14.773 Political Economy of Institutions and Development.
Lecture 6: Political Economy of Weak States

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February 28, 2017

Introduction

- As we will see in the next lecture, "weak states" are generally associated with worse economic outcomes.

- In fact, many of the ideas we have seen so far might have little relevance to the problem of economic development in some parts of the world where the state is notable in its absence.

- In Joel Migdal's words in *Strong Societies and Weak States*:

    "In parts of the Third World, the inability of state leaders to achieve predominance in large areas of their countries has been striking..."

- In traditional political science, much emphasis on "state capacity" and "weak states".

- But key question: why are states weak and remain weak?

Income and Taxes

Figure 1

Tax Revenue and Income 1990-2000
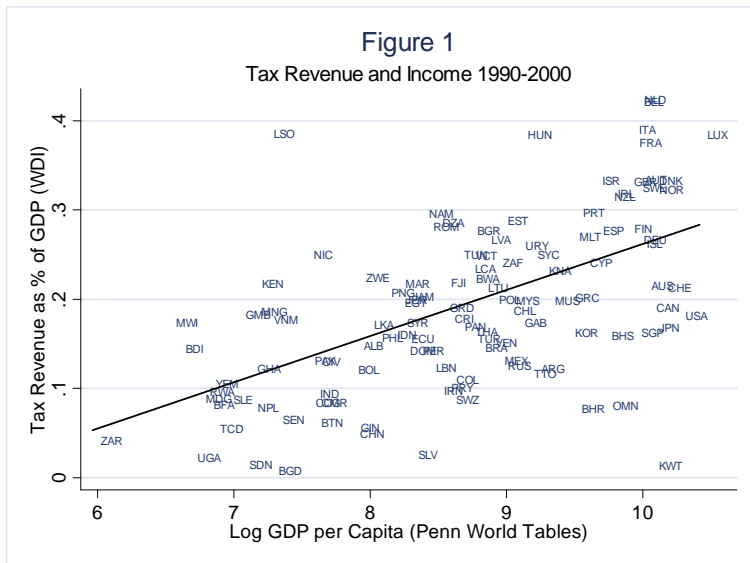
Ideas about State Building

- State building: the process of states building capacity and strength.
- Otto Hinze and later Charles Tilly emphasized the role of inter-state wars in the formation of the state.
- Tilly:

    "War made the state, and state made war."

- Based on this, Jeffrey Herbst in *States and Power in Africa* suggested that the weakness of the sub-Saharan African states is due to its difficult terrain and low population density that discouraged inter-state warfare.

A Simple Model of State Building

- Besley and Persson (2009) provided a simple formalization of Tilly.
- "State power" is a state variable (i.e., is persistent "stock"). It enables more efficient taxation.
- In an economy with two competing groups, the group in power may not want to invest in state power if it expects to lose power because then this power will be in the hands of its rival group.
- However, if there is a threat of war, then building state power becomes a necessity.

Evidence

Table 2: Economic and Political Determinants of Fiscal Capacity

	(1) One Minus Share of Trade Taxes in Total Taxes	(2) One Minus Share of Trade and Indirect Taxes in Total Taxes	(3) Share of Income Taxes in GDP	(4) Share of Taxes in GDP
Incidence of External Conflict up to 1975	0.921*** (0.229)	0.683*** (0.201)	0.747*** (0.246)	0.678*** (0.211)
Incidence of Democracy up to 1975	0.005 (0.085)	– 0.037 (0.096)	0.057 (0.062)	0.097 (0.064)
Incidence of Parliamentary Democracy up to 1975	0.123 (0.086)	0.208** (0.094)	0.231*** (0.074)	0.166** (0.069)
English Legal Origin	– 0.013 (0.069)	- 0.012 (0.061)	– 0.015 (0.056)	0.013 (0.051)
Socialist Legal Origin	0.051 (0.095)	– 0.332*** (0.084)	– 0.155** (0.065)	– 0.110 (0.082)
German Legal Origin	0.283*** (0.064)	0.290*** (0.093)	0.295*** (0.084)	0.206*** (0.065)
Scandinavian Legal Origin	0.333*** (0.068)	0.195** (0.078)	0.364** (0.141)	0.363*** (0.092)
Observations	104	104	104	104
R-squared	0.412	0.435	0.628	0.639

Robust standard errors in parentheses: * significant at 10%; ** significant at 5%; *** significant at 1%
All specifications include regional fixed effects (for eight regions).

But

- Osafo-Kwaako and Robinson (2013), using the Standard Cross-Cultural Sample, show that these ideas have limited explanatory power for Africa.

Table 3
Correlates of state formation (full SCCS sample).

	Dependent variable: jurisdictional hierarchy beyond local community (SCCS variable §237)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Population density	0.386***	0.363***					0.361***	0.295***
	(0.0407)	(0.0403)					(0.0609)	(0.0590)
Africa ∗ Population density	−0.317***	−0.325***					−0.217	−0.173
	(0.115)	(0.112)					(0.180)	(0.171)
Being attacked			0.293*	0.364**			0.263	0.253
			(0.168)	(0.146)			(0.170)	(0.175)
Africa ∗ Being attacked			−0.315	−0.472			−0.102	−0.220
			(0.382)	(0.379)			(0.366)	(0.382)
Trade					0.191*	0.172	0.0113	−0.0375
					(0.112)	(0.115)	(0.107)	(0.117)
Africa ∗ Trade					−0.000762	0.104	0.294	0.408*
					(0.258)	(0.263)	(0.227)	(0.227)
Africa dummy	1.631***	1.644***	0.975	1.064	0.863	0.253	0.595	0.263
	(0.442)	(0.413)	(0.780)	(0.793)	(0.823)	(0.862)	(1.271)	(1.326)
F-stat	0.41	0.13	0.00	0.10	0.67	1.37	1.06	1.46
p-Value	0.52	0.72	0.95	0.76	0.41	0.24	0.37	0.23
Geographic controls	No	Yes	No	Yes	No	Yes	No	Yes
Disease environment control	No	Yes	No	Yes	No	Yes	No	Yes
Observations	182	182	152	152	98	98	94	94
R-squared	0.342	0.369	0.040	0.206	0.099	0.272	0.381	0.440

But (continued)

Table 4
Correlates of state formation.

	Dependent variable: levels of sovereignty (SCCS variable §83)							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Population density	0.388***	0.379***					0.397***	0.325***
	(0.0353)	(0.0379)					(0.0542)	(0.0516)
Africa * Population density	−0.288**	−0.314**					−0.293	−0.272
	(0.131)	(0.129)					(0.220)	(0.199)
Being attacked			0.199	0.263*			0.106	0.0423
			(0.156)	(0.140)			(0.147)	(0.137)
Africa * Being attacked			−0.645*	−0.789**			−0.263	−0.397
			(0.347)	(0.327)			(0.323)	(0.331)
Trade					0.248**	0.260**	0.0255	0.0102
					(0.103)	(0.104)	(0.0898)	(0.0918)
Africa * Trade					−0.254	−0.165	0.0511	0.160
					(0.340)	(0.362)	(0.375)	(0.393)
Africa dummy	1.415**	1.445***	1.428**	1.456**	1.695	0.980	1.995	1.620
	(0.560)	(0.535)	(0.709)	(0.694)	(1.126)	(1.231)	(1.569)	(1.664)
F-stat	0.62	0.27	2.07	3.10	0.00	0.08	0.24	0.58
p-Value	0.43	0.60	0.15	0.08	0.99	0.78	0.87	0.63
Geographic controls	No	Yes	No	Yes	No	Yes	No	Yes
Disease environment control	No	Yes	No	Yes	No	Yes	No	Yes
Observations	182	182	152	152	99	99	95	95
R-squared	0.355	0.373	0.028	0.187	0.108	0.321	0.401	0.505

Stationary Bandits

- Mancur Olson, also based on some of Tilly's ideas, argued that the origins of the state lie in organized banditry (e.g., Olson, 1993, and McGuire and Olson, 1996).
- A roving bandit will apply maximal extraction (as in the model above in the MPE with δ high). Roving bandits arise when the bandits themselves don't have any security and for this or other reasons have a short horizon.
- A stationary bandit, with a longer horizon, will act like a state, encouraging production and taking more moderate taxes (as in the SPE of the models we have seen before).
- A stationary bandit ultimately becomes a state.

Evidence

- Sanchez de la Sierra (2014) provides evidence for this perspective by exploiting the differential increases in incentives of armed groups in the civil war of Eastern Congo to become "stationary bandits" because of the Colton price hike.

- Colton is easier to tax because it's much harder to conceal than gold, so he uses gold as a control.

- He therefore hypothesizes that "attempted conquests" should increase due to the higher interaction of Colton deposits and Colton price, but not the same for gold.

- But does this have anything to do with State building?

Evidence (continued)

Table 3: Effects of price shocks, conquests

| VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Coltan X Coltan Price	0.25*	0.61***	0.26*	0.30**	0.60**	0.59**	0.60**	0.30**	0.66***
	(0.14)	(0.14)	(0.14)	(0.14)	(0.24)	(0.24)	(0.25)	(0.14)	(0.20)
Coltan X Coltan Price X Far		-0.58***			-0.57*	-0.56*	-0.57*		-0.66**
		(0.19)			(0.29)	(0.29)	(0.29)		(0.27)
Gold X Gold Price	-0.01***	0.00	-0.01***	-0.02	-0.02	-0.02	-0.03		
	(0.00)	(0.01)	(0.00)	(0.02)	(0.03)	(0.03)	(0.04)		
Gold X Gold Price X Far		-0.01			0.01	0.02	0.02		
		(0.01)			(0.05)	(0.04)	(0.04)		
Congolese Army, Village			-0.06**						
			(0.03)						
Constant	0.12***	-70.08	0.12***	0.15*	-0.02	-0.04	-0.05	0.09***	0.09***
	(0.03)	(74.18)	(0.03)	(0.09)	(0.09)	(0.08)	(0.09)	(0.02)	(0.02)
Observations	1,342	1,342	1,342	732	732	678	732	244	244
R-squared	0.21	0.33	0.22	0.34	0.35	0.29	0.37	0.68	0.69
Year FE	YES	YES	YES	YES	YES	YES	YES	YES	YES
Village FE	YES	YES	YES	YES	YES	YES	YES	YES	YES
Region*Year FE	NO	NO	NO	NO	NO	NO	YES	NO	NO
Distance to road	NO	NO	NO	NO	NO	YES	NO	NO	NO
Sample	1998-2009	1998-2009	1998-2009	1998-2003	1998-2003	1998-2003	1998-2003	1999-2000	1999-2000

But

- Empirically, long-lived dictators (Mobutu, Mugabe, the Duvaliers in Haiti) are not more developmental, and if anything seem to be among the most kleptocratic.
- Conceptually, this equates the state with organized banditry. But is that right?
- In fact, the view, going back to Huntington, that state build capacity when they are strong against all other groups in society may be too simplistic.

Alternative Thesis

- Different types of stable states, which we call *Absent Leviathan* (weak state), *Despotic Leviathan* (despotic state), and *Shackled Leviathan* (inclusive state).

- Very different implications from different types of states:
  - For *conflict resolution*—in the Absent Leviathan, there is "dominance" of the strong against weak, Hobbesia "Warre"; in the Despotic and Paper Leviathans, there is dominance of the state against the rest.
  - For *public services*—in the Absent Leviathan, there is none; in the Paper Leviathan, very little; in the Despotic Leviathan, the public services that the state deems appropriate are provided).
  - For *economic growth*—very little of it in the Absent Leviathan; distorted, non-sustained growth in the Paper and Despotic Leviathans).

Alternative Thesis (continued)

- Perspectives on their evolution:
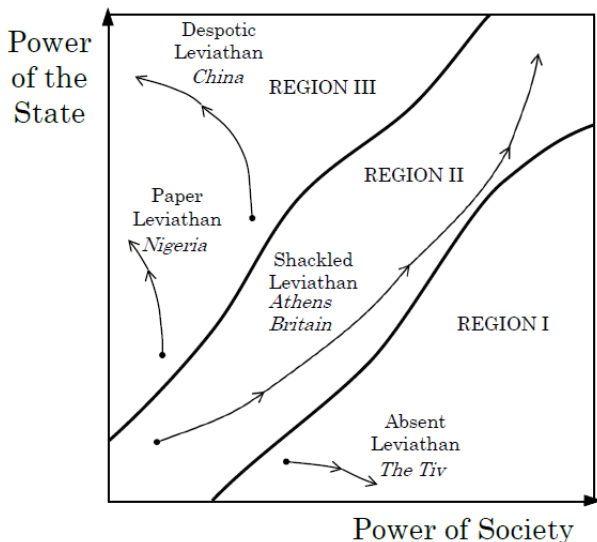
Alternative Thesis (continued)

- Contrary to structural views (e.g., Hintze, 1975, Tilly, 1990), great diversity within similar geographies and cultures. Contrary to (post-)Hegelian views similar to "End of History", no tendency for these states to converge into a unitary form.

- Most importantly, contrary to common views (e.g., Huntington), a state capable of providing effective conflict resolution, services and appropriate economic policy (our Shackled Leviathan) needs a strong society, not a strong leader or uncontested monopoly of violence—the opposite of the "state first" theses.

  - In other words, it needs to mobilize the *Red Queen Effect*, whereby state and society need to race to become stronger against the other, in the process of becoming more capable.

The Rest of This Lecture

- A model of the race (or the lack thereof) between state and society, and how this leads to a simplified form of the phase diagram above.
- Main mechanism: a contest between state and society.
  - If the state is too weak relative to society, it is dominated by society, and state building doesn't get off the ground—weak state or Absent Leviathan.
  - If the state is too strong relative to society, society gives up its attempts to control it—and how strong the state becomes, i.e., the balance between Despotic and Paper Leviathan, depends on the costs and benefits of state strength.
  - If the two are equally matched, they are both encouraged to invest in their "capabilities"—inclusive state or Shackled Leviathan.
- Note that this is just one aspect over a brother theory (one important aspect, relevant to this is the notion of "consensually strong state" from Acemoglu, 2005, also featuring in the Shackled Leviathan notion in the book, but not in this model).

An Example of Region III: The Tiv in Nigeria
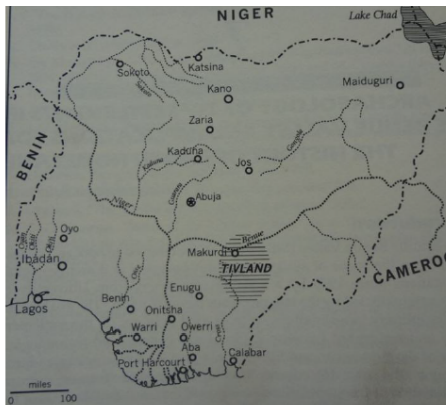
- *Social norms of societal control*—over political inequality and economic inequality.

The Tiv in Nigeria (continued)

- During the summer of 1939, social and economic activity came to a standstill in Tivland because of a cult called *Nyambua*. At the heart of the cult was a shrine and a man called Kokwa who sold charms to provide protection from *mbatsav* or "witches".

- *Tsav* means "power", particularly power over others. A person with tsav (it is a substance that grows on the heart of a person) can make others do what they want and kill them by using the power of fetishes and tsav can be increased by cannibalism.

  "A diet of human flesh makes the tsav, and of course the power, grow large. Therefore the most powerful men, no matter how much they are respected or liked, are never fully trusted. They are men of tsav - and who knows?" (Bohannon, 1958)

- The people with tsav belong to an organization — the mbatsav, which means a group of *witches*.

- Mbatsav also means: *Powerful people*.

The Tiv in Nigeria (continued)

- In 1939, the Nyambua cult had turned against the 'chiefs' created by British indirect rule (the Tiv had no chiefs before).
- In fact, turning against the powerful was a common occurrence:

  "... the Tiv have taken strong measures to overcome the mbatsav. These big movements have taken place over a period extending from the days of the ancestors into modern times" (Akiga, 1939).
  "Men who had acquired too much power ... were whittled down by means of witchcraft accusations.. Nyambua was one of a regular series of movements to which Tiv political action, with its distrust of power, gives rise so that the greater political institutions - the one based on the lineage system and a principle of egalitarianism - can be preserved" (Bohannon, 1958)

- But to have a state someone has to become powerful, start giving orders to others who accept their authority...

Why Stay in Region III?

- Why not start from a position of societal strength as with the Tiv, and then engineer the building of a Shackled Leviathan?

- Fear of *slippery slopes*: the Tiv did not have the institutions to check power once it started being accumulated.

  - Their social norms were predicated on the notion that they had to nip any accumulation of power in the bud. So once the social norms were broken and a group or individual allowed to become economically or politically powerful enough, there was nothing they could do to stop its domination.

- A lot of parallels to the norms and attitudes towards power in other stateless societies.

An Example of Region II: Ancient Greece

- Classical Greece: huge diversity of states—— Athenian democracy, despotism in Syracuse and Sparta, absence of effective state in Thrace, Paper Leviathan in Crete, etc.
- Recent scholarship (Ian Morris, Josh Ober) ties the roots of the economic boom from 700 BC onwards to the emergence of inclusive economic and political institutions starting with the reforms of Solon in 594 BC:
  - economic: made enserfing an Athenian citizen illegal, established freedom of movement within Attica, implemented an egalitarian land reform.
  - political: assembly which all Athenian citizens could attend; created a Council of 400 equally representing the 4 traditional tribes of Athens. Although the chief executive offices were reserved for elites, their decisions could be challenged by anyone in front of juries which were composed of all classes.

Shackled Leviathan in Greece

- Solon's reforms came out in the context of deep and sustained conflict between elites and masses.
- Consolidated by Cleisthenes in 508/7 BC, once again in the context of ongoing conflict.
  - New Council of 500 chosen at lot from all of Attica. You had to be older than 30 but could only serve for a year and at most twice in your life (almost every citizen ended up serving once in their life).
- The most interesting aspect of Solon and Cleisthenes reforms were the institutionalization of social norms for controlling elites.
  - It was these institutional prerequisites that were important for the building of a Shackled Leviathan in Greece that were absent in the Tiv.
- Solon's *Hubris Law* which made behavior aimed at humiliation and intimidation against any resident of Athens illegal.

Shackled Leviathan in Greece (continued)

- Cleisthenes *Ostracism Law*:
  - Every year the Assembly voted on whether there should be an ostracism. If at least 6,000 voted and 50% said yes then each citizen wrote a name on a fragment of broken pottery (an *ostrakon*, hence ostracism). Whoever got the most votes was banished from Athens for 10 years.
  - Fantastic device for disciplining elites who threatened to become too powerful and overthrow inclusive institutions (next slide).
  - A threat "off the equilibrium path" in the 180 years where the institutions functioned only 15 people were actually ostracized, but the threat was ever present.

The Model

- Consider a game with two types of players: civil society and an elite synonymous with the state.

- Let us assume that the game is played between non-overlapping generations of representatives of civil society and the state, and thus without forward-looking behavior.

- At time t, the state variables inherited from the previous period are $(x_{t-\Delta}, s_{t-\Delta}) \in [0, 1]^2$, where the first element corresponds to the strength (or conflict capacity) of civil society and the second to the strength of the state controlled by the elite

- We will take Δ to be small so as to work with differential, rather than difference equations.

Investment Decisions

- The players simultaneously make their investment decisions, $i_t^x \geq 0$ and $i_t^s \geq 0$ such that

$$x_t = x_{t-\Delta} + i_t^x \Delta - \delta \Delta$$

and

$$s_t = s_{t-\Delta} + i_t^s \Delta - \delta \Delta.$$

Production

- A state and society with strengths s_t and x_t produces output/surplus given by

$$f(s_t, x_t),$$

  where f is assumed to be nondecreasing and differentiable. Let us first simplify the discussion by imposing:

Assumption 0 $f(s, x) = 1$ for all $(x, s) \in [0, 1]^2$.

- This assumption simplifies the treatment by making the state and civil society symmetric as players.
- Generalizations discussed below.

Contests for Power

- There is conflict over the division of production.
- At date t, if the state and citizens decide to fight, then one side will win and capture all of the output of the economy, and the other side receives zero. Winning probabilities are functions of relative strengths. In particular, the state will win if

$$s_t \geq x_t + \sigma,$$

  where σ is drawn from the distribution H, and denote its density by h.
- The existence of the shock captures the stochastic nature of winning the conflict.

Costs

- The costs of investment of society and state are

$$\Delta \cdot C_x(i_t^x, x_{t-\Delta}) = \begin{cases} c_x(i_t^x) & \text{if } x_{t-\Delta} > \gamma_x, \\ c_x(i_t^x) + (\gamma_x - x_{t-\Delta}) \, i_t^x & \text{if } x_{t-\Delta} \le \gamma_x. \end{cases}$$

$$\Delta \cdot C_s(i_t^s, s_{t-\Delta}) = \begin{cases} c_s(i_t^s) & \text{if } s_{t-\Delta} > \gamma_s, \\ c_s(i_t^s) + (\gamma_s - s_{t-\Delta}) i_t^s & \text{if } s_{t-\Delta} \le \gamma_s. \end{cases}$$

- Because this cost is defined as per unit of time, it is multiplied by Δ.
- The term $\gamma_x > 0$ captures the "increasing returns" nature of conflict: once one of the players stops making investments in its conflict capacity, it faces greater costs to get started.

Assumptions

Assumption 1   **1** c_x and c_s are continuously differentiable, strictly increasing and weakly convex over $\mathbb{R}_+$, and satisfy $\lim_{x \to \infty} c_x(x) = \infty$ and $\lim_{s \to \infty} c_s(s) = \infty$.

2
$$\frac{|c_s''(\delta) - c_x''(\delta)|}{\min\{c_s''(\delta), c_x''(\delta)]} < \frac{1}{\sup_z |h'(z)|}$$

3
$$c_s'(\delta) + \gamma_s \geq c_x'(\delta) \text{ and } c_x'(\delta) + \gamma_x \geq c_s'(\delta).$$

Assumptions (continued)

Assumption 2 h is differentiable, single-peaked and symmetric around zero and satisfies for each $z \in \{x, s\}$:

$$c_z'(\delta) > h(1)$$

and

$$\min\{h(0) - \gamma_z; h(\gamma_z)\} > c_z'(\delta).$$

Objective Functions

- Under these assumptions at time t civil society maximizes

$$H(x_t - s_t) - \Delta \cdot C_x(x_t, x_{t-\Delta})$$

while the state maximizes

$$H(s_t - x_t) - \Delta \cdot C_s(s_t, s_{t-\Delta})$$

where we have used the investment equation to substitute in for the state variables.

Investment Decisions

- Given Assumptions 1 and 2, the investment decisions of both state and civil society are given by their respective first-order conditions.
- Take the limit $\Delta \to 0$, the optimality conditions for the state and society as

$$
\begin{aligned}
h(s_t - x_t) &\le c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} && \text{if } \dot{s}_t = -\delta \text{ or } s_t = 0, \\
h(s_t - x_t) &\ge c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} && \text{if } s_t = 1, \\
h(s_t - x_t) &= c_s'(\dot{s}_t + \delta) + \max\{0; \gamma_s - s_t\} && \text{otherwise,}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
h(x_t - s_t) &\le c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} && \text{if } \dot{x}_t = -\delta \text{ or } x_t = 0, \\
h(x_t - s_t) &\ge c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} && \text{if } x_t = 1, \\
h(x_t - s_t) &= c_x'(\dot{x}_t + \delta) + \max\{0; \gamma_x - x_t\} && \text{otherwise.}
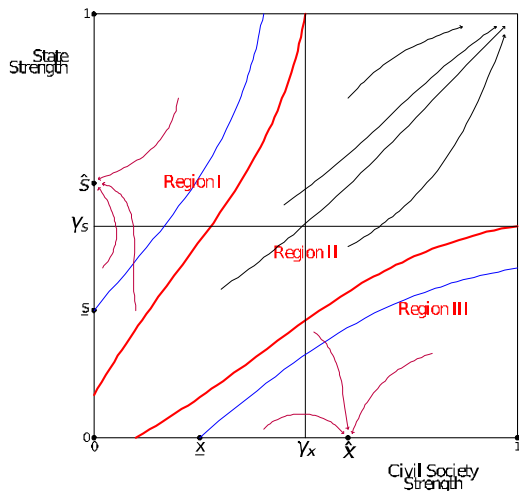\end{aligned}
\tag{2}
$$

The Main Result

Proposition

There are three locally asymptotically stable steady states

1. $x^* = s^* = 1$.
2. $x^* = 0$ and $s^* \in (\gamma_s, 1)$.
3. $x^* \in (\gamma_x, 1)$ and $s^* = 0$.
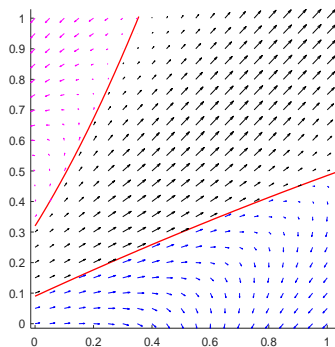
Global Dynamics

Numerical Results

- Take the following cost functions for state and society:

$$0.006 \times (i + \delta) + 0.001 \times \left(\frac{i}{\delta}\right)^2,$$

- $\gamma_x = 0.3$, $\gamma_s = 0.6$, and $\delta = 0.05$.

Interpretation

- These three asymptotically stable steady states correspond to very different types of states/governments
  - $x^* = s^* = 1$: here both state and society are strong and this results from a dynamic where each pushes the other in accumulating strength — this is the highest capacity.
  - $x^* = 0$ and $s^* \in (\gamma_s, 1)$: society is 'prostrate' (to use the terminology of James Scott) but as a consequence the state gives up and is weaker than the previous case — there is lower capacity even if the state is dominant in society.
  - $x^* \in (\gamma_x, 1)$ and $s^* = 0$: society dominates the state which gives up the fight.

Red Queen Effect

- We can see this from the dynamics of Region II.
- Also, note that investment incentives are highest when

$$h(x - s) = h(s - x) \approx h(0).$$

- Both parties are discouraged from investment when there is a big difference between their strengths.

Sketch of the Proof

- Let us outlined the proof for the first part.
- At $x^* = s^* = 1$, the marginal cost of investment for player $z \in \{x, s\}$ is $c_z'(\delta)$, while the marginal benefit starting from this point is $h(0)$.
- Assumption 2 $\Rightarrow$ marginal benefit $>$ marginal cost, and thus $x^* = s^* = 1$ is a steady state.
- For asymptotic stability, first note that the laws of motion of x and s in the neighborhood of $x^* = s^* = 1$ are given by

$$
\begin{aligned}
c_x'(\dot{x} + \delta) &= h(x - s) \\
c_s'(\dot{s} + \delta) &= h(s - x).
\end{aligned}
$$

- Why? We are away from the steady state and there cannot be an immediate jump and thus the first-order conditions have to hold in view of Assumption 1, and because we are in the neighborhood of the steady state $(1, 1)$, we must have $x > \gamma_x$ and $s > \gamma_s$.

Sketch of the Proof (continued)

- This dynamical system can then be written as

$$
\begin{aligned}
\dot{x} &= (c'_x)^{-1}(h(x - s)) - \delta \\
\dot{s} &= (c'_s)^{-1}(h(s - x)) - \delta.
\end{aligned}
\tag{3}
$$

- Now to establish asymptotic stability, we will show that

$$
L(x, s) = \frac{1}{2}(1 - x)^2 + \frac{1}{2}(1 - s)^2
$$

is a Lyapunov function in the neighborhood of the steady state $(1, 1)$.

- Indeed, $L(x, s)$ is continuous and differentiable, and has a unique minimum at $(1, 1)$.

- We will next verify that in is sufficiently small neighborhood of $(1, 1)$, $L(x, s)$ is decreasing along solution trajectories of the dynamical system given by (3).

Sketch of the Proof (continued)

- Since L is differentiable, for $x \in (\gamma_x, 1)$ and $s \in (\gamma_s, 1)$, we can write

$$\frac{dL(x,s)}{dt} = -(1-x)\dot{x} - (1-s)\dot{s}.$$

- First note that since $h(x - s) > c'_x(\delta)$ and $h(s - x) > c'_s(\delta)$ for x and s in a sufficiently small neighborhood of $(1,1)$, we have both $\dot{x} > 0$ and $\dot{s} > 0$.
- This implies that, in this range, both terms in $\frac{dL(x,s)}{dt}$ are negative, and thus $\frac{dL(x,s)}{dt} < 0$.
- Moreover, the same conclusion applies when $x = 1$ (respectively when $s = 1$), with the only modification that $\frac{dL(x,s)}{dt}$ will not only have the $\dot{s}$ (respectively the $\dot{x}$) term, which continues to be strictly negative.
- Then the asymptotic stability of $(1,1)$ follows from LaSalle's Theorem (which takes care of the fact that our steady state is on the boundary of the domain of the dynamical system in question).

Sketch of the Proof (continued)

- The argument for the existence and local stability of the other steady states is analogous.

- To show that there are no other locally stable steady states, we consider all different types of steady states, and either show that they do not exist or that they cannot be locally stable even if they existed.

Forward-Looking Model

- Let us now consider forward-looking players.
- To maximize the parallel with the model with short-lived players, we assume that both players again correspond to sequence of non-overlapping generations, but each generation has an exponentially-distributed lifetime or equivalently, a Poisson end date with parameter $\beta = e^{-\rho\Delta}$.
- We assume that this random end date is the only source of discounting.
- Clearly, as the period length Δ shrinks, discounting between periods will also decline (and the discount factor will approach 1).
- Again to maximize the parallel with our static model, we also assume that there is an expectation one conflict between the two players during the lifetime of each generation. Since with this Poisson specification, the expected lifetime of his generation is $1/(1-\beta)$, this implies that a conflict arrives at the rate $1 - \beta$.

Value Functions

- Now the maximization problem of each player is a solution to a recursive, dynamic programming problem, written as

$$V_x(x_{t-\Delta}, s_{t-\Delta}) = (1-\beta)H(x_{t-\Delta} - s_{t-\Delta})$$
$$+ \max_{x_t \geq 0} \; [-\Delta \cdot C_x(x_t, x_{t-\Delta}) + \beta V_x(x_t, s_\Delta^*(x_{t-\Delta}, s_{t-\Delta}))],$$

and

$$V_s(x_{t-\Delta}, s_{t-\Delta}) = (1-\beta)H(s_{t-\Delta} - x_{t-\Delta})$$
$$+ \max_{s_t \geq 0} \; [-\Delta \cdot C_s(s_t, s_{t-\Delta}) + \beta V_s(x_\Delta^*(x_{t-\Delta}, s_{t-\Delta}), s_t)].$$

- Here, we have evaluated the value functions after the cost of investment is incurred but before the conflict stage, which turns out to be slightly convenient for what follows.

Value Functions (continued)

- We again multiply the benefits and costs with Δ, since these are flow benefits, and we have conditioned on Δ in writing the value functions for emphasis.
- Also, $x_\Delta'^*(x, s)$ and $s_\Delta'^*(x, s)$ are the policy functions, which give the next period's values of the state variables as a function of this period's values.

Equilibrium

- A dynamic equilibrium in this setup is given by a pair of policy functions, $x'^*_\Delta(x, s)$ and $s'^*_\Delta(x, s)$ which give the next period's values of the state variables as a function of this period's values (for $\Delta > 0$), and each solves the its corresponding value function taking the policy function of the other party is given.

- Once these policy functions are determined, the dynamics of civil society and state strength can be obtained by iterating over these functions.

Continuous-Time Limit

- As $\Delta \to 0$, the value functions $V_x(x,s)$ and $V_s(x,s)$ (implicitly functions of Δ) converge to their continuous time limits $V_x(x,s)$ and $V_s(x,s)$, and the policy functions $x'^*_\Delta(x,s)$ and $s'^*_\Delta(x,s)$ converge to their limits $x'^*(x,s)$ and $s'^*(x,s)$.

- To obtain the continuous-time HJB equations, rearrange the above value functions evaluated at the optimal choices and divide both sides by Δ,

$$\frac{1-\beta}{\Delta} V_x(x_{t-\Delta}, s_{t-\Delta}) = \frac{1-\beta}{\Delta} H(x_{t-\Delta} - s_{t-\Delta}) -$$

$$\max_{x_t \geq 0} \left[C_x\left(x_t, x_{t-\Delta}\right) + (1-\beta) \frac{V_x(x_t, s^*_\Delta(x_{t-\Delta}, s_{t-\Delta})) - V_x(x_{t-\Delta}, s_{t-\Delta})}{\Delta} \right]$$

- Now note that as $\Delta \to 0$, $(1-\beta) \to 0$ and $(1-\beta)/\Delta \to \rho$.

- Moreover the last term in the previous expression tends to the total derivative of the value function with respect to time, which involves change because of the time derivatives of both x and s.

Continuous-Time HJB Equation

- Therefore, the continuous-time HJB equation for civil society is

$$\rho V_x(x,s) = \rho H(x-s)$$
$$+ \max_{\dot{x} \geq -\delta} \left\{ -C_x(x,\dot{x}) + \frac{\partial V_x(x,s)}{\partial x} \dot{x} \right\} + \frac{\partial V_x(x,s)}{\partial s} \dot{s}^*(x,s).$$

- Here we have used the notation $C_x(x,\dot{x})$ to denote the continuous-time cost function as a function of the change in the conflict capacity of civil society, and $\dot{x}^*(x,s)$ and $\dot{s}^*(x,s)$ are the continuous-time policy functions.

- With a similar argument,

$$\rho V_s(x,s) = \rho H(s-x)$$
$$+ \max_{\dot{s} \geq -\delta} \left\{ -C_s(s,\dot{s}) + \frac{\partial V_s(x,s)}{\partial s} \dot{s} \right\} + \frac{\partial V_s(x,s)}{\partial x} \dot{x}^*(x,s).$$

Optimality Conditions

- The first-order optimality conditions for civil society are given by

$$\frac{\partial C_x(x, \dot{x})}{\partial \dot{x}} = \frac{\partial V_x(x, s)}{\partial x} \qquad \text{if} \qquad -\delta < \dot{x}(x, s), \text{ and } x \in (0, 1),$$

$$\frac{\partial C_x(x, \dot{x})}{\partial \dot{x}} \leq \frac{\partial V_x(x, s)}{\partial x} \qquad \text{if} \qquad x = 1, \tag{4}$$

$$\frac{\partial C_x(x, \dot{x})}{\partial \dot{x}} \geq \frac{\partial V_x(x, s)}{\partial x} \qquad \text{if} \qquad \dot{x}(x, s) = -\delta \text{ or } x = 0.$$

Dynamical System with Forward-Looking Players

- In the first case, when we have an interior solution, we can also write

$$
\dot{x} = \begin{cases} (c'_x)^{-1} \left(\frac{\partial V_x(x,s)}{\partial x} - \gamma_x + x \right) & \text{if } x \leq \gamma_x \\ (c'_x)^{-1} \left(\frac{\partial V_x(x,s)}{\partial x} \right) & \text{if } x > \gamma_x \end{cases} .
$$

- The first-order conditions for state are also similar, and for interior solution, they yield

$$
\dot{s} = \begin{cases} (c'_s)^{-1} \left(\frac{\partial V_s(x,s)}{\partial s} - \gamma_s + s \right) & \text{if } s \leq \gamma_s \\ (c'_s)^{-1} \left(\frac{\partial V_s(x,s)}{\partial s} \right) & \text{if } s > \gamma_s \end{cases} .
$$

Envelope Theorem

- Now applying the envelope condition we obtain

$$\frac{\partial V_x(x,s)}{\partial x} = h(x-s)$$

$$+\frac{1}{\rho}\left\{-\frac{\partial C_x(x,\dot{x})}{\partial x} + \frac{\partial^2 V_x}{\partial x^2}\dot{x} + \frac{\partial^2 V_x}{\partial s \partial x}\dot{s}^*(x,s) + \frac{\partial V_x}{\partial s}\frac{\partial \dot{s}^*(x,s)}{\partial x}\right\}.$$

- We next show that each one of the four terms in the curly square brackets has a finite limit as $\rho \to \infty$, which I skipped here for everybody's benefit.

Limiting Dynamical System

- But then, as $\rho \to \infty$, the dynamical systems converge to

$$\dot{x} = \begin{cases} (c_x')^{-1} \left(h(x-s) - \gamma_x + x \right) & \text{if } x \leq \gamma_x \\ (c_x')^{-1} \left(h(x-s) \right) & \text{if } x > \gamma_x \end{cases}.$$

and

$$\dot{s} = \begin{cases} (c_s')^{-1} \left(h(s-x) - \gamma_s + s \right) & \text{if } s \leq \gamma_s \\ (c_s')^{-1} \left(h(s-x) \right) & \text{if } s > \gamma_s \end{cases}.$$

Main Result

Proposition

Suppose Assumptions 0, 1 and 2 hold. Then there exists a discount rate $\bar{\rho} > 0$ such that for all $\rho > \bar{\rho}$, there are three (locally) asymptotically stable steady states:
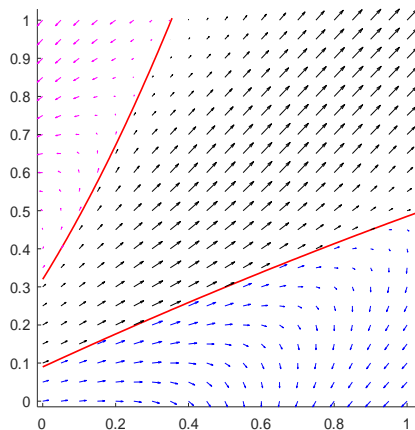
1. $x^* = s^* = 1$.
2. $x^* = 0$ and $s^* \in (\gamma_s, 1)$.
3. $x^* \in (\gamma_x, 1)$ and $s^* = 0$.

Moreover, for all $\rho < \bar{\rho}$, there exists a unique globally stable steady state $x^* = s^* = 1$.

- Therefore, with sufficient discounting, all of the same insights apply.

Numerical Results for the Forward-Looking Model

- Same parameters as above, but now also $\rho = 500$. The resulting vector field is identical to the static model:

Main Idea

- Comparative statics are *conditional*—they depend exactly where a society is and to which region it is being shifted by changes in parameters

Generalized Results

- Mathematically, we relax Assumption 0.

Assumption $0'$ $f(x, s) = \phi_0 + \phi_x x + \phi_s s$, where $\phi_0 > 0$, $\phi_x > 0$ and
            $\phi_s > 0$.

- In addition, we modify Assumptions 1 and 2 in minor ways, in particular, ensuring that at $x = s = 1$ the marginal benefit of investment exceeds the cost for both parties.

- Then, all of the results so far generalize.

Comparative Statics

- For comparative statics, let us also adopt:

Assumption 3 $h(y)(\phi_0 + \phi_z y) + H(y)\phi_z$ is a decreasing function of y for $z \in \{s, x\}$ and for $y \geq 0$.

- A sufficient condition for this is that the elasticity of the h function is greater than $1/2$.
- Then, we can show that:
    1. A small increase in ϕ_s has no impact on the steady states with $s = 1$ and $s = 0$, and increases the level of state strength in the steady state with $s = \hat{s}$.
    This implies that we can think of Paper Leviathan is a situation in which ϕ_s is very low, while the Despotic Leviathan is one in which it is high.
    2. A small increase in ϕ_x has analogous effects.
    3. Changes in cost functions also have similar effects.
- Furthermore, all of these parameter changes shift the boundaries of the basins of attraction.

Changes in the Basins of Attraction

- With an increase in ϕ_x, $\hat{x}$ increases, and its basin of attraction, Region III, expands. Region I tends to contract.
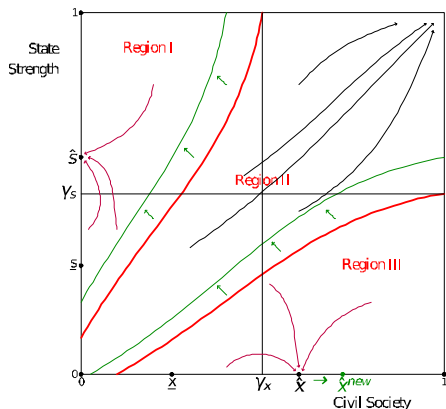
Comparative Statics in Action: Greece

- The transition from the use of bronze to iron in itself redistributed political power in society. As Gordon Childe put it

  "*cheap iron democratized agriculture and industry and warfare too*".

- Other technological revolutions.
  - The emergence of writing. Bronze Age Greece had Linear A and Linear B, restricted to the elite and use primarily for record keeping by the state. Around 800 BC a new type of writing emerged which spread much more broadly in society.
  - The perfection of hoplite warfare, perhaps connected to the spread of iron weaponry. Polities who could amass more hoplites in battle had a military advantage and it is possible that this helped to undermine a further empowerment of the mass of citizens.
  - Political leaders could not claim to rule by divine right and there was no fusion between the political elite and religion. Religious power, such as that of the oracle at Delphi, was not controlled by political elites.

Comparative Statics in Action: England

- Economic diversification intensified after the discovery of the Americas, broad participation in trade and mercantile activities
- Absence of natural resources creating very high rents from holding political power (as in Early Modern Spain with its' colonial extractions)
- Relatively weak Monarchy faced with constitutional constraints in principle since the Magna Carta of 1215
- Absence of labor coercion after the final collapse of feudalism in the wake of the Black Death in the 1340s allowed emergence of vibrant civil society

Persistence of Civil Wars

- Related idea in Acemoglu, Ticchi and Vindigni: civil wars, and thus the lack of power of the central state in many parts of the territory, persist in many African countries because the elites are afraid of increasing the power of the military.
  - Extreme example: Sierra Leone.
- The elite (often the politicians controlling the coffers) have a choice:
  - Increase the power of the military. This will end the civil war, but then will unleash a series of further political changes, either necessitating greater power sharing with the military, or other political reforms.
  - Keep the military weak. This will lead to the persistence of civil wars, but the elite can still grab a little rents.