Recalculating Sargent and Wallace's Unpleasant Arithmetic using Interest Rates[*]

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This note celebrates the seminal paper by Sargent and Wallace, simply recasting it in terms of interest rates rather than money growth rates. Their most surprising result that "lower money growth today creates more inflation today and in the future" can then be seen to rely on being on the wrong side of a Laffer curve for money.

Sargent and Wallace (1981) cautioned against the perils of thinking separately about monetary and fiscal policy. Indeed, their paper taught us that monetary policy may be severely constrained by fiscal policy. Armed with a truly minimalist model, featuring a money demand and government budget constraint, they derived two insightful results. First, they showed that efforts to lower current inflation by lowering money growth may succeed in the short run, but require higher inflation in the future. Second, more drastically, in some cases these same efforts may backfire completely, leading to higher inflation in the present and future!

In this simple note, I redo Sargent-Wallace in terms of interest rates (or inflation) directly, instead of money growth rates. My presentation attempts to faithfully capture the essence of the original, not really attempting anything novel. The hope is to provides another perpective that complements and compliments the original.

As is well understood thanks to Friedman, nominal interest rate can be seen as a tax on money holdings. One can then think of revenue from this tax on money in a conventional public finance manner. The first results in Sargent Wallace corresponds to the simple notion that when taxes are lowered in some periods, they may have to be raised in other periods. Their second more drastic result that "lower money growth today creates more inflation at all times" is seen to rely on a situation where one finds oneself on the wrong side of a Laffer curve.

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[*]This note was adapted from lecture notes I used in first-year graduate macro classes for years. Everything in this note is quite simple and I make no claim for its originality, although I have not encountered any other presentation emphasizing the Laffer perspective I do here.

1 Present Value of Taxes from Money

Sargent and Wallace's model is simply a nominal government budget constraint, a money demand function and the Fisher equation[1]

$$B_{t,t+1} + M_{t,t+1} = P_t d_t + (1 + i_t)B_{t-1,t} + M_{t-1,t},$$

$$\frac{M_{t,t+1}}{P_t} = L(i_{t+1}),$$

$$1 + i_t = (1 + r_t)(1 + \pi_t) \qquad t = 1, 2, \ldots,$$

where $\pi_t = P_t/P_{t-1}$ is (gross) inflation, i_t is the nominal interest rates d_t is an exogenous fiscal deficit in real terms, r_t is the real rate, P_t is the price level, finally $B_{t,t+1}$ and $M_{t,t+1}$ are bonds and money held by households between periods $t-1$ and t. The demand for money is denoted by a non-increasing function $L(i)$ of the interest rate. Note that $B_{-1,0}$, $M_{-1,0}$ and i_0 are all naturally exogenous predetermined variables in the initial period $t = 0$. In addition, Sargent and Wallace assume M_0 is also exogenously predetermined.

Following the monetarist perspective, assume the real interest rate sequence r_t is exogenous. Define the real discount $q_t \equiv \frac{1}{(1+r_1)(1+r_2)\cdots(1+r_t)}$ with $q_0 = 1$. Imposing the no-Ponzi condition $\lim_{t \to \infty} q_t \frac{B_{t,t+1}}{P_t} = 0$ gives the present-value constraint

$$\sum_{t=0}^{\infty} q_t \frac{M_{t,t+1} - M_{t-1,t}}{P_t} - \frac{(1+i_0)B_{-1,0}}{P_0} = D,$$

where $D \equiv \sum_{t=0}^{\infty} q_t d_t$ is exogenously given. This simply says that the present value of seignorage must cover the present value of deficits and the initial real value of debt. After some algebra this can be transformed into[2]

$$\sum_{t=0}^{\infty} q_t \frac{i_t}{1 + i_t} L(i_{t+1}) - \omega L(i_1) = D, \tag{1}$$

where $\omega \equiv \frac{M_{-1,0} + (1+i_0)B_{-1,0}}{M_{0,1}} > 0$ is an exogenously given constant.

It is useful to think of $\frac{i}{1+i} L(i)$ as a "Laffer curve": the tax revenue collected on money, with i playing the role of a tax (the $1 + i$ in the denominator adjusts for the fact that the tax

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[1]We consider a stable demand for money, abstracting from population growth to simplify the calculations.

[2]To see this, write

$$S = \sum_{t=0}^{\infty} q_t \frac{i_t}{1 + i_t} \frac{M_{t,t+1}}{P_t} - \frac{M_{-1}}{P_0}$$

where I have used the definition of q_t and the Fischer relation $1 + i_t = (1 + r_t)P_{t+1}/P_t$. Finally, substituting $M_{t,t+1}/P_t = L(i_t)$ gives the desired result.

is collected a period later). This function is initially increasing in the neighborhood of $i = 0$ but may be decreasing at high values of i.

2   Recalculating Unpleasant Arithmetic

The present value condition derived above invites us to work with interest rates, since the condition is additively separable in interest rates. Sargent and Wallace instead worked with money growth rates. However, there is a one-to-one mapping between them in equilibrium in this setting in the following sense. Recall that Sargent and Wallace take M_0 as given and fixed across policy exercises. Given this restriction, a sequence of nominal interest rates $\{i_t\}_{t=1}^{\infty}$ implies a unique sequence for prices and money.[3] The mapping between inflation and interest rates is especially straightforward, so one can also think in terms of sequence of inflation rates. Conversely, for any sequence of money $\{M_{t,t+1}\}$ that converges to a constant growth rate, there is a unique sequence of prices satisfying the natural refinement that inflation converges to the long run growth rate in money. This then gives a unique sequence of interest rates.

   We now make two observations using the present value condition expressed in terms of interest rates.

Result #1: Lower Inflation Today, Higher Inflation in Future.   Supposes first that the Laffer curve $\frac{i}{1+i}L(i_t)$ is an everywhere increasing function, then the left hand side of equation (1) is increasing in interest rates.[4] Then, starting from some sequence $\{i_t\}$ satisfying this equation, a decrease in interest rates over some periods must be accompanied by an increase in interest rate in other periods. The same is true for inflation. In particular, a decline in inflation in earlier periods must be followed by a rise in inflation in latter periods. This captures the essence of the intertemporal tradeoff in the first example by Sargent and Wallace.

   This conclusion can be extended even if the Laffer curve is not everywhere increasing. In particular, suppose it has a declining section for $i \geq \bar{i}$ for some $\bar{i} > 0$. Then the previous result clearly hold as long as we limit ourselves to the "good side" of the Laffer curve, where $i_t < \bar{i}$. In particular if we start with an equilibrium $\{i_t\}$ with $i_t < \bar{i}$ then marginal decreases in $\{i_t\}$ must come at a revenue cost that is offset in some other period by increases in revenue, which require increases in interest rates.

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[3]To see this first use $\{i_t\}$ to compute the inflation rates $P_{t+1}/P_t = \frac{1+i_t}{1+r}$ for all $t = 0, 1, \ldots$; then simply compute $P_0 = M_{0,1}/L(i_1)$ recalling that $M_{0,1}$ is exogenously given; finally, compute money in all other periods using $M_{t,t+1} = P_t L(i_t)$ for $t = 0, 1, \ldots$

[4]Note that the stray $-\omega L(i_1)$ is always increasing and, thus, only reinforces this conclusion.

We have focused on the implications for interest rates and thus inflation. But it is easy to find conditions that ensure that lower initial inflation rates are associated with lower money growth rates. In particular, this is true as long as money demand $L(i)$ is sufficiently insensitive to the interest rate. Indeed, Sargent and Wallace derive their first result in the quantity-theory case where money demand $L(i)$ is completely inelastic.

Result #2: Higher Inflation Everywhere. We now make our second observation. If the Laffer curve is not monotone, it is also possible for find sequences of interest rates (thus, inflation) that rise in all periods. In particular, increasing the interest rate may increase or decrease $\frac{i}{1+i}L(i)$ so one can find infinitely many new sequences $\{i'_t\}$ with $i_t \leq i'_t$ satisfying equation (1). This captures the essence of the second, more drastic, result in Sargent and Wallace.

The previous conclusion holds even if the original equilibrium is on the "good side" of the the Laffer curve $i_t < \bar{i}$. In this case, non-local discrete change in interest rate are required in particular $i_t < \bar{i} < i'_t$. However, if the original equilibrium features interest rates for some $t \geq 2$ on the "bad side" of the Laffer curve with $i_t > \bar{i}$, and at least one interest rate on the good side, $i_\tau < \bar{i}$ for some τ, then it is also possible to find a local infinitesimal change in interest rates $i_t \leq i'_t$.

Note that if $i'_1 > i_1$ then the price level at $t = 0$ must also rise (i.e. inflation at $t = 0$ between $t = -1$ and $t = 0$ rises). We have focused on the implications for interest rates, and thus inflation rates. But, again, it is possible to provide conditions that guarantee that money growth is lowered over initial periods.

3 Final Thoughts

Sargent and Wallace is rightly celebrated as outlining the constraints on monetary policy that fiscal policy may impose, depending on how it is conducted. In the following few decades, most advanced economies designed central bank institutions with independent mandates that may largely overcome these challenges, at least for now. However, the fiscal-monetary constraints are very real and still felt today in parts of Latin America and elsewhere.

References

Sargent, Thomas J. and Neil Wallace, "Some unpleasant monetarist arithmetic," *Quarterly Review*, 1981, *5* (Fall).