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*Impact of Public Announcements on Trade in Financial Markets**

Stephen Morris
Department of Economics
University of Pennsylvania

Hyun Song Shin
Department of Economics
University of Southampton

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Abstract

When traders with asymmetric information act strategically, public announcements can convey crucial information concerning the higher order beliefs of other traders. This is so even though the public announcement is uninformative concerning the fundamentals of the market. We exhibit a case of trade which is fragile to the announcement of a fact which is already known to all market participants.

Keywords: Volume of Trade, Bayesian Games, Common Knowledge.

1. Introduction

Our understanding of the workings of financial markets has been greatly enhanced by the significant advances made in the theory of asset pricing. However, the lack of any convincing theory of the volume of trade remains a serious gap in our understanding. Ross (1989) laments this state of affairs, and notes that any such theory of trading volume will be difficult to reconcile with existing theories of intertemporal asset pricing, since these theories are based on price taking behaviour aimed at incremental and gradual portfolio rebalancing in the face of news. Events such as the crash of 1987, as well as the magnitude of day to day fluctuations in trading volume sit uncomfortably with the elegant, yet inadequate theories of asset pricing.

The literature generated by the 1987 crash has had some success in showing how prices may be volatile, but has had little to say on trading volume. For Gennotte and

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Leland (1990), for example, the introduction of non-linear hedging strategies in an otherwise linear, price-taking world introduces discontinuities in prices. Grossman (1988) and Jacklin, Kleidon and Pfleiderer (1992) also emphasize the role of hedging strategies and program trading. For Romer (1993), a discrete probability distribution over types of traders with varying qualities of news means that one round of trading reveals information concerning the quality of traders' information. Bulow and Klemperer (1994) is an exception to the rule that competitive price taking behaviour is assumed. They examine price volatility in an auction market with fully sophisticated traders.

Our paper is a product of the view that the fluctuations in the volume of trade can only be addressed adequately if account is taken of the strategic nature of trade, and when the interaction of the beliefs of the traders is modelled explicitly. Our aims in this paper are modest, and our arguments are based on a specialized asset structure. Nevertheless, we point to suggestive features which deserve further investigation. In particular, we will show how the arrival of apparently uninformative news can propagate large fluctuations in the volume of trade. Indeed, we can claim that in some cases the public announcement of a fact which is already known by all traders can cause large changes in trading volume. Although these announcements may seem at first sight to be uninformative, they convey crucial information about the beliefs of others when placed in the context of a game.

To elucidate this point, it is instructive to contrast the role of uncertainty in games from that in single person decision theory. Suppose that the uncertainty concerning the fundamentals of the economy is represented by a state space Ω, and a trader has an information partition over this space. At some state ω, the trader knows that the true state is in the cell of his information partition which contains ω. Suppose now that there is a public announcement to the effect that the true state is in some subset A of Ω, but that A does not intersect with the cell of the trader's partition containing ω. Suppose now that there is a public announcement to the effect that the true state is in some subset A of Ω, but that A does not intersect witht the cell of the trader's partition containing ω. In single person decision theory, this announcement is completely uninformative to the decision maker, since the decision maker's private information dominates the information conveyed by the announcement.

However, if trade is strategic, the optimal action of a trader at one state depends on the actions of other traders at that state, and this raises the issue of what other traders believe at that state. With asymmetric information, the beliefs of traders in neighbouring states then become relevant. But then, the reasoning does not stop there, since the beliefs of traders at these neighbouring states will depend on their beliefs about the beliefs of other traders at a further set of states, and so on. Rubinstein's (1989) 'electronic mail game' is an example of such an effect, and Monderer and Samet (1990) and Geanakoplos (1992) discuss related examples.

The key to understanding the impact of a public announcement is that the announce-

ment conveys information by virtue of the fact that it is public, and hence its content becomes common knowledge among the traders. Thus, although the announcement may reveal nothing new in terms of the "fundamentals" to any of the traders, it often conveys information concerning the higher order beliefs concerning these fundamentals. In these circumstances, a public announcement which may seem uninformative can, nevertheless, generate a significant impact. We exhibit a case of trade which is fragile to such an 'apparently uninformative' public announcement and diagnose the reasons for the fragility in subsequent sections. We begin with our model.

2. The Model

Two manufacturers of lightbulbs are in competition to meet a sizeable order for long-life lightbulbs from a government department. The department has set down a fixed price, and has announced that it will conduct a durability trial of the two brands of lightbulbs. The brand which has the greater durability (in terms of the number of hours of operation) will win the order. The two firms have developed competing versions of a revolutionary design of lightbulb which, in principle, could last forever provided that the filament consists of pure tungsten. However, if there is even the tiniest amount of impurity in the filament, the bulb has a finite life. Although the purifying process for tungsten has come a long way, ti cannot be relied on to remove all the impurities. The durability of the lightbulb is an increasing function of the purity of the filament.

The tungsten refining industry is dominated by a monopoly, and both lightbulb manufacturers are supplied with the same grade of refined tungsten for their filaments. Although the two lightbulb manufacturers have developed competing designs, the durability of the two brands is highly correlated, due to their use of filaments of identical purity. We denote by θ the level of purity of the tungsten used by both manufacturers; θ lies in the open unit interval (0,1).

The state space which underlies our analysis is given by the set of all triples (θ, v_1, v_2), where v_i is the durability of the lightbulb of the ith manufacturer. We assume that the density function over the triples (θ, v_1, v_2) is atomless and continuous, and will denote it by $\pi(\cdot, \cdot, \cdot)$. We will formalize the idea that the durability of the lightbulbs increase without bound as θ approaches 1 in the following way. Assume that there is a strictly increasing function d which maps each $\theta \in (0, 1)$ to a positive real number such that $d(\theta) \to \infty$ as $\theta \to 1$. Then, for any given value of θ, both v_1 and v_2 can take any value within ϵ distance of $d(\theta)$. In other words,

$$\pi(\theta, v_1, v_2) > 0 \Leftrightarrow d(\theta) - \epsilon \leq v_1 \leq d(\theta) + \epsilon, \text{for all } i \in \{1, 2\}.$$

Let us denote by V_i the random variable whose realization is v_i, the durability of the ith lightbulb. We will assume that V_i can take any non-negative value. Each man-

ufacturer observes the durability of its own brand, but observes neither the durability of its rival's brand, nor the purity of the tungsten θ.

Of particular importance to our analysis is the density over the pairs (v_1, v_2) obtained from π by summing over the first component. We denote this density by μ. Thus,

$$\mu(v_1, v_2) \equiv \int_{\theta=0}^{1} \pi(\theta, v_1, v_2) d\theta. \tag{2.1}$$

Player 1 will form beliefs by conditioning on the realization of V_1, while player 2 will form beliefs by conditioning on the realization of V_2. From (1), μ inherits the feature that $\mu(v_1, v_2) > 0$ if and only if $\mid v_1 - v_2 \mid \leq 2\epsilon$. Hence, upon observing the realization $V_1 = v_1$, player 1 infers that the realization of V_2 lies within distance 2ϵ of v_1. The support of the density μ is indicated by the shaded region in figure 1. Also, μ is continuous since π is continuous.

[Figure 1 here]

We will assume that there is a number $\eta > 0$ such that for every realization of V_i, player i attaches conditional probability of at least η that his rival's realization is above his own. In otherwords, η is such that, for $j \neq i$ and all v_i,

$$\int_{z=v_i}^{\infty} \mu(z \mid V_i = v_i) dz \geq \eta. \tag{2.2}$$

This assumption has the effect that, however large v_i is, there is a uniform lower bound on the probability that the rival's durability is greater than v_i.

3. Trade

We now turn to the description of trade. The game is played between the owners of the two firms. There is a single consumption good. The two firms are assets which lay claim to the consumption good at specified states. The payoffs reflect the "all or nothing" nature of the competition between the two firms. The firm with the more durable lightbulb wins the order, and yields one unit of the consumption good at the end of the game. The losing firm is worth zero in terms of the consumption good. In other words, firm 1 is an asset whose payoff is a random variable x_1 defined on the pairs (v_1, v_2) such that:

$$x_1(v_1, v_2) = \begin{cases} 1 & \text{if } v_1 > v_2 \\ 0 & \text{otherwise} \end{cases} \tag{3.1}$$

Then, firm 2 is an asset $x_2(v_1, v_2)$ such that $x_2 = 1 - x_1$.

Both players are risk averse, and have identical preferences given by the von Neumann Morganstern utility function

$$u(c) = c^{\alpha}, \tag{3.2}$$

where $0 < \alpha < 1$, and c is the level of consumption.

A trade t is an element of the unit square. Trade $t \equiv (t_1, t_2)$ has the interpretation of an exchange in which proportion t_1 of firm 1 is given up in exchange for proportion t_2 of firm 2. Each trading game is indexed by a particular trade t. The action set of both players is {Accept, Reject}. We will contrast two distinct versions of the trading game.

(i) Trade in absence of announcement. For a given trade t, traders observe their respective signals before choosing an action. A strategy of player i is a function which maps each realization of his signal V_i to the action set {Accept, Reject}. We will denote by A_1 the set of realizations of V_1 at which player 1 accepts t. Call A_1 the acceptance set of player 1. Denote by A_2 the acceptance set for player 2. Note that a player's acceptance set depends on the proposed trade t.

The trade t takes place when both players accept. Thus, for the pair of messages (v_1, v_2), trade t takes place if and only if $(v_1, v_2) \in A_1 \times A_2$. The acceptance sets of the players provide a convenient shorthand for the strategies of the players. An equilibrium of the trading game in the absence of announcement for trade $t \equiv (t_1, t_2)$ is a pair of acceptance sets (A_1, A_2) such that each player maximizes his expected utility conditional on the realization of his signal. More formally, for any pair of strategies (A_1, A_2), denote by $y_1(v_1, v_2)$ the post-trade allocation of the consumption good for trader 1. It is given by:

$$y_1(v_1, v_2) = \begin{cases} (1 - t_1)x_1(v_1, v_2) + t_2 x_2(v_1, v_2) & \text{if } (v_1, v_2) \in A_1 \times A_2 \\ x_1(v_1, v_2) & \text{otherwise} \end{cases} \tag{3.3}$$

Analogously, the post-trade allocation of the consumption good at (v_1, v_2) for trader 2 given strategies (A_1, A_2) is given by:

$$y_2(v_1, v_2) = \begin{cases} t_1 x_1(v_1, v_2) + (1 - t_2)x_2(v_1, v_2) & \text{if } (v_1, v_2) \in A_1 \times A_2 \\ x_2(v_1, v_2) & \text{otherwise} \end{cases}$$

Apair of strategies (A_1, A_2) is an equilibrium of this game if, for all i,

$$E_{\mu}(u(y_i) \mid V_i) \geq E_{\mu}(u(x_i) \mid V_i). \tag{3.4}$$

where E_μ denotes the expectations operator with respect to the distribution μ.

(ii) Trade following announcement. In our alternative scenario of the trading game, the players not only observe their own signals, but also receive a public announcement from the supplier of the tungsten filament. The supplier of the filament announces the results of a test which measures the incidence of cobolt in the tungsten. Cobolt is one of a dozen or so possible impurities in the tungsten, but it happens to be the only impurity which can be measured. Thus, the announcement of the cobolt content is, in effect, an announcement of an upper bound $\bar{\theta}$ for the purity of the tungsten. From this, the players can infer that the value of d is at most $d(\bar{\theta})$, and from (1), that the durability of both brands of lightbulbs is at most $d(\bar{\theta}) + \epsilon$.

In terms of the players' beliefs over the pairs (v_1, v_2), the effect of the public announcement is to truncate the support of the joint distribution so that any realization of V_i above $d(\bar{\theta}) + \epsilon$ receives zero density. Assuming that both traders update by Bayes rule from the distribution μ, the traders' beliefs following the announcement are governed by a new density over the pairs (v_1, v_2) denoted by λ, where the density of a pair $\lambda(v_1, v_2)$ such that $(v_1, v_2) \leq (d(\bar{\theta}) + \epsilon, d(\bar{\theta}) + \epsilon)$ is given by

$$\lambda(v_1, v_2) = \frac{\int_{\theta=0}^{\bar{\theta}} \pi(\theta, v_1, v_2) d\theta}{\int_{\theta=0}^{\bar{\theta}} \int_{v_1=0}^{\infty} \int_{v_2=0}^{\infty} \pi(\theta, v_1, v_2) dv_2 dv_1 d\theta} \tag{3.5}$$

while the density for a pair (v_1, v_2) such that $(v_1, v_2) > (d(\bar{\theta}) + \epsilon, d(\bar{\theta}) + \epsilon)$ is zero. The support of λ is illustrated in figure 2.

[Figure 2 here]

An equilibrium of the trading game following the public announcement is defined in an analogous way to our first game, except that players now form beliefs from the density λ rather than the density μ. Thus, a pair of strategies (A_1, A_2) is an equilibrium of the trading game following the announcement if, for all i,

$$E_\lambda(u(y_i) \mid V_i) \geq E_\lambda(u(x_i) \mid V_i). \tag{3.6}$$

It should be noted that $\bar{\theta}$ may be much higher than the realized value of θ, in which case the public announcement of $\bar{\theta}$ will not be very informative to the traders as to the true value of d and their rival's duribility. The observation of their own signal may be far more informative concerning these "fundamentals". In this sense, this public announcement is apparently uninformative. However, by virtue of the fact that the announcement is public, the players have access to the fact that it is common knowledge

that θ is at most $\bar{\theta}$. In turn, it is now common knowledge that the durability of both brands of lightbulbs is at most $d(\bar{\theta}) + \epsilon$. This is the key insight which will allow us to understand the contrast between the set of propositions presented below.

In stating our results, we will rule out the trade $t = (0, 0)$. Let's say that a trade t is non-trivial if $t \neq (0, 0)$.

Proposition 1. If the traders are sufficiently risk averse, there is a non-trivial trade t and an equilibrium of the trading game in the absence of announcement in which $A_1 = A_2 = \Re_+$.

Proposition 2. For any $\alpha \in (0, 1)$, any non-trivial trade t, and any equilibrium (A_1, A_2) of the trading game following announcement, $A_1 \times A_2$ has measure zero.

Thus, in the absence of a public announcement, trade takes place if traders are sufficiently risk averse, but the public announcement precludes trade, however risk averse the traders are. Before diagnosing the reasons for the absence of trade in proposition 2, let us work through the proofs of both results. We shall prove the first proposition by exhibiting a non-trivial trade which is accepted in equilibrium.

Suppose player 1 has observed $V_1 = v_1$. Then, by (4),

$$E_\mu(u(x_1) \mid V_1 = v_1) \leq (1 - \eta)u(1) = 1 - \eta. \tag{3.7}$$

The conditional expected utility following the trade $(\frac{1}{2}, \frac{1}{2})$ is given by:

$$E_\mu \left(u\left(\frac{x_1 + x_2}{2}\right) \middle| v_1 \right) = u\left(\frac{1}{2}\right). \tag{3.8}$$

There is some $\alpha^* \in (0, 1)$ such that $(\frac{1}{2})^{\alpha^*} \geq 1 - \eta$. Thus, for all values of $\alpha < \alpha^*$, we have $u(\frac{1}{2}) > 1 - \eta$. In other words, the expected utility following trade is higher than the expected utility of the endowment. An exactly analogous argument holds for player 2. Hence, there is an equilibrium in which the trade $(\frac{1}{2}, \frac{1}{2})$ is accepted by both players at evert state. Clearly, $(\frac{1}{2}, \frac{1}{2})$ is a non-trivial trade. This proves proposition 1.

Let us now consider proposition 2. Let us suppose, contrary to proposition 2, that $A_1 \times A_2$ has positive measure for a non-trivial trade t. Then, there is an open set of pairs (v_1, v_2) at which both players accept this trade. Let us denote by $\{V_2 \in A_2\}$ the event that the realization of V_2 is in the acceptance set A_2 of player 2. We denote by $\text{Prob}_\lambda(\{V_2 \in A_2\} \mid V_1 = v_1)$ the probability of this event conditional on $V_1 = v_1$. In other words,

$$\text{Prob}_\lambda(\{V_2 \in A_2\} \mid V_1 = v_1) \equiv \frac{\int_{v_2 \in A_2}^{\infty} \lambda(v_1, v_2) dv_2}{\int_{v_2=0}^{\infty} \lambda(v_1, v_2) dv_2} \tag{3.9}$$

Consider the set of realizations of V_i for which player i accepts trade, and conditional on which i attaches positive probability to player j accepting trade, where $i \neq j$. Denote this set by $\bar{V}_i$. In other words,

$$\bar{V}_i \equiv \{v_i \mid v_i \in A_i \text{ and } \text{Prob}_\lambda(\{V_j \in A_j\} \mid V_i = v_i) > 0\} \tag{3.10}$$

Let us denote by $\bar{v}_i$ the least upper bound of the interior of this set. In other words,

$$\bar{v}_i = \sup \text{Int } \bar{V}_i.$$

Note that $\bar{v}_i$ is finite for both players, since it is bounded above by $d(\bar{\theta}) + \epsilon$.

Now, let us suppose that $\bar{v}_1 \geq \bar{v}_2$. The expected utility of player 1's endowment given $V_1 = v_1$ is $E_\lambda(u(x_1) \mid V_1 = v_1)$ which is :

$$\text{Prob}_\lambda(\{V_2 < v_1\} \mid V_1 = v_1)u(1) + \text{Prob}_\lambda(\{V_2 \leq v_1\} \mid V_1 = v_1)u(0). \tag{3.11}$$

We know that $u(0) = 0$ and $u(1) = 1$. Also, we know that λ inherits from μ the fact that it is continuous on the interior of its support. Hence, as v_1 approaches $\bar{v}_1$ from below,

$$E_\lambda(u(x_1) \mid V_1 = v_1) \to \text{Prob}_\lambda(\{V_2 < \bar{v}_1\} \mid V_1 = \bar{v}_1) \tag{3.12}$$

Now, consider the expected utility of the post-trade allocation y_1 given $V_1 = v_1$. It is:

$$\begin{aligned} E_\lambda(u(y_1) \mid V_1 = v_1) = \quad & \text{Prob}_\lambda(\{V_2 < v_1 \text{ and } V_2 \notin A_2\} \mid V_1 = v_1)u(1) \\ & + \text{Prob}_\lambda(\{V_2 < v_1 \text{ and } V_2 \in A_2\} \mid V_1 = v_1)u(1 - t_1) \\ & + \text{Prob}_\lambda(\{V_2 \geq v_1 \text{ and } V_2 \in A_2\} \mid V_1 = v_1)u(t_2) \\ & + \text{Prob}_\lambda(\{V_2 \geq v_1 \text{ and } V_2 \notin A_2\} \mid V_1 = v_1)u(0). \end{aligned}$$

By hypothesis, $\bar{v}_1 \geq \bar{v}_2$, so that

$$\{V_2 \geq \bar{v}_1 \text{ and } V_2 \in A_2\} \subseteq \{V_2 \geq \bar{v}_2 \text{ and } V_2 \in A_2\}. \tag{3.13}$$

But by the definition of $\bar{v}_2$, we have $\bar{v}_2 \leq \sup A_2$ so that the event on the right hand side of (16) has probability zero. Hence,

$$\text{Prob}_\lambda(\{V_2 \geq \bar{v}_1 \text{ and } V_2 \in A_2\} \mid V_1 = \bar{v}_1) = 0 \tag{3.14}$$

Since $u(0) = 0$ and $u(1) = 1$, the continuity of the conditional probabilities implies that, as v_1 approaches $\bar{v}_1$ from below, we have:

$$\begin{aligned} E_\lambda(u(y_1) \mid V_1 = v_1) \to \quad & \text{Prob}_\lambda(\{V_2 < \bar{v}_1 \text{ and } V_2 \notin A_2\} \mid V_1 = \bar{v}_1) \\ & + \text{Prob}_\lambda(\{V_2 < \bar{v}_1 \text{ and } V_2 \in A_2\} \mid V_1 = \bar{v}_1)u(1 - t_1) \\ & < \text{Prob}_\lambda(\{V_2 < \bar{v}_1\} V_1 = \bar{v}_1), \end{aligned} \tag{3.15}$$

where the strict inequality follows from the fact that $u(1-t_1) < 1$ and the fact that $\text{Prob}_\lambda(\{V_2 < \bar{v}_1 \text{ and } V_2 \in A_2\} \mid V_1 = \bar{v}_1)$ is strictly positive by construction. Thus, from (15) and (18), it follows that for an open set of values of v_1 close to $\bar{v}_1$, the endowment x_1 of player 1 yields strictly higher expected utility than the post-trade allocation y_1, which violates the rationality of trader 1. Hence, our hypothesis that $\bar{v}_1 \geq \bar{v}_2$ is inconsistent with equilibrium. Thus, in any equilibrium, $\bar{v}_1 < \bar{v}_2$. However, an exactly anaogous argument shows that the rationality of player 2 implies $\bar{v}_1 > \bar{v}_2$, which leads to the absurd conclusion that $\bar{v}_1 > \bar{v}_2 > \bar{v}_1$. Therefore, our initial supposition that $A_1 \times A_2$ has positive measure cannot hold. This proves proposition 2.

4. An Assessment

The arguments used in establishing our pair of results show that many features of the model may be relaxed with affecting the results. For proposition 1, the continuity of the conditional expectation and the uniform lower bound on the probability of a higher type, as expressed by (3), are the only essential ingredients of the proof. For proposition 2, the argument rests on there being an upper bound on the support of the distribution λ. We did not make any essential use of the relationship between the distributions μ and λ, apart from the fact that λ inherits continuity from μ.

In delving into the reasons behind the fragility of trade as witnessed by proposition 2, it is helpful to consider more systematically the beliefs of the two traders. In the space of pairs (v_1, v_2), the support of the density μ is given by the diagonal band of width 4ϵ as shown in figure 1. The public announcement informs the traders that the true state is in the event Q, where:

$$Q = \{(v_1, v_2) \mid v_i \leq d(\bar{\theta}) + \epsilon\} \tag{4.1}$$

Thus, prior to the announcement of the event Q, trader 1 believes Q with probability 1 only at those values of v_1 which are at least distance 2ϵ away from the highest value of v_1 consistent with Q. Thus, we may represent the set of points at which trader 1 believes Q as in figure 3. We denote this event by $B_1(Q)$.

[Figure 3 here]

In turn, trader 2 believes $B_1(Q)$ with probability 1 at those realizations of v_2 which are distance at least 4ϵ from the top edge of $B_1(Q)$. This allows us to represent the event in which 2 believes that 1 believes Q as in figure 3. We denote this event as $B_2(B_1(Q))$. Proceeding in this way, we can consider events at which there is even higher order belief concerning the event Q.

In particular, the sequence $X_0, X_1, X_2, \cdots$ define as $X_0 = Q$ and $X_n = B_2(B_1(X_{n-1}))$ converges to the empty set, since each application of the operator $B_2(B_1(\cdot))$ acts as a strict contraction of bounded size. One consequence of this is that the event in which the event Q is common belief is the empty set. In other words, prior to the public announcement of Q, the event Q cannot be common belief between the two traders. When ϵ is small, and the traders observe low realizations of V_i, the event Q may be believed to a high order of iterated belief. Nevertheless, Q will never be common belief.

However, the effect of the public announcement is to render this event common belief. It is precisely this feature which precludes trade, since a trader of a given type v_i enters the market only if he places positive probability on the trading partner being a higher type. Otherwise, trade will make him worse off, for sure. If there is common belief that the type of one's trading partner is at most $\hat{v}$, then for any given level of the risk aversion parameter α, a trader whose own type is sufficiently close to $\hat{v}$ will withdraw from the market, since the probability of the trading partner being a higher type is too small to justify trade. Since both traders can reason in this way, it then becomes common belief among the traders that the type of one's trading partner is at most $\hat{v} - \delta$, where δ is the size of the interval of traders who withdraw. But then, the traders can apply the reasoning again to conclude that it is common belief that the type of one's trading partner is at most $\hat{v} - 2\delta$. With each round of this reasoning, the highest types of traders who remain are driven out of the market. In equilibrium, all traders are driven out.

When explained in these terms, our model has more than a passing resemblance to the "lemons" example of Akerlof (1970). In both cases, traders with private information on the value of their own endowment assess the likely consequence of trade, and the traders endowed with the highest quality items withdraw from the market. The difference between our model and that of Akerlof is that, in our story the withdrawal of one side of the market leads to the withdrawal of the other side, which in turn precipitates a further withdrawal of the first side. At the risk of mixing untidy metaphors, we could dub the mechanism underlying proposition 2 as being a "double-sided lemon". Brams, Kilgour and Davis (1993) have discussed an example which also deals with such unravelling arguments.

It goes without saying that our model is but one special case of an exchange economy with differential information. It is an interesting open question as to how large the class of economies is which is susceptible to the sort of arguments developed in this paper. For the class of finite Bayesian games, Morris, Rob and Shin (1993) have shown that certain general features of the information structure and of the underlying game can interact to produce outcomes which magnify the effects of higher order beliefs. At a more technical level, we can pose the general question as that of identifying the appropriate topology over the set of information structures with respect to which outcomes are continuous. Monderer and Samet (1990) is one paper which addresses this issue for abstract games.

One of the goals of further research will be to distill those essential features of richer economic settings which make outcomes sensitive to higher order beliefs.

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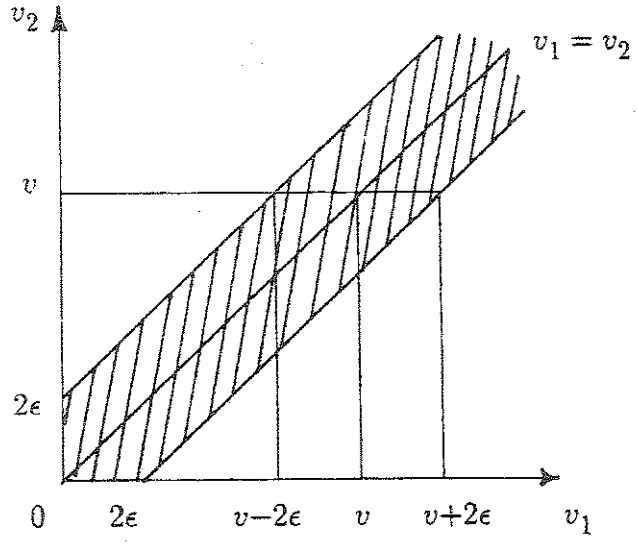
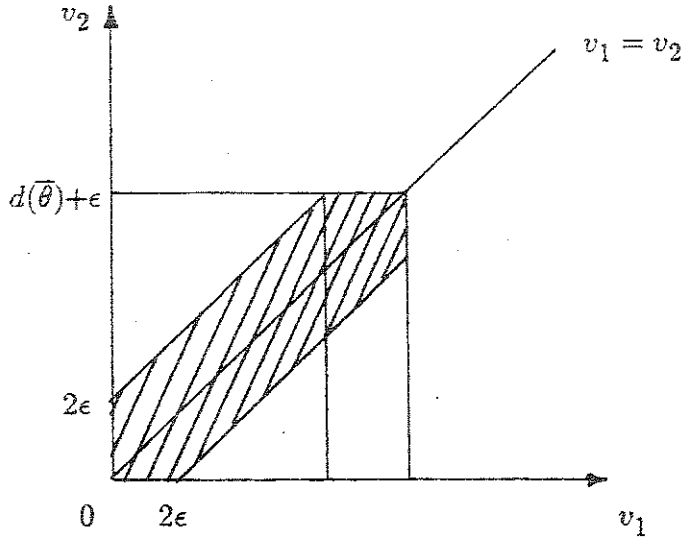
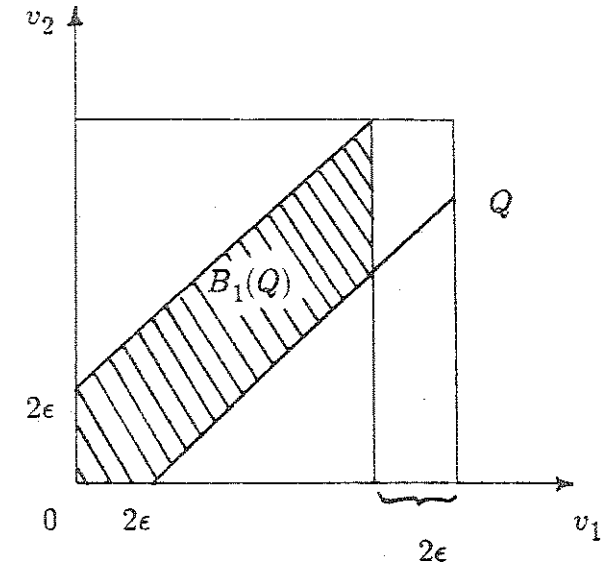
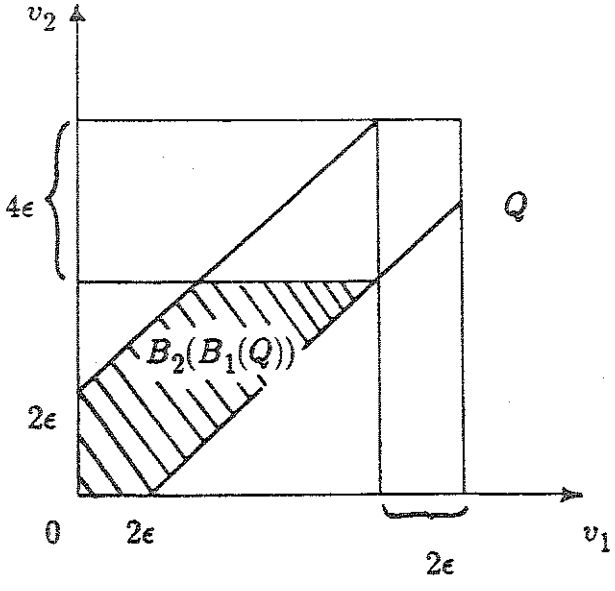


[Figure 1]



[Figure 2]


v_2
Q
$B_1(Q)$
2ϵ
0
2ϵ
2ϵ
v_1


v_2
4ϵ
Q
$B_2(B_1(Q))$
2ϵ
0
2ϵ
2ϵ
v_1

[Figure 3]

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