

CARESS Working Paper #92-22
Acceptance Games and Protocols*

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Abstract: A group of agents with private information must decide whether to stay with the status quo or move to some alternative. We consider the incomplete information *acceptance* game, where each agent must accept or reject, depending on the signal he has observed, and unanimous acceptance is required to implement change. Such an acceptance game underlies Holmström and Myerson's (1983) notion of *durability*. Agents have *conflicting ordinal interpretation of signals* if, whenever a signal represents good news about the alternative for the agent who observes it, it is bad news for all other agents. This occurs in a two agent trading problem with affiliated values. Under this assumption, there is a Pareto-dominant pure strategy Nash equilibrium of the acceptance game where each agent's strategy is to accept or reject depending on the signal he has observed. This equilibrium coincides with the set of signals where agents accept under a public acceptance *protocol* [following Sebenius and Geanakoplos (1983)] when agents publicly discuss their willingness to accept the trade, and converge to common knowledge of acceptance. I show this using two extensions of results about *games with strategic complementarities* [Milgrom and Roberts (1990)]. First, it is sufficient for existing results that complementary properties hold *locally*, i.e. on some subset of strategies containing all iteratively undominated strategies. Second, in games with incomplete information, a *single crossing property* [Milgrom and Shannon (1991)] on agents' actions and types implies that all undominated strategies are monotone strategies, and therefore complementarity need only hold on monotone strategies.

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Section 1: Introduction

Two agents are considering whether to trade an object. Both agents must agree in order for the trade to take place. Each agent has some information, and each agent's valuation depends on both agents' information signals. Suppose that there is some prevailing price for the object, and the agents have to decide whether to accept or reject *at that price* (there is no negotiation about price).

More generally, I will be concerned with situations where some set of agents must decide whether to stay with the status quo or move to some fixed alternative. All agents must agree in order for the change to take place. All that will matter is the difference between their expected utilities under the status quo and the alternative, conditional on the signals observed. It is assumed that agents have *conflicting interpretation of signals*: whenever a signal represents good news about the alternative for the agent who observes it, it is bad news for all the other agents. This occurs in a two agent trading problem with affiliated values.

In section 5, I consider an *acceptance game*. This is a game with incomplete information. Each agent must choose to accept or reject the alternative. If any agent rejects, the status quo remains. Such an acceptance game underlies Holmström and Myerson's (1983) concept of durability. In a refined version of that game, it is shown (theorem 3) that, for each agent, there is a largest and smallest iteratively undominated strategy in this game, and the profile of such largest and smallest strategies are pure Nash equilibria. A theorem of this form was proved by Milgrom and Roberts (1990) for games with strategic complementarities¹, although we will see below that this game only has *local ordinal strategic complementarities*.

An alternative approach is to consider the mechanism design problem, where a mechanism specifies a probability of moving to the alternative, as a function of signals that agents report. By the revelation principle, we can restrict attention to truth-telling mechanisms, which are equivalent to correlated equilibria of the acceptance game. But since only undominated strategies are played with positive probability in correlated equilibria, the largest and smallest Nash profiles are also largest and smallest correlated equilibria.

But both Nash and correlated equilibria rely on agents making choices simultaneously and in ignorance of other agents' choices. In many contexts, this is not a natural assumption to make. We imagine most situations of the type described being resolved by some discussion, during which information is implicitly or explicitly conveyed, at the end of which it is common knowledge that agents wish to accept the change. Notice that typically information must be conveyed: *ex ante*, common

1. Topkis (1979), Bulow, Geanakoplos and Klemperer (1985), Vives (1990), Milgrom and Roberts (1990), Milgrom and Shannon (1991).

knowledge that agents wanted to accept would require that both agents gained, conditional on every realization of the signals.

Sebenius and Geanakoplos (1983) proposed one stylized model of the process by which agreement to change is reached. Suppose a change is proposed, and "agent 1 is tentatively willing to take it. Knowing this, agent 2 may tentatively accept or decline. Agent 1 then has the same option if agent 2 has accepted, and the dialogue continues until both are finally satisfied or a rejection is encountered. These assumptions correspond to a situation where both agents discuss the change before making it and not the situation where a firm offer is made and simply accepted or refused".

A generalized version of such a *protocol* is proposed in section 6. It is shown (theorem 5) that under the conflicting ordinal interpretation of signals assumption, the alternative is never refused (and acceptance thus occurs) only on those profiles of signals where acceptance occurs in the largest Nash equilibrium of the acceptance game.

This result is related to learning results in (complete information) games with strategic complementarities [Milgrom and Roberts (1990, 1991), Krishna (1992)]. But how is the conflicting ordinal interpretation of signals property related to strategic complementarities? In the acceptance game, 1 will accept the alternative when it is *most* valuable to him. With conflicting interpretation of signals, this means that agent 1 will accept on a set of signals where the alternative is *least* valuable to agent 2, and vice versa. But now, the more that agent 1 is prepared to accept (the more signals he accepts on), the less bad the expected outcome is for agent 2, and the more agent 2 is prepared to accept. There appears to be a monotonic best response property holding.

Unfortunately, we cannot simply apply results about strategic complementarities in order to get our results. One problem is that if 1 rejects the alternative always, then any acceptance strategy by 2 is a best response (it doesn't make any difference). This implies that *all* actions are iteratively (strictly) undominated and rationalizable in any acceptance game - in contrast to the Milgrom and Roberts (1990) result. As noted above, it is necessary to first refine the acceptance game to ensure a comparable result.

But there is a much more serious problem. The natural ordering on agents' strategies in the acceptance game is set inclusion. But it is typically not the case in acceptance games that the best response function is monotonic. The argument above relied on the best response function being monotonic *locally*, i.e. at all the points that matter. If we restrict attention to those strategies where each agent only accepts on the signals which are best for him, the best response functions are indeed monotonic.

In section 3, I review results from Milgrom and Shannon (1991) about games with cardinal and ordinal complementarities, and show that they continue to hold even if the complementarity properties

hold only on some subset of strategies, as long as that subset contains all iteratively undominated strategies.

In section 4, I apply that result to games with incomplete information. A *single crossing property* in agents' actions and types (lemma 3) guarantees that all strategies are monotonic (actions increasing in types). Thus it is sufficient that agents' payoffs satisfy *ordinal complementarity* conditions on monotonic strategies (and not necessarily on other strategies) to ensure that the best response function is locally monotonic (theorem 2). I also show in section 4 that while cardinal complementarities in a class of complete information games translate into cardinal complementarities in the corresponding incomplete information game [Vives (1990)], the same is not true of ordinal complementarities.

These results of section 3 and 4 are all necessary for the acceptance game with conflicting ordinal interpretation of signals studied in section 5. These games do not satisfy cardinal complementarity conditions, and satisfy ordinal complementarity conditions only on monotonic strategies. But all undominated strategies are monotonic.

The results of section 5 and section 6 on acceptance protocols were described above. Section 7 considers the special case where the alternative is a feasible trade, so there are no gains from trade. It is shown that not only can there be no acceptance of trade in the static acceptance game or the dynamic acceptance protocol [Sebenius and Geanakoplos (1983)], but with conflicting ordinal interpretation of signals, rejection of the trade is the unique iteratively undominated (and thus the unique *rationalizable*) action for all agents in the refined acceptance game. Not only is there no common knowledge trade, there is also no rationalizable trade.

Section 2: Example

The example in this section illustrates each of the key ideas in the paper. Two agents must agree whether to trade an object. Agent 1 has observed a signal with realizations U, M or D; agent 2 has observed a signal with realizations L, C, or R. Their different valuations of the object, conditional on the signals which both of them have observed, are given in figure 1(a). Notice that, while agents have different values for the object, they agree ordinally on which signal makes the object most valuable. Agent 1 always values the object more, so he is the potential buyer, 2 the potential seller. Suppose that the prevailing price for the object is 4, and the agents have to decide whether to accept or reject *at that price* (there is no negotiation about price). Then the value of the trade to the agents is as shown in table 1(b).

Now we can order the signals of each agent [$U > M > D$ and $L > C > R$] so that each agent strictly prefers a higher realization of his signal and a lower realization of the other agent's signal. This is the conflicting ordinal interpretation of signals property.

Now consider the static acceptance game, where each agent decides whether to accept or reject the alternative, depending on the signal he has observed. His actions can therefore be thought of as subsets of the set of signals. The alternative is chosen only on those combinations of signals where both agents accept. Each agent's ex ante payoff is equal to the probability of acceptance times the expected value of the alternative. Suppose that there is a $1/9$ probability of each pair of signals. Then 1's expected payoff if 1 chooses action $\{U,M\}$ and 2 chooses action $\{L,C\}$ is $[1+3-1+2]/9 = 5/9$. Figure 1(c) gives the pure strategy best response function for both agents. Note that anything is a best response if the other agent always rejects the trade.

There are three pure strategy Nash equilibria: (\emptyset, \emptyset) , $(\{U\}, \{L\})$ and $(\{U,M\}, \{L,C\})$. There is also a class of mixed strategy Nash equilibria with 1 playing \emptyset with probability $1-2\alpha$, $\{U\}$ with probability α and $\{U,M\}$ with probability α , and 2 playing \emptyset with probability $1-2\beta$, $\{L\}$ with probability β and $\{L,C\}$ with probability β , for any $0 \leq \alpha, \beta \leq 0.5$. The pure strategy Nash equilibria are ranked by set inclusion, which in turn implies that they are Pareto-ranked: $(\{U,M\}, \{L,C\})$ is the largest and Pareto-dominant Nash equilibrium.

What would happen in this example with the protocol described in the introduction? Suppose 1 observes signal D and 2 observes L. First, 1 is asked if he is prepared to accept the alternative (on the assumption that 2 will accept). 1 is prepared to accept the alternative. This doesn't reveal anything to 2 however, since 1 would always be willing to accept the alternative, whatever signal he had observed. Now 2 is asked if he is prepared to accept. He is, given his signal L. However, this *does* reveal something to agent 1: if 2 had observed R, he would not be prepared to accept. Now 1 knows that 2 observed L or C. Now when 1 is asked again if he is prepared to accept, he says no, since he has observed D, and knowing that 2 observed L or C, it is not worthwhile accepting. It can be verified that the alternative will eventually be rejected if 1 observes D or if 2 observes R, and never rejected otherwise.

The set of signals where rejection does not occur ($\{U,M\} \times \{L,C\}$) exactly corresponds to the largest pure Nash equilibria, as predicted by theorem 3. Let us look at the best response function [figure 1(c)] to see in detail how this behavior is related to games with strategic complementarities and monotonic best response functions.

Notice first that since any strategy is a best response to never accepting, this acceptance game must be refined before we can even hope for monotonic best responses.

But there is a much more serious problem. The natural ordering on agents' strategies in the acceptance game is set inclusion. Yet the best response correspondence (see figure 1(c)) is *not* monotonic, even ignoring the problem at \emptyset . The best response function is monotonic *locally*, i.e. at all the points that matter. The argument in the introduction was that if conflicting ordinal interpretation of signals holds, agents will always accept on their best signals. Thus 1's best responses will be $\{U\}$, $\{U,M\}$ or $\{U,M,D\}$, while 2's best responses will be $\{L\}$, $\{L,C\}$ and $\{L,C,R\}$. The best response functions are indeed monotonic when restricted to those strategies. Sections 3 and 4 show why this is sufficient for strategic complementarity results.

It is worth noting that there does exist another ordering on strategies (i.e. not set inclusion) under which the best response is globally monotonic (except at \emptyset): consider the complete orderings $\{M,D\} \sim \{M\} \sim \{D\} > \{U,M,D\} > \{U,M\} \sim \{U,D\} > \{U\}$ and $\{L,C,R\} \sim \{L,R\} \sim \{C,R\} \sim \{R\} > \{L,C\} \sim \{C\} > \{L\}$. These orderings are constructed by taking all those strategies which are never best responses (which were excluded from the local complementarity conditions discussed in the introduction), and placing them within the set inclusion ordering on strategies which *are* best responses in such a way that monotonicity is maintained.

Section 3: Local Strategic Complementarities

In the differentiable setting of Bulow, Geanakoplos and Klemperer (1985), actions of agents were said to be strategic complements if the marginal profitability of an agent's action increases in the actions of other agents. Vives (1990) and Milgrom and Roberts (1991) built on the work of Topkis (1979) to give a notion of strategic complementarity with no differentiability assumptions. Milgrom and Shannon [MS] (1991) gave properties which depend only on ordinal properties of payoffs. The following section summarizes notions of complementarities in that paper, which can be consulted for a more detailed discussion.

Let X be a partially ordered set, with antisymmetric, transitive and reflexive order relation \geq . X is a lattice if, for every pair $\{x, y\} \subset X$, there exists a least upper bound ($x \vee y$) and a greatest lower bound ($x \wedge y$) in X .

Suppose $f : X \times Y \rightarrow \mathbb{R}$, $X_0 \subset X$ is a lattice and $Y_0 \subset Y$ is a partially ordered set. Then f is *supermodular* in x on (X_0, Y_0) if $f(x,y) - f(x \wedge x', y) \leq f(x \vee x', y) - f(x', y)$ for all $x, x' \in X_0$, $y \in Y_0$. Function f has *increasing differences* in (x,y) on (X_0, Y_0) if $x \geq x'$ and $y \geq y'$ imply $f(x,y) - f(x',y) \leq f(x,y') - f(x',y')$, for all $x, x' \in X_0$ and $y, y' \in Y_0$. Function f satisfies *cardinal complementarity* on (X_0, Y_0) if these two properties hold.

In the many comparative static applications of supermodularity, it is only ordinal properties of f that matter. Analogous ordinal versions of the two properties above are as follows. Function f is *quasisupermodular* in x on (X_0, Y_0) if $f(x, y) \geq f(x \wedge x', y)$ implies $f(x \vee x', y) \geq f(x', y)$ and $f(x, y) > f(x \wedge x', y)$ implies $f(x \vee x', y) > f(x', y)$ for all $x, x' \in X_0, y \in Y_0$. Function f has the *single crossing property [SCP]* in (x, y) on (X_0, Y_0) if $f(x, y') \geq f(x', y')$ implies $f(x, y) \geq f(x', y)$ and $f(x, y') > f(x', y')$ implies $f(x, y) > f(x', y)$, for all $x, x' \in X_0$ and $y, y' \in Y_0$, with $x > x'$ and $y > y'$. Function f satisfies *ordinal complementarity* on (X_0, Y_0) if these two properties hold.

A stronger version of SCP, also from MS, will be required later. Function f has the *strong single crossing property* in (x, y) on (X_0, Y_0) if $x > x', y > y'$, and $f(x, y') \geq f(x', y')$ implies $f(x, y) > f(x', y)$, for all $x, x' \in X_0$ and $y, y' \in Y_0$.

We now describe a finite game, written in a form so that notation can be easily extended to games of incomplete information in the next section. But the results which follow could easily be extended to arbitrary lattices.

There are I players, $\{1, \dots, I\}$. Each agent i has a finite set of possible basic actions A_i . His strategy in a game will be multidimensional: his strategy will be a vector of actions $s_i: T_i \rightarrow A_i$, where T_i is some finite set. We will call T_i agent i 's set of "types". Later this representation of strategies can be used for games of incomplete information. For now, however, we are considering payoffs as an arbitrary function of strategies. Let S_i be the set of strategies (functions from T_i to A_i) of agent i , and $S = S_1 \times \dots \times S_I$. Now agent i 's payoffs are given by a function $g_i: S \rightarrow \mathbb{R}$.

There is an asymmetric, complete, transitive order \geq on each agent's types and another asymmetric, complete, transitive order \geq on each agent's actions. These orderings will not be subscripted (by agent) in order to simplify notation. There is a natural ordering on the S_i : $s_i \geq s_i'$ if $s_i(t_i) \geq s_i'(t_i)$, for all $t_i \in T_i$. This relation is asymmetric, reflexive and transitive, but not complete unless T_i is a singleton.

We will be concerned with *local* restrictions on payoffs g : local in the sense that restrictions are placed only on subsets of strategies. In particular, we will be interested in imposing restrictions on subsets of strategies which contain all iteratively undominated strategies. Here and throughout the paper, "domination" refers to *strict* domination in *pure* strategies. Formally, R is a product set of agent strategies if $R = (R_1, \dots, R_I)$, each $R_i \subset S_i$. For any such product set, let $U_i[R] = \{ s_i \in R_i \mid g_i(s_i, s_{-i}) \geq g_i(s_i', s_{-i}) \forall s_i' \in R_i, \text{ for some } s_{-i} \in R_{-i} \}$. $U[R]$ is the collection of $U_i[R]$ for each i . Define the sequence U_i^k inductively by $U_i^{k+1} = U_i[U_i^k]$ and $U_i^0 = S_i$. A strategy s_i is iteratively undominated if it is contained in U_i^k , for all k . A product set of agent strategies R is said to be iteratively undominated if $U_i^k \subset R_i$, for all i, k . A product set of agent strategies R is said to be a lattice if each R_i is a lattice.

Theorem 1 *Suppose R is an iteratively undominated product set of agent strategies, and also a lattice. Suppose g_i satisfies the ordinal complementarity conditions in (s_i, s_{-i}) on R , for each i . Then, for each i , there exist largest and smallest iteratively undominated strategies s_i^* and s_i^* in R_i and the strategy profiles s^* and s_* , are pure strategy Nash equilibria.*

The theorem is proved in MS for the case where $R = S$. But since the set of iteratively undominated strategies and Nash equilibria are not changed by deletion of dominated strategies, the *local* version (where $R \neq S$) of the theorem must also be true. Of course, typically this extension will not help a lot: but in the class of games of incomplete information in the next section, the ordinal complementarity conditions hold after one round of deletion of dominated strategies, but not before.

This result could be extended to arbitrary complete lattices², with continuity restrictions on f as in MS.

Section 4: Games of Incomplete Information

Suppose there exist $p: T \rightarrow \mathbb{R}_{++}$ and, for each i , $u_i: A \times T \rightarrow \mathbb{R}$, such that:-

$$g_i(s) = \sum_{t \in T} p(t) u_i(s(t), t)$$

Then we have a game of incomplete information, where each agent chooses an action a_i as a function of his type t_i . Vives (1990) showed that if the underlying complete information games $u_i(\cdot, t)$ satisfy the cardinal complementarity conditions, then, for any p , the corresponding game with incomplete information satisfies cardinal complementarity conditions under the natural ordering of strategies. Milgrom and Roberts (1990) showed how this result underlies Hendricks and Kovenock's (1989) model of oil drilling.

Lemma 1 *Suppose $u_i(\cdot, t)$ has increasing differences in (a_i, a_{-i}) on A , for each fixed t . Then g_i has increasing differences in (s_i, s_{-i}) on S .*

2. A lattice X is complete if the least upper bound and greatest lower bound of any subset $X_0 \subset X$ are contained in X . All finite lattices are complete.

Proof $u_i(\cdot, t)$ has increasing differences in (a_i, a_{-i}) implies that for all $t \in T$, $s \geq s'$,

$$\begin{aligned} u_i[(s_i(t_i), s_{-i}(t_{-i})), (t_i, t_{-i})] - u_i[(s'_i(t_i), s_{-i}(t_{-i})), (t_i, t_{-i})] \geq \\ u_i[(s_i(t_i), s'_{-i}(t_{-i})), (t_i, t_{-i})] - u_i[(s'_i(t_i), s'_{-i}(t_{-i})), (t_i, t_{-i})] \end{aligned}$$

Summing across t gives:-

$$\begin{aligned} \sum_{t \in T} p(t) \{ u_i[(s_i(t_i), s_{-i}(t_{-i})), (t_i, t_{-i})] - u_i[(s'_i(t_i), s_{-i}(t_{-i})), (t_i, t_{-i})] \} \geq \\ \sum_{t \in T} p(t) \{ u_i[(s_i(t_i), s'_{-i}(t_{-i})), (t_i, t_{-i})] - u_i[(s'_i(t_i), s'_{-i}(t_{-i})), (t_i, t_{-i})] \} \end{aligned}$$

which can be re-written as:-

$$g_i(s_i, s_{-i}) - g_i(s'_i, s_{-i}) \geq g_i(s_i, s'_{-i}) - g_i(s'_i, s'_{-i})$$

Because a complete order was assumed on A_i , the complete information games $u_i(\cdot, t)$, for fixed t , are always supermodular (and thus quasisupermodular). The incomplete information game is always supermodular.

Lemma 2 g_i is supermodular

Proof

$$\begin{aligned} g_i(s_i, s_{-i}) - g_i(s_i \wedge s'_i, s_{-i}) &= \\ \sum_{t \in T} p(t) \{ u_i[(s_i(t_i), s_{-i}(t_{-i})), (t_i, t_{-i})] - u_i[(\min(s_i(t_i), s'_i(t_i)), s_{-i}(t_{-i})), (t_i, t_{-i})] \} &= \\ \sum_{\{t_i | s_i(t_i) \geq s'_i(t_i)\}} \sum_{t_{-i} \in T_{-i}} u_i[(s_i(t_i), s_{-i}(t_{-i})), (t_i, t_{-i})] - u_i[(s'_i(t_i), s_{-i}(t_{-i})), (t_i, t_{-i})] &= \\ \sum_{t \in T} p(t) \{ u_i[(\max(s_i(t_i), s'_i(t_i)), s_{-i}(t_{-i})), (t_i, t_{-i})] - u_i[(s'_i(t_i), s_{-i}(t_{-i})), (t_i, t_{-i})] \} &= \\ g_i(s_i \vee s'_i, s_{-i}) - g_i(s'_i, s_{-i}) & \end{aligned}$$

In the acceptance game to be studied in the next section (and in the example of section 2), the complete information payoffs $u_i(\cdot, t)$ and the incomplete information payoffs g_i do *not* satisfy the cardinal

complementarity conditions, globally or locally as discussed in the last section. The $u_i(\cdot, t)$ do have the single crossing property. Unfortunately, it is *not* the case that if $u_i(\cdot, t)$ has the single crossing property in (a_i, a_i) for each t , then g_i satisfies the single crossing property. Vives' (1990) result extending cardinal complementarity to games of incomplete information does not apply to ordinal complementarity.

We could directly impose the single crossing property globally on the incomplete information game. But this too is too strong for the acceptance game we will consider. Instead, the single crossing property will be required to hold only for monotone strategies, and conditions will be given for all undominated actions to be monotone.

Define the set of *monotone* strategies of agent i by:-

$$M_i = \{s_i \in S_i \mid s_i(t_i) \geq s_i(t'_i), \text{ for all } t_i \geq t'_i\}$$

Now say that game g is additively separable if there exist, for each agent, *interim* utility functions $w_i: A_i \times T_i \times S_{-i} \rightarrow \mathbb{R}_+$ such that

$$g_i(s) = \sum_{t_i \in T_i} w_i(s_i(t_i), t_i, s_{-i})$$

If g is a game of incomplete information, as described above, then g is additively separable with agent i 's interim utility function w_i given by:-

$$w_i(a_i, t_i, s_{-i}) = \sum_{t_{-i} \in T_{-i}} p(t_i, t_{-i}) u_i[(a_i, s_{-i}(t_{-i})), (t_i, t_{-i})]$$

With this notation, note that the set of (once) undominated strategies of i can be given by:-

$$U_i^1 = \{s_i \in S_i \mid w_i(s_i(t_i), t_i, s_{-i}) \geq w_i(a_i, t_i, s_{-i}), \text{ for all } a_i \in A_i, t_i \in T_i, \text{ for some } s_{-i} \in S_{-i}\}$$

Lemma 3 Suppose g is additively separable and each interim utility function w_i has the strong single crossing property [SSCP] in (a_i, t_i) , for fixed $s_{-i} \in S_{-i}$. Then all undominated strategies of player i are monotone [$U_i^1 \subset M_i, \forall i$].

Proof

Suppose (1) $s_i \in U_i^1$ and (2) $s_i \notin M_i$.

Then by (1) $\exists s_{-i}$ s.t. $w_i(s_i(t_i''), t_i'', s_{-i}) \geq w_i(a_i, t_i'', s_{-i})$, for all $a_i \in A_i$, $t_i'' \in T_i$.

By (2) and asymmetry of \succeq , $\exists t_i > t_i'$ s.t. $s_i(t_i') > s_i(t_i)$

But now by SSCP, $w_i(s_i(t_i'), t_i', s_{-i}) \geq w_i(s_i(t_i), t_i', s_{-i}) \rightarrow$

$w_i(s_i(t_i'), t_i, s_{-i}) > w_i(s_i(t_i), t_i, s_{-i})$, contradicting [1].

Milgrom and Shannon (1991) showed that a single crossing property on actions and types implies a monotone best response function in the auction model of Milgrom and Weber (1982).

Theorem 2 Suppose g is additively separable, each interim utility function w_i has the strong single crossing property in (a_i, t_i) , for each fixed $s_{-i} \in S_{-i}$, and g_i has the single crossing property in (s_i, s_{-i}) for monotone strategies ($s \in M$). Then, for each i , there exist largest and smallest iteratively undominated strategies s_i^* and s_{i*} . These strategies are monotone, as are all undominated strategies. The profiles s^* and s_* are pure strategy Nash equilibria.

Proof By lemma 3, all undominated strategies are monotone. So theorem 1 implies theorem 2.

The conditions of theorem 2, taken together, are typically weaker than requiring that each g_i satisfies ordinal complementarity conditions (globally). The acceptance game of the next section is an example where this is the case.

Section 5: The Acceptance Game

In this section, a problem of accepting a change from the status quo to some given alternative will be modelled as an unanimous acceptance game, a particular kind of game with imperfect information. The alternative occurs only if all agents agree to the change. Such games have appeared in numerous guises in the literature. Holmström and Myerson (1983) included such an acceptance game as part of the definition of *durability*.

Agent i 's valuation of the change (to the alternative from the status quo) is a real valued function of types: $v_i: T \rightarrow \mathbb{R}$. Given each of his possible types he must either say yes or no to the change. Thus

the action set has two elements, $A_i = \{Y, N\}$. We endow this two element set with the order $Y > N$. Now consider the following complete information game:-

$$u_i(a, t) = \begin{cases} v_i(t), & \text{if } a_j = Y, \text{ for all } j. \\ 0, & \text{otherwise.} \end{cases}$$

This gives us the incomplete information game g with

$$g_i(s) = \sum_{t \in T} p(t) u_i(s(t), t) = \sum_{t \in \{t' \in T | s_j(t') = Y, \text{ for all } j\}} p(t) v_i(t)$$

This game will be referred to as the *acceptance* game for v . Notice that the assumption is that agents must decide to accept or reject the alternative once and for all. In general, the outcome may be ex post inefficient and agents may have incentives to renegotiate (as in Holmström and Myerson (1983); see also Lagunoff (1992)). It is implicitly assumed that agents accept or reject, without taking into account the possibility of re-negotiation.

In this game, any strategy is a best response to any strategy profile where some agent rejects always [$s_j(t_j) = N$, for all $t_j \in T_j$]. So it is useful to consider a natural refinement of the acceptance game. Suppose that whenever some agent's strategy is to always reject, other agents, instead of simply being indifferent between all strategies, form a pessimistic conjecture about other agents' strategies. In particular, let t_j^* be the maximal element of T_j , for each j . Later assumptions will guarantee that this is the most pessimistic conjecture. Now define the *refined acceptance* game as follows:-

$$g_i^*(s) = \begin{cases} g_i(s), & \text{if } s_j(t_j) = Y, \text{ for some } t_j, \text{ for all } j. \\ \sum_{t_i \in \{t'_i \in T_i | s_i(t'_i) = Y\}} p(t_i, t_{-i}^*) v_i(t_i, t_{-i}^*), & \text{otherwise.} \end{cases}$$

This particular refinement allows us to see the connections with games with strategic complementarities. Theorem 4 below shows that the set of relevant equilibria of the acceptance and refined acceptance games are identical.

We will be concerned with a situation where a signal which is good news about the alternative for agent i is bad news for all other agents. This would be the case if there were two agents trading a

good for which they have common values. Define agent i 's payoff to acceptance at t_i if other agents accept on T_{-i}' , where $T_{-i}' = \{T_j\}_{j \neq i}$, each T_j' a subset of T_j , as:-

$$v_i^*(t_i, T_{-i}') = \sum_{t_{-i} \in T_{-i}'} p(t_i, t_{-i}) v_i(t_i, t_{-i}).$$

It is useful to identify each pure strategy of agent i , s_i , with the set of types where acceptance takes place, i.e. $T_i' = \{t_i \in T_i \mid s_i(t_i) = Y\}$. In particular, I will sometimes write \emptyset for the strategy s_i with $s_i(t_i) = N$, for all $t_i \in T_i$. The ordering on strategies is thus equivalent to set inclusion in this representation. T_i' is a monotone strategy of agent i if $T_i' = \emptyset$ or $\{t_i' \in T_i \mid t_i' \geq t_i, \text{ for some } t_i \in T_i\}$. Now agent i 's interim utility function is:-

$$w_i(Y, t_i, T_{-i}') = \begin{cases} v_i^*(t_i, T_{-i}'), & \text{if } T_j \neq \emptyset, \text{ for all } j. \\ p(t_i, t_{-i}^*) v_i(t_i, t_{-i}^*), & \text{otherwise.} \end{cases}$$

$$w_i(N, t_i, T_{-i}') = 0.$$

Lemma 4 In the refined acceptance game, interim utility function w_i has the strict single crossing property in (a_i, t_i) for fixed s_{-i} if

$$t_i > t_i' \text{ and } v_i^*(t_i', T_{-i}') \geq 0 \rightarrow v_i^*(t_i, T_{-i}') > 0, \text{ for all } T_{-i}', T_j' \neq \emptyset, \text{ for all } j. \quad [*]$$

Proof The strict single crossing property requires that $a_i > a_i'$, $t_i > t_i'$ and $w_i(a_i, t_i', s_{-i}) \geq w_i(a_i', t_i', s_{-i})$ imply $w_i(a_i, t_i, s_{-i}) > w_i(a_i', t_i, s_{-i})$. With $A_i = \{Y, N\}$, $Y > N$, we must consider two cases. If $T_j \neq \emptyset$, for all j , then the above condition is implied by condition [*] above. If not, we require that (a) $t_i > t_i'$ and $w_i(Y, t_i', s_{-i}) \geq w_i(N, t_i', s_{-i})$ imply $w_i(Y, t_i, s_{-i}) > w_i(N, t_i, s_{-i})$. Thus we require [1] $t_i > t_i'$, $v_i^*(t_i', T_{-i}') \geq 0$, $T_j' \neq \emptyset$ for all j , implies $v_i^*(t_i, T_{-i}') > 0$ and [2] $t_i > t_i'$ and $v_i(t_i', t_{-i}^*) \geq 0$ implies $v_i(t_i, t_{-i}^*) > 0$. But [*] implies [1] and $p(t_i, t_{-i}^*) v_i(t_i, t_{-i}^*) = v_i^*(t_i, T_{-i}')$, if each T_j is the singleton set $\{t_j\}$, so [1] implies [2].

Now agent i 's payoff to acceptance on some interval of types, given that other agents use monotonic strategies is

$$v_i^*(\underline{t}_i, \bar{t}_i, \underline{t}_{-i}') = \sum_{\bar{t}_i \geq t_i \geq \underline{t}_i} \sum_{t_{-i}' \geq \underline{t}_{-i}'} p(t_i, t_{-i}') v_i(t_i, t_{-i}')$$

Lemma 5 In the refined acceptance game, g_i has the single crossing property in (s_i, s_{-i}) on monotonic strategies if

$$t_{-i} \geq t'_{-i} \text{ and } v_i^+(t_i, \bar{t}_i, t'_{-i}) \geq [>] 0 \rightarrow v_i^+(t_i, \bar{t}_i, t_{-i}) \geq [>] 0 \quad [**]$$

Proof Single crossing property requires that $s > s'$ and $g_i^*(s_i, s'_{-i}) \geq [>] g_i^*(s')$ imply $g_i^*(s) \geq [>] g_i^*(s_i, s_{-i})$. If $s_j' \neq \emptyset$, for all $j \neq i$, this is equivalent to condition [**] above. For $s_j = \emptyset$ for some $j \neq i$ (and thus $s_j' = \emptyset$, for some $j \neq i$), [**] is trivially satisfied. If $s_j' = \emptyset$ for some $j \neq i$, but $s_j \neq \emptyset$ for all $j \neq i$, then we require

$$\sum_{\bar{t}_i > t_i \geq t'_i} p(t_i, t'_{-i}) v_i(t_i, t'_{-i}) \geq [>] 0 \rightarrow \sum_{\bar{t}_i > t_i \geq t'_i} \sum_{t_{-i} \geq t'_{-i}} p(t_i, t'_{-i}) v_i(t_i, t'_{-i}) \geq [>] 0.$$

But this is implied by [**].

We will say for short that *conflicting ordinal interpretation of signals* (COIS) holds if the above two restrictions ([*] and [**]) on p and v hold. A simpler set of sufficient conditions for COIS can also be given. Say that beliefs p are *independent* if $p(t) = p_1(t_1) \dots p_i(t_i)$, for some p_1, \dots, p_i . It is straightforward to verify the following lemma:

Lemma 6 COIS holds if beliefs are independent and, for each i , $v_i(t_i, t_{-i})$ is strictly increasing in t_i and weakly decreasing in t_{-i} .

Proof

$$\begin{aligned} v_i^+(t_i, T'_{-i}) &= \sum_{t_{-i} \in T'_{-i}} p(t_i, t_{-i}) v_i(t_i, t_{-i}) \\ &= p_i(t_i) \sum_{t_{-i} \in T'_{-i}} \left\{ \prod_{j \neq i} p_j(t_j) \right\} v_i(t_i, t_{-i}) \end{aligned}$$

Now $v_i(t_i, t_{-i})$ strictly increasing in t_i ensures that the interim payoff function satisfies the strict single crossing property.

$$\begin{aligned}
& v_i^+(t_i, \bar{t}_i, t'_{-i}) \geq [>] 0 \\
& \rightarrow v_i(t_i, t'_{-i}) \geq [>] 0 \\
& \rightarrow v_i(t_i, t''_{-i}) \geq [>] 0, \text{ for all } t'_{-i} \geq t''_{-i} \\
& \rightarrow v_i^+(t_i, \bar{t}_i, t''_{-i}) \geq [>] 0, \text{ for all } t'_{-i} \geq t''_{-i}.
\end{aligned}$$

Now lemmas 4 and 5 together with theorem 2 imply:-

Theorem 3 *If conflicting interpretation of signals holds in a refined acceptance game, then for each agent i , there exist largest and smallest iteratively undominated strategies T_i^* and T_{i*} . These are monotonic, as are all iteratively undominated strategies. The pure strategy profiles T^* and T_* are Nash equilibria.*

Milgrom and Roberts (1990) made the argument that since all rationalizable actions are iteratively undominated, and since only iteratively undominated actions are played with positive probability in correlated equilibria and mixed strategy Nash equilibria, this result characterizes those solution concepts as well.

The remainder of this section investigates the relation between the acceptance and refined acceptance games. The indeterminacy of the best response function of the acceptance game when some agent never accepts has been treated in different ways in the literature. For example, in Holmström and Myerson's (1983) notion of durability, sequential equilibria of an extensive form version of the acceptance game are studied, in order to rule out unsatisfactory equilibria. Theorem 4 shows that if we are only interested in Nash equilibria where acceptance takes place, the acceptance and refined acceptance games are equivalent.

Theorem 3 showed that there existed largest and smallest iteratively undominated strategies which are pure Nash equilibria. Any mixed Nash equilibria must assign positive probability only to strategies between the largest and smallest pure Nash equilibria. In the (unrefined) acceptance game, we can no longer appeal to complementarity results, so we must introduce notation for mixed strategy Nash equilibria. A mixed strategy π_i for player i is a simple probability distribution over S_i . We can extend g_i to mixed strategy profiles in the usual way. Strategy profile π is an *acceptance* profile if, for all i , $\pi_i(\emptyset) \neq 1$, where \emptyset is the pure strategy "always reject". Strategy profile π is a *rejection* profile if $\pi_i(\emptyset) = 1$, for at least two agents.

Theorem 4 *In any acceptance game,*

[1] All strategies are rationalizable (and thus iteratively undominated).

[2] The set of Nash acceptance profiles equals the set of Nash acceptance profiles of the refined acceptance game.

[3] All rejection profiles are Nash equilibria.

Proof In the acceptance game, all strategies are best responses to profiles of other agents' strategies where some agent always rejects. This immediately implies [1] and [3]. Elsewhere, the best response correspondence of the acceptance game is identical to the best response correspondence of the refined acceptance game, implying [2].

Section 6: Acceptance Protocols

Sebenius and Geanakoplos (1983) proposed an alternative model of when a change from the status quo is accepted or not. Their concern was with the learning process by which it becomes common knowledge that all agents favor change. In the acceptance game, it does not typically become common knowledge that agents wish to make the change. Suppose then that the change has been proposed and agents have the opportunity to withdraw. A *protocol* will specify at each date who has the opportunity to withdraw. If some agent withdraws, the status quo remains. If no agent ever withdraws, then the alternative is accepted³.

A *protocol* is a function $\gamma : N_{++} \rightarrow \{1, \dots, I\}$. The protocol is said to be *fair* if, for all agents i , and integers K , there exists $k \geq K$ such that $\gamma(k) = i$. In other words, each agent is chosen infinitely often under the protocol.

At each date k , agent $\gamma(k)$ makes an announcement, Y or N , as to whether he is prepared to accept the change, $\alpha : T \times N_+ \rightarrow \{Y, N\}$. This agent takes into account the previous history of announcements in making his choice. Say that $T(k) \subset T$ is the set of type profiles which have been

3. This is thus a *public* protocol where all information is publicly revealed. Similar results would go through for private protocols, where not all agents learn each agent's announcement when they are made. For the two person case of Sebenius and Geanakoplos (1983), the distinction was irrelevant. Cave (1983) and McKelvey and Page (1986) considered public protocols in related settings, Parikh and Krapuski (1990) private protocols.

revealed to be possible by previous announcements, at time k . Announcements α and period k information $T(k)$ can be defined inductively by:-

$$\begin{aligned}
 T(0) &= T, \quad \alpha(0,t) = Y \\
 i = \gamma(k+1) \rightarrow \alpha(k+1,t) &= \begin{cases} Y, & \text{if } v_i^*[t_i, T_{-i}(k)] \geq 0, \text{ and } \alpha(k',t) = Y, \text{ for all } k' \leq k \\ N, & \text{otherwise} \end{cases} \\
 T(k+1) &= \{t \in T(k) \mid \alpha(k+1,t) = Y\}.
 \end{aligned}$$

Theorem 5 *For every fair protocol γ , there exists K and product subset of types, $T^\infty \subset T$, such that for all $k \geq K$, $T(k) = T^\infty$ and $\alpha(k,t) = Y$ if and only if $t \in T^\infty$. If, in addition, conflicting interpretation of signals holds, then T^∞ is the largest pure strategy Nash acceptance profile of the acceptance game (if one exists) or T^∞ is the empty set (if not).*

Proof Convergence of T and α is guaranteed because T is finite and $T(k)$ is (by definition) monotonically decreasing under set inclusion. Recall that common ordinal interpretation of signals gives us two properties:-

[1] $\{t_i' \in T_i \mid v_i^*[t_i', T_{-i}'] \geq 0\} = \emptyset$ or $\{t_i' \in T_i \mid t_i' \geq t_i, \text{ for some } t_i\}$

[2] $\{t_i \in T_i \mid v_i^*[t_i, \{t_i' \mid t_i' \geq t_i\}] \geq 0\}$ is weakly decreasing in t_i

[1] and [2] imply, by induction on k , [3] $T(k) = \{t' \in T \mid t' \geq t, \text{ for some } t \in T\}$, for all k .

Now for all i , there exists $k \geq K$ such that $\gamma(k) = i$, so [5] $\{t_i \in T_i^\infty \mid v_i^*[t_i, T_{-i}^\infty] \geq 0\} = T_i^\infty$.

Now [2], [3] and [5] imply T^∞ is a pure strategy Nash equilibrium.

Suppose T^* is the largest pure strategy Nash equilibrium.

[2] and [3] imply $T^* \subset T(k)$ for each k , by induction on k . So $T^* = T^\infty$ ■

This is closely related to learning results in games with strategic complementarities. Milgrom and Roberts (1990, 1991) have shown that if there is a unique Nash equilibrium, then almost any learning process will converge to that equilibrium. Krishna (1992) has shown, under additional conditions, global convergence of fictitious play, even with multiple equilibria. Note that (refined and unrefined) acceptance games may have many equilibria. The fair protocol here is equivalent to best response dynamics, which will not typically converge globally. But the fair protocol only looks at one starting point: it is a local convergence result.

Despite the similarity to learning results, note that the interpretation is rather different. Here agents are learning not about strategies, but about the actions of a particular type: this is a mechanism by which agents learn others agents' information, or "types".

Two examples illustrate why conflicting ordinal interpretation of signals is required for the latter half of the theorem. Suppose agents' valuations are as given in figure 2. There are two agents, 1 and 2. 1 observes only one signal, 2 observes either L or R. Agent's have common valuations of outcomes and each of 2's signals are equally likely. Notice that there is certainly a pure strategy Nash equilibrium with acceptance where 1 accepts (always) and 2 accepts if he observes L. But consider the natural protocol such that $\gamma(k) = 1$ if k is odd, $\gamma(k) = 2$ if k is even. 1 will reject immediately in the first round, before 2 has a chance to reject on R.

On the other hand, suppose agents' valuations are as given in figure 3. Now there are two signals for each agent. Suppose all combinations are equally likely. There is no Nash equilibrium with acceptance in this game, since 2 will never accept if he observes R, 1 will always accept if 2 accepts only on L, and 2 will not accept on L if 1 always accepts.

But now consider the same odd/evens protocol. In the first round, 1 will accept only if he has observed U. In the second round, 2 will accept only if he has observed L. In the third round, 1 is still willing to accept on U, and the protocol has converged. But note that 1 would like to change his mind and accept on D (but cannot under the rules of the protocol).

It was somewhat arbitrary to say that agents accepted in the protocol, when they were indifferent between accepting and rejecting. Generically, there is no difference (as in the example of section 2). But for *feasible trades* (discussed in the next section) it will be useful to have the analogous result for *strict acceptance protocols*, where the alternative is accepted only if it is strictly advantageous. Recall that a *strict Nash equilibrium* is a pure strategy Nash equilibrium where each agent's action is *strictly* preferred to any other. Define a strict protocol as follows:-

$$\begin{aligned}
 T^+(0) &= T, & \alpha^+(0,t) &= Y \\
 i = \gamma(k+1) \Rightarrow \alpha^+(k+1,t) &= \begin{cases} Y, & \text{if } v_i^*[t_i, T_{-i}^+(k)] > 0, \text{ and } \alpha^+(k',t) = Y, \text{ for all } k' \leq k \\ N, & \text{otherwise} \end{cases} \\
 T^+(k+1) &= \{t \in T^+(k) \mid \alpha^+(k+1,t) = Y\}.
 \end{aligned}$$

Theorem 6 For every strict fair protocol γ , there exists K and product subset of types, $T^{+\infty} \subset T$, such that for all $k \geq K$, $T^+(k) = T^{+\infty}$ and $\alpha^+(k,t) = Y$ if and only if $t \in T^{+\infty}$. If, in addition, conflicting interpretation of signals holds, then $T^{+\infty}$ is the largest pure strategy strict Nash acceptance profile of the acceptance game (if one exists) or $T_+^{+\infty}$ is the empty set (if not).⁴

4. It is easy to verify that if there exist strict Nash equilibria, there is a largest strict Nash equilibrium.

Section 7: No Trade Results

A special case of an "alternative" is some feasible trade contingent on agents' signals. We know that in this case common knowledge of acceptance of trade implies no strict gains from the trade. Sebenius and Geanakoplos (1983) showed that trade cannot be accepted in a strict acceptance protocol, and that there can be no acceptance in a static acceptance game with strict gains from trade. In this section, I give (theorem 7) a version of that result and also show (theorem 8) that with conflicting interpretation of signals, rejection of the trade is also the only iteratively undominated action for all agents.

The conflicting ordinal interpretation of signals assumptions that have driven results in the previous two sections imply that there is some degree of conflicting interest among the agents. A more extreme case of conflicting interest is when the move from the status quo to the alternative is a feasible trade:-

$$v \text{ is feasible if } \sum_{i=1}^I v_i(t) = 0, \text{ for all } t \in T.$$

The following result is a slight generalization of Sebenius and Geanakoplos (1983) concerning such feasible trades:-

Theorem 7 *If v is feasible, then*

[1] *Every (possibly mixed) Nash equilibrium π of the acceptance game has $g_i(\pi) = 0$, for all i .*

[2] *The alternative is always rejected for all types under all fair strict protocols.*

Proof [1] Every agent has a strategy "never accept" guaranteeing him 0, regardless of other agents' actions. [2] By theorem 6, *either* rejection always occurs under the strict protocol, *or* there is convergence to a strict Nash equilibrium. But by [1], there is no strict Nash equilibrium ■

v is *non-trivial* if $v_i(t^*) \neq 0$, for some i . Recall that t_i^* is the maximal element of T_i , for each i . A game is said to be *dominance solvable* [Moulin (1979)] if there is a unique iterative undominated action for each agent. If the game is dominance solvable, that unique profile of iteratively undominated actions is the unique Nash equilibrium of the game.

Theorem 8 *If v is feasible, non-trivial and satisfies conflicting ordinal interpretation of signals, then the refined acceptance game is dominance solvable, with "always reject" the unique iteratively undominated (and thus rationalizable) action for each player.*

Proof Suppose COIS is satisfied, and agent i accepts with positive probability on t_i and t_i' with $t_i > t_i'$, then he must have a strict gain from acceptance on t_i . v feasible implies $g_i(\pi) = 0$, for every Nash equilibrium π (by theorem 7), so he must have strictly negative gain from accepting on t_i' , contradicting the best response property. So each agent i can accept on at most one signal. By COIS, this would have to be t_i^* , but by non-triviality and feasibility $v_i(t_i^*) < 0$ for some i ■

It is useful to consider the significance of this result if $I = 2$. Then $v_2 = -v_1$ and conflicting ordinal interpretation of signals is equivalent to the requirement that $v_1(t_1, t_2)$ strictly increasing in t_1 and strictly decreasing in t_2 .

Let us give an example which makes clear that it is possible for *all* strategies to be iteratively undominated, in a *refined* acceptance game, for a non-trivial, feasible v , when COIS fails. Consider the valuations in figure 4, with each combination of signals equally likely. This looks very much like the matching pennies game, although remember the strategies for each agent are \emptyset , $\{H\}$, $\{T\}$ and $\{H, T\}$.

As in the matching pennies game, every action is iteratively undominated even in the refined acceptance game⁵.

Theorems 7 and 8 can be extended if we are interested in the relation between efficiency and the acceptance of trade [see Morris (1992)]. We can characterize conditions under which the status quo satisfies different notions of efficiency under asymmetric information [Holmström and Myerson (1983)]⁶. Clearly v feasible implies that the status quo is *ex ante* efficient. Theorem 7 will go through under the assumption that the status quo is interim incentive compatible efficient. Indeed, the status quo is interim incentive compatible efficient if and only if there is no correlated equilibrium π of the acceptance game, with $g_i(\pi) > 0$ for some agent. Thus under the COIS assumption, a partial converse of theorem 7 holds: if all Nash equilibria π of the acceptance game have $g_i(\pi) = 0$, for all i , then the status quo is interim

5. The refined acceptance game depends on profile of "largest" types t^* , and thus on an ordering of types. But ever action is iteratively undominated in the refined acceptance game, *whatever* the profile of largest types, t^* .

6. The status quo is *ex ante efficient* if, for each i , there exists $\lambda_i \in \mathbf{R}_{++}$ s.t. $\sum_i \lambda_i v_i(t) \leq 0$, for all $t \in T$. It is *interim efficient* if, for each i , there exists $\lambda_i: T_1 \rightarrow \mathbf{R}_{++}$ s.t. $\sum_i \lambda_i(t) v_i(t, t_i) \leq 0$, for all $t \in T$. Linear algebraic conditions on v to characterize *interim incentive compatible efficiency* of the status quo can be derived using methods in Myerson (1991) chapter 10.

incentive compatible efficient. This is true because under COIS, no Nash equilibrium with acceptance implies no correlated equilibrium with acceptance⁷.

Section 8: Conclusion

In the presence of asymmetric information, when are agents able to unanimously agree to (interim) welfare improving transactions? Any answer to this question will involve answering the following narrower question. Given some proposed transaction, would agents be prepared to unanimously accept it at some state of the world? Any voluntary transaction has the characteristic that at some point agents must decide whether to accept it, or not, and acceptance requires unanimity.

A natural model for thinking about acceptance is as a static game with incomplete information. Nash equilibria and other game theoretic solution concepts will typically have a complicated structure. This paper identified a class of acceptance games, satisfying conflicting ordinal interpretation of signals, where local strategic complementarities in agents' evaluations of the transaction generate a simple structure of equilibria.

But voluntary transactions do not typically take place by agents making simultaneous, unrevokable conditional commitments to accept or reject. Yet this is the model implicit in the static acceptance game. The problem with considering more realistic dynamic stories is that it is hard to specify, in general, the exact extensive form mechanism by which agreement is reached. In this paper, one simple dynamic mechanism for reaching agreement was studied. Under the conflicting ordinal interpretation of signals assumption, acceptance took place exactly when acceptance would have taken place in the largest, Pareto-dominant, Nash equilibrium of the static acceptance game.

The dynamic protocol by which agreement was reached had the characteristic that it eventually becomes common knowledge that agents are accepting the trade. Note that it need not be the case that common knowledge of acceptance occurs in the static acceptance game. Indeed, it is possible (in games not satisfying COIS) for there to exist a Nash equilibrium of the static acceptance game, where acceptance occurs, when it is impossible for there to be common knowledge acceptance (see Morris (1992) for an example).

The protocol was specified in a non-strategic way. Agents announced whether they accepted the trade or not, and they were assumed to tell the truth. But notice that, under the common ordinal interpretation of signal assumption, agents do not have an incentive to lie. The protocol can easily be re-interpreted as an (infinite) extensive form game. Each agent announces, as in the protocol, acceptance

7. Morris (1992) gives an example (where COIS does not hold) where there is no Nash equilibrium with acceptance from an interim incentive compatible inefficient status quo.

or rejection. If any agent rejects the trade, the payoff is zero. The payoff if all agents always accept is the utility they get from the transaction. Then acceptance will occur at some profile of types in some subgame perfect equilibrium if and only if acceptance occurs given that profile of types in some Nash equilibrium of the static acceptance game. By imposing small costs for accepting when you would have rejected under the protocol, the extensive form can be manipulated to mimic the dynamic protocol. Indeed, it can be conjectured that the conflicting ordinal interpretation of signals assumption will imply the same unique outcome in a wide class of extensive form representations of the dynamic acceptance problem.

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Figure 1a: Agents' Valuations of the Object
(1's on left, 2's on right)

		2's signal		
		1	L	C
1's signal	U	5	7	10
	M	3	6	9
	D	2	3	8

		2's signal		
		2	L	C
1's signal	U	3	5	8
	M	1	2	3
	D	0	1	2

Figure 1b: Agents' Valuations of the Trade (at price 4)
(1's on left, 2's on right)

		2's signal		
		1	L	C
1's signal	U	1	3	6
	M	-1	2	5
	D	-2	-1	4

		2's signal		
		2	L	C
1's signal	U	1	-1	-4
	M	3	2	1
	D	4	3	2

Figure 1c: Best Response Functions

1's action	2's best response
\emptyset	all
{U}	{L}
{M}	{L,C,R}
{D}	{L,C,R}
{U,M}	{L,C}
{U,D}	{L,C}
{M,D}	{L,C,R}
{U,M,D}	{L,C}

2's action	1's best response
\emptyset	all
{L}	{U}
{C}	{U,M}
{R}	{U,M,D}
{L,C}	{U,M}
{L,R}	{U,M,D}
{C,R}	{U,M,D}
{L,C,R}	{U,M,D}

Figure 2: Agents' Valuations
(1's on the left, 2's on the right)

		2's signal		
		1	L	R
1's signal	U	1	-2	

		2's signal		
		2	L	R
1's signal	U	1	-2	

Figure 3: Agents' Valuations
(1's on the left, 2's on the right)

		2's signal		
		1	L	R
1's signal	U	1	0	
	D	1	-2	

		2's signal		
		2	L	R
1's signal	U	1	-2	
	M	-2	-2	

Figure 4: Agents' Valuations
(1's on the left, 2's on the right)

		2's signal		
		1	H	T
1's signal	H	1	-1	
	T	-1	1	

		2's signal		
		2	H	T
1's signal	H	-1	1	
	T	1	-1	

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