Labor Economics, 14.661. Lecture 5: Career Concerns and Multitasking

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Introduction

- Labor economics typically dealing with supply, demand and allocations in the market.
- Much of labor is transacted within firms.
- Potential new frontier of labor economics: understand what is happening within firms.
- Two aspects:
  1. Incentives within firms
  2. Allocation of workers to firms
- We start with incentives within firms.

Recap: Basic Moral Hazard

- Imagine a single worker (agent) is contracting with a single employer (principal).
- The agent's utility function is

$$H(w, a) = U(w) - c(a)$$

- w =wage,
- $a \in \mathbb{R}_+ =$ action/effort,
- $U(\cdot) =$ concave utility function
- $c(\cdot)$ =convex cost of effort/action.
- $\bar{H} =$ outside option of the agent.
- $x =$ output/performance.

Basic Moral Hazard Framework (continued)

- Output a function of effort a and random variable $\theta \in \mathbb{R}$

$$x(a, \theta).$$

- Greater effort$\longrightarrow$higher output, so

$$x_a \equiv \frac{\partial x}{\partial a} > 0$$

- Typically, x is publicly observed, but a and θ *private information* of the worker.
- The principal cares about output minus costs:

$$V(x - w)$$

- V typically increasing concave utility function.
- Special case: V linear (risk neutral principal).

Contracts

- Let Ω be the set of observable and contractible events, so when only x is observable, $\Omega = \mathbb{R}$.
    - what is the difference between observable in contractible events?
- When any two of x, a, and θ are observable, then $\Omega = \mathbb{R}_+ \times \mathbb{R}$ (why only two?).
- A contract is a mapping

$$s : \Omega \to \mathbb{R}$$

  specifying how much the agent will be paid as a function of contractible variables.
- When there is limited liability, then

$$s : \Omega \to \mathbb{R}_+$$

Timing of Events

- This is a dynamic game of asymmetric or incomplete information (though incompleteness of information not so important here, why?)
- *Timing:*

1. The principal offers a contract $s : \Omega \to \mathbb{R}$ to the agent.
2. The agent accepts or rejects the contract. If he rejects the contract, he receives his outside utility $\overline{H}$.
3. If the agent accepts the contract $s : \Omega \to \mathbb{R}$, then he chooses effort a.
4. Nature draws θ, determining $x(a, \theta)$.
5. Agent receives the payment specified by contract s.

- Look for a Perfect Bayesian Equilibrium.

Incentives-Insurance Tradeoff

- The problem is

$$
\begin{array}{lll}
 & \max\limits_{s(x),a} \mathbb{E}\left[V(x - s(x))\right] & \\
\text{s.t.} & \mathbb{E}\left[H(s(x), a)\right] \geq \overline{H} & \text{Participation Constraint (PC)} \\
\text{and} & a \in \arg\max\limits_{a'} \mathbb{E}\left[H(s(x), a')\right] & \text{Incentive Constraint (IC)}
\end{array}
$$

Incentives-Insurance (continued)

- Suppressing θ, we work directly with $F(x \mid a)$.
- Natural assumption:

$$F_a(x \mid a) < 0,$$

(implied by $x_a > 0$)
$\rightarrow$an increase in a leads to a *first-order stochastic-dominant* shift in F.

- Recall that F first-order stochastically dominates another G, if

$$F(z) \leq G(z)$$

for all z.

Basic Moral Hazard Problem

- Canonical problem:

$$
\begin{aligned}
\max_{s(x),a} \quad & \int V(x - s(x)) dF(x \mid a) \\
\text{s.t.} \quad & \int \left[U(s(x) - c(a)) \right] dF(x \mid a) \geq \overline{H} \\
a \in \arg\max_{a'} \quad & \int \left[U(s(x)) - c(a') \right] dF(x \mid a')
\end{aligned}
$$

- Considerably more difficult, because the *incentive compatibility*, IC, constraint is no longer an inequality constraint, but an abstract constraint requiring the value of a function,

$$
\int \left[U(s(x)) - c(a') \right] dF(x \mid a'),
$$

to be highest when evaluated at $a' = a$.

- Difficult to make progress on this unless we take some shortcuts.

The First-Order Approach

- The standard shortcut is the "*first-order approach*,".
- It involves replacing the IC constraint with the first-order conditions of the agent, that is, with

$$\int U(s(x)) f_a(x \mid a) dx = c'(a).$$

- Why is this a big assumption?
  - *Incorrect argument*: suppose that

$$\max_{a'} \int \left[U(s(x)) - c(a') \right] dF(x \mid a')$$

    is strictly concave
  - Why is this argument in correct?
- The first-order approach is a very strong assumption and often invalid.
- **Special care necessary.**

Solution to the Basic Moral Hazard Problem

- Now using the first-order approach the principal's problem becomes

$$\min_{\lambda,\mu}\max_{s(x),a}\mathcal{L}=\int\left\{V(x-s(x))\quad+\lambda\left[U(s(x))-c(a)-\overline{H}\right]\qquad+\right.$$
$$\left.\mu\left[U(s(x))\frac{f_a(x\mid a)}{f(x\mid a)}-c'(a)\right]\right\}f(x\mid a)dx$$

- Now carrying out "point-wise maximization" with respect to $s(x)$:

$$\begin{aligned}0 &= \frac{\partial\mathcal{L}}{\partial s(x)}\\ &= -V'(x-s(x))+\lambda U'(s(x))+\mu U'(s(x))\frac{f_a(x\mid a)}{f(x\mid a)}\text{ for all } x\end{aligned}$$

Solution to the Basic Moral Hazard Problem (continued)

- Therefore:

$$\frac{V'(x-s(x))}{U'(s(x))}=\lambda+\mu\frac{f_a(x\mid a)}{f(x\mid a)}. \tag{1}$$

- What happens when $\mu=0$?
- Also, we must have $\lambda>0$. Why?

Incentive-Insurance Tradeoff Again

- Can we have perfect risk sharing?
- This would require

$$V'(x - s(x)) / U'(s(x)) = \text{constant}.$$

- Since V' is constant, this is only possible if U' is constant.
- Since the agent is risk-averse, so that U is strictly concave, this is only possible if $s(x)$ is constant.
- But if $s(x)$ is constant and effort is costly, the incentive compatibility constraint will be violated (unless the optimal contract asks for $a = 0$).

Back to the Optimal Contract

- Let the level of effort that the principal wants to implement be $\bar{a}$.
- Then the optimal contract solves:

$$\frac{V'(x - s(x))}{U'(s(x))} = \lambda + \mu \frac{f_a(x \mid \bar{a})}{f(x \mid \bar{a})}.$$

- If $a(x) > \bar{a}$, then

$$f_a(x \mid \bar{a}) / f(x \mid \bar{a}) > 0$$

  so V' / U' has to be greater, which means that U' has to be lower.
- Therefore $s(x)$ must be increasing in x.
- Intuitively, when the realization of output is *good news* relative to what was expected, the agent is rewarded, when it is *bad news*, he is punished.

Recap Continued: Robustness of Contracts

- Basic moral hazard problem captures some nice intuitions about insurance-incentive trade-offs.
- But little prediction about the form of equilibrium contracts, and what's worse is that an even very simple problems, the form of contracts is very complex and highly nonlinear.
- Is this a good prediction? Perhaps not because these contracts are not "robust"?
- What does "robust" mean?

Robustness (continued)

- Holmstrom and Milgrom: no manipulation in *dynamic principal-agent* problems
- Consider a model in continuous time.
- The interaction between the principal and the agent take place over an interval normalized to $[0, 1]$.
- The agent chooses an effort level $a_t \in A$ at each instant after observing the relaxation of output up to that instant.
- The output process is given by the continuous time random walk, that is, the following *Brownian motion process*:

$$dx_t = a_t dt + \sigma dW_t$$

  where W is a standard Brownian motion (Wiener process).
- This implies that its increments are independent and normally distributed, that is, $W_{t+\tau} - W_t$ for any t and τ is distributed normally with variance equal to τ.

Robustness (continued)

- Let
$$X^t = (x_\tau; 0 \leq \tau < t)$$
be the entire history of the realization of the increments of output x up until time t (or alternatively a "sample path" of the random variable x).
- Effort choice
$$a_t : X^t \longrightarrow A.$$
- Similarly, the principal also observes the realizations of the increments (though obviously not the effort levels and the realizations of W_t).
- Therefore, contract
$$s_t : X^t \longrightarrow \mathbb{R}.$$

Robustness (continued)

- Holmstrom and Milgrom assume that utility of the agent is

$$u\left(C_1 - \int_0^1 c(a_t)d_t\right)$$

- C_1 is consumption at time $t = 1$, while $c(\cdot)$ is a strictly convex cost function.
- Two special assumptions:
  1. the individual only derives utility from consumption at the end (at time $t = 1$) and
  2. the concave utility function applies to consumption minus the total cost of effort between 0 and 1.
- A further special assumption constant absolute risk aversion (CARA) utility:

$$u(z) = -\exp(-rz) \tag{2}$$

Robustness: Key Result

- In this case, optimal contracts are only a function of (cumulative) output x_1 and are *linear*.
- Independent of the exact sample path leading to the cumulative output.
- Moreover, in response to this contract the optimal behavior of the agent is to choose a constant level of effort, which is also independent of the history of past realizations of the stochastic shock.
- Loose intuition: with any nonlinear contract there will exist an event, i.e., a sample path, after which the incentives of the agent will be distorted, whereas the linear contract achieves a degree of "robustness".

Recap Continued: Linear Contracts

- Motivated by this result, many applied papers look at the following *static* problem:
  1. The principal chooses a linear contract, of the form $s = \alpha + \beta x$.
  2. The agents chooses $a \in A \equiv [0, \infty]$.
  3. $x = a + \varepsilon$ where $\varepsilon \sim \mathcal{N}\left(0, \sigma^2\right)$
- The principal is risk neutral
- The utility function of the agent is

$$U(s, a) = -\exp\left(-r\left(s - c\left(a\right)\right)\right)$$

with

$$c(a) = ca^2/2$$

Linear Contracts (continued)

- Loose argument: a linear contract is approximately optimal here.
- Is this true?

Linear Contracts (continued)

- Even if linear contracts are not optimal in the static model, they are attractive for their simplicity and can be justified as thinking of the dynamic model.
- They are also easy to characterize.
- The first-order approach works in this case.
- The maximization problem of the agent is

$$\begin{aligned} & \max_a \mathbb{E}\left\{-\exp\left(-r\left(s\left(a\right)-c\left(a\right)\right)\right)\right\} \\ = \quad & \max_a \left\{-\exp\left(-r\mathbb{E}s\left(a\right)+\frac{r^2}{2}\mathrm{Var}\left(s\left(a\right)\right)-rc\left(a\right)\right)\right\} \end{aligned}$$

- Where is the second line coming from?

Linear Contracts (continued)

- Therefore, the agent's problem is

$$\max_a \left\{ \mathbb{E}s(a) - \frac{r}{2}\text{Var}(s(a)) - \frac{c}{2}a^2 \right\}$$

- Substituting for the contract:

$$\max_a \beta a - \frac{c}{2}a^2 - \frac{r}{2}\beta^2\sigma^2$$

- The first-order condition for the agent's optimal effort choice is:

$$a = \frac{\beta}{c}$$

Linear Contracts (continued)

- The principal will then maximize

$$\max_{a,\alpha,\beta} \mathbb{E}\left((1-\beta)(a+\varepsilon)-\alpha\right)$$

subject to

$$\begin{aligned} a &= \frac{\beta}{c} \\ \alpha+\frac{\beta^2}{2}\left(\frac{1}{c}-r\sigma^2\right) &\geq \bar{h} \end{aligned}$$

- First equation is the incentive compatibility constraint in the second is the participation constraint (with $\bar{h}=-\ln\left(-\bar{H}\right)$).

Linear Contracts: Solution

- Solution:

$$\beta^* = \frac{1}{1 + rc\sigma^2} \tag{3}$$

and

$$\alpha^* = \bar{h} - \frac{1 - rc\sigma^2}{2c^2 \left(1 + rc\sigma^2\right)^2},$$

- Because negative salaries are allowed, the participation constraint is binding.
- The equilibrium level of effort is

$$a^* = \frac{1}{c\left(1 + rc\sigma^2\right)}$$

- Always lower than the first-best level of effort which is $a^{fb} = 1/c$.

Linear Contracts: Comparative Statics

- Incentives are *lower powered*—i.e., β^* is lower, when
  - the agent is more risk-averse is the agent, i.e., the greater is r,
  - effort is more costly, i.e., the greater is c,
  - there is greater uncertainty, i.e., the greater is σ^2.

Evidence

- The evidence on the basic principal-agent model is mixed.
- Evidence in favor of the view that *incentives matter.*
- Lazear: data from a large auto glass installer, high incentives lead to more effort.
    - For example, Lazear's evidence shows that when this particular company went from fixed salaries to piece rates productivity rose by 35% because of greater effort by the employees (the increase in average wages was 12%), but part of this response might be due to selection, as the composition of employees might have changed.

Evidence (continued)

TABLE 3—REGRESSION RESULTS

Regression number	Dummy for PPP person-month observation	Tenure	Time since PPP	New regime	R^2	Description
1	0.368 (0.013)				0.04	Dummies for month and year inc
2	0.197 (0.009)				0.73	Dummies for month and year; wo specific dummies included (2,7 individual workers)
3	0.313 (0.014)	0.343 (0.017)	0.107 (0.024)		0.05	Dummies for month and year inc
4	0.202 (0.009)	0.224 (0.058)	0.273 (0.018)		0.76	Dummies for month and year; wo specific dummies included (2,7 individual workers)
5	0.309 (0.014)	0.424 (0.019)	0.130 (0.024)	0.243 (0.025)	0.06	Dummies for month and year inc

Notes: Standard errors are reported in parentheses below the coefficients.
Dependent variable: ln output-per-worker-per-day.
Number of observations: 29,837.

Evidence (continued)

- Similar evidence is reported in other papers.
  - For example, Kahn and Sherer, using the personnel files of a large company, show that employees (white-collar office workers) whose pay depends more on the subjective evaluations obtain better evaluations and are more productive.
- Incentives also matter in extreme situations.
  - John McMillan on the responsibility system in Chinese agriculture
  - Ted Groves similar effects from the Chinese industry.

Evidence (continued)

- But various pieces of evidence that high-powered incentives might backfire.
    - Ernst Fehr and Simon Gachter: that incentive contracts might destroy voluntary cooperation.
- More standard examples are situations in which high-powered incentives lead to distortions that were not anticipated by the principals.
    - e.g., consequences of Soviet incentive schemes specifying "performance" by number of nails or the weight of the materials used, leading to totally unusable products.
- Less standard but more consequential examples: Acemoglu et al. (2016)—high-powered incentives for Colombian military led to murder of civilians dressed up as guerrilla.

Evidence (continued)

- Closer to labor economics—Pascal Courty and Gerard Marschke: Job training centers, whose payments are based on performance incentives, manipulating the reporting of the time of training termination.

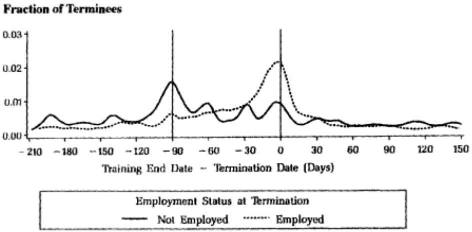
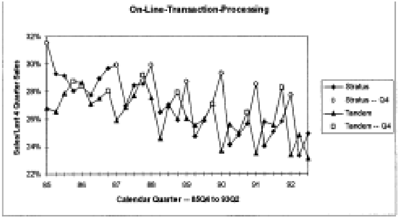


FIGURE 1. INCIDENCE OF DELAYED TERMINATION BY EMPLOYMENT PERFORMANCE OUTCOME

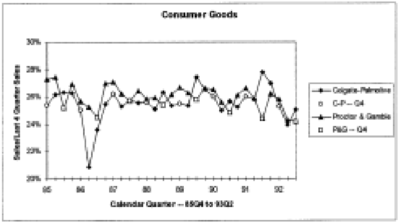
- If the outcome is good, report immediately, and if not, report late (hoping for an improvement in outcome in the meantime).

Evidence (continued)

- Paul Oyer: Managers increasing effort or shifting sales to their last fiscal quarter to improve their bonuses:



Fiscal Year Ends: Tandem - September; Stratus - December



Fiscal Year Ends: Colgate-Palmolive - December; Proctor and Gamble - June

Evidence (continued)

- The most negative evidence against the standard moral hazard models is that they do not predict the form of performance contracts.
- Prendergast: there is little association between riskiness and noisiness of tasks and the types of contracts when we look at a cross section of jobs.
- In many professions performance contracts are largely absent.
- Why could this be?
  - Again robustness.
  - Multitask issues.
  - Career concerns.

Multitask Models

- Let us now modify the above linear model so that there are two efforts that the individual chooses, a_1 and a_2, with a cost function
$$c\left(a_1, a_2\right)$$
which is increasing and convex as usual.
- These efforts lead to two outcomes:
$$x_1 = a_1 + \varepsilon_1$$
and
$$x_2 = a_2 + \varepsilon_2,$$
where ε_1 and ε_2 could be correlated.
- The principal cares about both of these inputs with potentially different weights, so her return is
$$\phi_1 x_1 + \phi_2 x_2 - s$$
where s is the salary paid to the agent.

Multitask Models (continued)

- Main difference: only x_1 is observed, while x_2 is unobserved.
- Example: the agent is a home contractor where x_1 is an inverse measure of how long it takes to finish the contracted work, while x_2 is the quality of the job, which is not observed until much later, and consequently, payments cannot be conditioned on this.
- Another example: the behavior of employees in the public sector, where quality of the service provided to citizens is often difficult to contract on.
- High-powered incentives may distort the composition of effort.

Multitask Models: Solution

- Let us focus on linear contracts of the form

$$s(x_1) = \alpha + \beta x_1$$

since x_1 is the only observable output.

- The first-order condition of the agent now gives:

$$\begin{aligned} \beta &= \frac{\partial c(a_1, a_2)}{\partial a_1} \quad (4) \\ 0 &\leq \frac{\partial c(a_1, a_2)}{\partial a_2} \text{ and } \frac{\partial c(a_1, a_2)}{\partial a_2} \times a_2 = 0. \end{aligned}$$

- So if

$$\frac{\partial c(a_1, a_2)}{\partial a_2} > 0$$

whenever $a_2 > 0$, then the agent will choose $a_2 = 0$, and there is no way of inducing him to choose $a_2 > 0$.

Multitask Models: Solution (continued)

- However, suppose that

$$\frac{\partial c\left(a_1, a_2 = 0\right)}{\partial a_2} < 0.$$

- Then, without "incentives" the agent will exert some positive effort in the second task.
- Now providing stronger incentives in task 1 can *undermine* the incentives in task 2;
  - this will be the case when the two efforts are substitutes, i.e.,

$$\partial^2 c\left(a_1, a_2\right) / \partial a_1 \partial a_2 > 0.$$

Multitask Models: Solution (continued)

- More formally, imagine that the first-order conditions in (4) have an interior solution (why is an interior solution important?).
- Then differentiate these two first-order conditions with respect to β.
- Using the fact that these two first-order conditions correspond to a maximum (i.e., the second order conditions are satisfied), we obtain

$$\frac{\partial a_1}{\partial \beta} > 0.$$

- This has the natural interpretation that high-powered incentives lead to stronger incentives as the evidence discussed above suggests.

Multitask Models: Solution (continued)

- However, in addition provided that $\partial^2 c(a_1, a_2)/\partial a_1 \partial a_2 > 0$, we also have
$$\frac{\partial a_2}{\partial \beta} < 0,$$
- Therefore, high-powered incentives in one task adversely affect the other task.
- What are the implications for interpreting empirical evidence?

Multitask Models: Solution (continued)

- What about the optimal contract?
- If the second task is sufficiently important for the principal, then she will "shy away" from high-powered incentives; if you are afraid that the contractor will sacrifice quality for speed, you are unlikely to offer a contract that puts a high reward on speed.
- In particular, the optimal contract will have a slope coefficient of

$$\beta^{**} = \frac{\phi_1 - \phi_2 \left(\partial^2 c\left(a_1, a_2\right) / \partial a_1 \partial a_2\right) / \left(\partial^2 c\left(a_1, a_2\right) / \partial a_2^2\right)}{1 + r\sigma_1^2 \left(\partial^2 c\left(a_1, a_2\right) / \partial a_1^2 - \left(\partial^2 c\left(a_1, a_2\right) / \partial a_1 \partial a_2\right)^2 / \partial^2 c\left(a_1, a_2\right)\right.}$$

- As expected β^{**} is declining in ϕ_2 (the importance of the second task) and in $-\partial^2 c\left(a_1, a_2\right) / \partial a_1 \partial a_2$ (degree of substitutability between the efforts of the two tasks).

Career Concerns

- "Career concerns" $\approx$reasons to exert effort unrelated to current compensation.
- These could be social effects.
- Or more standard: anticipation of future compensation
- Question: is competition in market for managers sufficient to give them sufficient incentives without agency contracts?

The Basic Model of Career Concerns

- Basic model due to Holmstrom.
- The original Holmstrom model is infinite horizon, but useful to start with a 2-period model.
- Output produced is equal to

$$x_t = \underbrace{\eta}_{\text{ability}} + \underbrace{a_t}_{\text{effort}} + \underbrace{\varepsilon_t}_{\text{noise}} \quad t = 1, 2$$

- Since the purpose is to understand the role of career concerns, let us go to the extreme case where there are no performance contracts.
- As before $a_t \in [0, \infty)$.

Career Concerns (continued)

- Also assume that

$$\varepsilon_t \sim \mathcal{N}(0, 1/h_\varepsilon)$$

where h is referred to as "precision" (inverse of the variance)

- Also, the prior on η has a normal distribution with mean m_0, i.e.,

$$\eta \sim \mathcal{N}(m_0, 1/h_0)$$

and $\eta, \varepsilon_1, \varepsilon_2$ are independent.

- What does it mean for the prior to have distribution?

Career Concerns (continued)

- Differently from the basic moral hazard model this is an *equilibrium* model, in the sense that there are other firms out there who can hire this agent. This is the source of the career concerns.
- Loosely speaking, a higher perception of the market about the ability of the agent, η, will translate into higher wages.
- This class of models are also referred to as “signal jamming” models, since the agent might have an interest in working harder in order to improve the perception of the market about his ability.

Career Concerns: Timing and Information Structure

- Information structure:
    - the firm, the worker, and the market all share prior belief about η (thus there is no asymmetric information and adverse selection; is this important?).
    - they all observe x_t each period.
    - only worker sees a_t (moral hazard/hidden action).
- In equilibrium firm and market correctly conjecture a_t (Why?)
    - $\rightarrow$along-the-equilibrium path despite the fact that there is hidden action, information will stay symmetric.
- The labor market is competitive, and all workers are paid their expected output.
- Recall: no contracts contingent on output (and wages are paid at the beginning of each period).

Career Concerns: Wage Structure

- Competition in the labor market→ the wage of the worker at a time t is equal to the mathematical expectation of the output he will produce given the history of its outputs

$$w_t(x^{t-1}) = \mathbb{E}(x_t \mid x^{t-1})$$

where $x^{t-1} = \{x_1, ..., x_{t-1}\}$ is the history of his output realizations.

- Alternatively,

$$\begin{aligned} w_t(x^{t-1}) &= \mathbb{E}(x_t \mid x^{t-1}) \\ &= \mathbb{E}(\eta \mid x^{t-1}) + a_t(x^{t-1}) \end{aligned}$$

where $a_t(x^{t-1})$ is the effort that the agent will exert given history x^{t-1}

- Important: $a_t(x^{t-1})$ is perfectly anticipated by the market along the equilibrium path.

Career Concerns: Preferences

- Instantaneous utility function of the agent is

$$u(w_t, a_t) = w_t - c(a_t)$$

- With horizon equal to T, preferences are

$$U(w, a) = \sum_{t=1}^{T} \beta^{t-1} [w_t - c(a_t)]$$

- For now $T = 2$.
- Finally,

$$c'(\cdot) > 0, \quad c''(\cdot) > 0$$
$$c'(0) = 0$$

- First best level of effort a^{fb} again solves

$$c'(a^{fb}) = 1.$$

Career Concerns: Summary

- Recall that all players, including the agent himself, have prior on $\eta \sim \mathcal{N}(m_0, 1/h_0)$
- So the world can be summarized as:

$$\text{period 1:} \begin{cases} \text{wage } w_1 \\ \text{effort } a_1 \text{ chosen by the agent (unobserved)} \\ \text{output is realized } x_1 = \eta + a_1 + \varepsilon_1 \end{cases}$$

$$\text{period 2:} \begin{cases} \text{wage } w_2(x_1) \\ \text{effort } a_2 \text{ chosen} \\ \text{output is realized } x_2 = \eta + a_2 + \varepsilon_2 \end{cases}$$

- Appropriate equilibrium concept: Perfect Bayesian Equilibrium.

Career Concerns: Equilibrium

- Backward induction immediately implies

$$a_2^* = 0$$

- Why?
- Therefore:

$$\begin{aligned} w_2(x_1) &= \mathbb{E}(\eta \mid x_1) + a_2(x_1) \\ &= \mathbb{E}(\eta \mid x_1) \end{aligned}$$

- The problem of the market is the estimation of η given information $x_1 = \eta + a_1 + \varepsilon_1$.
- The only difficulty is that x_1 depends on first period effort.
- In equilibrium, the market will anticipate the correct level of effort a_1.

Career Concerns: Equilibrium (continued)

- Let the conjecture of the market be $\bar{a}_1$.
- Define

$$z_1 \equiv x_1 - \bar{a}_1 = \eta + \varepsilon_1$$

as the deviation of observed output from this conjecture.
- Once we have z_1, standard normal updating formula implies that

$$\eta \mid z_1 \sim \mathcal{N}\left(\frac{h_0 m_0 + h_\varepsilon z_1}{h_0 + h_\varepsilon}, \frac{1}{h_0 + h_\varepsilon}\right)$$

- Interpretation: we start with prior m_0, and update η according to the information contained in z_1. How much weight we give to this new information depends on its precision relative to the precision of the prior. The greater its h_ε relative to h_0, the more the new information matters.
- Also important: the variance of this posterior will be less than the variance of both the prior and the new information (Why?).

The Basic Model of Career Concerns: Equilibrium (continued)

- Combining these observations

$$\mathbb{E}(\eta \mid z_1) = \frac{h_0 m_0 + h_\varepsilon z_1}{h_0 + h_\varepsilon}$$

- Or equivalently:

$$\mathbb{E}(\eta \mid x_1) = \frac{h_0 m_0 + h_\varepsilon (x_1 - \bar{a}_1)}{h_0 + h_\varepsilon}$$

- Therefore, equilibrium wages satisfy

$$w_2(x_1) = \frac{h_0 m_0 + h_\varepsilon (x_1 - \bar{a}_1)}{h_0 + h_\varepsilon}$$

- To complete the characterization of equilibrium we have to find the level of a_1 that the agent will choose as a function of $\bar{a}_1$, and make sure that this is indeed equal to $\bar{a}_1$, that is, this will ensure a *fixed point*.

Career Concerns: Equilibrium (continued)

- Let us first write the optimization problem of the agent:

$$\max_{a_1} [w_1 - c(a_1)] + \beta[\mathbb{E}\{w_2(x_1) \mid \bar{a}_1\}]$$

where we have used the fact that $a_2 = 0$.

- Substituting from above and dropping w_1:

$$\max_{a_1} \beta\, \mathbb{E}\left\{ \frac{h_0 m_0 + h_\varepsilon(x_1 - \bar{a}_1)}{h_0 + h_\varepsilon} \middle| \bar{a}_1 \right\} - c(a_1)$$

- Important: both η and ε_1 are uncertain to the agent as well as to the market.

Career Concerns: Equilibrium (continued)

- Therefore

$$\max_{a_1} \beta \mathbb{E}\left\{ \frac{h_0 m_0 + h_\varepsilon(\eta + \varepsilon_1 + a_1 - \bar{a}_1)}{h_0 + h_\varepsilon} \middle| a_1 \right\} - c(a_1)$$

- And since a_1 is not stochastic (the agent is choosing it), we have

$$\max_{a_1} \beta \frac{h_\varepsilon}{h_0 + h_\varepsilon} a_1 - c(a_1) + \beta\, \mathbb{E}\left\{ \frac{h_0 m_0 + h_\varepsilon(\eta + \varepsilon_1 - \bar{a}_1)}{h_0 + h_\varepsilon} \right\}$$

- The first-order condition is:

$$c'(a_1^*) = \beta \frac{h_\varepsilon}{h_0 + h_\varepsilon} < 1 = c'(a_{fb})$$

- This does not depend on $\bar{a}_1$, so the fixed point problem is solved immediately by setting $\bar{a}_1 = a_1^*$.
- First result: equilibrium effort is always less than first this.

Career Concerns: Equilibrium (continued)

- Why?: because there are two "leakages" (increases in output that the agent does not capture): the payoff from higher effort only occurs next period, therefore its value is discounted to β, and the agent only gets credit for a fraction $h_\varepsilon / (h_0 + h_\varepsilon)$ of her effort, the part that is attributed to ability.
- The characterization of the equilibrium is completed by imposing $\bar{a}_1 = a_1^*$
- This was not necessary for computing a_1^*, but is needed for computing the equilibrium wage w_1.
- Recall that

$$\begin{aligned} w_1 &= \mathbb{E}(y_1 \mid \text{prior}) \\ &= \mathbb{E}(\eta) + \bar{a}_1 \\ &= m_0 + a_1^* \end{aligned}$$

Career Concerns: Comparative Statics

- We immediately obtain:

$$\frac{\partial a_1^*}{\partial \beta} > 0$$
$$\frac{\partial a_1^*}{\partial h_\varepsilon} > 0$$
$$\frac{\partial a_1^*}{\partial h_0} < 0$$

- Greater β means that the agent discounts the future less, so exerts more effort because the first source of leakage is reduced.
- Greater h_ε implies that there is less variability in the random component of performance. This, from the normal updating formula, implies that any given increase in performance is more likely to be attributed to ability, so the agent is more tempted to jam the signal by exerting more effort.
- The intuition for the negative effect of h_0 is similar.

Multiperiod Career Concerns

- Considered the same model with three periods.
- This model can be summarized by the following matrix

$$\begin{array}{ll} w_1 & a_1^* \\ w_2(x_1) & a_2^* \\ w_3(x_1, x_2) & a_3^* \end{array}$$

- With similar analysis to before, the first-order conditions for the agent are

$$c'(a_1^*) = \beta \frac{h_\varepsilon}{h_0 + h_\varepsilon} + \beta^2 \frac{h_\varepsilon}{h_0 + 2h_\varepsilon}$$

$$c'(a_2^*) = \beta \frac{h_\varepsilon}{h_0 + 2h_\varepsilon}.$$

Multiperiod Career Concerns (continued)

- First result:

$$a_1^* > a_2^* > a_3^* = 0.$$

- Why?
- More generally, in the T period model, the relevant first-order condition is

$$c'(a_t^*) = \sum_{\tau=t}^{T-1} \beta^{\tau-t+1} \frac{h_\varepsilon}{h_0 + \tau h_\varepsilon}.$$

Multiperiod Career Concerns: Overeffort

- With T sufficiently large, it can be shown that there exists a period $\overline{\tau}$ such that

$$a^*_{t<\overline{\tau}} \geq a_{fb} \geq a^*_{t>\overline{\tau}}.$$

- In other words, workers work too hard when young and not hard enough when old—
  - compare assistant professors to tenured faculty.
  - important: these effort levels depend on the horizon (time periods), but not on past realizations.

Multiperiod Career Concerns: Generalizations

- Similar results hold when ability is not constant, but evolves over time (as long as it follows a normal process).
- For example, we could have

$$\eta_t = \eta_{t-1} + \delta_t$$

with

$$\begin{array}{rcl} \eta_0 & \sim & \mathcal{N}(m_0, 1/h_0) \\ \delta_t & \sim & \mathcal{N}(0, 1/h_\delta)\ \forall\, t \end{array}$$

- In this case, it can be shown that the updating process is stable, so that the process and therefore the effort level converge, and in particular as $t \longrightarrow \infty$, we have

$$a_t \longrightarrow \bar{a}$$

but as long as $\beta < 1$, $\bar{a} < a_{fb}$.
- Also, the same results apply when the agent knows his ability.
    - Why is this? In what ways it special?

Evidence

- Chevalier and Ellison look at the behavior of fund managers in the early 1990s, and find that the possibility of termination (as a function of their performance) creates career concerns for these managers. In particular, the prevailing termination policies make the probability of termination a convex function of performance for young managers.
- Highlighting that the setting matters for that form of career concerns (in this case driven by convexity of termination), their results suggest that younger fund managers avoid (unsystematic) risk and hold more conventional portfolios