Advanced Economic Growth: Lectures 15 and 17, Trade, Growth and Innovation

Daron Acemoglu

MIT

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Introduction

- Models of world equilibria with international trade in commodities (or intermediate goods) and economic growth.
- Interactions between trade and growth depend on nature of trade:
  - Heckscher-Ohlin type: trade from differences in factor abundance.
  - Ricardian type: trade driven by technological comparative advantage.
  - Main difference: whether prices of goods a country supplies are affected by its own production and accumulation.
- Also study whether international trade encourages growth
  - Depends on how trade is modeled and source of growth (learning by doing versus innovation).

Growth and Financial Capital Flows

- KEY: In globalized economy, if rates of return to capital differ across countries, capital should flow where return is higher.
- Implies very different pattern of growth in financially integrated world; changes transitional dynamics in neoclassical growth model.
- Raises puzzles:
    - Lucas (1990): "Why Does Capital Not Flow from Rich to Poor Countries?".
    - Free flows of capital lead to a pattern of growth that appears counterfactual.
    - There is much less *net* flows of capital from countries with high saving rates towards those with lower saving rates than a theory of frictionless international capital markets would suggest (Feldstein and Horioka, 1980)

A World Equilibrium with Free Financial Flows I I

- $j = 1, ..., J$ countries, with production function for unique final good:

$$Y_j(t) = F(K_j(t), A_j(t) L_j(t)),$$

- Each country is "small" and ignores its effects on world aggregates.
- Common constant rate of technological change, but there may be level differences:

$$A_j(t) = A_j \exp(gt),$$

- Each country's representative household:

$$U_j = \int_0^\infty \exp(-(\rho - n)t) \left[\frac{\tilde{c}_j(t)^{1-\theta} - 1}{1-\theta} \right] dt, \qquad (1)$$

A World Equilibrium with Free Financial Flows II

- Note all countries have same time discount and population growth rate.
- Moreover, assume $L_j(0) = 1$ for all $j = 1, ..., J$:

$$L_j(t) = L(t) = \exp(nt),$$

- Assume $\rho - n > (1 - \theta) g$.

International Borrowing and Lending

- Key feature:
  - Allows smoother consumption profile for households in each country.
  - Influences dynamics of capital accumulation and growth, since consumption smoothing was one reason why capital stock did not adjust immediately to steady-state.
- $B_j(t) \in \mathbb{R}$: net borrowing of country j from the world at time t.
- Free capital flows:
  - $r(t)$,world interest rate, independent of which country is borrowing and whether it is borrowing or lending.
- Small country assumption:
  - countries are price takers and can borrow or lend as much as they like at $r(t)$.

Resource and Flow International Budget Constraints

- Resource constraint:

$$\dot{k}_j(t) = f(k_j(t)) - c_j(t) + b_j(t) - (n + g + \delta) k_j(t), \qquad (2)$$

where:

- $k_j(t) \equiv K_j(t) / A_j(t) L_j(t)$, $c_j(t) \equiv C_j(t) / A_j(t) L_j(t)$,
  $b_j(t) \equiv \frac{B_j(t)}{A_j(t) L_j(t)}$
- $y_j(t) \equiv \frac{Y_j(t)}{L_j(t)} \equiv A_j(t) f(k_j(t))$

- Consumption and investment need not be equal to domestic production: transfers $B_j(t)$.
- Flow international budget constraint for country j at time t :

$$\dot{\mathcal{A}}_j(t) = r(t) \mathcal{A}_j(t) - B_j(t), \qquad (3)$$

where $\mathcal{A}_j(t)$: international asset position of country j at time t (positive if the country is a net lender).

A World Equilibrium with Free Financial Flows I

- No-Ponzi game condition applies to international asset position of a country:

$$\lim_{t\to\infty} \mathcal{A}_j(t) \exp\left(-\int_0^t r(s)\,ds\right) = 0.$$

- Define $\mathrm{a}_j(t) \equiv \frac{\mathcal{A}_j(t)}{A_j(t)L_j(t)}$,then (3) can be rewritten as

$$\dot{\mathrm{a}}_j(t) = (r(t) - g - n)\,\mathrm{a}_j(t) - b_j(t) \tag{4}$$

and no-Ponzi game condition becomes

$$\lim_{t\to\infty} \mathrm{a}_j(t) \exp\left(-\int_0^t (r(s) - g - n_j)\,ds\right) = 0. \tag{5}$$

A World Equilibrium with Free Financial Flows II

- World capital market clearing condition:

$$\sum_{j=1}^{J} B_j(t) = 0$$

- Or, dividing and multiplying by $A_j(t) L_j(t)$, and recalling $A_j(t) = A_j \exp(gt)$ and $L_j(t) = L(t)$ for all j:

$$\sum_{j=1}^{J} A_j b_j(t) = 0 \tag{6}$$

for all $t \geq 0$.

Equilibrium

- Problem of representative household in each country is maximizing (1) subject to (2), (4) and (5).
- World equilibrium: sequence $\left\{ \left[k_j(t), c_j(t), a_j(t) \right]_{t \geq 0} \right\}_{j=1}^{J}$ and time path $\left[r(t) \right]_{t \geq 0}$, such that each country's allocation maximizes the utility of the representative household in each country, and the world financial market clears, that is, (6) is satisfied.
- Steady-state (balanced growth path) world equilibrium: world equilibrium in which $k_j(t)$ and $c_j(t)$ are constant and output in each country grows at a constant rate.

Some Results I

Proposition In the world equilibrium of the economy with free flows of capital, we have that

$$k_j(t) = k(t) = f'^{-1}(r(t) + \delta) \text{ for all } j = 1, ..., J,$$

where $f'^{-1}(\cdot)$ is the inverse function of $f'(\cdot)$ and $r(t)$ is the world interest rate.

- Intuition:
  - Each firm stops renting capital when its marginal product is equal to the opportunity cost, the world rental rate $r(t) + \delta$.
  - Hence effective capital-labor ratios are equalized across countries.
- Does *not* imply equalization of capital-labor ratios.
  - If $A_{j'}(t) \neq A_j(t)$, the capital-labor ratios of j and j' are not, and *should not*, be equalized.

Some Results II

Proposition Suppose that $\rho > n$. In the world economy with free flows of capital, there exists a unique steady-state world equilibrium in which output, capital and consumption per capita in all countries grow at the rate g and effective capital-labor ratios are:

$$k_j^* = k^* = f'^{-1}(\rho + \delta + \theta g) \text{ for all } j = 1, ..., J.$$

Moreover, in the steady-state equilibrium:

$$\lim_{t \to \infty} \dot{\mathcal{A}}_j(t) = 0 \text{ for all } j = 1, ..., J.$$

- i.e., with free capital flows, *integrated* world economy with unique steady-state equilibrium similar to standard neoclassical growth model.
- Intuitive, but proof requires ensuring that no country runs a Ponzi scheme and that this implies the *normalized* asset position of each country (and each household), i.e., $a_j(t)$ for each j, must asymptote to a constant.

Transitional Dynamics I

Proposition In the world equilibrium of the economy with free flows of capital, there exists a unique equilibrium path $\left\{ \left[k_j(t), c_j(t), \mathrm{a}_j(t) \right]_{t \geq 0} \right\}_{j=1}^{J}$ that converges to the steady-state world equilibrium. Along this equilibrium path $k_j(t) / k_{j'}(t) = 1$ and $c_j(t) / c_{j'}(t) = \text{constant}$ for any two countries j and j'.

Corollary Consider the world economy with free flows of capital. Suppose that at time t, a fraction λ of the capital stock of country j is destroyed. Then capital flows immediately to this country (i.e., $\dot{\mathrm{a}}_j(t) \to -\infty$) to ensure that $k_j(t') / k_{j'}(t') = 1$ for all $t' \geq t$ and for all $j' \neq j$.

Transitional Dynamics II

- Corollary follows from propositions above: there exists a unique globally stable equilibrium, and at any point in it $k_j(t)/k_{j'}(t) = 1$. Only possible by an immediate inflow of capital into country j.
- There are only transitional dynamics for the aggregate world economy, but not separately for each country ($k_j(t)/k_{j'}(t) = 1$ for all t ,j and j').

Transitional Dynamics III

- International capital flows ensure each country has the same effective capital-labor ratio:
  - No dynamics resulting from slow capital accumulation.
  - Any theory emphasizing transitional dynamics must limit extent or speed of international capital flows.
- Mixed evidence:
  - Large amount of gross capital flows but Feldstein-Horioka puzzle remains—countries that save more tend to invest more rather than lending internationally.
  - One reason might be the potential risk of sovereign default by countries that borrow significant amounts.
- Implications for cross-regional convergence:
  - Cannot be related to slow capital accumulation as in baseline neoclassical growth model.

Why Doesn't Capital Flow from Rich to Poor Countries?

- In basic Solow and neoclassical growth models, *only* reason why one country would be richer than another is differences in capital-labor ratios.
- But if two countries with same production possibilities set differ in capital-labor ratios, rate of return to capital will be lower in rich and incentives for capital to flow to poor.
- Discuss reasons why capital may not flow from societies with higher capital-labor ratios to those with greater capital scarcity.

Capital Flows under Perfect International Capital Markets I

- One potential answer: with perfect capital markets, capital flows will equalize effective capital-labor ratios but not capital-labor ratios.

Proposition Consider a world economy with identical neoclassical preferences across countries and free flows of capital. Suppose that countries differ according to their productivities, the A_j's. Then there exists a unique steady state equilibrium in which capital-labor ratios differ across countries (in particular, effective capital-labor ratios, the k_j's, are equalized), and there are no capital flows across countries.

- i.e., capital need not flow from rich to poor countries, because rich countries are more "productive".

Capital Flows under Perfect International Capital Markets II

- Similar to explanation in Lucas (1990), but he linked differences in A_j's to differences in human capital and human capital externalities.
- Only a "proximate" answer: takes productivity differences across countries as given..
- But suggests range of explanations that do not depend on details of financial system, but on productivity differences.

Capital Flows under Imperfect International Financial Markets I

- Other reason: rate of return to capital is higher in poor countries, but financial market frictions or sovereign risk prevent flows.
- Evidence is mixed. Three types of evidence:
- 1. Differences in the return to capital across countries are limited (Trefler, 1993; Caselli and Feyrer, 2007).
  - Directly relevant but computed under a variety of assumptions:
    - Trefler: data on factor contents of trade and assumptions on the impact of trade on factor prices;
    - Caselli and Feyrer: require comparable measures of quality-adjusted differences in capital stocks.

Capital Flows under Imperfect International Financial Markets II

- 2. Microdata (e.g., summarized in Banerjee and Duflo, 2005): rate of return for additional investment in some firms in less-developed countries could be as high as 100%.
  - But may be generated by within-country credit market imperfections:
  - Maybe return is very high for credit-rationed firms, but incentive problems make it impossible to lend to them.
  - Rate of return would given by much lower return of unconstrained firms.

Capital Flows under Imperfect International Financial Markets II

- 3. Evidence related to Feldstein-Horioka puzzle: differences in savings and investment rates across countries are highly correlated.
  - Regression of the form:

$$\Delta\left(\frac{I_j(t)}{Y_j(t)}\right)=\alpha_0+\alpha_1\Delta\left(\frac{S_j(t)}{Y_j(t)}\right),$$

  - In a world with free capital flows, if saving to GDP changes from saving shocks and other reasons, $\alpha_1 \approx 0$.
  - Feldstein and Horioka estimated α_1 close to 1 (around 0.9) for OECD economies.
  - Similar results for other samples, though Taylor (1994) argues including additional controls removes the puzzle.
  - Econometric issues: e.g. correlation between investment and savings can arise without imperfections in international financial markets, when major difference across countries is in investment opportunities.

Economic Growth in a Heckscher-Ohlin World I

- Heckscher-Ohlin benchmark:
  - Countries have same (or similar) technologies, and main source of trade is differences in factor proportions.
- Assume each country has access to aggregate production function:

$$Y_j(t) = F\left(X_j^K(t), X_j^L(t)\right), \tag{7}$$

- F: constant returns to scale, with usual characteristics, in particular Inada conditions.
- $X_j^L(t)$ and $X_j^K(t)$: Intermediate inputs, respectively labor and capital intensive.
- X: amounts *used in production* rather than *produced* in country j (denoted by Y).
- Production of final good is competitive.

Economic Growth in a Heckscher-Ohlin World II

- Assume the two intermediate inputs are each produced by one factor:

$$Y_j^L(t) = A_j L_j(t) \tag{8}$$

and

$$Y_j^K(t) = K_j(t), \tag{9}$$

- $L_j(t)$ is total labor input in country j at time t, supplied inelastically, and $K_j(t)$ is the total capital stock of the country.
- Productivity differences only in the production of the labor-intensive good.
  - Leads to well-behaved world equilibrium,in the spirit of only labor-augmenting technological progress.
  - May think of differences in A_j's as reflecting differences in the human capital embodied in labor.
- No technological progress to simplify the exposition.

Economic Growth in a Heckscher-Ohlin World III

- Free international trade in commodities—in intermediate goods.
  - Extreme assumption but main insights for growth do not depend on trading costs.
  - Prices of traded commodities, here intermediate goods, are the same in all countries, "world prices".
- World supply and demand determine prices, $p^L(t)$ and $p^K(t)$.
- Final good in the world market as numeraire.
- Given the production technologies in (8) and (9):

$$\begin{aligned} w_j(t) &= A_j p^L(t) \\ R_j(t) &= p^K(t). \end{aligned}$$

Economic Growth in a Heckscher-Ohlin World IV

- Important insights:
  - Factor prices shape incentives to accumulate capital and are typically determined by the capital-labor ratio.
  - Here factor prices are determined by world prices.
  - In particular, since capital is used only in capital-intensive intermediate and there is free trade, $R_j(t) = p^K(t)$.
  - Similar for wage rate, but effective wage rates, $w_j(t) / A_j$'s, are equalized (*Conditional factor price equalization,* Trefler, 1993).
  - Key implication:
    - ★ Factor prices in each country *entirely independent* of its capital stock and labor (provided country is "small").
    - ★ Independence of factor prices from accumulation decisions.

Economic Growth in a Heckscher-Ohlin World V

- Here equalization is immediate since each factor is only used in a single traded intermediate, but results apply in general:
  - Trading commodities is trading factors: with sufficient trade in commodities with different factor intensities, countries more abundant in one factor will sell enough of goods embedding that factor to equalize factor prices.
  - There will exist *a cone of diversification*: when factor proportions are within this cone there will be (conditional) factor price equalization.

Economic Growth in a Heckscher-Ohlin World VI

- Capital depreciates at an exponential rate δ in each country, so that:

$$\begin{aligned} r_j(t) &= R_j(t) - \delta \\ &= p^K(t) - \delta. \end{aligned} \tag{10}$$

- No international trade in assets, to isolate the effects of international trade.
- *Balanced international trade*: the value country's net sales of the capital-intensive good should be made up by purchases of labor-intensive good:

$$p^K(t)\left[X_j^K(t) - Y_j^K(t)\right] + p^L(t)\left[X_j^L(t) - Y_j^L(t)\right] = 0, \tag{11}$$

- Resource constraint in each country:

$$\dot{K}_j(t) = F\left(X_j^K(t), X_j^L(t)\right) - C_j(t) - \delta K_j(t), \tag{12}$$

Economic Growth in a Heckscher-Ohlin World VII

- World market clearing:

$$\sum_{j=1}^{J} X_j^L(t) = \sum_{j=1}^{J} Y_j^L(t) \text{ and } \sum_{j=1}^{J} X_j^K(t) = \sum_{j=1}^{J} Y_j^K(t) \text{ for all } t. \quad (13)$$

- Consumption and capital goods technology: one unit of the final good can be transformed into one unit of consumption good or of capital good.
- Representative household:

$$U_j = \int_0^\infty \exp\left(-(\rho - n)t\right)\left[\frac{c_j(t)^{1-\theta} - 1}{1-\theta}\right] dt, \quad (14)$$

where $c_j(t) \equiv C_j(t) / L_j(t)$.
- Assume that $\rho > n$ to ensure positive discounting and finite lifetime utilities.

Economic Growth in a Heckscher-Ohlin World VIII

- Define

$$x_j(t) \equiv \frac{X_j^K(t)}{X_j^L(t)},$$

so that

$$\begin{aligned} Y_j(t) &= F\left(X_j^K(t), X_j^L(t)\right) \\ &= X_j^L(t) F\left(\frac{X_j^K(t)}{X_j^L(t)}, 1\right) \\ &\equiv X_j^L(t) f(x_j(t)), \end{aligned} \tag{15}$$

- $x_j(t)$ =capital intermediate intensity of country j.
- $k_j(t) \equiv K_j(t) / L_j(t)$ as the capital labor ratio in country j at time t.

Economic Growth in a Heckscher-Ohlin World IX

- *World equilibrium*: $\left[\{c_j(t), k_j(t), x_j(t)\}_{j=1}^{J}, p^K(t), p^L(t)\right]_{t \geq 0}$ such that $[c_j(t), k_j(t), x_j(t)]_{t \geq 0}$ maximizes the utility of the representative household in country j subject to (11) and (12) given $\left[p^K(t), p^L(t)\right]_{t \geq 0}$, and world prices are such that world markets clear, i.e., the equations in (13) hold.
- *Steady-state world equilibrium*: equilibrium in which all of these quantities are constant.

Equilibrium Capital Intermediate Intensity

Proposition Consider the above-described model. In any world equilibrium we have

$$x_j(t) = x_{j'}(t) = \frac{\sum_{j=1}^{J} k_j(t)}{\sum_{j=1}^{J} A_j} \text{ for any } j \text{ and } j' \text{ and any } t.$$

- i.e., irrespective of differences in capital-labor ratios, ratio of capital-intensive to labor-intensive intermediates will be equalized.
- Enables to aggregate production and capital stocks of different countries to obtain behavior of world aggregates.
- Let $c(t)$ be average consumption per capita in the world and $k(t)$ be average capital-labor ratio in the world:

$$c(t) \equiv \frac{1}{J}\sum_{j=1}^{J} c_j(t) \text{ and } k(t) \equiv \frac{1}{J}\sum_{j=1}^{J} k_j(t).$$

Proof of Proposition: Equilibrium Capital Intermediate Intensity I

- Given world prices at time t, the representative household in each country maximizes $F\left(X_j^L(t), X_j^K(t)\right)$ subject to (11). This implies

$$\frac{F_K\left(X_j^K(t), X_j^L(t)\right)}{F_L\left(X_j^K(t), X_j^L(t)\right)} = \frac{p^K(t)}{p^L(t)} \text{ for any } j \text{ and any } t.$$

- Using the definition in (15) and the linear homogeneity of F, this can be written as

$$\frac{f'(x_j(t))}{f(x_j(t)) - x_j(t) f'(x_j(t))} = \frac{p^K(t)}{p^L(t)} \text{ for any } j \text{ and any } t,$$

- The left-hand side is strictly decreasing in $x_j(t)$, thus defines a unique $x_j(t)$ given the world price ratio.

Proof of Proposition: Equilibrium Capital Intermediate Intensity II

- Since $x_j(t)$'s are equal across countries, they must all be equal to the ratio of capital-intensive intermediates to labor-intensive intermediates in the world, i.e.,

$$x_j(t) = \frac{\sum_{j=1}^{J} K_j(t)}{\sum_{j=1}^{J} A_j L_j(t)}.$$

- Using the fact that $k_j(t) = K_j(t) / L_j(t) = K_j(t) / L(t)$ completes proof.

Law of Motion of World Aggregates

Proposition In any world equilibrium, world averages follow the laws of motion:

$$\begin{aligned}\frac{\dot{c}(t)}{c(t)} &= \frac{1}{\theta}\left(f'\left(\frac{k(t)}{A}\right)-\delta-\rho\right)\\ \dot{k}(t) &= Af\left(\frac{k(t)}{A}\right)-c(t)-(n+\delta)k(t),\end{aligned}$$

where $r(t) = p^K(t)$ is the world interest rate at time t and

$$A = \frac{1}{J}\sum_{j=1}^{J} A_j$$

is average labor productivity.

- With (conditional) factor price equalization, world behaves as an integrated closed economy, and thus obeys the two key differential equations of neoclassical model.

Proof of Proposition: Law of Motion of World Aggregates I

- Using (11), (12) and previous Proposition:

$$\dot{K}_j(t) = p^K(t) K_j(t) + p^L(t) A_j L(t) - C_j(t) - \delta K_j(t).$$

- Now define $K(t) \equiv \frac{1}{J}\sum_{j=1}^{J} K_j(t)$, sum over $j = 1, ..., J$, and use definitions of $p^K(t)$ and $p^L(t)$, the previous Proposition and the linear homogeneity of F (together with Euler Theorem) to obtain

$$\sum_{j=1}^{J} \dot{K}_j(t) = F\left(\sum_{j=1}^{J} K_j(t), \sum_{j=1}^{J} A_j L(t)\right) - \sum_{j=1}^{J} C_j(t) - \delta \sum_{j=1}^{J} K_j(t).$$

Proof of Proposition: Law of Motion of World Aggregates II

- Dividing both sides by $JL(t)$ and using the Euler Theorem once more:

$$\frac{\dot{K}(t)}{L(t)} = Af\left(\frac{K(t)}{AL(t)}\right) - c(t) - \delta\frac{K(t)}{L(t)}.$$

- Using the definition of $k(t)$ gives second differential equation.
- To obtain the differential equation for $c(t)$, aggregate the Euler equation for the representative household in each county, $\dot{c}_j(t)/c_j(t) = (r(t) - \rho)/\theta$, for each j.

Steady State Equilibrium I

Proposition Consider the above-described model. There exists a unique steady-state equilibrium whereby

$$f'\left(x_j^*\right) = f'\left(\frac{k^*}{A}\right) = \rho + \delta \text{ for all } j, \tag{16}$$

where

$$x_j^* = x^* = \frac{\sum_{j=1}^{J} K_j(t)}{L(t)\sum_{j=1}^{J} A_j} \text{ and } k^* = \frac{\sum_{j=1}^{J} K_j(t)}{JL(t)}. \tag{17}$$

Moreover

$$p^{K*} = \rho + \delta. \tag{18}$$

Steady State Equilibrium II

- Ratio of capital-intensive to labor-intensive intermediates pinned down purely by F (or its transform, f) and by the ratio of total capital to total labor in the world.
- Steady-state production structure determined by world supplies of capital and labor: with (conditional) factor price equalization, world economy is *effectively integrated*.
- Heckscher-Ohlin trade with conditional factor price equalization leads to same result as financial integration.

Stability of Steady State World Equilibrium

- Transitional dynamics for individual economies can be rich and complicated, but steady-state world equilibrium is globally stable.

Proposition Consider the above-described economy. The steady-state equilibrium characterized in the Proposition above is globally saddle-path stable.

Proof of Proposition: Stability of Steady State

- With the arguments in the proof of Proposition on the Law of Motion of World Aggregates, for any sequence of world prices $\left[p^L(t), p^K(t)\right]_{t\geq 0}$, the problem of the representative household satisfies:
$$\frac{\dot{c}_j(t)}{c_j(t)} = \frac{1}{\theta}\left(p^K(t) - \delta - \rho\right)$$
$$\dot{k}_j(t) = \left[p^K(t) - (n+\delta)\right] k_j(t) + p^L(t) A_j - c_j(t).$$
- Standard arguments imply world averages converge to the unique world steady state equilibrium and $\left[p^K(t)\right]_{t\geq 0}$ converges to $\rho + \delta$.
- This implies law of motion for the consumption and capital-labor ratio of each country also converges.
- With $p^{K*} = \rho + \delta$, the convergence is necessarily to the unique steady-state world equilibrium.

Contrast with Closed Economy Growth Models I

- Model generates a pattern of growth similar to neoclassical growth model, with each country converging to unique steady state.
- But important difference, as in model with international borrowing and lending: nature of the transitional dynamics.
  - Despite no international capital flows, the rate of return to capital is equalized.
  - Hence no transitional dynamics from a country with a higher rate of return to capital accumulating capital faster than the rest.
  - Points to pitfalls of using closed-economy growth model for analysis across countries and regions.

Contrast with Closed Economy Growth Models II

- Here each country faces an "*AK*" technology:
  - The world economy has standard neoclassical technology satisfying standard assumptions.
  - But each country can accumulate as much as it wishes without running into diminishing returns.
    - ★ For every additional unit of capital, country receives a return of $p^K(t)$, independently of its own capital stock.
- But the world does not generate endogenous growth:
  - Accumulation by all countries drives down $p^K(t)$ to a level that is consistent with steady state.
  - $p^K(t)$ will adjust to ensure the steady state equilibrium where capital, output and consumption per capita are constant.

Short-Run Dynamics I

- Very different short-run (or "medium run") dynamics possible, especially for countries with different saving rates than others.

Proposition Consider the above-described model. Suppose J is arbitrarily large and the world starts in steady state at time $t = 0$, then the discount rate of country 1 declines to $\rho' < \rho$. After this change, there exists some $T > 0$ such that for all $t \in [0, T)$, country one grows at the rate

$$g_1 = \frac{\dot{c}_1(t)}{c_1(t)} = \frac{1}{\theta}(\rho - \rho').$$

Short-Run Dynamics II

- Proof:
  - In steady state, Proposition on Stability of Steady State and equation (18) imply $p^{K*} = \rho + \delta$.
  - We are in the AK world, with country 1 facing this price as the return on capital and having lower discount factor ρ'.
- Intuition:
  - Each country faces AK technology, thus can accumulate capital and grow without running into diminishing returns.
  - p^K and thus rate of return to capital is pinned down by the discount rate of other countries in the world.
  - Country 1 with lower discount rate will save faster than the rest of the world and achieve positive growth.
- Can rationalize bouts of rapid growth ("growth miracles") by countries that change their policies or their savings rates (or discount rates).

Short-Run Dynamics III

- Ventura (1997): potential explanation for why in the 1970s East Asian tigers may have grown rapidly without running into diminishing returns.
- East Asian economies were more open and have accumulated capital rapidly (e.g., Young, 1992, 1995, Vogel, 2006).
- International trade can temporarily prevent the diminishing returns and enable sustained growth.
- But such behavior cannot go on forever: world output cannot grow in the long run.
- Growth for $t \in [0, T)$: at some point, country 1 will become so large that the rate of return on capital will fall so that accumulation by this country comes to an end.

Economic Growth in a Heckscher-Ohlin World

- Lesson: growth miracles in this model can only apply in the "medium run".
- Also model does *not* admit steady-state equilibrium when discount rates differ across countries.
- Well behaved world equilibrium relies on knife-edge case with same discount rate (and also same productivity of the capital-intensive intermediates).
- General shortcoming of Heckscher-Ohlin approach:
  - No comparative advantage from technology: each country is either small and takes prices as given, or large and influences world prices for all commodities.
  - Models with more Ricardian features richer and more tractable.

Trade, Specialization and the World Income Distribution

- Model with Ricardian features: each country will specialize in subset of available goods and affect their prices.
- Hence each country's terms of trade will be endogenous and depend on the rate at which it accumulates capital.
- Model can allow for differences in discount (and saving) rates and has richer comparative static results.
- Also now exhibit endogenous growth, determined by the investment decisions of all countries.
- International trade (without any technological spillovers) will create sufficient interactions to ensure a common long-run growth rate.

Basics I

- J of "small" countries, $j = 1, ..., J$.
- Continuum of intermediate products $\nu \in [0, N]$.
- Two final products used for consumption and investment.
- Free trade in intermediate goods and no trade in final products or assets (rule out international borrowing and lending).
- Each country has constant population normalized to 1.
- Country j will be defined by (μ_j, ρ_j, ζ_j),vary across countries but constant over time:
  - μ: indicator of how advanced the technology of the country is,
  - ρ: rate of time preference, and
  - ζ: measure the effect of policies and institutions on the incentives to invest.

Basics II

- All countries admit a representative household with utility function:

$$\int_0^{\infty} \exp\left(-\rho_j t\right) \ln C_j(t) dt \,, \tag{19}$$

- Country j starts with a capital stock of $K_j(0) > 0$ at time $t = 0$.
- Budget constraint of representative household in country j at time t:

$$\begin{aligned} p_j^I(t)\dot{K}_j(t) + p_j^C C_j(t) &= Y_j(t) \\ &= r_j(t) K_j(t) + w_j(t) \,, \end{aligned} \tag{20}$$

- Because consumption and investment goods are not traded, their prices might differ across countries.
- Notice equation (20) imposes no depreciation.
- Consumption and investment goods have different production technologies and thus their prices will differ.

Basics III

- *Armington* preferences or technology: N intermediates partitioned such that each intermediate can only be produced by one country.
- While each country is small in import markets, it will affect its own terms of trades by the amount of the goods it exports.
- Denoting the measure of goods produced by country j by μ_j:

$$\sum_{j=1}^{J} \mu_j = N. \tag{21}$$

- A higher level of μ_j implies country j has the technology to produce a larger variety of intermediates.

Basics IV

- Intermediates produced competitively.
- In each country one unit of capital produces one unit of any of the intermediates that the country is capable of producing.
- Free entry to the production of intermediates.
- Hence prices of all intermediates

$$p_j(t) = r_j(t), \tag{22}$$

The AK Model I

- Simplified version where capital is the only factor of production.
- In (20) we have $w_j(t) = 0$:

$$Y_j(t) = r_j(t) K_j(t).$$

- Consumption and investment goods produced using domestic capital and a bundle of all the intermediate goods in the world.
- Production function for consumption goods:

$$C_j(t) = \chi K_j^C(t)^{1-\tau} \left(\int_0^N x_j^C(t,\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\tau\varepsilon}{\varepsilon-1}}. \tag{23}$$

The AK Model II

- Note:
  - K_j^C= "non-traded" component; if a country has low K_j^C, relative price of capital will be high and less of it will be used
  - Term in parentheses represents bundle of intermediates purchased from the world economy.
  - Throughout assume
    $$\varepsilon > 1,$$
    which avoids the counterfactual and counterintuitive pattern of "immiserizing growth".
  - Exponent τ ensures constant returns to scale. τ is also the share of trade in GDP for all countries.
  - χ is introduced for normalization.
- Production function for investment goods:

$$I_j(t) = \zeta_j^{-1} \chi K_j^I(t)^{1-\tau} \left(\int_0^N x_j^I(t,\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\tau\varepsilon}{\varepsilon-1}}, \qquad (24)$$

The AK Model III

- Term ζ_j allows differential levels of productivity in production of investment goods across countries:
  - Consistent with results on relative prices of investment goods
  - May think of greater distortions as higher ζ_j (higher ζ_j reduce output and increase relative price of investment goods).
- Market clearing for capital:

$$K_j^C(t) + K_j^I(t) + K_j^\mu(t) \leq K_j(t), \tag{25}$$

  where $K_j^\mu(t)$ capital used in the production of intermediates and $K_j(t)$ is total capital stock of country j at time t.
- AK version: production goods uses capital and intermediates that are produced from capital. Doubling capital stock will double the output of intermediates and of consumption and investment goods.

The AK Model IV

- *Unit cost functions*: cost of producing one unit of consumption and investment goods in terms of the numeraire.
- Production functions (23) and (24) are equivalent to unit cost functions for consumption and production:

$$B_j^C\left(r_j(t), [p(t,\nu)]_{\nu\in[0,N]}\right) = r_j(t)^{1-\tau}\left[\left(\int_0^N p(t,\nu)^{1-\varepsilon}d\nu\right)^{\frac{\tau}{1-\varepsilon}}\right], \tag{26}$$

$$B_j^I\left(r_j(t), [p(t,\nu)]_{\nu\in[0,N]}\right) = \zeta_j r_j(t)^{1-\tau}\left[\left(\int_0^N p(t,\nu)^{1-\varepsilon}d\nu\right)^{\frac{\tau}{1-\varepsilon}}\right], \tag{27}$$

  where $p(t,\nu)$ is the price of the intermediate ν at time t and the constant χ in (23) and (24) is chosen appropriately.
- These prices not indexed by j, since there is free trade in intermediates.

The AK Model V

- World equilibrium: sequence of prices, capital stock levels and consumption levels for each country, such that all markets clear and the representative household in each country maximizes his utility given the price sequences,

$$\left[\left\{p_j^C(t), p_j^I(t), r_j(t), K_j(t), C_j(t)\right\}_{j=1}^J, [p(t,\nu)]_{\nu\in[0,N]}\right]_{t\geq 0}.$$

- Steady-state world equilibrium defined as usual, in particular, requiring that all prices are constant.
- Maximization of the representative household, i.e. of (19) subject to (20) for each j yields Euler equation:

$$\frac{r_j(t) + \dot{p}_j^I(t)}{p_j^I(t)} - \frac{\dot{p}_j^C(t)}{p_j^C(t)} = \rho_j + \frac{\dot{C}_j(t)}{C_j(t)} \tag{28}$$

The AK Model VI

- Euler requires (net) rate of return to capital to be equal to rate of time preference plus slope of the consumption path.
- Difference from standard Euler stems: potentially different technologies for producing consumption and investment, thus change in their relative price—term $\dot{p}_j^I(t)/p_j^I(t) - \dot{p}_j^C(t)/p_j^C(t)$.
- Transversality condition:

$$\lim_{t\to\infty} \exp\left(-\rho_j t\right) \frac{p_j^I(t) K_j(t)}{p_j^C(t) C_j(t)} = 0, \tag{29}$$

for each j.

- Integrating budget constraint and using the Euler and transversality conditions, consumption function:

$$p_j^C(t) C_j(t) = \rho_j p_j^I(t) K_j(t), \tag{30}$$

The AK Model VII

- Individuals spend a fraction ρ_j of their wealth on consumption at every instant.
- Define the numeraire for this world economy as the ideal price index for the basket of all the (traded) intermediates:

$$
\begin{aligned}
1 &= \left[\int_0^N p(t,\nu)^{1-\varepsilon} d\nu \right]^{\frac{1}{1-\varepsilon}} \\
&= \sum_{j=1}^{J} \mu_j p_j \left(t\right)^{1-\varepsilon}.
\end{aligned}
\tag{31}
$$

- Since each country is small it exports practically all of its production of intermediates and imports the ideal basket of intermediates.
- Thus $p_j(t) = r_j(t)$ is not only the price of intermediates produced by j, but also its *terms of trade*.

The AK Model VIII

- Using the price normalization in (31), (26) and (27) imply:

$$p_j^C(t) = r_j(t)^{1-\tau} \text{ and } p_j^I(t) = \zeta_j r_j(t)^{1-\tau}. \quad (32)$$

- To compute the rate of return to capital, need to impose market clearing for capital in each country and have a trade balance equation for each country.
- By Walras' law enough to use the trade balance equation:

$$Y_j(t) = \mu_j r_j(t)^{1-\varepsilon} Y(t), \quad (33)$$

where $Y(t) \equiv \sum_{j=1}^{J} Y_j(t)$ is total world income at time t. Here,

  - Each country spends τ of its income on intermediates, and, since it is small, on imports.
  - The rest of the world spends a fraction $\tau\mu_j r_j(t)^{1-\varepsilon}$ of its income on intermediates produced by country j (follows from CES and that $p_j(t) = r_j(t)$ is the relative price of each country j intermediate and there are μ_j of them).

The AK Model IX

- (22), (30), (32) and (33) together with the resource constraint, (20), characterize the world equilibrium fully.
- Distribution of capital stocks across the J economies, combining (20), (30) and (32) on the one hand, and (20) and (33) on the other:

$$\frac{\dot{K}_j(t)}{K_j(t)} = \frac{r_j(t)^\tau}{\zeta_j} - \rho_j, \tag{34}$$

$$r_j(t) K_j(t) = \mu_j r_j(t)^{1-\varepsilon} \sum_{i=1}^{J} r_i(t) K_i(t). \tag{35}$$

Proposition: Steady State Equilibrium

There exists a unique steady-state world equilibrium where

$$\frac{\dot{K}_j(t)}{K_j(t)} = \frac{\dot{Y}_j(t)}{Y_j(t)} = g^* \tag{36}$$

for $j = 1, ..., J$, and the world steady-state growth rate g^* is the unique solution to

$$\sum_{j=1}^{J} \mu_j \left[\zeta_j \left(\rho_j + g^*\right)\right]^{(1-\varepsilon)/\tau} = 1. \tag{37}$$

The steady-state rental rate of capital and the terms of trade in country j are given by

$$r_j^* = p_j^* = \left[\zeta_j \left(\rho_j + g^*\right)\right]^{1/\tau}. \tag{38}$$

This unique steady-state equilibrium is globally saddle-path stable.

Proof of Proposition (Sketch): Steady State Equilibrium I

- A steady-state equilibrium must have constant prices, thus constant r_j^*.
- This implies that in any state state, for each $j = 1, ..., J$, $\dot{K}_j(t) / K_j(t)$ must grow at some constant rate g_j.
- Suppose these rates are not equal for two countries j and j'.
  - Taking the ratio of equation (35) for these two countries yields a contradiction, establishing that $\dot{K}_j(t) / K_j(t)$ is constant for all countries.
- Equation (33) then implies that all countries also grow at this common rate, say g^*. Given this common growth rate, (34) immediately implies (38). Substituting this back into (35) gives (37).

Proof of Proposition (Sketch): Steady State Equilibrium II

- Since these equations are all uniquely determined and (37) is strictly decreasing in g^*, thus has a unique solution, the steady-state world equilibrium is unique.
- To establish global stability, it suffices to note that (35) implies that $r_j(t)$ is decreasing in $K_j(t)$.
- Thus whenever a country has a high capital stock relative to the world, it has a lower rate of return on capital, which from (34) slows down the process of capital accumulation in that country.

Discussion of Proposition: Steady State Equilibrium I

1. Despite the high degree of interaction among the various economies, there exists a unique globally stable steady-state world equilibrium.
2. Equilibrium takes a relatively simple form.
3. All countries grow at the same rate g^*.
   - Surprising, since each economy has AK technology, and without any international trade, each country would grow at a different rate (e.g., those with lower ζ_j's or ρ_j's would have higher growth rates).
   - International trade keeps countries together, and leads to a stable world income distribution.

- Intuition of third result: terms of trade effects encapsulated in equation (35).

Discussion of Proposition: Steady State Equilibrium II

- Consider special case $\mu_j = \mu$ for all j and j has lower ζ_j and ρ_j than the rest of the world.
  - Then (34) implies j will tend to accumulate more capital than others.
  - But (35) implies this cannot go on forever and j, being richer than the world average, will have a lower rate of return on capital.
  - This will compensate the greater incentive to accumulate and accumulation in j converges back to the rate of the world.
- Each country is "small" relative to the world, but has market power in the goods that it supplies.
- Hence when a country accumulates faster it will face *worsening terms of trades*.
  - This will reduce the income of the country that is accumulating faster.
  - Dynamic effects: (22) shows it also experiences a decline in the rate of return the capital and in the interest rate, that slows down its rate of capital accumulation.

Discussion of Proposition: Steady State Equilibrium III

- Let $y_j^* \equiv Y_j(t) / Y(t)$ the the relative income of country j in steady state. Then equations (33) and (38) immediately imply that

$$y_j^* = \mu_j \left[\zeta_j \left(\rho_j + g^* \right) \right]^{(1-\varepsilon)/\tau}. \qquad (39)$$

- Growth at a common rate does not imply same level of income:
  - Countries with better technology (high μ_j), lower distortions (low ζ_j) and lower discount rates (low ρ_j) will be relatively richer.
  - Elasticity of income with respect to ζ_j and ρ_j depends on elasticity of substitution between the intermediates, ε, and degree of openness, τ.
  - When ε is high and τ is relatively low, small differences in ζ_j's and ρ_j's can lead to very large differences in income across countries.
- Recall that in a world with a Cobb-Douglas aggregate production function and no human capital differences, the Solow model implies:

$$y_j^* = A_j \left(\frac{s_j}{g^*} \right)^{\alpha/(1-\alpha)}, \qquad (40)$$

Discussion of Proposition: Steady State Equilibrium IV

- Equation (39) shows similar implications, except that:
  1. the role of the labor-augmenting technologies is played by the technological capabilities of the country, which determine the range of goods in which it has a comparative advantage;
  2. the role of the saving rate is played by the discount rate ρ_j and the policy parameter affecting the distortions on the production of investment goods, ζ_j;
  3. instead of the share of capital in national income, the elasticity of substitution between intermediates and the degree of trade openness affects how spread out the world income distribution is.

The General Model I

- Preferences, demographics, production functions for intermediates and production function for investment goods are the same.
- Production function for consumption goods:

$$C_j(t) = \chi K_j^C(t)^{(1-\tau)(1-\gamma)} (L_j(t))^{(1-\tau)\gamma} \left(\int_0^N x_j^C(t,\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\frac{\tau\varepsilon}{\varepsilon-1}}$$

for some $\gamma \in (0,1)$.

- $L_j(t)$ $(=1)$ is the total labor supply, supplied inelastically by the representative household in the economy
- Hence in terms of (20), $w_j(t)$ stands both for the wage rate per unit of labor and total labor income.
- Unit cost function for the consumption good is

$$B_j^C(\cdot) = w_j(t)^{(1-\tau)(1-\gamma)} r_j(t)^{(1-\tau)\gamma} \left[\left(\int_0^N p(t,\nu)^{1-\varepsilon} d\nu \right)^{\frac{\tau}{1-\varepsilon}} \right].$$

The General Model II

- Using the same price normalization, i.e., (31), we continue to have (22) for intermediate prices and

$$p_j^I(t) = \zeta_j r_j(t)^{1-\tau}$$

for the price of the investment good.

- The price of the consumption good is obtained:

$$p_j^C(t) = w_j(t)^{(1-\tau)(1-\gamma)} r_j(t)^{(1-\tau)\gamma}. \quad (41)$$

- The maximization problem of the representative agent again leads to the necessary and sufficient conditions given by (28) and (29).

The General Model III

- Combining these two equations, again obtain consumption expenditure is fraction of the lifetime wealth:

$$p_j^C(t)\,C_j(t) = \rho_j \left(\begin{array}{c} p_j^I(t)\,K_j(t) \\ +\int_t^\infty \exp\left(-\int_t^z \frac{r_j(s)+\dot{p}_j^I(s)}{p_j^I(s)}ds\right) w(z)\,dz \end{array} \right). \tag{42}$$

- (33) still gives the necessary trade balance equation for each country.
- Labor demand comes only from the consumption goods sector, and given the Cobb-Douglas assumption, this demand is $(1-\gamma)(1-\tau)$ times consumption expenditure, $p_j^C C_j$, divided by the wage rate, w_j.
- Hence market clearing condition for labor is:

$$1 = (1-\gamma)(1-\tau)\frac{p_j^C(t)\,C_j(t)}{w_j(t)}. \tag{43}$$

The General Model IV

- Finally, because (43) implies labor income, $w_j(t)$, is always proportional to consumption expenditure, the optimal consumption rule, (42), can be simplified:

$$p_j^C(t)\, C_j(t) = \frac{\rho_j}{1-(1-\gamma)(1-\tau)} p_j^I(t)\, K_j(t). \qquad (44)$$

- Households again consume a constant fraction of the value of the capital stock, but this fraction now depends not only on their discount rate, ρ_j, but also on the technology parameters, τ and γ.

The General Model V

Proposition In the general model with labor, the world equilibrium is characterized by (34) for each j and t, as well as

$$r_j(t) K_j(t) + w_j(t) = \mu_j r_j(t)^{1-\varepsilon} \sum_{i=1}^{J} [r_i(t) K_i(t) + w_i(t)], \tag{45}$$

$$\frac{w_j(t)}{r_j(t) K_j(t) + w_j(t)} = \frac{(1-\gamma)(1-\tau)\rho_j}{[\gamma + (1-\gamma)\tau]\zeta_j^{-1} r_j(t) + (1-\gamma)(1-\tau)\rho_j} \tag{46}$$

Proposition There exists a unique steady-state world equilibrium. Capital stock and output in each country grows at constant rate g^* as in (36), and g^* is the unique solution to (37). This unique steady-state equilibrium is globally stable.

The General Model VI

- Stable income distribution continues to apply.
- Relative prices of investment and consumption goods:

$$\frac{p_j^I(t)}{p_j^C(t)} = \zeta_j \left(\frac{r_j(t)}{w_j(t)} \right)^{(1-\gamma)(1-\tau)},$$

- Hence relative price of investment goods will be higher in countries that have high ζ_j and low wages.
- That countries with high ζ_j's (high distortions on investment good sectors) have higher relative prices of investment goods, is consistent with literature.

The General Model VII

- (46) above shows countries with worse technology (low μ_j) and higher discount rates (high ρ_j) have lower wages and, via this channel, higher relative prices of investment goods.
- Thus cross section of the relative prices of investment and consumption goods that is consistent with the patterns in the data.
- Also note relative price of investment goods may vary for reasons different from distortions on the investment sector, as literature implicitly assumes.

The General Model VIII

- Contrast to Heckscher-Ohlin world, where country takes factor prices as given and then accumulates without running into diminishing returns to scale.
- Can the two approaches be reconciled? One way is view them as applying at different stages of development and for different kinds of goods.
  - "standardized" goods (as in East Asian tigers in the 1970s and 80s e.g., Vogel, 2006) vs. more specialized goods of differentiated varieties in later stages of development.

The International Division of Labor I

- Trade enriches process of technology diffusion:
    - "international product cycle," technology diffusion goes hand-in-hand with certain products previously produced by technologically advanced economies migrating to less-developed nations.
- Two sets of economies, the North and the South.
- It does not matter whether there is one or many countries within each group.
- Free international trade, without any trading costs.
- All individuals in all countries have CES preferences with love for variety defined over a consumption index.

The International Division of Labor II

- Consumption index for country $j \in \{n, s\}$ at time t:

$$C_j(t) = \left(\int_0^{N(t)} c_j(t, z)^{\frac{\varepsilon - 1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon - 1}} \tag{47}$$

  where:

  - $c_j(t, z)$=consumption of the zth good in country $j \in \{n, s\}$ at time t,
  - $N(t)$=total number of goods in the world economy at time t , determined endogenously and traded freely,
  - $\varepsilon > 1$=elasticity of substitution between these goods.

- Do not need to specify what dynamic preferences are; for concreteness assume CRRA as in (1).
- Goods fall into two categories:
  - new goods: just invented in the North and can only be produced there;
  - old goods: invented in the past and production technology imitated by South, can be producedin South and North.

The International Division of Labor III

- Technology of production: one worker produces one unit of any good to which the country in which he is located has access to.
- When producing old goods, Northern workers have no productive advantage, their only advantage is they have access to a larger set of goods.
- Total labor supply: L^n in the North and L^s in the South. All labor supplied inelastically.
- Two types of equilibria.
  1. *Equalization equilibrium*: sufficiently few new goods that both South and North produce some of old goods; both goods will command the same price.
  2. *Specialization equilibrium*: the South specializes in old goods, the North in new goods.

The International Division of Labor IV

- Take a given set of new and old goods, $N^n(t)$ and $N^o(t)$, where $N(t) = N^n(t) + N^o(t)$.
- Ratio $N^n(t) / N^o(t)$ (or $N(t) / N^o(t)$)=measure of the technology gap between North and South.
- Suppose that the world is in a specialization equilibrium.
- Prices of all new goods and the prices of all old goods will be equalized, denote them by $p^n(t)$ and $p^o(t)$.
- Let the wage rate in the North be $w^n(t)$ and that in the South $w^s(t)$.
- A specialization equilibrium implies

$$\begin{aligned} p^n(t) &= w^n(t) \\ p^o(t) &= w^s(t). \end{aligned} \tag{48}$$

The International Division of Labor V

- Must have $w^n(t) \geq w^s(t)$, otherwise Northern workers would prefer to produce old goods.
- Thus specialization equilibrium can exist only if when all old goods are produced in the South, the implied equilibrium wage rate in the South is lower than that in the North.
- CES preferences specified in (47) imply:

$$\frac{c^n(t)}{c^o(t)} = \left(\frac{p^n(t)}{p^o(t)}\right)^{-\varepsilon}. \tag{49}$$

- Specialization implies all labor force of South is used to produce old goods, and all of North in production of new goods.
- Therefore

$$c^n(t) = \frac{L^n}{N^n(t)} \text{ and } c^o(t) = \frac{L^s}{N^o(t)}. \tag{50}$$

The International Division of Labor VI

- Combining:

$$\frac{w^n(t)}{w^s(t)} \equiv \omega(t) = \left(\frac{N^n(t)}{N^o(t)} \frac{L^s}{L^n} \right)^{1/\varepsilon}. \tag{51}$$

- A specialization equilibrium will exist only if $\omega(t) > 1$.
- If $\omega(t) < 1$, equalization equilibrium. Therefore,

$$c^n(t) = \frac{\phi L^n}{N^n(t)} \text{ and } c^o(t) = \frac{L^s + (1-\phi) L^n}{N^o(t)},$$

  where $\phi \in (0,1)$ is chosen such that $c^n(t) = c^o(t)$.
- such a $\phi \in (0,1)$ exists, since $\omega(t) < 1$, which implies that $c^n(t) > c^o(t)$ at $\phi = 1$.

The International Division of Labor VII

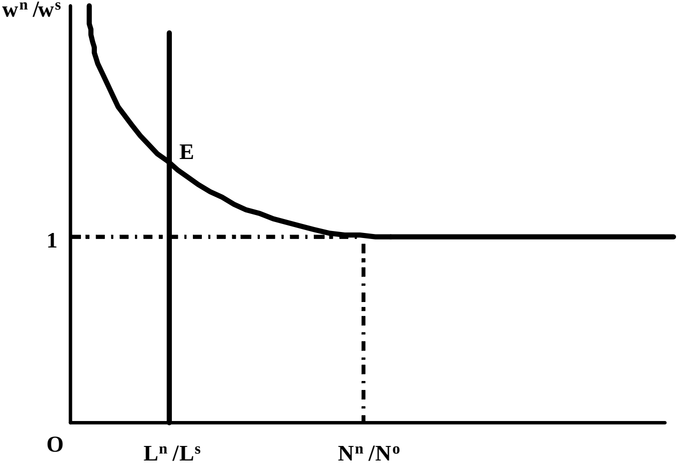

Figure: Determination on the relative wages in the North and the South in the basic product cycle model.

The International Division of Labor VIII

- Note even when there is a technology gap, Northern and Southern incomes may be equalized.
- Income gap between the North and the South when the technology gap is relatively large or when the labor supply in the South, L^s, is sufficiently large (e.g. India and China as potential low-cost producers of "old" goods).
- Positive income gap more realistic, but the possibility that such a gap may not exist is of theoretical interest:
    - With international trade South may achieve the same consumption bundle and the same level of income as North.
    - International trade is a powerful force limiting the extent of cross-country income inequality, but it is not always so.

Product Cycles and Technology Transfer I

- Follow Krugman (1979) to endogenize number of new and old goods using model of exogenous technological change.
- New goods are created in the North according to:

$$\dot{N}(t) = \eta N(t),$$

with some $N(0) > 0$ and innovation parameter $\eta > 0$.

- Goods invented in the North can be imitated by the South:

$$\dot{N}^n(t) = \iota N^n(t),$$

where $\iota > 0$ is the imitation parameter.

- Unique globally stable steady-state ratio of new to old goods:

$$\frac{N^n(t)}{N^o(t)} = \frac{\eta}{\iota}. \tag{52}$$

Product Cycles and Technology Transfer II

- i.e., the ratio of new to old goods will be high when the rate of innovation in the North, η, is high relative to the rate of imitation from the South, ι.
- Combining with (51):

$$\frac{w^n(t)}{w^s(t)} = \max\left\{\left(\frac{\eta}{\iota}\frac{L^s}{L^n}\right)^{1/\varepsilon}, 1\right\}. \tag{53}$$

- Since $w^n(t)/w^s(t)$ corresponds to the ratio of income between North and South,high rate of innovation by North makes the South relatively poor (though not absolutely so), while a higher rate of imitation by South makes it relatively richer.

Product Cycles and Technology Transfer III

- Product cycle: focus on the specialization equilibrium.
  - New goods are invented in the North and $w^n(t) > w^s(t)$.
  - After a while, new good is imitated by the South, so its production shifts South, where labor costs are lower.
- Rate of imitation ι can also be considered as an inverse measure of the international protection of IPR.
- Stronger international IPR protection will always increase the income gap between North and South, but does not always lead to a welfare improvement in the North.

Trade and Endogenous Technological Change I

- Most believe trade promotes growth. Micro and macro evidence:
  - Dollar (1992) and Sachs and Warner (1995): positive correlation between openness and growth.
  - Bernard and Jensen (1997), Bernard, Eaton, Jensen and Kortum (2004): firms that engage in exporting are typically more productive, might be "learning by exporting" but could be selection (Melitz, 2003).
  - Firms in developing countries that import machinery from advanced economies more productive (e.g., Goldberg and Pavnik, 2007).
- Instrumental-variables evidence:
  - Frankel and Romer (1999): gravity equations as source of variation to estimate effect of trade on long-run income differences.
  - Is this credible?
- Skeptics:
  - Rodrik (1997) and Rodriguez and Rodrik (1999): empirical evidence that trade promotes growth is not compelling.
  - Matsuyama (1992) and Young (1993): models in which international trade can slow down growth in some countries.

Trade and Endogenous Technological Change II

- Start with a model illustrating how trade opening may change the pace of endogenous technological change.
- Two independent economies approximated by the baseline endogenous technological change model with expanding input varieties (Grossman and Helpman, 1991b).
- Two economies, 1 and 2, with identical technologies, identical preferences, and identical labor forces normalized to 1 (no population growth).
- No knowledge spillovers, so no problem about this occurring at the same time as trade opening.
- Compare equilibrium growth rates under no trade and costless trade.

Trade and Endogenous Technological Change III

Proposition Suppose that condition

$$\eta\beta > \rho \text{ and } 2\,(1-\theta)\,\eta\beta < \rho \tag{54}$$

holds. Then in autarky there exists a unique equilibrium in which starting from any level of technology, both countries innovate and grow at the same rate

$$g^{A} = \frac{1}{\theta}\,(\eta\beta - \rho)\,. \tag{55}$$

- Implications of trade will depend on whether, before trade opening, the two countries were producing some of the same inputs or not (there is a continuum that can be produced).
  - If producing the same inputs, the static gains from trade will be limited.
  - If producing different inputs, there will be larger static gains.

Trade and Endogenous Technological Change IV

Proposition Suppose that condition (54) holds. Then after trade opening, the world economy and both countries produce new technologies and grow at the rate

$$g^T = \frac{1}{\theta}(2\eta\beta - \rho) > g^A,$$

where g^A is the autarky growth rate given by (55).

- i.e., opening to international trade encourages technological change and increases the growth rate of world economy.
- Trade enables each input producer to access a larger market, and this makes inventing new inputs more profitable.
- Greater profitability translates into higher rate of innovation and more rapid growth.

Trade and Endogenous Technological Change V

- Main effect captured and economic force, a version of market size effect, leading to the innovation gains from trade are reasonably robust.
- Caveats:
  - If the R&D sector competes with production, there will be powerful offsetting effects, because trade will also increase the demand for production workers.
  - If the full scale effect is removed and we focus on an economy with semi-endogenous growth, trade opening will increase innovation temporarily, but not in the long run.

Learning-by-Doing, Trade and Growth I

- Static gains can also encourage economic growth.
- Some economists remain skeptical of the positive growth effects
- Popular argument, used to justify infant industry protection:
  - static gains come at the cost of dynamic gains.
  - international trade induces some countries to specialize in industries with relatively low growth potential.
  - the mechanism for potential dynamic losses is learning-by-doing externalities in some sectors.
- Two blocks of countries, North and South, and each block consists of many identical countries.
- Thought experiment: move from autarky to full international trade integration between two blocks.
- Assume all countries are "almost identical".

Learning-by-Doing, Trade and Growth II

- Each country has a total labor force of 1, and labor can be used to produce one of two intermediate goods with:

$$Y_j^1(t) = A_j(t) L_j^1(t) \text{ and } Y_j^2(t) = L_j^2(t),$$

- Labor market clearing condition:

$$L_j^1(t) + L_j^2(t) \leq 1$$

  for $j \in \{n, s\}$

- Assume total number of Northern and Southern countries are equal, and total number of countries in the world is $2J$.
- Final good produced as a CES aggregate of two intermediates.

Learning-by-Doing, Trade and Growth III

- Distinguishing between production of intermediates and their use in the final good sector:

$$Y_j(t) = \left[\gamma X_j^1(t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) X_j^2(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

  where ε is the elasticity of substitution between the two intermediates.
- Assume intermediates are gross substitutes, so that $\varepsilon > 1$.
- However, $\varepsilon = 1$ (Cobb-Douglas) also of special-interest.
- To simplify the algebra set $\gamma = 1/2$.
- Learning-by-doing:

$$\frac{\dot{A}_j(t)}{A_j(t)} = \eta L_j^1(t), \tag{56}$$

- No learning-by doing opportunities in sector 2.
- Each producer ignores the positive externality that it creates.

Learning-by-Doing, Trade and Growth IV

- Difference between North and South is a small "comparative advantage" for the North in the production of sector 1:

$$A_n(0) = 1 \text{ and } A_s(0) = 1 - \delta, \tag{57}$$

where δ is a small number.

- The value of the marginal product of labor ("wage rates") in two sectors have to be equalized or only one sector will be active.
- Start with closed economy, and suppose both sectors have to be active at t.
- Then marginal products have to be equalized:

$$p_j^1(t) A_j(t) = p_j^2(t), \tag{58}$$

where $p_j^1(t)$ and $p_j^2(t)$=prices of the two intermediates, and $A_j(t)$ the level of productivity in sector 1.

Learning-by-Doing, Trade and Growth V

- Prices are indexed by j since we are in the closed economy.
- Profit-maximization by the final good producers implies:

$$\frac{p_j^1(t)}{p_j^2(t)} = \left(\frac{X_j^1(t)}{X_j^2(t)}\right)^{-\frac{1}{\varepsilon}} = \left(\frac{A_j(t)\,L_j^1(t)}{1-L_j^1(t)}\right)^{-\frac{1}{\varepsilon}},$$

where $L_j^1(t)$ denotes the amount of labor allocated the sector 1, and to sector 2 is $L_j^2(t) = 1 - L_j^1(t)$.

- Combining with (58),

$$L_j^1(t) = \frac{A_j(t)^{\varepsilon-1}}{1+A_j(t)^{\varepsilon-1}}. \tag{59}$$

- The evolution of productivity of sector 1 is given by (56).

Learning-by-Doing, Trade and Growth VI

Proposition Consider the above-described model and suppose that $\varepsilon > 1$. Then in the absence of international trade the equilibrium involves the allocation of labor given by (59) for all j and t. In particular, we have $L_j^1 (t = 0) = 1/2$, and $L_j^1 (t)$ monotonically converges to 1. The growth rate of each country $g_j (t)$ converges to $g^* = \eta$.

If, on the other hand, $\varepsilon = 1$, then $L_j^1 (t) = 1/2$ for all t, and the long-run growth rate of each country is $g^{**} = \eta/2$.

- Next consider free international trade starting at time $t = 0$.
- For each intermediate good, there is now only a single world price, $p^1 (t)$ for good 1 and $p^2 (t)$ for good 2.

Learning-by-Doing, Trade and Growth VII

- These prices satisfy

$$\begin{aligned}\frac{p^1(t)}{p^2(t)} &= \left(\frac{X_n^1(t)+X_s^1(t)}{X_s^2(t)+X_s^2(t)}\right)^{-\frac{1}{\varepsilon}} \\ &= \left(\frac{A_n(t)\,L_n^1(t)+A_s(t)\,L_s^1(t)}{2-L_n^1(t)-L_s^1(t)}\right)^{-\frac{1}{\varepsilon}},\end{aligned}$$

  where the subscripts n and s denote Northern and Southern countries.
- As a result of the slight comparative advantage in (57):
  - At $t = 0$, Northern workers in sector 1 have higher marginal product, all labor force in North will be in sector 1, and all of South in sector 2.
  - Moreover, all of sector 1 production will be in North and all sector 2 production in South.
  - In subsequent periods, productivity of Northern workers in sector 1 is even higher, and productivity of South remains stagnant.

Learning-by-Doing, Trade and Growth VIII

Proposition Consider model above. With free international trade, equilibrium is as follows: $L_n^1(t) = 1$ and $L_s^1(t) = 0$ for all t. In this equilibrium,

$$\frac{\dot{A}_n(t)}{A_n(t)} = \eta \text{ and } \frac{\dot{A}_s(t)}{A_s(t)} = 0.$$

The world economy converges to a growth rate of $g^* = \eta$ in the long run. Throughout, the ratio of income in the North and the South is

$$\frac{Y_n(t)}{Y_s(t)} = A_n(t)^{\frac{\varepsilon-1}{\varepsilon}}.$$

Consequently, if $\varepsilon > 1$, the North becomes progressively richer relative to the South, i.e., $\lim_{t\to\infty} Y_n(t) / Y_s(t) = \infty$. If $\varepsilon = 1$, the relative incomes of the North and the South remain constant, i.e., $Y_n(t) / Y_s(t) =$ constant.

Learning-by-Doing, Trade and Growth IX

- International trade can harm when there are learning-by-doing externalities in some sectors:
  - South has a slight comparative disadvantage in sector 1.
  - If no trade, it devotes enough of its resources to that sector and achieves same growth rate as North.
  - If free trade, South specializes in sector 2 and fails to benefit from the learning-by-doing.
  - Thus South becomes poorer relative to the North.
- Shortcomings:
  - e.g, if $\varepsilon = 1$ (or close to 1), specialization in sector 2 does not hurt the South.
    - Increase in productivity of sector 1 in North creates a negative terms of trade effect against North.
    - Effect is always present, but when $\varepsilon = 1$ sufficiently powerful to prevent the impoverishment of the South despite specialized in sector with low growth potential.
  - Infant industry protection will not help South.
    - Even if no trade for some infant industry, with protection period of duration $T > 0$ the ultimate outcome will be the same as above.

Learning-by-Doing, Trade and Growth X

- Effect of trade on growth is empirical issue, but theoretical perspectives are useful.
- The effect of trade integration on the rate of endogenous technological progress may be limited because of the factors discussed above.
- But the benefits of the greater market size for firms involved in innovation must be present according to any model of endogenous technological change.

Learning-by-Doing, Trade and Growth XI

- Potential negative effects of trade on growth because "incorrect" specialization are exaggerated.
  1. No strong evidence that learning-by-doing externalities are important in general and much more important in some sectors than in others.
  2. In most situations specialization is not perfect, thus some amount of learning-by-doing takes place in all economies.
  3. International flows of information imply improvements in productivity in some countries will affect productivity in others that were not initially specializing in those sectors (e.g., Korea was an importer of cars, and now a net exporter, its productivity increased with technology transfer).
  4. Terms of trade effects ameliorate any negative impact of specialization in some countries.

Conclusions

- International trade in assets (international borrowing and lending) and in commodities change both the dynamics and potentially long-run implications of the closed-economy neoclassical growth models.
- Nature of international trade interacts with the process of economic growth.
- Impact of international trade on growth ultimately an empirical question, though theoretical analysis highlighted important mechanisms and suggested negative effects of trade on growth unlikely to be important.