

## The Distributional Consequences of Public School Choice<sup>†</sup>

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*School choice systems aspire to delink residential location and school assignments by allowing children to apply to schools outside of their neighborhood. However, choice programs also affect incentives to live in certain neighborhoods, and this feedback may undermine the goals of choice. We investigate this possibility by developing a model of public school and residential choice. School choice narrows the range between the highest and lowest quality schools compared to neighborhood assignment rules, and these changes in school quality are capitalized into equilibrium housing prices. This compressed distribution generates an ends-against-the-middle trade-off with school choice compared to neighborhood assignment. Paradoxically, even when choice results in improvement in the lowest-performing schools, the lowest type residents need not benefit. (JEL H75, I21, I28, R23, R31)*

A central fault line in debates about K–12 education involves how students access public schools. In most of the United States, students are assigned by neighborhood assignment rules based on residential location. An alternative is school choice, in which pupils can apply to schools outside of their neighborhood and residence plays little or no role in determining access. Proponents argue that choice would result in a more equitable distribution of school access and lead to improvements in school productivity.<sup>1</sup> Notwithstanding, choice plans remain controversial. In recent years, there has been a backlash against choice in several districts and active discussions to return back to neighborhood-based assignment.<sup>2</sup>

This paper provides a simple model to explore how the link between school assignment rules, house prices, and the residential choices of families affect the distributional consequences of public school choice. It is motivated by empirical

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<sup>1</sup>See, for instance, Friedman (1962), Chubb and Moe (1990), and Hoxby (2003). These arguments have been central to recent policy efforts to expand choice (DeVos 2017).

<sup>2</sup>For instance, a well-known advocate of Boston's 1970s busing plan recently called for a return to neighborhood assignment: see Ted Landsmark, "It's Time to End Busing in Boston," *Boston Globe*, June 31, 2009. Former Boston Mayor Thomas Menino encouraged the Boston school committee to adopt a plan that assigns pupils closer to home, and a plan restricting the amount of choice outside of neighborhoods was adopted in 2014 (for more details, see Pathak and Shi forthcoming). Other districts have also severely scaled back their choice plans (see Pathak and Sönmez 2013 for details about Seattle).

evidence showing that the housing market and residential choices reflect school assignment rules (see, e.g., Black 1999; Kane, Riegg, and Staiger 2006; Reback 2005; and Bayer, Ferreira, and McMillan 2007). In contrast to work that emphasizes the connection between assignment rules and the incentives for schools to improve their quality (see, e.g., Hoxby 2003; MacLeod and Urquiola 2009; Barseghyan, Clark, and Coate 2014; and Hatfield, Kojima, and Narita 2016), we focus on the effect of outside options in nearby towns on locational decisions of families living in a town that adopts school choice.

When a town with multiple school districts uses a neighborhood assignment rule, endogenous differentiation of housing prices and school qualities emerge in equilibrium, suggesting that public education may widen rather than narrow existing inequalities.<sup>3</sup> Our primary question is whether school choice rules truly increase access to high-quality schools in the context of general equilibrium pricing and self-selection of housing choices by families.

The incentive for flight of high types from a town that adopts school choice has been discussed in the literature on the residential consequences of school desegregation or busing. For instance, Baum-Snow and Lutz (2011) attribute the decline in white public school enrollment in urban centers to court-ordered desegregation decrees, finding that migration to other districts plays a larger role than private school enrollment. In the context of our model, withholding the option of paying for a high-quality school will drive high types to other towns that offer that option. But this same logic applies inexorably to predict flight of low types when a town adopts school choice. In fact, any model that predicts that school choice results in a narrowing of the range between highest and lowest quality schools in a town and allows for changes in school qualities to be capitalized into housing prices will generate a prediction that the adoption of school choice will produce incentives for types at both extremes to move. Yet to our knowledge, ours is the first paper to model how narrowing the gap between highest and lowest quality schools provides equilibrium incentives for flight of low types (in addition to high types) from the public schools in that town.

Our approach is also inspired by past studies of the effects of private school vouchers, especially Epple and Romano (1998) and Nechyba (2000). These papers develop ambitious models that include multidimensional student types, define school quality as a function of tax funding and average peer quality, and allow for tax regimes, housing prices, enrollment in private schools, and residential choices of families within a town to be determined endogenously in equilibrium. They characterize some aspects of any equilibrium outcome while focusing attention on simulations that assume a specific form of utility function (generally Cobb-Douglas) to assess the welfare implications of different voucher plans.

<sup>3</sup>These ideas have their roots in Tiebout (1956) and Schelling (1971, 1978), and have been explored extensively by (among many others) Bénabou (1993, 1996), Durlauf (1996), and Loury (1977) in studies of intergenerational mobility, by Fernandez and Rogerson (1996) and Nechyba (2003b) in studies of the effects of different tax regimes for funding public schools, and by Epple and Romano (1998, 2003) and Nechyba (2000, 2003a) in studies of school vouchers. Epple and Sieg (1999) empirically examine the relationship between locational equilibrium and community income distribution, while Rothstein (2006) provides empirical evidence of the relationship between neighborhood sorting and school quality. Epple and Romano (2015) analyze efficient allocations in a multi-community model with peer effects.

Our model is distinct from the framework of this earlier literature because we allow for endogenous selection of the families who choose to live in a town in equilibrium in addition to the choice of where to reside within that town. Since these previous papers typically assume that each family must purchase a house in a given town, the outside option of enrolling in a private school requires an additional tuition payment and is only appealing to high types. Our model allows for a new and natural mechanism, moving to a different town, for both high and low types to opt out of an existing public school regime.<sup>4</sup> In addition, Epple and Romano (2003), the prior paper that is closest in spirit to this paper, assumes that there is a fixed price for houses attached to the lowest quality school in a town. This is not an innocuous assumption, as it implies that changes in the quality of the worst school in the town are not capitalized into market prices, and thus improvements in the quality of the worst school are necessarily beneficial to low types. These distinctions from the prior literature allow for rich and intuitive welfare analysis, which we emphasize throughout the paper, but also add complexity to equilibrium analysis.

The paper is organized as follows. Section I presents the model, the equilibrium concept, and describes a simple example. Section II provides a characterization of equilibrium and shows that a relatively strong assumption, “Strong Assortative Matching,” is required to ensure the result that school choice induces more flight than neighborhood assignment. Section III examines welfare and discusses some extensions, while Section IV concludes.

## I. The Model

### A. Primitives

Each family has one child who will enroll in school as a student. We assume that there is a unitary actor for each household and refer to the student as the decision maker for the family. Each student has a two-dimensional type. The first dimension is binary and identifies “partisans” who derive a distinct benefit from living in the town.<sup>5</sup> Partisanship could represent the cost of commuting to work, a frictional moving cost, or general affinity for the amenities of the town. The second dimension is “student type,” which is independent and identically distributed according to density  $f(x)$  which is continuous and differentiable with  $f(x) > 0$  for each  $x \in [0, 1]$ . To ease exposition, we refer to the value of  $x$  as the one-dimensional type of a student, neglecting partisanship. We sometimes focus on symmetric and single-peaked distributions. The function  $f$  is *symmetric* if  $f((1/2) + d) = f((1/2) - d)$  for all  $d$ . The function  $f$  is *single-peaked* if for any  $x_0 < x_1 < 1/2$ ,  $f(x_0) \leq f(x_1) \leq f(1/2)$  and for any  $1/2 > x_1 > x_0$ ,  $f(1/2) \geq f(x_1) \geq f(x_0)$ .

<sup>4</sup>Epple and Romano (2003) and Nechyba (2003a) consider the effects of public school choice; Epple and Romano (2003, pp. 273–74) provide an example in their concluding remarks where a public school choice rule induces exit by either low- or high-income households, but do not conduct a formal analysis along those lines as the framework of that example is quite distinct from the models they analyze in the main section of their paper.

<sup>5</sup>In a richer model families could vary continuously in their partisanship, we assume that it is binary to keep the model tractable. Epple, Romano, and Sarpa (2018) study a model of income-targeted vouchers with families with two-dimensional types, where income is continuous and preference for religious instruction is binary. We provide an example where partisanship is continuous in online Appendix Section C.

Each family  $i$  has a separable utility function that takes the type,  $x_i$ , the quality of school  $j$  attended by the student,  $y_j$ , and the price of attending that school,  $p_j$ , as arguments. Since we study rules for assigning students to public schools which are freely provided,  $p_j$  is simply the cost of housing associated with school  $j$  with corresponding quality  $y_j$ . We write this utility function as

$$u(x_i, y_j, p_j) = \theta_{ij} + v(x_i, y_j) - p_j,$$

where  $\theta_{ij} = \theta > 0$  if family  $i$  is partisan to the town and school  $j$  is in the town, and  $\theta_{ij} = 0$  otherwise. As in Epple and Romano (1998), the quality of a school is equal to the average type of pupils attending the school, and so  $y \in [0, 1]$ ; partisanship plays no direct role in determining school quality. Moreover, there is no capacity constraint at a school; a school can accommodate as many pupils as needed. A separable utility function facilitates interpretation of “marginal utility” and “marginal cost” of changes in school quality at given prices.

**ASSUMPTION 1 (Increasing Differences):**  $v$  is continuous, differentiable, strictly increasing in each argument,  $v(0, 0) = 0$ , and there is a positive constant  $\kappa > 0$ , such that  $\partial^2 v / \partial x \partial y > \kappa$  for each  $(x, y)$ .

Assumption 1 implies that  $v$  satisfies the property of strictly increasing differences in  $(x, y)$ .<sup>6</sup> That is, if  $x^H > x^L$  and  $y^H > y^L$ , then

$$v(x^H, y^H) - v(x^H, y^L) > v(x^L, y^H) - v(x^L, y^L).$$

This assumption follows much of the literature on local public economics (e.g., Epple and Romano 1998 and Rothstein 2006).<sup>7</sup> Since high types are willing to pay more for an increase in school quality than low types, this assumption induces assortative matching of students to schools.<sup>8</sup> The assumption that  $v(0, 0) = 0$  is simply a normalization.

**ASSUMPTION 2 (Competitive Outside Option):** *There is a competitive market for schools outside of the town, such that schools of quality  $y$  are available at competitive price  $p(y)$  for each  $y$ .*

Assumption 2 guarantees the option to reside outside the town and obtain school quality  $y$  at price  $p(y)$  for each  $y$ . We relax this assumption in analysis of a two-town model in online Appendix Section D, where the outside option to living in one town is to move to the other town: the results are qualitatively similar to the results of

<sup>6</sup>See, for example, van Zandt (2002).

<sup>7</sup>Although this assumption is standard in models of sorting, it is difficult to provide a rigorous defense of it without strong assumptions on unobservables.

<sup>8</sup>If the one-dimensional type in the model is initial wealth, then it is natural to use a slightly different formulation of utility, as is standard in the prior literature, namely  $u(x_i, y_j, p_j) = h(x_i - p_j, y_j)$  for some function  $h$ . Then, as long as  $p_j$ , the price for attending school  $j$ , is an increasing function of the quality of that school,  $h_{11} < 0$  and  $h_{12} > 0$  are jointly sufficient for  $u$  to exhibit strictly increasing differences in  $(x, y)$ . Since  $h_{ij}$  refers to the second derivative of  $h$  with respect to  $i$  and  $j$ , these sufficient conditions correspond to assumptions of decreasing marginal utility in net wealth and higher marginal utility for school quality as net wealth increases.

the one-town model.<sup>9</sup> The competitive market for public schools outside the given town is quite similar to the nature of private schools in Nechyba (2000, 2003a). One important distinction is that students who opt for an outside option in our model do not also have to pay for a house in the town, whereas students who choose a private school in Nechyba (2003a) also have to reside in the original town and pay for a house there.

LEMMA 1: *The competitive pricing rule  $p(y) = \int_{z=0}^y \frac{\partial v}{\partial y}(z, z) dz$  induces a nonpartisan student of type  $x$  to choose a school of quality  $x$ .*

Lemma 1 identifies a unique pricing rule for self-sorting of all nonpartisans into homogeneous schools. Nonpartisans are willing to pay a competitive price to live in the town, but will not choose a house in the town with a price greater than that. Under Assumption 2, schools of every quality level  $y$  are available outside town  $t$ , so we denote the (outside option) value available in equilibrium to partisan type  $x$  as

$$\pi(x) = v(x, x) - p(x).$$

We denote  $m$  as the measure of families who are town partisans and  $M$  as the measure of homogeneous housing stock available in the town. We make the normalizing assumption  $M = 1$  throughout the analysis.

ASSUMPTION 3 (Housing Market Clearing): *There are enough houses for all partisans to live in the town:  $M \geq m$ . The measure of nonpartisan families of each type  $x$  is greater than the measure of housing stock  $M$ .*

The assumption that  $M \geq m$  facilitates the comparison across school assignment rules and allows us to focus on conditions for equilibria whereby all partisans choose the town. Assumption 3 also guarantees that there are sufficiently many nonpartisans so that there are no vacant houses in the town. In an allocation in which the measure of town partisans is less than the measure of houses, nonpartisan families occupy the remaining houses in the town and are indifferent between residing in the town and living outside the town and obtaining a school with quality equal to their type.

## B. Equilibrium

We define a neighborhood assignment rule as one where the town's houses are exogenously partitioned into districts with measure of houses  $M_d$  in district  $d$  (where  $\sum_{d=1}^D M_d = M = 1$  by our previous normalizing assumption), house prices vary by district, and all students living in district  $d$  are assigned to the single school in that district.

<sup>9</sup>Epple, Newlon, and Romano (2002) study a model of tracking and competition between public and private schools. One possible equilibrium of this model produces an outcome whereby only lowest and highest ability students attend public schools, as those highest ability students are attracted by the ability to enroll in the advanced track. This outcome is broadly similar to a Type 2 equilibrium of the two-town model.

**DEFINITION 1: A Neighborhood School (NS) equilibrium** in the town with  $D$  districts consists of prices  $p_1, p_2, \dots, p_D$  and sets of partisan types  $T_1, T_2, \dots, T_D$  enrolling in these districts with measures  $m_{T_1}, m_{T_2}, \dots, m_{T_D}$  and average types  $y_1, y_2, \dots, y_D$ , such that  $y_d = E[x|x \in T_d]$  for each  $d$  and

- (i)  $v(x, y_d) + \theta - p_d \geq \pi(x)$  for each  $d$  and each  $x \in T_d$ ,
- (ii) If  $x \in T_d$ , then  $v(x, y_d) - p_d \geq v(x, y_k) - p_k$  for each  $k \in \{1, 2, \dots, D\}$ ,
- (iii)  $m_{T_d} \leq M_d$  for each  $d$ , where
  - if  $m_{T_d} = M_d$ , then  $p_d \geq p(y_d)$ , and
  - if  $m_{T_d} < M_d$ , then  $p_d = p(y_d)$ .

The first condition of this definition is an individual rationality constraint, which ensure that a partisan family of given type chooses to live in district  $d$  in the town if and only if that yields higher utility than the option outside of the town. The second condition is an incentive compatibility constraint, which ensures that partisan families choose their most preferred district if they choose to live in town  $t$ . Finally, the last condition involves housing market clearing: partisan demand for housing in district  $d$  is no greater than the supply of housing in district  $d$ . If partisan housing demand is less than housing supply in district  $d$ , then the price in that district must equal the competitive price  $p(y_d)$  so that nonpartisans are willing to reside in the remaining houses.

We define a school choice rule as one where there is a lottery that assigns students to schools. We assume that there are no informational frictions or priorities in the lottery, so that all of the district's residents submit identical rank-order lists of schools in descending order of anticipated quality.<sup>10</sup> Therefore, under a school choice rule, all schools in the town have equal quality levels and all houses have the same price.

**DEFINITION 2: A School Choice (SC) equilibrium** in the town is a Neighborhood School equilibrium with  $D = 1$ . Let  $p_{SC}$  denote prices,  $T_{SC}$  denote partisan types residing in the town with measure  $m_{SC}$ , and  $y_{SC}$  denote school quality given by  $y_{SC} = E[x|x \in T_{SC}]$ .

If  $m_{SC} = 1$ , then partisans fill all available housing in the town and prices satisfy  $p_{SC} \geq p(y_{SC})$ , which discourages nonpartisans from living in the town. If  $m_{SC} < M$  then partisan demand does not exhaust the town's housing supply, so in equilibrium the remaining houses in the town are filled by nonpartisans of type  $x = y_{SC}$ , maintaining the condition  $p_{SC} = p(y_{SC})$ .

<sup>10</sup>Xu (2019) develops an extension of the model in this paper where school choice takes place via deferred acceptance with residential priorities. Her model has two important differences: the quality of a school is exogenous and not a function of student types, and there is a capacity constraint at schools, so neighborhood priorities play a role in those cases. Xu (2019) shows that the possibility that some students are unassigned may lead some high types to opt for lower quality schools where they face a lower risk of being unassigned.

**Example 1:** Assume that  $v(x, y) = xy$  and that the distribution of types is Uniform on  $(0, 1)$ .<sup>11</sup>

A partisan of type  $x$  achieves utility  $v(x, y) - p(y) + \theta$  by locating in a district in the town with school quality  $y$  and housing price  $p(y)$  or  $\pi(x)$  by choosing the outside option. With  $v(x, y) = xy$ , the competitive price is given by  $p(y) = y^2/2$  and the outside option yields utility  $\pi(x) = x^2/2$ . Thus, with these two options, a partisan of type  $x$  will prefer to locate in the town if  $xy - (y^2/2) + \theta \geq x^2/2$ , which is equivalent to  $2\theta \geq y^2 - 2xy + x^2$ , or  $\theta \geq (y - x)^2/2$ . Defining  $\Delta_d = y_d - x$ , a partisan prefers district  $d$  in the town under competitive pricing to the outside option if  $\Delta_d \leq \sqrt{2\theta}$ . In words, partisans will locate in the town if it is possible to choose a school with quality close to their ideal point, where the range of acceptable schools is determined by the magnitude of partisan bonus  $\theta$ .

Define  $\theta_N$  and  $\theta_{SC}$  to be the thresholds for  $\theta$ , such that all partisans choose to live in the town under the neighborhood and school choice rules respectively. If all partisans choose to live in the town under school choice, then, given the assumption of uniform distribution of types,  $y_{SC} = 1/2$ . In this case, the incentive condition for choosing the town is most restrictive at the extreme values  $x = 0$ ,  $x = 1$ , where  $\Delta = 1/2$  and so  $\theta_{SC} = \Delta^2/2 = 1/8$ .

Next, consider a symmetric NS equilibrium with two districts each with measure  $1/2$  of houses in the town and where for some constant  $b$ , partisans of types  $[(1/2) - b, 1/2]$  enroll in district 1 while partisans of types  $[1/2, (1/2) + b]$  enroll in district 2. Then  $y_1 = (1/2) - (b/2)$  and  $y_2 = (1/2) + (b/2)$ . A type- $x$  partisan achieves utility  $xy_2 - (y_2^2/2) + \theta$  by enrolling in district 2 or utility  $xy_1 - (y_1^2/2) + \theta$  by enrolling in district 1. Comparing these values, a type- $x$  student prefers district 2 to district 1 if  $xy_2 - (y_2^2/2) \geq xy_1 - (y_1^2/2)$  or  $x \geq (y_1 + y_2)/2 = 1/2$ . Thus, partisans with the lowest types either enroll in district 1 or take the outside option; similarly, partisans with the highest types either enroll in district 2 or take the outside option. If all partisans choose to live in the town in a symmetric NS equilibrium with two districts, then  $b = 1/2$ ,  $\Delta = 1/4$  in each district and so  $\theta_N = \Delta^2/2 = 1/32$ .

There are also equilibria for both assignment rules when  $\theta$  is too small to support an equilibrium with all partisans choosing the town. If  $\theta < \theta_{SC} = 1/8$ , there is a school choice equilibrium with partisan types  $[(1/2) - \sqrt{2\theta}, (1/2) + \sqrt{2\theta}]$  in the town. Similarly, if  $\theta < \theta_N = 1/32$ , there is a neighborhood equilibrium with partisan types  $[(1/2) - \sqrt{8\theta}, 1/2]$  in district 1 and  $[1/2, (1/2) + \sqrt{8\theta}]$  in district 2.<sup>12</sup> As illustrated in Figure 1, a school choice rule typically induces more “flight” from the town than a neighborhood assignment rule in this example: in equilibrium with if  $\theta < \theta_{SC} = 1/8$ , more partisans choose to live in the town with a NS assignment rule than with a SC assignment rule.

<sup>11</sup> This function does not satisfy Assumption 1 because  $\partial v/\partial x = 0$  when  $y = 0$  and  $\partial v/\partial y = 0$  when  $x = 0$ , but this does not affect the analysis.

<sup>12</sup> The incentive conditions only determine the width of the range of types in a given district. If  $\theta < 1/8$ , then for each  $s \in [\sqrt{2\theta}, 1 - \sqrt{2\theta}]$ , there is a school choice equilibrium with partisan types  $[s - \sqrt{2\theta}, s + \sqrt{2\theta}]$  in town  $t$ . If  $\theta < 1/32$ , then for each pair  $(s_1, s_2)$ , such that  $\sqrt{8\theta} \leq s_1 \leq s_2 \leq 1 - \sqrt{8\theta}$ , there is a neighborhood equilibrium with partisan types  $[s_1 - \sqrt{8\theta}, s_1]$  in district 1 and  $[s_2, s_2 + \sqrt{8\theta}]$  in district 2; if  $s_1 < s_2$ , there is a gap between the districts.

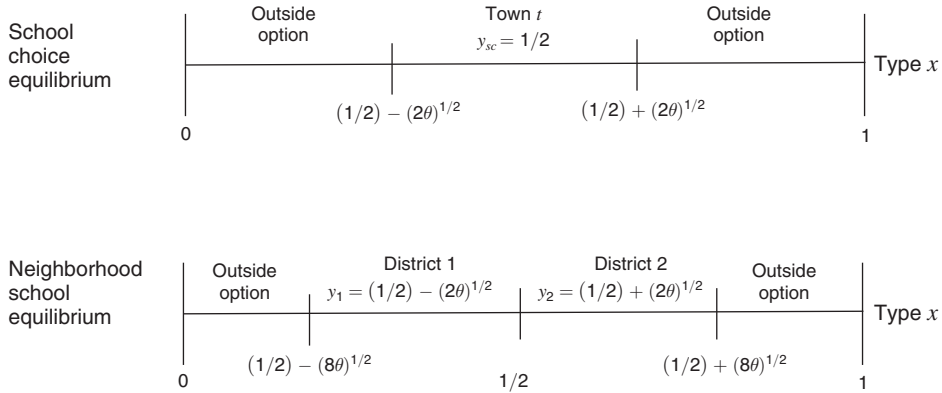


FIGURE 1. SCHOOL ASSIGNMENTS IN EXAMPLE 1 WHEN  $\theta \leq 1/32$

## II. Equilibrium Analysis

### A. Interval Characterization

In the model, students of all types have a preference for high-quality schools. But since higher types are willing to pay more on the margin for increases in quality, competitive pricing induces assortative matching. Competitive pricing has the following implication: a student with type  $x < y$  who selects a school of quality  $y$  “overpays” on the margin for school quality, while a student with type  $x > y$  who selects a school of quality  $y$  values marginal school quality more than its marginal cost, but forsakes additional gains by choosing a school with quality  $y$ .

To make these ideas precise, we define the cost function  $C(x, y)$  as the “cost” for type  $x$  to choose a district in the town with quality  $y$  at competitive price  $p(y)$  instead of an outside option with quality  $y = x$  and competitive price  $p(x)$ . That is,

$$C(x, y) = [v(x, x) - p(x)] - [v(x, y) - p(y)] = p(y) - p(x) - (v(x, y) - v(x, x)).$$

Since Lemma 1 shows that  $p(y) = \int_{z=0}^y \frac{\partial v}{\partial y}(z, z) dz$ , we can express  $C(x, y)$  in integral form:

$$C(x, y) = \int_x^y \int_x^z \frac{\partial^2 v}{\partial x \partial y}(a, z) da dz.$$

In Example 1,  $C(x, y) = (y - x)^2/2$ . The assumption that  $v$  exhibits increasing differences in  $x$  and  $y$  ensures that the integrand in this formula is nonnegative for all  $(x, y)$ , implying  $C(x, y) \geq 0$  and that  $C(x, y)$  is decreasing in  $x$  for  $x < y$  and increasing in  $x$  for  $x > y$ .<sup>13</sup> We use these facts to show that the set of types in district  $d$  in the town form an interval in any NS or SC equilibrium.

<sup>13</sup> Alternatively, we can study the properties of the surplus for type  $x$  and observe that that the marginal price for a school of quality  $y$  is equal to the marginal benefit of school quality for a student of type  $x = y$  at  $(x = y, y)$ . A student with type  $x < y$  who selects a school of quality  $y$  “overpays” on the margin for school quality, whereas a



**PROPOSITION 1:** *In any Neighborhood School or School Choice equilibrium, the set of types in district  $d$  in the town,  $T_{N_d}$ , is an interval  $[\underline{x}_{N_d}, \bar{x}_{N_d}]$ , where  $d = 1$  in a School Choice equilibrium.*

In either a Neighborhood School or School Choice equilibrium, partisans of the town therefore face a trade-off between their partisan interest in residing in the town and choosing a school with quality exactly equal to their type given competitive pricing for their types. Since  $C(x, y)$  increases as  $x$  moves farther from  $y$ , each district in the town will only attract partisans with types close to the quality of that school, and thus an interval of partisan types containing the school’s quality enroll in equilibrium. As a result, a NS equilibrium consists of ordered intervals, where lower type students choose lower quality schools within the town.<sup>14</sup>

If all partisan students enroll in the town, then districts can be ordered according to enrollment  $\{[x_0 = 0, x_1], (x_1, x_2), \dots, (x_{D-1}, x_D = 1]\}$ , where partisan students with types  $x \in (x_{d-1}, x_d]$  choose district  $d$  in the town. In this case, the marginal type at the boundary of the two intervals must be indifferent between the two districts so that partisan students with type  $x$  just below  $x_d$  will choose district  $d$ , while those with  $x$  just above  $x_d$  will choose district  $d + 1$ . Following the terminology of Epple and Romano (2003), we refer to this constraint as a *boundary indifference condition*:

$$\Delta p_{d+1} = p_{d+1} - p_d = v(x_d, y_{d+1}) - v(x_d, y_d),$$

for each  $d$ . Taken together, the  $D - 1$  boundary indifference conditions yield a general formula for the prices of all  $D$  districts in a NS equilibrium:

$$p_d = p_1 + \sum_{j=2}^d \Delta p_j.$$

The set of boundary conditions leave one degree of freedom, which is the price in district 1. There is a unique choice of this price  $p_1$  to meet the equilibrium conditions that all prices must be at least equal to competitive prices for schools of given quality,  $p_d \geq p(y_d)$ , and that at least one price is exactly equal to the competitive price to attract nonpartisans to the remaining supply of houses in the town. In sum, as expressed formally in Corollary 1, there is a unique set of (potential) equilibrium prices for any partition of partisan types into intervals assigned to districts in the town and an associated minimum value of  $\theta$  to induce those decisions by partisans.

**COROLLARY 1:** *For any partition of  $(0, 1)$  into  $D$  intervals  $(0 = x_0, x_1), (x_1, x_2), \dots, (x_{D-1}, x_D = 1)$  where  $0 < x_1 < x_2 < \dots < x_{D-1} < 1$  there is a cutoff  $\theta^*$*

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student with type  $x > y$  who selects a school of quality  $y$  values marginal school quality more than its cost at that point. We emphasize that students of all types have an ex ante preference for high-quality schools. But since higher types are willing to pay more on the marginal for increases in quality, competitive pricing results in assortative matching.

<sup>14</sup>Epple and Romano (2003) establish an analogous result in a model where school quality depends on expenditures and peer quality, and the residential choice problem is combined with voting over the tax schedule. This result can also be shown using techniques from optimal transportation. If we apply Theorem 4.3 of Galichon (2016) for a discrete distribution on the school side, then the corresponding c.d.f. in the optimal assignment is a staircase function, and therefore so is its inverse, which results in the interval characterization.

such that  $\theta \geq \theta^*$ , there is a Neighborhood School equilibrium with  $D$  districts and appropriate measures of houses  $M_d$  in district  $d$  so that partisan types in interval  $d = (x_{d-1}, x_d)$  choose to live in district  $d$  in the town.

### B. Comparing Neighborhood School and School Choice Equilibrium

To facilitate initial comparison of Neighborhood School and School Choice equilibrium, we focus on the neighborhood assignment rule with  $D = 2$  to eliminate the structural advantage whereby neighborhood assignment provides a larger and larger menu of options for partisans as  $D$  increases. We further focus on symmetric equilibria and therefore define  $\theta_N$  as the minimum value of the partisan bonus that can sustain a equilibrium with neighborhood assignment and  $D = 2$  where all below-median partisan types enroll in district 1 and all above-median types enroll in district 2. Compared to a School Choice equilibrium, a Neighborhood School equilibrium with two districts imposes the additional incentive condition that a student at the boundary between the two districts must be willing to enroll in the town rather than to choose the outside option. In Example 1, this additional constraint is redundant. With  $v(x, y) = xy$ , the cross-partial derivative  $\partial^2 v / \partial x \partial y$  is constant in both  $x$  and  $y$ , so the cost function  $C(x, y)$  depends only on the distance from  $x$  to  $y$ . With a Uniform distribution of types, school quality falls exactly at the middle of the range of partisan types in a school. Given the combination of these assumptions in Example 1, the incentive condition for highest and lowest types who choose the town also guarantees that types at the boundary between the two districts in a proposed neighborhood equilibrium prefer the town to the outside option.

Beyond Example 1, the assumption of increasing differences does not place any restriction on the relative magnitudes of the cross-partials  $\partial^2 v / \partial x \partial y$  over the entire region of possible pairs  $(x, y)$ . Note,  $C(x, y)$  is determined both by the distance from  $x$  to  $y$  and the magnitude of the cross-partial  $\partial^2 v / \partial x \partial y$  in the relevant range. If, for instance, this cross-partial is unusually large on the range around  $1/2$ , then middle types tend to require the greatest partisan bonus to enroll in the town and tend to prefer the school quality offered with a school choice rule than under neighborhood assignment. Proposition 2 formalizes this intuition and shows that the assumption of increasing differences in the value function is not sufficient to ensure the phenomenon of greater flight from the town under school choice than neighborhood assignment. The proof of Proposition 2 is by construction: we provide an example in the Appendix where  $\theta_N > \theta_{SC}$  with a Uniform(0,1) distribution of types; this example can be readily adapted to produce the same result for other distributions of types.

**PROPOSITION 2:** *For any symmetric distribution of types, there is a continuous value function  $v$  with increasing differences in  $(x, y)$  such that  $\theta_N > \theta_{SC}$ .*

### C. When Does School Choice Induce More Flight?

We next turn to additional conditions on function  $v$  that restrict the relative magnitudes of its second-order cross partial derivatives since Proposition 2 indicates that the result of Example 1 ( $\theta_{SC} \geq \theta_N$ ) does not hold more generally. We allow for a

general number of districts and define  $\theta_N$  to be the minimum value of  $\theta$  to sustain a neighborhood equilibrium with an equal measure of partisan types in each district and all partisans enrolling in the town. We further assume single-peaked distribution of types to rule out cases where the level of partisanship required to sustain school choice is lower than that for neighborhood assignment because of bimodal distributions where the peaks are far from the center.

Our main result identifies a sufficient condition for a school choice rule to induce more flight than a neighborhood assignment rule. We state this condition as a separate assumption.

**ASSUMPTION 4 (Strong Assortative Matching):** *For all pairs  $(x, y)$ , either  $\partial^3 v / \partial x^2 y \geq 0$  and  $\partial^3 v / \partial xy^2 \geq 0$ , or  $\partial^3 v / \partial x^2 y \leq 0$  and  $\partial^3 v / \partial xy^2 \leq 0$ .*

**THEOREM 1:** *Suppose the value function satisfies Strong Assortative Matching. If the distribution of types is symmetric and single-peaked, then  $\theta_N < \theta_{SC}$ .*

When the third-order partial derivatives are both weakly positive (for example, if  $v(x, y) = x^\alpha y^\beta$  with  $\alpha \geq 1$  and  $\beta \geq 1$ ), the match between school quality and type has greatest effect on the value function for highest student types and school qualities, so it is hardest to attract these highest types to the town. Given equilibrium pricing, it is easier to attract these highest types for a neighborhood assignment rule than with a school choice rule, and so  $\theta_N < \theta_{SC}$ . When third-order partial derivatives are weakly negative, it is hardest to attract lowest-types to enroll, and the same argument implies that  $\theta_N < \theta_{SC}$ .

The proof illustrates two instructive features of the model. First, with weakly positive third-order partial derivatives, the highest district  $D$  has equilibrium price equal to the competitive price for its school quality  $y_D$ . Second, defining  $\theta_D$  as the minimum partisan value required for type  $x_D = 1$  to enroll in district  $D$  rather than choosing the outside option,  $\theta_N = \theta_D$  (as discussed above, the highest types are the hardest to attract to enroll in this case). The strict inequality  $\theta_N < \theta_{SC}$  follows by the same logic as in Example 1: the marginal partisan type is the same under each assignment rule and this type can find a closer match in the choice of school under neighborhood assignment than under the school choice rule.

**COROLLARY 2:** *If the distribution of types is symmetric and single-peaked and the value function satisfies Strong Assortative Matching, then  $\theta_N$  is strictly decreasing in  $D$ .*

This result follows directly from the observation that  $\theta_N = C(1, y_D)$  where  $y_D = E[x|x \geq F^{-1}((D-1)/D)]$ . Since  $y_D$  is strictly increasing in  $D$  given our assumption of no point mass in the distribution, type  $x_D = 1$  chooses an option closer to its ideal school quality as  $D$  increases and so  $C(1, y_D)$  and thus  $\theta_N$  is strictly decreasing in  $D$ . That is, when there are more districts, the value of partisanship needed to support a Neighborhood School equilibrium decreases. For that reason, the comparison between  $D = 2$  and  $D = 1$  provides the best case for school choice.

*Cases Where Partisans Do Not Divide Equally across Districts.*—The formal proof of Theorem 1 relies on the assumption that  $\theta_N$  is defined as the minimum value of partisanship needed to sustain a neighborhood equilibrium in which partisan types divide equally across the  $D$  districts, with measure  $m/D$  in each one. When  $m = 1$ , an assumption of equal-sized districts implies equal measures of partisans across districts when all partisans enroll in the town. When  $m < 1$ , however, an assumption of equal-sized districts need not imply equal measures of partisans across districts: a neighborhood equilibrium with *equal measures of partisans* in each district typically requires a larger district that attract nonpartisans (as only one district is guaranteed to offer a competitive price given the boundary indifference conditions). By contrast, a neighborhood equilibrium with *equal-sized districts* typically involves an imbalance of partisans across the districts when  $m < 1$ .

Online Appendix Section C.5 provides detailed analysis of cases with  $D = 2$  and  $m < 1$  (since  $D = 2$  limits the set of options available to partisans in a neighborhood equilibrium). When  $m < 1$ , it is possible to construct counterexamples to Theorem 1 when we drop the assumption that partisan types divide equally across districts. In these counterexamples, almost all partisans enroll in a single district so that a neighborhood equilibrium approximates the school choice equilibrium. These counterexamples require coordinated choices of  $m$  and district sizes as well as specialized conditions on distribution function  $f$ . In particular,  $f$  must be relatively tightly distributed about  $x = 1/2$  (so that a neighborhood equilibrium can yield an outcome similar to the school choice equilibrium), but must also be relatively flat for values farther away from the peak at  $x = 1/2$  so that the boundary indifference condition has substantive effect on equilibrium prices. These conditions on  $f$  are broadly inconsistent with each other and are not satisfied by standard distribution functions such as the Normal distribution.

### III. Welfare Analysis

#### A. Comparing Equilibria

We extend equilibrium analysis to cases of partial enrollment of partisan types in the town to support welfare comparisons.

**PROPOSITION 3:** *Suppose the value function satisfies Strong Assortative Matching. For any  $\theta < \theta_{SC}$ , there is a School Choice equilibrium with partial enrollment of partisans in the town. If there is a unique School Choice equilibrium and the distribution of types is symmetric and single-peaked, then there is a two-district Neighborhood School equilibrium for the same value of  $\theta$  such that a superset of the partisan types who enroll in the town under the school choice rule enroll under the neighborhood assignment rule.*

The first part of Proposition 3 relates to equilibrium existence under the school choice rule. The second part of Proposition 3 extends Theorem 1 to the case of the partial enrollment of all partisan types in a unique School Choice equilibrium,

where the proof demonstrates that it is possible to use a constructive approach to extend that equilibrium to a two-district neighborhood equilibrium.<sup>15</sup>

Since the number of partisans enrolling in the town is less than the measure of houses in the School Choice equilibrium when  $\theta < \theta_{SC}$ , the equilibrium price in this case must be the competitive price  $p(y_{SC})$  to allow nonpartisans to fill the remaining houses in the town. Therefore, Proposition 3 implies a clear ordering of school qualities in these equilibria with  $y_1 \leq y_{SC} \leq y_2$ . Otherwise, if  $y_{SC} > y_1$  and  $y_{SC} > y_2$ , the highest partisan type living in the town under school choice would not live in the town in this neighborhood equilibrium (and similarly if  $y_{SC} < y_1$  and  $y_{SC} < y_2$ , the lowest partisan type enrolling in the town under school choice would not live in the town in the neighborhood equilibrium).

Corollary 3 shows that the school choice rule produces the greatest relative advantage (in terms of the absolute difference in payoffs) over the neighborhood assignment rule for the partisan type at the cutoff between enrolling in districts 1 and 2 in equilibrium under neighborhood assignment. Further, type  $x = y_{SC}$  prefers school choice over neighborhood assignment because that type obtains its preferred school quality at competitive price in the School Choice equilibrium. Therefore, types near the middle among those enrolling under school choice rule must obtain a higher equilibrium payoff under school choice rule than under neighborhood assignment rule. Similarly, types near the extreme in the district with a price equal to the competitive price for its school quality obtain a higher payoff under neighborhood assignment than under school choice. These types choose a school quality closer to their type under neighborhood assignment than with school choice, but they pay the competitive price for school quality in either case.

**COROLLARY 3** (“Ends against the Middle”): *Denote the equilibrium payoffs for partisan type  $x$  as  $\pi_{SC}(x)$  and  $\pi_N(x)$ . For any partisan type enrolling in the town under school choice,  $\pi_{SC}(x) - \pi_N(x)$  is increasing in  $x$  for types enrolling in district 1 and  $\pi_{SC}(x) - \pi_N(x)$  is decreasing in  $x$  for types enrolling in district 2 under the neighborhood assignment rule.*

Corollary 3 follows from simple differentiation. A partisan of type  $x$  enrolling under school choice receives payoff  $\pi_{SC}(x) = v(x, y_{SC}) - p(y_{SC}) + \theta$ . Proposition 3 implies that the same partisan type also enrolls in the corresponding Neighborhood School equilibrium, so  $\pi_N(x) = v(x, y_N) - p_N + \theta$ . Subtracting one from the other and then differentiating with respect to  $x$  gives  $(\partial/\partial x)(\pi_{SC}(x) - \pi_N(x)) = (\partial v/\partial x)(x, y_{SC}) - (\partial v/\partial x)(x, y_N) = \int_{y_N}^{y_{SC}} (\partial^2 v/\partial x \partial y)(x, z) dz$ . If partisan type  $x$  enrolls in the town under the school choice rule and enrolls in district 1 in the town under neighborhood assignment, then  $y_N = y_1 \leq y_{SC}$  and so the integral is positive. Similarly, if partisan type  $x$  enrolls district 2 in the town under neighborhood assignment, then  $y_N = y_2 \geq y_{SC}$  and so the integral is negative.

<sup>15</sup>Proposition 3 relies on the existence of a unique School Choice equilibrium. If there are multiple non-overlapping School Choice equilibria, it is possible that there is no neighborhood equilibrium that contains each School Choice equilibrium. Examples 3 and 4 in the online Appendix illustrate cases with a unique School Choice equilibrium.

TABLE 1—WELFARE COMPARISON FOR EXAMPLE 1 WHEN  $\theta = 1/72$

Type $x$	NS Eqm. School	SC Eqm. School	NS Eqm. Utility	SC Eqm. Utility	Family Prefers
$[0, \frac{1}{6}]$	Outside opt.	Outside opt.	$x^2/2$	$x^2/2$	Indifferent
$[\frac{1}{6}, \frac{1}{3}]$	$y_1 = \frac{1}{3}$	Outside opt.	$\frac{x}{3} - \frac{1}{24}$	$x^2/2$	Neighborhood
$[\frac{1}{3}, \frac{5}{12}]$	$y_1 = \frac{1}{3}$	$y_{SC} = \frac{1}{2}$	$\frac{x}{3} - \frac{1}{24}$	$\frac{x}{2} - \frac{1}{9}$	Neighborhood
$[\frac{5}{12}, \frac{1}{2}]$	$y_1 = \frac{1}{3}$	$y_{SC} = \frac{1}{2}$	$\frac{x}{3} - \frac{1}{24}$	$\frac{x}{2} - \frac{1}{9}$	School choice
$[\frac{1}{2}, \frac{7}{12}]$	$y_2 = \frac{2}{3}$	$y_{SC} = \frac{1}{2}$	$\frac{2x}{3} - \frac{5}{24}$	$\frac{x}{2} - \frac{1}{9}$	School choice
$[\frac{7}{12}, \frac{2}{3}]$	$y_2 = \frac{2}{3}$	$y_{SC} = \frac{1}{2}$	$\frac{2x}{3} - \frac{5}{24}$	$\frac{x}{2} - \frac{1}{9}$	Neighborhood
$[\frac{2}{3}, \frac{5}{6}]$	$y_2 = \frac{2}{3}$	Outside opt.	$\frac{2x}{3} - \frac{5}{24}$	$x^2/2$	Neighborhood
$[\frac{5}{6}, 1]$	Outside opt.	Outside opt.	$x^2/2$	$x^2/2$	Indifferent

Note: NS means Neighborhood School and SC means School Choice.

Table 1 illustrates the mechanics of the “ends against the middle” result with  $\theta = 1/72$  in Example 1. In equilibrium given this case, partisans of types  $[1/6, 1/2]$  enrolling in district 1 and partisans of types  $[1/2, 5/6]$  enrolling in district 2 in a two-district Neighborhood School equilibrium. By contrast, partisans of types  $[1/3, 2/3]$  enroll in the town in the School Choice equilibrium. The distinction between these results illustrate the “ends against the middle” conflict, whereby the benefit to the school choice rule is greatest for middle types than for types than at either end of the interval. This phenomenon was originally observed by Epple and Romano (1996) in a model where low-income voters have little ability to pay for essentials and place relatively low value on a public good, while high-income voters plan to opt out and pay separately for a privately provided version of that public good. Similarly, in our model low types have relatively limited willingness to pay for school quality, while high types benefit from the creation of a low-cost, low-quality school in a Neighborhood School equilibrium, because that implies provision of a second high-cost, high-quality public school that they can choose instead of the low-quality school.

From the perspective of realized school quality, the neighborhood assignment rule offers a surprising advantage over school choice for the lowest types enrolling in district 1. As shown in Table 1, partisan types between  $x = 1/6$  and  $1/3$  stay in the town in the neighborhood assignment rule, but choose the outside option under the school choice rule. As a result, these types achieve higher utility and also attend higher quality schools under the neighborhood assignment rule than under the school choice rule.

Welfare comparisons are more ambiguous for the next lowest range of partisan types, starting with the lowest partisan type that enrolls in the town under the school choice rule. These partisan types (e.g., from  $x = 1/3$  to  $5/12$  in the example shown in Table 1) attend a higher quality school in the town under the school choice rule than the school they choose in a neighborhood assignment rule, but they achieve higher utility in the Neighborhood School equilibrium. In response to this

observation, advocates of school choice might advance a paternalistic argument that it is beneficial to reduce the difference in quality between schools attended by high and low types and especially to increase the quality of schools attended by low types even if those low types achieve higher utility with a neighborhood assignment rule.

These welfare comparisons have parallels in the debate between the merits of unconditional and conditional cash transfers. Like a conditional cash transfer, the adoption of a school choice rule offers a benefit to lowest types in exchange for taking a costly action. Under school choice, the benefit is a higher quality school and the costly action is living in the town. As Das, Do, and Özler (2005, p. 63) summarize:

*By imposing conditions, the policymaker provides incentives for households to take an action that they would not ordinarily take on their own (otherwise why have the condition in the first place?). But if that action is different from what households would have chosen on their own, their resulting welfare must be lower—by distorting the consumption choices of households, conditional cash transfer programs reduce welfare compared with unconditional cash grants.*

### B. Aggregate Welfare

Given the assumption that  $v(x, y)$  satisfies increasing differences in  $(x, y)$ , assortative matching maximizes the average (realized) value of  $v(x, y)$ . A change from neighborhood assignment to school choice eliminates sorting of types into ordered intervals and thus represents a step away from assortative matching. Combining these observations, if all partisans enroll in the town under either assignment rule, neighborhood assignment should produce greater average values of  $v(x, y)$  than a school choice rule. For instance, in Example 1, if all partisans enroll in the town, then, the average value of  $v(x, y)$  is  $1/16$  in district 1 and  $9/16$  in district 2, for an overall average of  $5/16$  with neighborhood assignment. By contrast, with school choice,  $y_{SC} = 1/2$  and so the average value of  $v(x, y)$  is  $1/2 \times 1/2 = 1/4$ .<sup>16</sup>

The apparent advantage of neighborhood assignment over school choice (in terms of aggregate utility) as a result of assortative matching can be overturned if not all partisans choose to live in the town. Example C.4 in the online Appendix illustrates a case where the existence of the outside option makes high and low types effectively indifferent between the school choice and neighborhood assignment rules. Since middle types prefer the school choice rule, aggregate utility is higher under school choice than the neighborhood assignment rule.

<sup>16</sup>One complication with this comparison is that the average housing price in the town may differ across the two assignment rules. If all partisans enroll in the town in this example, the housing price in the town under school choice is  $1/8$ , while under neighborhood assignment,  $p_1 = 1/32$  and  $p_2 = 9/32$ , for an average price of  $5/32$ . Here, both the average value of  $v(x, y)$  and the average housing price are greater with neighborhood assignment than with school choice, but the net utility remains greater with neighborhood assignment than with school choice. Moreover, changes in housing rents that accrue to some agents are just a transfer.

### C. Extensions

One of our primary goals in this paper was to develop a tractable and transparent model that links school assignment rules and residential sorting patterns. We now discuss briefly the implications of several factors that would be natural to include in the model.

First, we assume that the only relevant characteristic of a house is the quality of the school associated with that house. If houses have additional inherent qualities, then in equilibrium we might expect sorting by type with highest types locating choosing the nicest houses in a given district. However, we would still expect to see a reduction in housing price dispersion after a switch from neighborhood assignment to a school choice rule, which (depending partly on the nature of outside options) would likely result in the same qualitative patterns of flight as in the existing model, with both highest and lowest types moving to other towns under a school choice rule.

Second, we assume that the school choice process necessarily equalizes the qualities of all schools in the town. But differences in school quality could persist if their qualities are determined (at least partly) by exogenously fixed factors or if there are frictions in the school choice process (e.g., through transportation costs, priorities in the school assignment mechanism, or behavioral responses by students in a school choice lottery). With persistent differences in school quality, some high types might plan to enroll in the town if assigned in a lottery to a top quality school, but to move (or choose private school) if assigned to a less desirable school. Adoption of this strategy by high types would likely yield systematic demographic differences in enrollment across schools, undoing to some degree the purpose of the school choice rule.

Third, if housing prices are sticky and/or low-type families are immobile in their residential choices, then a school choice rule could, in fact, equalize the quality of schools in a town without displacing those low types. For example, families in public housing would likely remain in place and would (presumably) see no difference in their housing costs as a result of a change in school assignment rules. Even in this case, however, low-type families not living in public housing could still be displaced from the town by a school choice rule.

In sum, these three extensions tend to reduce but not eliminate the predicted negative effects of a switch from neighborhood assignment to school choice rules for lowest type students, sometimes by suggesting that school choice outcomes will simply mimic neighborhood assignment. For example, as discussed by Epple and Romano (2003), transportation costs could induce residential and school sorting in equilibrium under a school choice rule with low types ranking a nearby low-quality school as their top choice to avoid the costs of attending a distant high-quality school.

Finally, our model also abstracts away from the potential productive effects of choice. Competition appears in the model through the outside option. If a student wishes to attend a school providing a given level of quality, that option is always available outside of the town, and pricing is determined competitively. We do not model any additional competitive effects because the theoretical (e.g., Friedman 1962 and McMillan 2004) and empirical literature (e.g., Hoxby 2003, 2007;



Rothstein 2006, 2007; Card, Dooley, and Payne 2010) on the productive effects of school choice has not reached a firm consensus. Abdulkadiroğlu et al. (2020) show that parental demand in a choice plan is driven by peer quality rather than school effectiveness. MacLeod and Urquiola (2015) develop a model that shows when student composition serves as a signal of effectiveness, it may result in an “anti-lemons” effect where schools invest in screening technologies over productivity improvements.

#### IV. Conclusion

Our analysis contributes to a recent literature on school choice mechanisms, which has focused on the best way to assign pupils to schools given their residential location in a centralized assignment scheme. In particular, research has examined the best way to fine-tune socioeconomic or income-based criteria in choice systems. Cities have now experimented with complex school choice tie-breakers in an effort to achieve a stable balance (Kahlenberg 2003).<sup>17</sup> By incorporating feedback between residential and school choices, our model suggests that analysis of school assignment that does not account for possible residential resorting may lead to an incomplete understanding about the distributional consequences of school choice.

A common rationale for school choice is to improve the quality of school options for disadvantaged students. But, our analysis shows that feedback from residential choice can undercut this approach, for if a school choice plan succeeds in narrowing the range between the lowest and highest quality schools, that change should compress the distribution of house prices in that town, thereby providing incentives for the lowest and highest types to exit from the town’s public schools. This intuition extends to the idealized case of a symmetric model of many towns and partisans, where each town adopts school choice and all schools within a given town have the same quality. Although there is an equilibrium in this idealized model where schools in all towns have the same quality, this equilibrium would likely be unstable, and instead we would expect to observe an equilibrium with differentiation of school qualities and housing prices across towns. That is, the within-town diversity observed in equilibrium under neighborhood assignment could be replicated in cross-town diversity under school choice.

A broader implication of our model is that systemic changes beyond the details of the school assignment system may be necessary to reduce inequalities in educational opportunities. One such approach addresses the residential choice problem directly by transferring low-income families to better neighborhoods. For instance, the US Department of Housing and Urban Development’s Moving to Opportunity Program offered vouchers to low-income families to move to low-poverty neighborhoods. The evidence on the effects of this experiment on educational outcomes is mixed (Kling, Liebman, and Katz 2007), though a recent literature suggests there may be some positive effects (Chetty, Hendren, and Katz 2016). A second approach involves directly influencing the quality of schools available to low-income families.

<sup>17</sup> Also see Richard D. Kahlenburg, “Elite, Separate, Unequal,” *New York Times*, June 23, 2014.

There is growing evidence that some urban charter schools generate large achievement effects and more disadvantaged children benefit more (Abdulkadiroğlu et al. 2011; Angrist, Pathak, and Walters 2013; Walters 2018). Our model suggests that the general approach of attacking the roots of schooling inequities likely has more promise than efforts solely designed to change the rules by which students are assigned to schools.

#### APPENDIX A: PROOF OF THEOREM 1

Assume that the type distribution is symmetric and single-peaked on  $[0, 1]$ . Let  $F$  be the cumulative distribution;  $F(0) = 0$ ,  $F(0.5) = 0.5$ ,  $F(1) = 1$ , and  $F(1 - x) = 1 - F(x)$  for  $x \in [0, 1]$ .

Suppose that there are  $D$  districts of equal size in a Neighborhood School equilibrium. Write the associated cutoffs in ascending order  $x_0 = 0 < x_1 < x_2 < \dots < x_D = 1$ , so that a student of type  $x$  chooses district  $d$  if  $x_{d-1} \leq x \leq x_d$ . By construction,  $F(x_d) - F(x_{d-1}) = 1/D$ , since each district enrolls the same number of students. Since the distribution is symmetric about  $1/2$ ,  $x_d = 1 - x_{D-d}$ , for each  $d \leq D$ . If  $D$  is even, then types  $x \leq 0.5$  enroll in districts 1 to  $D/2$  and types  $x \geq 0.5$  enroll in districts  $(D/2) + 1$  to  $D$  with  $x_{D/2} = 0.5$ . If  $D$  is odd, then types  $x$  just below and just above  $0.5$  enroll in district  $(D + 1)/2$ . In fact, if  $D$  is odd, district  $(D + 1)/2$  is symmetric about the median, so the average type in that district is  $y_{\frac{D+1}{2}} = 0.5$ .

Define  $\Delta_d = v(x_d, y_{d+1}) - v(x_d, y_d)$ , for values  $d = 1, \dots, D - 1$ . In a Neighborhood School equilibrium, the boundary indifference condition implies that the difference in prices for adjacent districts  $d$  and  $d + 1$  is  $\Delta_d = p_{d+1} - p_d$ . Define the price differential that would occur at competitive prices for these districts as  $\Delta p_d = p(y_{d+1}) - p(y_d)$ .

**LEMMA 2:** *If the distribution of types is symmetric and single-peaked and the third-order partials of the value function are weakly positive for all pairs  $(x, y)$ , then  $p_D = p(y_D)$ .*

In any Neighborhood School equilibrium, at least one district is priced at the competitive price on the outside market for its equilibrium quality. The lemma asserts that when the third-order partials are weakly positive, then the highest-quality district must be at the competitive price with  $p_D = p(y_D)$ ; this district provides a reference price that, together with the indifference condition and the  $\Delta_d$  values, determines the equilibrium prices for the other districts.<sup>18</sup>

**CLAIM 1:** *If  $x_d \geq 1/2$ , then  $y_{d+1} - x_d \geq x_d - y_d$ .*

We first consider the special case where  $x_d = 1/2$ , which requires  $D$  to be even with  $d = D/2$ . In this case, districts  $d$  and  $d + 1$  lie just above (district  $d + 1$ ) and

<sup>18</sup>By symmetry, when the third-order partials are weakly negative, then the lowest-quality district must be the reference price district with  $p_1 = p(y_1)$ .

below (district  $d$ ) the median value  $x = 1/2$ . By symmetry,  $y_d$  and  $y_{d+1}$  are equidistant from the median at  $x = 1/2$ , so we obtain the desired result  $y_{d+1} - x_d = x_d - y_d$ .

Next, if  $x_d > 1/2$ , then  $y_d \geq 1/2$ . All districts  $d$  to the right of the median have  $y_d > 1/2$  and  $x_d > 1/2$ . Since  $f$  is symmetric and single-peaked,  $f$  is weakly decreasing in districts  $d$  through  $D$ . Suppose  $f(x_d) = K$ . Given that measure  $1/D$  students enroll in each district and  $f$  is weakly decreasing, the minimum possible value for  $y_d$  occurs if  $f(x)$  is constant and equal to  $K$  throughout district  $d$  (where  $x_{d-1} \leq x \leq x_d$ ), in which case,  $x_d = x_{d-1} + (K/D)$  and  $y_d^{\min} = x_d - (K/2D)$ . Similarly, the minimum possible value for  $y_{d+1}$  occurs if  $f(x) = K$  through district  $d + 1$ , in which case,  $x_{d+1} = x_d + (K/D)$  and  $y_{d+1}^{\min} = x_d + (K/2D)$ . That is,  $y_d \geq y_d^{\min} = x_d - (K/2D)$  and  $y_{d+1} \geq y_{d+1}^{\min} = x_d + (K/2D)$ , implying the result.

CLAIM 2: *If  $x_d \geq 1/2$ , then  $\Delta_d \leq \Delta p_d$ .*

First, note that

$$\begin{aligned} (1) \quad \Delta p_d - \Delta_d &= (p(y_{d+1}) - p(y_d)) - (p_{d+1} - p_d) \\ &= \int_{y_d}^{y_{d+1}} \frac{\partial v}{\partial y}(z, z) dz - \int_{y_d}^{y_{d+1}} \frac{\partial v}{\partial y}(x_d, z) dz. \end{aligned}$$

Since we assume the second-order cross-partial derivatives of  $v$  are nonnegative, we rewrite equation (1) as the difference of two positive-valued integrals:

$$\Delta p_d - \Delta_d = \underbrace{\int_{x_d}^{y_{d+1}} \left[ \frac{\partial v}{\partial y}(z, z) - \frac{\partial v}{\partial y}(x_d, z) \right] dz}_{>0} - \underbrace{\int_{y_d}^{x_d} \left[ \frac{\partial v}{\partial y}(x_d, z) - \frac{\partial v}{\partial y}(z, z) \right] dz}_{>0}$$

Applying the fundamental theorem of calculus, We rewrite each term as a double integral of a second-order mixed partial derivative of  $v$ :

$$(2) \quad \Delta p_d - \Delta_d = \int_{x_d}^{y_{d+1}} \int_{x_d}^z \frac{\partial^2 v}{\partial x \partial y}(a, z) da dz - \int_{y_d}^{x_d} \int_z^{x_d} \frac{\partial^2 v}{\partial x \partial y}(a, z) da dz.$$

From Claim 1, we know that for  $x_d \geq 1/2$ ,  $y_{d+1} - x_d \geq x_d - y_d$ . Therefore, the first integral in equation (2) covers a (weakly) larger range of pairs  $(a, z)$  than the second integral in equation (2) and the arguments take on systematically higher values in the first integral than in the second. Since the third-order mixed partial derivatives of  $v$  are nonnegative, the smallest value of the integrand in the first integral is at least as large as the largest value of the integrand in the second integral in equation (2). Taking these facts together, the first integral cannot be smaller than the second one, which shows  $\Delta p_d \geq \Delta_d$ .

CLAIM 3: *If  $x_d < 1/2$ , then  $\Delta p_d + \Delta p_{D-d} \geq \Delta_d + \Delta_{D-d}$ .*

As in the proof of Claim 2,

$$(3) \quad \Delta p_d - \Delta_d = \underbrace{\int_{x_d}^{y_{d+1}} \int_{x_d}^z \frac{\partial^2 v}{\partial x \partial y}(a, z) \, dadz}_{=C} - \underbrace{\int_{y_d}^{x_d} \int_z^{x_d} \frac{\partial^2 v}{\partial x \partial y}(a, z) \, dadz}_{=D}.$$

With  $x_d < 1/2$ , it is not necessarily true that  $\Delta p_d \geq \Delta_d$  because the positive integral ranges over values closer to the center than the first integral in equation (3) and so covers a (weakly) smaller range of values than the second integral in equation (3). Instead, we apply a version of that argument to the combination of terms on opposite sites of the median,  $\Delta p_d + \Delta p_{D-d}$ , compared to  $\Delta_d + \Delta_{D-d}$ .

Using the analysis from Claim 2,

$$(4) \quad \Delta p_{D-d} - \Delta_{D-d} = \underbrace{\int_{x_{D-d}}^{y_{D-d+1}} \int_{x_{D-d}}^z \frac{\partial^2 v}{\partial x \partial y}(a, z) \, dadz}_{=A} - \underbrace{\int_{y_{D-d}}^{x_{D-d}} \int_z^{x_{D-d}} \frac{\partial^2 v}{\partial x \partial y}(a, z) \, dadz}_{=B}.$$

Note that

$$(\Delta p_d - \Delta_d) + (\Delta p_{D-d} - \Delta_{D-d}) = A - B + C - D.$$

We will show that the values of the integrands can be strictly ordered (because the ranges of  $(x, y)$  values are strictly ordered for these integrals) with highest values in Integral A, next highest in Integral B, third highest in Integral C, and lowest in Integral D.

To see why, first note that by symmetry,  $x_d = 1 - x_{D-d}$  and  $y_d = 1 - y_{D-d+1}$ , for each  $d$ . As a result,  $x_d - y_d = y_{D-d+1} - x_{D-d}$ . Similarly,  $y_{d+1} - x_d = x_{D-d} - y_{D-d}$ . Therefore, Integrals A and D cover equal-sized triangles of  $(x, y)$  values, and Integrals B and C also cover equal-sized triangles of  $(x, y)$  values. In addition, from Claims 1 and 2, Integrals A and D cover larger-sized triangles of  $(x, y)$  values than Integrals B and C. This means that we can divide Integrals A and D into sub-integrals  $A_1, A_2$  and  $D_1, D_2$ , where  $A_1, B, C$ , and  $D_1$  all cover equal-sized ranges of  $(x, y)$  values. Since we can strictly order the integrands in these terms,  $A_1 \geq B \geq C \geq D_1$  and  $A_2 \geq D_2$ , so  $A - B + C - D = (A_1 - B) + (C - D_1) + (A_2 - D_2) \geq 0$  because each term is weakly positive.

CLAIM 4:  $p_D = p(y_D)$ .

There are  $D$  different possible values for  $p_D$ : (1)  $p(y_D)$ , (2)  $p(y_{D-1}) + \Delta_{D-1}$ , (3)  $p(y_{D_2}) + \Delta_{D-2} + \Delta_{D-1}, \dots$ , (d)  $p(y_{D-d+1}) + \sum_{j=D-d+1}^{D-1} \Delta_j, \dots$ , or (D)  $p(y_1) + \sum_{j=1}^{D-1} \Delta_j$ . In equilibrium, the price for district  $D$  must be the maximum of these values. We show that  $p(y_D)$  is greater than or equal to each of the other  $D - 1$  possible prices.

For each value of  $d$ , we want to show that  $p(y_D) \geq p(y_{D-d+1}) + \sum_{j=D-d+1}^{D-1} \Delta_j$ , or equivalently  $p(y_D) - p(y_d) - \sum_{j=d}^{D-1} \Delta_j \geq 0$ . To simplify this expression, we write  $p(y_D)$  as a telescoping sum of differences:  $p(y_D) = p(y_d) +$

$[p(y_{d+1}) - p(y_d)] + [p(y_{d+2}) - p(y_{d+1})] + \dots + [p(y_D) - p(y_{D-1})]$  or equivalently  $p(y_D) = p(y_d) + \sum_{j=d}^{D-1} \Delta p_j$ . The requirement that  $p(y_D) - p(y_d) - \sum_{j=d}^{D-1} \Delta_j \geq 0$  is therefore equivalent to  $\sum_{j=d}^{D-1} (\Delta p_j - \Delta_j) \geq 0$ .

If  $x_d \geq 1/2$ , then each pair of terms  $\Delta p_d - \Delta_d$  is nonnegative by Claim 2. Thus,  $\sum_{j=D-d}^{D-1} (\Delta p_j - \Delta_j) \geq 0$ , or  $p(y_D) \geq p(y_d) + \sum_{j=D-d+1}^{D-1} \Delta_j$ , as desired. If  $x_d < 1/2$ , then we can apply a combination of Claims 2 and 3 to reach the same conclusion.

If  $D$  is even, then for each  $d \leq D/2$ ,  $\Delta p_d + \Delta p_{D-d} - \Delta_d - \Delta p_{D-d} \geq 0$ , by Claim 3. Therefore,  $\sum_{j=d}^{D/2} \Delta p_j + \Delta p_{D-j} - \Delta_j - \Delta p_{D-j} \geq 0$ . Then we can rewrite  $\sum_{j=d}^{D-1} (\Delta p_j - \Delta_j) = \sum_{j=d}^{D/2} (\Delta p_d + \Delta p_{D-d} - \Delta_d - \Delta p_{D-d}) + \sum_{j=\frac{D-d}{2}+1}^{D-d+1} (\Delta p_j - \Delta_j)$ . Similarly, if  $D$  is odd, we can rewrite  $\sum_{j=d}^{D-1} (\Delta p_j - \Delta_j) = \sum_{j=d}^{\frac{D-1}{2}} (\Delta p_d + \Delta p_{D-d} - \Delta_d - \Delta p_{D-d}) + \sum_{j=\frac{D+1}{2}}^{D-d+1} (\Delta p_j - \Delta_j)$ . In each case, the first sum is nonnegative by Claim 3 and the second sum is nonnegative by Claim 2, so  $\sum_{j=d}^{D-1} (\Delta p_j - \Delta_j) \geq 0$ , as desired for Claim 4, which in turn establishes the Lemma.

**LEMMA 3:** *If the distribution of types is symmetric and single-peaked and the third-order partials of the value function are weakly positive for all pairs  $(x, y)$ , then  $\theta_N = \theta_D$ , where  $\theta_D$  is the partisan bonus required for types from  $x_D$  to  $x = 1$  to prefer district  $D$  in the town at price  $p(y_D)$  to the outside option.*

Denote  $\theta_d$  as the minimum value required for type  $x_d$  to enroll in the town under neighborhood assignment. For  $d = 1, 2, \dots, D - 1$ , type  $x_d$  is indifferent between enrolling in district  $d + 1$  and district  $d$ , and so  $\theta_d$  is computed by finding the minimum partisan value required for  $x_d$  to enroll in either of these districts. For the extreme types, there is only one adjacent district, so  $\theta_D$  is the minimum partisan value required for type  $x_D = 1$  to enroll in district  $D$  and similarly,  $\theta_0$  is the minimum partisan value required for type  $x_0 = 0$  to enroll in district 1.

For each  $d$  from  $d = 1$  to  $d = D$ , we compare  $\theta_{d-1}$  and  $\theta_d$  by finding the separate partisan values required for these types to enroll in the same district  $d$ . That is,  $v(x_d, y_d) - p_d + \theta_d = v(x_d, x_d) - p(x_d)$  and  $v(x_{d-1}, y_d) - p_d + \theta_{d-1} = v(x_{d-1}, x_{d-1}) - p(x_{d-1})$ . Solving these equations for  $\theta_d$  and  $\theta_{d-1}$  gives  $\theta_d = v(x_d, x_d) - v(x_d, y_d) - p(x_d) + p_d$  and  $\theta_{d-1} = v(x_{d-1}, x_{d-1}) - v(x_{d-1}, y_d) - p(x_{d-1}) + p_d$ . Subtracting the two previous equations from one another, we obtain  $\theta_d = \theta_{d-1} + [v(x_d, x_d) - v(x_d, y_d)] + [v(x_{d-1}, y_d) - v(x_{d-1}, x_{d-1})] - [p(x_d) - p(x_{d-1})]$ , which can be written in integral form as

$$\theta_d = \theta_{d-1} + \int_{y_d}^{x_d} \frac{\partial v}{\partial y}(x_d, z) dz + \int_{x_{d-1}}^{y_d} \frac{\partial v}{\partial y}(x_{d-1}, z) dz - \int_{x_{d-1}}^{x_d} \frac{\partial v}{\partial y}(z, z) dz.$$

In double-integral form, we have

$$(5) \quad \theta_d = \theta_{d-1} + \underbrace{\int_{y_d}^{x_d} \int_z^{y_d} \frac{\partial^2 v}{\partial x \partial y}(a, z) da dz}_{=\text{Integral 1}} - \underbrace{\int_{x_{d-1}}^{y_d} \int_{x_{d-1}}^z \frac{\partial^2 v}{\partial x \partial y}(a, z) da dz}_{\text{Integral 2}}.$$

We apply the following extended versions of Claims 1 to 4 from the proof of Lemma 2 to this equation to conclude that  $\theta_D \geq \theta_d$  for each  $d$ .

*Extended Version of Claim 1.*—If  $d > D/2$  then  $y_d \geq 0.5$ . If  $y_d = 0.5$ , then district  $d$  is symmetric about  $y_d = 0.5$ , so  $y_d - x_{d-1} = x_d - y_d$ . If  $y_d > 0.5$ , then since the distribution of types is symmetric and single-peaked, the mean in the range of types from  $x_{d-1}$  to  $x_d$  (which is  $y_d$ ) falls at or below the midpoint between  $x_{d-1}$  and  $x_d$ , so  $x_d - y_d \geq y_d - x_{d-1}$ .

*Extended Version of Claim 2.*—This result indicates that if  $y_d \geq 0.5$  (i.e.,  $d > D/2$ ), Integral 1 in the double-integral equation (5) above covers a larger range of pairs  $(x, y)$  than Integral 2 in equation (5). Once again, the integrand in Integral 1 is always larger than the integrand in Integral 2. This shows that if  $d > D/2$ , then  $\theta_d \geq \theta_{d-1}$ . Iterating this reasoning, if  $d > D/2$ , then  $\theta_D \geq \theta_d$ .

*Extended Version of Claim 3.*—If  $d \leq D/2$ , the integrand in the double-integral formula is larger for Integral 1 than for Integral 2, but Integral 2 covers a wider range of pairs of values than does Integral 1. Following the logic of Claim 3 above, we pair two sets of integrals based on the observation that  $x_d - y_d = y_{D-d+1} - x_{D-d}$  and  $y_d - x_{d-1} = x_{D-d+1} - y_{D-d+1}$  to produce the conclusion  $\theta_{D-d+1} - \theta_{D-d} + \theta_d - \theta_{d-1} \geq 0$  or  $\theta_d + \theta_{D-d+1} \geq \theta_{d-1} + \theta_{D-d}$ .

*Extended Version of Claim 4.*—If  $d \leq D/2$ , then we write  $\theta_D - \theta_d$  as a telescoping sum of first differences:  $\theta_D - \theta_d = \sum_{j=d}^{D-1} (\theta_j - \theta_{j-1})$ .

**Case 1:** If  $D$  is even, then for each  $j = d$  to  $j = D/2$ , we can pair the term  $\theta_j - \theta_{j-1}$  with  $\theta_{D-j+1} - \theta_{D-j}$ . That is,  $\theta_D - \theta_d = \sum_{j=d}^{D/2} [(\theta_j - \theta_{j-1}) + (\theta_{D-j+1} - \theta_{D-j})] + \sum_{j=D-d+2}^D (\theta_j - \theta_{j-1})$ .<sup>19</sup> By Claim 3, each term in the first sum is nonnegative. By Claim 2, each term in the second sum is nonnegative.

**Case 2:** If  $D$  is odd, then for each  $j = d$  to  $j = (D-1)/2$ , we pair the term  $\theta_j - \theta_{j-1}$  with  $\theta_{D-j+1} - \theta_{D-j}$ . That is,  $\theta_D - \theta_d = \sum_{j=d}^{(D-1)/2} [(\theta_j - \theta_{j-1}) + (\theta_{D-j+1} - \theta_{D-j})] + \sum_{j=D-d+2}^D (\theta_j - \theta_{j-1})$ . Once again, by Claim 3, each term in the first sum is nonnegative and by Claim 2, each term in the second sum is nonnegative. In either case, the conclusion is that  $\theta_D - \theta_d \geq 0$ , so  $\theta_N = \theta_D$ , as desired. This proves the Extended Version of Claim 4 and completes the proof of the lemma.

#### PROOF OF THEOREM GIVEN THE LEMMAS:

We now use the lemmas to prove the theorem. We know from Lemma 3 that  $\theta_D = \theta_N$ , where  $\theta_D$  is the partisan bonus required for type  $x = 1$  to enroll in district  $D$

<sup>19</sup>Note that if  $d = 1$ ,  $D - d + 2 > D$  and so this second sum is empty.

with school quality  $y_D$  under market pricing at price  $p(y_D)$ . The value  $\theta_{SC}$  is the partisan bonus required for type  $x = 1$  to enroll under school choice with school quality  $y_{SC} = 1/2$  and price  $p(1/2)$ . With two or more districts,  $y_D > 1/2$ , so a partisan of type 1 prefers district  $D$  in the town with full enrollment of partisan types to the town under school choice with full enrollment of partisan types, so  $\theta_N = \theta_D < \theta_{SC}$ , where the strict inequality follows from the assumption of that  $v$  exhibits strictly increasing differences (so its second-order mixed partials are strictly positive). ■

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