Advanced Economic Growth: Lecture 5, Directed Technological Change

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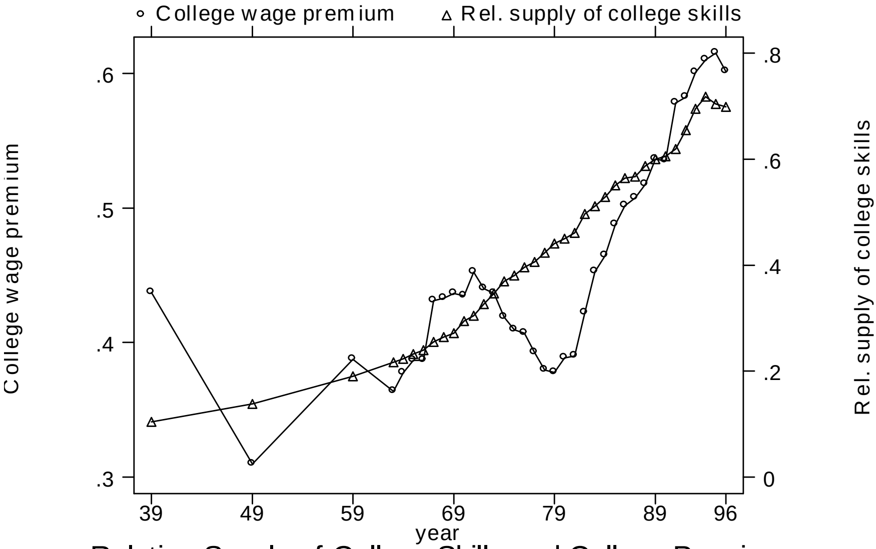
Introduction

- Thus far have focused on a single type of technological change (e.g., Hicks-neutral).
- But, technological change is often not neutral:
  1. Benefits some factors of production and some agents more than others. Distributional effects imply some groups will embrace new technologies and others oppose them.
  2. Limiting to only one type of technological change obscures the competing effects that determine the nature of technological change.
- *Directed technological change*: endogenize the direction and bias of new technologies that are developed and adopted.

Skill-biased technological change

- Over the past 60 years, the U.S. relative supply of skills has increased, but:
  1. there has also been an increase in the college premium, and
  2. this increase accelerated in the late 1960s, and the skill premium increased very rapidly beginning in the late 1970s.
- Standard explanation: skill bias technical change, and an acceleration that coincided with the changes in the relative supply of skills.
- Important question: skill bias is endogenous, so, why has technological change become more skill biased in recent decades?

Skill-biased technological change



Relative Supply of College Skills and College Premium

Unskill-biased technological change

- Late 18th and early 19th *unskill-bias:*
  "First in firearms, then in clocks, pumps, locks, mechanical reapers, typewriters, sewing machines, and eventually in engines and bicycles, interchangeable parts technology proved superior and replaced the skilled artisans working with chisel and file." (Mokyr 1990, p. 137)
- Why was technological change unskilled-biased then and skilled-biased now?

Wage push and capital-biased technological change

- First phase. Late 1960s and early 1970s: unemployment and share of labor in national income increased rapidly continental European countries.
- Second phase. 1980s: unemployment continued to increase, but the labor share declined, even below its initial level.
- Blanchard (1997):
  - Phase 1: wage-push by workers
  - Phase 2: *capital-biased* technological changes.

  Is there a connection between capital-biased technological changes in European economies and the wage push preceding it?

Importance of Biased Technological Change: more examples

- Balanced economic growth:
  - Only possible when technological change is asymptotically Harrod-neutral, i.e., purely labor augmenting.
  - Is there any reason to expect technological change to be endogenously labor augmenting?
- Globalization:
  - Does it affect the types of technologies that are being developed and used?

Directed Technological Change: Basic Arguments I

- Two factors of production, say L and H (unskilled and skilled workers).
- Two types of technologies that can complement either one or the other factor.
- Whenever the profitability of H-augmenting technologies is greater than the L-augmenting technologies, more of the former type will be developed by profit-maximizing (research) firms.
- What determines the relative profitability of developing different technologies? It is more profitable to develop technologies...
  1. when the goods produced by these technologies command higher prices (*price effect*);
  2. that have a larger market (*market size effect*).

Equilibrium Relative Bias

- Potentially counteracting effects, but the market size effect will be more powerful often.
- Under fairly general conditions:
  - *Weak Equilibrium (Relative) Bias*: an increase in the relative supply of a factor always induces technological change that is biased in favor of this factor.
  - *Strong Equilibrium (Relative) Bias*: if the elasticity of substitution between factors is sufficiently large, an increase in the relative supply of a factor induces sufficiently strong technological change biased towards itself that the endogenous-technology relative demand curve of the economy becomes *upward-sloping.*

Equilibrium Relative Bias in More Detail I

- Suppose the (inverse) relative demand curve:

$$w_H / w_L = D(H/L, A)$$

  where w_H / w_L is the relative price of the factors and A is a technology term.

- A is H-biased if D is increasing in A, so that a higher A increases the relative demand for the H factor.
- D is *always* decreasing in H/L.
- Equilibrium bias: behavior of A as H/L changes,

$$A(H/L)$$

Equilibrium Relative Bias in More Detail II

- Weak equilibrium bias:
  - $A(H/L)$ is increasing (nondecreasing) in H/L.
- Strong equilibrium bias:
  - $A(H/L)$ is sufficiently responsive to an increase in H/L that the total effect of the change in relative supply H/L is to increase w_H/w_L.
  - i.e., let the endogenous-technology relative demand curve be

$$w_H/w_L = D(H/L, A(H/L)) \equiv \tilde{D}(H/L)$$

$\rightarrow$*Strong equilibrium bias:* $\tilde{D}$ **increasing in** H/L.

Factor-augmenting technological change

- Production side of the economy:

$$Y(t) = F(L(t), H(t), A(t)),$$

where $\partial F / \partial A > 0$.

- Technological change is *L-augmenting* if

$$\frac{\partial F(L, H, A)}{\partial A} \equiv \frac{L}{A} \frac{\partial F(L, H, A)}{\partial L}.$$

- Equivalent to:
  - the production function taking the special form, $F(AL, H)$.
  - Harrod-neutral technological change when L corresponds to labor and H to capital.
- H-augmenting defined similarly, and corresponds to $F(L, AH)$.

Factor-biased technological change

- Technological change change is *L-biased*, if:

$$\frac{\partial \frac{\partial F(L,H,A)/\partial L}{\partial F(L,H,A)/\partial H}}{\partial A} \geq 0.$$

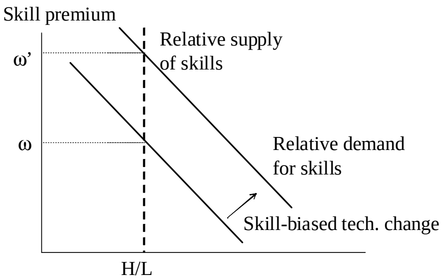


Figure: The effect of *H*-biased technological change on relative demand and relative factor prices.

Constant Elasticity of Substitution Production Function I

- CES production function case:

$$Y(t) = \left[\gamma_L (A_L(t) L(t))^{\frac{\sigma-1}{\sigma}} + \gamma_H (A_H(t) H(t))^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where

- $A_L(t)$ and $A_H(t)$ are two separate technology terms.
- γ_is determine the importance of the two factors, $\gamma_L + \gamma_H = 1$.
- $\sigma \in (0, \infty)$=elasticity of substitution between the two factors.
  - $\sigma = \infty$, perfect substitutes, linear production function is linear.
  - $\sigma = 1$, Cobb-Douglas,
  - $\sigma = 0$, no substitution, Leontieff.
  - $\sigma > 1$, "gross substitutes,"
  - $\sigma < 1$, "gross complements".

- Clearly, $A_L(t)$ is L-augmenting, while $A_H(t)$ is H-augmenting.
- Whether technological change that is L-augmenting (or H-augmenting) is L-biased or H-biased depends on σ.

Constant Elasticity of Substitution Production Function II

- Relative marginal product of the two factors:

$$\frac{MP_H}{MP_L} = \gamma \left(\frac{A_H(t)}{A_L(t)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H(t)}{L(t)} \right)^{-\frac{1}{\sigma}}, \tag{1}$$

  where $\gamma \equiv \gamma_H / \gamma_L$.
- *substitution effect:* the relative marginal product of H is decreasing in its relative abundance, $H(t)/L(t)$.
- The effect of $A_H(t)$ on the relative marginal product:
  - If $\sigma > 1$, an increase in $A_H(t)$ (relative to $A_L(t)$) increases the relative marginal product of H.
  - If $\sigma < 1$, an increase in $A_H(t)$ reduces the relative marginal product of H.
  - If $\sigma = 1$, Cobb-Douglas case, and neither a change in $A_H(t)$ nor in $A_L(t)$ is biased towards any of the factors.
- Note also that since σ is the elasticity of substitution between the two factors:

$$\sigma = -\left(\frac{d \log (MP_H / MP_L)}{d \log (H/L)} \right)^{-1}$$

Constant Elasticity of Substitution Production Function III

- Intuition for why, when $\sigma < 1$, H-augmenting technical change is L-biased:
  - with gross complementarity ($\sigma < 1$), an increase in the productivity of H increases the demand for labor, L, by more than the demand for H, creating "excess demand" for labor.
  - the marginal product of labor increases by more than the marginal product of H.
  - Take case where $\sigma \to 0$ (Leontieff): starting from a situation in which $\gamma_L A_L(t) L(t) = \gamma_H A_H(t) H(t)$, a small increase in $A_H(t)$ will create an excess of the services of the H factor, and its price will fall to 0.

Equilibrium Bias

- **Weak equilibrium bias** of technology: an increase in H/L, induces technological change biased towards H. i.e., given (1):

$$\frac{d\left(A_H\left(t\right)/A_L\left(t\right)\right)^{\frac{\sigma-1}{\sigma}}}{dH/L} \geq 0,$$

  so $A_H\left(t\right)/A_L\left(t\right)$ is biased towards the factor that has become more abundant.

- **Strong equilibrium bias:** an increase in H/L induces a sufficiently large change in the bias so that the relative marginal product of H relative to that of L increases following the change in factor supplies:

$$\frac{dMP_H/MP_L}{dH/L} > 0,$$

- The major difference is whether the relative marginal product of the two factors are evaluated at the initial relative supplies (weak bias) or at the new relative supplies (strong bias).

Baseline Model of Directed Technical Change I

- Framework: expanding varieties model with lab equipment specification of the innovation possibilities frontier (so none of the results here depend on technological externalities).
- Constant supply of L and H.
- Representative household with the standard CRRA preferences:

$$\int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt, \tag{2}$$

- Aggregate production function:

$$Y(t) = \left[\gamma_L Y_L(t)^{\frac{\varepsilon-1}{\varepsilon}} + \gamma_H Y_H(t)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where intermediate good $Y_L(t)$ is L-intensive, $Y_H(t)$ is H-intensive.

Baseline Model of Directed Technical Change II

- Resource constraint (define $Z(t) = Z_L(t) + Z_H(t)$):

$$C(t) + X(t) + Z(t) \leq Y(t), \tag{4}$$

- Intermediate goods produced competitively with:

$$Y_L(t) = \frac{1}{1-\beta}\left(\int_0^{N_L(t)} x_L(\nu, t)^{1-\beta} d\nu\right) L^{\beta} \tag{5}$$

and

$$Y_H(t) = \frac{1}{1-\beta}\left(\int_0^{N_H(t)} x_H(\nu, t)^{1-\beta} d\nu\right) H^{\beta}, \tag{6}$$

where machines $x_L(\nu, t)$ and $x_H(\nu, t)$ are assumed to depreciate after use.

Baseline Model of Directed Technical Change III

- Differences with baseline expanding product varieties model:
  1. These are production functions for intermediate goods rather than the final good.
  2. (5) and (6) use different types of machines–different ranges $[0, N_L(t)]$ and $[0, N_H(t)]$.
- All machines are supplied by monopolists that have a fully-enforced perpetual patent, at prices $p_L^x(\nu, t)$ for $\nu \in [0, N_L(t)]$ and $p_H^x(\nu, t)$ for $\nu \in [0, N_H(t)]$.
- Once invented, each machine can be produced at the fixed marginal cost ψ in terms of the final good.
- Normalize to $\psi \equiv 1 - \beta$.

Baseline Model of Directed Technical Change IV

- Total resources devoted to machine production at time t are

$$X(t) = (1-\beta)\left(\int_0^{N_L(t)} x_L(\nu,t)\,d\nu + \int_0^{N_H(t)} x_H(\nu,t)\,d\nu\right).$$

- Innovation possibilities frontier:

$$\dot{N}_L(t) = \eta_L Z_L(t) \text{ and } \dot{N}_H(t) = \eta_H Z_H(t), \quad (7)$$

- Value of a monopolist that discovers one of these machines is:

$$V_f(\nu,t) = \int_t^{\infty} \exp\left[-\int_t^s r(s')\,ds'\right] \pi_f(\nu,s)ds, \quad (8)$$

where $\pi_f(\nu,t) \equiv p_f^x(\nu,t)x_f(\nu,t) - \psi x_f(\nu,t)$ for $f = L$ or H.

- Hamilton-Jacobi-Bellman version:

$$r(t)\,V_f(\nu,t) - \dot{V}_f(\nu,t) = \pi_f(\nu,t). \quad (9)$$

Baseline Model of Directed Technical Change V

- Normalize the price of the final good at every instant to 1, which is equivalent to setting the ideal price index of the two intermediates equal to one, i.e.,

$$\left[\gamma_L^{\varepsilon}\left(p_L\left(t\right)\right)^{1-\varepsilon}+\gamma_H^{\varepsilon}\left(p_H\left(t\right)\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}=1 \text{ for all } t, \qquad (10)$$

  where $p_L(t)$ is the price index of Y_L at time t and $p_H(t)$ is the price of Y_H.
- Denote factor prices by $w_L(t)$ and $w_H(t)$.

Equilibrium I

- Allocation. Time paths of
  - $[C(t), X(t), Z(t)]_{t=0}^{\infty}$,
  - $[N_L(t), N_H(t)]_{t=0}^{\infty}$,
  - $\left[p_L^x(\nu,t), x_L(\nu,t), V_L(\nu,t)\right]_{\nu\in[0,N_L(t)]}^{\infty\ \ t=0,}$ and

    $\left[\chi_H(\nu,t), x_H(\nu,t), V_H(\nu,t)\right]_{\nu\in[0,N_H(t)]}^{\infty\ \ t=0,}$, and
  - $[r(t), w_L(t), w_H(t)]_{t=0}^{\infty}$.
- Equilibrium. An allocation in which
  - All existing research firms choose $\left[p_f^x(\nu,t), x_f(\nu,t)\right]_{\nu\in[0,N_f(t)]}^{\infty\ \ t=0,}$ for $f = L, H$ to maximize profits,
  - $[N_L(t), N_H(t)]_{t=0}^{\infty}$ is determined by free entry
  - $[r(t), w_L(t), w_H(t)]_{t=0}^{\infty}$, are consistent with market clearing, and
  - $[C(t), X(t), Z(t)]_{t=0}^{\infty}$ are consistent with consumer optimization.

Equilibrium II

- Maximization problem of producers in the two sectors:

$$\max_{L,[x_L(\nu,t)]_{\nu\in[0,N_L(t)]}} p_L(t)\,Y_L(t) - w_L(t)\,L - \int_0^{N_L(t)} p_L^x(\nu,t)\,x_L(\nu,t)\,d\nu, \tag{11}$$

and

$$\max_{H,[x_H(\nu,t)]_{\nu\in[0,N_H(t)]}} p_H(t)\,Y_H(t) - w_H(t)\,H - \int_0^{N_H(t)} p_H^x(\nu,t)\,x_H(\nu,t)\,d\nu. \tag{12}$$

- Note the presence of $p_L(t)$ and $p_H(t)$, since these sectors produce intermediate goods.

Equilibrium III

- Thus, demand for machines in the two sectors:

$$x_L(\nu, t) = \left[\frac{p_L(t)}{p_L^x(\nu, t)}\right]^{1/\beta} L \quad \text{for all } \nu \in [0, N_L(t)] \text{ and all } t, \tag{13}$$

and

$$x_H(\nu, t) = \left[\frac{p_H(t)}{p_H^x(\nu, t)}\right]^{1/\beta} H \quad \text{for all } \nu \in [0, N_H(t)] \text{ and all } t. \tag{14}$$

- Maximization of the net present discounted value of profits implies a constant markup:

$$p_L^x(\nu, t) = p_H^x(\nu, t) = 1 \text{ for all } \nu \text{ and } t.$$

Equilibrium IV

- Substituting into (13) and (14):

$$x_L(\nu, t) = p_L(t)^{1/\beta} L \quad \text{for all } \nu \text{ and all } t,$$

and

$$x_H(\nu, t) = p_H(t)^{1/\beta} H \quad \text{for all } \nu \text{ and all } t.$$

- Since these quantities do not depend on the identity of the machine profits are also independent of the machine type:

$$\pi_L(t) = \beta p_L(t)^{1/\beta} L \text{ and } \pi_H(t) = \beta p_H(t)^{1/\beta} H. \qquad (15)$$

- Thus the values of monopolists only depend on which sector they are, $V_L(t)$ and $V_H(t)$.

Equilibrium V

- Combining these with (5) and (6), *derived* production functions for the two intermediate goods:

$$Y_L(t) = \frac{1}{1-\beta} p_L(t)^{\frac{1-\beta}{\beta}} N_L(t) L \tag{16}$$

and

$$Y_H(t) = \frac{1}{1-\beta} p_H(t)^{\frac{1-\beta}{\beta}} N_H(t) H. \tag{17}$$

Equilibrium VI

- For the prices of the two intermediate goods, (3) imply

$$
\begin{aligned}
p(t) &\equiv \frac{p_H(t)}{p_L(t)} = \gamma \left(\frac{Y_H(t)}{Y_L(t)} \right)^{-\frac{1}{\varepsilon}} \\
&= \gamma \left(p(t)^{\frac{1-\beta}{\beta}} \frac{N_H(t)\, H}{N_L(t)\, L} \right)^{-\frac{1}{\varepsilon}} \\
&= \gamma^{\frac{\varepsilon\beta}{\sigma}} \left(\frac{N_H(t)\, H}{N_L(t)\, L} \right)^{-\frac{\beta}{\sigma}}, \qquad (18)
\end{aligned}
$$

where $\gamma \equiv \gamma_H / \gamma_L$ and

$$
\begin{aligned}
\sigma &\equiv \varepsilon - (\varepsilon - 1)(1 - \beta) \\
&= 1 + (\varepsilon - 1)\beta.
\end{aligned}
$$

Equilibrium VII

- We can also calculate the relative factor prices:

$$\begin{aligned}\omega(t) &\equiv \frac{w_H(t)}{w_L(t)} \\ &= p(t)^{1/\beta}\frac{N_H(t)}{N_L(t)} \\ &= \gamma^{\frac{\varepsilon}{\sigma}}\left(\frac{N_H(t)}{N_L(t)}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{H}{L}\right)^{-\frac{1}{\sigma}}. \qquad (19)\end{aligned}$$

- σ is the (derived) elasticity of substitution between the two factors, since it is exactly equal to

$$\sigma = -\left(\frac{d\log\omega(t)}{d\log(H/L)}\right)^{-1}.$$

Equilibrium VIII

- Free entry conditions:

$$\eta_L V_L(t) \leq 1 \text{ and } \eta_L V_L(t) = 1 \text{ if } Z_L(t) > 0. \tag{20}$$

and

$$\eta_H V_H(t) \leq 1 \text{ and } \eta_H V_H(t) = 1 \text{ if } Z_H(t) > 0. \tag{21}$$

- Consumer side:

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}(r(t) - \rho), \tag{22}$$

and

$$\lim_{t\to\infty}\left[\exp\left(-\int_0^t r(s)\,ds\right)(N_L(t)\,V_L(t) + N_H(t)\,V_H(t))\right] = 0, \tag{23}$$

where $N_L(t)\,V_L(t) + N_H(t)\,V_H(t)$ is the total value of corporate assets in this economy.

Balanced Growth Path I

- Consumption grows at the constant rate, g^*, and the relative price $p(t)$ is constant. From (10) this implies that $p_L(t)$ and $p_H(t)$ are also constant.
- Let V_L and V_H be the BGP net present discounted values of new innovations in the two sectors. Then (9) implies that

$$V_L = \frac{\beta p_L^{1/\beta} L}{r^*} \text{ and } V_H = \frac{\beta p_H^{1/\beta} H}{r^*}, \tag{24}$$

- Taking the ratio of these two expressions, we obtain

$$\frac{V_H}{V_L} = \left(\frac{p_H}{p_L}\right)^{\frac{1}{\beta}} \frac{H}{L}.$$

Balanced Growth Path II

- Note the two effects on the direction of technological change:
  1. The price effect: V_H / V_L is increasing in p_H / p_L. Tends to favor technologies complementing scarce factors.
  2. The market size effect: V_H / V_L is increasing in H / L. It encourages innovation for the more abundant factor.
- The above discussion is incomplete since prices are endogenous. Combining (24) together with (18):

$$\frac{V_H}{V_L} = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{\varepsilon}{\sigma}} \left(\frac{N_H}{N_L}\right)^{-\frac{1}{\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{\sigma}} . \qquad (25)$$

- Note that an increase in H / L will increase V_H / V_L as long as $\sigma > 1$ and it will reduce it if $\sigma < 1$. Moreover,

$$\sigma \gtreqless 1 \iff \varepsilon \gtreqless 1.$$

- Hence the two factors will be gross substitutes when the two intermediate goods are gross substitutes in the production of the final good.

Balanced Growth Path III

- Next, using the two free entry conditions (20) and (21) as equalities, we obtain the following BGP "technology market clearing" condition:

$$\eta_L V_L = \eta_H V_H. \tag{26}$$

- Combining this with (25), BGP ratio of relative technologies is

$$\left(\frac{N_H}{N_L}\right)^* = \eta^\sigma \gamma^\varepsilon \left(\frac{H}{L}\right)^{\sigma-1}, \tag{27}$$

where $\eta \equiv \eta_H / \eta_L$.

- Note that relative productivities are determined by the innovation possibilities frontier and the relative supply of the two factors. In this sense, this model totally endogenizes technology.

Summary of Balanced Growth Path

Proposition Consider the directed technological change model described above. Suppose

$$\beta \left[\gamma_H^{\varepsilon} (\eta_H H)^{\sigma-1} + \gamma_L^{\varepsilon} (\eta_L L)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} > \rho \quad (28)$$

$$\text{and} \quad (1-\theta) \beta \left[\gamma_H^{\varepsilon} (\eta_H H)^{\sigma-1} + \gamma_L^{\varepsilon} (\eta_L L)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} < \rho.$$

Then there exists a unique BGP equilibrium in which the relative technologies are given by (27), and consumption and output grow at the rate

$$g^* = \frac{1}{\theta} \left(\beta \left[\gamma_H^{\varepsilon} (\eta_H H)^{\sigma-1} + \gamma_L^{\varepsilon} (\eta_L L)^{\sigma-1} \right]^{\frac{1}{\sigma-1}} - \rho \right). \quad (29)$$

Transitional Dynamics

- Differently from the baseline endogenous technological change models, there are now transitional dynamics (because there are two state variables).
- Nevertheless, transitional dynamics simple and intuitive:

Proposition Consider the directed technological change model described above. Starting with any $N_H(0) > 0$ and $N_L(0) > 0$, there exists a unique equilibrium path. If $N_H(0)/N_L(0) < (N_H/N_L)^*$ as given by (27), then we have $Z_H(t) > 0$ and $Z_L(t) = 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$. If $N_H(0)/N_L(0) < (N_H/N_L)^*$, then $Z_H(t) = 0$ and $Z_L(t) > 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$.

- Summary: the dynamic equilibrium path always tends to the BGP and during transitional dynamics, there is only one type of innovation.

Directed Technological Change and Factor Prices

- In BGP, there is a positive relationship between H/L and N_H^*/N_L^* only when $\sigma > 1$.
- But this does not mean that depending on σ (or ε), changes in factor supplies may induce technological changes that are biased in favor or against the factor that is becoming more abundant.
- Why?
  - N_H^*/N_L^* refers to the ratio of factor-augmenting technologies, or to the ratio of *physical* productivities.
  - What matters for the bias of technology is *the value of marginal product* of factors, affected by relative prices.
  - The relationship between factor-augmenting and factor-biased technologies is reversed when σ is less than 1.
  - When $\sigma > 1$, an increase in N_H^*/N_L^* is relatively biased towards H, while when $\sigma < 1$, a *decrease* in N_H^*/N_L^* is relatively biased towards H.

Weak Equilibrium (Relative) Bias Result

Proposition Consider the directed technological change model described above. There is always **weak equilibrium (relative) bias** in the sense that an increase in H/L always induces relatively H-biased technological change.

- The results reflect the strength of the market size effect: it always dominates the price effect.
- But it does not specify whether this induced effect will be strong enough to make the endogenous-technology relative demand curve for factors upward-sloping.

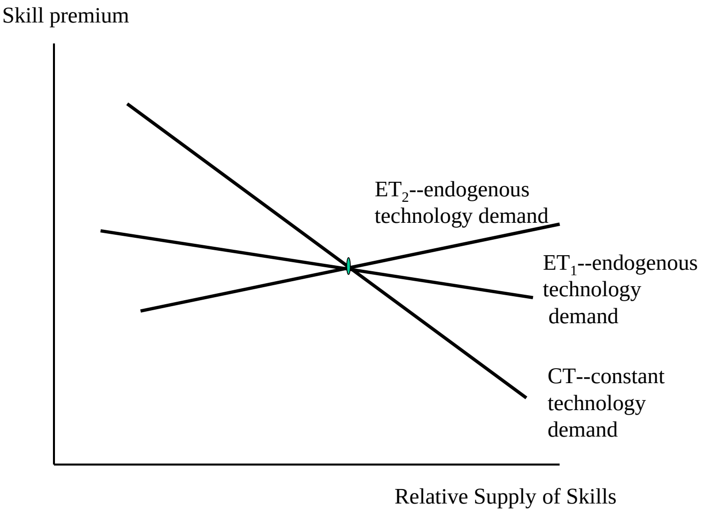
Strong Equilibrium (Relative) Bias Result

- Substitute for $(N_H/N_L)^*$ from (27) into the expression for the relative wage given technologies, (19), and obtain:

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\sigma-1}\gamma^{\varepsilon}\left(\frac{H}{L}\right)^{\sigma-2}. \tag{30}$$

Proposition Consider the directed technological change model described above. Then if $\sigma > 2$, there is **strong equilibrium (relative) bias** in the sense that an increase in H/L raises the relative marginal product and the relative wage of the H factor compared to factor L.

Relative Supply of Skills and Skill Premium



Discussion

- Analogous to Samuelson's *LeChatelier principle*: think of the endogenous-technology demand curve as adjusting the "factors of production" corresponding to technology.
- But, the effects here are caused by general equilibrium changes, not on partial equilibrium effects.
- Moreover ET_2, which applies when $\sigma > 2$ holds, is upward-sloping.
- A complementary intuition: importance of non-rivalry of ideas:
  - leads to an aggregate production function that exhibits increasing returns to scale (in all factors including technologies).
  - the market size effect can create sufficiently strong induced technological change to increase the relative marginal product and the relative price of the factor that has become more abundant.

Implications I

- Recall we have the following stylized facts:
  - Secular skill-biased technological change increasing the demand for skills throughout the 20th century.
  - Possible acceleration in skill-biased technological change over the past 25 years.
  - A range of important technologies biased against skill workers during the 19th century.
- The current model gives us a way to think about these issues.
  - The increase in the number of skilled workers should cause steady skill-biased technical change.
  - Acceleration in the increase in the number of skilled workers should induce an acceleration in skill-biased technological change.
  - Available evidence suggests that there were large increases in the number of unskilled workers during the late 18th and 19th centuries.

Implications II

- The framework also gives a potential interpretation for the dynamics of the college premium during the 1970s and 1980s.
  - It is reasonable that the equilibrium skill bias of technologies, N_H / N_L, is a sluggish variable.
  - Hence a rapid increase in the supply of skills would first reduce the skill premium as the economy would be moving along a constant technology (constant N_H / N_L).
  - After a while technology would start adjusting, and the economy would move back to the upward sloping relative demand curve, with a relatively sharp increase in the college premium.

Implications III

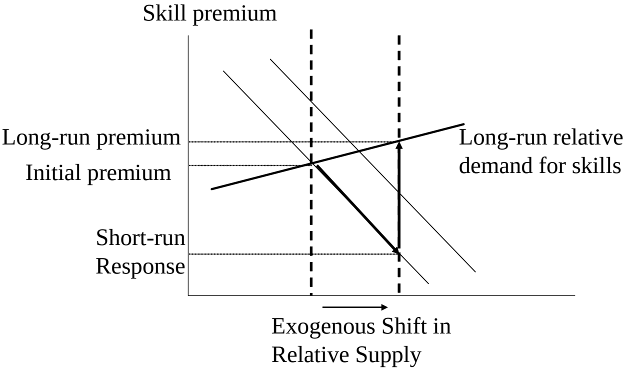


Figure: Dynamics of the skill premium in response to an exogenous increase in the relative supply of skills, with an upward-sloping endogenous-technology relative demand curve.

Implications IV

- If instead $\sigma < 2$, the long-run relative demand curve will be downward sloping, though again it will be shallower than the short-run relative demand curve.
- An increase in the relative supply of skills leads again to a decline in the college premium, and as technology starts adjusting the skill premium will increase.
- But it will end up below its initial level. To explain the larger increase in the college premium in the 1980s, in this case we would need some exogenous skill-biased technical change.

Implications V

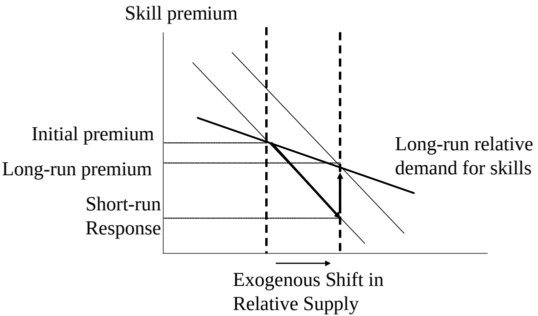


Figure: Dynamics of the skill premium in response to an increase in the relative supply of skills, with a downward-sloping endogenous-technology relative demand curve.

Implications VI

- Other remarks:
  - Upward-sloping relative demand curves arise only when $\sigma > 2$. Most estimates put the elasticity of substitution between 1.4 and 2. One would like to understand whether $\sigma > 2$ is a feature of the specific model discussed here
  - Results on induced technological change are not an artifact of the scale effect (exactly the same results apply when scale effects are removed, see below).

Pareto Optimal Allocations I

- The social planner would not charge a markup on machines:

$$
\begin{aligned}
x_L^S(\nu, t) &= (1-\beta)^{-1/\beta} p_L(t)^{1/\beta} L \\
\text{and } x_H^S(\nu, t) &= (1-\beta)^{-1/\beta} p_H(t)^{1/\beta} H.
\end{aligned}
$$

- Thus:

$$
\begin{aligned}
Y^S(t) &= (1-\beta)^{-1/\beta} \beta [\gamma_L^{\varepsilon} \left(N_L^S(t) L \right)^{\frac{\sigma-1}{\sigma}} \\
&\quad + \gamma_H^{\varepsilon} \left(N_H^S(t) H \right)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}.
\end{aligned} \tag{31}
$$

Pareto Optimal Allocations II

- The current-value Hamiltonian is:

$$\begin{aligned}\mathcal{H}(\cdot) &= \frac{C^S(t)^{1-\theta}-1}{1-\theta} \\ &+\mu_L(t)\,\eta_L Z_L^S(t)+\mu_H(t)\,\eta_H Z_H^S(t),\end{aligned}$$

subject to

$$\begin{aligned}C^S(t) &= (1-\beta)^{-1/\beta}\left[\gamma_L^{\varepsilon}\left(N_L^S(t)\,L\right)^{\frac{\sigma-1}{\sigma}}+\gamma_H^{\varepsilon}\left(N_H^S(t)\,H\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\ &-Z_L^S(t)-Z_H^S(t).\end{aligned}$$

Summary of Pareto Optimal Allocations

Proposition The stationary solution of the Pareto optimal allocation involves relative technologies given by (27) as in the decentralized equilibrium. The stationary growth rate is higher than the equilibrium growth rate and is given by

$$g^S = \frac{1}{\theta}\left((1-\beta)^{-1/\beta}\beta\left[(1-\gamma)^{\varepsilon}(\eta_H H)^{\sigma-1} + \gamma^{\varepsilon}(\eta_L L)^{\sigma-1}\right]^{\frac{1}{\sigma-}}\right.$$

where g^* is the BGP growth rate given in (29).

Directed Technological Change with Knowledge Spillovers I

- The lab equipment specification of the innovation possibilities does not allow for *state dependence*.
- Assume that R&D is carried out by scientists and that there is a constant supply of scientists equal to S
- With only one sector, sustained endogenous growth requires $\dot{N}/N$ to be proportional to S.
- With two sectors, there is a variety of specifications with different degrees of state dependence, because productivity in each sector can depend on the state of knowledge in both sectors.
- A flexible formulation is

$$\begin{aligned} \dot{N}_L(t) &= \eta_L N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} S_L(t) \qquad (32) \\ \text{and } \dot{N}_H(t) &= \eta_H N_L(t)^{(1-\delta)/2} N_H(t)^{(1+\delta)/2} S_H(t), \end{aligned}$$

where $\delta \leq 1$.

Directed Technological Change with Knowledge Spillovers II

- Market clearing for scientists requires that

$$S_L(t) + S_H(t) \leq S. \tag{33}$$

- δ measures the degree of state-dependence:
  - $\delta = 0$. Results are unchanged. No state-dependence:

$$\left(\partial \dot{N}_H / \partial S_H\right) / \left(\partial \dot{N}_L / \partial S_L\right) = \eta_H / \eta_L$$

    irrespective of the levels of N_L and N_H.
    Both N_L and N_H create spillovers for current research in both sectors.
  - $\delta = 1$. Extreme amount of state-dependence:

$$\left(\partial \dot{N}_H / \partial S_H\right) / \left(\partial \dot{N}_L / \partial S_L\right) = \eta_H N_H / \eta_L N_L$$

    an increase in the stock of L-augmenting machines today makes future labor-complementary innovations cheaper, but has no effect on the cost of H-augmenting innovations.

Directed Technological Change with Knowledge Spillovers III

- State dependence adds another layer of "increasing returns," this time not for the entire economy, but for specific technology lines.
- Free entry conditions:

$$\begin{aligned} \eta_L N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} V_L(t) &\leq w_S(t) \quad (34)\\ \text{and } \eta_L N_L(t)^{(1+\delta)/2} N_H(t)^{(1-\delta)/2} V_L(t) &= w_S(t) \text{ if } S_L(t) > 0. \end{aligned}$$

and

$$\begin{aligned} \eta_H N_L(t)^{(1-\delta)/2} N_H(t)^{(1+\delta)/2} V_H(t) &\leq w_S(t) \quad (35)\\ \text{and } \eta_H N_L(t)^{(1-\delta)/2} N_H(t)^{(1+\delta)/2} V_H(t) &= w_S(t) \text{ if } S_H(t) > 0, \end{aligned}$$

where $w_S(t)$ denotes the wage of a scientist at time t.

Directed Technological Change with Knowledge Spillovers IV

- When both of these free entry conditions hold, BGP technology market clearing implies

$$\eta_L N_L(t)^{\delta} \pi_L = \eta_H N_H(t)^{\delta} \pi_H, \tag{36}$$

- Combine condition (36) with equations (15) and (18), to obtain the equilibrium relative technology as:

$$\left(\frac{N_H}{N_L}\right)^* = \eta^{\frac{\sigma}{1-\delta\sigma}} \gamma^{\frac{\varepsilon}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{1-\delta\sigma}}, \tag{37}$$

where $\gamma \equiv \gamma_H/\gamma_L$ and $\eta \equiv \eta_H/\eta_L$.

Directed Technological Change with Knowledge Spillovers V

- The relationship between the relative factor supplies and relative physical productivities now depends on δ.
- This is intuitive: as long as $\delta > 0$, an increase in N_H reduces the relative costs of H-augmenting innovations, so for technology market equilibrium to be restored, π_L needs to fall relative to π_H.
- Substituting (37) into the expression (19) for relative factor prices for given technologies, yields the following long-run (endogenous-technology) relationship:

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\frac{\sigma-1}{1-\delta\sigma}} \gamma^{\frac{(1-\delta)\varepsilon}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-2+\delta}{1-\delta\sigma}}. \tag{38}$$

Directed Technological Change with Knowledge Spillovers VI

- The growth rate is determined by the number of scientists. In BGP we need $\dot{N}_L(t)/N_L(t) = \dot{N}_H(t)/N_H(t)$, or

$$\eta_H N_H(t)^{\delta-1} S_H(t) = \eta_L N_L(t)^{\delta-1} S_L(t).$$

- Combining with (33) and (37), BGP allocation of researchers between the two different types of technologies:

$$\eta^{\frac{1-\sigma}{1-\delta\sigma}} \left(\frac{1-\gamma}{\gamma}\right)^{-\frac{\varepsilon(1-\delta)}{1-\delta\sigma}} \left(\frac{H}{L}\right)^{-\frac{(\sigma-1)(1-\delta)}{1-\delta\sigma}} = \frac{S_L^*}{S-S_L^*}, \tag{39}$$

- Notice that given H/L, the BGP researcher allocations, S_L^* and S_H^*, are uniquely determined.

Balanced Growth Path with Knowledge Spillovers

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Suppose that

$$(1-\theta)\frac{\eta_L \eta_H (N_H/N_L)^{(\delta-1)/2}}{\eta_H (N_H/N_L)^{(\delta-1)} + \eta_L} S < \rho,$$

where N_H/N_L is given by (37). Then there exists a unique BGP equilibrium in which the relative technologies are given by (37), and consumption and output grow at the rate

$$g^* = \frac{\eta_L \eta_H (N_H/N_L)^{(\delta-1)/2}}{\eta_H (N_H/N_L)^{(\delta-1)} + \eta_L} S. \tag{40}$$

Transitional Dynamics with Knowledge Spillovers

- Transitional dynamics now more complicated because of the spillovers.
- The dynamic equilibrium path does not always tend to the BGP because of the additional increasing returns to scale:
  - With a high degree of state dependence, when $N_H(0)$ is very high relative to $N_L(0)$, it may no longer be profitable for firms to undertake further R&D directed at labor-augmenting (L-augmenting) technologies.
  - Whether this is so or not depends on a comparison of the degree of state dependence, δ, and the elasticity of substitution, σ.

Summary of Transitional Dynamics

Proposition Suppose that

$$\sigma < 1/\delta.$$

Then, starting with any $N_H(0) > 0$ and $N_L(0) > 0$, there exists a unique equilibrium path. If $N_H(0)/N_L(0) < (N_H/N_L)^*$ as given by (37), then we have $Z_H(t) > 0$ and $Z_L(t) = 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$. $N_H(0)/N_L(0) < (N_H/N_L)^*$, then $Z_H(t) = 0$ and $Z_L(t) > 0$ until $N_H(t)/N_L(t) = (N_H/N_L)^*$.

If

$$\sigma > 1/\delta,$$

then starting with $N_H(0)/N_L(0) > (N_H/N_L)^*$, the economy tends to $N_H(t)/N_L(t) \to \infty$ as $t \to \infty$, and starting with $N_H(0)/N_L(0) < (N_H/N_L)^*$, it tends to $N_H(t)/N_L(t) \to 0$ as $t \to \infty$.

Equilibrium Relative Bias with Knowledge Spillovers I

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Then there is always **weak equilibrium (relative) bias** in the sense that an increase in H/L always induces relatively H-biased technological change.

Proposition Consider the directed technological change model with knowledge spillovers and state dependence in the innovation possibilities frontier. Then if

$$\sigma > 2 - \delta,$$

there is **strong equilibrium (relative) bias** in the sense that an increase in H/L raises the relative marginal product and the relative wage of the H factor compared to the L factor.

Equilibrium Relative Bias with Knowledge Spillovers II

- Intuitively, the additional increasing returns to scale coming from state dependence makes strong bias easier to obtain, because the induced technology effect is stronger.
- Note the elasticity of substitution between skilled and unskilled labor significantly less than 2 may be sufficient to generate strong equilibrium bias.
- How much lower than 2 the elasticity of substitution can be depends on the parameter δ. Unfortunately, this parameter is not easy to measure in practice.

Directed Technological Change without Scale Effects I

- The market size effect and its implications for the direction of technological change are independent of whether or not there are scale effects.
- The market size effect here refers to the relative market sizes of the users of two different types of technologies, not necessarily to the scale of the entire economy.
- The scale effect concerns the impact of the size of the population on the equilibrium growth rate.
- Suppose that equation (32) is modified to

$$\dot{N}_L = \eta_L N_L^{\lambda} S_L \text{ and } \dot{N}_L = \eta_H N_H^{\lambda} S_H, \tag{41}$$

  where $\lambda \in (0, 1]$.
- In the case $\lambda = 1$, we have the knowledge-based R&D formulation of the previous section, but with no state dependence.

Directed Technological Change without Scale Effects II

- When $\lambda < 1$, the extent of spillovers from past research are limited, and this economy will not have steady growth in the absence of population growth.
- Assume that total population, in particular, the population of scientists, grows at the exponential rate n.
- Aggregate output in this economy will grow at the rate:

$$g^* = \frac{n}{1-\lambda}. \tag{42}$$

- Consequently, even with limited knowledge spillovers there will be income per capita growth at the rate $\lambda n / (1-\lambda)$.
- Note that the technology market clearing condition implied by (41) is:

$$\eta_L N_L^{\lambda} \pi_L = \eta_H N_H^{\lambda} \pi_H, \tag{43}$$

Directed Technological Change without Scale Effects III

- Equilibrium relative technology can be derived as

$$\left(\frac{N_H}{N_L}\right)^* = \eta^{\frac{\sigma}{1-\lambda\sigma}} \gamma^{\frac{\varepsilon}{1-\lambda\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-1}{1-\lambda\sigma}} . \tag{44}$$

  Now combining this with (19):

$$\omega^* \equiv \left(\frac{w_H}{w_L}\right)^* = \eta^{\frac{\sigma-1}{1-\lambda\sigma}} \gamma^{\frac{(1-\lambda)\varepsilon}{1-\lambda\sigma}} \left(\frac{H}{L}\right)^{\frac{\sigma-2+\lambda}{1-\lambda\sigma}} . \tag{45}$$

- Thus even without scale effects we obtain exactly the same results as before.

Directed Technological Change without Scale Effects: Summary

Proposition Consider the directed technological change model with no scale effects described above. Then there is always **weak equilibrium (relative) bias**, meaning that an increase in H/L always induces relatively H-biased technological change.

Proposition Consider the directed technological change model with no scale effects described above. If

$$\sigma > 2 - \lambda,$$

then there is **strong equilibrium (relative) bias** in the sense that an increase in H/L raises the relative marginal product and the relative wage of the H factor compared to the L factor.

Endogenous Labor-Augmenting Technological Change I

- Models of directed technological change create a natural reason for technology to be more labor augmenting than capital augmenting.
- Under most circumstances, the resulting equilibrium is not purely labor augmenting and as a result, a BGP fails to exist.
- But in one important special case, the model delivers long-run purely labor augmenting technological changes exactly as in the neoclassical growth model.
- Consider a two-factor model with H corresponding to capital, that is, $H(t) = K(t)$.
- Assume that there is no depreciation of capital.
- Note that in this case the price of the second factor, $K(t)$, is the same as the interest rate, $r(t)$.
- Empirical evidence suggests $\sigma < 1$ and is also economically plausible.

Endogenous Labor-Augmenting Technological Change II

- Recall that when $\sigma < 1$ labor-augmenting technological change corresponds to capital-biased technological change.
- Hence the questions are:
  1. Under what circumstances would the economy generate relatively capital-biased technological change?
  2. When will the equilibrium technology be sufficiently capital biased that it corresponds to Harrod-neutral technological change?

Endogenous Labor-Augmenting Technological Change III

- To answer 1, note that what distinguishes capital from labor is the fact that it accumulates.
- The neoclassical growth model with technological change experiences continuous capital-deepening as $K(t)/L$ increases.
- This implies that technological change should be *more labor-augmenting than capital augmenting.*

Proposition In the baseline model of directed technological change with $H(t) = K(t)$ as capital, if $K(t)/L$ is increasing over time and $\sigma < 1$, then $N_L(t)/N_K(t)$ will also increase over time.

Endogenous Labor-Augmenting Technological Change IV

- But the results are not easy to reconcile with purely-labor augmenting technological change. Suppose that capital accumulates at an exogenous rate, i.e.,

$$\frac{\dot{K}(t)}{K(t)} = s_K > 0. \tag{46}$$

Proposition Consider the baseline model of directed technological change with the knowledge spillovers specification and state dependence. Suppose that $\delta < 1$ and capital accumulates according to (46). Then there exists no BGP.

- Intuitively, even though technological change is more labor augmenting than capital augmenting, there is still capital-augmenting technological change in equilibrium.
- Moreover it can be proved that in any asymptotic equilibrium, $r(t)$ cannot be constant, thus consumption and output growth cannot be constant.

Endogenous Labor-Augmenting Technological Change V

- Special case that justifies the basic structure of the neoclassical growth model: extreme state dependence ($\delta = 1$).
- In this case:

$$\frac{r(t)\,K(t)}{w_L(t)\,L} = \eta^{-1}. \tag{47}$$

- Thus, directed technological change ensures that the share of capital is constant in national income. .
- Recall from (19) that

$$\frac{r(t)}{w_L(t)} = \gamma^{\frac{\varepsilon}{\sigma}} \left(\frac{N_K(t)}{N_L(t)}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{K(t)}{L}\right)^{-\frac{1}{\sigma}},$$

where $\gamma \equiv \gamma_K/\gamma_L$ and γ_K replaces γ_H in the production function (3).

Endogenous Labor-Augmenting Technological Change VI

- Consequently,

$$\frac{r(t) K(t)}{w_L(t) L(t)} = \gamma^{\frac{\varepsilon}{\sigma}} \left(\frac{N_K(t)}{N_L(t)} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{K(t)}{L} \right)^{\frac{\sigma-1}{\sigma}}.$$

- In this case, (47) combined with (46) implies that

$$\frac{\dot{N}_L(t)}{N_L(t)} - \frac{\dot{N}_K(t)}{N_K(t)} = s_K. \tag{48}$$

- Moreover:

$$r(t) = \beta \gamma_K N_K(t) \left[\gamma_L \left(\frac{N_L(t) L}{N_K(t) K(t)} \right)^{\frac{\sigma-1}{\sigma}} + \gamma_L) \right]^{\frac{1}{\sigma-1}}. \tag{49}$$

Endogenous Labor-Augmenting Technological Change VII

- From (22), a constant growth path which consumption grows at a constant rate is only possible if $r(t)$ is constant.
- Equation (48) implies that $(N_L(t)L) / (N_K(t)K(t))$ is constant, thus $N_K(t)$ must also be constant.
- Therefore, equation (48) implies that technological change must be purely labor augmenting.

Summary of Endogenous Labor-Augmenting Technological Change

Proposition Consider the baseline model of directed technological change with the two factors corresponding to labor and capital. Suppose that the innovation possibilities frontier is given by the knowledge spillovers specification and **extreme state dependence**, i.e., $\delta = 1$ and that capital accumulates according to (46). Then there exists a constant growth path allocation in which there is only labor-augmenting technological change, the interest rate is constant and consumption and output grow at constant rates. Moreover, there cannot be any other constant growth path allocations.

Stability

- The constant growth path allocation with purely labor augmenting technological change is globally stable if $\sigma < 1$.
- Intuition:
  - If capital and labor were gross substitutes ($\sigma > 1$), the equilibrium would involve rapid accumulation of capital and capital-augmenting technological change, leading to an asymptotically increasing growth rate of consumption.
  - When capital and labor are gross complements ($\sigma < 1$), capital accumulation would increase the price of labor and profits from labor-augmenting technologies and thus encourage further labor-augmenting technological change.
  - $\sigma < 1$ forces the economy to strive towards a balanced allocation of effective capital and labor units.
  - Since capital accumulates at a constant rate, a balanced allocation implies that the productivity of labor should increase faster, and the economy should converge to an equilibrium path with purely labor-augmenting technological progress.

More General Results (not included in the book)

- Do functional forms and the endogenous growth restrictions matter for these results?
- The answer is no.
- But there are limits to how much generalization is possible unless we look at *absolute* rather than *relative* bias.

More General Results on Relative Bias

Theorem Consider an economy with two-factors and factor-augmenting technologies, i.e., a continuously differentiable production function $F(A_L L, A_Z L)$, and suppose that the costs of producing technologies are $C_Z(A_Z)$ and $C_L(A_L)$. Let σ be the *local elasticity of substitution* between L and Z, defined by $\sigma = -\left.\frac{\partial \ln(Z/L)}{\partial \ln(w_Z/w_L)}\right|_{\frac{A_Z}{A_L}}$, let $\delta = \frac{\partial \ln(C_Z'(A_Z)/C_L'(A_L))}{\partial \ln(A_Z/A_L)}$, and suppose that factor supplies are given by $(\bar{Z}, \bar{L})$. Then:

$$\frac{d \ln(A_Z/A_L)}{d \ln(Z/L)} = \frac{\sigma - 1}{1 + \sigma\delta}$$

and

$$\frac{d \ln\left(w_Z\left(A_L \bar{L}, A_Z \bar{Z}\right) / w_L\left(A_L \bar{L}, A_Z \bar{Z}\right)\right)}{d \ln(Z/L)} = \frac{\sigma - 2 - \delta}{1 + \sigma\delta},$$

so that there is always **(weak) relative equilibrium bias** and there is **strong relative equilibrium bias** if

$$\sigma - 2 - \delta > 0.$$

More General Results on Absolute Bias (Weak Bias)

- Consider absolute bias, where we only look at the price of a particular factor (rather than relative price).

Theorem Suppose that $\Theta \subset \mathbb{R}^K$ and $F(Z, L, \theta)$ is twice continuously differentiable in (Z, θ) for all $\theta \in \Theta$. Let the equilibrium technology at factor supplies $(\bar{Z}, \bar{L})$ be $\theta(\bar{Z}, \bar{L})$ and assume that $d\theta_j(\bar{Z}, \bar{L})/dZ$ exists at $(\bar{Z}, \bar{L})$ for all $j = 1, ..., K$. Then, there is **absolute equilibrium bias** for all $(\bar{Z}, \bar{L}) \in Z \times L$, i.e.,

$$\sum_{j=1}^{K} \frac{\partial w_Z(\bar{Z}, \bar{L}, \theta(\bar{Z}, \bar{L}))}{\partial \theta_j} \frac{d\theta_j(\bar{Z}, \bar{L})}{dZ} \geq 0 \text{ for all } (\bar{Z}, \bar{L}) \in \mathcal{Z} \times \mathcal{L}.$$

Moreover, if $d\theta_j(\bar{Z}, \bar{L})/dZ \neq 0$ for some j, then

$$\sum_{j=1}^{K} \frac{\partial w_Z(\bar{Z}, \bar{L}, \theta(\bar{Z}, \bar{L}))}{\partial \theta_j} \frac{d\theta_j(\bar{Z}, \bar{L})}{dZ} > 0.$$

More General Results on Absolute Bias (Strong Bias)

Theorem Suppose that Θ is a convex subset of $\mathbb{R}^K$, let $\theta(\bar{Z}, \bar{L})$ be the equilibrium technology at factor supplies $(\bar{Z}, \bar{L})$ and assume that $d\theta(\bar{Z}, \bar{L})/dZ$ exists at $(\bar{Z}, \bar{L})$. Then there is **strong absolute bias** at $(\bar{Z}, \bar{L})$ **if and only if** $F(Z, L, \theta)$ is *not jointly concave* in (Z, θ) at $(\bar{Z}, \bar{L}, \theta(\bar{Z}, \bar{L}))$.

- Relatively weak condition for strong bias.
- Equilibrium not in maximum for everybody, but maximum for each (separately)—like a saddle point.
  - This is the usual situation in game-theoretic environments.
- *When there is strong absolute bias, factor curves* **slope up rather than down**.

Choice of Techniques and Direction of Technological Change (not included in the book)

- Based on recent paper by Chad Jones.
- The basic premise of this model is that we have to think of the production function as the outer envelope of the existing ideas and ways of producing output.
  - Similar to old-fashioned activity analysis.
  - In fact, method of analysis similar to Houthakker's classic paper on aggregation of Leontieff production functions with a Pareto distribution of Leontieff coefficients. In this situation, Houthakker showed that the aggregate production function is Cobb-Douglas.
- Under some conditions, the long-run envelope will look like Cobb-Douglas, but the way that this is achieved is by some type of "labor-augmenting technological change"

Model of Choice of Techniques I

- An idea in this economy is a technique for combining capital and labor to produce output.
- Ideas are indexed by a vector (a_i, b_i), which essentially denote the productivity of capital and labor.
- The production technique associated with idea i is

$$\tilde{F}(b_i K, a_i L).$$

- "Local" production function; meaning if we only had this idea, we would be moving along this locus.
- Jones assumes that the local production technology is Leontieff:

$$Y = \tilde{F}(b_i K, a_i L) = \min\{b_i K, a_i L\}.$$

- Thus: an idea or a production technique is parameterized simply by its labor-augmenting and capital-augmenting parameters, a_i and b_i.

The Pareto Distribution I

- In the same way as we needed the strong assumption of $\delta = 1$ in the directed technical change model above, here we need a very specific assumption as well.
- In particular, we need to assume that ideas are distributed "**Pareto**".
- The parameters (a_i, b_i) describing ideas are drawn from *independent Pareto distributions.*
- This implies that:

$$\Pr[a_i \leq a] = 1 - \left(\frac{a}{\gamma_a}\right)^{-\alpha}, \text{ for } a \geq \gamma_a > 0$$
$$\Pr[b_i \leq b] = 1 - \left(\frac{b}{\gamma_b}\right)^{-\beta}, \text{ for } b \geq \gamma_b > 0$$

where $\alpha, \beta > 0$, and $\alpha + \beta > 1$.

- Why Pareto?

The Pareto Distribution II

- What is special about the Pareto distribution is that it has *thick tails.*
  - In particular, if x is Pareto with parameter greater than 1 (i.e., α or β $>$1), then its expectation of conditional on being larger than some $\bar{x}$ is proportional to $\bar{x}$, thus at some loose level making it quite appropriate for "proportional type behavior".
  - If the parameter is less than 1, the mean is infinite.
- Suppose a_i is Pareto as specified above, and $\alpha > 1$.
- Then, its density is $\alpha \left(a_i / \gamma_a \right)^{-\alpha}$ defined over $[\gamma_a, \infty)$. Thus

$$E \left[a_i \mid a_i \geq a \right] = \frac{\int_a^\infty \alpha \left(x/\gamma_a \right)^{1-\alpha} dx}{\Pr \left[a_i \geq a \right]} = \frac{\frac{\alpha}{1-\alpha} \left(\frac{x}{\gamma_a} \right)^{1-\alpha} \Big|_{x=a}^{x=\infty}}{\left(\frac{a}{\gamma_a} \right)^{-\alpha}} = \frac{\alpha}{\alpha - 1} \frac{a}{\gamma_a}.$$

The Pareto Distribution III

- Therefore

$$G(b,a) \equiv \Pr\left[a_i \geq a \text{ and } b_i \geq b\right] = \left(\frac{b}{\gamma_b}\right)^{-\beta}\left(\frac{a}{\gamma_a}\right)^{-\alpha}.$$

- Let

$$Y_i(K,L) \equiv \tilde{F}(b_i K, a_i L)$$

denote output using technique i with capital K and labor L.

- Since $\tilde{F}$ is Leontief, the distribution of Y_i is given by

$$\begin{aligned} H(\tilde{y}) \equiv \Pr\left[Y \geq \tilde{y}\right] &= \Pr\left[a_i L \geq \tilde{y} \text{ and } b_i K \geq \tilde{y}\right] \\ &= G\left(\frac{\tilde{y}}{K}, \frac{\tilde{y}}{L}\right) \\ &= \gamma K^{\beta} L^{\alpha} \tilde{y}^{-(\alpha+\beta)}, \end{aligned}$$

where $\gamma \equiv \gamma_a^{\alpha}\gamma_b^{\beta}$. That is, the distribution of Y_i is itself Pareto, with $\tilde{y} \geq \min\{\gamma_b K, \gamma_a L\}$.

The "Equilibrium" Production Function I

- Now let's look at the "global production function," which describes the maximum amount of output that can be produced using any combination of existing production techniques is a function of inputs.
- This will generally involve the use of multiple techniques as in activity analysis.
- Jones simplifies the theoretical analysis by allowing only a single technique to be used at each point in time.
    - How reasonable is this?
- Let N denote the total number of production techniques that are available, which means that there have been N ideas discovered so far.
- Let's assume that these N ideas are drawn independently.
- Then, the global production function is defined as $Y = F(K, L; N)$ such that

$$F(K, L; N) \equiv \max_{i=1,\ldots,N} \tilde{F}(b_i K, a_i L)$$

The "Equilibrium" Production Function II

- Since the N draws are independent, the distribution of the global production function satisfies

$$\begin{aligned} \Pr\left[Y \leq \tilde{y}\right] &= \left(1 - H(\tilde{y})\right)^N . \\ &= \left(1 - \gamma K^\beta L^\alpha \tilde{y}^{-(\alpha+\beta)}\right)^N . \end{aligned}$$

- As the number of ideas N gets large, this probability for any given level of $\tilde{y}$ goes to zero, i.e., output is growing without a limit as in most growth models.
- What we wish to derive is the distribution of output at the limit, so a version of the *Central Limit Theorem* needs to be used.

The "Equilibrium" Production Function III

- It can be shown that output "grows" at the rate $\left(\gamma N K^{\beta} L^{\alpha}\right)^{\frac{1}{\alpha+\beta}}$, in the sense that

$$\Pr\left[Y \leq \left(\gamma N K^{\beta} L^{\alpha}\right)^{\frac{1}{\alpha+\beta}} \tilde{y}\right] = \left(1 - \gamma K^{\beta} L^{\alpha} (z_N \tilde{y})^{-(\alpha+\beta)}\right)^{N}$$
$$= \left(1 - \frac{\tilde{y}^{-(\alpha+\beta)}}{N}\right)^{N}.$$

- Now using the result that

$$\lim_{N\to\infty} (1 - x/N)^{N} = exp(-x),$$

we have

$$\lim_{N\to\infty} \Pr\left[Y \leq \left(\gamma N K^{\beta} L^{\alpha}\right)^{\frac{1}{\alpha+\beta}} \tilde{y}\right] = \exp(-\tilde{y}^{-(\alpha+\beta)})$$

for $\tilde{y} > 0$.

- This distribution is the **Fréchet distribution.**

Long-Run Envelope Production Function I

- Therefore, the long-run distribution of output will take the form

$$\frac{Y}{(\gamma N K^{\beta} L^{\alpha})^{1/\alpha+\beta}} \sim \text{Fréchet}(\alpha+\beta),$$

  so that the global production function, appropriately normalized, converges asymptotically to a Fréchet distribution.

- This means that as N gets large, the production function behaves like

$$Y \approx \varepsilon \left(\gamma N K^{\beta} L^{\alpha}\right)^{\frac{1}{\alpha+\beta}}$$

  where ε is a random variable drawn from a Fréchet distribution with shape parameter $\alpha+\beta$ and a scale parameter equal to unity.

- This model therefore implies that as the number of ideas goes to infinity, the global production function becomes Cobb-Douglas, despite the fact that the local production function is Leontieff.

Long-Run Envelope Production Function II

- Therefore, the long-run (endogenous technology) production function is Cobb-Douglas, but the actual production function has a lower elasticity of substitution.
- One issue of interpretation here is what "the economy" is in this model?
  - It could be an industry, a region, a country? So should we expect all production functions to be Cobb-Douglas?

Conclusions I

- The bias of technological change is potentially important for the distributional consequences of the introduction of new technologies (i.e., who will be the losers and winners?); important for political economy of growth.
- Models of directed technological change enable us to investigate a range of new questions:
    - the sources of skill-biased technological change over the past 100 years,
    - the causes of acceleration in skill-biased technological change during more recent decades,
    - the causes of unskilled-biased technological developments during the 19th century,
    - the relationship between labor market institutions and the types of technologies that are developed and adopted,
    - why technological change in neoclassical-type models may be largely labor-augmenting.

Conclusions II

- The implications of the class of models studied for the empirical questions mentioned above stem from the *weak equilibrium bias* and *strong equilibrium bias* results.
- Technology should not be thought of as a black box. Profit incentives will play a major role in both the aggregate rate of technological progress and also in the biases of the technologies.