Clarendon Lectures, Lecture 3
General Theory of Directed Technical Change

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Summary of Results so Far

- Importance of directed technical change.
- Relatively strong results on the equilibrium direction of technical change.
- Implications for the evolution of skill bias of technology.
- But results derived under two sets of special assumptions:
  1. Constant elasticity of substitution production functions.
  2. "Standard baggage" of endogenous growth (implicit linearity, Dixit-Stiglitz preferences, factor-augmenting technologies).
- How general are the insights?

This Lecture

- Main insights will hold very generally.
- Useful to distinguish between:
  1. relative bias: about shifts of relative demand curves
  2. absolute bias: about shifts of factor demands
- Results so far about relative bias.
- Main results:
  1. Theorems on relative bias can be generalized, but only to some degree.
  2. Much more general theorems on absolute bias.

Plan

- First introduce a class of environments where we can study bias of technology.
- Then generalize results on relative bias and show their limitations.
- Most important results: weak and strong theorems on absolute bias.
- Main takeaway message: under fairly reasonable conditions, factor demand curves will be **upward sloping**!

Basic Environment

- Static economy consisting of a unique final good (dynamics not central to the message here).
- $N + 1$ factors of production, Z and $L = (L_1, ..., L_N)$.
- Inelastic supplies: $\bar{Z} \in \mathbb{R}_+$ and $\bar{L} \in \mathbb{R}_+^N$.
- Main comparative statics: changing $\bar{Z}$.
- Representative household with preferences defined over the consumption of the final good.
- A continuum of firms (final good producers) denoted by the set $\mathcal{F}$, each with an identical production function.
- Normalize the measure of $\mathcal{F}$, $|\mathcal{F}|$, to 1.
- The price of the final good is also normalized to 1.

Alternative Economies

- Consider four different environments:
  1. Economy D: Fully decentralized. Technologies chosen by firms themselves.
  2. Economy C: Centralized. Technology decided by a centralized agency (taking firms' profit maximization is given).
  3. Economy M: Monopoly. Technology decided by a profit-maximizing technology monopolist.
  4. Economy O: Oligopoly. Technology decided by a set of (potentially competing) oligopolist.

Economy D

- For benchmark (not the most realistic economy for technology choice).
- Each firm $i \in \mathcal{F}$ has access to a production function

$$Y^i = G(Z^i, L^i, \theta^i),$$

- $Z^i \in \mathcal{Z} \subset \mathbb{R}_+, L^i \in \mathcal{L} \subset \mathbb{R}_+^N$
- $\theta^i \in \Theta \subset \mathbb{R}^K$ is the measure of technology.
- G: production function (throughout assumed to be twice differentiable).
- The cost of technology $\theta \in \Theta$ in terms of final goods is $C(\theta)$.

Economy D (continued)

- Each final good producer (firm) maximizes profits:

$$\max_{Z^i \in \mathcal{Z}, L^i \in \mathcal{L}, \theta_i \in \Theta} \pi(Z^i, L^i, \theta^i) = G(Z^i, L^i, \theta^i) - w_Z Z^i - \sum_{j=1}^{N} w_{Lj} L^i_j - C\left(\theta^i\right),$$

- w_Z is the price of factor Z and w_{Lj} is the price of factor L_j for $j = 1, ..., N$.
- All factor prices taken as given by firms.
- The vector of prices for factors L denoted by w_L.
- Market clearing:

$$\int_{i \in \mathcal{F}} Z^i di \leq \bar{Z} \text{ and } \int_{i \in \mathcal{F}} L^i_j di \leq \bar{L}_j \text{ for } j = 1, ..., N.$$

Economy D (continued)

Definition 1 An equilibrium in Economy D is a set of decisions $\left\{Z^i, L^i, \theta^i\right\}_{i\in\mathcal{F}}$ and factor prices (w_Z, w_L) such that $\left\{Z^i, L^i, \theta^i\right\}_{i\in\mathcal{F}}$ maximize profits given prices (w_Z, w_L) and market clearing conditions hold.

- Any θ^i that is part of the set of equilibrium allocations, $\left\{Z^i, L^i, \theta^i\right\}_{i\in\mathcal{F}}$, is an **equilibrium technology**.

- Let us also define the net production function :

$$F(Z^i, L^i, \theta^i) \equiv G(Z^i, L^i, \theta^i) - C\left(\theta^i\right).$$

Economy D (continued)

Assumption 1 Either $F(Z^i, L^i, \theta^i)$ is jointly strictly concave in (Z^i, L^i, θ^i) and increasing in (Z^i, L^i), and $\mathcal{Z}$, $\mathcal{L}$ and Θ are convex; or $F(Z^i, L^i, \theta^i)$ is increasing in (Z^i, L^i) and exhibits constant returns to scale in (Z^i, L^i, θ^i), and we have $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$.

- Main problem with Economy D: Assumption 1 overly restrictive.
- It requires concavity (strict concavity or constant returns to scale) jointly in the factors of production and technology.

Economy D (continued)

- Equilibrium characterization and welfare theorems:

Proposition 1 Suppose Assumption 1 holds. Then any equilibrium technology θ in Economy D is a solution to

$$\max_{\theta' \in \Theta} F(\bar{Z}, \bar{L}, \theta'), \tag{1}$$

and any solution to this problem is an equilibrium technology.

- Equilibrium factor prices given by the marginal products of G or F.

$$w_Z = \partial G(\bar{Z}, \bar{L}, \theta)/\partial Z = \partial F(\bar{Z}, \bar{L}, \theta)/\partial Z$$

and

$$w_{Lj} = \partial G(\bar{Z}, \bar{L}, \theta)/\partial L_j = \partial F(\bar{Z}, \bar{L}, \theta)/\partial L_j$$

for $j = 1, ..., N$

Economy C

- Now assume that firms maximize profits, but technologies chosen by a "welfare-maximizing" centralized research firm.
- Useful as an introduction to the more realistic models with monopoly and oligopoly technology suppliers.
- The research firm chooses a single technology θ and makes it available to all firms (single technology for simplicity).
- Notice that this will typically not give the social (Pareto) optimum, since employment decisions controlled by different agents.

Economy C (continued)

- The maximization problem of each final good producer is

$$\max_{Z^i \in \mathcal{Z}, L^i \in \mathcal{L}} \pi(Z^i, L^i, \theta) = G(Z^i, L^i, \theta) - w_Z Z^i - \sum_{j=1}^{N} w_{Lj} L_j^i.$$

- Notice: in contrast to Economy D, final good producers are only maximizing with respect to $\left(Z^i, L^i\right)$, not with respect to θ^i.

- The objective of the research firm is to maximize total net output:

$$\max_{\theta \in \Theta} \Pi(\theta) = \int_0^1 G(Z^i, L^i, \theta) di - C(\theta).$$

Economy C (continued)

Definition 2 An equilibrium in Economy C is a set of firm decisions $\{Z^i, L^i\}_{i\in\mathcal{F}}$, technology choice θ and factor prices (w_Z, w_L) such that $\{Z^i, L^i\}_{i\in\mathcal{F}}$ maximize profits given (w_Z, w_L) and θ, market clearing conditions hold, and the technology choice for the research firm, θ, maximizes its objective function.

- Major difference: we only need a weaker version of Assumption 1
- Concavity only in (Z, L):

  Assumption 2 Either $G(Z^i, L^i, \theta^i)$ is jointly strictly concave and increasing in (Z^i, L^i) and $\mathcal{Z}$ and $\mathcal{L}$ are convex; or $G(Z^i, L^i, \theta^i)$ is increasing and exhibits constant returns to scale in (Z^i, L^i), and we have $(\bar{Z},\bar{L}) \in \mathcal{Z} \times \mathcal{L}$.

Economy C (continued)

Proposition 2 Suppose Assumption 2 holds. Then any equilibrium technology θ in Economy C is a solution to

$$\max_{\theta' \in \Theta} F(\bar{Z}, \bar{L}, \theta') \equiv G(\bar{Z}, \bar{L}, \theta') - C(\theta')$$

and any solution to this problem is an equilibrium technology.

- Most important novel feature: while in Economy D the function $F(\bar{Z}, \bar{L}, \theta)$ is jointly concave in (Z, θ), the same is **not** true in Economy C.

- As in Economy D, equilibrium factor prices are given by

  $$w_Z = \partial G(\bar{Z}, \bar{L}, \theta) / \partial Z = \partial F(\bar{Z}, \bar{L}, \theta) / \partial Z$$

  and

  $$w_{Lj} = \partial G(\bar{Z}, \bar{L}, \theta) / \partial L_j = \partial F(\bar{Z}, \bar{L}, \theta) / \partial L_j$$

  for $j = 1, ..., N$.

Economy M

- Now a profit-maximizing monopolist sells technologies to final good producers.
- To facilitate analysis, assume that

$$Y^i = \alpha^{-\alpha} (1-\alpha)^{-1} \left[G(Z^i, L^i, \theta^i)\right]^{\alpha} q\left(\theta^i\right)^{1-\alpha}.$$

- Here $G(Z^i, L^i, \theta^i)$ is a subcomponent of the production function.
- Productivity depends on the technology used, θ^i.
- The subcomponent G needs to be combined with an intermediate good embodying technology θ^i, denoted by $q\left(\theta^i\right)$.
- This intermediate good will be sold by the monopolist.
- The term $\alpha^{-\alpha} (1-\alpha)^{-1}$ is a convenient normalization.

Economy M (continued)

- The monopolist can create technology θ at cost $C(\theta)$ from the technology menu.
- Once θ is created, the technology monopolist can produce the intermediate good embodying technology θ at constant per unit cost normalized to $1-\alpha$ unit of the final good.
- It can then set a (linear) price per unit of the intermediate good of type θ, denoted by χ.
- All factor markets are again competitive, and each firm takes the available technology, θ, and the price of the intermediate good embodying this technology, χ, as given.

Economy M (continued)

- Final good producers' maximization problem:

$$\max_{\substack{Z^i \in \mathcal{Z}, L^i \in \mathcal{L}, \\ q(\theta) \geq 0}} \alpha^{-\alpha} (1-\alpha)^{-1} \left[G(Z^i, L^i, \theta) \right]^{\alpha} q(\theta)^{1-\alpha} - w_Z Z^i - \sum_{j=1}^{N} w_{Lj} L_j^i - \chi q(\theta),$$

- Inverse demand for intermediates of type θ as a function of its price, χ:

$$q^i \left(\theta, \chi, Z^i, L^i \right) = \alpha^{-1} G(Z^i, L^i, \theta) \chi^{-1/\alpha}.$$

- Isoelastic inverse demand.

Economy M (continued)

- The monopolist's maximization problem:

$$\max_{\theta,\chi,[q^i(\theta,\chi,Z^i,L^i)]_{i\in\mathcal{F}}} \Pi = (\chi - (1-\alpha)) \int_{i\in\mathcal{F}} q^i\left(\theta,\chi,Z^i,L^i\right) di - C(\theta)$$

  subject to the inverse demand curve.

  Definition 3 An equilibrium in Economy M is a set of firm decisions $\left\{Z^i, L^i, q^i\left(\theta,\chi,Z^i,L^i\right)\right\}_{i\in\mathcal{F}}$, technology choice θ, and factor prices (w_Z, w_L) such that $\left\{Z^i, L^i, q^i\left(\theta,\chi,Z^i,L^i\right)\right\}_{i\in\mathcal{F}}$ maximizes profits given (w_Z, w_L) and technology θ, market clearing conditions hold, and the technology choice and pricing decision of the monopolist, (θ,χ), maximize monopoly profits subject to the inverse demand curve.

- As in Economy C, factor demands and technology are decided by **different agents**; the former by the final good producers, the latter by the technology monopolist.

Economy M (continued)

- Note that the inverse demand function has constant elasticity.
- Profit-maximizing price will be a constant markup over marginal cost

$$\chi = 1$$

- Consequently, $q^i(\theta) = q^i\left(\theta, \chi = 1, \bar{Z}, \bar{L}\right) = \alpha^{-1} G(\bar{Z}, \bar{L}, \theta)$ for all $i \in \mathcal{F}$.
- Therefore, the monopolist's problem becomes

$$\max_{\theta \in \Theta} \Pi(\theta) = G(\bar{Z}, \bar{L}, \theta) - C(\theta).$$

Proposition 3 Suppose Assumption 2 holds. Then any equilibrium technology θ in Economy M is a solution to

$$\max_{\theta' \in \Theta} F(\bar{Z}, \bar{L}, \theta') \equiv G(\bar{Z}, \bar{L}, \theta') - C(\theta')$$

and any solution to this problem is an equilibrium technology.

- Relative to Economies D and C, the presence of the monopoly markup implies greater distortions in this economy.

Economy M (continued)

- However, qualitatively equilibrium similar to that in Economy C.
- It is given by the maximization of

$$F(\bar{Z},\bar{L},\theta) \equiv G(\bar{Z},\bar{L},\theta) - C(\theta)$$

- Most important: as in Economy C, $F(\bar{Z},\bar{L},\theta)$ need **not be concave** in (Z,θ), even in the neighborhood of the equilibrium.
- Factor prices again given by:

$$w_Z = \partial G(\bar{Z},\bar{L},\theta)/\partial Z = \partial F(\bar{Z},\bar{L},\theta)/\partial Z$$

and

$$w_{Lj} = \partial G(\bar{Z},\bar{L},\theta)/\partial L_j = \partial F(\bar{Z},\bar{L},\theta)/\partial L_j$$

for $j = 1, ..., N$.

Economy O

- Same as Economy M, except that multiple technologies supplied by competing oligopolists.
- Let θ^i be the vector $\theta^i \equiv \left(\theta^i_1, ..., \theta^i_S\right)$.
- Suppose that output is now given by

$$Y^i = \alpha^{-\alpha} (1-\alpha)^{-1} \left[G(Z^i, L^i, \theta^i)\right]^{\alpha} \sum_{s=1}^{S} q_s \left(\theta^i_s\right)^{1-\alpha},$$

- $\theta^i_s \in \Theta_s \subset \mathbb{R}^{K_s}$: technology supplied by technology producer $s = 1, ..., S$;
- $q_s \left(\theta^i_s\right)$: intermediate good produced and sold by technology producer s, embodying technology θ^i_s.

Economy O (continued)

- Essentially the same result as in Economy M.

Proposition 4 Suppose Assumption 2 holds. Then any equilibrium technology in Economy O is a vector $(\theta_1^*, ..., \theta_S^*)$ such that θ_s^* is solution to

$$\max_{\theta_s \in \Theta_s} G(\bar{Z}, \bar{L}, \theta_1^*, ..., \theta_s, ..., \theta_S^*) - C_s(\theta_s)$$

for each $s = 1, ..., S$, and any such vector gives an equilibrium technology.

- Main difference: equilibrium technology no longer given by maximization, but by a fixed point problem.

- Nevertheless, general insights continue to apply.

Relative Bias

- Let us first study relative bias.
- Two factors Z and L.
- Defined factor prices as:

$$w_Z\left(Z,L,\theta\right)=\frac{\partial G\left(Z,L,\theta\right)}{\partial Z} \text{ and } w_L\left(Z,L,\theta\right)=\frac{\partial G\left(Z,L,\theta\right)}{\partial L},$$

Definitions

Definition 4 An increase in technology θ_j for $j = 1, ..., K$ is **relatively biased** towards factor Z at $\left(\bar{Z}, \bar{L}, \theta\right) \in \mathcal{Z} \times \mathcal{L} \times \Theta$ if $\partial\left(w_Z/w_L\right)/\partial\theta_j \geq 0$.

Definition 5 Denote the equilibrium technology at factor supplies $\left(\bar{Z}, \bar{L}\right) \in \mathcal{Z} \times \mathcal{L}$ by $\theta^*\left(\bar{Z}, \bar{L}\right)$, and assume that $\partial\theta_j^*/\partial Z$ exists at $\left(\bar{Z}, \bar{L}\right)$ for all for all $j = 1, ..., K$. Then there is **weak relative equilibrium bias** at $\left(\bar{Z}, \bar{L}, \theta^*\left(\bar{Z}, \bar{L}\right)\right)$ if

$$\sum_{j=1}^{K} \frac{\partial\left(w_Z/w_L\right)}{\partial\theta_j}\frac{\partial\theta_j^*}{\partial Z} \geq 0.$$

Definition 6 Denote the equilibrium technology at factor supplies $\left(\bar{Z}, \bar{L}\right) \in \mathcal{Z} \times \mathcal{L}$ by $\theta^*\left(\bar{Z}, \bar{L}\right)$, and assume that $\partial\theta_j^*/\partial Z$ exists at $\left(\bar{Z}, \bar{L}\right)$ for all $j = 1, ..., K$. Then there is **strong relative equilibrium bias** at $\left(\bar{Z}, \bar{L}, \theta^*\left(\bar{Z}, \bar{L}\right)\right)$ if

$$\frac{d\left(w_Z/w_L\right)}{dZ} = \frac{\partial\left(w_Z/w_L\right)}{\partial Z} + \sum_{j=1}^{K} \frac{\partial\left(w_Z/w_L\right)}{\partial\theta_j}\frac{\partial\theta_j^*}{\partial Z} > 0.$$

Generalized Relative Bias Theorem

Theorem 1 Consider Economy C, M or O with two factors, Z, L, and two factor-augmenting technologies, A_Z, A_L. Assume that $G(A_Z Z, A_L L)$ is twice differentiable, concave and homothetic, and the cost of producing technologies $C(A_Z, A_L)$, is twice differentiable, strictly convex and homothetic. Let $\sigma = -\left.\frac{\partial \ln(Z/L)}{\partial \ln(w_Z/w_L)}\right|_{\frac{A_Z}{A_L}}$ be the (local) elasticity of substitution between Z and L, and $\delta = \frac{\partial \ln(C_Z/C_L)}{\partial \ln(A_Z/A_L)}$. Then:

$$\frac{\partial \ln (A_Z/A_L)^*}{\partial \ln (Z/L)} = \frac{\sigma - 1}{1 + \sigma\delta}, \text{ and}$$

$$\frac{\partial \ln (w_Z/w_L)}{\partial \ln (A_Z/A_L)} \frac{\partial \ln (A_Z/A_L)^*}{\partial \ln (Z/L)} \geq 0,$$

so that there is always **weak relative equilibrium bias**. Moreover,

$$\frac{d \ln (w_Z/w_L)}{d \ln (Z/L)} = \frac{\sigma - 2 - \delta}{1 + \sigma\delta},$$

so that there is **strong relative equilibrium bias** if and only if $\sigma - 2 - \delta > 0$.

Idea of the Proof

- Essentially the same as the simple example in Lecture 1.
- Locally, the economy behaves as if the elasticity of substitution is constant.
- Important that the result is for Economy, C, M or O, since the maximization problem choosing all of Z, L, A_Z and A_L is **not concave**.
- In fact, this non-concavity is essential for strong bias as we will see shortly.

Can This Result Be Generalized Further?

- None of the assumptions of Theorem 1 can be relaxed (for sufficiency).
- In particular, with non-factor augmenting technologies, increase in relative supply of Z can induced technological changes biased against Z.
- This does not mean that this "contrarian" result will apply in general.
- But it does mean that we cannot guarantee induced biased to go in the "right direction".

Counterexample 1

- Suppose

$$G\left(Z,L,\theta\right)=\left[Z^{\theta}+L^{\theta}\right]^{1/\theta}$$

  and $C\left(\theta\right)$ convex and differentiable.

- The choice of θ again maximizes $F\left(Z,L,\theta\right)\equiv G\left(Z,L,\theta\right)-C\left(\theta\right)$:

$$\partial G\left(\bar{Z},\bar{L},\theta^{*}\right)/\partial\theta-\partial C\left(\theta^{*}\right)/\partial\theta=0$$

  and

$$\partial^{2}G\left(\bar{Z},\bar{L},\theta^{*}\right)/\partial\theta^{2}-\partial^{2}C\left(\theta^{*}\right)/\partial\theta^{2}<0$$

- A counterexample would correspond to a situation where

$$\Delta\left(w_{Z}/w_{L}\right)\equiv\frac{\partial\left(w_{Z}/w_{L}\right)}{\partial\theta}\frac{\partial\theta^{*}}{\partial Z}=-\frac{\partial\left(w_{Z}/w_{L}\right)}{\partial\theta}\frac{\partial^{2}F/\partial\theta\partial Z}{\partial^{2}F/\partial\theta^{2}}<0.$$

Counterexample 1 (continued)

- Here:

$$w_Z/w_L = (Z/L)^{\theta-1}$$

increasing in θ as long as $Z > L$, so that higher θ is relatively biased towards Z.

- Now choose $C(\cdot)$ such that θ^* is sufficiently small, e.g., $\bar{L} = 1$, $\bar{Z} = 2$, and $\theta^* = 0.1$.
- In this case, it can be verified that $\partial^2 F\left(\bar{Z}, \bar{L}, \theta^*\right)/\partial\theta\partial Z < 0$.
- From the second-order conditions $\partial^2 F/\partial\theta^2 < 0$.
- Therefore $\left(\partial^2 F/\partial\theta\partial Z\right) \times \left(\partial^2 F/\partial\theta^2\right) > 0$.
- Conclusion: an increase in Z/L reduces θ^* and induces technological change technology relatively biased against Z.

Counterexample 2

- Suppose
$$G(Z, L, \theta) = Z\theta + L\theta^2,$$
and
$$C(\theta) = C_0\theta^2/2$$
for all $\theta \in \Theta = \mathbb{R}$ and $L \in \mathcal{L} \subset (0, C_0/2)$.

- The equilibrium technology θ^* is given by
$$\theta^*\left(\bar{Z}, \bar{L}\right) = \frac{\bar{Z}}{C_0 - 2\bar{L}},$$

- This is increasing in $\bar{Z}$ for any $\bar{L} \in \mathcal{L}$.

- The relative price of factor Z is decreasing in θ:
$$w_Z(\theta)/w_L(\theta) = \theta^{-1}$$

- $\bar{Z} \uparrow \Rightarrow$ technological change relatively biased against Z.

Why the Counterexamples?

- In both cases, the increase in $\bar{Z}$ increases w_Z (at given factor proportions).
- But it increases w_L even more so that w_Z/w_L declines at given factor proportions.
- Perhaps looking at **absolute bias** more natural.

Absolute Bias: Definitions

- Straightforward definitions of absolute bias (in light of the definitions for relative bias above).

Definition 7 An increase in technology θ_j for $j = 1, ..., K$ is **absolutely biased** towards factor Z at $\left(\bar{Z}, \bar{L}\right) \in \mathcal{Z} \times \mathcal{L}$ if $\partial w_Z / \partial \theta_j \geq 0$.

Definition 8 Denote the equilibrium technology at factor supplies $\left(\bar{Z}, \bar{L}\right) \in \mathcal{Z} \times \mathcal{L}$ by $\theta^* \left(\bar{Z}, \bar{L}\right)$ and assume that $\partial \theta_j^* / \partial Z$ exists at $\left(\bar{Z}, \bar{L}\right)$ for all $j = 1, ..., K$. Then there is **weak absolute equilibrium bias** at $\left(\bar{Z}, \bar{L}, \theta^* \left(\bar{Z}, \bar{L}\right)\right)$ if

$$\sum_{j=1}^{K} \frac{\partial w_Z}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial Z} \geq 0.$$

Absolute Bias: Local Theorem

Theorem 2 Consider Economy D, C or M. Suppose that Θ is a convex subset of $\mathbb{R}^K$ and $F(Z, L, \theta)$ is twice continuously differentiable in (Z, θ). Let the equilibrium technology at factor supplies $(\bar{Z}, \bar{L})$ be $\theta^*(\bar{Z}, \bar{L})$ and assume that $\theta^*(\bar{Z}, \bar{L})$ is in the interior of Θ and that $\partial\theta_j^*/\partial Z$ exists at $(\bar{Z}, \bar{L})$ for all $j = 1, ..., K$. Then, there is **weak absolute equilibrium bias** at all $(\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}$, i.e.,

$$\sum_{j=1}^{K} \frac{\partial w_Z}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial Z} \geq 0 \text{ for all } (\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L},$$

with strict inequality if $\partial\theta_j^*/\partial Z \neq 0$ for some $j = 1, ..., K$.

Sketch of the Proof

- The result follows from the Implicit Function Theorem.
- Consider the special case where $\theta \in \Theta \subset \mathbb{R}$.
- Since θ^* is in the interior of Θ, we have $\partial F / \partial \theta = 0$ and $\partial^2 F / \partial \theta^2 \leq 0$.
- The Implicit Function Theorem then implies:

$$\frac{\partial \theta^*}{\partial Z} = -\frac{\partial^2 F / \partial \theta \partial Z}{\partial^2 F / \partial \theta^2} = -\frac{\partial w_Z / \partial \theta}{\partial^2 F / \partial \theta^2}, \tag{2}$$

- Therefore:

$$\frac{\partial w_Z}{\partial \theta} \frac{\partial \theta^*}{\partial Z} = -\frac{\left(\partial w_Z / \partial \theta\right)^2}{\partial^2 F / \partial \theta^2} \geq 0, \tag{3}$$

  establishing the weak inequality.

- Moreover, if $\partial \theta^* / \partial Z \neq 0$, then $\partial w_Z / \partial \theta \neq 0$, so the strict inequality applies.
- The general result somewhat more involved, but a similar intuition.

Intuition

- Again the **market size effect**.
- Locally, an increase in Z makes technologies that the value of marginal product of Z more profitable.
- The result applies in all four economies.
- Once again, similarity to LeChatelier Principle.
- Major differences to come soon.

Local Bias Does Not Imply Global Bias

- Theorem 2 is for small changes.
- A natural question is whether it also holds for "large" (non-infinitesimal) changes.
- Interestingly, the answer is No.
- The reason is intuitive: technological change biased towards an particular factor at some factor proportion may be biased against that factor at some other (not too far) factor proportion.
- The next example illustrates this.

No Global Bias without Further Assumptions

- Suppose that $F(Z,\theta) = Z + \left(Z - Z^2/8\right)\theta - C(\theta)$ and $Z \in \mathcal{Z} = [0,6]$ and $\Theta = [0,2]$ so that F is everywhere increasing in Z.

- Suppose also that $C(\theta)$ is a strictly convex and differentiable function with $C'(0) = 0$ and $C'(2) = \infty$.

- Note that $F(Z,\theta)$ satisfies all the conditions of Theorem 2 at all points $Z \in \mathcal{Z} = [0,6]$ (since F is strictly concave in θ everywhere on $\mathcal{Z} \times \Theta = [0,6] \times [0,2]$).

No Global Bias without Further Assumptions (continued)

- Now consider $\bar{Z} = 1$ and $\bar{Z}' = 5$ as two potential supply levels of factor Z.
- It can be easily verified that $\theta^* (1)$ satisfies $C' (\theta^* (1)) = 7/8$ while $\theta^* (5)$ is given by $C' (\theta^* (5)) = 15/8$
- The strict convexity of $C (\theta)$ implies that $\theta^* (5) > \theta^* (1)$.
- Moreover, $w_Z (Z, \theta) = 1 + (1 - Z/4) \theta$, therefore $w_Z (5, \theta^* (5)) = 1 - \theta^* (5) /4 < 1 - \theta^* (1) /4 = w_Z (5, \theta^* (1))$.
- Intuition: reversal in the meaning of bias.

A Global Theorem

- For a global result, we need to rule out "reversals in the meaning of bias"
- Somewhat stronger assumptions are necessary.
- Fortunately, reasonable assumptions suffice for this purpose.
- What we need to ensure is that "complements" do not become "substitutes".
- Natural assumption: supermodularity.

Globality

Definition 9 Let θ^* be the equilibrium technology choice in an economy with factor supplies $(\bar{Z}, \bar{L})$. Then there is **global absolute equilibrium bias** if for any $\bar{Z}', \bar{Z} \in \mathcal{Z}$, $\bar{Z}' \geq \bar{Z}$ implies that

$$w_Z\left(\tilde{Z}, \bar{L}, \theta^*\left(\bar{Z}', \bar{L}\right)\right) \geq w_Z\left(\tilde{Z}, \bar{L}, \theta^*\left(\bar{Z}, \bar{L}\right)\right) \text{ for all } \tilde{Z} \in \mathcal{Z} \text{ and } \bar{L} \in \mathcal{L}.$$

- Two notions of globality.
  1. the increase from $\bar{Z}$ to $\bar{Z}'$ is not limited to small changes;
  2. the change in technology induced by this increase is required to raise the price of factor Z for all $\tilde{Z} \in \mathcal{Z}$.
- The same economic forces will take care of both types of globality.

Supermodularity and Increasing Differences

Definition 10 Let $x = (x_1, ..., x_n)$ be a vector in $X \subset \mathbb{R}^n$, and suppose that the real-valued function $f(x)$ is twice continuously differentiable in x. Then $f(x)$ is **supermodular** on X if and only if $\partial^2 f(x) / \partial x_i \partial x_{i'} \geq 0$ for all $x \in X$ and for all $i \neq i'$.

Definition 11 Let X and T be partially ordered sets. Then a function $f(x,t)$ defined on a subset S of $X \times T$ has **increasing differences (strict increasing differences)** in (x,t), if for all $t'' > t$, $f(x,t'') - f(x,t)$ is nondecreasing (increasing) in x.

Absolute Bias: The Global Theorem

Theorem 3 Suppose that Θ is a lattice, let $\bar{\mathcal{Z}}$ be the convex hull of $\mathcal{Z}$, let $\theta^*\left(\bar{Z}, \bar{L}\right)$ be the equilibrium technology at factor proportions $\left(\bar{Z}, \bar{L}\right)$, and suppose that $F\left(Z, L, \theta\right)$ is continuously differentiable in Z, supermodular in θ on Θ for all $Z \in \bar{\mathcal{Z}}$ and $L \in \mathcal{L}$, and exhibits strictly increasing differences in (Z, θ) on $\bar{\mathcal{Z}} \times \Theta$ for all $L \in \mathcal{L}$, then there is **global absolute equilibrium bias**, i.e., for any $\bar{Z}', \bar{Z} \in \mathcal{Z}$, $\bar{Z}' \geq \bar{Z}$ implies

$$\theta^*\left(\bar{Z}', \bar{L}\right) \geq \theta^*\left(\bar{Z}, \bar{L}\right) \text{ for all } \bar{L} \in \mathcal{L},$$

and

$$w_Z\left(\tilde{Z}, \bar{L}, \theta^*\left(\bar{Z}', \bar{L}\right)\right) \geq w_Z\left(\tilde{Z}, \bar{L}, \theta^*\left(\bar{Z}, \bar{L}\right)\right) \text{ for all } \tilde{Z} \in \mathcal{Z} \text{ and } \bar{L} \in \mathcal{L},$$

with strict inequality if $\theta^*\left(\bar{Z}', \bar{L}\right) \neq \theta^*\left(\bar{Z}, \bar{L}\right)$.

Proof Idea

- The proof basically follows from Topkis's Monotone Comparative Statics Theorem.
- An increase in Z is complementary to technologies that are biased towards Z.
- Therefore, the increase in Z will cause globally (weak) absolute bias.

Global Absolute Bias with Multiple Factors

- The same result generalizes to the case where the supply of a subset of complementary factors increases.
- In this case, technology becomes biased towards all of these factors.
- Let now Z denote a vector of inputs.

Theorem 4 Consider Economy D, C or M. Suppose that $\mathcal{Z}$ and Θ are lattices, let $\bar{\mathcal{Z}}$ be the convex hull of $\mathcal{Z}$, let $\theta\left(\bar{Z},\bar{L}\right)$ be the equilibrium technology at factor proportions $\left(\bar{Z},\bar{L}\right)$, and suppose that $F\left(Z,L,\theta\right)$ is continuously differentiable in Z, supermodular in θ on Θ for all $Z\in\bar{\mathcal{Z}}$ and $L\in\mathcal{L}$, and exhibits strictly increasing differences in (Z,θ) on $\bar{\mathcal{Z}}\times\Theta$ for all $L\in\mathcal{L}$, then there is **global absolute equilibrium bias**, i.e., for any $\bar{Z}',\bar{Z}\in\mathcal{Z}$, $\bar{Z}'\geq\bar{Z}$ implies

$$\theta\left(\bar{Z}',\bar{L}\right)\geq\theta\left(\bar{Z},\bar{L}\right)\text{ for all }\bar{L}\in\mathcal{L}$$

and

$$w_{Zj}\left(\tilde{Z},\bar{L},\theta\left(\bar{Z}',\bar{L}\right)\right)\geq w_{Zj}\left(\tilde{Z},\bar{L},\theta\left(\bar{Z},\bar{L}\right)\right)\text{ for all }(\tilde{Z},\bar{L})\in\mathcal{Z}\times\mathcal{L}\text{ and for all }j.$$

Strong Bias

- Much more interesting and surprising are the results on **strong bias**.
- The main result will show that strong bias is quite ubiquitous.

Definition of Strong Bias

Definition 12 Denote the equilibrium technology at factor supplies $\left(\bar{Z}, \bar{L}\right) \in \mathcal{Z} \times \mathcal{L}$ by $\theta^* \left(\bar{Z}, \bar{L}\right)$ and suppose that $\partial\theta_j^*/\partial Z$ exists at $\left(\bar{Z}, \bar{L}\right)$ for all $j = 1, ..., K$. Then there is **strong absolute equilibrium bias** at $\left(\bar{Z}, \bar{L}\right) \in \mathcal{Z} \times \mathcal{L}$ if

$$\frac{dw_Z}{dZ} = \frac{\partial w_Z}{\partial Z} + \sum_{j=1}^{K} \frac{\partial w_Z}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial Z} > 0.$$

Main Theorem

Theorem 5 Consider Economy D, C or M. Suppose that Θ is a convex subset of $\mathbb{R}^K$, F is twice continuously differentiable in (Z, θ), let $\theta^*\left(\bar{Z}, \bar{L}\right)$ be the equilibrium technology at factor supplies $\left(\bar{Z}, \bar{L}\right)$ and assume that θ^* is in the interior of Θ and that $\partial\theta_j^*\left(\bar{Z}, \bar{L}\right)/\partial Z$ exists at $\left(\bar{Z}, \bar{L}\right)$ for all $j = 1, ..., K$. Then there is **strong absolute equilibrium bias** at $\left(\bar{Z}, \bar{L}\right)$ if and only if $F\left(Z, L, \theta\right)$'s Hessian in (Z, θ), $\nabla^2 F_{(Z,\theta)(Z,\theta)}$, **is not negative semi-definite** at $\left(\bar{Z}, \bar{L}, \theta^*\left(\bar{Z}, \bar{L}\right)\right)$.

Sketch of the Proof

- Let us again focus on the case where $\Theta \subset \mathbb{R}$.
- By hypothesis, $\partial F/\partial\theta = 0$, $\partial^2 F/\partial\theta^2 \leq 0$.
- Then the condition for strong absolute equilibrium bias can be written as:

$$\begin{aligned} \frac{dw_Z}{dZ} &= \frac{\partial w_Z}{\partial Z} + \frac{\partial w_Z}{\partial \theta}\frac{\partial \theta^*}{\partial Z}, \\ &= \frac{\partial^2 F}{\partial Z^2} - \frac{\left(\partial^2 F/\partial\theta\partial Z\right)^2}{\partial^2 F/\partial\theta^2} > 0. \end{aligned}$$

- From Assumption 1 or 2, F is concave in Z, so $\partial^2 F/\partial Z^2 \leq 0$, and from the fact that θ^* is a solution to the equilibrium maximization problem

$$\partial^2 F/\partial\theta^2 < 0.$$

Sketch of the Proof (continued)

- Then the fact that F's Hessian, $\nabla^2 F_{(Z,\theta)(Z,\theta)}$, is not negative semi-definite at $\left(\bar{Z}, \bar{L}, \theta^* \left(\bar{Z}, \bar{L}\right)\right)$ implies that

$$\frac{\partial^2 F}{\partial Z^2} \times \frac{\partial^2 F}{\partial \theta^2} < \left(\frac{\partial^2 F}{\partial Z \partial Z \theta}\right)^2,$$

- Since at the optimal technology choice $\partial^2 F / \partial \theta^2 < 0$, this immediately yields $dw_Z / dZ > 0$, establishing strong absolute bias at $\left(\bar{Z}, \bar{L}, \theta \left(\bar{Z}, \bar{L}\right)\right)$.
- Conversely, if $\nabla^2 F_{(Z,\theta)(Z,\theta)}$ is negative semi-definite at $\left(\bar{Z}, \bar{L}, \theta^* \left(\bar{Z}, \bar{L}\right)\right)$, then the previous relationship does not hold and this together with $\partial^2 F / \partial \theta^2 < 0$ implies that $dw_Z / dZ \leq 0$.

Intuition

- When $F(Z, L,\theta)$ is not jointly concave in Z and θ, the equilibrium corresponds to a saddle point of F in the Z, θ space.
- This implies that there exists direction in which output and hence monopoly profits for technology suppliers can be increased.
- Nevertheless, the saddle point is an equilibrium, since Z and θ are chosen by different agents.
- When Z changes by a small amount, then θ can be changed in the direction of ascent.
- This not only increases output but also the marginal product of factor Z that has become more abundant.
- The result is an **upward-sloping** demand curve for Z.

Simple Example

- Let us suppose $\Theta = \mathbb{R}$ and $F(Z, L, \theta) = 4Z^{1/2} + Z\theta - \theta^2/2 + B(L)$ with the cost of creating new technologies incorporated into this function.

- Clearly F is not jointly concave in Z and θ (for $Z > 1$) but is strictly concave in Z and θ individually.

- Consider a change from $\bar{Z} = 1$ to $\bar{Z} = 4$.

- The first-order necessary and sufficient condition for technology choice gives $\theta\left(\bar{Z}, \bar{L}\right) = \theta\left(\bar{Z}\right) = \bar{Z}$.

- Therefore, $\theta\left(\bar{Z} = 1\right) = 1$ while $\theta\left(\bar{Z} = 4\right) = 4$.

- Moreover, for any $\bar{L} \in \mathcal{L}$, $w_Z\left(\bar{Z}, \bar{L}, \theta\right) = 2Z^{-1/2} + \theta$

- Therefore, $w_Z\left(\bar{Z} = 1, \bar{L}, \theta(1)\right) = 3 < w_Z\left(\bar{Z} = 4, \bar{L}, \theta(4)\right) = 5$, establishing strong (absolute) equilibrium bias between $\bar{Z} = 1$ to $\bar{Z} = 4$.

How Likely Is This?

- The key requirement is that technologies and factor demands are not decided by the same agent.
- Once we are in such an equilibrium situation, there is no guarantee that the equilibrium point corresponds to a global maximum.
- Thus the requirements are not very restrictive.
- However, naturally, F cannot be globally concave in all of its arguments.
- Thus some degree of increasing returns is necessary.

How Likely Is This? (continued)

- Therefore an immediate corollary:

Corollary 1 Suppose that Θ is a convex subset of $\mathbb{R}^K$, F is twice continuously differentiable in (Z, θ), let the equilibrium technology at factor supplies $(\bar{Z}, \bar{L})$ be $\theta^* (\bar{Z}, \bar{L})$, and assume that $\partial \theta_j^* / \partial Z$ exists at $(\bar{Z}, \bar{L})$ for all $j = 1, ..., K$. Then there cannot be strong absolute equilibrium bias in Economy D.

- Intuitively, in Economy D, F must be negative semi-definite in Z and θ, since the same firms choose both Z and θ.

- However, interestingly, one can construct examples where there is strong bias in Economy D if Θ is a finite set.

How Likely Is This? (continued)

- However, outside of Economy D, strong equilibrium bias easily possible.
- Let $\mathcal{C}^2[B]$ denote the set of twice continuously differentiable functions over B.
- Let $\mathcal{C}^2_+[B] \subset \mathcal{C}^2[B]$ be the set of such functions that are strictly convex.
- Let $\mathcal{C}^2_-[B] \subset \mathcal{C}^2[B]$ be the set of such functions that are strictly concave in each of their arguments (though not necessarily jointly so).

Theorem 6 Suppose that $\Theta \subset \mathbb{R}$ and $\mathcal{Z} \subset \mathbb{R}_+$ are compact, and denote the equilibrium technology by θ^*, and for fixed $\bar{L} \in \mathcal{L}$, let $G\left(\bar{Z}, \bar{L}, \theta\right) \in \mathcal{C}^2_-[\mathcal{Z}\times\Theta]$. For each $C(\cdot) \in \mathcal{C}^2_+[\Theta]$, let $\mathcal{D}_C \subset \mathcal{C}^2_-[\Theta]$ be such that for all $G\left(\bar{Z}, \bar{L}, \theta\right) \in \mathcal{D}_C$ there is strong absolute equilibrium bias. Then we have:

1. For each $C(\cdot) \in \mathcal{C}^2_+[\Theta]$, $\mathcal{D}_C$ is a nonempty open subset of $\mathcal{C}^2_-[\Theta]$.
2. Suppose that θ^* is an equilibrium technology for both $C_1(\cdot), C_2(\cdot) \in \mathcal{C}^2_+[\Theta]$ and that $\partial^2 C_1(\theta^*)/\partial\theta^2 < \partial^2 C_2(\theta^*)/\partial\theta^2$, then $\mathcal{D}_{C_2} \subset \mathcal{D}_{C_1}$ (and $\mathcal{D}_{C_2} \neq \mathcal{D}_{C_1}$).

Global Strong Bias

- In contrast to the weak bias absolute theorem, not much more is necessary for a global version of the strong absolute bias theorem.
- Technical intuition: Fundamental Theorem of Calculus.

Global Strong Bias Theorem

Theorem 7 Suppose that Θ is a convex subset of $\mathbb{R}^K$ and that F is twice continuously differentiable in (Z, θ). Let $\bar{Z}, \bar{Z}' \in \mathcal{Z}$, with $\bar{Z}' > \bar{Z}$, $\bar{L} \in \mathcal{L}$, and let $\theta^* \left(\tilde{Z}, \bar{L}\right)$ be the equilibrium technology at factor supplies $\left(\tilde{Z}, \bar{L}\right)$ and assume that $\theta^* \left(\tilde{Z}, \bar{L}\right)$ is in the interior of Θ and that $\partial\theta_j^* / \partial Z$ exists at $\left(\tilde{Z}, \bar{L}\right)$ for all $j = 1, ..., K$ and all $\tilde{Z} \in \left[\bar{Z}, \bar{Z}'\right]$. Then there is **strong absolute equilibrium bias** at $\left(\left\{\bar{Z}, \bar{Z}'\right\}, \bar{L}\right)$ if $F(Z, L, \theta)$'s Hessian, $\nabla^2 F_{(Z,\theta)(Z,\theta)}$, fails to be negative semi-definite at $\left(\tilde{Z}, \bar{L}, \theta^* \left(\tilde{Z}, \bar{L}\right)\right)$ for all $\tilde{Z} \in \left[\bar{Z}, \bar{Z}'\right]$.

Conclusions

- Study of direction and bias of technology important both for practical and theoretical reasons.
- Surprisingly tractable framework and many strong results are possible.
- Most interestingly:
  1. In contrast to previous non-micro-founded models, a strong force towards induced bias in favor of factors becoming more abundant (weak bias theorems).
  2. Under fairly reasonable conditions, demand curves can slope upward (strong bias theorems).

Conclusions (continued)

- Many applications of endogenous bias:
  1. Endogenous skill bias (both recently and industry).
  2. Why is long-run technological change labor augmenting?
  3. Technological sources of unemployment persistence in Europe.
  4. Demographics and evolution on innovations in the pharmaceutical industry.
  5. A theory of cross-country income differences.
  6. Possible perspectives on "lost decades".
  7. The effect of international trade on the nature of innovation and on cross-country income differences.