14.452 Economic Growth: Lecture 6, Overlapping Generations

Daron Acemoglu

MIT

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Growth with Overlapping Generations

- In many situations, the assumption of a *representative household* is not appropriate because
 - In households do not have an infinite planning horizon
 - 2 new households arrive (or are born) over time.
- New economic interactions: decisions made by older "generations" will affect the prices faced by younger "generations".
- Overlapping generations models
 - Capture potential interaction of different generations of individuals in the marketplace;
 - Provide tractable alternative to infinite-horizon representative agent models;
 - Some key implications different from neoclassical growth model;
 - Dynamics in some special cases quite similar to Solow model rather than the neoclassical model;
 - Generate new insights about the role of national debt and Social Security in the economy.

Problems of Infinity I

- Static economy with countably infinite number of households, $i \in \mathbb{N}$
- Countably infinite number of commodities, $j \in \mathbb{N}$.
- All households behave competitively (alternatively, there are *M* households of each type, *M* is a large number).
- Household *i* has preferences:

$$u_i=c_i^i+c_{i+1}^i,$$

- c_j^i denotes the consumption of the *j*th type of commodity by household *i*.
- Endowment vector ω of the economy: each household has one unit endowment of the commodity with the same index as its index.
- Choose the price of the first commodity as the numeraire, i.e., $p_0 = 1$.

Problems of Infinity II

Proposition In the above-described economy, the price vector \bar{p} such that $\bar{p}_j = 1$ for all $j \in \mathbb{N}$ is a competitive equilibrium price vector and induces an equilibrium with no trade, denoted by \bar{x} .

Proof:

- At \bar{p} , each household has income equal to 1.
- Therefore, the budget constraint of household *i* can be written as

$$c_i^i + c_{i+1}^i \leq 1.$$

- This implies that consuming own endowment is optimal for each household,
- Thus \bar{p} and no trade, \bar{x} , constitute a competitive equilibrium.

Problems of Infinity III

- However, this competitive equilibrium is not Pareto optimal. Consider alternative allocation, \tilde{x} :
 - Household i = 0 consumes its own endowment and that of household 1.
 - All other households, indexed i > 0, consume the endowment of than neighboring household, i + 1.

 - Individual i = 0 is strictly better-off.

Proposition In the above-described economy, the competitive equilibrium at (\bar{p}, \bar{x}) is not Pareto optimal.

Problems of Infinity IV

- Source of the problem must be related to the infinite number of commodities.
- Extended version of the First Welfare Theorem covers infinite number of commodities, but only assuming ∑_{j=0}[∞] p_j^{*}ω_j < ∞ (written with the aggregate endowment ω_j).
- Here the only endowment is labor, and thus $p_j^* = 1$ for all $j \in \mathbb{N}$, so that $\sum_{j=0}^{\infty} p_j^* \omega_j = \infty$ (why?).
- This abstract economy is "isomorphic" to the baseline overlapping generations model.
- The Pareto suboptimality in this economy will be the source of potential inefficiencies in overlapping generations model.

Problems of Infinity V

- Second Welfare Theorem did not assume $\sum_{i=0}^{\infty} p_i^* \omega_j < \infty$.
- Instead, it used convexity of preferences, consumption sets and production possibilities sets.
- This exchange economy has convex preferences and convex consumption sets:
 - Pareto optima must be decentralizable by some redistribution of endowments.

Proposition In the above-described economy, there exists a reallocation of the endowment vector $\boldsymbol{\omega}$ to $\tilde{\boldsymbol{\omega}}$, and an associated competitive equilibrium (\bar{p}, \tilde{x}) that is Pareto optimal where \tilde{x} is as described above, and \bar{p} is such that $\bar{p}_j = 1$ for all $j \in \mathbb{N}$.

Proof of Proposition

- Consider the following reallocation of ω : endowment of household
 - $i \geq 1$ is given to household i 1.
 - At the new endowment vector $\tilde{\boldsymbol{\omega}}$, household i = 0 has one unit of good j = 0 and one unit of good j = 1.
 - Other households i have one unit of good i + 1.
- At the price vector \bar{p} , household 0 has a budget set

$$c_0^0 + c_1^1 \leq 2$$
,

thus chooses $c_0^0 = c_1^0 = 1$.

• All other households have budget sets given by

$$c_i^i+c_{i+1}^i\leq 1$$
,

- Thus it is optimal for each household i > 0 to consume one unit of the good cⁱ_{i+1}
- Thus \tilde{x} is a competitive equilibrium.

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The Baseline Overlapping Generations Model

- Time is discrete and runs to infinity.
- Each individual lives for two periods.
- Individuals born at time t live for dates t and t + 1.
- Assume a general (separable) utility function for individuals born at date *t*,

$$U(t) = u(c_1(t)) + \beta u(c_2(t+1)),$$
 (1)

- $u: \mathbb{R}_+ \to \mathbb{R}$ satisfies the usual Assumptions on utility.
- $c_1(t)$: consumption of the individual born at t when young (at date t).
- $c_2(t+1)$: consumption when old (at date t+1).
- $\beta \in (0, 1)$ is the discount factor.

Demographics, Preferences and Technology I

• Exponential population growth,

$$L(t) = (1+n)^{t} L(0).$$
 (2)

• Production side same as before: competitive firms, constant returns to scale aggregate production function, satisfying Assumptions 1 and 2:

$$Y\left(t
ight)=F\left(K\left(t
ight)$$
 , $L\left(t
ight)
ight)$.

- Factor markets are competitive.
- Individuals can only work in the first period and supply one unit of labor inelastically, earning w (t).

Demographics, Preferences and Technology II

- Assume that $\delta = 1$.
- $k \equiv K/L$, $f(k) \equiv F(k, 1)$, and the (gross) rate of return to saving, which equals the rental rate of capital, is

$$1 + r(t) = R(t) = f'(k(t)), \qquad (3)$$

• As usual, the wage rate is

$$w(t) = f(k(t)) - k(t) f'(k(t)).$$
(4)

Consumption Decisions I

• Savings by an individual of generation t, s(t), is determined as a solution to

$$\max_{c_{1}(t),c_{2}(t+1),s(t)} u(c_{1}(t)) + \beta u(c_{2}(t+1))$$

subject to

$$c_{1}(t) + s(t) \leq w(t)$$

and

$$c_{2}\left(t+1
ight)\leq R\left(t+1
ight)s\left(t
ight)$$
 ,

- Old individuals rent their savings of time t as capital to firms at time t + 1, and receive gross rate of return R(t + 1) = 1 + r(t + 1)
- Second constraint incorporates notion that individuals only spend money on their own end of life consumption (no altruism or bequest motive).

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Consumption Decisions II

- No need to introduce $s(t) \ge 0$, since negative savings would violate second-period budget constraint (given $c_2(t+1) \ge 0$).
- Since $u(\cdot)$ is strictly increasing, both constraints will hold as equalities.
- Thus first-order condition for a maximum can be written in the familiar form of the consumption Euler equation,

$$u'(c_{1}(t)) = \beta R(t+1) u'(c_{2}(t+1)).$$
(5)

- Problem of each individual is strictly concave, so this Euler equation is sufficient.
- Solving for consumption and thus for savings,

$$s(t) = s(w(t), R(t+1)), \qquad (6)$$

Consumption Decisions III

- s: ℝ²₊ → ℝ is strictly increasing in its first argument and may be increasing or decreasing in its second argument.
- Total savings in the economy will be equal to

$$S(t) = s(t) L(t)$$
,

- L(t) denotes the size of generation t, who are saving for time t + 1.
- Since capital depreciates fully after use and all new savings are invested in capital,

$$K(t+1) = L(t) s(w(t), R(t+1)).$$
 (7)

Equilibrium I

Definition A competitive equilibrium can be represented by a sequence of aggregate capital stocks, individual consumption and factor prices,

$$\begin{split} & \{K\left(t\right),c_{1}\left(t\right),c_{2}\left(t\right),R\left(t\right),w\left(t\right)\}_{t=0}^{\infty}, \text{ such that the factor} \\ & \text{price sequence } \{R\left(t\right),w\left(t\right)\}_{t=0}^{\infty} \text{ is given by (3) and (4),} \\ & \text{individual consumption decisions } \{c_{1}\left(t\right),c_{2}\left(t\right)\}_{t=0}^{\infty} \text{ are} \\ & \text{given by (5) and (6), and the aggregate capital stock,} \\ & \{K\left(t\right)\}_{t=0}^{\infty}, \text{ evolves according to (7).} \end{split}$$

- Steady-state equilibrium defined as usual: an equilibrium in which $k \equiv K/L$ is constant.
- To characterize the equilibrium, divide (7) by L(t+1) = (1+n) L(t),

$$k(t+1) = rac{s(w(t), R(t+1))}{1+n}.$$

Equilibrium II

• Now substituting for R(t+1) and w(t) from (3) and (4),

$$k(t+1) = \frac{s(f(k(t)) - k(t)f'(k(t)), f'(k(t+1)))}{1+n}$$
(8)

- This is the fundamental law of motion of the overlapping generations economy.
- A steady state is given by a solution to this equation such that $k(t+1) = k(t) = k^*$, i.e.,

$$k^{*} = \frac{s\left(f\left(k^{*}\right) - k^{*}f'\left(k^{*}\right), f'\left(k^{*}\right)\right)}{1+n}$$
(9)

• Since the savings function $s(\cdot, \cdot)$ can take any form, the difference equation (8) can lead to quite complicated dynamics, and multiple steady states are possible.

Restrictions on Utility and Production Functions I

• Suppose that the utility functions take the familiar CRRA form:

$$U(t) = \frac{c_1(t)^{1-\theta} - 1}{1-\theta} + \beta\left(\frac{c_2(t+1)^{1-\theta} - 1}{1-\theta}\right), \quad (10)$$

where $\theta > 0$ and $\beta \in (0, 1)$.

Technology is Cobb-Douglas,

$$f(k) = k^{\alpha}$$

- The rest of the environment is as described above.
- The CRRA utility simplifies the first-order condition for consumer optimization,

$$rac{c_{2}\left(t+1
ight)}{c_{1}\left(t
ight)}=\left(eta R\left(t+1
ight)
ight)^{1/ heta}.$$

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Restrictions on Utility and Production Functions II

• This Euler equation can be alternatively expressed in terms of savings as $(x) = \theta + 2\pi (x + z)^{1-\theta} + (x + z)^{1-\theta}$

$$s(t)^{-\theta}\beta R(t+1)^{1-\theta} = (w(t) - s(t))^{-\theta},$$
 (11)

• Gives the following equation for the saving rate:

$$s(t) = \frac{w(t)}{\psi(t+1)},$$
(12)

where

$$\psi(t+1) \equiv [1+\beta^{-1/\theta}R(t+1)^{-(1-\theta)/\theta}] > 1,$$

• Ensures that savings are always less than earnings.

Restrictions on Utility and Production Functions III

• The impact of factor prices on savings is summarized by the following and derivatives:

$$\begin{split} s_{w} &\equiv \frac{\partial s\left(t\right)}{\partial w\left(t\right)} = \frac{1}{\psi\left(t+1\right)} \in \left(0,1\right), \\ s_{R} &\equiv \frac{\partial s\left(t\right)}{\partial R\left(t+1\right)} = \left(\frac{1-\theta}{\theta}\right) \left(\beta R\left(t+1\right)\right)^{-1/\theta} \frac{s\left(t\right)}{\psi\left(t+1\right)}. \end{split}$$

- Since $\psi(t+1) > 1$, we also have that $0 < s_w < 1$.
- Moreover, in this case $s_R > 0$ if $\theta > 1$, $s_R < 0$ if $\theta < 1$, and $s_R = 0$ if $\theta = 1$.
- Reflects counteracting influences of income and substitution effects.
- Case of $\theta = 1$ (log preferences) is of special importance, may deserve to be called the *canonical overlapping generations model*.

Restrictions on Utility and Production Functions IV

• Equation (8) implies

$$k(t+1) = \frac{s(t)}{(1+n)}$$
(13)
= $\frac{w(t)}{(1+n)\psi(t+1)}$,

Or more explicitly,

$$k(t+1) = \frac{f(k(t)) - k(t)f'(k(t))}{(1+n)\left[1 + \beta^{-1/\theta}f'(k(t+1))^{-(1-\theta)/\theta}\right]}$$
(14)

 The steady state then involves a solution to the following implicit equation:

$$k^{*} = \frac{f(k^{*}) - k^{*}f'(k^{*})}{(1+n)\left[1 + \beta^{-1/\theta}f'(k^{*})^{-(1-\theta)/\theta}\right]}.$$

Restrictions on Utility and Production Functions V

• Now using the Cobb-Douglas formula, steady state is the solution to the equation

$$(1+n)\left[1+\beta^{-1/\theta}\left(\alpha(k^*)^{\alpha-1}\right)^{(\theta-1)/\theta}\right] = (1-\alpha)(k^*)^{\alpha-1}.$$
 (15)

• For simplicity, define $R^* \equiv \alpha(k^*)^{\alpha-1}$ as the marginal product of capital in steady-state, in which case, (15) can be rewritten as

$$(1+n)\left[1+\beta^{-1/\theta}\left(R^*\right)^{(\theta-1)/\theta}\right]=\frac{1-\alpha}{\alpha}R^*.$$
 (16)

- Steady-state value of R^* , and thus k^* , can now be determined from equation (16), which always has a unique solution.
- To investigate the stability, substitute for the Cobb-Douglas production function in (14)

$$k(t+1) = \frac{(1-\alpha) k(t)^{\alpha}}{(1+n) \left[1 + \beta^{-1/\theta} \left(\alpha k(t+1)^{\alpha-1}\right)^{-(1-\theta)/\theta}\right]}.$$
 (17)

Restrictions on Utility and Production Functions VI

Proposition In the overlapping-generations model with two-period lived households, Cobb-Douglas technology and CRRA preferences, there exists a unique steady-state equilibrium with the capital-labor ratio k^* given by (15), this steady-state equilibrium is globally stable for all k(0) > 0.

- In this particular (well-behaved) case, equilibrium dynamics are very similar to the basic Solow model
- Figure shows that convergence to the unique steady-state capital-labor ratio, *k*^{*}, is monotonic.

Canonical Model I

- Even the model with CRRA utility and Cobb-Douglas production function is relatively messy.
- Many of the applications use log preferences (heta=1).
- Income and substitution effects exactly cancel each othe: changes in the interest rate (and thus in the capital-labor ratio of the economy) have no effect on the saving rate.
- Structure of the equilibrium is essentially identical to the basic Solow model.
- Utility of the household and generation t is,

$$U(t) = \log c_1(t) + \beta \log c_2(t+1),$$
 (18)

- $\beta \in (0, 1)$ (even though $\beta \ge 1$ could be allowed).
- Again $f(k) = k^{\alpha}$.

Canonical Model II

• Consumption Euler equation:

$$\frac{c_{2}\left(t+1\right)}{c_{1}\left(t\right)}=\beta R\left(t+1\right)$$

• Savings should satisfy the equation

$$s(t) = \frac{\beta}{1+\beta} w(t), \qquad (19)$$

• Constant saving rate, equal to $\beta / (1 + \beta)$, out of labor income for each individual.

Canonical Model III

• Combining this with the capital accumulation equation (8),

$$k(t+1) = \frac{s(t)}{(1+n)}$$
$$= \frac{\beta w(t)}{(1+n)(1+\beta)}$$
$$= \frac{\beta (1-\alpha) [k(t)]^{\alpha}}{(1+n)(1+\beta)},$$

- Second line uses (19) and last uses that, given competitive factor markets, $w(t) = (1 \alpha) [k(t)]^{\alpha}$.
- There exists a unique steady state with

$$k^* = \left[\frac{\beta \left(1-\alpha\right)}{\left(1+n\right)\left(1+\beta\right)}\right]^{\frac{1}{1-\alpha}}.$$
(20)

• Equilibrium dynamics are identical to those of the basic Solow model and monotonically converge to k^* .

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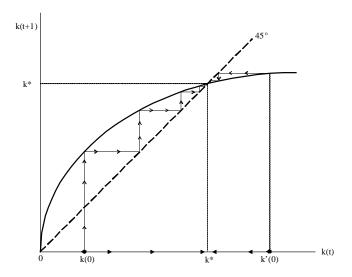


Figure: Equilibrium dynamics in the canonical overlapping generations model.

Canonical Model IV

Proposition In the canonical overlapping generations model with log preferences and Cobb-Douglas technology, there exists a unique steady state, with capital-labor ratio k^* given by (20). Starting with any $k(0) \in (0, k^*)$, equilibrium dynamics are such that $k(t) \uparrow k^*$, and starting with any $k'(0) > k^*$, equilibrium dynamics involve $k(t) \downarrow k^*$.

Overaccumulation I

- Compare the overlapping-generations equilibrium to the choice of a social planner wishing to maximize a weighted average of all generations' utilities.
- Suppose that the social planner maximizes

$$\sum_{t=0}^{\infty}\beta_{S}^{t}U(t)$$

• β_S is the discount factor of the social planner, which reflects how she values the utilities of different generations.

Overaccumulation II

• Substituting from (1), this implies:

$$\sum_{t=0}^{\infty}\beta_{S}^{t}\left(u\left(c_{1}\left(t\right)\right)+\beta u\left(c_{2}\left(t+1\right)\right)\right)$$

subject to the resource constraint

$$F(K(t), L(t)) = K(t+1) + L(t)c_1(t) + L(t-1)c_2(t).$$

• Dividing this by L(t) and using (2),

$$f(k(t)) = (1+n)k(t+1) + c_1(t) + \frac{c_2(t)}{1+n}$$

Overaccumulation III

• Social planner's maximization problem then implies the following first-order necessary condition:

$$u'(c_1(t)) = \beta f'(k(t+1)) u'(c_2(t+1)).$$

- Since R(t+1) = f'(k(t+1)), this is identical to (5).
- Not surprising: allocate consumption of a given individual in exactly the same way as the individual himself would do.
- No "market failures" in the over-time allocation of consumption at given prices.
- However, the allocations across generations may differ from the competitive equilibrium: planner is giving different weights to different generations
- In particular, competitive equilibrium is **Pareto suboptimal** when $k^* > k_{gold}$,

Overaccumulation IV

- When $k^* > k_{gold}$, reducing savings can increase consumption for every generation.
- More specifically, note that in steady state

$$f(k^*) - (1+n)k^* = c_1^* + (1+n)^{-1} c_2^*$$

$$\equiv c^*,$$

- First line follows by national income accounting, and second defines c^* .
- Therefore

$$\frac{\partial c^*}{\partial k^*} = f'(k^*) - (1+n)$$

• k_{gold} is defined as

$$f'(k_{gold}) = 1 + n.$$

Overaccumulation V

- Now if k* > k_{gold}, then ∂c*/∂k* < 0: reducing savings can increase (total) consumption for everybody.
- If this is the case, the economy is referred to as *dynamically inefficient*—it involves overaccumulation.
- Another way of expressing dynamic inefficiency is that

$$r^{*} < n$$
,

- Recall in infinite-horizon Ramsey economy, transversality condition required that r > g + n.
- Dynamic inefficiency arises because of the heterogeneity inherent in the overlapping generations model, which removes the transversality condition.
- Suppose we start from steady state at time T with $k^* > k_{gold}$.

Overaccumulation VI

- Consider the following variation: change next period's capital stock by -Δk, where Δk > 0, and from then on, we immediately move to a new steady state (clearly feasible).
- This implies the following changes in consumption levels:

$$egin{array}{rcl} \Delta c\left(T
ight) &=& \left(1+n
ight) \Delta k>0 \ \Delta c\left(t
ight) &=& -\left(f'\left(k^{st}-\Delta k
ight) -\left(1+n
ight)
ight) \Delta k ext{ for all }t>T \end{array}$$

- The first expression reflects the direct increase in consumption due to the decrease in savings.
- In addition, since $k^* > k_{gold}$, for small enough Δk , $f'(k^* - \Delta k) - (1 + n) < 0$, thus $\Delta c(t) > 0$ for all $t \ge T$.
- The increase in consumption for each generation can be allocated equally during the two periods of their lives, thus necessarily increasing the utility of all generations.

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Pareto Optimality and Suboptimality in the OLG Model

Proposition In the baseline overlapping-generations economy, the competitive equilibrium is not necessarily Pareto optimal. More specifically, whenever $r^* < n$ and the economy is dynamically inefficient, it is possible to reduce the capital stock starting from the competitive steady state and increase the consumption level of all generations.

• Pareto inefficiency of the competitive equilibrium is intimately linked with *dynamic inefficiency*.

Interpretation

- Intuition for dynamic inefficiency:
 - Individuals who live at time *t* face prices determined by the capital stock with which they are working.
 - Capital stock is the outcome of actions taken by previous generations.
 - Pecuniary externality from the actions of previous generations affecting welfare of current generation.
 - Pecuniary externalities typically second-order and do not matter for welfare.
 - But not when an infinite stream of newborn agents joining the economy are affected.
 - It is possible to rearrange in a way that these pecuniary externalities can be exploited.

Further Intuition

- Complementary intuition:
 - Dynamic inefficiency arises from overaccumulation.
 - Results from current young generation needs to save for old age.
 - However, the more they save, the lower is the rate of return and may encourage to save even more.
 - Effect on future rate of return to capital is a pecuniary externality on next generation
 - If alternative ways of providing consumption to individuals in old age were introduced, overaccumulation could be ameliorated.

Role of Social Security in Capital Accumulation

- Social Security as a way of dealing with overaccumulation
- Fully-funded system: young make contributions to the Social Security system and their contributions are paid back to them in their old age.
- Unfunded system or a *pay-as-you-go:* transfers from the young directly go to the current old.
- Pay-as-you-go (unfunded) Social Security discourages aggregate savings.
- With dynamic inefficiency, discouraging savings may lead to a Pareto improvement.

Fully Funded Social Security I

- Government at date t raises some amount d(t) from the young, funds are invested in capital stock, and pays workers when old R(t+1) d(t).
- Thus individual maximization problem is,

$$\max_{c_{1}(t),c_{2}(t+1),s(t)}u\left(c_{1}\left(t\right)\right)+\beta u\left(c_{2}\left(t+1\right)\right)$$

subject to

$$c_{1}(t) + s(t) + d(t) \le w(t)$$

and

$$c_{2}\left(t+1
ight)\leq R\left(t+1
ight)\left(s\left(t
ight)+d\left(t
ight)
ight)$$
 ,

for a given choice of d(t) by the government.

• Notice that now the total amount invested in capital accumulation is s(t) + d(t) = (1 + n) k (t + 1).

Fully Funded Social Security II

- No longer the case that individuals will always choose s(t) > 0.
- As long as s(t) is free, whatever $\{d(t)\}_{t=0}^{\infty}$, the competitive equilibrium applies.
- When $s(t) \ge 0$ is imposed as a constraint, competitive equilibrium applies if given $\{d(t)\}_{t=0}^{\infty}$, privately-optimal $\{s(t)\}_{t=0}^{\infty}$ is such that s(t) > 0 for all t.

Fully Funded Social Security III

Proposition Consider a fully funded Social Security system in the above-described environment whereby the government collects d(t) from young individuals at date t.

- Suppose that s (t) ≥ 0 for all t. If given the feasible sequence {d(t)}[∞]_{t=0} of Social Security payments, the utility-maximizing sequence of savings {s(t)}[∞]_{t=0} is such that s(t) > 0 for all t, then the set of competitive equilibria without Social Security are the set of competitive equilibria with Social Security.
- Without the constraint s (t) ≥ 0, given any feasible sequence $\{d(t)\}_{t=0}^{\infty}$ of Social Security payments, the set of competitive equilibria without Social Security are the set of competitive equilibria with Social Security.
- Moreover, even when there is the restriction that $s(t) \ge 0$, a funded Social Security program cannot lead to the Pareto improvement.

Unfunded Social Security I

- Government collects d(t) from the young at time t and distributes to the current old with per capita transfer b(t) = (1 + n) d(t)
- Individual maximization problem becomes

$$\max_{c_{1}(t),c_{2}(t+1),s(t)} u(c_{1}(t)) + \beta u(c_{2}(t+1))$$

subject to

$$c_{1}\left(t
ight)+s\left(t
ight)+d\left(t
ight)\leq w\left(t
ight)$$

and

$$c_{2}\left(t+1
ight)\leq R\left(t+1
ight)s\left(t
ight)+\left(1+n
ight)d\left(t+1
ight)$$
 ,

for a given feasible sequence of Social Security payment levels $\left\{ d\left(t\right)
ight\}_{t=0}^{\infty}.$

Rate of return on Social Security payments is n rather than
r (t+1) = R (t+1) - 1, because unfunded Social Security is a pure
transfer system.

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Unfunded Social Security II

- Only s(t)—rather than s(t) plus d(t) as in the funded scheme—goes into capital accumulation.
- It is possible that s (t) will change in order to compensate, but such an offsetting change does not typically take place.
- Thus unfunded Social Security reduces capital accumulation.
- Discouraging capital accumulation can have negative consequences for growth and welfare.
- In fact, empirical evidence suggests that there are many societies in which the level of capital accumulation is suboptimally low.
- But here reducing aggregate savings may be good when the economy exhibits dynamic inefficiency.

Unfunded Social Security III

Proposition Consider the above-described overlapping generations economy and suppose that the decentralized competitive equilibrium is dynamically inefficient. Then there exists a feasible sequence of unfunded Social Security payments $\{d(t)\}_{t=0}^{\infty}$ which will lead to a competitive equilibrium starting from any date t that Pareto dominates the competitive equilibrium without Social Security.

- Similar to way in which the Pareto optimal allocation was decentralized in the example economy above.
- Social Security is transferring resources from future generations to initial old generation.
- But with *no* dynamic inefficiency, any transfer of resources (and any unfunded Social Security program) would make some future generation worse-off.

Conclusions

- Overlapping generations of the more realistic than infinity-lived representative agents.
- Models with overlapping generations fall outside the scope of the First Welfare Theorem:
 - they were partly motivated by the possibility of Pareto suboptimal allocations.
- Equilibria may be "dynamically inefficient" and feature overaccumulation: unfunded Social Security can ameliorate the problem.
- Declining path of labor income important for overaccumulation, and what matters is not finite horizons but arrival of new individuals.
- Overaccumulation and Pareto suboptimality: pecuniary externalities created on individuals that are not yet in the marketplace.
- Not overemhasize dynamic inefficiency: major question of economic growth is why so many countries have so little capital.

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Economic Growth Lecture 6