14.773 Political Economy of Institutions and Development.
Lecture 4. Economic Institutions under Elite Domination

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Introduction

- So far the focus has been on democratic decision-making (voting, electoral control, dynamics of constituencies)
- But most countries in the world throughout the 20th century and today are not democratic
- Even when they have democratic institutions, the democratic system seems rather "dysfunctional".
- We now turn to an analysis of how economic and political decisions are made in non-democratic situations with a given social group, the elite, in power.
- This will then open the way for a more systematic analysis of institutional change

Simple Model of Elite Control

- Consider an infinite horizon economy populated by a continuum $1 + \theta_e + \theta_m$ of risk neutral agents, each with a discount factor equal to $\beta < 1$.
- Unique non-storable final good denoted by y.
- The expected utility of agent j at time 0 is given by:

$$U_0^j = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t^j, \tag{1}$$

where $c_t^j \in \mathbb{R}$ denotes the consumption of agent j at time t and $\mathbb{E}_t$ is the expectations operator conditional on information available at time t.

Environment

- Agents are in three groups.
  1. workers, mass 1, supplying labor inelastically.
  2. elite (denoted by e), total mass θ^e (set S^e); initially hold political power in this society and engage in entrepreneurial activities
  3. middle class (denoted by m), total mass θ^m (set S^m); engage in entrepreneurial activities
- Each member of the elite and middle class has access to production opportunities, represented by the production function

$$y_t^j = \frac{1}{1-\alpha}(A_t^j)^\alpha (k_t^j)^{1-\alpha}(l_t^j)^\alpha, \quad (2)$$

  where k denotes capital and l labor.
- Capital is assumed to depreciate fully after use.
- Productivity of each elite agent is A^e in each period, and that of each middle class agent is A^m.
- In addition, natural resource rents R at each date.

Policies

- Taxes: activity-specific tax rates on production, $\tau^e \geq 0$ and $\tau^m \geq 0$.
- No other fiscal instruments to raise revenue. (in particular, no lump-sum non-distortionary taxes).
- The proceeds of taxes and revenues from natural resources can be redistributed as nonnegative lump-sum transfers targeted towards each group, $T^w \geq 0$, $T^m \geq 0$ and $T^e \geq 0$.
- $\phi \in [0,1]$ reduced form measure of "state capacity,"
- Government budget constraint:

$$T_t^w + \theta^m T_t^m + \theta^e T_t^e \leq \phi \int_{j \in S^e \cup S^m} \tau_t^j y_t^j dj + R. \tag{3}$$

Employment

- Maximum scale for each firm, so that

$$l_t^j \leq \lambda \text{ for all } j \text{ and } t.$$

- This prevents the most productive agents in the economy from employing the entire labor force.
- Market clearing:

$$\int_{j \in S^e \cup S^m} l_t^j dj \leq 1. \tag{4}$$

- Since $l_t^j \leq \lambda$, (4) implies that if

$$\theta^e + \theta^m \leq \frac{1}{\lambda}, \tag{ES}$$

there can never be full employment.
- Depending on whether Condition (ES) holds, there will be excess demand or excess supply of labor in this economy. Also assume

$$\theta^e \leq \frac{1}{\lambda} \text{ and } \theta^m \leq \frac{1}{\lambda}.$$

Economic Equilibrium

- An *economic equilibrium* is defined as a sequence of wages $\{w_t\}_{t=0,1,\dots,\infty}$, and investment and employment levels for all producers, $\left\{\left[k_t^j, l_t^j\right]_{j\in S^e\cup S^m}\right\}_{t=0,1,\dots,\infty}$ such that given $\{\tau_t^e, \tau_t^m\}_{t=0,1,\dots,\infty}$ and $\{w_t\}_{t=0,1,\dots,\infty}$, all producers choose their investment and employment optimally and the labor market clears.
- Each producer takes wages, w_t, as given, and maximizes

$$\max_{k_t^j, l_t^j} \frac{1-\tau_t^j}{1-\alpha}(A^j)^\alpha (k_t^j)^{1-\alpha}\left(l_t^j\right)^\alpha - w_t l_t^j - k_t^j.$$

- Solution:

$$k_t^j = (1-\tau_t^j)^{1/\alpha} A^j l_t^j, \text{ and} \tag{5}$$

$$l_t^j \begin{cases} = 0 & \text{if } w_t > \frac{\alpha}{1-\alpha}(1-\tau_t^j)^{1/\alpha}A^j \\ \in [0,\lambda] & \text{if } w_t = \frac{\alpha}{1-\alpha}(1-\tau_t^j)^{1/\alpha}A^j \\ = \lambda & \text{if } w_t < \frac{\alpha}{1-\alpha}(1-\tau_t^j)^{1/\alpha}A^j \end{cases}. \tag{6}$$

Comments

- $\alpha(1-\tau_t^j)^{1/\alpha}A^j / (1-\alpha)$ is the net marginal product of a worker employed by a producer of group j.
- If the wage is above this amount, this producer would not employ any workers, and if it is below, he or she would prefer to hire as many workers as possible (i.e., up to the maximum, λ).
- Potential distortion: producers invest in physical capital but only receive a fraction $(1-\tau_t^j)$ of the revenues.
- Therefore, taxes discourage investments, creating potential "inefficiencies"
- But are these Pareto inefficiencies?

Equilibrium Wages

- Combining (6) with (4), equilibrium wages are obtained as follows:
  (i) If Condition (ES) holds, there is excess supply of labor and $w_t = 0$.
  (ii) If Condition (ES) does not hold, then there is "excess demand" for labor and the equilibrium wage is

$$w_t = \min \left\langle \frac{\alpha}{1-\alpha}(1-\tau_t^e)^{1/\alpha}A^e, \frac{\alpha}{1-\alpha}(1-\tau_t^m)^{1/\alpha}A^m \right\rangle. \tag{7}$$

- Note that when Condition (ES) does not hold, the equilibrium wage is equal to the net productivity of one of the two groups of producers, so either the elite or the middle class will make zero profits in equilibrium.

Summary of Economic Equilibrium

- Finally, equilibrium level of aggregate output is

$$
\begin{aligned}
Y_t \quad = \quad & \frac{1}{1-\alpha}(1-\tau_t^e)^{(1-\alpha)/\alpha} A^e \int_{j \in S^e} l_t^j dj \\
& + \frac{1}{1-\alpha}(1-\tau_t^m)^{(1-\alpha)/\alpha} A^m \int_{j \in S^m} l_t^j dj + R.
\end{aligned} \tag{8}
$$

Proposition: For a given sequence of taxes $\{\tau_t^e, \tau_t^m\}_{t=0,1,\dots,\infty}$, the equilibrium takes the following form: if Condition (ES) holds, then $w_t = 0$, and if Condition (ES) does not hold, then w_t is given by (7). Given the wage sequence, factor demands are given by (5) and (6), and aggregate output is given by (8).

"Inefficient" Policies

- Let us now look at sources of inefficient policies under the dictatorship of the elite.
- Key distortionary policy, tax on the middle class
- Three reasons to use this tax:
    1. Revenue Extraction;
    2. Factor Price Manipulation;
    3. Political Consolidation.

Simplifying Assumptions

- Upper bound on taxation, so that

$$\tau_t^m \leq \bar{\tau} \text{ and } \tau_t^e \leq \bar{\tau},$$

  where $\bar{\tau} \leq 1$.

- *The timing of events within each period*
  1. taxes are set;
  2. investments are made.

- This removes an additional source of inefficiency related to the *holdup problem*.

- To start with, equilibrium concept: Markov Perfect Equilibria (MPE)—the elite set the tax rate today without commitment to future tax rates (but in the baseline model we start with this is equivalent to choosing the entire future sequences of tax rates).

Revenue Extraction

- To highlight this mechanism, suppose that Condition (ES) holds, so wages are constant at zero. T
- his removes any effect of taxation on factor prices.
- In this case, from (6), we also have $l_t^j = \lambda$ for all producers.
- Also assume that $\phi > 0$ (for example, $\phi = 1$).
- Tax revenues to be distributed back to the elite

$$\text{Revenue}_t = \frac{\phi}{1-\alpha}\tau_t^m (1-\tau_t^m)^{(1-\alpha)/\alpha} A^m \lambda \theta^m + R. \tag{9}$$

- Clearly this is maximized at

$$\tau_t^m = \tau^{RE} \equiv \min\{\alpha, \bar{\tau}\}. \tag{10}$$

Revenue Extraction (continued)

- No intertemporal linkages

Proposition: Suppose Condition (ES) holds and $\phi > 0$, then the unique MPE features $\tau_t^m = \tau^{RE} \equiv \min\{\alpha, \bar{\tau}\}$ for all t.

- Taxing at the top of the Laffer curve
- High taxes distortionary, but fiscal policies are not used to harm the middle class.

Factor Price Manipulation

- To highlight this mechanism in the simplest possible way, let us first assume that $\phi = 0$ so that there are no direct benefits from taxation for the elite.
- There are indirect benefits, because of the effect of taxes on factor prices, which will be present as long as the equilibrium wage is positive.
- Suppose that Condition (ES) does not hold, so that equilibrium wage is given by (7).
- Therefore, choose taxes to minimize equilibrium wages.

Factor Price Manipulation (continued)

Proposition: Suppose Condition (ES) does not hold, and $\phi = 0$, then the unique MPE features $\tau_t^m = \tau^{FPM} \equiv \bar{\tau}$ for all t.

- Higher taxes in order to harm the middle class
- Because of competition in the labor market.
- *Implication:* factor price manipulation much more damaging to output.
- Naturally, $\phi = 0$ important

Combined Effects

- Now let us combine the two effects.
- Main results: the factor price manipulation effect will push the economy *beyond the peak* of the Laffer curve
- The elite's problem can be written as

$$\max_{\tau_t^m} \left[\frac{\alpha}{1-\alpha} A^e - w_t \right] l_t^e + \frac{1}{\theta^e} \left[\frac{\phi}{1-\alpha} \tau_t^m (1-\tau_t^m)^{(1-\alpha)/\alpha} A^m l_t^m \theta^m + R \right], \tag{11}$$

subject to (7) and

$$\theta^e l_t^e + \theta^m l_t^m = 1, \text{ and} \tag{12}$$

$$l_t^m = \lambda \text{ if } \quad (1-\tau_t^m)^{1/\alpha} A^m \geq A^e. \tag{13}$$

- Assume

$$A^e \geq \phi (1-\alpha)^{(1-\alpha)/\alpha} A^m \frac{\theta^m}{\theta^e}$$

so that the elite do not wish to stop producing altogether.

Combined Effects (continued)

- Then the equilibrium will be $w_t = \alpha(1-\tau_t^m)^{1/\alpha} A^m \ \tau_t^m / \ (1-\alpha)$, and the elite's problem simply boils down to choosing τ_t^m to maximize

$$\frac{1}{\theta^e}\left[\frac{\phi}{1-\alpha}\tau_t^m(1-\tau_t^m)^{(1-\alpha)/\alpha}A^m l^m \theta^m + R\right] - \frac{\alpha}{1-\alpha}(1-\tau_t^m)^{1/\alpha}A^m\lambda, \tag{14}$$

  where we have used the fact that all elite producers will employ λ employees, and from (12), $l_m = (1-\lambda\theta^e)/\theta^m$.
- The maximization of (14) gives

$$\frac{\tau_t^m}{1-\tau_t^m} = \kappa\left(\lambda, \theta^e, \alpha, \phi\right) \equiv \frac{\alpha}{1-\alpha}\left(1 + \frac{\lambda\theta^e}{\left(1-\lambda\theta^e\right)\phi}\right).$$

- τ_t^m is always less than 1, which is the desired tax rate in the case of pure factor price manipulation.
- But $\kappa\left(\lambda, \theta^e, \alpha, \phi\right)$ is also strictly greater than $\alpha / \ (1-\alpha)$, so that τ_t^m is always greater than α, the desired tax rate with pure revenue extraction.

Combined Effects (continued)

- In summary, combined effects lead to desired tax rate:

$$\tau_t^m = \tau^{COM} \equiv \min\left\{\frac{\kappa\left(\lambda, \theta^e, \alpha, \phi\right)}{1+\kappa\left(\lambda, \theta^e, \alpha, \phi\right)}, \bar{\tau}\right\}. \tag{15}$$

- *Comparative Statics:*
  1. ϕ reduces τ^{COM} because increased state capacity makes revenue extraction more important.declines.
  2. θ^e increases τ^{COM} because revenue extraction becomes less important and factor price manipulation becomes more important.
  3. α increases taxes.

Proposition: Suppose Condition (ES) does not hold, and $\phi > 0$. Then the unique MPE features $\tau_t^m = \tau^{COM}$ as given by (15) for all t. Equilibrium taxes are increasing in θ^e and α and decreasing in ϕ.

Political Consolidation

- Same results if competition for political power other than in the labor market.
- Imagine that if the middle class become richer, then they are more likely to gain political power.
- Then:

Proposition: Consider the economy with political replacement. Suppose Condition (ES) holds and $\phi > 0$, then the unique MPE features $\tau_t^m = \tau^{PC} > \tau^{RE}$ for all t. This tax rate is increasing in R and ϕ.

- New result: tax rate is increasing in R and ϕ.
- This is because political stakes are higher.
- The "dark side" of state capacity.

Subgame Versus Markov Perfect Equilibria

- What happens if you look at subgame perfect equilibria?

Proposition: The MPEs characterized above are the unique SPEs.

- Why? Because unique best responses within each period, and no intertemporal linkages.
- More interestingly, this is because there is no "political failure".
- All of the equilibria above (with the exception of political consolidation effect depending on details) are *Pareto optimal*.

Holdup

- Political failures are introduced if investments are "long term" so that tax decisions are made partly after investments are sunk.
- Change the timing of events such that:
  1. individual producers undertake their investments;
  2. the elite set taxes.
- The elite will no longer take the discourage of taxes on investment into account in the MPE.
- Therefore

Proposition: With holdup, there is a unique MPE with $\tau_t^m = \tau^{HP} \equiv \bar{\tau}$ for all t.

- Now greater distortions and potential Pareto inefficiencies.

Subgame Perfect Equilibria

- Now imagine trigger-strategy equilibria.
- Suppose that Condition (ES) holds and $\phi > 0$, so that most preferred tax rate for the elite is $\tau^m = \alpha$.
- Suppose also that $\bar{\tau} = 1$.
- Consider the strategy profile where the elite set $\tau^m = \alpha$ at each date and the middle class choose investment levels according to this tax rate.
- If the elite ever set a higher tax rate, then the middle class expect $\tau^m = 1$ in all future dates, and choose zero production.

Subgame Perfect Equilibria (continued)

- With this strategy profile, the elite will raise

$$\frac{\phi}{(1-\beta)(1-\alpha)}\alpha(1-\alpha)^{(1-\alpha)/\alpha}A^m\lambda\theta^m \tag{16}$$

if they set α at the state.

- If, in contrast, they deviate at any point, the most profitable deviation for them is to set $\tau^m = 1$, and they will raise

$$\frac{\phi}{1-\alpha}(1-\alpha)^{(1-\alpha)/\alpha}A^m\lambda\theta^m. \tag{17}$$

- The trigger-strategy profile will be an equilibrium as long as (16) is greater than or equal to (17), which requires $\beta \geq 1-\alpha$. Therefore:

Proposition: Consider the holdup game, and suppose that Conditions (ES) hold and $\bar{\tau} = 1$. Then for $\beta \geq 1-\alpha$, there exists a subgame perfect equilibrium where $\tau_t^m = \alpha$ for all t.

Technology Adoption and Holdup

- Suppose now that taxes are set before investments, so the source of holdup above is absent.
- Instead, suppose that at time $t = 0$ before any economic decisions or policy choices are made, middle class agents can invest to increase their productivity.
- There is a cost $\Gamma\left(A^m\right)$ of investing in productivity A^m.
- Once investments in technology are made, the game proceeds as before.
- Since investments in technology are sunk after date $t = 0$, the equilibrium allocations are the same as in the results presented above.
- *Question:* if they could, the elite would prefer to commit to a tax rate sequence at time $t = 0$.

Technology Adoption: Factor Price Manipulation

Proposition: Consider the game with technology adoption and suppose that Condition (ES) does not hold, and $\phi = 0$, then the unique MPE and unique SPE feature $\tau_t^m = \tau^{FPM} \equiv \bar{\tau}$ for all t. Moreover, if the elite could commit to a tax sequence at time $t = 0$, then they would still choose $\tau_t^m = \tau^{FPM} \equiv \bar{\tau}$.

- Intuition: this is the case of pure factor price manipulation, so the only objective of the elite is to reduce the middle class' labor demand.
- Therefore, they have no interest in increasing the productivity of middle class producers.

Technology Adoption: Revenue Extraction

- Let us next consider the pure revenue extraction case with Condition (ES) satisfied.
- Once again, the MPE is identical to before with $\tau^m = \tau^{RE} \equiv \min\{\alpha, \bar{\tau}\}$.
- As a result, the first-order condition for an interior solution to the middle class producers' technology choice is:

$$\Gamma'(A^m) = \frac{1}{1-\beta}\frac{\alpha}{1-\alpha}(1-\tau^m)^{1/\alpha}. \quad (18)$$

- This is also the unique SPE, since no punishments are possible.
- But, if the elite could commit to a tax rate sequence at time $t = 0$, they would choose lower taxes in order to increase investment by the middle class and thus tax revenues.

Technology Adoption: Revenue Extraction (continued)

- To illustrate this, suppose that the elite can commit to a constant tax rate.
- Then, the optimization problem of the elite is to maximize tax revenues taking the relationship between taxes and technology as in (18) as given. In other words, they will solve:

$$\max \phi\tau^m (1-\tau^m)^{(1-\alpha)/\alpha} A^m \lambda\theta^m / (1-\alpha)$$

subject to (18).

- The first-order condition for an interior solution can be expressed as

$$A^m - \frac{1-\alpha}{\alpha}\frac{\tau^m}{1-\tau^m}A^m + \tau^m \frac{dA^m}{d\tau^m} = 0$$

where

$$\frac{dA^m}{d\tau^m} = -\frac{1}{1-\beta}\frac{1}{1-\alpha}\frac{(1-\tau^m)^{(1-\alpha)/\alpha}}{\Gamma''(A^m)} < 0$$

takes into account the effect of future taxes on technology choice at time $t=0$.

Technology Adoption: Revenue Extraction (continued)

Proposition: Consider the game with technology adoption, and suppose that Condition (ES) holds and $\phi > 0$, then the unique political equilibrium features $\tau_t^m = \tau^{RE} \equiv \min\{\alpha, \bar{\tau}\}$ for all t. If the elite could commit to a tax policy at time $t = 0$, they would prefer to commit to $\tau^{TA} < \tau^{RE}$.

- Therefore, in contrast to the pure holdup problem where SPE could prevent the additional inefficiency (when $\beta \geq 1 - \alpha$), with the technology adoption game, the inefficiency survives the SPE.
- The reason is that, since middle class producers invest only once at the beginning, there is no possibility of using history-dependent punishment strategies.
- This illustrates the limits of implicit agreements to keep tax rates low.
- Such agreements not only require a high discount factor ($\beta \geq 1 - \alpha$), but also frequent investments by the middle class, so that there is a credible threat against the elite if they deviate from the promised policies.

Conclusion

- Distributional conflicts will lead to distortionary policies.
- The extent of distortions depends on whether groups in power wish to manipulate factor prices.
- Factor price manipulation could lead to higher taxes, insecure property rights, and barriers against technology adoption
- These equilibria not necessarily Pareto suboptimal—the set of instruments is restricted.
- However, Pareto inefficiencies arise when there are nontrivial dynamic interactions (as in holdup or technology adoption)
- Also note that simply changing the identity of the group in power may not improve the allocation of resources as we discuss in greater detail next.

Simple Model of Elite Control

- Infinite horizon economy populated by a continuum 1 of risk neutral agents, with discount factor equal to $\beta < 1$.
- Unique non-storable final good denoted by y.
- The expected utility of agent j at time 0 is given by:

$$U_0^j = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t^j, \tag{19}$$

  where $c_t^j \in \mathbb{R}$ denotes the consumption of agent j at time t and $\mathbb{E}_t$ is the expectations operator conditional on information available at time t.
- Suppose that each individual dies with a small probability ε in every period, and a mass ε of new individuals are born (with the convention that after death there is zero utility and β is the discount factor inclusive of the probability of death).
- We will consider the limit of this economy with $\varepsilon \to 0$.

Occupations

- production workers versus capitalists/entrepreneurs.
- All agents have the same productivity as workers, their productivity in entrepreneurship differs.
- Agent j at time t has entrepreneurial talent/skills $a_t^j \in \{A^L, A^H\}$ with $A^L < A^H$.
- To become an entrepreneur, an agent needs to set up a firm, if he does not have an active firm already.
- Setting up a new firm may be costly because of entry barriers created by existing entrepreneurs.

States

- Each agent therefore starts period t with two state variables:
  - skill level $a_t^j \in \{A^H, A^L\}$
  - $s_t^j \in \{0, 1\}$ denoting whether the individual has an active firm.
- We refer to an agent with $s_t^j = 1$ as a member of the "elite," since he will have an advantage in becoming an entrepreneur (when there are entry barriers), and in an oligarchic society, he may be politically more influential than non-elite agents.

Decisions

- Within each period, each agent makes the following decisions:
  - an occupation choice $e_t^j \in \{0, 1\}$, and in addition if $e_t^j = 1$, i.e., if he becomes an entrepreneur,
  - investment, employment, and hiding decisions, k_t^j, l_t^j and h_t^j, where h_t^j denotes whether he decides to hide his output in order to avoid taxation (since the final good is not storable, the consumption decision is simply given by the budget constraint).
- Agents also make the policy choices in this society.
- Three policy choices:
  - a tax rate $\tau_t \in [0, 1]$ on output,
  - lump-sum transfers to all agents denoted by $T_t \in [0, \infty)$,
  - cost $B_t \in [0, \infty)$ to set up a new firm.

Production

- An entrepreneur with skill level a_t^j can produce

$$y_t^j = \frac{1}{1-\alpha}(a_t^j)^{\alpha}(k_t^j)^{1-\alpha}(l_t^j)^{\alpha} \tag{20}$$

- Suppose that all firms have to operate at the same size, λ, so

$$l_t^j = \lambda.$$

- Suppose also that the entrepreneur himself can work in his firm as one of the workers, which implies that the opportunity cost of becoming an entrepreneur is 0.

Profits

- Given a tax rate τ_t and a wage rate $w_t \geq 0$ and using the fact that $l_t^j = \lambda$, the net profits of an entrepreneur with talent a_t^j at time t are:

$$\pi\left(k_t^j \mid a_t^j, w_t, \tau_t\right) = \frac{1-\tau_t}{1-\alpha}(a_t^j)^\alpha (k_t^j)^{1-\alpha}\lambda^\alpha - w_t\lambda - k_t^j. \tag{21}$$

- If taxes are too high, he can choose to hide his output, $h_t^j = 1$. In this case, he avoids the tax, but loses a fraction $\delta < 1$ of his revenues, so his profits are:

$$\tilde{\pi}\left(k_t^j \mid a_t^j, w_t, \tau_t\right) = \frac{1-\delta}{1-\alpha}(a_t^j)^\alpha (k_t^j)^{1-\alpha}\lambda^\alpha - w_t\lambda - k_t^j.$$

- This implies that taxes are always constrained to be:

$$0 \leq \tau_t \leq \delta.$$

Profit Maximization

- The (instantaneous) gain from entrepreneurship for an agent of talent $z \in \{L, H\}$ as a function of the tax rate τ_t, and the wage rate, w_t, is:

$$\Pi^z(\tau_t, w_t) = \max_{k_t^j} \pi\left(k_t^j \mid a_t^j = A^z, w_t, \tau_t\right). \tag{22}$$

- Note that this is the *net gain* to entrepreneurship since the agent receives the wage rate w_t irrespective (either working for another entrepreneur when he is a worker, or working for himself—thus having to hire one less worker—when he is an entrepreneur).
- The gain to becoming an entrepreneur for an agent with $s_t^j = 0$ and ability $a_t^j = A^z$ is

$$\Pi^z(\tau_t, w_t) - B_t = \Pi^z(\tau_t, w_t) - \lambda b_t,$$

where $b_t \equiv B_t / \lambda$ is the cost imposed by the entry barriers.

Market Clearing

- Market clearing condition:

$$\int_0^1 e_t^j l_t^j dj = \int_{j \in \mathbf{S}_t^E} \lambda dj \leq 1, \tag{23}$$

where $\mathbf{S}_t^E$ is the set of entrepreneurs at time t.

Evolution of State Variables

- Law of motion of the vector $\left(s_t^j, a_t^j\right)$ given by

$$s_{t+1}^j = e_t^j, \tag{24}$$

  with $s_0^j = 0$ for all j, and also $s_t^j = 0$ if an individual j is born at time t.

- And

$$a_{t+1}^j = \begin{cases} A^H & \text{with probability} \quad \sigma^H & \text{if } a_t^j = A^H \\ A^H & \text{with probability} \quad \sigma^L & \text{if } a_t^j = A^L \\ A^L & \text{with probability } 1-\sigma^H & \text{if } a_t^j = A^H \\ A^L & \text{with probability } 1-\sigma^L & \text{if } a_t^j = A^L \end{cases}, \tag{25}$$

  where σ^H, $\sigma^L \in (0,1)$.

- Suppose that $\sigma^H \geq \sigma^L > 0$, so that skills are persistent and low skill is not an absorbing state.

Evolution of State Variables (continued)

- Fraction of high skill agents in the stationary distribution is

$$M \equiv \frac{\sigma^L}{1-\sigma^H+\sigma^L} \in (0,1) .$$

- Suppose that

$$M\lambda > 1,$$

so that, without entry barriers, high-skill entrepreneurs generate more than sufficient demand to employ the entire labor supply.

Timing of Events

- Entrepreneurial talents/skills, $\left[a_t^j\right]$, are realized.
- The entry barrier for new entrepreneurs b_t is set.
- Agents make occupational choices, $\left[e_t^j\right]$, and entrepreneurs make investment decisions, $\left[k_t^j\right]$.
- The labor market clearing wage rate, w_t, is determined.
- The tax rate on entrepreneurs, τ_t, is set.
- Entrepreneurs make hiding decisions, $\left[h_t^j\right]$.

Policy Choices

- Entry barriers and taxes will be set by different agents in different political regimes.
- Taxes are set after the investment decisions, which can be motivated by potential commitment problems whereby entrepreneurs can be "held up" after they make their investments decision.
- Once these investments are sunk, it is in the interest of the workers to tax and redistribute entrepreneurial income.

Equilibrium Concept

- Focus on the Markov Perfect Equilibrium (MPE), where strategies are only a function of the payoff relevant states.
- For individual j the payoff relevant state at time t includes his own state $\left(s_t^j, a_t^j\right)$, and potentially the fraction of entrepreneurs that are high skill, denoted by μ_t, and defined as

$$\mu_t = \Pr\left(a_t^j = A^H \mid e_t^j = 1\right) = \Pr\left(a_t^j = A^H \mid j \in \mathbf{S}_t^E\right).$$

- $x_t^j = \left(e_t^j, k_t^j, , h_t^j\right)$: the vector of choices of agent j at time t,
- $x_t = \left[x_t^j\right]_{j\in[0,1]}$: the choices for all agents,
- $p_t = (b_t, \tau_t)$: vector of policies at time t.
- $\mathbf{p}^t = \{p_n\}_{n=t}^{\infty}$: the infinite sequence of policies from time t onwards,
- $\mathbf{w}^t$ and $\mathbf{x}^t$: sequences of wages and choices from t onwards.

Economic Equilibrium

- $s_0^j = 0$ for all j, and suppose $b_0 = 0$, so that in the initial period there are no entry barriers (since $s_0^j = 0$ for all j, any positive entry barrier would create waste, but would not affect who enters entrepreneurship).
- Since $l_t^j = \lambda$ for all $j \in \mathbf{S}_t^E$, profit-maximizing investments are given by:

$$k_t^j = (1 - \tau_t)^{1/\alpha} a_t^j \lambda. \tag{26}$$

- Investment increasing in the skill level of the entrepreneur, a_t^j, and decreasing in the tax rate, τ_t.
- Net current gain to entrepreneurship, as a function of entry barriers, taxes, equilibrium wages, for an agent of type $z \in \{L, H\}$ is then

$$\Pi^z(\tau_t, w_t) = \frac{\alpha}{1-\alpha}(1 - \tau_t)^{1/\alpha} A^z \lambda - w_t \lambda. \tag{27}$$

Value Functions

- Let us denote the value of an entrepreneur with skill level $z \in \{L, H\}$ as a function of the sequence of future policies and equilibrium wages, $(\mathbf{p}^t, \mathbf{w}^t)$, by $V^z(\mathbf{p}^t, \mathbf{w}^t)$, and the value of a worker of type z in the same situation by $W^z(\mathbf{p}^t, \mathbf{w}^t)$.
- Then,

$$W^z\left(\mathbf{p}^t, \mathbf{w}^t\right) = w_t + T_t + \beta CW^z\left(\mathbf{p}^{t+1}, \mathbf{w}^{t+1}\right), \tag{28}$$

where

$$\begin{aligned} & CW^z\left(\mathbf{p}^{t+1}, \mathbf{w}^{t+1}\right) = \qquad (29) \\ & \sigma^z \max\left\{W^H\left(\mathbf{p}^{t+1}, \mathbf{w}^{t+1}\right), V^H\left(\mathbf{p}^{t+1}, \mathbf{w}^{t+1}\right) - \lambda b_{t+1}\right\} \\ & + (1-\sigma^z) \max\left\{W^L\left(\mathbf{p}^{t+1}, \mathbf{w}^{t+1}\right), V^L\left(\mathbf{p}^{t+1}, \mathbf{w}^{t+1}\right) - \lambda b_{t+1}\right\}. \end{aligned}$$

- Intuition: a worker of type z receives a wage income of w_t (independent of his skill), a transfer of T_t, and the continuation value $CW^z\left(\mathbf{p}^{t+1}, \mathbf{w}^{t+1}\right)$.

Value Functions (continued)

- To understand this continuation value, note that the worker stays high skill with probability σ^z, and in this case, he can either choose to remain a worker, receiving value $W^H\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right)$, or decide to become an entrepreneur by incurring the entry cost λb_{t+1}, receiving the value of a high-skill entrepreneur, $V^H\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right)$.
- The max operator makes sure that he chooses whichever option gives higher value.
- With probability $1-\sigma^z$, he transitions from high skill to low skill, and receives the corresponding values.

Value Functions (continued)

- For entrepreneurs:

$$V^z\left(\mathbf{p}^t,\mathbf{w}^t\right) = w_t + T_t + \Pi^z\left(\tau_t, w_t\right) + \beta CV^z\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right), \quad (30)$$

where Π^z is given by (27) and

$$\begin{aligned} CV^z\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right) &= \sigma^z \max\left\{W^H\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right), V^H\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right)\right\} \quad (31) \\ &+ \left(1-\sigma^z\right)\max\left\{W^L\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right), V^L\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right)\right\} \end{aligned}$$

Value Functions (continued)

- Finally, let us define the *net value* of entrepreneurship as a function of an individual's skill a and ownership status, s,

$$
\begin{aligned}
& NV\left(\mathbf{p}^t, \mathbf{w}^t \mid a_t^j = A^z, s_t^j = s\right) \\
& = V^z\left(\mathbf{p}^t, \mathbf{w}^t\right) - W^z\left(\mathbf{p}^t, \mathbf{w}^t\right) \qquad - (1-s)\,\lambda b_t,
\end{aligned}
$$

  where the last term is the entry cost incurred by agents with $s = 0$.
- The max operators in (29) and (31) imply that if $NV > 0$ for an agent, then he prefers to become an entrepreneur.

Entrepreneurship Choices

- Straightforward to see that

$$NV\left(\mathbf{p}^t, \mathbf{w}^t \mid a_t^j = A^H, s_t^j = 0\right) \geq NV\left(\mathbf{p}^t, \mathbf{w}^t \mid a_t^j, s_t^j = s\right) \geq NV\left(\mathbf{p}^t, \mathbf{w}^t \mid a_t^j = A^L, s_t^j = 1\right)$$

- In other words, the net value of entrepreneurship is highest for high-skill existing entrepreneurs, and lowest for low-skill workers. However, it is unclear ex ante whether

$$NV\left(\mathbf{p}^t, \mathbf{w}^t \mid a_t^j = A^H, s_t^j = 0\right) > NV\left(\mathbf{p}^t, \mathbf{w}^t \mid a_t^j = A^L, s_t^j = 0\right)$$

or the other way round.

Entrepreneurship Choices (continued)

- Two different types of equilibria:
  1. *Entry equilibrium* where all entrepreneurs have $a_t^j = A^H$.
  2. *Sclerotic equilibrium* where agents with $s_t^j = 1$ become entrepreneurs irrespective of their productivity.
- An entry equilibrium requires the net value of entrepreneurship to be greater for a non-elite high skill agent than for a low-skill elite, i.e.,

$$NV\left(\mathbf{p}^t, \mathbf{w}^t \mid a_t^j = A^H, s_t^j = 0\right) \geq NV\left(\mathbf{p}^t, \mathbf{w}^t \mid a_t^j = A^L, s_t^j = 1\right).$$

- Define w_t^H such that at this wage rate, $NV\left(\mathbf{p}^t, \left[w_t^H, \mathbf{w}^{t+1}\right] \mid a_t^j = A^H, s_t^j = 0\right) = 0$, that is,

$$w_t^H \equiv \max\{\frac{\alpha}{1-\alpha}(1-\tau_t)^{1/\alpha}A^H - b_t + \frac{\beta\left(CV^H\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right) - CW^H\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right)\right)}{\lambda}; 0\}, \quad (32)$$

Entrepreneurship Choices (continued)

- Similarly, let w_t^L be such that $NV\left(\mathbf{p}^t, \left[w_t^L, \mathbf{w}^{t+1}\right] \mid a_t^j = A^L, s_t^j = 1\right) = 0$, that is,

$$w_t^L \equiv \max\{\frac{\alpha}{1-\alpha}(1-\tau_t)^{1/\alpha}A^L + \frac{\beta\left(CV^L\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right) - CW^L\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right)\right)}{\lambda}; 0\}. \tag{33}$$

Entry Equilibrium

- Given these definitions, the condition for an entry equilibrium to exist at time t can simply be written as

$$w_t^H \geq w_t^L. \tag{34}$$

- A sclerotic equilibrium emerges, on the other hand, only if the converse of (34) holds.

Equilibrium Wages

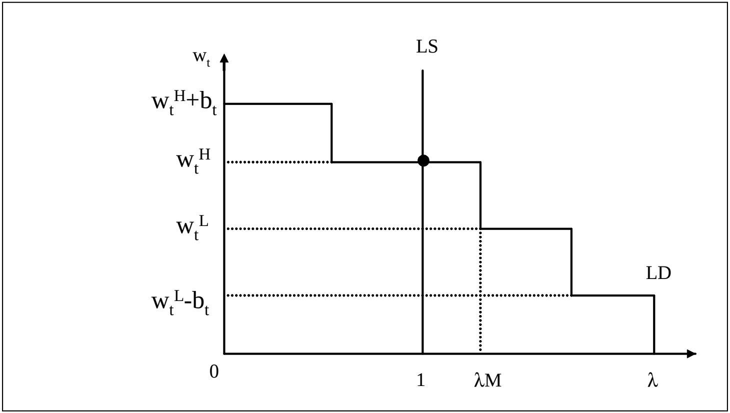
- In an entry equilibrium, i.e., when (34) holds, we must have that

$$NV\left(\mathbf{p}^{t}, \mathbf{w}^{t} \mid a_t^j = A^z, s_t^j = 0\right) = 0.$$

- Why?
- This implies that the equilibrium wage must be

$$w_t^e = w_t^H. \tag{35}$$

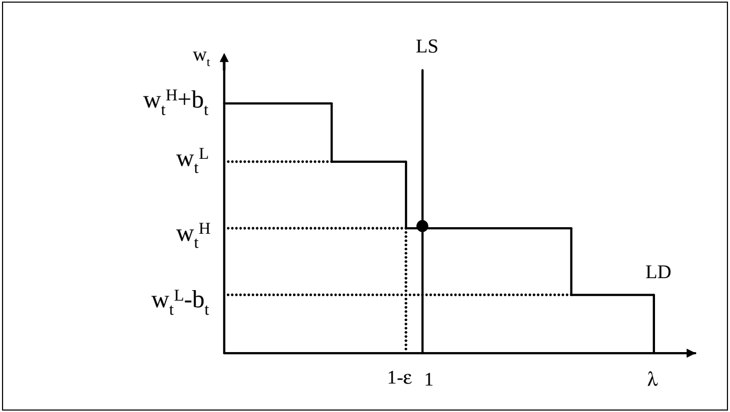
Entry Equilibrium (continued)



Labor supply and labor demand when (34) holds and there exists an entry equilibrium.

Sclerotic Equilibrium

- In this case, wages are still given by $w_t^e = w_t^H$ because of $\varepsilon > 0$.



Labor supply and labor demand when (34) does not hold and there exists a sclerotic equilibrium.

Composition of Entrepreneurs

- Law of motion of the fraction of entrepreneurs with high skills is

$$\mu_t = \begin{cases} \sigma^H \mu_{t-1} + \sigma^L (1 - \mu_{t-1}) & \text{if (34) does not hold} \\ 1 & \text{if (34) holds} \end{cases} . \quad (36)$$

starting with $\mu_0 = 1$.

Democratic Equilibrium

- In democracy, policies made by majoritarian voting.
- In MPE, after investments are made, the median voter, a worker, wishes through distribute as much as possible, thus

$$\tau_t = \delta.$$

- Moreover, entry barriers reduce wages (from (32)), thus

$$b_t = 0.$$

- Than in equilibrium:

$$V^H = W^H = W^L = W = \frac{w^D + T^D}{1-\beta}, \tag{37}$$

where w^D is the equilibrium wage in democracy, and T^D is the level of transfers, given by δY^D.

Democratic Equilibrium (continued)

Proposition: A democratic equilibrium always features $\tau_t = \delta$ and $b_t = 0$. Moreover, we have $e_t^j = 1$ if and only if $a_t^j = A^H$, so $\mu_t = 1$. The equilibrium wage rate is given by

$$w_t^D = w^D \equiv \frac{\alpha}{1-\alpha}(1-\delta)^{1/\alpha} A^H, \tag{38}$$

and the aggregate output is

$$Y_t^D = Y^D \equiv \frac{1}{1-\alpha}(1-\delta)^{\frac{1-\alpha}{\alpha}} A^H. \tag{39}$$

- Aggregate output is constant over time
- Also perfect equality because the excess supply of high-skill entrepreneurs ensures that they receive no rents.

Oligarchy Equilibrium

- Policies are determined by majoritarian voting among the elite.
- At the time of voting over the entry barriers, b_t, the elite are those with $s_t = 1$, and at the time of voting over the taxes, τ_t, the elite are those with $e_t = 1$.
- Let us start with the taxation decision among those with $e_t = 1$.
- It can be proved that as long as

$$\lambda \geq \frac{1}{2}\frac{A^H}{A^L} + \frac{1}{2}, \tag{40}$$

  then both high-skill and low-skill entrepreneurs prefer zero taxes, i.e., $\tau_t = 0$.
- Condition (40) requires the productivity gap between low and high-skill elites not to be so large that low-skill elites wish to tax profits in order to indirectly transfer resources from high-skill entrepreneurs to themselves.
- When condition (40) holds, the oligarchy will always choose $\tau_t = 0$.

Oligarchy Equilibrium (continued)

- Then anticipating this tax choice, at the stage of deciding the entry barriers, high-skill entrepreneurs would like to maximize $V^H\left(\left[b_t, 0, \mathbf{p}^{t+1}\right], \left[w_t, \mathbf{w}^{t+1}\right]\right)$, while low-skill entrepreneurs would like to maximize $V^L\left(\left[b_t, 0, \mathbf{p}^{t+1}\right], \left[w_t, \mathbf{w}^{t+1}\right]\right)$.
- Both of these are maximized by setting a level of the entry barrier that ensures the minimum level of equilibrium wages.
- Equilibrium wage, given in (35), will be minimized at $w_t^H = 0$, by choosing any

$$b_t \geq b_t^E \equiv \frac{\alpha}{1-\alpha}A^H + \beta\left(\frac{CV^H\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right) - CW^H\left(\mathbf{p}^{t+1},\mathbf{w}^{t+1}\right)}{\lambda}\right). \tag{41}$$

- Without loss of any generality, set $b_t = b_t^E$.

Oligarchy Equilibrium (continued)

- Aggregate output in equilibrium is:

$$Y_t^E = \mu_t \frac{1}{1-\alpha} A^H + (1-\mu_t) \frac{1}{1-\alpha} A^L, \tag{42}$$

where $\mu_t = \sigma^H \mu_{t-1} + \sigma^L (1 - \mu_{t-1})$ as given by (36), with $\mu_0 = 1$.

- Since μ_t is a decreasing sequence converging to M, aggregate output Y_t^E is also decreasing over time with:

$$\lim_{t \to \infty} Y_t^E = Y_\infty^E \equiv \frac{1}{1-\alpha} \left(A^L + M(A^H - A^L) \right). \tag{43}$$

- The reason for this is that as time goes by, the comparative advantage of the members of the elite in entrepreneurship gradually disappears because of the imperfect correlation between ability over time.

Oligarchy Equilibrium (continued)

- Also high degree of (earnings) inequality.
    - Wages are equal to 0, while entrepreneurs earn positive profits

Proposition: Suppose that condition (40) holds. Then an oligarchic equilibrium features $\tau_t = 0$ and $b_t = b^E$, and the equilibrium is sclerotic, with equilibrium wages $w_t^e = 0$, and fraction of high-skill entrepreneurs $\mu_t = \sigma^H \mu_{t-1} + \sigma^L (1 - \mu_{t-1})$ starting with $\mu_0 = 1$. Aggregate output is given by (**??**) and decreases over time starting at $Y_0^E = \frac{1}{1-\alpha} A^H$ with $\lim_{t \to \infty} Y_t^E = Y_\infty^E$ as given by (43).

Comparison between Democracy and Oligarchy

- First, as long as $\delta > 0$, then

$$Y^D = \frac{1}{1-\alpha}(1-\delta)^{\frac{1-\alpha}{\alpha}} A^H < Y_0^E = \frac{1}{1-\alpha} A^H.$$

- Therefore, for all $\delta > 0$, oligarchy initially generates greater output than democracy, because it is protecting the property rights of entrepreneurs.
- However, the analysis also shows that Y_t^E declines over time, while Y^D is constant, the oligarchic economy may subsequently fall behind the democratic society.
- Whether it does so or not depends on whether Y^D is greater than Y_∞^E as given by (43).

Comparison between Democracy and Oligarchy (continued)

- This will be the case if $(1-\delta)^{\frac{1-\alpha}{\alpha}} A^H / (1-\alpha) > \left(A^L + M(A^H - A^L)\right) / (1-\alpha)$, or if

$$(1-\delta)^{\frac{1-\alpha}{\alpha}} > \frac{A^L}{A^H} + M\left(1 - \frac{A^L}{A^H}\right). \tag{44}$$

- If condition (44) holds, then at some point the democratic society will overtake ("leapfrog") the oligarchic society.

Comparison between Democracy and Oligarchy (continued)

Proposition: Suppose that condition (40) holds. Then at $t = 0$, aggregate output is higher in an oligarchic society than in a democratic society, i.e., $Y_0^E > Y^D$. If (44) does not hold, then aggregate output in oligarchy is always higher than in democracy, i.e., $Y_t^E > Y^D$ for all t. If (44) holds, then there exists $t' \in N$ such that for $t \leq t'$, $Y_t^E \geq Y^D$ and for $t > t'$, $Y_t^E < Y^D$, so that the democratic society leapfrogs the oligarchic society. Leapfrogging is more likely when δ, A^L / A^H and M are low.

- Oligarchies are more likely to be relatively inefficient in the long run:
  - when δ is low, meaning that democracy is unable to pursue highly populist policies
  - when A^H is high relative to A^L, so that high-skill comparative advantage is important
  - M is low, so that a random selection of agents contains a small fraction of high-skill agents, making oligarchic sclerosis highly distortionary.

Comparison between Democracy and Oligarchy (continued)

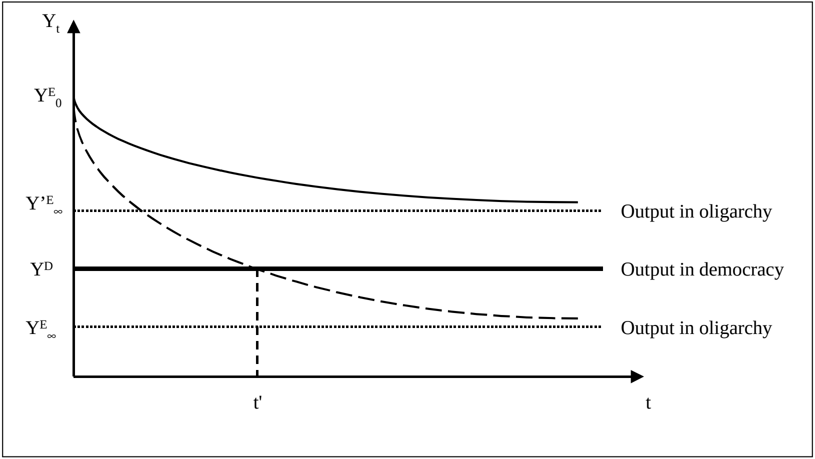


Figure 3: Comparison of aggregate output in democracy and oligarchy. The dashed curve depicts output in oligarchy when (44) holds, and the solid line when it does not.

Other Systems?

- Can other political systems do better?
- Yes, for example, delegate taxes to entrepreneurs and entry barriers to workers
- But, generally not feasible.
- Political power "indivisible": if the system is democratic, the party in power can also decides taxes.

New Technologies and Institutional Flexibility

- Democracies also more flexible.
- Suppose that at some date $t' > 0$, there is an unanticipated and exogenous arrival of a new technology, enabling entrepreneur j to produce:

$$y_t^j = \frac{1}{1-\alpha}(\psi \hat{a}_t^j)^\alpha (k_t^j)^{1-\alpha}(l_t^j)^\alpha,$$

  where $\psi > 1$ and $\hat{a}_t^j$ is the talent of this entrepreneur with the new technology.
- Suppose $l_t^j = \lambda$ for the new technology as well, entrepreneur j's output can be written as

$$\max\left\{\frac{1}{1-\alpha}(\psi \hat{a}_t^j)^\alpha (k_t^j)^{1-\alpha}\lambda^\alpha, \frac{1}{1-\alpha}(a_t^j)^\alpha (k_t^j)^{1-\alpha}\lambda^\alpha\right\}.$$

New Technologies and Institutional Flexibility (continued)

- Also to simplify the discussion, assume that the law of motion of $\hat{a}_t^j$ is similar to that of a_t^j, given by

$$\hat{a}_{t+1}^j = \begin{cases} A^H & \text{with probability } \sigma^H \quad \text{if } \hat{a}_t^j = A^H \\ A^H & \text{with probability } \sigma^L \quad \text{if } \hat{a}_t^j = A^L \\ A^L & \text{with probability } 1-\sigma^H \quad \text{if } \hat{a}_t^j = A^H \\ A^L & \text{with probability } 1-\sigma^L \quad \text{if } \hat{a}_t^j = A^L \end{cases} \tag{45}$$

- Comparative advantage shifts to a new set of entrepreneurs.

New Technologies and Institutional Flexibility (continued)

- Democracy will immediately switched to the new technology, thus

$$\hat{Y}^D \equiv \frac{\psi}{1-\alpha}(1-\delta)^{\frac{1-\alpha}{\alpha}} A^H.$$

- In contrast, switch to new technology will be delayed in oligarchy in oligarchy.

Conclusion

- We have seen in this lecture how different types of economic institutions emerge when political power is largely uncontested in the hands of a single group with broadly homogeneous interests but competing with others in the economy.
- In the next lecture, we will investigate in greater theoretical and empirical detail the economics and politics of a specific and very common economic institutions that emerges under elite control—labor coercion.