (Relative) Bias Result

Proposition Consider the directed technological change model described above. There is always **weak equilibrium (relative) bias** in the sense that an increase in H/L always induces relatively H-biased technological change.

- The results reflect the strength of the market size effect: it always dominates the price effect.
- But it does not specify whether this induced effect will be strong enough to make the endogenous-technology relative demand curve for factors upward-sloping.

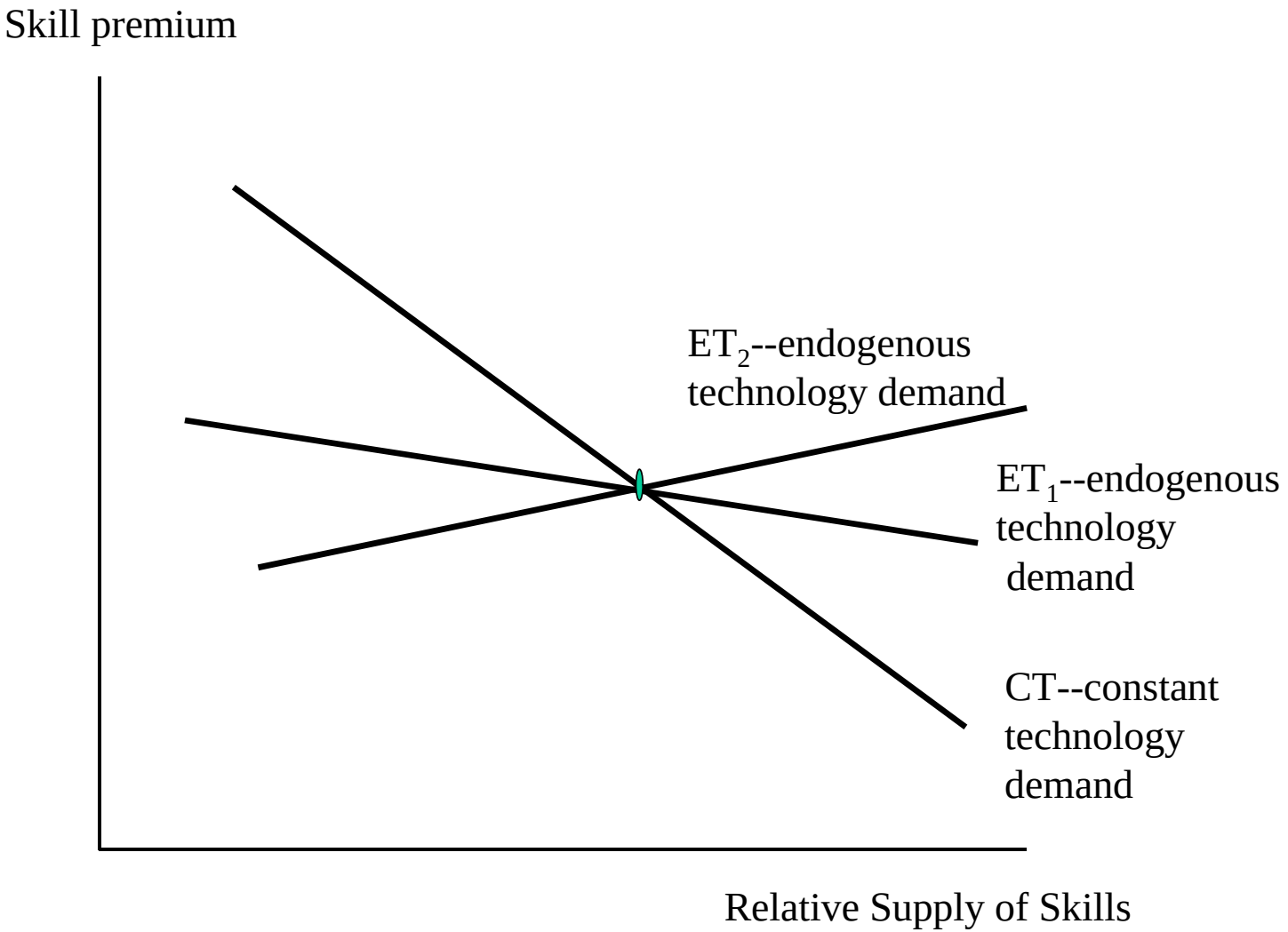
Strong Equilibrium (Relative) Bias Result

- Substitute for $(N_H/N_L)^*$ from (27) into the expression for the relative wage given technologies, (19), and obtain:

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\sigma-1}\gamma^{\varepsilon}\left(\frac{H}{L}\right)^{\sigma-2}. \tag{30}$$

Proposition Consider the directed technological change model described above. Then if $\sigma > 2$, there is **strong equilibrium (relative) bias** in the sense that an increase in H/L raises the relative marginal product and the relative wage of the factor H compared to factor L.

Relative Supply of Skills and Skill Premium



Discussion

- Analogous to Samuelson's *LeChatelier principle*: think of the endogenous-technology demand curve as adjusting the "factors of production" corresponding to technology.
- But, the effects here are caused by general equilibrium changes, not on partial equilibrium effects.
- Moreover ET_2, which applies when $\sigma > 2$ holds, is upward-sloping.
- A complementary intuition: importance of non-rivalry of ideas:
  - leads to an aggregate production function that exhibits increasing returns to scale (in all factors including technologies).
  - the market size effect can create sufficiently strong induced technological change to increase the relative marginal product and the relative price of the factor that has become more abundant.

Implications I

- Recall we have the following stylized facts:
  - Secular skill-biased technological change increasing the demand for skills throughout the 20th century.
  - Possible acceleration in skill-biased technological change over the past 25 years.
  - A range of important technologies biased against skill workers during the 19th century.
- The current model gives us a way to think about these issues.
  - The increase in the number of skilled workers should cause steady skill-biased technical change.
  - Acceleration in the increase in the number of skilled workers should induce an acceleration in skill-biased technological change.
  - Available evidence suggests that there were large increases in the number of unskilled workers during the late 18th and 19th centuries.

Implications II

- The framework also gives a potential interpretation for the dynamics of the college premium during the 1970s and 1980s.
  - It is reasonable that the equilibrium skill bias of technologies, N_H / N_L, is a sluggish variable.
  - Hence a rapid increase in the supply of skills would first reduce the skill premium as the economy would be moving along a constant technology (constant N_H / N_L).
  - After a while technology would start adjusting, and the economy would move back to the upward sloping relative demand curve, with a relatively sharp increase in the college premium.

Implications III

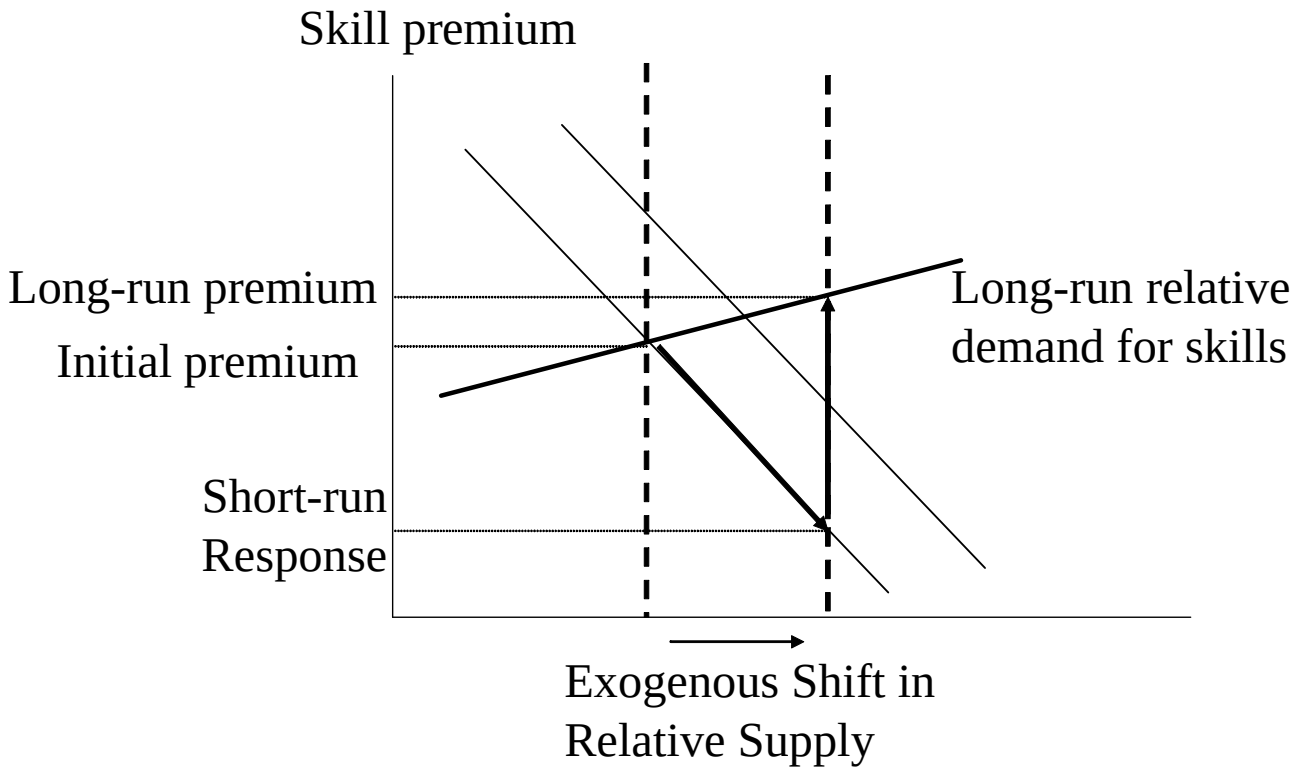


Figure: Dynamics of the skill premium in response to an exogenous increase in the relative supply of skills, with an upward-sloping endogenous-technology relative demand curve.

Implications IV

- If instead $\sigma < 2$, the long-run relative demand curve will be downward sloping, though again it will be shallower than the short-run relative demand curve.
- An increase in the relative supply of skills leads again to a decline in the college premium, and as technology starts adjusting the skill premium will increase.
- But it will end up below its initial level. To explain the larger increase in the college premium in the 1980s, in this case we would need some exogenous skill-biased technical change.

Implications V

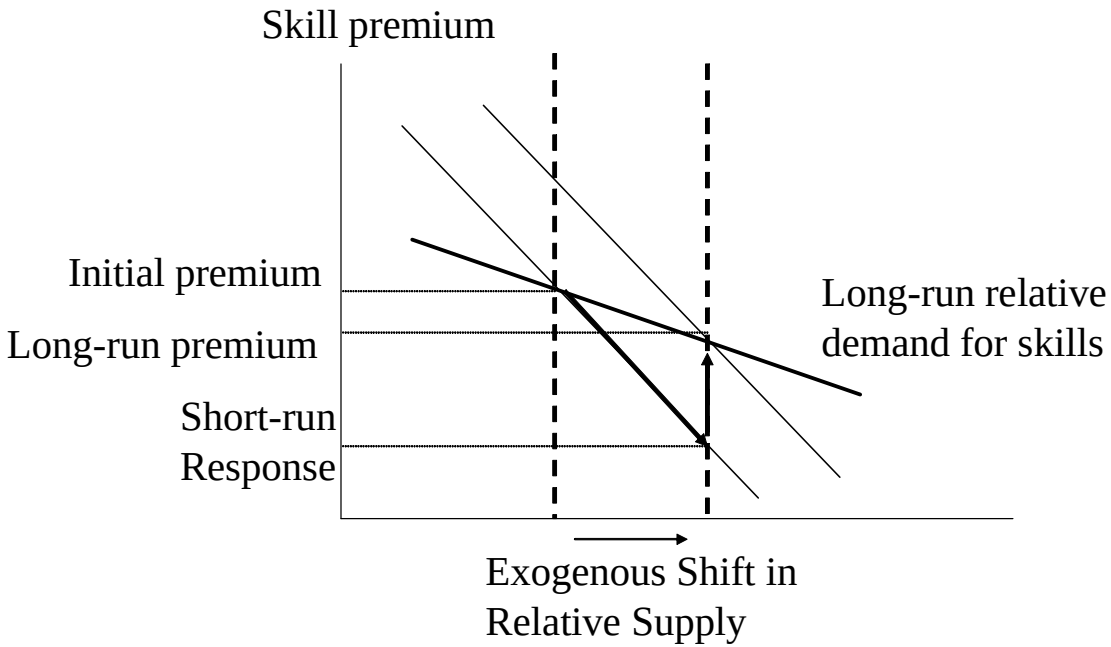


Figure: Dynamics of the skill premium in response to an increase in the relative supply of skills, with a downward-sloping endogenous-technology relative demand curve.

Implications VI

- Other remarks:
  - Upward-sloping relative demand curves arise only when $\sigma > 2$. Most estimates put the elasticity of substitution between 1.4 and 2. One would like to understand whether $\sigma > 2$ is a feature of the specific model discussed here
  - Results on induced technological change are not an artifact of the scale effect (exactly the same results apply when scale effects are removed, see below).

Pareto Optimal Allocations I

- The social planner would not charge a markup on machines:

$$\begin{aligned} x_L^S(\nu,t) &= (1-\beta)^{-1/\beta} p_L(t)^{1/\beta} L \\ \text{and } x_H^S(\nu,t) &= (1-\beta)^{-1/\beta} p_H(t)^{1/\beta} H. \end{aligned}$$

- Thus:

$$\begin{aligned} Y^S(t) &= (1-\beta)^{-1/\beta} \beta [\gamma_L^{\varepsilon} \left(N_L^S(t) L \right)^{\frac{\sigma-1}{\sigma}} \\ &\quad + \gamma_H^{\varepsilon} \left(N_H^S(t) H \right)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}. \end{aligned} \tag{31}$$

Pareto Optimal Allocations II

- The current-value Hamiltonian is:

$$\mathcal{H}(\cdot) = \frac{C^S(t)^{1-\theta}-1}{1-\theta} + \mu_L(t)\eta_L Z_L^S(t) + \mu_H(t)\eta_H Z_H^S(t),$$

subject to

$$C^S(t) = (1-\beta)^{-1/\beta}\left[\gamma_L^{\varepsilon}\left(N_L^S(t)L\right)^{\frac{\sigma-1}{\sigma}} + \gamma_H^{\varepsilon}\left(N_H^S(t)H\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - Z_L^S(t) - Z_H^S(t).$$

Summary of Pareto Optimal Allocations

Proposition The stationary solution of the Pareto optimal allocation involves relative technologies given by (27) as in the decentralized equilibrium. The stationary growth rate is higher than the equilibrium growth rate and is given by

$$g^S = \frac{1}{\theta}\left((1-\beta)^{-1/\beta}\beta\left[(1-\gamma)^{\varepsilon}(\eta_H H)^{\sigma-1}+\gamma^{\varepsilon}(\eta_L L)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}\right.$$

where g^* is the BGP growth rate given in (29).

Directed Technological Change with Knowledge Spillovers I

- The lab equipment specification of the innovation possibilities does not allow for *state dependence*.
- Assume that R&D is carried out by scientists and that there is a constant supply of scientists equal to S
- With only one sector, sustained endogenous growth requires $\dot{N}/N$ to be proportional to S.
- With two sectors, there is a variety of specifications with different degrees of state dependence, because productivity in each sector can depend on the state of knowledge in both sectors.
- A flexible formulation is

$$\begin{aligned} \dot{N}_L(t) &= \eta_L N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} S_L(t) \qquad (32) \\ \text{and } \dot{N}_H(t) &= \eta_H N_L(t)^{(1-\delta)/2} N_H(t)^{(1+\delta)/2} S_H(t), \end{aligned}$$

where $\delta \leq 1$.

Directed Technological Change with Knowledge Spillovers II

- Market clearing for scientists requires that

$$S_L(t) + S_H(t) \leq S. \tag{33}$$

- δ measures the degree of state-dependence:
  - $\delta = 0$. Results are unchanged. No state-dependence:

$$\left(\partial \dot{N}_H / \partial S_H\right) / \left(\partial \dot{N}_L / \partial S_L\right) = \eta_H / \eta_L$$

    irrespective of the levels of N_L and N_H.
    Both N_L and N_H create spillovers for current research in both sectors.
  - $\delta = 1$. Extreme amount of state-dependence:

$$\left(\partial \dot{N}_H / \partial S_H\right) / \left(\partial \dot{N}_L / \partial S_L\right) = \eta_H N_H / \eta_L N_L$$

    an increase in the stock of L-augmenting machines today makes future labor-complementary innovations cheaper, but has no effect on the cost of H-augmenting innovations.

Directed Technological Change with Knowledge Spillovers III

- State dependence adds another layer of "increasing returns," this time not for the entire economy, but for specific technology lines.
- Free entry conditions:

$$\begin{aligned} \eta_L N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} V_L(t) &\leq w_S(t) \qquad (34) \\ \text{and } \eta_L N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} V_L(t) &= w_S(t) \text{ if } S_L(t) > 0. \end{aligned}$$

and

$$\begin{aligned} \eta_H N_L(t)^{(1-\delta)/2} N_H(t)^{(1+\delta)/2} V_H(t) &\leq w_S(t) \qquad (35) \\ \text{and } \eta_H N_L(t)^{(1-\delta)/2} N_H(t)^{(1+\delta)/2} V_H(t) &= w_S(t) \text{ if } S_H(t) > 0, \end{aligned}$$

where $w_S(t)$ denotes the wage of a scientist at time t.

Directed Technological Change with Knowledge Spillovers IV

- When both of these free entry conditions hold, BGP technology market clearing implies

$$\eta_L N_L (t)^{\delta} \pi_L = \eta_H N_H (t)^{\delta} \pi_H, \tag{36}$$

- Combine condition (36) with equations (15) and (18), to obtain the equilibrium relative technology as:

$$\left(\frac{N_H}{N_L}\right)^* = \eta^{\frac{\sigma}{1-\delta\sigma}} \gamma^{\frac{\varepsilon}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{1-\delta\sigma}}, \tag{37}$$

where $\gamma \equiv \gamma_H/\gamma_L$ and $\eta \equiv \eta_H/\eta_L$.

Directed Technological Change with Knowledge Spillovers V

- The relationship between the relative factor supplies and relative physical productivities now depends on δ.
- This is intuitive: as long as $\delta > 0$, an increase in N_H reduces the relative costs of H-augmenting innovations, so for technology market equilibrium to be restored, π_L needs to fall relative to π_H.
- Substituting (37) into the expression (19) for relative factor prices for given technologies, yields the following long-run (endogenous-technology) relationship:

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\frac{\sigma-1}{1-\delta\sigma}} \gamma^{\frac{(1-\delta)\varepsilon}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-2+\delta}{1-\delta\sigma}}. \tag{38}$$

Directed Technological Change with Knowledge Spillovers VI

- The growth rate is determined by the number of scientists. In BGP we need $\dot{N}_L(t)/N_L(t) = \dot{N}_H(t)/N_H(t)$, or

$$\eta_H N_H(t)^{\delta-1} S_H(t) = \eta_L N_L(t)^{\delta-1} S_L(t).$$

- Combining with (33) and (37), BGP allocation of researchers between the two different types of technologies:

$$\eta^{\frac{1-\sigma}{1-\delta\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{-\frac{\varepsilon(1-\delta)}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{-\frac{(\sigma-1)(1-\delta)}{1-\delta\sigma}} = \frac{S_L^*}{S-S_L^*}, \tag{39}$$

- Notice that given H/L, the BGP researcher allocations, S_L^* and S_H^*, are uniquely determined.

Balanced Growth Path with Knowledge Spillovers

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Suppose that

$$(1-\theta)\frac{\eta_L\eta_H\left(N_H/N_L\right)^{(\delta-1)/2}}{\eta_H\left(N_H/N_L\right)^{(\delta-1)}+\eta_L}S<\rho,$$

where N_H/N_L is given by (37). Then there exists a unique BGP equilibrium in which the relative technologies are given by (37), and consumption and output grow at the rate

$$g^*=\frac{\eta_L\eta_H\left(N_H/N_L\right)^{(\delta-1)/2}}{\eta_H\left(N_H/N_L\right)^{(\delta-1)}+\eta_L}S. \tag{40}$$

Transitional Dynamics with Knowledge Spillovers

- Transitional dynamics now more complicated because of the spillovers.
- The dynamic equilibrium path does not always tend to the BGP because of the additional increasing returns to scale:
  - With a high degree of state dependence, when $N_H(0)$ is very high relative to $N_L(0)$, it may no longer be profitable for firms to undertake further R&D directed at labor-augmenting (L-augmenting) technologies.
  - Whether this is so or not depends on a comparison of the degree of state dependence, δ, and the elasticity of substitution, σ.

Summary of Transitional Dynamics

Proposition Suppose that

$$\sigma < 1/\delta.$$

Then, starting with any $N_H(0) > 0$ and $N_L(0) > 0$, there exists a unique equilibrium path. If $N_H(0)/N_L(0) < (N_H/N_L)^*$ as given by (37), then we have $Z_H(t) > 0$ and $Z_L(t) = 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$. $N_H(0)/N_L(0) < (N_H/N_L)^*$, then $Z_H(t) = 0$ and $Z_L(t) > 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$.
If

$$\sigma > 1/\delta,$$

then starting with $N_H(0)/N_L(0) > (N_H/N_L)^*$, the economy tends to $N_H(t)/N_L(t) \to \infty$ as $t \to \infty$, and starting with $N_H(0)/N_L(0) < (N_H/N_L)^*$, it tends to $N_H(t)/N_L(t) \to 0$ as $t \to \infty$.

Equilibrium Relative Bias with Knowledge Spillovers I

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Then there is always **weak equilibrium (relative) bias** in the sense that an increase in H/L always induces relatively H-biased technological change.

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Then if

$$\sigma > 2 - \delta,$$

there is **strong equilibrium (relative) bias** in the sense that an increase in H/L raises the relative marginal product and the relative wage of the H factor compared to the L factor.

Equilibrium Relative Bias with Knowledge Spillovers II

- Intuitively, the additional increasing returns to scale coming from state dependence makes strong bias easier to obtain, because the induced technology effect is stronger.
- Note the elasticity of substitution between skilled and unskilled labor significantly less than 2 may be sufficient to generate strong equilibrium bias.
- How much lower than 2 the elasticity of substitution can be depends on the parameter δ. Unfortunately, this parameter is not easy to measure in practice.

Endogenous Labor-Augmenting Technological Change I

- Models of directed technological change create a natural reason for technology to be more labor augmenting than capital augmenting.
- Under most circumstances, the resulting equilibrium is not purely labor augmenting and as a result, a BGP fails to exist.
- But in one important special case, the model delivers long-run purely labor augmenting technological changes exactly as in the neoclassical growth model.
- Consider a two-factor model with H corresponding to capital, that is, $H(t) = K(t)$.
- Assume that there is no depreciation of capital.
- Note that in this case the price of the second factor, $K(t)$, is the same as the interest rate, $r(t)$.
- Empirical evidence suggests $\sigma < 1$ and is also economically plausible.

Endogenous Labor-Augmenting Technological Change II

- Recall that when $\sigma < 1$ labor-augmenting technological change corresponds to capital-biased technological change.
- Hence the questions are:
  1. Under what circumstances would the economy generate relatively capital-biased technological change?
  2. When will the equilibrium technology be sufficiently capital biased that it corresponds to Harrod-neutral technological change?

Endogenous Labor-Augmenting Technological Change III

- To answer 1, note that what distinguishes capital from labor is the fact that it accumulates.
- The neoclassical growth model with technological change experiences continuous capital-deepening as $K(t)/L$ increases.
- This implies that technological change should be *more labor-augmenting than capital augmenting.*

Proposition In the baseline model of directed technological change with $H(t) = K(t)$ as capital, if $K(t)/L$ is increasing over time and $\sigma < 1$, then $N_L(t)/N_K(t)$ will also increase over time.

Endogenous Labor-Augmenting Technological Change IV

- But the results are not easy to reconcile with purely-labor augmenting technological change. Suppose that capital accumulates at an exogenous rate, i.e.,

$$\frac{\dot{K}(t)}{K(t)} = s_K > 0. \tag{41}$$

Proposition Consider the baseline model of directed technological change with the knowledge spillovers specification and state dependence. Suppose that $\delta < 1$ and capital accumulates according to (41). Then there exists no BGP.

- Intuitively, even though technological change is more labor augmenting than capital augmenting, there is still capital-augmenting technological change in equilibrium.
- Moreover it can be proved that in any asymptotic equilibrium, $r(t)$ cannot be constant, thus consumption and output growth cannot be constant.

Endogenous Labor-Augmenting Technological Change V

- Special case that justifies the basic structure of the neoclassical growth model: extreme state dependence ($\delta = 1$).
- In this case:

$$\frac{r(t) K(t)}{w_L(t) L} = \eta^{-1}. \tag{42}$$

- Thus, directed technological change ensures that the share of capital is constant in national income. .
- Recall from (19) that

$$\frac{r(t)}{w_L(t)} = \gamma^{\frac{\varepsilon}{\sigma}} \left(\frac{N_K(t)}{N_L(t)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{K(t)}{L} \right)^{-\frac{1}{\sigma}},$$

where $\gamma \equiv \gamma_K / \gamma_L$ and γ_K replaces γ_H in the production function (3).

Endogenous Labor-Augmenting Technological Change VI

- Consequently,

$$\frac{r(t)K(t)}{w_L(t)L(t)} = \gamma^{\frac{\varepsilon}{\sigma}} \left(\frac{N_K(t)}{N_L(t)}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{K(t)}{L}\right)^{\frac{\sigma-1}{\sigma}}.$$

- In this case, (42) combined with (41) implies that

$$\frac{\dot{N}_L(t)}{N_L(t)} - \frac{\dot{N}_K(t)}{N_K(t)} = s_K. \tag{43}$$

- Moreover:

$$r(t) = \beta\gamma_K N_K(t) \left[\gamma_L \left(\frac{N_L(t)L}{N_K(t)K(t)}\right)^{\frac{\sigma-1}{\sigma}} + \gamma_L)\right]^{\frac{1}{\sigma-1}}. \tag{44}$$

Endogenous Labor-Augmenting Technological Change VII

- From (22), a constant growth path which consumption grows at a constant rate is only possible if $r(t)$ is constant.
- Equation (43) implies that $(N_L(t) L) / (N_K(t) K(t))$ is constant, thus $N_K(t)$ must also be constant.
- Therefore, equation (43) implies that technological change must be purely labor augmenting.

Summary of Endogenous Labor-Augmenting Technological Change

Proposition Consider the baseline model of directed technological change with the two factors corresponding to labor and capital. Suppose that the innovation possibilities frontier is given by the knowledge spillovers specification and **extreme state dependence**, i.e., $\delta = 1$ and that capital accumulates according to (41). Then there exists a constant growth path allocation in which there is only labor-augmenting technological change, the interest rate is constant and consumption and output grow at constant rates. Moreover, there cannot be any other constant growth path allocations.

Stability

- The constant growth path allocation with purely labor augmenting technological change is globally stable if $\sigma < 1$.
- Intuition:
  - If capital and labor were gross substitutes ($\sigma > 1$), the equilibrium would involve rapid accumulation of capital and capital-augmenting technological change, leading to an asymptotically increasing growth rate of consumption.
  - When capital and labor are gross complements ($\sigma < 1$), capital accumulation would increase the price of labor and profits from labor-augmenting technologies and thus encourage further labor-augmenting technological change.
  - $\sigma < 1$ forces the economy to strive towards a balanced allocation of effective capital and labor units.
  - Since capital accumulates at a constant rate, a balanced allocation implies that the productivity of labor should increase faster, and the economy should converge to an equilibrium path with purely labor-augmenting technological progress.

Conclusions I

- The bias of technological change is potentially important for the distributional consequences of the introduction of new technologies (i.e., who will be the losers and winners?); important for political economy of growth.
- Models of directed technological change enable us to investigate a range of new questions:
  - the sources of skill-biased technological change over the past 100 years,
  - the causes of acceleration in skill-biased technological change during more recent decades,
  - the causes of unskilled-biased technological developments during the 19th century,
  - the relationship between labor market institutions and the types of technologies that are developed and adopted,
  - why technological change in neoclassical-type models may be largely labor-augmenting.

Conclusions II

- The implications of the class of models studied for the empirical questions mentioned above stem from the *weak equilibrium bias* and *strong equilibrium bias* results.
- Technology should not be thought of as a black box. Profit incentives will play a major role in both the aggregate rate of technological progress and also in the biases of the technologies.

Introduction

- We still know relatively little about determinants of technology adoption and innovation.
- A classic question: *does shortage of labor encourage innovation*?
- Related: *do high wages encourage innovation?*
- Answers vary.

Different Answers?

- Neoclassical growth model: *No*, with technology embodied in capital and constant returns to scale, labor shortage and high wages always discourage technology adoption.
- Endogenous growth theory: *No*, it discourages innovation because of scale effects. True also in "semi-endogenous" growth models such as Jones (1995), Young (1999) or Howitt (1999).
- Ester Boserup: *No*, population pressure is a major factor in innovations.

Different Answers? (continued)

- John Hicks: *Yes*,
  "A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind—directed to economizing the use of a factor which has become relatively expensive..." (*Theory of Wages*, p. 124).
- Habakkuk: *Yes*, in the context of 19th-century US-UK comparison

  "... it was scarcity of labor 'which laid the foundation for the future continuous progress of American industry, by obliging manufacturers to take every opportunity of installing new types of labor-saving machinery.' " (quoted from Pelling),

  "It seems obvious— it certainly seemed so to contemporaries— that the dearness and inelasticity of American, compared with British, labour gave the American entrepreneur ... a greater inducement than his British counterpart to replace labour by machines." (Habakkuk, 1962, p. 17).

Different Answers? (continued)

- Robert Allen: *Yes*, high British wages are the reason why the major technologies of the British Industrial Revolution got invented.

  "... Nottingham, Leicester, Birmingham, Sheffield etc. must long ago have given up all hopes of foreign commerce, if they had not been constantly counteracting the advancing price of manual labor, by adopting every ingenious improvement the human mind could invent." (T. Bentley).

- Zeira; Hellwig-Irmen: *Yes*, high wages encourage switch to capital-intensive technologies.
- Alesina-Zeira and others: *Yes*, high wages may have encouraged adoption of certain capital-intensive technologies in Europe

Why the Different Answers?

- Different models, with different assumptions about technology adoption and market structure
- But which assumptions drive these results is not always clear
  - Thus, unclear which different historical accounts and which models we should trust more.
- In fact, theory leads to quite general results and clarifies conditions under which labor shortages will encourage technology adoption.
  - Partly building on Acemoglu (2007).

Framework

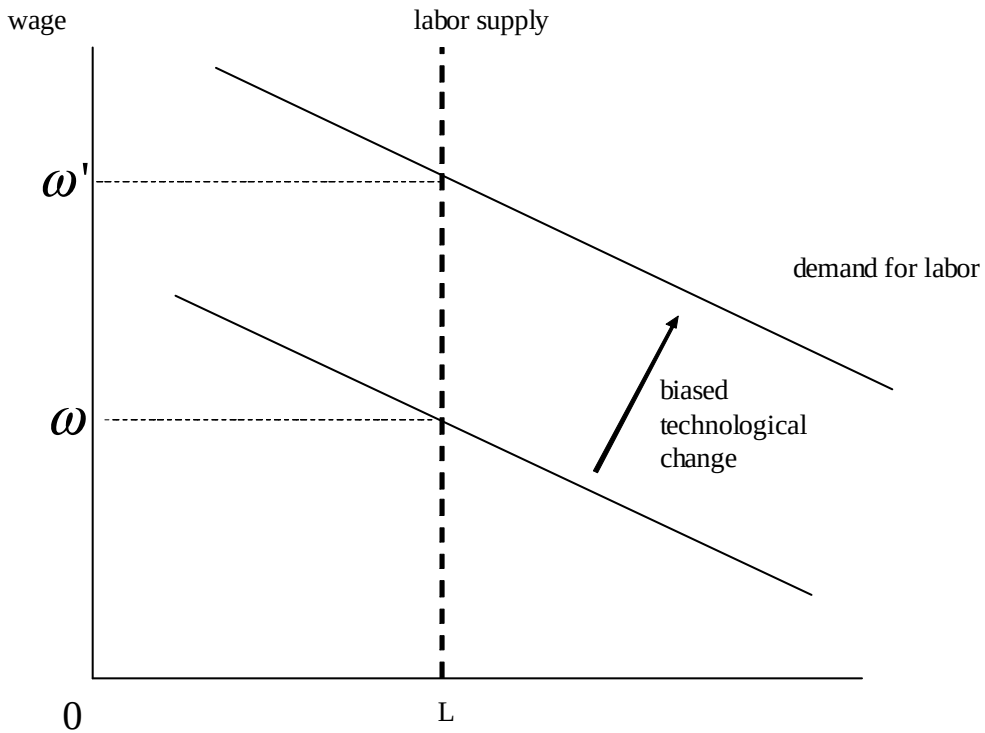
- Which framework for technology adoption?
- **Answer:** it does not matter too much.
- **First step**: a general tractable model of technology adoption
  - Competitive factor markets.
  - Technology could be one dimensional, represented by a scaler as in canonical neoclassical growth model or endogenous growth models, or multidimensional.
  - Technology could correspond to discrete choices (in many settings, more realistic, switch from one organizational form to another; adoption of a new general purpose technology)
- Wage push vs. labor scarcity: generally no difference
  - provided that factor prices proportional the marginal product
  - provided that equilibrium demand curves are downward sloping
  - we will see conditions under which this will be the case

General Results

- We know quite a bit about relationship between labor scarcity and **bias** of technology.
- In particular:
  Theorem (Acemoglu, 2007): Under some weak regularity conditions (to be explained below), a decrease in labor supply will change technology in a way that is *biased* against labor.
  Theorem (Acemoglu, 2007): Under some weak regularity conditions, a decrease in labor supply will *decrease* wages *if and only if* the aggregate production possibilities set of the economy is *locally nonconvex.*

What Is (Absolute) Bias?

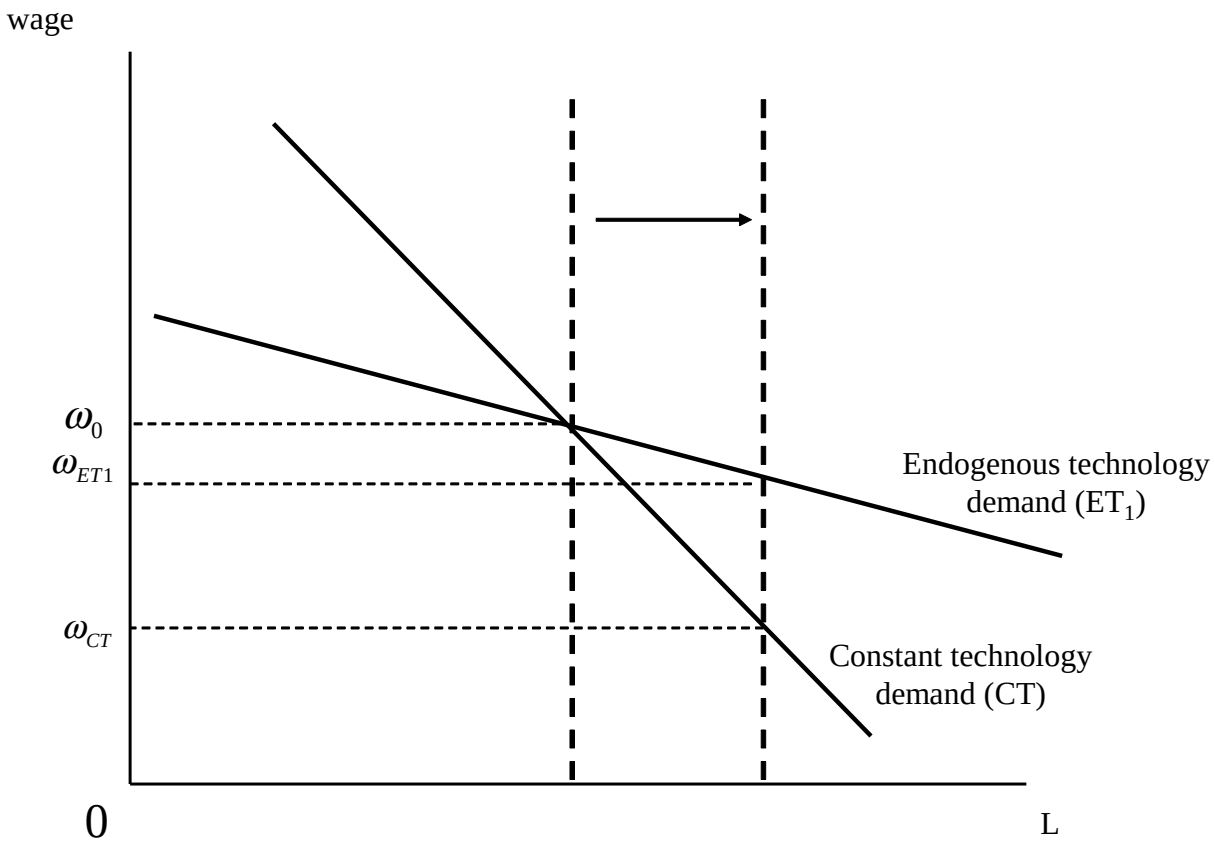
- Same as relative bias; but now "absolute," i.e., shift of the usual demand curve.



Intuition For Bias

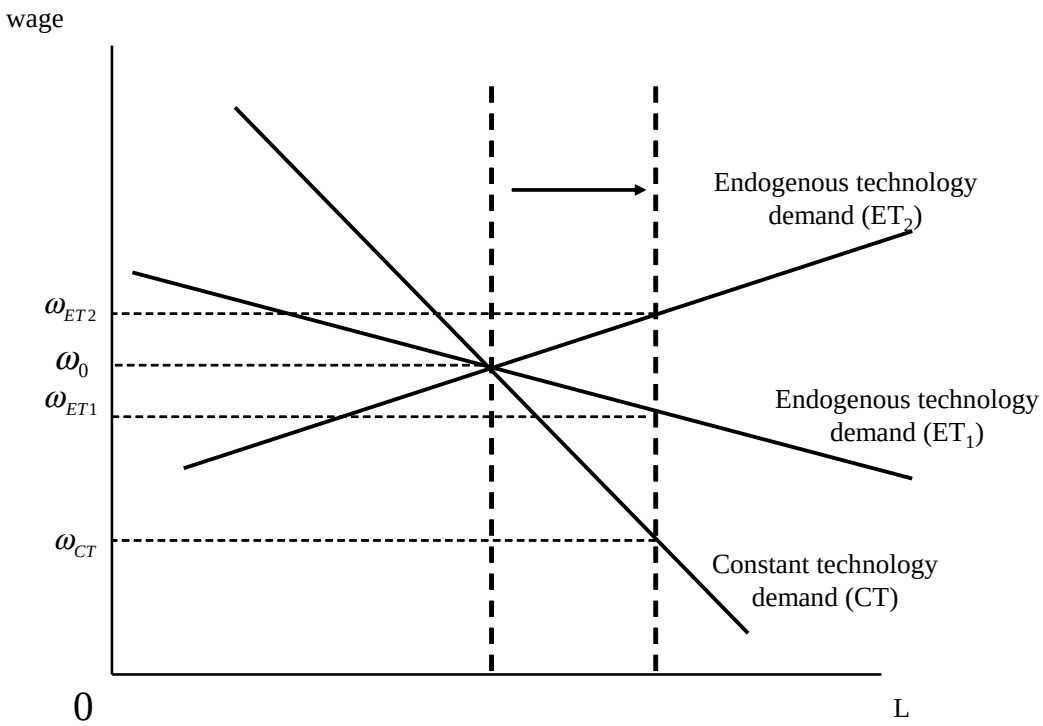
- An increase in employment (L), at the margin, increases the the value of technologies that are "complementary" to L.
  - Denote technology by θ.
  - Suppose that L and θ are complements, then an increase in L increases the incentives to improve θ, but then this *increases* the marginal contribution of L to output and thus wages$\longrightarrow$*biased change.*
  - Suppose that L and θ are substitutes,then an increase in L reduces the incentives to improve θ, but then this *increases* the marginal contribution of L to output and thus wages$\longrightarrow$*biased change*
- But this intuition also shows that an increase in L could lead to an increase or decrease in θ.
- Thus implications for "technological advances" unclear.

Induced (Absolute) Bias



Upward Sloping Demand Curves?

- Impossible in producer theory. But in general equilibrium, quite usual.



Labor Scarcity and Technological Advances

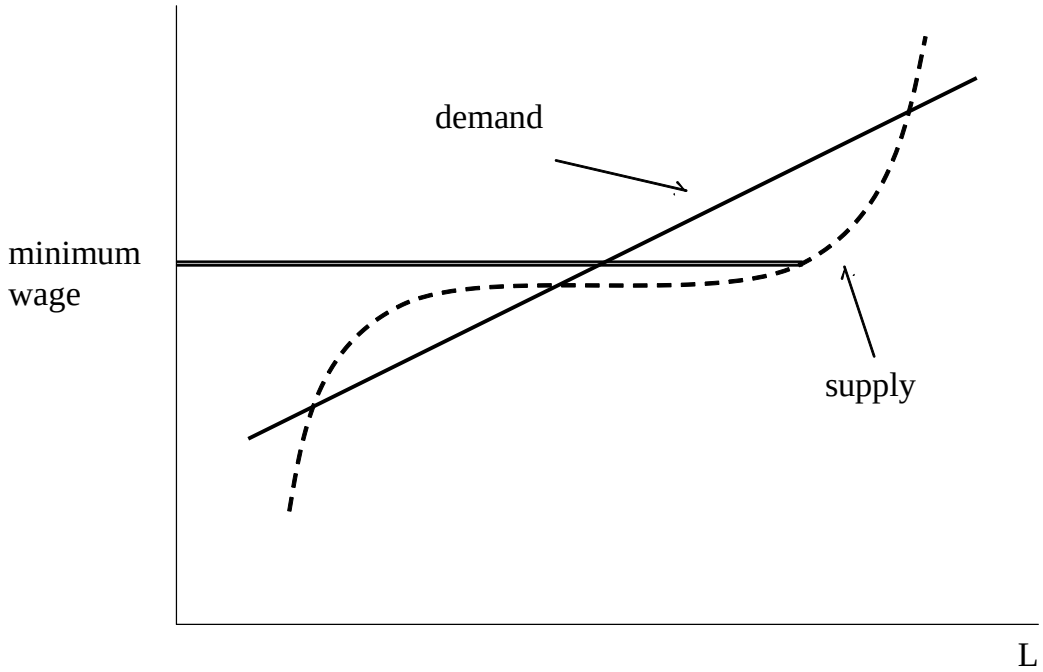
- The above discussion suggests that we should not look for an unambiguous relationship.
- Is there a simple unifying theme?
- Suppose that aggregate output can be represented as $F(L, Z, \theta)$, where Z is a vector of other inputs.
- Let us say that technological change is strongly labor saving if F exhibits *decreasing differences* in L and θ.
- Conversely, technological change is strongly labor complementary if F exhibits *increasing differences* in L and θ.
- **Answer:** labor scarcity will lead to technological advances if technology is strongly labor saving and will lead to technological regress if technology is strongly labor complementary.

What Does This Mean?

- At the margin, labor and the relevant technologies need to be "substitutes".
- This is generally not the case in neoclassical models or endogenous growth models, but not unusual.
- Examples of models where technology is likely to be strongly labor saving:
  - CES model with the decreasing returns to scale and technology loosely "labor saving".
  - Models in which "machines" replace workers (Zeira, 1998; Hellwig-Irmen; 2001).

Labor Scarcity vs. Wages

- What happens if there is "local nonconvexity": then, the relationship between labor scarcity and wages reversed.
- Wage push can increase wages, labor supply, and technology.



Basic Framework

- Consider a static economy consisting of a unique final good and $N+1$ factors of production, $Z = (Z_1, ..., Z_N)$ and labor L.
- All agents' preferences are defined over the consumption of the final good.
- Suppose, for now, that all factors are supplied inelastically, with supplies denoted by $\bar{Z} \in \mathbb{R}_+^N$ and $\bar{L} \in \mathbb{R}_+$.
- The economy consists of a continuum of firms (final good producers) denoted by the set $\mathcal{F}$, each with an identical production function.
- Without loss of any generality let us normalize the measure of $\mathcal{F}$, $|\mathcal{F}|$, to 1.
- The price of the final good is also normalized to 1.

Four Different Economies

1. Economy D (for *decentralized*) is a decentralized competitive economy in which technologies are chosen by firms themselves.
2. Economy E (for *externalities*), where firms are competitive but there is a technological externality.
3. Economy M (for *monopoly*), where technologies are created and supplied by a profit-maximizing monopolist.
4. Economy O (for *oligopoly*), where technologies are created and supplied by a set of oligopolistically (or monopolistically) competitive firms.

- Our main focus on Economies M and O.

Economy D

- All markets are competitive and technology chosen by firms.
- Each firm $i \in \mathcal{F}$ has access to a production function

$$Y^i = G(L^i, Z^i, \theta^i), \tag{45}$$

- Here $L^i \in \mathcal{L} \subset \mathbb{R}_+$ and $Z^i \in \mathcal{Z} \subset \mathbb{R}_+^N$.
- Most importantly, $\theta^i \in \Theta \subset \mathbb{R}^K$ is the measure of technology.
- Suppose that G is twice continuously differentiable in (L^i, Z^i, θ^i)—to be relaxed later.
- Thus factor prices are well defined and denote them by w_L and w_{Z_j} (vector w_Z).
- The cost of technology $\theta \in \Theta$ in terms of final goods is $C(\theta)$, convex and twice differentiable
  - but $C(\theta)$ could be increasing or decreasing.

Economy D (continued)

- Each final good producer maximizes profits, or in other words, solves:

$$\max_{L^i \in \mathcal{L}, Z^i \in \mathcal{Z}, \theta_i \in \Theta} \pi(L^i, Z^i, \theta^i) = G(L^i, Z^i, \theta^i) - w_L L^i - \sum_{j=1}^{N} w_{Zj} Z_j^i - C(\theta^i). \tag{46}$$

- Factor prices taken as given by the firm.
- Market clearing:

$$\int_{i \in \mathcal{F}} L^i di \leq \bar{L} \text{ and } \int_{i \in \mathcal{F}} Z_j^i di \leq Z_j \text{ for } j = 1, ..., N. \tag{47}$$

Definition

An equilibrium in Economy D is a set of decisions $\left\{L^i, Z^i, \theta^i\right\}_{i \in \mathcal{F}}$ *and factor prices* (w_L, w_Z) *such that* $\left\{L^i, Z^i, \theta^i\right\}_{i \in \mathcal{F}}$ *solve (46) given prices* (w_L, w_Z) *and (47) holds.*

Economy D (continued)

- Let us refer to any θ^i that is part of the set of equilibrium allocations, $\left\{L^i, Z^i, \theta^i\right\}_{i\in\mathcal{F}}$, as *equilibrium technology*.
- Also for future use, let us define the "net production function":

$$F(L^i, Z^i, \theta^i) \equiv G(L^i, Z^i, \theta^i) - C(\theta^i). \tag{48}$$

- For the competitive equilibrium to be well-defined, we introduce:

Assumption

Either $F(L^i, Z^i, \theta^i)$ is jointly strictly concave in (L^i, Z^i, θ^i) and increasing in (L^i, Z^i), and $\mathcal{L}$, $\mathcal{Z}$ and Θ are convex; or $F(L^i, Z^i, \theta^i)$ is increasing in (L^i, Z^i) and exhibits constant returns to scale in (L^i, Z^i, θ^i), and we have $(\bar{L}, \bar{Z}) \in \mathcal{L} \times \mathcal{Z}$.

Economy D (continued)

Proposition

Suppose Assumption 1 holds. Then any equilibrium technology θ in Economy D is a solution to

$$\max_{\theta' \in \Theta} F(\bar{L}, \bar{Z}, \theta'), \tag{49}$$

and any solution to this problem is an equilibrium technology.

- Therefore, to analyze equilibrium technology choices, we can simply focus on a simple maximization problem.
- Moreover, the equilibrium is a Pareto optimum (and vice versa).
- Equilibria factor prices given by marginal products, in particular:

$$w_L = \frac{\partial G(\bar{L}, \bar{Z}, \theta)}{\partial L} = \frac{\partial F(\bar{L}, \bar{Z}, \theta)}{\partial L}.$$

Economy E

- We can also consider a variant on Economy D, where

$$Y^i = G(L^i, Z^i, \theta^i, \bar{\theta}), \tag{50}$$

- Here $\bar{\theta}$ is some aggregate of the technology choices of all other firms in the economy.
- For simplicity, we can take $\bar{\theta}$ to be the sum of all firms' technologies.
- In particular, if θ is a K-dimensional vector, then $\bar{\theta}_k = \int_{i \in \mathcal{F}} \theta^i_k di$ for each component of the vector (i.e., for $k = 1, 2, ..., K$).
- Results here will be very similar to Economy O below
  - important differences from Economy D to be explained below.

Economy M

- Let us next consider a more usual environment for models of technological progress (similar to, but more general than Romer, 1990, Aghion-Howitt, 1992, Grossman-Helpman, 1991).
- The final good sector is competitive with production function

$$Y^i = \alpha^{-\alpha} (1-\alpha)^{-1} G(L^i, Z^i, \theta^i)^{\alpha} q(\theta^i)^{1-\alpha}. \tag{51}$$

- Now $G(L^i, Z^i, \theta^i)$ is a subcomponent of the production function, which depends on θ^i, the technology used by the firm.
- Assumption 2 now applies to this subcomponent.
- The subcomponent G needs to be combined with an intermediate good embodying technology θ^i, denoted by $q(\theta^i)$—conditioned on θ^i to emphasize that it embodies technology θ^i.
- This intermediate good is supplied by the monopolist.
- The term $\alpha^{-\alpha} (1-\alpha)^{-1}$ for normalization.

Economy M (continued)

- The monopolist can create technology θ at cost $C(\theta)$ from the technology menu.
- Suppose that $C(\theta)$ is convex, but for now, it could be increasing or decreasing in θ;
  - There is as yet no sense that the higher θ corresponds to "better technology".
- Once θ is created, the technology monopolist can produce the intermediate good embodying technology θ at constant per unit cost normalized to $1-\alpha$ unit of the final good (this is also a convenient normalization).
- It can then set a (linear) price per unit of the intermediate good of type θ, denoted by χ.

Economy M (continued)

- Each final good producer takes the available technology, θ, and the price of the intermediate good embodying this technology, χ, as given and maximizes

$$\max_{\substack{L^i \in \mathcal{L}, Z^i \in \mathcal{Z}, \\ q(\theta) \geq 0}} \pi(L^i, Z^i, q(\theta) \mid \theta, \chi) = \frac{1}{(1-\alpha)\alpha^{-\alpha}} G(L^i, Z^i, \theta)^{\alpha} q(\theta)^{1-\alpha} - w_L L^i - \sum_{j=1}^{N} w_{Zj} Z_j^i - \chi q(\theta), \tag{52}$$

- This problem gives the following simple inverse demand for intermediates of type θ:

$$q^i(\theta, \chi, L^i, Z^i) = \alpha^{-1} G(L^i, Z^i, \theta) \chi^{-1/\alpha}. \tag{53}$$

Economy M (continued)

- The problem of the monopolist is then to maximize its profits:

$$\max_{\theta,\chi,[q^i(\theta,\chi,L^i,Z^i)]_{i\in\mathcal{F}}} \Pi = (\chi - (1-\alpha)) \int_{i\in\mathcal{F}} q^i\left(\theta,\chi,L^i,Z^i\right) di - C(\theta) \tag{54}$$

  subject to (53).

Definition

An equilibrium in Economy M is a set of firm decisions $\left\{L^i, Z^i, q^i\left(\theta,\chi,L^i,Z^i\right)\right\}_{i\in\mathcal{F}}$, *technology choice* θ, *and factor prices* (w_L,w_Z) *such that* $\left\{L^i, Z^i, q^i\left(\theta,\chi,L^i,Z^i\right)\right\}_{i\in\mathcal{F}}$ *solve (52) given* (w_L,w_Z) *and technology* θ, *(47) holds, and the technology choice and pricing decision for the monopolist,* (θ,χ), *maximize (54) subject to (53).*

- Equilibrium easy to characterize because (53) defines a constant elasticity demand curve.

Economy M (continued)

- Profit-maximizing price of the monopolist is given by the standard monopoly markup over marginal cost and is equal to $\chi = 1$.
- Consequently, $q^i(\theta) = q^i(\theta, \chi = 1, \bar{L}, \bar{Z}) = \alpha^{-1} G(\bar{L}, \bar{Z}, \theta)$ for all $i \in \mathcal{F}$.
- Substituting this into (54), the profits and the maximization problem of the monopolist can be expressed as

$$\max_{\theta \in \Theta} \Pi(\theta) = G(\bar{L}, \bar{Z}, \theta) - C(\theta). \tag{55}$$

- Assumption 1 is no longer necessary. Instead only concavity in (L, Z):

Assumption

Either $G(L^i, Z^i, \theta^i)$ is jointly strictly concave and increasing in (L^i, Z^i) and $\mathcal{L}$ and $\mathcal{Z}$ are convex; or $G(L^i, Z^i, \theta^i)$ is increasing and exhibits constant returns to scale in (L^i, Z^i), and we have $(\bar{L}, \bar{Z}) \in \mathcal{L} \times \mathcal{Z}$.

Economy M (continued)

Proposition

Suppose Assumption 2 holds. Then any equilibrium technology θ in Economy M is a solution to

$$\max_{\theta' \in \Theta} F(\bar{L}, \bar{Z}, \theta') \equiv G(\bar{L}, \bar{Z}, \theta') - C(\theta')$$

and any solution to this problem is an equilibrium technology.

Economy M (continued)

- Relative to Economies D and C, the presence of the monopoly markup implies greater distortions in this economy.
- But equilibrium technology is still a solution to a problem identical to that in Economy D or C, that of maximizing

$$F(\bar{L}, \bar{Z}, \theta) \equiv G(\bar{L}, \bar{Z}, \theta) - C(\theta).$$

- Aggregate (net) output in the economy can be computed as

$$Y(\bar{L}, \bar{Z}, \theta) \equiv \frac{2-\alpha}{1-\alpha} G(\bar{L}, \bar{Z}, \theta) - C(\theta).$$

- Note that if $C'(\theta) > 0$, then $\partial F(\bar{L}, \bar{Z}, \theta^*)/\partial\theta = 0$ implies $\partial Y(\bar{L}, \bar{Z}, \theta^*)/\partial\theta > 0$ (since $(2-\alpha)/(1-\alpha) > 1$).
- Factor prices slightly different, but no effect on comparative statics:

$$w_L = \frac{1}{1-\alpha}\frac{\partial G(\bar{L}, \bar{Z}, \theta)}{\partial L} = \frac{1}{1-\alpha}\frac{\partial F(\bar{L}, \bar{Z}, \theta)}{\partial L} = \frac{1}{2-\alpha}\frac{\partial Y(\bar{L}, \bar{Z}, \theta)}{\partial L}.$$

Economy O

- Similar results can also be obtained when a number of different firms supply complementary or competing technologies. In this case, some more structure needs to be imposed to ensure tractability.
- Let θ^i be the vector $\theta^i \equiv (\theta^i_s)$, and suppose that output is now given by

$$Y^i = \alpha^{-\alpha}(1-\alpha)^{-1} G(L^i, Z^i, \theta^i)^{\alpha} \sum_{s=1}^{S} q_s \left(\theta^i_s\right)^{1-\alpha}, \tag{56}$$

where $\theta^i_s \in \Theta_s \subset \mathbb{R}^{K_s}$ is a technology supplied by technology producer $s = 1, ..., S$, and $q_s\left(\theta^i_s\right)$ is an intermediate good (or machine) produced and sold by technology producer s, which embodies technology θ^i_s.

Economy O (continued)

- Factor markets are again competitive.
- The inverse demand functions for intermediates is

$$q_s^i\left(\theta,\chi_s,L^i,Z^i\right)=\alpha^{-1}G(L^i,Z^i,\theta)\chi_s^{-1/\alpha}, \tag{57}$$

where χ_s is the price charged for intermediate good $q_s\left(\theta_s^i\right)$ by technology producer $s=1,...,S$.

Definition

An equilibrium in Economy O is a set of firm decisions $\left\{L^i,Z^i,\left[q_s^i\left(\theta,\chi_s,L^i,Z^i\right)\right]_{s=1}^S\right\}_{i\in\mathcal{F}}$, *technology choices* $(\theta_1,...,\theta_S)$, *and factor prices* (w_L,w_Z) *such that* $\left\{L^i,Z^i,\left[q_s^i\left(\theta,\chi_s,L^i,Z^i\right)\right]_{s=1}^S\right\}_{i\in\mathcal{F}}$ *maximize firm profits given* (w_L,w_Z) *and the technology vector* $(\theta_1,...,\theta_S)$, *(47) holds, and the technology choice and pricing decision for technology producer* $s=1,...,S$, (θ_s,χ_s), *maximize its profits subject to (57).*

Economy O (continued)

Proposition

Suppose Assumption 2 holds. Then any equilibrium technology in Economy O is a vector $(\theta_1^*, ..., \theta_S^*)$ *such that* θ_s^* *is solution to*

$$\max_{\theta_s \in \Theta_s} G(\bar{L}, \bar{Z}, \theta_1^*, ..., \theta_s, ..., \theta_S^*) - C_s(\theta_s)$$

for each $s = 1, ..., S$*, and any such vector gives an equilibrium technology.*

- Difference: Nash equilibrium; equilibrium now solution to fixed point problem
  - but this is not important for the results here.

Equilibrium Bias: Definitions

- With this framework, now we can derive the basic results on equilibrium bias.
- Take any of Economies D, M or O.
- For simplicity, let us suppose that all of the functions introduced above are twice differentiable.

Definition

An increase in technology θ_j *for* $j = 1, ..., K$ *is* absolutely biased *towards factor* L *at* $(\bar{L}, \bar{Z},) \in \mathcal{L} \times \mathcal{Z}$ *if* $\partial w_L / \partial \theta_j \geq 0$.

- Note the definition at *current* factor proportions.

Equilibrium Bias: Definitions (continued)

Definition

Denote the equilibrium technology at factor supplies $(\bar{L}, \bar{Z}) \in \mathcal{L} \times \mathcal{Z}$ *by* $\theta^* (\bar{L}, \bar{Z})$ *and assume that* $\partial \theta_j^* / \partial L$ *exists at* $(\bar{L}, \bar{Z})$ *for all* $j = 1, ..., K$. *Then there is* weak absolute equilibrium bias *at* $(\bar{L}, \bar{Z})$ *if*

$$\sum_{j=1}^{K} \frac{\partial w_L}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial L} \geq 0. \tag{58}$$

- Note that what is important is "the sum of" all technological effects.

Main Result on Weak Bias

Theorem

(Weak Absolute Equilibrium Bias) *Let the equilibrium technology at factor supplies* $(\bar{L}, \bar{Z})$ *be* $\theta^* (\bar{L}, \bar{Z})$ *and assume that* $\theta^* (\bar{L}, \bar{Z})$ *is in the interior of* Θ *and that* $\partial \theta_j^* / \partial L$ *exists at* $(\bar{L}, \bar{Z})$ *for all* $j = 1, ..., K$. *Then, there is weak absolute equilibrium bias at all* $(\bar{L}, \bar{Z}) \in \mathcal{L} \times \mathcal{Z}$, *i.e.,*

$$\sum_{j=1}^{K} \frac{\partial w_L}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial L} \geq 0 \text{ for all } (\bar{L}, \bar{Z}) \in \mathcal{L} \times \mathcal{Z}, \tag{59}$$

with strict inequality if $\partial \theta_j^* / \partial L \neq 0$ *for some* $j = 1, ..., K$.

Why Is This True?

- The result is very intuitive.
- Consider the case where $\theta \in \Theta \subset \mathbb{R}$ (the general case is similar with more notation).
- In equilibrium we have $\partial F/\partial \theta = 0$ and $\partial^2 F/\partial \theta^2 \leq 0$.
- Then from the Implicit Function Theorem

$$\frac{\partial \theta^*}{\partial L} = -\frac{\partial^2 F/\partial \theta \partial L}{\partial^2 F/\partial \theta^2} = -\frac{\partial w_L/\partial \theta}{\partial^2 F/\partial \theta^2}, \tag{60}$$

- And therefore,

$$\frac{\partial w_L}{\partial \theta}\frac{\partial \theta^*}{\partial L} = -\frac{\left(\partial w_L/\partial \theta\right)^2}{\partial^2 F/\partial \theta^2} \geq 0. \tag{61}$$

- Moreover, if $\partial \theta^*/\partial L \neq 0$, then from (60), $\partial w_L/\partial \theta \neq 0$, so (61) holds with strict inequality.

Intuition

- Similarity to the LeChatelier principle
  - but in *general equilibrium*, which is important as we will see.
- More detailed intuition:
  - Suppose that L and θ are complements (i.e., $\partial^2 F/\partial\theta\partial L \geq 0$), then an increase in L increases the incentives to improve θ, but then this raises the marginal contribution of to L output and thus wages$\rightarrow$*biased change.*
  - Suppose that L and θ are substitutes (i.e., $\partial^2 F/\partial\theta\partial L < 0$), then an increase in L reduces the incentives to improve θ, but then this increases the marginal contribution of L to output and thus wages$\rightarrow$*biased change*

Equilibrium Bias: Further Results

- The main result above is "local" in the sense that it is true only for small changes.
- Interestingly, it may not be true for large changes, because technological change that is biased *towards* labor at certain factor proportions may be biased *against* labor at certain other factor proportions.
- We thus need to ensure that such "reversals" not happen.
- These will be "supermodularity" type conditions.

Equilibrium Bias: Further Results (continued)

- Let us define:

Definition

Let θ^* be the equilibrium technology choice in an economy with factor supplies $(\bar{L}, \bar{Z})$. Then there is global absolute equilibrium *bias if for any $\bar{L}', \bar{L} \in \mathcal{L}$, $\bar{L}' \geq \bar{L}$ implies that*

$$w_L\left(\tilde{L}, \bar{Z}, \theta^*\left(\bar{L}', \bar{Z}\right)\right) \geq w_L\left(\tilde{L}, \bar{Z}, \theta^*\left(\bar{L}, \bar{Z}\right)\right) \text{ for all } \tilde{L} \in \mathcal{L} \text{ and } \bar{Z} \in \mathcal{Z}.$$

- Note: two notions of "globality" in this definition:
  - Large changes
  - Statement about factor prices at all intermediate factor proportions.

Global Results

Theorem

(Global Equilibrium Bias) *Suppose that Θ is a lattice, let $\bar{\mathcal{L}}$ be the convex hull of $\mathcal{L}$, let $\theta^*(\bar{L}, \bar{Z})$ be the equilibrium technology at factor proportions $(\bar{L}, \bar{Z})$, and suppose that $F(Z, L, \theta)$ is continuously differentiable in Z, supermodular in θ on Θ for all $Z \in \bar{\mathcal{Z}}$ and $L \in \mathcal{L}$, and exhibits strictly increasing differences in (Z, θ) on $\bar{\mathcal{L}} \times \Theta$ for all $Z \in \mathcal{Z}$, then there is global absolute equilibrium bias, i.e., for any $\bar{L}', \bar{L} \in \mathcal{L}$, $\bar{L}' \geq \bar{L}$ implies*

$$\theta^*(\bar{L}', \bar{Z}) \geq \theta^*(\bar{L}, \bar{Z}) \text{ for all } \bar{Z} \in \mathcal{Z}, \text{ and}$$

$$w_L(\tilde{L}, \bar{Z}, \theta^*(\bar{L}', \bar{Z})) \geq w_L(\tilde{L}, \bar{Z}, \theta^*(\bar{L}, \bar{Z})) \text{ for all } \tilde{L} \in \mathcal{L} \text{ and } \bar{Z} \in \mathcal{Z}, \tag{62}$$

with strict inequality if $\theta^*(\bar{L}', \bar{Z}) \neq \theta^*(\bar{L}, \bar{Z})$.

- This result follows from Topkis's Monotonicity Theorem.

Labor Scarcity and Technology

- Let us now turn to the effect of labor scarcity on "technological advances".
- Results so far silent on this, since either an "increase" or a "decrease" in θ may correspond to technology advances.
- Let us focus on labor scarcity for simplicity, but the results apply to "wage push" provided that equilibrium labor demand downward is sloping (more on this below).

Definitions

- Suppose that $C(\theta)$ strictly increasing in θ everywhere, so that higher θ corresponds to *technological advances*.
- We write $\theta \geq \theta'$ when all components of θ are at least as large as those of θ'.

Assumption

(Supermodularity) *$G(L, Z, \theta)$ [$Y(L, Z, \theta)$] is supermodular in θ on Θ for all $L \in \mathcal{L}$ and $Z \in \mathcal{Z}$.*

Definitions (continued)

Definition

Technological progress is strongly labor saving *at* $\bar{\theta}$, $\bar{L}$ *and* $\bar{Z}$ *if there exist neighborhoods* $\mathcal{B}_\Theta$, $\mathcal{B}_L$ *and* $\mathcal{B}_Z$ *of* $\bar{\theta}$, $\bar{L}$ *and* $\bar{Z}$ *such that* $Y(L, Z, \theta)$ *exhibits decreasing differences in* (L, θ) *on* $\mathcal{B}_L \times \mathcal{B}_Z \times \mathcal{B}_\Theta$.
Technological progress is strongly labor complementary *at* $\bar{\theta}$, $\bar{L}$ *and* $\bar{Z}$ *if there exist neighborhoods* $\mathcal{B}_\Theta$, $\mathcal{B}_L$ *and* $\mathcal{B}_Z$ *of* $\bar{\theta}$, $\bar{L}$ *and* $\bar{Z}$ *such that* $Y(L, Z, \theta)$ *exhibits increasing differences in* (L, θ) *on* $\mathcal{B}_L \times \mathcal{B}_Z \times \mathcal{B}_\Theta$.

- Note that $Y(L, Z, \theta)$ is increasing in θ on $\mathcal{B}_L \times \mathcal{B}_Z \times \mathcal{B}_\Theta$ if θ is an equilibrium technology at $\bar{L}$ and $\bar{Z}$, since $C(\theta)$ is strictly increasing.

Main Result

Theorem

Suppose that Y is supermodular in θ. Then labor scarcity starting from $\bar{\theta}$, $\bar{L}$ and $\bar{Z}$ will induce technological advances if technology is strongly labor saving at $\bar{\theta}$, $\bar{L}$ and $\bar{Z}$.
Conversely, labor scarcity will discourage technological advances if technology is strongly labor complementary.

Why Is This True?

- In Economy M, the result from Topkis's Monotonicity Theorem.
- In Economy O, equilibrium technology results from the Nash equilibrium of a supermodular game.
  - Use comparative statics for supermodular games to obtain the result.
- Similar results can also be obtained for Economy E, with additional mild assumptions.
- Throughout, important ingredient is that in Economies M, O or E, the equilibrium condition $\partial F\left(\bar{L}, \bar{\bar{Z}}, \theta^*\right)/\partial\theta = 0$ implies

$$\partial Y\left(\bar{L}, \bar{\bar{Z}}, \theta^*\right)/\partial\theta > 0.$$

Interpretation

- But this result is not possible in Economy D.
  - Because by construction in this economy,

$$\partial F\left(\bar{L}, \bar{Z}, \theta^*\right) / \partial \theta = \partial Y\left(\bar{L}, \bar{Z}, \theta^*\right) / \partial \theta = 0,$$

    so there cannot be "local technology advances" starting in equilibrium.
- Note also that even when technologies strongly labor saving, this does not imply that an exogenous increase in wages will lead to a Pareto improvement.
  - But it is also possible to construct examples, in Economies M, and O, where this is the case.

Implications

- What does this theorem imply?
  1. Wage push and labor scarcity can easily induce technological advances
  2. But there is no guarantee that they will and the opposite is equally (or more) likely.
- We need to understand what the condition "strongly labor saving" entails.

Cobb-Douglas Production Functions

- Suppose
$$G(L, Z, \theta) = H(Z)(\theta L)^{\beta}.$$
- Then, there are always increasing differences between θ and L and wage push will always discourage technological advances.
- **But**, if technology affects "substitution," then it can be strongly labor saving.
- An example is
$$G(L, Z, \theta) = A(\theta) L^{1-\theta},$$
with $A(\theta)$ increasing.
  - Then $A'(\theta^*)\bar{L}^{1-\theta^*} - A(\theta^*)\bar{L}^{1-\theta^*}\ln\bar{L} = C'(\theta^*)$ and thus
$$G_{L\theta} = -A(\theta^*)\bar{L}^{1-\theta^*} + (1-\theta^*)C'(\theta^*) < 0$$
provided that θ^*close to 1 or $C'(\theta^*)$ small.

CES and Factor Augmenting Technology

- Consider CES (type 1) with Z one-dimensional.
- Technology θ is Z-augmenting:

$$G = \left[(1-\alpha) (\theta Z)^{\frac{\sigma-1}{\sigma}} + \alpha L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\gamma\sigma}{\sigma-1}}.$$

- Straightforward differentiation gives

$$\begin{aligned} G_L &= \gamma\alpha L^{-\frac{1}{\sigma}} \left[(1-\alpha) (\theta Z)^{\frac{\sigma-1}{\sigma}} + \alpha L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\gamma\sigma}{\sigma-1}-1} \\ G_{L\theta} &= \frac{\gamma\sigma+1-\sigma}{\sigma} \gamma\alpha (1-\alpha) Z^{\frac{\sigma-1}{\sigma}} (\theta L)^{-\frac{1}{\sigma}} \\ &\quad \times \left[(1-\alpha) (\theta Z)^{\frac{\sigma-1}{\sigma}} + \alpha L^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\gamma\sigma}{\sigma-1}-2} \end{aligned}$$

- Thus $G_{L\theta} < 0$ if $(\gamma-1)\sigma + 1 < 0$.

CES and Factor Augmenting Technology (continued)

- In this case, there are increasing differences between θ and L if either of the following two conditions are satisfied:

1. $\gamma = 1$ (constant returns to scale)
2. $\sigma \leq 1$ (gross complements)

- Therefore, in this case we would need $\gamma < 1$ (decreasing returns) and $\sigma > 1$ (capital and labor substitutable).
- In fact, both inequalities need to be "sufficiently slack", so that

$$1 - \gamma > \frac{1}{\sigma}.$$

- This result generalizes to any G with Z-augmenting θ and homothetic in Z and L.
- Intuitively, since θ is Z-augmenting, we need $\sigma > 1$ so that it is "labor saving".

CES and Factor Augmenting Technology (continued)

- Consider CES (type 2) again with Z one-dimensional.
- Now technology L-augmenting:

$$G = \left[(1-\eta) Z^{\frac{\sigma-1}{\sigma}} + \eta (\theta L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\gamma\sigma}{\sigma-1}}.$$

- Straightforward differentiation now gives

$$\begin{aligned} G_L &= \gamma\eta\theta^{\frac{\sigma-1}{\sigma}} L^{-\frac{1}{\sigma}} \left[(1-\eta) Z^{\frac{\sigma-1}{\sigma}} + \eta (\theta L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\gamma\sigma}{\sigma-1}-1} \\ G_{L\theta} &= \left\{ \gamma\eta (\theta L)^{\frac{\sigma-1}{\sigma}} + \frac{\sigma-1}{\sigma} (1-\eta) Z^{\frac{\sigma-1}{\sigma}} \right\} \\ &\quad \times \gamma\eta (\theta L)^{-\frac{1}{\sigma}} \left[(1-\eta) Z^{\frac{\sigma-1}{\sigma}} + \eta (\theta L)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\gamma\sigma}{\sigma-1}-2} \end{aligned}$$

CES and Factor Augmenting Technology (continued)

- Now defining the relative labor share as

$$s_L \equiv \frac{w_L L}{w_Z Z} = \frac{\eta \left(\theta L\right)^{\frac{\sigma-1}{\sigma}}}{\left(1-\eta\right) Z^{\frac{\sigma-1}{\sigma}}},$$

- The condition that $G_{L\theta} < 0$ is equivalent to

$$s_L < \frac{1-\sigma}{\sigma\gamma}.$$

- The smaller is γ—meaning the further away from constant returns we are—the more likely is this to be true.
- But, in this case, technology cannot be strongly labor saving if $\sigma \geq 1$.
  - Only possible when $\sigma < 1$.
  - This is the opposite of the results for CES (type 1).
- Intuitively, since θ is now L-augmenting, we need $\sigma < 1$ so that it is "labor saving".

Machines Replacing Labor

- Consider the following generalization of Zeira (1998).
- All markets are competitive.
- Output is produced as

$$Y = \left[\int_0^1 y(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $y(\nu)$ is product of type ν produced as

$$y(\nu) = \begin{cases} \frac{k(\nu)}{\eta(\nu)} & \text{if } \nu \text{ uses new technology} \\ \frac{l(\nu)}{\beta(\nu)} & \text{if } \nu \text{ uses old technology,} \end{cases}$$

- Suppose that $\eta(\nu)$ is a continuous strictly increasing function, $\beta(\nu)$ is a continuous and decreasing function, and $k(\nu)$ is capital used in the production of intermediate ν.

Machines Replacing Labor (continued)

- Firms are competitive and can choose which product to produce with the new technology and which one with the old technology.
- Total labor supply is L and total supply of capital is K.
- Let the price of the final good be normalized to 1 and that of each intermediate good be $p(\nu)$.
- We write $n(\nu) = 1$ if ν is using the new technology. Clearly, $n(\nu) = 1$ whenever

$$R\eta(\nu) < w\beta(\nu),$$

where w is the wage rate and R is the endogenously determined rate of return on capital.

- Given the structure of the problem, it is clear that

$$n(\nu) = 1 \text{ for all } \nu \leq \theta$$

for some θ.

Machines Replacing Labor (continued)

- Therefore:

$$p(\nu) = \begin{cases} \eta(\nu) R & \text{if } \nu \leq \theta \\ \beta(\nu) w & \text{if } \nu > \theta \end{cases}$$

- Profit maximization of final good producers is

$$\max_{[y(\nu)]_{\nu \in [0,1]}} \left[\int_0^1 y(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right]^{\frac{\varepsilon-1}{\varepsilon}} - \int_0^\theta \eta(\nu) R y(\nu) d\nu - \int_\theta^1 \beta(\nu) w y(\nu) d\nu$$

so

$$y(\nu) = \begin{cases} (\eta(\nu) R)^{-\varepsilon} Y & \text{if } \nu \leq \theta \\ (\beta(\nu) w)^{-\varepsilon} Y & \text{if } \nu > \theta \end{cases}$$

Machines Replacing Labor (continued)

- Market clearing for capital and labor implies

$$\begin{aligned}\int_0^{\theta} k(\nu)\, d\nu &= \int_0^{\theta} \eta(\nu)\, y(\nu)\, d\nu \\ &= \int_0^{\theta} \eta(\nu)^{1-\varepsilon} R^{-\varepsilon} Y d\nu = K\end{aligned}$$

and similarly,

$$\int_{\theta}^{1} \beta(\nu)^{1-\varepsilon} w^{-\varepsilon} Y d\nu = L.$$

Machines Replacing Labor (continued)

- Let us define

$$A(\theta) \equiv \int_0^{\theta} \eta(\nu)^{1-\varepsilon} d\nu \text{ and } B(\theta) \equiv \int_{\theta}^{1} \beta(\nu)^{1-\varepsilon} d\nu$$

- Then we have

$$R^{1-\varepsilon} = \left(\frac{Y}{K} A(\theta)\right)^{\frac{1-\varepsilon}{\varepsilon}} \text{ and } w^{1-\varepsilon} = \left(\frac{Y}{L} B(\theta)\right)^{\frac{1-\varepsilon}{\varepsilon}}$$

- Then total output can be written as

$$\begin{aligned} Y &= \left[\int_0^{\theta} \left((\eta(\nu) R)^{-\varepsilon} Y\right)^{\frac{\varepsilon-1}{\varepsilon}} d\nu + \int_{\theta}^{1} \left(\beta(\nu) w^{-\varepsilon} Y\right)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right]^{\frac{\varepsilon}{\varepsilon-1}} \\ &= \left[A(\theta)^{\frac{1}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} + B(\theta)^{\frac{1}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned}$$

Machines Replacing Labor (continued)

- Now in equilibrium, we have

$$\begin{aligned}\frac{\partial Y}{\partial \theta} &= \frac{1}{\varepsilon - 1}\left[\eta\left(\theta^*\right)^{1-\varepsilon} A\left(\theta^*\right)^{\frac{1-\varepsilon}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} - \beta\left(\theta^*\right)^{1-\varepsilon} B\left(\theta^*\right)^{\frac{1-\varepsilon}{\varepsilon}} L^{\frac{\varepsilon-1}{\varepsilon}}\right] Y^{\frac{1}{\varepsilon}} \\ &= 0,\end{aligned}$$

  which implies that the term square brackets must be equal to zero.

- Therefore

$$\frac{\partial^2 Y}{\partial \theta \partial L} = -\frac{1}{\varepsilon}\beta\left(\theta^*\right)^{1-\varepsilon} B\left(\theta^*\right)^{\frac{1-\varepsilon}{\varepsilon}} L^{-\frac{1}{\varepsilon}} Y^{\frac{1}{\varepsilon}} < 0.$$

- Thus a decrease in $\bar{L}$ will increase θ.
- But in this case an increase in θ is **not** a technological advance.

Machines Replacing Labor (continued)

- Now consider a related monopolistic economy, where

$$G\left(L,Z,\theta\right)=\left[A\left(\theta\right)^{\frac{1}{\varepsilon}}K^{\frac{\varepsilon-1}{\varepsilon}}+B\left(\theta\right)^{\frac{1}{\varepsilon}}L^{\frac{\varepsilon-1}{\varepsilon}}\right],$$

  with cost $C\left(\theta\right)$ and $\varepsilon>1$.
- Then an increase in θ is a technological advance.
- Moreover,

$$G_{L\theta}=-\frac{\varepsilon-1}{\varepsilon^{2}}\beta\left(\theta^{*}\right)^{1-\varepsilon}B\left(\theta^{*}\right)^{\frac{1-\varepsilon}{\varepsilon}}L^{-\frac{1}{\varepsilon}}<0,$$

  so that technology is strongly labor saving and a decrease in $\bar{L}$ will induce technological advances.

Machines Replacing Labor (continued)

- However, this is not a general result even when machines replace labor.
- If

$$G(L,Z,\theta)=\left[A(\theta)^{\frac{1}{\varepsilon}}K^{\frac{\varepsilon-1}{\varepsilon}}+B(\theta)^{\frac{1}{\varepsilon}}L^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

  then, answer depends on ε, $\eta(\theta)$, $\beta(\theta)$ and $C(\theta)$.
- In particular, in this case, we obtain

$$\begin{aligned} G_{L\theta} \quad \propto \quad & -\frac{1}{\varepsilon}\beta(\theta^*)^{1-\varepsilon}B(\theta^*)^{\frac{1-\varepsilon}{\varepsilon}}L^{\frac{\varepsilon-1}{\varepsilon}}G(L,Z,\theta)^{\frac{1}{\varepsilon}} \\ & +(2-\alpha)C'(\theta^*)S_L, \end{aligned}$$

  with S_L as the labor share of income.
- Therefore, technology will be strongly labor saving if labor share S_L or $C'(\theta^*)$ is small.

A Simple Example

- The following is a simpler example along the same lines.

$$G(L, Z, \theta) = 3\theta Z^{1/3} + 3(1-\theta) L^{1/3}.$$

- $C(\theta) = 3\theta^2/2$.
- Normalize $Z = 1$.
- Equilibrium technology

$$\theta^*(L) = 1 - L^{1/3}.$$

- Equilibrium wage

$$w(L, \theta) = (1-\theta) L^{-2/3}.$$

- Labor scarcity and wage work in the same direction, since

$$w(L, \theta^*(L)) = L^{-1/3}.$$

Scarcity vs. Wage Push

- So far we have equated labor scarcity and wage push.
- But because bias of technology is endogenous, this need not be the case.
- This has important implications both
  - in its own right—general equilibrium demand curves very different from those implied by producer theory
  - for the relationship between wage push and technological advances.

Definitions

Definition

Denote the equilibrium technology at factor supplies $(\bar{L}, \bar{Z}) \in \mathcal{L} \times \mathcal{Z}$ *by* $\theta^* (\bar{L}, \bar{Z})$ *and suppose that* $\partial \theta_j^* / \partial Z$ *exists at* $(\bar{L}, \bar{Z})$ *for all* $j = 1, ..., K$. *Then there is* strong absolute equilibrium bias *at* $(\bar{L}, \bar{Z}) \in \mathcal{L} \times \mathcal{Z}$ *if*

$$\frac{dw_L}{dL} = \frac{\partial w_L}{\partial L} + \sum_{j=1}^{K} \frac{\partial w_L}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial L} > 0.$$

- In this definition, dw_L / dL denotes the total derivative, while $\partial w_L / \partial L$ denotes the partial derivative holding $\theta = \theta^* (\bar{L}, \bar{Z})$.
- Recall also that if F is jointly concave in (L, θ) at $(L, \theta^* (\bar{L}, \bar{Z}))$, its Hessian with respect to (L, θ), $\nabla^2 F_{(L,\theta)(L,\theta)}$, is negative semi-definite at this point (though negative semi-definiteness is not sufficient for local joint concavity).

Main Result

Theorem

(Nonconvexity and Strong Bias) *Suppose that Θ is a convex subset of $\mathbb{R}^K$, F is twice continuously differentiable in (L,θ), let $\theta^*(\bar{L},\bar{Z})$ be the equilibrium technology at factor supplies $(\bar{L},\bar{Z})$ and assume that θ^* is in the interior of Θ and that $\partial\theta_j^*(\bar{L},\bar{Z})/\partial L$ exists at $(\bar{L},\bar{Z})$ for all $j=1,...,K$. Then there is strong absolute equilibrium bias at $(\bar{L},\bar{Z})$ if and only if $F(L,Z,\theta)$'s Hessian in (L,θ), $\nabla^2 F_{(L,\theta)(L,\theta)}$, is not negative semi-definite at $(\bar{L},\bar{Z})$.*

Corollary: There cannot be strong bias in a fully competitive economy such as Economy D.

- This is because competitive equilibrium exists only when the production possibilities set is locally convex.

Why Is This True?

- Again, for simplicity, take the case where $\Theta \subset \mathbb{R}$.
- The fact that θ^* is the equilibrium technology implies that $\partial F/\partial\theta = 0$ and that $\partial^2 F/\partial\theta^2 \leq 0$.
- Moreover, we still have

$$\frac{\partial\theta^*}{\partial L} = -\frac{\partial^2 F/\partial\theta\partial L}{\partial^2 F/\partial\theta^2} = -\frac{\partial w_L/\partial\theta}{\partial^2 F/\partial\theta^2}.$$

- Substituting this into the definition for dw_L/dL and recalling that $\partial w_L/\partial L = \partial^2 F/\partial L^2$, we have the condition for strong absolute equilibrium bias as

$$\begin{aligned}\frac{dw_L}{dL} &= \frac{\partial w_L}{\partial L} + \frac{\partial w_L}{\partial\theta}\frac{\partial\theta^*}{\partial L}, \\ &= \frac{\partial^2 F}{\partial L^2} - \frac{\left(\partial^2 F/\partial\theta\partial L\right)^2}{\partial^2 F/\partial\theta^2} > 0.\end{aligned}$$

Why Is This True?

- From Assumption 1 or 2, F is concave in Z, so $\partial^2 F/\partial L^2 \leq 0$, and from the fact that θ^* is an equilibrium and $\partial\theta^*/\partial L$ exists, we also have $\partial^2 F/\partial\theta^2 < 0$.
- Then the fact that F's Hessian, $\nabla^2 F_{(L,\theta)(L,\theta)}$, is not negative semi-definite at $(\bar{L}, \bar{Z})$ implies that

$$\frac{\partial^2 F}{\partial L^2} \times \frac{\partial^2 F}{\partial \theta^2} < \left(\frac{\partial^2 F}{\partial L \partial L \theta}\right)^2, \tag{63}$$

- Since at the optimal technology choice, $\partial^2 F/\partial\theta^2 < 0$, this immediately yields $dw_L/dZ > 0$, establishing strong absolute bias at $(\bar{L}, \bar{Z})$ as claimed in the theorem.
- Conversely, if $\nabla^2 F_{(L,\theta)(L,\theta)}$ is negative semi-definite at $(\bar{L}, \bar{Z})$, then (63) does not hold and this together with $\partial^2 F/\partial\theta^2 < 0$ implies that $dw_L/dL \leq 0$.

Intuition

- Induced bias can be strong enough to overwhelm the standard "substitution effect" leading to downward sloping demand curves.
- Why is "local nonconvexity" sufficient?
- If there is local nonconvexity, then we are not at a global maximum but at a **saddle point.**
  - with technology and factor demands chosen by different firms/agents;
  - note that this is all that equilibrium requires.
- Then there exists a direction in which output can be increased locally.
- A change in L induces technology firms to move θ in that direction, and locally this increases the marginal contribution of L to all put (and thus wages).

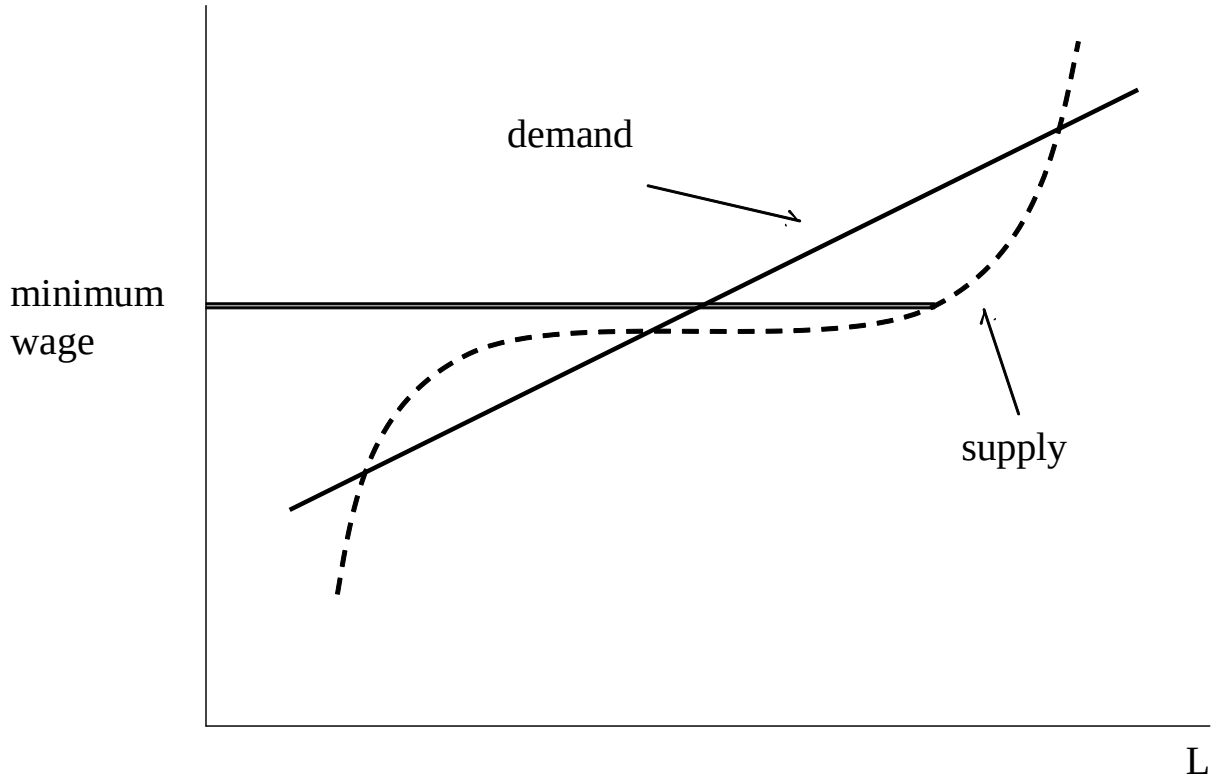
Implications I

- Labor scarcity and wage push can have very different effects.
- Suppose the local nonconvexity condition is satisfied and also that F exhibits decreasing differences in (L, θ).
- Then labor scarcity will induce technological advances but *reduce* wages.
- In contrast, wage push will lead to technological regress.

Implications II

- Suppose that labor is supplied elastically and assume that it takes the form $L(w)$.
- Then multiple equilibria are possible.
- The higher technology equilibrium also has higher wages.
- Exogenous wage push, for example, minimum wages, can eliminate the "worse" equilibrium.

Implications II (continued)



An Example

- The following is a modification of the example provided above:

$$G(L, Z, \theta) = \frac{3}{2}\theta Z^{2/3} + \frac{3}{2}(1-\theta) L^{2/3}.$$

- $C(\theta) = -3\theta^2/4$.
- Again normalize $Z = 1$.
- Equilibrium technology

$$\theta^*(L) = 1 - L^{2/3}.$$

- Equilibrium wage

$$w(L, \theta) = (1-\theta) L^{-1/3}.$$

- Thus both strongly labor saving technologies and strong absolute bias:

$$w(L, \theta^*(L)) = L^{1/3}.$$

An Example (continued)

- Now suppose that labor supply is endogenous.
- In particular:
$$L^S(w) = 6w^2 - 11w + 6.$$
- Combining this with $w(L, \theta^*(L))$ shows that there are three equilibrium wages, with different levels of labor supply and technology, $w = 1$, 2 and 3.
- Technology is most advanced and labor supply is highest at $w = 3$.
- A minimum wage between 2 and 3 may destroy the other equilibria.
- However, caution: it may also introduce a no-activity equilibrium depending on whether technology and factor demands are chosen by the same firms or by different firms.

Wage Growth

- Are strongly labor saving technologies inconsistent with technology leading to secular wage increases?
- The answer is no.
- Consider a dynamic environment similar to Economy E.
- Suppose that all firms are fully competitive and the production function of each at time t is firm

$$y_t^i\left(L_t^i, Z_t^i, \theta_t^i\right) = \bar{A}_t\left[\left(\theta_t^i\right)^{1+\gamma}\left(Z_t^i\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(1-\theta_t^i\right)^{1+\gamma}\left(L_t^i\right)^{\frac{\varepsilon-1}{\varepsilon}}\right], \tag{64}$$

  where $\gamma < 0$
- Also assume that $\bar{Z} < 2^{\gamma\varepsilon/(\varepsilon-1)}\bar{L}$.
- Higher θ corresponds to substituting factor Z, which may be capital or other human or nonhuman factors, for tasks performed by labor.

Wage Growth (continued)

- An equilibrium technology $\theta^*\left(\bar{L},\bar{Z}\right)$ will be independent of $\bar{A}_t$ and will satisfy

$$\frac{\partial Y_t^i\left(\bar{L},\bar{Z},\theta^*\left(\bar{L},\bar{Z}\right)\right)}{\partial\theta}=0,$$

- This gives

$$\theta^*\left(\bar{L},\bar{Z}\right)=\frac{1}{1+\left(\bar{Z}/\bar{L}\right)^{\frac{\varepsilon-1}{\gamma\varepsilon}}}.$$

- *Strongly labor saving technology:* $\theta^*\left(\bar{L},\bar{Z}\right)$ is decreasing in $\bar{L}$, so labor scarcity increases $\theta^*\left(\bar{L},\bar{Z}\right)$.

Wage Growth (continued)

- Next, suppose that the intertemporal externalities take the form

$$\bar{A}_t = \left(1 + g\left(\bar{\theta}_{t-1}\right)\right) \bar{A}_{t-1},$$

where g is an increasing function and

$$\bar{\theta}_t \equiv \int_{i \in \mathcal{F}} \theta_t^i di$$

is the average technology choice of firms at time t.

- Similar to that in Romer (1986).
- Higher equilibrium level of $\theta^*\left(\bar{L}, \bar{Z}\right)$ will lead to faster growth of output and wages, even though at the margin, labor scarcity increases $\theta^*\left(\bar{L}, \bar{Z}\right)$ and substitutes for tasks previously performed by labor.

Conclusion

- Labor scarcity and wage push can have major implications for technological progress
- Discussed in the economic history literature and other contexts.
- Different explanations and hypotheses boiled down to whether there are "decreasing differences" or "increasing differences" between labor and technology.
- Functional forms matter, so that theory is useful in clarifying the main countervailing forces.
    - Empirical evidence on the impact of labor scarcity and wage push on technology adoption necessary.

Empirical Evidence?

- Acemoglu and Finkelstein (2008):
  - Move from retrospective Medicare reimbursements to prospective payment system
  - Increase in labor costs, particularly for hospitals with a high share of Medicare patients.
  - Impact: an increase in technology adoption, again particularly in hospitals with high share of Medicare patients.
  - Various possible channels, but potentially related to the impact of changes in factor prices on technology adoption.
- Lewis (2007):
  - Impact of skill mix choice of technology across US metropolitan areas.

Environmental Issues

- Growing concern that economic growth is not "sustainable" because of
  - Negative impact on the environment (pollution, global warming)
  - Depletion of natural resources (in particular, oil).
- Discussion among climate scientists focusing on effects of environmental change on health, conflict,... and on effect of environmental regulation.
- Economic analyses using computable general equilibrium models with exogenous technology (and climatological constraints; e.g., Nordhaus, 1994, 2002).
- Key issues for economic analyses: (1) economic costs and benefits of environmental policy; (2) costs of delaying intervention (3) role of discounting and risk aversion

Context

- Most of previous literature (with exogenous technological change) involves three different type of answers:

1. **Nordhaus approach:** intervention should be limited and gradual; small long-run growth costs.
2. **Stern/Al Gore approach:** intervention needs to be large, immediate and maintained permanently; large long-run growth costs.
3. **Greenpeace approach:** only way to avoid disaster is zero growth.

- Our paper: yet another approach.

Importance of Technology

- All these approaches essentially ignore the essence of technological responses.
- Recalled be evidence that technological change and technology adoption respond to profit incentives
  - **Newell, Jaffe and Stavins (1999)**: energy prices on direction of technological change in air conditioning
  - **Popp (2002)**: relates energy prices and energy saving innovation

This paper

- Once directed technical change is factored in, a very different answer.
  1. Immediate and decisive intervention is necessary (in contrast to Nordhaus)
  2. Temporary intervention may be sufficient (in contrast to Stern/Al Gore)
  3. Long-run growth costs very limited or zero (in contrast to all of them).
  4. Two instruments—not one—necessary for optimal environmental regulation.
- Therefore, more optimistic than all, and as proactive as any.

Why?

- Two sector model with "clean" and "dirty" inputs with two key externalities
- *Environmental externality*: production of dirty inputs creates environmental degradation.
- Researchers work to improve the technology depending on expected profits and "build on the shoulders of giants" in their sector.
  - → *Knowledge externality*: advances in dirty (clean) inputs make their future use more profitable.
- Policy interventions can redirect technical change towards clean technologies.
- **New important parameter:** elasticity of substitution between clean and dirty inputs
  → e.g., whether clean energy replaces fossil fuel (high elasticity) or whether producing components for clean cars entails CO2 emissions (low elasticity)

Why? (Continued)

1. Immediate and decisive intervention is necessary (in contrast to Nordhaus)
   - → without intervention, innovation is directed towards dirty sectors; thus gap between clean and dirty technology widens; thus cost of intervention (reduced growth when clean technologies catch up with dirty ones) increases
2. Temporary intervention may be sufficient (in contrast to Stern/Al Gore), long-run growth costs limited (in contrast to all of them)
   - → once government intervention has induced a technological lead in clean technologies, firms will spontaneously innovate in clean technologies (if clean and dirty inputs are sufficiently substitutes).
3. Two instruments, not one:
   - → optimal policy involves both a carbon tax and a subsidy to clean research to redirect innovation to green technologies
   - → too costly in terms of foregone short-run consumption to use carbon tax alone

Factoring in Exhaustible Resource

- Baseline model without exhaustible resources.
- Oil will likely become more expensive over the next 20 years.
- Directed technical change implies that this will also redirect innovations towards clean technologies.
- Environmental disaster may be avoided without government intervention
  - The market may reallocate research away from dirty technologies by itself
  - ... but it need not be the case
- Interestingly, structure of optimal environmental regulation fairly similar with exhaustible resources.

Model (1): production

- Infinite horizon in discrete time (suppress time dependence for now)
- Final good Y produced competitively with a clean intermediary input Y_c, and a dirty input Y_d

$$Y = \left(Y_c^{\frac{\varepsilon-1}{\varepsilon}} + Y_d^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

- Most of the analysis: $\varepsilon > 1$, the two inputs are substitute.
- For $j \in \{c, d\}$, input Y_j produced with labor L_j and a continuum of machines x_{ji}:

$$Y_j = L_j^{1-\alpha} \int_0^1 A_{ji}^{1-\alpha} x_{ji}^{\alpha} di$$

- Machines produced **monopolistically** using the final good

Model (2): consumption

- Constant mass 1 of infinitely lived representative consumers with intertemporal utility:

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t)$$

where u increasing and concave, with

$$\lim_{S \to 0} u(C, S) = -\infty; \frac{\partial u}{\partial S}(C, \bar{S}) = 0$$

Model (3): environment

- Production of dirty input depletes environmental stock S:

$$S_{t+1} = -\zeta Y_{dt} + (1+\delta) S_t \quad \text{if} \quad S \in (0, \bar{S}) . \tag{65}$$

- Reflecting at the upper bound $\bar{S}$ ($< \infty$): baseline (unpolluted) level of environmental quality.
- Absorbing at the lower bound $S = 0$.
- $\delta > 0$: rate of "environmental regeneration" (measures amount of pollution that can be absorbed without extreme adverse consequences)
- S is general quality of environment, inversely related to CO2 concentration (what we do below for calibration).

Model (4): innovation

- At the beginning of every period scientists (of mass $s = 1$) work either to innovate in the clean or the dirty sector.
- Given sector choice, each randomly allocated to one machine in their target sector.
- Every scientist has a probability η_j of success (without congestion).
  - if successful, proportional improvement in quality by $\gamma > 0$ and the scientist gets monopoly rights for one period, thus

$$A_{jit} = (1 + \gamma) A_{jit-1};$$

  - if not successful, no improvement and monopoly rights in that machine randomly allocated to an entrepreneur who uses technology

$$A_{jit} = A_{jit-1}.$$

  - simplifying assumption, mimicking structure in continuous time models.

Model (5): innovation (continued)

- Therefore, law of motion of quality of input in sector $j \in \{c, d\}$ is:

$$A_{jt} = \left(1 + \gamma \eta_j s_{jt}\right) A_{jt-1}$$

- **Note:** knowledge externality; "building on the shoulders of giants," but importantly "giants in the same sector"
  - Intuition: Fuel technology improvements do not directly facilitate discovery of alternative energy sources

Assumption

A_{d0} sufficiently higher than A_{c0}.

- Capturing the fact that currently fossil-fuel technologies are more advanced than alternative energy/clean technologies.

Laissez-faire equilibrium: direction of innovation

- Scientists choose the sector with higher expected profits Π_{jt}:

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \underbrace{\left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{1}{1-\alpha}}}_{\text{price effect}} \underbrace{\frac{L_{ct}}{L_{dt}}}_{\text{market size effect}} \underbrace{\frac{A_{ct-1}}{A_{dt-1}}}_{\text{direct productivity effect}}$$

  - The direct productivity effect pushes towards innovation in the more advanced sector
  - The price effect towards the less advanced, price effect stronger when ε smaller
  - The market size effect towards the more advanced when $\varepsilon > 1$

Laissez-faire equilibrium (continued)

- Use equilibrium machine demands and prices in terms of technology levels (state variables) and let $\varphi \equiv (1-\alpha)(1-\varepsilon)$ (<0 if $\varepsilon > 1$):

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d}\left(\frac{1+\gamma\eta_c s_{ct}}{1+\gamma\eta_d s_{dt}}\right)^{-\varphi-1}\left(\frac{A_{ct-1}}{A_{dt-1}}\right)^{-\varphi}.$$

- Implications: innovation in relatively advanced sector if $\varepsilon > 1$

Laissez-faire equilibrium production levels

- Equilibrium input production levels

$$Y_d = \frac{1}{\left(A_c^{\varphi} + A_d^{\varphi}\right)^{\frac{\alpha+\varphi}{\varphi}}} A_c^{\alpha+\varphi} A_d;$$

$$Y = \frac{A_c A_d}{\left(A_c^{\varphi} + A_d^{\varphi}\right)^{\frac{1}{\varphi}}}$$

- Recall that $\varphi \equiv (1-\alpha)(1-\varepsilon)$.
- In particular, given the assumption that A_{d0} sufficiently higher than A_{c0}, Y_d will always grow without bound under laissez-faire
  - If $\varepsilon > 1$, then all scientists directed at dirty technologies, thus $g_{Y_d} \to \gamma \eta_d$

Environmental disaster

- An environmental "**disaster**" occurs if S_t reaches 0 in finite time.

Proposition

Disaster.

The laissez-faire equilibrium always leads to an environmental disaster.

Proposition

The role of policy.

1. *when the two inputs are strong substitutes ($\varepsilon > 1/(1-\alpha)$) and $\bar{S}$ is sufficiently high, a temporary clean research subsidy will prevent an environmental disaster;*
2. *in contrast, when the two inputs are weak substitutes ($\varepsilon < 1/(1-\alpha)$), a temporary clean research subsidy cannot prevent an environmental disaster.*

Sketch of proof

- Look at effect of a temporary clean research subsidy
- Key role: redirecting technical change; innovation can be redirected towards clean technology
- If $\varepsilon > 1$, then subsequent to an extended period of taxation, innovation will remain in clean technology
- Is this sufficient to prevent an environmental disaster?

Sketch of proof (continued)

- Even with innovation only in the clean sector, production of dirty inputs may increase
  - *on the one hand:* innovation in clean technology reduces labor allocated to dirty input $\Rightarrow Y_d \downarrow$
  - *on the other hand*: innovation in clean technology makes final good cheaper an input to production of dirty input $\Rightarrow Y_d \uparrow$
  - which of these two effects dominates, will depend upon ε.
- With clean research subsidy (because $\varepsilon > 1$ and thus $\varphi < 0$):

$$Y_d = \frac{1}{\left(A_c^{\varphi} + A_d^{\varphi}\right)^{\frac{\alpha+\varphi}{\varphi}}} A_c^{\alpha+\varphi} A_d \rightarrow A_c^{\alpha+\varphi}$$

- If $\alpha + \varphi > 0$ or $\varepsilon < 1/(1-\alpha)$, then second effect dominates, and long run growth rate of dirty input is positive equal to $(1+\gamma\eta_c)^{\alpha+\varphi} - 1$
- If $\alpha + \varphi < 0$ or $\varepsilon > 1/(1-\alpha)$, then first effect dominates, so that Y_d decreases over time.

Cost of intervention and delay

- Concentrate on strong substitutability case ($\varepsilon > 1/(1-\alpha)$)
- While A_{ct} catches up with A_{dt}, growth is reduced.
- T: number of periods necessary for the economy under the policy intervention to reach the same level of output as it would have done within one period without intervention
- If intervention delayed, not only the environment gets further degraded, but also technology gap A_{dt-1}/A_{ct-1} increases, growth is reduced for a longer period.

Complementary case

- Suppose instead that clean and dirty inputs are complements, i.e., $\varepsilon < 1$.
- Innovation is directed towards the more backward sector
  - price effect dominates the direct productivity effect and market size effect now favors innovation in the more backward sector
  - typically innovation first occurs in clean, then in both, asymptotically balanced between the two sectors.
- Asymptotic growth rate of dirty input: $g_{Y_d} \to \gamma\eta_c\eta_d / (\eta_c + \eta_d) < \gamma\eta_d$ (growth rate in substitute case): disaster occurs sooner than in the substitute case.
- ... but it is unavoidable using only a temporary clean research subsidy.
  - If the clean sector is the more advanced, innovation will take place in dirty once the subsidy is removed, and long-run growth rate of dirty input remains the same.

Undirected technical change

- Compare with a model where scientists randomly allocated across sectors so as to ensure equal growth in the qualities of clean and dirty machines, thus $g_{Y_d} \to \gamma\eta_c\eta_d/(\eta_c+\eta_d) < \gamma\eta_d$

Proposition

The role of directed technical change.
When $\varepsilon > 1/(1-\alpha)$*:*

1. *An environmental disaster under laissez-faire arises earlier with directed technical change than in the equivalent economy with undirected technical change.*
2. *However, a temporary clean research subsidy can prevent an environmental disaster with directed technical change, but not in the equivalent economy with undirected technical change.*

Optimal environmental regulation

Proposition

Optimal environmental regulation.
The social planner can implement the social optimum through a "carbon tax" on the use of the dirty input, a clean research subsidy and a subsidy for the use of all machines (all taxes/subsidies are financed lump sum).

1. *If $\varepsilon > 1$ and the discount rate ρ is sufficiently small, then in finite time innovation ends up occurring only in the clean sector, the economy grows at rate $\gamma\eta_c$ and the optimal subsidy on profits in the clean sector, q_t, is temporary.*
2. *The optimal carbon tax, τ_t, is temporary if $\varepsilon > 1/(1-\alpha)$ but not if $1 < \varepsilon < 1/(1-\alpha)$.*

- Parts 1 and 2 $\sim$ Us vs. Stern/Al Gore.

Carbon tax

- Optimal carbon tax schedule is given by

$$\tau_t = \frac{\omega_{t+1}\xi}{\lambda_t p_{dt}},$$

  - λ_t is the marginal utility of a unit of consumption at time t
  - ω_{t+1} is the shadow value of one unit of environmental quality at time $t+1$, equal to the discounted marginal utility of environmental quality as of period $t+1$

- If $\varepsilon > 1/(1-\alpha)$, dirty input production tends towards 0 and environmental quality S_t reaches $\overline{S}$ in finite time, carbon tax becomes null in finite time.
- If gap between the two technologies is high, relying on carbon tax to redirect technical change would reduce too much consumption.

Calibration: preferences and technology

- 1 period = 5 years
- $\alpha = 1/3$: share of capital
- $\eta_c = \eta_d = 0.1$, $\gamma = 1$: long-run growth 2% per year
- A_{c-1}, A_{d-1} to match Y_{c-1}, Y_{d-1} with 2002 - 2006 production of non fossil fuel, fossil fuel energy

Calibration: environmental quality

- Relate S with the atmospheric concentration of carbon:
  1. Relate atmospheric concentration of carbon dioxide (ppm), C_{CO_2} to increase in temperature since preindustrial times (°C), Δ. Common approximation:
  $$\Delta \approx 3 \log_2 \left(C_{CO_2} / 280 \right).$$
  2. Choose a "disaster temperature" $\Delta_{disaster} = 6.9°\text{C}$ which corresponds to twice the temperature increase that would lead to the melting of the Greenland icesheet.
  3. Relate S to Δ through previous equation and:
  $$S = 280 \times 2^{\Delta_{disaster}/3} - \max\left\{ C_{CO_2}, 280 \right\}.$$
  so that $S = 0 \Leftrightarrow \Delta = \Delta_{disaster} = 6.9°\text{C}$
- ξ from the observed value of Y_d and the annual emission of CO_2 in 2002-2006
- δ such that only half of the amount of emitted carbon contributes to increasing C_{CO2}

Calibration: utility

- Choose

$$u(C_t, S_t) = \frac{(\phi(S_t) C_t)^{1-\sigma}}{1-\sigma}.$$

With $\sigma = 2$. Same as previous literature.

$$\phi(S) = \frac{(\Delta_{disaster} - \Delta(S))^{\lambda} - \lambda \Delta_{disaster}^{\lambda-1} (\Delta_{disaster} - \Delta(S))}{(1-\lambda) \Delta_{disaster}^{\lambda}},$$

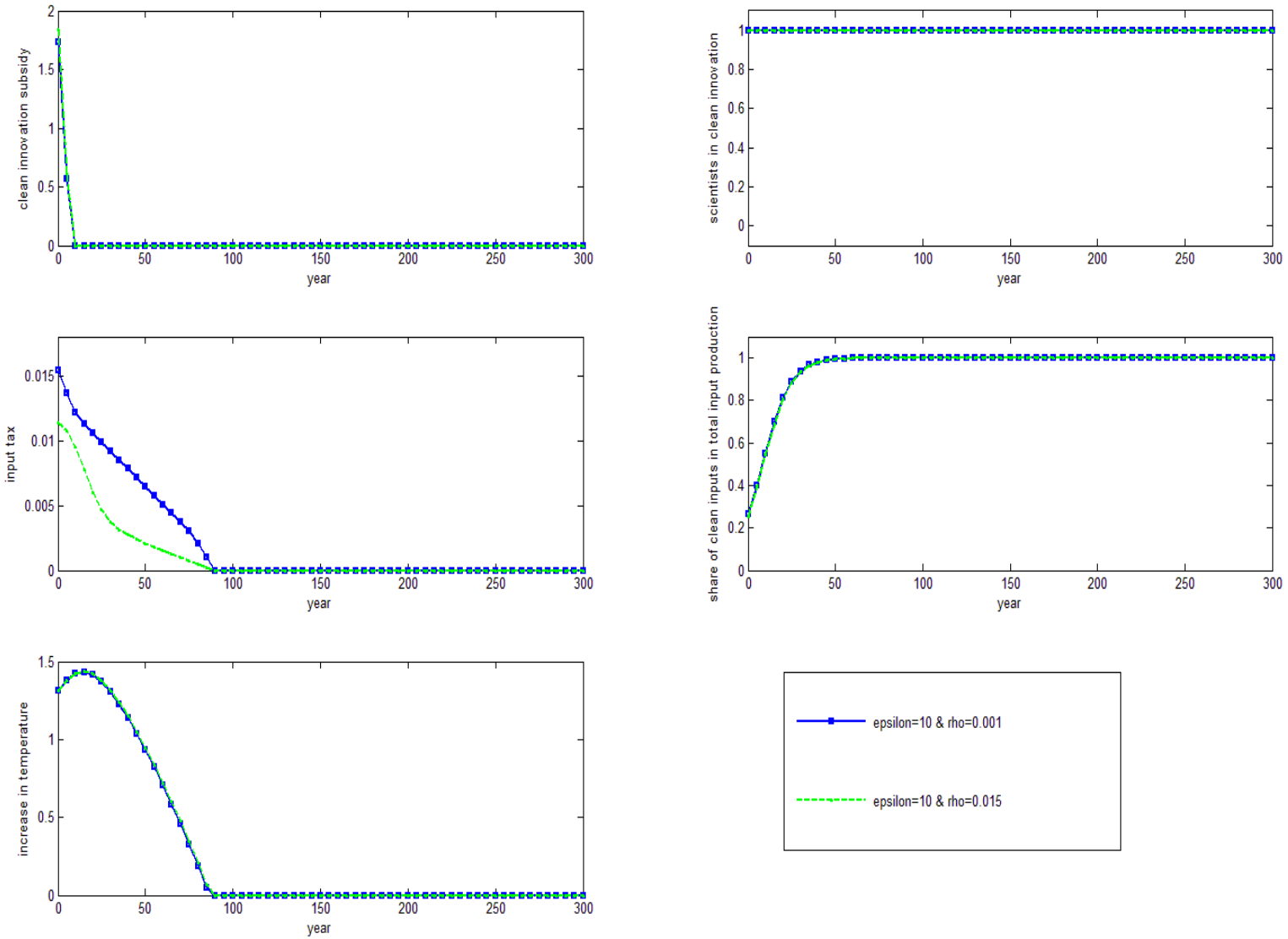
where ϕ is strictly increasing and concave in S.

- This defines a flexible family of continuous functions parameterized by λ, such that $\phi(0) = 0$
- Compute λ to match ϕ with the mapping between temperature and final output in Nordhaus' DICE 2007 model over the range of temperature increases up to 3°C.

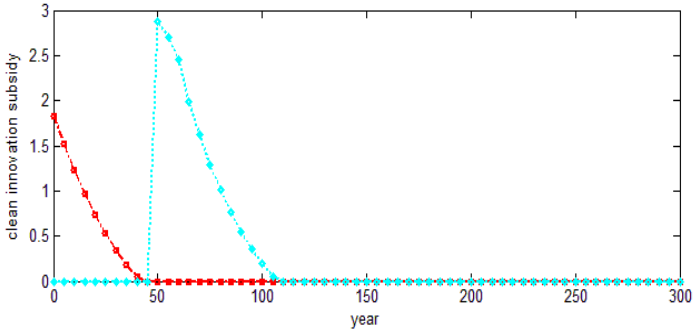
Calibration: 2 important parameters

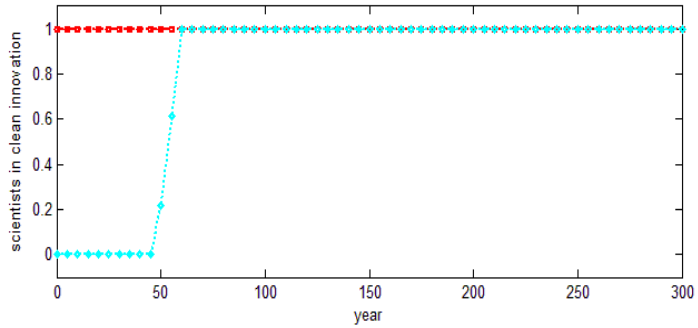
- Choose the elasticity of substitution between clean and dirty input as $\varepsilon = 3$ or 10 (low or high).
- Choose ρ, time discount rate (/year here) as $\rho = 0.001$ (Stern; discount factor$\simeq 0.999$) and $\rho = 0.015$ (Nordhaus; discount factor$\simeq 0.985$).

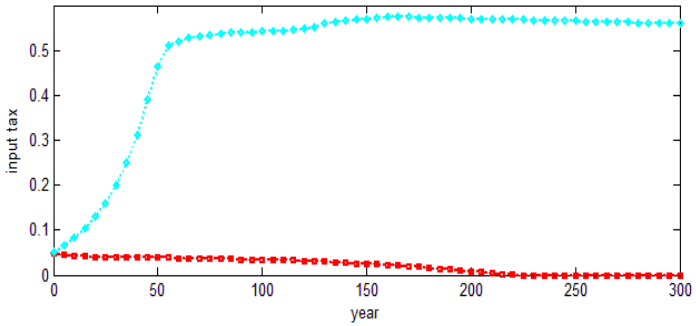
Simulation: optimal policy, high elasticity of substitution

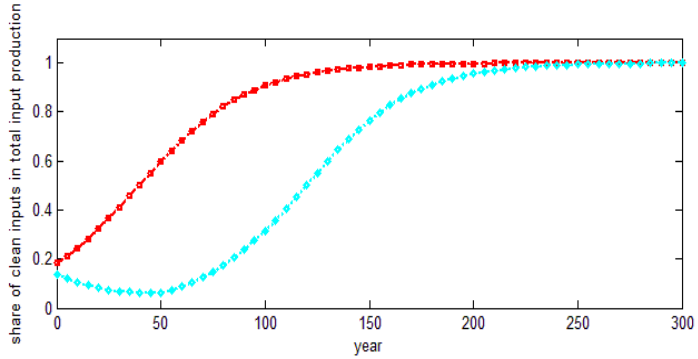


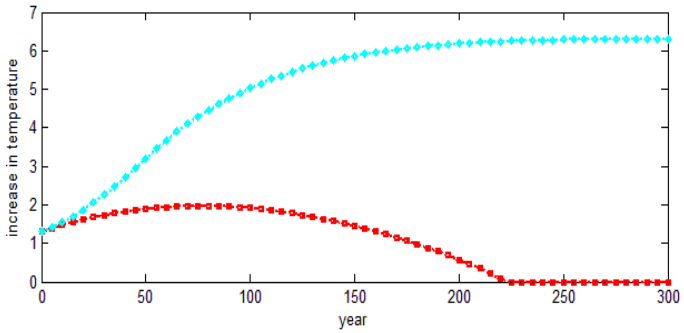
Simulation: optimal policy, low elasticity of substitution

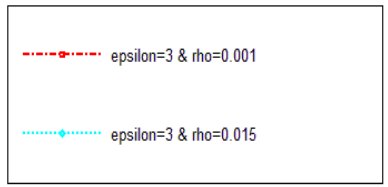












Simulation: cost of delaying optimal policy implementation

Welfare costs of delayed intervention as functions of ε and ρ
(Percentage reductions in consumption relative to immediate optimal policy)

Elasticity of substitution ε	10		3	
Discount rate ρ	0.001	0.015	0.001	0.015
delay = 10 years	8.75	1.87	2.71	0.05
delay = 20 years	14.02	1.92	4.79	0.12
delay = 30 years	17.65	1.99	6.88	0.23

Simulation: cost of using only carbon tax

Welfare costs of relying solely on the carbon tax as functions of ε and ρ

(Percentage reductions in consumption relative to optimal policy)

Elasticity of substitution ε	10		3	
Discount rate ρ	0.001	0.015	0.001	0.015
Welfare cost	0.95	1.58	1.74	2.70

Exhaustible resources

- Polluting activities (CO2 emissions) often use an exhaustible resource (most importantly, oil).
- Implications for evolution of production and direction of research.
- Questions:
  - Does this make environmental disaster less likely? *Yes.*
  - Does it change the structure of optimal environmental regulation? *No.*

Model

- Dirty input produced with some exhaustible resource R:

$$Y_d = R^{\alpha_2} L_d^{1-\alpha} \int_0^1 A_{di}^{1-\alpha_1} x_{di}^{\alpha_1} di,$$

  with $\alpha_1 + \alpha_2 = \alpha$.
- The resource stock Q_t evolves according to

$$Q_{t+1} = Q_t - R_t$$

- Extracting 1 unit of resource costs $c(Q_t)$ (with $c' \leq 0$, $c(0)$ finite). As Q_t decreases, extracting the resource becomes increasingly costly.
- Consider two polar cases:
  1. First, suppose the resource can be directly extracted ("tragedy of commons case")
  2. In a few slides, the resource is owned by infinitely-lived agents ("Hotelling case")

Direction of innovation

- Ratio of expected profits from innovation in clean versus dirty is modified to:

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \text{constant} \times \frac{\eta_c c(Q_t)^{\alpha_2(\varepsilon-1)}}{\eta_d} \frac{(1+\gamma\eta_c s_{ct})^{-\varphi-1}}{(1+\gamma\eta_d s_{dt})^{-\varphi_1-1}} \frac{A_{ct-1}^{-\varphi}}{A_{dt-1}^{-\varphi_1}},$$

  where $\varphi_1 \equiv (1-\alpha_1)(1-\varepsilon)$

- Provided that $\varepsilon > 1$, increasing cost of extraction helps switching towards clean innovation (again price effect vs market size effect).

Environmental disaster in the laissez-faire equilibrium

Proposition

When the two inputs are substitutes ($\varepsilon > 1$), innovation in the long-run will be directed towards the clean sector only and the economy will grow at a rate $\gamma\eta_c$. Provided that $\bar{S}$ is sufficiently high, an environmental disaster is avoided under laissez-faire.

- Either the increase in the cost of dirty input production due to depletion of exhaustible resources or the full depletion of the resource create enough incentives for research to switch to clean technologies.
- This prevents an environmental disaster provided that initial environmental stock is large enough.
- (In complement case ($\varepsilon < 1$), dirty input and so the resource are essential. Resource stock is depleted in finite time and economic growth is not sustainable).

Optimal environmental regulation

- How does optimal environmental regulation look like with exhaustible resources?
- *Answer:* generally similar to that without exhaustible resources, but also a resource tax so that the exhaustible resource does not get depleted completely.

Proposition

The social planner can implement the social optimum through a tax on the use of the dirty input, a subsidy on clean research, a subsidy on the use of all machines and a resource tax (all taxes/subsidies are imposed as a lump sum way to the corresponding agents). The resource tax must be maintained forever.

The Hotelling case (1)

- Firms in perfect competition own infinitesimal amount of the resource. Cost of extraction constant $c(Q_t) = c$; P_t price of the resource determined by the Hotelling rule:

$$\frac{P_{t+1} - c}{P_t - c} = 1 + r_t = (1 + \rho) \frac{\partial u / \partial C(C_t, S_t)}{\partial u / \partial C(C_{t+1}, S_{t+1})}$$

  net price raises at a rate equal to the interest rate of the economy.
- Case where utility is both separable in consumption and environment quality and CRRA with respect to consumption with a constant coefficient of relative risk aversion σ. Price of resource asymptotically grows at rate

$$r = \rho + \sigma g$$

  where g is the asymptotic growth rate of the economy.
- Importantly price of the resource is higher (there are rents) but resource is never exhausted (marginal product of the resource is unbounded).

The Hotelling case (2)

Proposition

A disaster is more easily avoided than without the resource, but not necessarily avoided.

1. *If the discount rate ρ and the elasticity of substitution ε are both sufficiently high ($\rho > [(1-\alpha_1)/\alpha_2]\gamma \max(\eta_d, \eta_c)$ and $\varepsilon > (2-\alpha_1-\alpha)^{-1}$) then innovation switches to the clean sector only and a disaster is avoided under laissez-faire provided that the initial environmental quality ($\overline{S}$) is sufficiently high.*
2. *If the discount rate and the elasticity of substitution are sufficiently low ($\varepsilon^{-1} - (1-\alpha) > \alpha_2(\rho/(\gamma\eta_c) + \sigma)$) a disaster cannot be avoided under laissez-faire.*

The Hotelling case (3)

- Intuition:
  - If price of the resource increases more slowly than the dirty productivity A_{dt}, innovation keeps occurring in the dirty sector forever and the economy runs into disaster.
  - If price increases sufficiently fast, innovation shifts to clean sectors. For high elasticity of substitution production of dirty input decreases.
  - High ρ pushes towards higher prices.
- As before temporary subsidy to clean research can implement the switch.
  - Threshold on ε from which a disaster can be avoided is lower and decreasing in the share α_2 of the resource in the production of dirty input (producing dirty involves incurring a resource price that grows at rate $\rho + \sigma\gamma\eta_c$).
- Optimal policy remains identical but the resource tax now becomes useless

Two-country case

- Two countries: North (N), identical to the economy studied so far, and that the South (S) imitating Northern technologies.
- Thus there are two externalities:
  1. *environmental externality*: dirty input productions by both contribute to global environmental degradation

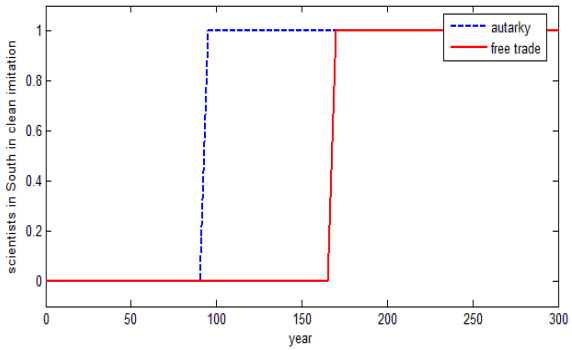
$$S_{t+1} = -\zeta \left(Y_{dt}^N + Y_{dt}^S \right) + (1+\delta) S_t \text{ for } S \in (0, \bar{S}).$$

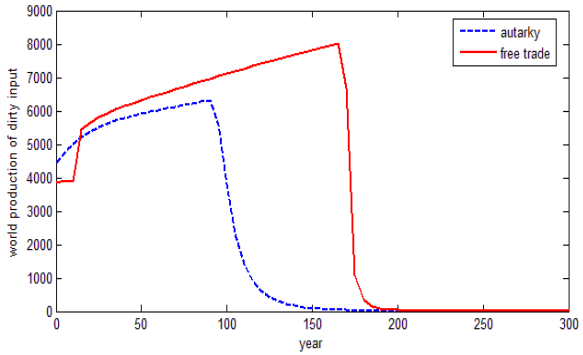
  2. *knowledge externality*: South imitates North' technologies
  $\Longrightarrow$ ratio of expected profits from imitation in the two sectors in the South

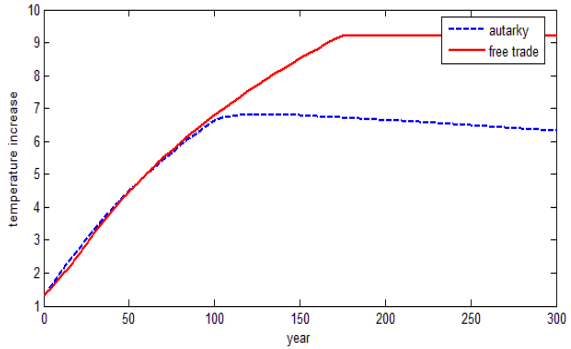
$$\frac{\Pi_{ct}^S}{\Pi_{dt}^S} = \frac{\kappa_c \left(p_{ct}^S\right)^{\frac{1}{1-\alpha}} L_{ct}^S A_{ct}^N}{\kappa_d \left(p_{dt}^S\right)^{\frac{1}{1-\alpha}} L_{dt}^S A_{dt}^N}$$

Global Interactions

- Do we need global coordination to avoid disasters?
  - In autarky, the answer is no again because advances in the North will induce the South to also switch to clean technologies.
  - But free trade may undermine this result by creating **pollution havens.**







Summary: preventing an environmental disaster

- Starting with more advanced dirty technology, the laissez-faire economy typically ends up in an environmental disaster.
  - Temporary policy intervention can redirect technical change and prevents a disaster (for high elasticity of substitution)
- Exhaustible resources sometimes make it possible for the laissez-faire economy to avoid an environmental disaster.
- Optimal policy involves:
  - the use of two instruments, carbon tax (for environmental externality) and clean research subsidy (for knowledge externality)
  - acting now, even with Nordhaus' discount rate for reasonable degree of substitutability between clean and dirty inputs

Future work

- Multicountries version with analysis of interactions between global environmental externalities, knowledge spillovers and international trade.
- More quantitative work to evaluate elasticity of substitution and therefore importance of endogenous and directed technical change.
- Introduce uncertainty (about the likelihood of environmental disaster, possibility of advances in clean technology, etc.).