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Customer poaching and brand switching

Drew Fudenberg*

and

Jean Tirole**

Firms sometimes try to "poach" the customers of their competitors by offering them inducements to switch. We analyze duopoly poaching under both short-term and long-term contracts assuming either that each consumer's brand preferences are fixed over time or that preferences are independent over time. With fixed preferences, short-term contracts lead to poaching and socially inefficient switching. The equilibrium with long-term contracts has less switching than when only short-term contracts are feasible, and it involves the sale of both short-term and long-term contracts. With independent preferences, short-term contracts are efficient, but long-term contracts lead to inefficiently little switching.

Is Rosaline, that thou didst love so dear,
So soon forsaken? young men's loves then lies
Not truly in their hearts, but in their eyes.

—*Romeo and Juliet,* act 2, scene 3

Take my wife—please.

—Henny Youngman

1. Introduction

■ If firms know the purchasing habits of potential customers, and trade between consumers is either infeasible or impractical, the firms can try to "poach" the current customers of their competitors by offering them special discounts or other inducements to switch. For example, in 1994 about 20% of all U.S. households changed their long-distance providers,[1] and in 1995–1996 several of the long-distance providers offered one-time bonuses for switching to their service from a competitor. The information required for firms to attempt this and other forms of "behavior-based" price discrimination is becoming increasingly available in many markets, yet behavior-based price discrimination has received little theoretical scrutiny.[2]

* Harvard University; dfudenberg@harvard.edu.

** IDEI, Université de Toulouse, CERAS, Paris, and MIT; tirole@cict.fr.

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[1] See Schwartz (1997).

[2] We discuss previous work at the end of this section. See Rossi, McCulloch, and Allenby (1996) for

Our goal in this article is to lay out the conceptual issues that arise in analyzing poaching, and to determine the nature of equilibrium poaching in the context of a particular model. We will mention several examples for motivation, but the model we have chosen is at best a crude approximation of any of them, and there are reasons to believe that some of the most striking aspects of recent attempts at poaching were "mistakes" rather than equilibrium phenomena.[3]

To analyze poaching, we consider a simple Hotelling model of duopoly with horizontal differentiation, where firms A and B sell their goods directly to consumers whose relative preferences for the two firms are indexed by their position along an interval. We suppose that consumers have 0-1 demand, so that each period each consumer uses either one unit of good A, one unit of good B, or neither. Moreover, we assume that the value of the goods exceeds their cost by a large enough amount so that in equilibrium, all customers will in fact use one of the goods in each period, that switching costs are zero, and that all consumers are familiar with the properties of both goods.

Taken together, our assumptions may best approximate the provision of services such as long-distance telecommunications that are delivered directly to the consumer's residence. Many services that are delivered directly to the home have until recently been regulated as "natural monopolies," the prospect that these services will become provided competitively suggests that the number of such examples may grow. Additionally, we hope our analysis can shed some light on what to expect in other markets, such as those for credit cards, mortgages, and skilled workers. Another example is that of "commuter tickets" on airlines, where several units of the service are bundled together.[4]

Throughout the article, we suppose that there are only two periods, so that poaching can only occur in the second period, if it occurs at all. In the simplest case, firms can offer only short-term contracts, and consumers' preferences are fixed from one period to the next. Here we find that in equilibrium firms do offer second-period discounts to customers who purchased from their opponent in the first period, and moreover that a positive fraction of consumers switch suppliers from one period to the next, even though customers who switch shift to the supplier they like less, so that the switching is inefficient. The intuition for this is clear: in the second period each firm faces two separate markets, that of its own past customers and that of its competitor's. Customers who bought from the competitor are revealed to have a lower relative preference for the firm's product, and so profit maximization requires that they be offered a lower price. Moreover, in equilibrium customers have correct expectations, and they take this second-period poaching into account when making their first-period decisions. When a firm has a larger first-period market, it is more of a "fat cat" in defending that market in the second period, so its opponent's "poaching" price is an increasing function of the "defender's" market share. When consumers' preferences are uniformly distributed (so demand is linear), this feedback makes first-period demand less elastic

a discussion of available purchase-history data. Note also that when software companies offer discounts on "upgrades," they are effectively price discriminating against new computer users. Whether the company is also discriminating for or against users of rivals' products depends on whether the company offers a "competitive upgrade" and, if so, how that price compares to the one charged to current users.

[3] In March 1997, AT&T decided to stop offering $100 inducements for consumers to switch to their service, since it had found that the customers who were attracted this way tended to switch back and forth between providers.

[4] When Air Liberté entered the Paris-Toulouse market, it offered a free subscription to Air Inter's (the incumbent's) subscribers while charging nonsubscribers a positive price.

than it would be if price discrimination were banned, and equilibrium poaching leads to first-period prices that are higher than those of the static model.

In a model with switching costs, firms would also offer discounts to their competitor's customers, but the equilibrium outcome is quite different. For example, Chen (1997) analyzes the duopoly equilibrium in short-term contracts of a homogeneous-good model in which consumers learn their switching costs at the start of the second period. As in other models of switching costs, and unlike our findings, firms price below static levels in the first period (indeed, in Chen's model they price below marginal cost) and then raise their prices once consumers are locked in.[5]

We then analyze the case where firms can offer binding long-term contracts at date 1.[6] Here it is *possible* for a firm to prevent any poaching of its customers by its rival, simply by choosing a contract that specifies two periods of sales with infinite penalties for breach. However, we show that a firm would not choose to offer such contracts, since it does better by offering a higher price for the long-term contract and a lower breach penalty, so that customers who have only a weak preference for the firm will buy its product in the first period and then pay the breach penalty in the second period to take advantage of the competitor's low "poaching" price. In effect, the firm "invites" second-stage poaching to make it more attractive in the first period. Put differently, and more precisely, a firm that locks in all of its first-period customers cannot use the second-period market to discriminate between those customers with a strong preference for its brand and the customers who have only a weak preference.

We derive our basic results about equilibrium long-term contracting in a model where firms offer consumers the choice of purchasing either a short-term contract or a contract for two periods of supply. After doing this, we check that the equilibrium remains an equilibrium when more general long-term contracts are allowed. We suspect that this is in fact the only equilibrium of that more general model, but we have not established this conjecture.

When consumers' preferences are the same in every period, social efficiency requires no switching at all, so it is perhaps not surprising that the equilibrium long-term contracts involve a lower penalty for breach and more switching than is socially optimal. To test the robustness of our conclusions, we also examine the case where each consumer's preferences are i.i.d. over time, so that there are efficiency losses if long-term contracts lock consumers in to using the same supplier in each period. Here too the equilibrium contracts involve a positive but finite penalty for breach, since these penalties effectively transfer surplus from the other firm to the current one in states where the consumer's preferences change enough to induce her to switch. In this case, long-term contracts are used to strengthen the bargaining power of the firm's customers vis-à-vis the poacher. This result is thus reminiscent of the contracts between an incumbent monopolist and a consumer studied by Aghion and Bolton (1987), and it provides some evidence for the robustness of the basic forces underlying their conclusions.

Our model is related to several strands of the literature. One is the classic literature on static price discrimination. More recently, Lee (1997) studies poaching in a one-period model where some customers are already known to be "loyal" to the brand;

[5] Taylor (1999) extends Chen's model in several ways. See Nilssen (1992), Klemperer (1995), and Kim and Koh (1997) for other discussions of poaching in models with switching costs. With identical consumers and identical switching costs, firms still offer discounts to induce switching but no switching actually occurs.

[6] Employers can use delayed stock options and delayed vesting of pension benefits to impose switching penalities on their employees. Mobile phone companies often impose a six-month or one-year minimum contract. Another example is Deutsche Telekom's 1997 imposition of a penalty on customers who switched suppliers, a practice that was later prohibited by German regulators.

since he does not study how consumers came to be identified as loyal, his model roughly corresponds to the second period of ours. Another strand is the literature on "ratcheting" in one-principal, one-agent models, which studies how a player's inability to commit herself about how she will use some information will lead other players to avoid revealing the information in the first place.[7] Yet another is the literature on how two players can use a contract to extract surplus from a third, as in Diamond and Maskin (1979) and the Aghion and Bolton (1987) article mentioned above.

This article is also closely related to our article "Upgrades, Tradeins and Buybacks" (1998). There we studied a similar sort of "observationally based" price discrimination, in which a consumer's purchase of the older version of a good could reveal information the monopolist might use in setting that consumer's price for the new version.[8] The present article differs from our previous one in several respects: we now consider duopoly instead of monopoly, nondurable rather than durable goods, and long-term as well as short-term contracts. Despite these differences, the two articles have a common theme: there are more forms of price discrimination than the standard typology suggests, and these more complex forms appear to have increasing relevance for mass-market consumer goods and services.

The most similar previous work is that of Caminal and Matutes (1990), who consider the i.i.d. preferences that we study in Section 6. They consider only the uniform distribution, and they proceed by computation; our treatment of the i.i.d. case attempts to present a clearer and more general intuition.[9]

2. The model

■ There are two firms, A and B, who produce (nondurable) goods A and B, respectively, with constant marginal cost of c per unit. The firms act to maximize the expected discounted value of their profits, using common discount factor $\delta < 1$. There are two periods, 1 and 2; each period a consumer can use either a unit of good A or a unit of good B or neither, and can make payments to either or both firms. There is a continuum of consumers, whose preferences are quasi-linear in money and who are indexed by $\theta \in [\underline{\theta}, \overline{\theta}]$, where $\underline{\theta} = -\overline{\theta} < 0$ and θ is a measure of a consumer's relative preferences for B over A. We assume for now that each consumer's preferences are constant over time; Section 5 explores the alternative case of i.i.d. types. There is a known cumulative distribution function F over θ. Throughout the article we will impose the following "regularity" condition on the distribution of preferences:

Assumption 1 (*"nice" demand curves*). The cumulative distribution function F is smooth, with strictly positive density f. Moreover, F is symmetric about zero and satisfies the monotone hazard rate (MHR) condition that $f(\theta)/[1 - F(\theta)]$ is strictly increasing in θ.[10]

[7] See, e.g., Freixas, Guesnerie, and Tirole (1985) and Laffont and Tirole (1988).

[8] As in this article, that sort of price discrimination required the firm to have information about the consumer's past decisions, and in fact we studied three cases corresponding to three different information conditions, the "identified," "semi-anonymous," and "anonymous" cases. Lee and Lee (1998) and Nahm (1999) also discuss the anonymous case.

[9] Padilla and Pagano (1997) consider poaching in a less similar model. Their point is that a lender may want to commit to actions that facilitate second-period poaching by other lenders in order to reduce the borrower's fear that the lender will expropriate the returns on the borrower's first-period investment. The poaching in their model is not the result of behavior-based price discrimination, and it is closer to the literature on second sourcing than to our model.

[10] The symmetry of F then implies that $f(\theta)/F(\theta)$ is strictly decreasing.

Consumer θ's behavior is chosen to maximize the expected discounted sum of per-period utilities $u_t(\theta)$, where for a given period-t payment p_t, $u_t(\theta) = v - \theta/2 - p_t$, if the consumer uses good A at date t, $u_t(\theta) = v + \theta/2 - p_t$ if the consumer uses good B at date t, and $u_t(\theta) = -p_t$ if the consumer doesn't use either good at date t. All consumers use the same discount factor δ as the firms.

To avoid solutions at the boundaries and cut down on the number of cases we analyze, we assume throughout that all customers use the good in both periods. Intuitively, this will be an equilibrium outcome if the value of the goods is sufficiently high.

We will consider equilibrium behavior in this model under different specifications of the space of feasible contracts and on the types of commitments that can be enforced. In all cases, we suppose that firms act simultaneously within each period, and that at the beginning of the first period firms have no information about the preferences of individual consumers. Before proceeding to the study of duopoly poaching, we first analyze the equilibrium of a duopoly where there are only short-term contracts and no price discrimination so we can measure the effects of behavior-based price discrimination in a duopoly.

3. Short-term contracts without price discrimination

■ We now examine the benchmark case where there are only short-term contracts and there is no price discrimination, either because firms do not observe the first-period decisions of individual consumers or because price discrimination is illegal. Here the model reduces to two replications of the static Hotelling equilibrium. To solve for this equilibrium, consider the one-period model, and let p^A and p^B be the prices set by firms A and B respectively. Since neither firm will set price below cost, and since each firm can make a positive profit when its opponent's price does equal the cost, in equilibrium both firms must have positive profits and positive market share. The market outcome is that consumers $\theta < \theta^*(p^A, p^B) = p^B - p^A$ buy from firm A, and consumers with types greater than θ^* purchase from firm B. Then in the relevant range of prices, firm A's profit $\pi^A(p^A, p^B)$ is given by

$$\pi^A(p^A, p^B) = F(p^B - p^A)(p^A - c), \tag{1}$$

so the first-order condition for firm A's maximization yields

$$p^A = \frac{F(p^B - p^A)}{f(p^B - p^A)} + c. \tag{2}$$

Similarly, firm B's profit function in the relevant range is

$$\pi^B(p^A, p^B) = [1 - F(p^B - p^A)](p^B - c),$$

yielding the first-order condition

$$p^B = \frac{1 - F(p^B - p^A)}{f(p^B - p^A)} + c. \tag{3}$$

The MHR property implies that the objective functions are strictly quasi-concave, so that the first-order conditions are sufficient for both of these maximizations, and that

(2) and (3) implicitly define reaction functions as opposed to reaction correspondences. Moreover, it also implies that the reaction curves implicitly defined by (2) and (3) both have positive slope less than one. It follows that there is at most one intersection; this intersection exists and is given by

$$p^A = p^B = \frac{F(0)}{f(0)} + c, \tag{4}$$

which gives the unique equilibrium of the game.

4. Poaching under short-term contracts

■ The simplest contracting environment that allows poaching is the case where firms can observe the first-period actions of consumers, but only short-term contracts are allowed, and firms cannot commit themselves in period 1 to behavior in period 2. In this case, each firm will offer a single first-period price, which we denote a and b respectively, but in the second period each firm can offer one price to its own past customers and a different one to those who purchased from their rival. We let α and β denote the second-period prices that firms A and B charge to their own past customers, and we let $\hat{\alpha}$ and $\hat{\beta}$ denote the prices they offer to the past customers of the opposing firm.

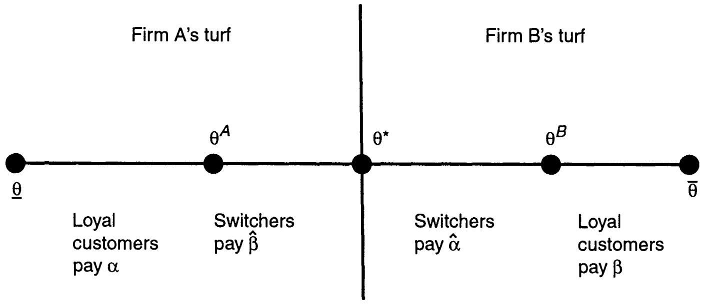
Because consumers have private information, we will use perfect Bayesian equilibrium as the solution concept. As a preliminary step, a standard revealed-preference argument implies that at any pair of first-period prices such that all consumers purchase and both firms have positive sales, there will be a first-period cutoff θ^* such that all consumers with $\theta < \theta^*$ buy from firm A in the first period, and all consumers with $\theta > \theta^*$ buy from firm B. Throughout the article, we extend this cutoff property to out-of-equilibrium prices by assuming that if the offered contracts are such that no consumer is expected to purchase from firm A, and some consumer does purchase, the consumer is believed to be type $\underline{\theta}$, and that a consumer who unexpectedly purchases from firm B is believed to be type $\overline{\theta}$.[11] We show below that in the overall equilibrium this cutoff is at $\theta^* = 0$, as in the static model, at least if the discount factor is not too large.

□ **Second-period pricing and poaching.** With that in mind, we now work backward from the second period. Given the existence of a first-period cutoff, the second-period situation is as depicted in Figure 1: Consumers to the left of θ^* lie in firm A's "turf" and those to the right lie in firm B's.

We will show that, provided $|\theta^*|$ is not too large, the second-period equilibrium has this form: Both firms poach some of their rival's first-period customers, so that some consumers do switch providers. These switchers are in the middle band of valuations, types between θ^A and θ^* switch from A to B, and types between θ^* and θ^B switch from B to A. For larger values of $|\theta^*|$, only the firm with the lower first-period market share poaches customers from its rival. The welfare effect of these switches depends on $|\theta^*|$ as well: switching lowers welfare if θ^* is at or near the efficient value of zero, which will be the case in the overall equilibrium we derive in the next section, while switching increases welfare if $|\theta^*|$ is large.

[11] We do not have a counterexample when this assumption is dropped, and we conjecture that the results might obtain without it, but because the assumption seems reasonable we have not pursued this possibility.

FIGURE 1



Since the two markets are so similar, it will be sufficient to determine the equilibrium on "firm A's turf." Denote firm A's price in this market by α and firm B's by $\hat{\beta}$, so that a consumer will be just indifferent between the two goods if her relative valuation is equal to $\hat{\beta} - \alpha$. Let $\theta^{\sim} \in (\underline{\theta}, 0)$ be the unique solution of

$$F(\theta^{\sim}) + f(\theta^{\sim})\theta^{\sim} = 0, \tag{5}$$

where uniqueness follows from MHR.

We will call this the "unit elastic point" on the demand curve. It will play an important role in several places in the article.

Proposition 1. If $\theta^* > \theta^{\sim}$, there is a unique second-period equilibrium on firm A's turf. In this equilibrium, both firms have positive sales. If $\theta^* < \overline{\theta}$, so firm A's turf is not the entire market, then the incumbent's market share is strictly larger than the poacher's, the poacher's price is strictly lower than the incumbent's, and both firms' prices are lower than in the static Hotelling equilibrium in the original whole market.

If $\theta^* < \theta^{\sim}$, firm B offers to sell at its cost but does not attract any of firm A's customers.

Proof. See the Appendix.

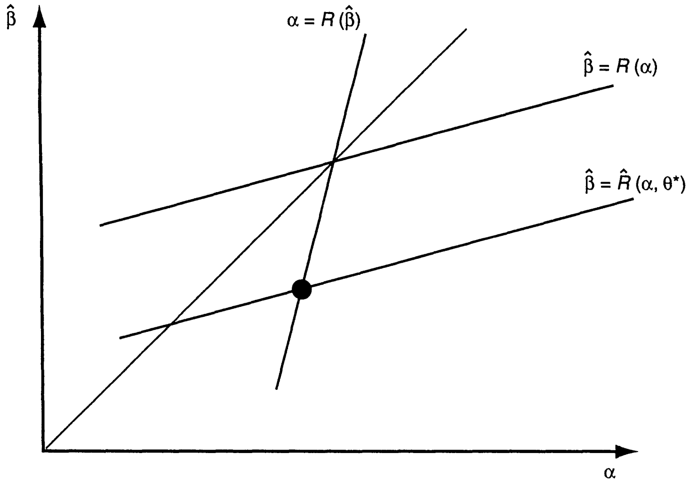
The formal analysis of the equilibrium is complicated by the need to keep track of boundary conditions, since for extreme values of θ^* the solution will be on the boundary where firm B has zero sales, but the intuition for the nature of the interior solution is illustrated in Figure 2. On the interior of firm A's turf, its reaction curve, $\alpha = R(\hat{\beta})$, reflects a tradeoff between losing marginal customers at θ^A and inframarginal rents on $[\underline{\theta}, \theta^A]$. This tradeoff is independent of the cutoff θ^*, and so firm A's reaction curve is independent of θ^* as well.[12] In contrast, the set of inframarginal consumers for firm B on firm A's turf is $[\theta^A, \theta^*]$, and in particular lowering θ^* makes firm B more aggressive. Thus, in the interior of A's turf, A's reaction curve is obtained by applying (2) to the demand curve on A's turf, yielding

$$\alpha = \frac{F(\hat{\beta} - \alpha)}{f(\hat{\beta} - \alpha)} + c. \tag{6}$$

Similarly, B's reaction curve on the interior of A's turf is given by

[12] With switching costs, we would expect firm A's reaction curve on its turf to vary with θ^*.

FIGURE 2



$$\hat{\beta} = \frac{F(\theta^*) - F(\hat{\beta} - \alpha)}{f(\hat{\beta} - \alpha)} + c. \tag{7}$$

To understand the role played by the "unit elasticity point" $\theta^{\sim}$ in determining the nature of second-period competition, suppose that firm B sets price $\hat{\beta} = c$ on A's turf. Then firm A's profit on its turf is $(\alpha - c)F(c - \alpha)$, which has the form $-xF(x)$. Hence holding firm B's price fixed at c, firm A will choose to serve all types up to θ^* whenever θ^* is less than the unit-elastic point $\theta^{\sim}$. If $\theta^* > \theta^{\sim}$, then firm A prefers to raise its price and lose marginal customers to the poacher; in that case it is not a best response for firm B to price as low as c.

□ **First-period pricing.** To evaluate the overall impact of the information that makes poaching possible, we now consider equilibrium first-period pricing and consumption decisions. Because we have assumed that firms lack commitment power, the size of the two first-period markets will influence second-period pricing and profits, and firms take this into account in setting first-period prices. Moreover, in equilibrium consumers correctly anticipate that they will be offered a lower second-period price by the "poaching" firm than by the firm that they purchase from in the first period. As we will see, the result will be that first-period prices are higher than if poaching were not allowed or, equivalently, if firms were unable to offer poaching prices because they did not have the necessary information about consumers' past purchases.

Let firm A's first-period price be a and let firm B's first-period price be b. If first-period prices lead to a cutoff θ^* that is in the interior of $[\underline{\theta}, \overline{\theta}]$, type θ^* must be indifferent between buying good A in period 1 at price a and then buying B in period 2 at the poaching price $\hat{\beta}$, or buying B in period 1 at price b and then buying A at the poaching price $\hat{\alpha}$. Thus, at an interior solution,

$$-\theta^*/2 - a + \delta(\theta^*/2 - \hat{\beta}) = \theta^*/2 - b + \delta(-\theta^*/2 - \hat{\alpha}), \tag{8}$$

so

$$\theta^* = \frac{b - a + \delta(\hat{\alpha}(\theta^*) - \hat{\beta}(\theta^*))}{1 - \delta}. \tag{9}$$

Since $\hat{\alpha} = \hat{\beta}$ when $\theta^* = 0$, (9) shows that $\theta^* = 0$ exactly when $a = b$, which is as it should be given the symmetry of the problem. Since $\hat{\alpha}(\theta^*)$ is weakly decreasing in θ^* and $\hat{\beta}(\theta^*)$ is weakly increasing, with at least one of the two functions having nonzero slope at every θ^*, the right-hand side of equality (9) is strictly decreasing, and so (9) has at most one solution. Moreover, a solution exists provided that the first-period prices aren't too far apart.

Note that if price discrimination is not allowed, or if the discount factor is zero, then $\partial\theta^*/\partial a = -1$, while with price discrimination we have

$$\frac{\partial\theta^*}{\partial a} = \frac{-1}{(1 - \delta) + \delta(\hat{\beta}'(\theta^*) - \hat{\alpha}'(\theta^*))}. \tag{10}$$

Thus demand will be less elastic in the first period if and only if $\hat{\beta}'(\theta^*) - \hat{\alpha}'(\theta^*) > 1$.

Now consider the equilibrium choices of a and b. Firm A's overall objective function is

$$\begin{aligned}[][a - c]F(\theta^*(a, b)) + \delta([\alpha(\theta^*(a, b)) - c]F(\theta^A(\theta^*(a, b)))\\ + [\hat{\alpha}(\theta^*(a, b)) - c][F(\theta^B(\theta^*(a, b))) - F(\theta^*(a, b))]),\end{aligned} \tag{11}$$

and firm B's objective function is

$$\begin{aligned}&[b - c][1 - F(\theta^*(a, b))]\\ &\quad + \delta\{[\beta(\theta^*(a, b)) - c][1 - F(\theta^B(\theta^*(a, b)))]\\ &\qquad + [\hat{\beta}(\theta^*(a, b)) - c](F(\theta^*(a, b)) - F(\theta^A(\theta^*(a, b))))\}.\end{aligned} \tag{12}$$

If the discount factor is close to zero, then this is approximately the same objective function as in the static case, so the objective function is again strictly quasi-concave. We have not been able to establish that this is true for all discount factors and all distributions F, but we do not have a counterexample, and in the case where F is the uniform distribution, the objective function is strictly quasi-concave for all δ.

Proposition 2. With a uniform distribution ($f' \equiv 0$),

(i) $\theta^* = 3(b - a)/(3 + \delta)$, so $\partial\theta^*/\partial a = -3/(3 + \delta)$, and first-period demand is less elastic than when price discrimination is banned.[13]

(ii) there is a symmetric equilibrium in which $a = b = (1 + \delta/3)\overline{\theta} + c$, as opposed to $\overline{\theta} + c$ in the static case or when price discrimination is banned. In this equilibrium $\theta^A = \underline{\theta}/3$ and $\theta^B = \overline{\theta}/3$, so one-third of all consumers switch brands from one period to the next.

Proof. See the Appendix.

The intuition for the higher prices is simply the decreased elasticity of demand established in part (i). In general there is a second effect: changing first-period prices

[13] A referee has provided a proof that this is true under the weaker conditions that F be ρ-concave with $\rho \geq 1/4$ and f quasi-concave.

changes the first-period cutoff and thus changes the nature of the second-period competition. However, in the uniform case these changes cancel out in the neighborhood of $\theta^* = 0$: the firm's marginal gains in one second-period market are exactly offset by losses in the other. Thus, in the uniform case the decreased elasticity of first-period demand determines the result.

Note that, once again, the implications of poaching are the reverse of those in models with switching costs: here prices are above the static levels in the first period and below them in the second, while in the standard switching-cost models firms price below static levels in the first period (in some cases even below marginal cost) and then raise their prices once consumers are locked in.

□ **Exogenous factors that reduce the intensity of poaching.** Before proceeding to the next section, which considers the possibility that firms may choose to offer long-term contracts in an attempt to ensure the loyalty of their customers and decrease poaching, we should acknowledge some exogenous factors that tend to decrease poaching even when only short-term contracts are used. One of these is switching costs, which we have already mentioned. Another is the possibility that some customers may cost more to service than others, and a customer's current supplier has superior information about the customer's type, so that the poacher faces a "lemons problem" in attracting the incumbent's customers. Related to this is the idea that customers may differ in their willingness to switch suppliers, so that (in a model with more than two periods) a poacher has to worry that the customers it attracts away from a rival will not remain customers for long.[14] Finally, loyal customers may feel resentment if they know that others receive better terms.

5. Simple long-term contracts

■ **Motivation.** Although long-term contracts are not always practical, they are offered in some of the markets that motivate this study, such as the market for cellular phone services. Moreover, as we will see, each individual firm can gain by using long-term contracts, holding the behavior of its opponent fixed. Thus it seems both natural and important to analyze competition in long-term contracts.

Of course, there are various sorts of long-term contracts. We start by analyzing contracts of the following simple form. In the first period, firms A and B offer to sell the good this period at "spot" prices a and b respectively; they also offer long-term contracts that promise to supply the good in both periods at prices A and B respectively. These first-period price offers are made simultaneously and are then observed both by consumers and by the firms. In the second period, firms know the first-period prices announced by their rivals, and they also know from whom each consumer purchased, but if a consumer purchased from a rival they do not know which contract that consumer chose. Firms then compete over the two "turfs" in the manner of Section 4, with the difference that firms know that some of the customers on their opponent's turf are locked in to a long-term contract.

Although this strikes us as a sensible-seeming class of contracts, it is not a very general one, which raises the question of the robustness of our results. We therefore proceed to check that the equilibrium we derive here is also an equilibrium of the more general model in which firms use "revelation schemes" where the price depends on the consumer's "announcement" of a type $\hat{\theta}$.

Even with the simple contracts we consider here, it is *possible* for a firm to prevent

[14] This seems to have been the case in the U.S. market for long-distance telephone service.

any poaching of its customers by its rival, simply by refusing to offer a contract for first-period sales alone. However, such contracts are not optimal; the firm does better by "allowing" second-stage poaching to make it a more attractive first-period "husband." Put differently, and more precisely, a firm that locks in all of its first-period customers cannot use the second-period market to discriminate between those customers with a strong preference for its brand and those with only a weak preference.

□ **Monopoly with commitment to long-term contracts.** Before analyzing the implications of long-term contracts in a duopoly setting, it is helpful to consider the use of long-term contracts by a monopolist who can commit not to renegotiate. Different forms of the monopoly problem are useful for understanding different aspects of the duopoly outcome. We will begin with the case of a monopoly supplier of good A when good B is supplied by a competitive fringe at the same price b in both periods; this will help us understand the effect of facing a rival who can try to "poach" one's customers. After that we consider a monopoly that supplies both goods; this case serves as a benchmark for evaluating the welfare loss caused by inefficient switching in the duopoly outcome.

When firm B's good is supplied at the same price in both periods to all customers (so there is no "poaching" by the suppliers of good B), then firm A faces a stationary mechanism-design problem. Hence, as shown by Baron and Besanko (1984), the optimal multiperiod contract for firm A is simply to commit to offer the optimal static contract in each period: in the context of this article, this means to commit to the price $a = \{F(b - \alpha)/[f(b - \alpha)]\} + c$ given by its static reaction curve. Implementing this contract requires firm A to commit itself not to use the information revealed by first-period purchases to offer "poaching discounts" to firm B's first-period customers.

The situation faced by firm A in our duopoly model differs from this Baron and Besanko setup in at least two important ways. First of all, we do not allow firm A to commit not to poach, which introduces several new considerations: its first-period sales will be influenced by the expected level of its own second-period poaching, the extent of its poaching will in turn be influenced by its first-period sales, and it must consider how the level of its first-period sales influences second-period profits in the two markets corresponding to the two firms' first-period customers. Secondly, firm B will not charge the same price in each period, and indeed firm B will also engage in poaching and offer two distinct second-period prices. These complications will make it difficult for us to identify a single factor as "the explanation" for our results that the duopoly equilibrium will involve poaching and the simultaneous use of short-run and long-run contracts.

To evaluate the welfare losses caused by switching in the duopoly equilibrium, we find it helpful to compare them with the losses incurred when there is a monopolist who produces both good A and good B.[15] Assume first that the monopolist can offer only deterministic contracts; then, leaving aside options to not consume in either or both periods, the monopolist can offer four "products," namely AA, AB, BA, and BB. From the symmetry of the problem, it is enough to analyze consumption by types in the interval $[\underline{\theta}, 0]$. Incentive compatibility implies that consumers in an interval $[\underline{\theta}, \hat{\theta}]$ choose to consume A both periods; we denote this consumption path as AA. Consumers in $[\hat{\theta}, 0]$ choose the switching option AB; symmetry implies that types $[0, -\hat{\theta}]$ choose BA, and types in $[-\hat{\theta}, \overline{\theta}]$ choose BB. The monopolist's profit is then

$$\pi^m = 2[p_{AA}F(\hat{\theta}) + p_{AB}(F(0) - F(\hat{\theta}))] - (1 + \delta)c.$$

[15] We thank Lars Stole for suggesting this benchmark.

The monopolist will set the price of the "switching" bundles AB and BA so that type 0 is indifferent about purchasing or not, so $p_{AB} = p_{BA} = (1 + \delta)v$. Type $\hat{\theta}$ must be indifferent between AA and AB, so $-p_{AA} - (\hat{\theta}/2) - \delta\hat{\theta}/2 = -p_{AB} - \hat{\theta}/2 + \delta\hat{\theta}/2$, and hence $p_{AA} - p_{AB} = -\delta\hat{\theta}$.

Substituting these expressions for the prices into the profit function yields $\pi^m = (1 + \delta)(v - c) - 2\delta\hat{\theta}F(\hat{\theta})$. Thus, it is as if the monopolist first sold all consumers a "switching bundle" at price $(1 + \delta)v$, and then offered an "upgrade" to AA or BB for a premium of $-\delta\hat{\theta}$. Since this upgrade has no additional production cost for the monopolist, maximizing over $\hat{\theta}$ yields $\hat{\theta} = \theta^{\sim}$, where $\theta^{\sim} \in (\underline{\theta}, 0)$ is the unit-elastic point defined in equation (5).[16] For the uniform distribution $\theta^{\sim} = \underline{\theta}/2$, so at the monopoly solution each of the four options is sold to one-quarter of the population. Since consumer preferences are the same in both periods, this switching is inefficient; it is used to extract more rents for the privilege of not switching.[17]

Inefficiency per se in a monopoly model is not surprising, but note that forcing customers to switch products seems to conflict with the Baron and Besanko result that the optimal allocation in a stationary environment is to repeat the static optimum. The reason for this apparent discrepancy is that the Baron and Besanko result covers the optimal stochastic mechanism, while above we restricted to deterministic allocations. It can be shown that the optimal stochastic mechanism here is for the monopolist to offer good A to types below $\theta^{\sim}$, good B to types above $-\theta^{\sim}$, and to offer all types in the interval $[\theta^{\sim}, -\theta^{\sim}]$ a $(½, ½)$, randomization between goods A and B. Thus the switching in the deterministic dynamic model arises as a substitute for the use of randomization, and unless $\delta = 1$, the monopolist is strictly worse off when restricted to using deterministic contracts.

To see the intuition for this result, let x be the probability of receiving good B, so type θ's utility before transfers is $\theta(x - ½) + v$. Since we are assuming that all agents will be supplied with one of the two goods, the monopolist's cost of production does not enter into the first-order conditions. Thus if we considered the interval $[0, \bar{\theta}]$ in isolation, the optimal static stochastic contract would be for types $\theta < -\theta^{\sim}$ to be assigned $x = 0$ and types $\theta > -\theta^{\sim}$ to be assigned $x = 1$. But the symmetric outcome on the interval $[-\underline{\theta}, 0]$ is for types $\theta > \theta^{\sim}$ to have $x = 1$ and types $\theta < \theta^{\sim}$ to have $x = 0$, and when these two solutions are combined the resulting allocation is not monotonic in types. So even though the constant-sign condition holds ($\partial^2 u/\partial\theta\partial x = 1$), we have a "bunching" problem brought about by the fact that that $\partial u/\partial x$ changes sign at $\theta = 0$.[18] Because A and B have the same cost, there is no cost implication of producing one rather than the other, which is why the switchpoints are at $\pm\theta^{\sim}$, and from the symmetry of the problem the only value of x that is consistent with full surplus extraction from type 0 and nonnegative surplus from all types is $x = ½$.[19]

Observation. A monopoly producer of both products who can use deterministic long-term contracts will induce types in the interval $[\theta^{\sim}, -\theta^{\sim}]$ to switch products. In the

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[16] The MHR condition implies that the second-order conditions for this maximization are satisfied.

[17] Because switching is inefficient, in the second period the monopolist would like to renegotiate with types in $[\theta^{\sim}, -\theta^{\sim}]$ and induce a subset of them not to switch. We conjecture that the result is Coasian dynamics in the style of Hart and Tirole (1988) and Laffont and Tirole (1990), but we have not verified this due to the complications caused by asymmetric first-period allocations.

[18] For a general treatment of bunching that includes the discontinuous controls considered here, see Rochet and Chone (1998), in particular their Proposition 8. Bias, Martimort, and Rochet (2000) consider a problem where bunching arises due to the individual-rationality constraint binding in the middle of the distribution of types.

[19] Other values of x give the same rent to the monopolist, but when $x \neq ½$ a price that extracts all of type 0's surplus would give a negative surplus to some nearby types.

uniform case, this implies that half of the consumers switch. A monopolist who can use stochastic long-term contracts will commit to repeated use of the static mechanism that offers a (½, ½) randomization between A and B to all types in the interval $[\theta^{\sim}, -\theta^{\sim}]$.

□ **Duopoly with simple long-term contracts.** Our goals in this subsection are to explain the general form of the equilibrium with both long-term and short-term contracts, and to show that in equilibrium both sorts of contracts will be purchased. As we shall see, the equilibrium with simple long-term contracts has the form depicted in Figure 3: the consumers who most prefer A buy a long-term contract, the next interval purchase A on a short-term basis in both periods, and those who are closer to indifferent purchase A in period 1 and then switch to B.

Thus, as in the case where only short-term contracts are feasible, the equilibrium involves socially inefficient switching, but the availability of long-term contracts does reduce the extent of this inefficiency.

To formally establish this result, we assume that when a firm sells both long-term and short-term contracts, the long-term contracts are purchased by the customers who most prefer that firm's product. This "tie-breaking" assumption is needed because our model is deterministic, and so a customer who plans to purchase from firm A in both periods will choose between a long-term contract and a sequence of short-run purchases solely on the basis of their cost. Hence, whenever a firm sells long-term contracts to some customers and a sequence of short-run contracts to others, the two ways of purchasing the same consumption flow must have the same cost ($A = a + \delta\alpha$), so that all the customers in question are indifferent between the two contracts. However, it is intuitive that the customers who value A more highly will be relatively more willing to commit themselves to consuming A and, conversely, that those who are less keen on A have a higher value for flexibility; this is what we assume here. Thus, the (possibly empty) sets of consumers who purchase long-term and short-term contracts from firm A will be intervals that we denote $[\underline{\theta}, \underline{\theta}^A]$ and $[\underline{\theta}^A, \theta^*]$; the sets who purchase long-term and short-term contracts from firm B are the intervals $[\underline{\theta}^B, \bar{\theta}]$ and $[\theta^*, \underline{\theta}^B]$ respectively. This assumption can be justified as capturing the limit of various sorts of stochastic models.[20]

To see how the cutoffs $\underline{\theta}^A$ and $\underline{\theta}^B$ are determined, note that expected second-period spot prices must satisfy the "no-arbitrage" conditions $\delta\alpha = A - a$ and $\delta\beta = B - b$ whenever both cutoffs are in the interior of the type space. Of course this is only possible if the implied spot prices can be generated by second-period competition, so the implied spot prices must be in the range $[c, c + \bar{\theta}]$; otherwise one form of contract will not be viable and at least one of the cutoffs will be on the boundary of the type space.

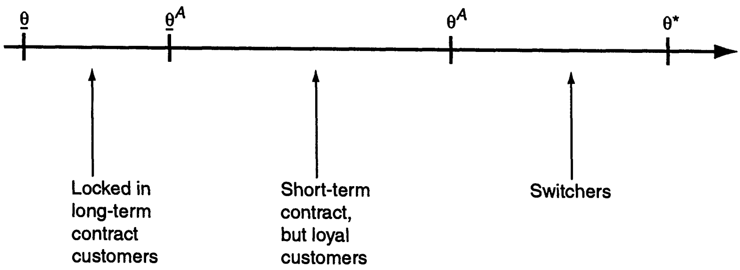
A key fact about selling long-term contracts is that locking in the interval $[\underline{\theta}, \underline{\theta}^A]$ changes firm A's second-period profit function on its own turf from $F(\hat{\beta} - \alpha)(\alpha - c)$ to $[F(\hat{\beta} - \alpha) - F(\underline{\theta}^A)](\alpha - c)$, which shifts its reaction curve from

$$\alpha = \frac{F(\hat{\beta} - \alpha)}{f(\hat{\beta} - \alpha)} + c \quad \text{to} \quad \alpha = \frac{F(\hat{\beta} - \alpha) - F(\underline{\theta}^A)}{f(\hat{\beta} - \alpha)} + c$$

and so leads to more aggressive pricing. In contrast, $\underline{\theta}^A$ has no effect on firm B's reaction curve so long as the second-period equilibrium has firm B successfully poaching some of firm A's customers. As in the case of short-term contracts, the second-period equilibrium on A's turf will be at the intersection of these two curves provided

[20] For example, the cutoff property holds if consumers' second-period valuations are drawn from smooth distributions that are ranked according to first-order stochastic dominance in the first-period type.

FIGURE 3



that that intersection is at least $\theta^{\sim}$; otherwise, firm B sets $\hat{\beta} = c$ and firm A prices to retain all of the customers on its turf.

Lemma 1. Any profile of first-period contracts (either on or off of the path of play) induces a unique first-period cutoff θ^*.

Proof. See the Appendix.

Proposition 3. If a firm's first-period sales exceed $F(\theta^{\sim})$, then a nonzero fraction of its customers purchase a short-term contract in the first period.

A partial intuition for Proposition 3 can be obtained from considering the case of a monopoly seller of good A when good B is supplied by a competitive fringe whose cost (and hence price) is lower in period 2. In this case firm A will find it optimal to sell to fewer consumers in period 2, that is, to allow some "poaching." This is only a partial intuition, as in our model consumers are eligible for the reduced second-period price from B only if they buy from firm A in the first period, so that firm B's poaching has implications for firm A's sales in the first period as well as in the second one.

Proof. Suppose this is not true for firm A, so that types in the interval $[\underline{\theta}, \theta^*]$ all buy a long-term contract from A. Let $\hat{\beta}(\theta^*)$ be the poaching price that firm B would set in the second period if the cutoff were θ^* and none of firm A's customers had a long-term contract, as characterized in the last section.

Now suppose that firm A deviates from the equilibrium by only offering a short-term contract in the first period, where the first-period price a is set so that type θ^* gets the same utility from buying good A in the first period and then switching as it did from the long-term contract. Since the original contract involved consuming A in both periods, it has type θ^* utility $v - A - \{[(1 + \delta)\theta^*]/2\}$. If θ^* buys A in the first period on a short-term contract, it will buy B in the second period, with overall utility of $v - \theta^*/2 - a + \delta\theta^*/2 - \delta\hat{\beta}(\theta^*)$; these two expressions for the utility are equal if $a = A - \delta\hat{\beta}(\theta^*) + \delta\theta^*$. We claim (i) that this will result in the same first-period cutoff as before, namely θ^*, and (ii) that firm A's profit will increase from this deviation.

Claim (i) follows from the uniqueness of the cutoff established in the lemma and the fact that the same cutoff will clearly still be *an* equilibrium. To prove (ii), we note first that firm A can obtain exactly the same profit as with the long-term contract by setting its second-period price α on its own turf to equal $\hat{\beta}(\theta^*) - \theta^*$. In this case it retains all of its customers, so its profit on its turf is

$$F(\theta^*)[(a - c) + \delta(\hat{\beta}(\theta^*) - \theta^* - c)] = F(\theta^*)[A - (1 + \delta)c],$$

which is its profit from the long-term contract. Since $\theta^* > \theta^{\sim}$, this choice of α is not a best response to $\hat{\beta}(\theta^*)$, so firm A can do strictly better with a different (higher) choice of α. *Q.E.D.*

The next step is to show that firms want to use long-term contracts as well as short-term ones. The intuition for this is that by locking in some of its "most captive" customers, the firm can commit itself to more aggressive second-period pricing. This commitment helps firm A because it induces firm B to lower its second-period poaching price, which makes it more attractive for consumers to purchase good A in the first period, and thus lets firm A charge a higher first-period price.

Proposition 4. If a firm's first-period sales exceed $F(\theta^{\sim})$, a nonzero fraction of its customers purchase a long-term contract.

Proof. Suppose to the contrary that there is an equilibrium in which firm A offers only a short-term contract at price a, and that it then offers price α to its old customers in period 2. Firm A's first-period customers will be an interval $[\underline{\theta}, \theta^*]$, with a subinterval $[\underline{\theta}, \theta^A]$ that stays loyal to A in the second period, and a subinterval $[\theta^A, \theta^*]$ of switchers that is nonempty because $\theta^* > \theta^{\sim}$. If firm A deviates from this equilibrium and introduces a long-term contract that it sells to an interval $[\underline{\theta}, \underline{\theta}^A]$ of consumers, $\underline{\theta}^A < \theta^A$, the second-period prices on A's turf will be α' and $\hat{\beta}'$ respectively, with $\alpha' < \alpha$ and $\hat{\beta}' < \hat{\beta}$. Suppose that firm A simultaneously increases its first-period "spot" price by just enough to make the marginal customers indifferent, so that $a' = a + \delta(\hat{\beta} - \hat{\beta}')$, or in the obvious notation $\Delta a + \delta\Delta\hat{\beta} = 0$,[21] and that the long-term contract is priced at $A = a' + \delta\alpha'$ to have the same cost as a sequence of spot purchases. This will keep firm A's first-period sales unchanged, so the second-period equilibrium will indeed generate the prices α' and $\hat{\beta}'$ we assumed it would.

We claim that firm A's profit is strictly higher under the new contract. To see this, note that we can get a lower bound on the firm's profit by supposing that it charges the suboptimal price $\alpha + \Delta\hat{\beta}$ in the second period instead of charging α'.[22] By exactly matching firm B's price reduction, firm A ensures that the cutoff between staying loyal and switching to B is the same θ^A as in the original contract, so that firm A has exactly the same sales in each period as before, and the only differences in profit are on the revenue side:

$$\begin{aligned} \Delta\Pi^A &= F(\theta^*)(\Delta a) + \delta[F(\theta^A) - F(\underline{\theta}^A)](\Delta\hat{\beta}) + \delta F(\underline{\theta}^A)(\Delta\alpha) \\ &= -\delta[F(\theta^*) - F(\theta^A)](\Delta\hat{\beta}) + \delta F(\underline{\theta}^A)(\Delta\alpha - \Delta\hat{\beta}). \end{aligned} \tag{13}$$

Now, since the second-period equilibrium is regular,[23] both $\Delta\alpha$ and $\Delta\hat{\beta}$ are of order $(\underline{\theta}^A - \underline{\theta})$, so for $\underline{\theta}^A$ near enough to $\underline{\theta}$, the first term in the last line of (13) dominates, and firm A can increase its profit by offering a sufficiently small number of long-term contracts. *Q.E.D.*

Propositions 3 and 4 show that firms sell both long-term and short-term contracts when the first-period cutoff θ^* isn't too extreme. Thus, in the neighborhood of the symmetric equilibrium outcome, the three first-period variables $\underline{\theta}^A$, θ^*, and $\underline{\theta}^B$ are determined by the following fixed-point problem:

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[21] Remember that the marginal first-period purchaser plans to switch to good B in the next period.

[22] Consumers make their first-period decisions expecting that firm A's second-period price on its turf will be α'.

[23] That is, the two reaction curves are smooth and cross at their intersection.

$$\delta\alpha = A - a, \qquad \delta\beta = B - b, \qquad \delta(\hat{\beta} - \hat{\alpha}) + (1 - \delta)\theta^* = b - a. \tag{14}$$

In the case of a uniform distribution, we have verified that this system has a unique fixed point and that the solution is a smooth function of the first-period prices. We believe this should be true under fairly general regularity conditions, but we have not verified it.

Proposition 5. (i) In a symmetric equilibrium, there is less switching than if only short-run contracts are allowed, so the ability to use long-term contracts increases efficiency.

(ii) With the uniform distribution there exists a unique symmetric equilibrium, and the equilibrium strategies are differentiable. In it, consumers in the intervals $[\underline{\theta}, 3\underline{\theta}/4]$ and $[3\overline{\theta}/4, \overline{\theta}]$ purchase long-term contracts from firms A and B respectively; consumers in the intervals $[3\underline{\theta}/4, \underline{\theta}/4]$ and $[\overline{\theta}/4, 3\overline{\theta}/4]$ buy from A and B in both periods using a series of short-term contracts; and consumers in the interval $[\underline{\theta}/4, \overline{\theta}/4]$ switch brands. Profits are lower than in a duopoly with short-term contracts, there is less switching, and consumer surplus and welfare are higher. There is also less switching than with a monopoly supplier of both goods using long-term contracts.

Proof. To prove part (i), note that selling some long-term contracts (e.g., $\underline{\theta}_A > \underline{\theta}$) makes firm A tougher and shifts its second-period reaction curve on its own turf outward as compared to the static equilibrium, while firm B's reaction curve is unchanged. The MHR assumption implies that $d\hat{\beta}/d\alpha < 1$ and $d\alpha/d\hat{\beta} < 1$; the conclusion follows. Part (ii) is obtained by computation and is verified in the Appendix. *Q.E.D.*

□ **What is the appropriate space of long-term contracts?** In this subsection we show that the equilibrium we derived in Section 4 is also an equilibrium of the more general model in which firms use "revelation schemes" where the price depends on the consumer's "announcement" $\hat{\theta}$ of her type. Firm A, for example, offers a contract of the following form: it charges first-period customers who announce $\hat{\theta}$ a first-period price $a_1(\hat{\theta})$, it commits to sell to them at a second-period price $a_2(\hat{\theta})$, and the customer agrees to pay a breach penalty $p(\hat{\theta})$ if she switches supplier. Thus we think of firm A as offering a menu of contracts of the form $\{a_1(\cdot), a_2(\cdot), p(\cdot)\}$. As in the previous section, we suppose that firms simultaneously announce their menus of contracts, which are observed by all consumers and by both firms. Consumers then choose which firm to contract with (consumers cannot accept contracts from both of them) and announce their type to the firm they selected. Firms observe if a customer purchased from their opponent, but not the specific terms she chose (equivalently, firms do not observe the announcement that a consumer makes to the other firm). Note that this more general framework does not allow the firm to implement a "most-favored-customer" promise that the second-period price will not exceed the poaching price $\hat{\alpha}$ it offers to firm B's first-period customers. Moreover, the setup does not allow for stochastic mechanisms, which are known to involve particular subtleties in the context of competition between mechanism designers.[24] Also, the setup doesn't allow a firm to sign up customers in period 1 for period 2 only, but this last restriction is not binding in equilibrium: A promise to sell to new customers at a price above the equilibrium poaching price is not time-consistent and will be ignored, while a promise to sell below the equilibrium poaching price lowers second-period profits holding first-period sales fixed (a Stackelberg leader in the second period would want to commit to a higher-than-equilibrium price, not a lower one) and also lowers the first-period price that induces that level of

[24] See Martimort and Stole (1997).

sales (because it makes purchasing from the other firm in the first period more attractive).[25]

Proposition 6. The equilibrium outcome described in the last subsection remains an equilibrium when the firms can use the more general contracts of the preceding paragraph.

Proof. Suppose that firm B offers the short-term and long-term contracts derived in the last section, and consider firm A's problem in maximizing over the more general space of menus of contracts. Whatever menu firm A offers, all of its first-period customers know whether they will remain loyal to A or switch to B in period 2. All types who consume A both periods will pay the same total amount; call this amount $A = \min_\theta a_1(\theta) + \delta a_2(\theta)$. Similarly, all of the switchers will choose a contract that minimizes the effective price $a_1(\theta) + \delta p(\theta)$ of consuming A in the first period only; call this minimum $a = \min_\theta a_1(\theta) + \delta p(\theta)$. Suppose moreover that all of the switchers choose the same pair (a_1, p).[26] Then revealed preference implies that the switchers will be an interval $[\theta^A, \theta^*]$, and Proposition 3 shows that it is not optimal for firm A to make this interval empty. Therefore, we can without loss of generality suppose that θ^A is indifferent between consuming A both periods and paying A, or consuming A and then B, paying $a + \delta\hat{\beta}$,[27] so $A = a + \delta\hat{\beta} - \delta\theta^A$.

If we then write firm A's total profit as

$$F(\theta^A)(A - (1 + \delta)c) + (F(\theta^*) - F(\theta^A))(a - c) + \delta(F(\theta^B) - F(\theta^*))(\hat{\alpha} - c),$$

and substitute for A, we obtain

$$F(\theta^*)(a + \delta\hat{\beta} - \delta\theta^A - (1 + \delta)c) - \delta(F(\theta^*) - F(\theta^A))(\hat{\beta} - \theta^A - c)$$
$$+ \delta(F(\theta^B) - F(\theta^*))(\hat{\alpha} - c).$$

Holding θ^* fixed requires that $a + \delta\hat{\beta}$ is constant, so firm A's profit for a fixed level of θ^* is decreasing in $\hat{\beta}$ as we claimed. The last step in the proof is to show that firm A cannot induce firm B to offer a lower poaching price for fixed (θ^A, θ^*) than the price $\hat{\beta}^*$ firm B chooses when firm A uses a simple long-term contract. Since $\theta^A < \theta^*$, any small increase in $\hat{\beta}$ above $\hat{\beta}^*$ shifts θ^A by the same amount. Thus the first-order effect on firm B's second-period profit of increasing $\hat{\beta}$ is the same as it was under simple contracts, so $\hat{\beta}$ must be at least as high. *Q.E.D.*

6. Changing preferences

■ In the model we have used so far, each consumer's brand preferences remain fixed over time, so in the social optimum consumers never switch brands. This section considers the extreme opposite case where each consumer's preferences are not just random but independent over time. Here, knowledge of a consumer's first-period choices carries no information about second-period preferences, so the equilibrium with short-term

[25] That is, the combination of a poaching price $\hat{\alpha}$ below the best-response level and a first-period price a_1 that leads to some cutoff θ^* is dominated by a pair $(\hat{\alpha}', a_1')$ that induces the same θ^* and such that $\hat{\alpha}'$ is the price that arises in the second-period equilibrium on firm B's turf.

[26] Since we are constructing *an* equilibrium, we are free to make this assumption.

[27] This is immediate if $\theta^A > \underline{\theta}$; if $\theta^A = \underline{\theta}$ and θ^A prefers to switch, firm A could as well offer a different contract with a price A that makes $\underline{\theta}$ indifferent.

contracts is simply two repetitions of the static equilibrium, and moreover this equilibrium is socially efficient. Hence long-term contracts cannot increase efficiency, and in fact we will show that they lead to a strictly less efficient outcome, in contrast to the case of fixed preferences where long-term contracts increase efficiency.[28]

In addition to having different efficiency implications here, long-term contracts have a quite different appeal for the firms. With fixed preferences, long-term contracts help firms price discriminate between consumers; that motivation is absent with independent preferences. Instead, as in Aghion and Bolton (1987), long-term contracts can help transfer surplus to the firm-consumer pair and away from the competing firm. The Aghion and Bolton model differs in several respects from ours, most notably because they considered a situation where only one firm can offer a long-term contract, so their analysis is not applicable here.[29] However, our results show that the intuitions from their model do extend to at least some situations of symmetric contracting.

To model changing preferences, we suppose that the first- and second-period preferences θ_1, θ_2 of each consumer are independent draws from the c.d.f.'s F and G, respectively; each of these c.d.f.'s is assumed to be smooth (with densities denoted f and g) and to satisfy Assumption 1. We suppose that firms offer contracts of the form studied in the last section, so that firm A names prices a_1 and a_2 for consumption in the two periods, and a penalty p for breach. Thus the effective price (in period-2 units) for consuming a second period of good A is $\alpha = a_2 - p$, and the total cost of consuming A in both periods is $A = a_1 + \delta a_2$. Firm A will also offer a poaching price $\hat{\alpha}$ to firm B's first-period customers; as in the last section, firm A cannot gain by precommitting to a poaching price at date 1.

Caminal and Matutes (1990) analyze essentially this model for the uniform distribution; they do not consider breach penalties, so that the preannounced second-period prices p_{2i} in their model correspond to α and β here.[30] Our analysis for general distributions is less computational; we believe it provides some additional insight.

Since firm A's first-period customers face an implicit price of α for consuming A in period 2, firm B's poaching price is given by a special case of equation (7), $\hat{\beta} = \{[1 - G(\hat{\beta} - \alpha)]/[g(\hat{\beta} - \alpha)]\} + c$. Thus firm A's choice of α determines the poaching price $\hat{\beta}(\alpha)$ and the cutoff $\hat{\theta} = \hat{\beta}(\alpha) - \alpha$, so a fraction $1 - G(\hat{\theta}) = 1 - G(\hat{\beta}(\alpha) - \alpha)$ of firm A's customers switch to firm B in period 2.

Firm A's average profit per customer on its first-period clients is then

$$A - (1 + \delta)c - \delta(\alpha - c)[1 - G(\hat{\theta})], \tag{15}$$

and the expected utility of one of firm A's first-period clients is the utility of consuming A in both periods, plus the "option value" of being able to switch:

$$u_A(\theta_1) = (1 + \delta)v - A - \frac{\theta_1}{2} + \delta \int_{\hat{\theta}}^{\bar{\theta}} [\theta_2 - (\hat{\beta} - \alpha)]g(\theta_2)\, d\theta_2, \tag{16}$$

where we have used the fact that $E(\theta_2) = 0$.

[28] Note that under both preference structures, long-term contracts reduce switching and result in greater "customer loyalty" as compared to short-term contracts.

[29] Aghion and Bolton consider long-term contracts between an incumbent monopolist and a single consumer when an entrant with unknown cost will arive in the next period. By agreeing to a penalty for breach, the consumer can force the entrant to lower its price to make a sale, thus transferring surplus from the entrant to the contracting pair. Diamond and Maskin (1979) made a similar point about breach penalties in a matching model.

[30] The size of breach penalties matters if there is a probability that consumers will choose not to consume in the second period, but in our model the breach penalties are indeterminate and can be set equal to zero without changing the results.

Firm A's equilibrium contract will maximize its profit per customer for a fixed level of utility corresponding to first-period sales.[31] Using the constraint to solve for A and substituting, firm A's problem reduces to

$$\max_{\alpha} \delta \int_{\hat{\theta}(\alpha)}^{\overline{\theta}} [\theta_2 - (\hat{\beta}(\alpha) - \alpha)]g(\theta_2)\, d\theta_2 - \delta(\alpha - c)[1 - G(\hat{\theta}(\alpha))],$$

which simplifies to

$$\max_{\hat{\theta}} \int_{\hat{\theta}}^{\overline{\theta}} \theta_2 g(\theta_2)\, d\theta_2 - (1 - G(\hat{\theta}))(\hat{\beta} - c) \quad \text{such that} \quad \hat{\beta} = \frac{1 - G(\hat{\theta})}{g(\hat{\theta})} + c. \tag{17}$$

Note that this maximand is simply the efficiency gain from the consumer switching minus the profits accruing to firm B; the difference is thus the net surplus to be shared by the consumer and firm A. If firm B's poaching price equalled its cost, the net surplus and the total surplus would coincide and thus the optimal choice would be $\hat{\theta} = 0$, but since in general $\hat{\beta} > c$, we expect that the solution will have $\hat{\theta} > 0$. This can be confirmed by substituting for $\hat{\beta}$ in the maximization, yielding

$$\max_{\hat{\theta}} \int_{\hat{\theta}}^{\overline{\theta}} \theta_2 g(\theta_2)\, d\theta_2 - \frac{(1 - G(\hat{\theta}))^2}{g(\hat{\theta})},$$

and by noting that $(1 - G(\hat{\theta}))^2/g(\hat{\theta})$ is a positive and decreasing function from the MHR assumption.

More strongly, we can show that $\alpha < c$, i.e., the implicit price for sticking with firm A is below marginal cost. Intuitively, if firm B's poaching price $\hat{\beta}$ were constant, then the first-order conditions imply that $\alpha = \hat{\beta} - \hat{\theta} = c$. When $\hat{\beta}$ is endogenous, firm A wants to lower it, resulting in $\alpha < c$.

Proposition 7. (i) With independent preferences, there is inefficiently little switching in period 2, so long-term contracts decrease total surplus, and each firm offers its first-period customers its good in the second period at an effective price below marginal cost.

(ii) (Caminal and Matutes, 1990). With independent uniformly distributed preferences, profits are lower than with static contracts.

Proof. (i) It remains only to show that $\alpha < c$. The first-order condition implies that

$$\hat{\theta} = (\hat{\beta} - c)\left(1 - \frac{d}{d\hat{\theta}}\left(\frac{1 - G(\hat{\theta})}{g(\hat{\theta})}\right)\right),$$

which is greater than $\hat{\beta} - c$ from the MHR condition. Thus $\alpha - c = \hat{\beta} - c - \hat{\theta} < 0$. *Q.E.D.*

7. Concluding remarks

■ This article has been a first look at the implications of poaching that is purely information-based and not driven by switching costs. As we have seen, the major

[31] More precisely, the equilibrium first-period cutoff θ^* will be given by $u_A(\theta^*) = u_B^*$ for some level of u_B^* determined by the equilibrium.

qualitative difference is that in our model firms price below static equilibrium levels when trying to poach from their competitors, while switching costs lead firms to raise prices above the static level. The article is also, as far as we know, the first study in which competing firms offer a menu of long-term contracts and short-term contracts.[32] We feel that our results provide some insight and intuition, but we also recognize that there is ample room for extensions and refinements of our conclusions, so we will conclude by sketching a few of these.

Villas-Boas (1999) extends our model of short-term contracts to an infinite-horizon, overlapping-generations model, in which a firm knows its own past customers but cannot distinguish between new customers and those who purchased from its rival. He finds that in Markov-perfect equilibrium, prices and market shares converge to a symmetric steady state; it would be interesting to know if the same is true in a model with long-term contracts.

It might also be interesting to consider alternate demand structures, such as vertical differentiation; we conjecture that the qualitative conclusions would be similar to those of this article, but we have not tried to confirm this. The case of network externalities in demand seems more different and more intriguing: In our model firms would like to induce their opponent to have a low poaching price, to increase the attractiveness of their good on the first-period spot market, but with network externalities firms might prefer to commit to low poaching prices to make their good more attractive to long-term purchasers. More generally, there may be other situations of interest where competing firms use both long-term and short-term contracts; the general topic might be studied as a contribution to the theory of common agency.

Appendix

■ This Appendix collects the proofs that were omitted from the text.

Proof of Proposition 1. Note first that depending on the values of the two prices, the "cutoff" type $\hat{\beta} - \alpha$ may lie outside the feasible set, which is now $[\underline{\theta}, \theta^*]$. Since $\hat{\beta}$ will not be less than c, we know that firm A will never choose to have zero sales in this market, since it can make a strictly positive profit by charging a price slightly higher than c. However, if θ^* is very close to $\underline{\theta}$, so this market consists only of customers with a strong preference for firm A, firm B's sales here may be zero for any price that exceeds its cost; if this occurs, $\hat{\beta} - \alpha$ will exceed θ^*. In either case, firm A's profit in this market is $F(\hat{\theta}(\alpha, \hat{\beta}))(\alpha - c)$ and firm B's profit is $[F(\theta^*) - F(\hat{\theta}(\alpha, \hat{\beta}))](\hat{\beta} - c)$, where $\hat{\theta}(\alpha, \hat{\beta}) = \min[(\hat{\beta} - \alpha), \theta^*]$.

It is easy to see that firm A's maximization problem has exactly the same first-order condition as in the static case, and that the second-order conditions are again satisfied. The first-order conditions are

$$\alpha = \hat{\beta} - \theta^* \tag{A1}$$

if $F(\theta^*) - f(\theta^*)(\hat{\beta} - \theta^* - c) < 0$ and $\hat{\beta} - \theta^* \leq v - \theta^*/2$,

$$\alpha = v - \theta^*/2$$

if $F(\theta^*) - 2f(\theta^*)(v - \theta^*/2 - c) < 0$ and $\hat{\beta} - \theta^* > v - \theta^*/2$,[33] and otherwise

$$\alpha = \frac{F(\hat{\beta} - \alpha)}{f(\hat{\beta} - \alpha)} + c.$$

[32] Hart and Tirole (1988) and Laffont and Tirole (1990) can be interpreted as models in which a monopolist offers a menu of long-term and short-term contracts, where the short-term contract is the rental of a lower-quality good. (In Hart and Tirole, the rental contract is identified with not consuming, but that is not needed for their results.)

[33] The first of the two inequalities on this line will be satisfied if v is large enough so that when firm B is not a factor on firm A's turf, A will choose to serve the entire market. Note that the first two cases in (A1) are mutually exclusive due to MHR.

In contrast, firm B's maximization problem on A's turf differs from the static one unless A's turf is the entire market. Firm B chooses $\hat{\beta}$ to maximize $[F(\theta^*) - F(\hat{\theta}(\alpha, \hat{\beta}))](\hat{\beta} - c)$; substituting in the definition of $\hat{\theta}(\alpha, \hat{\beta}) = \min[(\hat{\beta} - \alpha), \theta^*]$, we have the program

$$\max_{\hat{\beta}} \, [F(\theta^*) - F(\hat{\beta} - \alpha)](\hat{\beta} - c) \quad \text{subject to} \quad \underline{\theta} + \alpha \leq \hat{\beta} \leq \theta^* + \alpha.$$

Since we know that firm B will not capture this entire market, the relevant first-order condition is

$$\hat{\beta} = \frac{F(\theta^*) - F(\hat{\beta} - \alpha)}{f(\hat{\beta} - \alpha)} + c \tag{A2}$$

if $\theta^* + \alpha > c$.

If not, there is no choice for firm B that yields a positive profit on firm A's turf; any price exceeding $\theta^* + \alpha$ yields zero sales and is thus a best response. We will assume here that whatever price firm B chooses is at least its cost; that is, we rule out the weakly dominated strategies corresponding to pricing below cost.

The system (A1)–(A2) implies that firm B has zero sales on A's turf if and only if $F(\theta^*) + f(\theta^*)\theta^* < 0$. From MHR, there is a unique $\theta^{\sim}$ such that $F(\theta^{\sim}) + f(\theta^{\sim})\theta^{\sim} = 0$, with $\underline{\theta} < \theta^{\sim} < 0$; if θ^* is less than this $\theta^{\sim}$, the equilibrium is $\alpha = -\theta^* + c$, $\hat{\beta} = c$.[34] Otherwise, the equilibrium is in the region where both firms have positive sales. As in the static case, there can be at most one such equilibrium.

To see that an equilibrium exists, subtract (A1) from (A2) and let $y = \hat{\beta} - \alpha$, yielding

$$y = \frac{F(\theta^*) - 2F(y)}{f(y)} \equiv H(y). \tag{A3}$$

Using the MHR condition, we can show that H is decreasing in y.[35] The facts that $H(\underline{\theta}) > \underline{\theta}$ and $H(\theta^*) < \theta^*$ then imply that (A3) has a unique solution. Plugging this solution into (A1) and (A2) gives the equilibrium prices and also the second-period cutoff θ^A in firm A's market, with $\underline{\theta} \leq \theta^A \leq \theta^*$. Moreover, since $F(0) = 1/2$, we also know that θ^A is strictly negative if $\theta^* < \overline{\theta}$, i.e., if firm A's turf is not the whole market. This verifies the claim that the poacher always undercuts the incumbent. The second-period equilibrium on firm B's turf is very similar. If θ^* isn't too close to $\overline{\theta}$, the boundary conditions will not bind, and we have

$$\beta = \frac{1 - F(\beta - \hat{\alpha})}{f(\beta - \hat{\alpha})} + c \quad \text{and} \tag{A4}$$

$$\hat{\alpha} = \frac{F(\beta - \hat{\alpha}) - F(\theta^*)}{f(\beta - \hat{\alpha})} + c. \tag{A5}$$

This yields the second-period prices and a cutoff θ^B that lies strictly between θ^* and $\overline{\theta}$. Since the poacher acts more aggressively than in the static solution and reaction curves are upward sloping, both firms' second-period prices are lower than in the static solution. (This is the reverse of the conclusion with switching costs.) Note also that the ratio of the incumbent's market share to the poacher's in market A is $F(\theta^A)/[F(\theta^*) - F(\theta^A)] = (\alpha - c)/(\hat{\beta} - c) > 1$, so that the incumbent is dominant in its own market. *Q.E.D.*

Proof of Proposition 2. We note first that solving the second-period reaction functions shows that

[34] If we allowed firm B to set prices below c, there could be equilibria in this region with lower prices, but there cannot be an equilibrium in which firm B has zero sales yet its price is greater than c, since firm A will set its price to make type θ^* just indifferent, and firm B could then cut its price and have positive profit.

[35] We want to show that $H'(y) \leq 0$, where $H'(y) = -2 - \{[f'(y)(F(\theta^*) - 2F(y))]/f^2(y)\}$. MHR implies that $1 - (f'F/f^2) \geq 0$; MHR and symmetry imply that $-1 - [f'(1 - F)/f^2] \leq 0$. Subtracting the first inequality from the second yields $-2 - f'(1 - 2F)/f^2 \leq 0$. Thus it is immediate that $H'(y) \leq 0$ at points where $f' < 0$, with strict inequality whenever $F(\theta^*) < 1$, and it is also immediate that $H'(y) \leq 0$ when $f' > 0$ and $F(\theta^*) \geq 2F(y)$. If $f' > 0$ and $F(\theta^*) < 2F(y)$, the conclusion follows from $1 \geq f'F/f^2$.

$$\theta^A = (\theta^* + \underline{\theta})/3, \qquad \alpha = \frac{\theta^* - 2\underline{\theta}}{3} + c, \qquad \hat{\beta} = \frac{2\theta^* - \underline{\theta}}{3} + c, \qquad \beta = \frac{2\overline{\theta} - \theta^*}{3} + c, \quad \text{and}$$

$$\hat{\alpha} = \frac{\overline{\theta} - 2\theta^*}{3} + c.$$

Part (i) of the proposition then follows by solving for θ^*.

To prove part (ii), it will be convenient to rewrite equation (11) for firm A's overall present value using the functions π_2^A and $\hat{\pi}_2^A$ to represent firm A's second-period profit on its own turf and on firm B's, respectively. Thus firm A's objective is to maximize

$$[a - c]F(\theta^*(a, b)) + \delta(\pi_2^A(a, b) + \hat{\pi}_2^A(a, b, \theta^*)), \tag{A6}$$

where

$$\pi_2^A(a, b) = (\alpha(\theta^*(a, b)) - c)F(\theta^A(a, b)), \qquad \theta^A(a, b) = \hat{\beta}(a, b) - \alpha(a, b),$$

$$\hat{\pi}_2^A(a, b) = (\hat{\alpha}(\theta^*(a, b)) - c)(F(\theta^B(a, b)) - F(\theta^*(a, b))), \quad \text{and} \quad \theta^B(a, b) = \beta(a, b) - \hat{\alpha}(a, b).$$

Since firm A's own second-period prices are set to maximize A's second-period profit, we can use the envelope theorem to write the first-order conditions for this maximization as

$$F(\theta^*) + (a - c)f(\theta^*)\frac{\partial\theta^*}{\partial a} + \delta\left[\frac{\partial\pi_2^A}{\partial\hat{\beta}}\frac{d\hat{\beta}}{d\theta^*} + \frac{\partial\hat{\pi}_2^A}{\partial\beta}\frac{d\beta}{d\theta^*} + \frac{\partial\hat{\pi}_2}{\partial\theta^*}\right]\frac{\partial\theta^*}{\partial a} = 0, \tag{A7}$$

where we have used the fact that $\partial\pi_2^A/\partial\theta^* = 0$.

Now the effect of firm B's prices on A's profits, holding A's prices fixed, is simply A's markup times the marginal change in sales:

$$\frac{\partial\pi_2^A}{\partial\hat{\beta}} = (\alpha - c)f(\theta^A)\frac{\partial\theta^A}{\partial\hat{\beta}} = (\alpha - c)f(\theta^A), \quad \text{and} \quad \frac{\partial\hat{\pi}_2^A}{\partial\beta} = (\hat{\alpha} - c)f(\theta^B),$$

while $\partial\hat{\pi}_2^A/\partial\theta^* = -(\hat{\alpha} - c)f(\theta^*)$.

In the uniform case, $\hat{\beta}(a, b) = [(2\theta^* - \underline{\theta})/3] + c$, and $\beta(a, b) = [(2\overline{\theta} - \theta^*)/3] + c$, so $d\hat{\beta}/d\theta^* = 2/3$ and $d\beta/d\theta^* = -1/3$. And when $\theta^* = 0$,

$$\alpha(a, b) - c = \frac{-2\underline{\theta}}{3} \quad \text{and} \quad \hat{\alpha} - c = \frac{\overline{\theta}}{3} = \frac{-\underline{\theta}}{3}.$$

Thus evaluated at $\theta^* = 0$,

$$\frac{\partial\pi_2^A}{\partial\hat{\beta}}\frac{d\hat{\beta}}{d\theta^*} + \frac{\partial\hat{\pi}_2^A}{\partial\beta}\frac{d\beta}{d\theta^*} + \frac{\partial\hat{\pi}_2^A}{\partial\theta^*} = \left[\frac{-4\underline{\theta}}{9} + \frac{\underline{\theta}}{9} + \frac{3\underline{\theta}}{9}\right]f = 0,$$

so that changes in θ^* (and hence in a) have no net effect on firm A's second-period profit: Gains in one second-period market are exactly offset by losses in the other. From part (i), $\partial\theta^*/\partial a = -3/(3 + \delta)$ in the uniform case.

Hence the first-order condition at $\theta^* = 0$ simplifies to $1/2 = 3(a - c)/[(3 + \delta)2\overline{\theta}]$, or $a = (1 + \delta/3)\overline{\theta} + c$, as opposed to $\overline{\theta} + c$ in the static case or when price discrimination is banned.

Note also that $\hat{\beta} - \alpha = \underline{\theta}/3$, so that one-third of firm A's first-period customers switch to firm B.

We have verified (details available on request) that the second-order condition in this problem, evaluated at $\theta^* = 0$, simplifies to $(-18 + 4\delta)/2\overline{\theta}(3 + \delta)^2$, which is negative for all δ between zero and one. *Q.E.D.*

Proof of Lemma 1. Let θ' denote the firms' conjectures about the purchase decision of consumers. That is, in the second period, firms believe that the consumers who bought from A in the first period have types in the interval $[\underline{\theta}, \theta']$. Let $u_A(\theta, \theta')$ be the utility that type θ gets from buying from firm A in the first period and playing optimally in the second, and let $u_B(\theta, \theta')$ be type θ's utility if it purchases from firm B in the first period.

The continuation equilibrium requires either that a positive fraction of consumers buy from both firms, or that all consumers buy from one of them. The first type of equilibrium requires that there be $\theta^* \in (\underline{\theta}, \overline{\theta})$ such that consumers in the interval $[\underline{\theta}, \theta^*]$ buy from A, with $u_A(\theta^*, \theta^*) = u_B(\theta^*, \theta^*)$. The other possible equilibria are that all consumers buy from B, with $\theta' = \underline{\theta}$, and that all consumers buy from A, with $\theta' = \overline{\theta}$.

For any conjectures θ', second-period play always makes type θ' at least weakly prefer to switch brands, so $u_A(\theta^*, \theta^*) - u_B(\theta^*, \theta^*) = b - a + \delta(\hat{\alpha} - \hat{\beta}) - (1 - \delta)\theta^*$. Moreover, the poaching price $\hat{\beta}$ is weakly increasing in θ^*. If firm A sells only short-term contracts, the monotonicity of $\hat{\beta}$ follows from the analysis in Section 3; if firm A sells both long-term and short-term contracts, α is determined from the equation $\delta\alpha = A - a$, and $\hat{\beta}$ is an increasing function of θ^* from MHR. A similar argument shows that poaching price $\hat{\alpha}$ is weakly decreasing in θ^*. These monotonicity properties imply that there is at most a single $\theta^* \in (\underline{\theta}, \overline{\theta})$ that satisfies $u_A(\theta^*, \theta^*) = u_B(\theta^*, \theta^*)$ and that an interior solution exists if and only if $u_A(\underline{\theta}, \underline{\theta}) - u_B(\underline{\theta}, \underline{\theta}) > 0$ and $u_A(\overline{\theta}, \overline{\theta}) - u_B(\overline{\theta}, \overline{\theta}) < 0$. In this case, the interior solution is thus the unique continuation equilibrium; otherwise, the continuation equilibrium has all consumers purchase from one of the firms. *Q.E.D.*

Proof of Proposition 5 (ii). In equilibrium, both firms know that consumers in the interval $[\underline{\theta}, \underline{\theta}^A]$ are committed to A in the second period, so second-period competition on A's turf is over the types in the interval $[\underline{\theta}^A, \theta^*]$. Because neither firm can commit itself to second-period prices for uncommitted consumers, we compute the second-period reaction curves $\hat{\beta}(\alpha) = ½(\alpha + c + \theta^*)$ and $\alpha(\hat{\beta}) = ½(\hat{\beta} + c - \underline{\theta}^A)$; second-period prices are given by the intersection of these curves, yielding $\hat{\beta} = [(2\theta^* - \underline{\theta}^A)/3] + c$, $\alpha = [(\theta^* - 2\underline{\theta}^A)/3] + c$, and $\theta^A = \hat{\beta} - \alpha = (\theta^* + \underline{\theta}^A)/3$. A similar argument shows that $\hat{\alpha} = [(\underline{\theta}^B - 2\theta^*)/3] + c$ and $\beta = [(2\underline{\theta}^B - \theta^*)/3] + c$.

Our next step is to substitute these equations into the equation that determines θ^*. Thus

$$\hat{\alpha} - \hat{\beta} = \frac{-4\theta^* + \underline{\theta}^A + \underline{\theta}^B}{3},$$

and so $a - b + (1 - \delta)\theta^* = \delta(\hat{\alpha} - \hat{\beta}) = (\delta/3)(-4\theta^* + \underline{\theta}^A + \underline{\theta}^B)$. Now $3(\beta - \alpha) = 2(\underline{\theta}^A + \underline{\theta}^B - \theta^*)$, so a further substitution yields

$$\theta^* = \frac{b + B - (a + A)}{2}. \tag{A8}$$

Thus, first-period sales are unaffected if firm A increases a by a small amount ϵ while decreasing A by the same amount, so that $d\alpha = (-2/\delta)\epsilon$. Intuitively, decreasing A increases sales of long-term contracts, which makes firm A tougher on its own turf, and lowers both α and $\hat{\beta}$, and the decrease in $\hat{\beta}$ just compensates consumer θ^* for the increase in first-period price. Moreover, such a change has no effect on $\underline{\theta}^B$ and so has no effect on competition of B's turf. Since A's profit on its own turf is

$$(a - c)F(\theta^*) + \delta(\alpha - c)F(\theta^A),$$

a first-order condition for equilibrium is that

$$F(\theta^*) + \delta[F(\theta^A)d\alpha + (\alpha - c)fd\theta^A] = 0, \quad \text{or} \quad F(\theta^*) - 2F(\theta^A) = -\delta(\alpha - c)fd\theta^A.$$

Now $\theta^A = \hat{\beta} - \alpha = (\theta^* + \underline{\theta}^A)/3$, and $\underline{\theta}^A = [\theta^* - 3(\alpha - c)]/2$, so

$$\theta^A = \frac{\theta^*}{2} - \frac{1}{2}(\alpha - c). \tag{A9}$$

Hence $d\theta^A = -½d\alpha = 1/\delta$, and the first-order condition simplifies to $2(\alpha - c) = \overline{\theta}$, so $\theta^A = (\theta^*/2) - (\overline{\theta}/4)$.

When $\theta^* = 0$, this reduces to $\underline{\theta}^A = -3\overline{\theta}/4$ and $\theta^A = -\overline{\theta}/4$; comparing with the case of short-term contracts shows that the fraction of agents who switch firms falls from ⅓ to ¼.

The last step of the proof is to determine the first-period spot prices a and b. To do this, suppose that firm A considers increasing both a and A by the same amount ϵ, which will leave α unchanged but (from (A8)) will decrease θ^* by ϵ.

Write firm A's payoff as $(a - c)F(\theta^*) + \delta(\alpha - c)F(\theta^A) + \delta(\hat{\alpha} - c)(F(\beta - \hat{\alpha}) - F(\theta^*))$, where we have substituted $\underline{\theta}^B = \beta - \hat{\alpha}$. The first-order condition for a simultaneous and equal increase in a and A is [36]

[36] We can ignore the derivative with respect to $\hat{\alpha}$ from the envelope theorem because this is chosen to maximize second-period profit on B's turf.

$$F(\theta^*) - (a - c)f = \delta[(\alpha - c)d\theta^A - (\hat{\alpha} - c)d\theta^*]f, \quad \text{or} \quad \theta^* + \overline{\theta} - (a - c) = \delta[(\alpha - c)d\theta^A - (\hat{\alpha} - c)].$$

Since $\theta^A = (\theta^*/2) - ½(\alpha - c)$ (from (A9)) and α is not changing, $d\theta^A = d\theta^*/2 = ½$, and the first-order condition reduces to $\theta^* + \overline{\theta} - (a - c) = \delta([(\alpha - c)/2] - (\hat{\alpha} - c))$. In a symmetric equilibrium the right-hand side of this expression is zero, so $a = \overline{\theta} + c$. *Q.E.D.*

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